

Efficient and Accurate Analytical Performance Models for Translucent Optical Networks

Juzi Zhao, Suresh Subramaniam, and Maite Brandt-Pearce

Abstract—This paper presents new analytical models for computing the blocking performance of translucent optical networks with sparsely located optical–electrical–optical regeneration nodes. Due to physical layer impairments such as noise and crosstalk, there is a maximum distance [called the transmission reach (TR)] that optical signals can travel before they have to be regenerated. Regeneration nodes that are allocated to a lightpath can also be used for wavelength conversion besides mitigating PLIs. The quality of transmission of a lightpath is satisfied if the length of each segment between two allocated regeneration nodes is less than the TR. We develop analytical models to estimate the call blocking probability for two typical regeneration node allocation policies. The first policy allocates regeneration nodes when the lightpath needs either regeneration or wavelength conversion, while the other allocation policy allocates regenerators only for signal regeneration. The analytical models strike a balance between computational complexity and prediction accuracy. The models are validated using extensive simulation results for a variety of network topologies and parameters.

Index Terms—Analytical models; Blocking probability; Physical impairments; Regenerator allocation; Translucent optical networks; Transmission reach.

I. INTRODUCTION

Optical networks based on dense wavelength-division multiplexing (DWDM) are the dominant technology in today's transport networks due to their large fiber bandwidths. Two of the main challenges of resource allocation in optical networks are wavelength continuity and physical layer impairments (PLIs). Wavelength continuity means that the same wavelength has to be assigned on all the links of a lightpath. PLIs such as noise and crosstalk cause the quality of the optical signals to degrade as they traverse the network. For a given system, there is a transmission reach (TR), beyond which the quality of a signal is

considered to be not good enough to be detected correctly by the receiver [1]. In other words, the quality of transmission (QoT) of the signal is not adequate if a signal is allowed to propagate over distances longer than the TR. As the number of wavelengths and wavelength bit rates continues to increase, the QoT constraint becomes increasingly important when compared to wavelength continuity. 3R regenerators (that perform reamplification, reshaping, and retiming of signals) that use optical–electrical–optical (OEO) converters can be used to deal with these two constraints by converting the wavelengths and cleaning up the accumulated impairments. Optical networks with sparsely allocated 3R regenerators are called *translucent optical networks*.

The network performance of optical networks has traditionally been quantified using the connection blocking probability. This is the probability that a new connection or lightpath request is blocked due to the unavailability of resources and is a measure of the resources in the network as well as the quality of the resource allocation algorithm. In translucent networks, a connection can be blocked if there are not enough regenerators available on the connection's path so that the TR constraint is violated or if there are not enough wavelengths to satisfy the wavelength continuity constraint (on each segment between two regeneration points). Clearly, different resource allocation policies have different performance. Since OEO converters are precious network resources, they need to be carefully allocated. The OEO converters may be allocated to regenerate signals and/or convert wavelengths, and it is important to understand the performance of different OEO allocation policies. In this paper, blocking performance analysis models for two policies to allocate the OEO converters at 3R regenerators are developed, and numerical results are presented to demonstrate their accuracy. Analytical models have two advantages over simulation: 1) they can provide insight into the effect of various parameters on performance, and 2) they can significantly reduce the computational complexity in approximating the blocking performance compared to simulation, especially for low values of blocking probability (B_N), where one may need to simulate, say, $10/B_N$ connection arrivals. For networks with many resources (e.g., regenerators and wavelengths) subjected to a low traffic load, B_N could be as low as 10^{-7} , and the simulation would require a substantially longer running time than the analysis. In addition, several

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simulations with different seeds must be executed in order to ensure the necessary confidence in the results, adding to the computational burden. The analytical approach, once validated, only requires one computation.

A. Related Work

Analytical performance modeling of optical networks without PLIs (i.e., considering only wavelength continuity) has been the subject of study for about two decades, and many models exist, e.g., [2–8]. However, modeling of translucent optical networks considering PLIs is much scarcer in the literature since the problem becomes much more difficult. Recently, some papers have addressed PLI-aware blocking probability analysis. In [9], an analytical method is presented to evaluate blocking probability accounting for PLIs; fixed routing and random wavelength assignment are used. The authors of [10] improve that work by using first-fit and first-fit with ordering wavelength assignment methods. However, these papers consider transparent networks without 3R regenerators.

B. Contributions

To the best of our knowledge, this is the first work that presents complete analytical models for translucent optical networks considering the TR and limited regeneration resources. The models take into account realistic algorithms for wavelength conversion and regenerator allocation. The introduction of regenerator allocation to the problem of routing and wavelength assignment (RWA) brings about new and significant challenges in modeling. Our rigorously derived models strike a balance between complexity and accuracy, as demonstrated through extensive simulation results. The input to the models is the optical network topology with a limited number of OEO converters at selected 3R nodes (which can be used as wavelength converters and/or regenerators when necessary), and the number of wavelengths per fiber. Connections arrive at and depart from the network according to a stochastic process. The output of the analytical performance models is the blocking probability of a connection request.

The paper is organized as follows: In Section II, we present the network model. The analytical models are presented in Section III. Section IV presents and discusses the numerical results. We conclude the paper and present possible extensions to this work in Section V.

II. NETWORK AND TRAFFIC MODEL

The network is modeled as a set of nodes and links, where each link consists of two fibers, one in each direction, with W wavelengths each.

There are two kinds of nodes in the network: *transparent nodes* that do not have regenerators and *regeneration (3R) nodes*, and the locations of these nodes are given. Each 3R node has a given limited number of OEO converters (called

just OEOs for brevity, henceforth) for each outgoing link, which can be used for signal regeneration and/or wavelength conversion; these OEOs are shared among all connections using that outgoing link.¹ An OEO that is allocated to a connection cannot be used by another connection until the first connection is released. If an OEO is allocated to a connection, it can also provide wavelength conversion through retransmission on any desired available output wavelength. We assume that a connection's QoT requirement can be met as long as there is no OEO segment (subpath between regeneration points) longer than the TR on the connection's path.

Unidirectional connection requests (of full wavelength capacity) arrive at and depart from the network according to a Poisson process.² We assume that the holding time of connections is one unit, and so the offered load is the arrival rate of connections. A connection can be blocked due to either of two reasons: (a) the RWA algorithm cannot find an available wavelength on an OEO segment of the path (*wavelength blocking*), or (b) the TR constraint cannot be satisfied (*QoT blocking*) because of unavailability of OEOs.

We assume static fixed routing and random wavelength assignment, in which one of the available wavelengths on a path is randomly chosen to set up the connection. We recognize the importance of considering alternate paths and better wavelength assignment methods such as first fit, and techniques to model these methods already exist [6,10]. However, since our focus is on accounting for PLIs and limited regeneration in this work, we make these assumptions in order to keep the analysis (relatively) simple.

We assume the following realistic path selection algorithm. One candidate path is precomputed for each pair of nodes as follows. A logical graph is created for each source–destination pair. The logical nodes include all the 3R nodes and the source and destination nodes. A logical link exists between two logical nodes if the shortest path between those nodes in the original network is not longer than the TR. The cost of each logical link is set as the distance of its corresponding shortest path. The candidate path for the source–destination pair is chosen as the shortest path in the logical graph (if more than one shortest path has the same cost, choose the one with the least number of logical links).

III. ANALYTICAL MODELS

The models for the two resource allocation policies are presented in this section. The models are based on the analytical model for transparent networks (without PLI consideration) originally presented in [11]. As will be seen, significant extensions to this model are necessary to accurately compute the performance of translucent networks.

¹This is the so-called share-per-link model. The alternative, share-per-node, is left for future work.

²This is by far the most common traffic model in the optical networking literature.

A. Assumptions

The main model assumptions (for analytical tractability) are the following:

- The offered load to a 3R node (i.e., the rate at which regenerators are requested from a 3R node, for either regeneration or wavelength conversion or both) is independent of the offered loads to other 3R nodes.
- Wavelength usages on adjacent links are assumed to be correlated (as in [11]), except when the two links are separated by an OEO converter that is allocated to the connection. Nonadjacent links are assumed independent. Accordingly, we call this model the “two-link model.” The simpler assumption of wavelength usage independence on links (leading to the “independence model”) is shown below to be too inaccurate in certain cases.

These are reasonable simplifying assumptions that are shown later in the paper to have minimal impact on the accuracy of the model.

B. Notation

Some basic notation used in the models is presented in Table I. Additional notation is introduced below as needed.

C. Reduced Load Approximation

Since our analyses are link-based, and loads are offered to paths rather than links, we adopt the well-known iterative reduced load approximation technique [8,12] as follows. First, for all source–destination pairs, we calculate the initial blocking probability on a connection’s path p with the offered load value $\lambda_p^{(0)}$ (the superscript indicates the iteration number) with the assumption that $b_n^l = 0$ for all n, l . Then, for every iteration i , we calculate the new blocking values with the adjusted values of the offered load as $\lambda_p^{(i)} = \lambda_p^{(0)}(1 - B_p^{(i-1)})$, where $B_p^{(i-1)}$ is the blocking probability on path p in iteration $i-1$. The network blocking probability for iteration i can be calculated as $B_N^{(i)} = \sum_p \lambda_p^{(i)} B_p^{(i)} / \sum_p \lambda_p^{(i)}$. This process is repeated until $(B_N^{(i)} - B_N^{(i-1)})/B_N^{(i-1)} \leq 1\%$ after a few iterations. In the next few sections, we develop the models and describe the blocking probability computation for a generic iteration, dropping the superscript indicating the iteration number for notational simplicity.

D. OEO Allocation Policy I: Reach and Wavelength

First, we develop the analytical model for what we call the *reach and wavelength* (RW) OEO allocation policy. This most general policy allocates an OEO circuit when either the signal needs regeneration or the wavelength needs to be converted (or both). The algorithm can be explained in a straightforward manner. Starting at the source,

TABLE I
BASIC NOTATION

Symbol	Meaning
n	an arbitrary node
l	an arbitrary link ($l_{a,b}$ is a link from node a to node b)
p	an arbitrary path ($p_{s,d}$ is the path from node s to node d)
$D(p)$	the length of path p
R	the total number of OEOs per 3R node’s outgoing link
W	the total number of wavelengths per link per direction
Ω	the transmission reach
$\lambda_{s,d}$	the arrival rate of connection s, d
λ_p	the arrival rate on path p , $\lambda_{p_{s,d}} = \lambda_{s,d}$
$B_{s,d}$	the blocking probability of connection s, d
$A_{s,d}$	the acceptance probability of connection s, d
A_p	the acceptance probability of path p , $A_{p_{s,d}} = A_{s,d}$
B_N	the network blocking probability
b_n^l	the probability that there is no available OEO at 3R node n on outgoing link l
ρ_l	the offered load of link l
μ	$= 1$, the average duration of each connection
$E_{u,v}$	an indicator function; $= 1$ if there are available wavelengths on path $p_{u,v}$ without OEO converter allocation
H_n	an indicator function; $= 1$ if there are available OEOs at node n
$G_{u,v}$	an indicator function; $= 1$ if there is no blocking on path $p_{u,v}$ (some OEO converters may be allocated to make it true)
$F_k(u, v)$	the probability that there are exactly k available wavelengths on the path $p_{u,v}$ without OEO converter allocation
$F(u, v)$	$= \sum_{k=1}^W F_k(u, v)$, the probability that there are available wavelengths on path $p_{u,v}$ without OEO converter allocation

proceed along the connection’s path as far as possible, i.e., until either a continuous wavelength is not available or the TR is exceeded. Allocate an OEO at the furthest possible node to the connection so these constraints are met. Note that it is possible that all OEOs are being used at the furthest 3R node, so the algorithm may have to backtrack to the previous 3R node. After an OEO is allocated at a 3R node, the constraints are reset, and the algorithm starts afresh from that node; the algorithm terminates when the destination node is reached or the call is blocked. As simple as this algorithm may appear to be, modeling its performance is not easy. The main challenge comes from the fact that OEOs are allocated only when necessary (i.e., not allocated just because they are available at a node) and OEOs may not be available at a 3R node when needed, thus causing the backtrack mentioned above. This makes it necessary to evaluate the probabilities of a large set of (not always independent) events. The following example illustrates some of the possible steps of the algorithm.

Let us look at the example path in Fig. 1 that shows the source and destination nodes of a connection and some 3R nodes on the path. Consider the 3R node i ; suppose the next 3R node with available OEOs is node g ; if $D(p_{s,g}) \leq \Omega$, $E_{s,i} = 1$, and $E_{s,g} = 0$, then node i is allocated to convert wavelength. If $D(p_{s,g}) > \Omega$, $D(p_{s,i}) \leq \Omega$, $D(p_{i,g}) \leq \Omega$, and

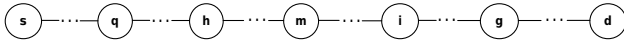


Fig. 1. Example showing OEO allocation.

$E_{s,i} = 1$, then node i is allocated to regenerate the signal. Similarly, suppose the previously allocated 3R node is node h ; if $D(p_{h,g}) \leq \Omega$, $E_{h,i} = 1$, and $E_{h,g} = 0$, then node i is allocated to convert wavelength. If $D(p_{h,g}) > \Omega$, $D(p_{h,i}) \leq \Omega$, $D(p_{i,g}) \leq \Omega$, and $E_{h,i} = 1$, then node i is allocated to regenerate the signal. The reason for the condition $D(p_{i,g}) \leq \Omega$ is that the call is blocked due to bad QoT when $D(p_{i,g}) > \Omega$; in this case, there is no load on OEO i . (In other words, when $D(p_{i,g}) > \Omega$, there is no need to allocate OEO i to the call, since the call is blocked.)

1) *Single-Hop Path*: For a connection (s, d) , if its candidate path $p_{s,d}$ has only one link $l_{s,d}$, the blocking probability $B_{s,d}$ can be found by the Erlang B formula with the offered load $\rho_{l_{s,d}}$ and total capacity W . Since $\mu = 1$, $\rho_{l_{s,d}} = \lambda_{l_{s,d}}$, and

$$\rho_l = \sum_{p: l \in p} \lambda_p, \quad (1)$$

we have

$$B_{s,d} = F_0(s, d) = \frac{\rho_{l_{s,d}}^W}{\sum_{i=0}^W \frac{\rho_{l_{s,d}}^i}{i!}}. \quad (2)$$

2) *Two-Link Model*: Since, by assumption, wavelength dependencies are only limited to adjacent links of a path, let us consider a generic two-hop path $a-b-c$. Some additional notation is defined in Tables II and III. Then, we have [11]

$$\begin{aligned} \mathcal{P}_1(h|i, j, k) &= \frac{\binom{W}{k} \binom{W-k}{i} \binom{i}{h} \binom{W-i-k}{j-h}}{\binom{W}{k} \binom{W-k}{i} \binom{W-k}{j}} \\ &= \frac{\binom{i}{h} \binom{W-i-k}{j-h}}{\binom{W-k}{j}}, \end{aligned} \quad (3)$$

TABLE II
NOTATION FOR A TWO-HOP SUBPATH $a-b-c$

Symbol	Meaning
C_e	the number of connections that use link $a-b$ only
C_f	the number of connections that use link $b-c$ only
C_c	the number of connections that use both links $a-b$ and $b-c$, but do not use OEOs at node b
C_r	the number of connections that use both links $a-b$ and $b-c$, and OEOs at node b are allocated to regenerate signals
C_w	the number of connections that use both links $a-b$ and $b-c$, and OEOs at node b are allocated to convert wavelengths
$\rho_e, \rho_f, \rho_c, \rho_r, \rho_w$	the offered loads for the above connection types

TABLE III
NOTATION FOR PATHS WITH MORE THAN TWO LINKS

Symbol	Meaning
$\mathcal{P}_1(h i, j, k)$	the probability that there are h available wavelengths on a two-hop path given that there are i available wavelengths on the first hop, there are j available wavelengths on the second hop, and k connections use the same wavelengths on both hops
$\mathcal{P}_2(i)$	the probability that there are i available wavelengths on a link
$\mathcal{P}_3(j i)$	the probability that there are j available wavelengths on a link of a path given that there are i available wavelengths on the previous link of the path
$\mathcal{P}_4(k i, j)$	the probability that there are k connections that use the same wavelengths on the current link and the previous link of a path given that there are i available wavelengths on the previous link, and j available wavelengths on the current link
$T^m(h, e)$	the probability that there are h available wavelengths on a path with m hops, and there are e available wavelengths on the last hop of the path
$\hat{x}_n^l(p)$	the offered load to 3R node n on outgoing link l for path p to convert wavelengths
$\tilde{x}_n^l(p)$	the offered load of 3R node n on outgoing link l for path p to regenerate signals
$x_n^l(p)$	the offered load of 3R node n on outgoing link l for path p . $= \hat{x}_n^l(p) + \tilde{x}_n^l(p)$
ρ_n^l	the offered load of 3R node n on outgoing link l

for $\min(i, j) \geq h \geq \max(0, i + j + k - W)$. Further, we have

$$F_k(u, v) = \sum_{e=k}^W T^m(k, e), \quad (4)$$

where $m \geq 2$ is the number of hops on $p_{u,v}$. If $m = 1$, $F_k(u, v)$ can be found by the Erlang B formula as

$$F_k(u, v) = \frac{\frac{\rho_{u,v}^{W-k}}{(W-k)!}}{\sum_{i=0}^W \frac{\rho_{u,v}^i}{i!}}. \quad (5)$$

$T^m(k, e)$ is computed in the following sections for $m > 1$.

The set of connections traversing a two-link subpath (where the middle node may be a 3R node) is partitioned into five subsets, whose cardinalities are C_e , C_f , C_c , C_r , and C_w . Note that C_r and C_w -type calls,³ respectively, are allocated OEOs to regenerate signals and convert wavelengths.⁴ However, once allocated, these calls also avail of the other function, namely, convert wavelengths or clean up impairments, respectively. There is no reason to assume otherwise.

The reason we differentiate between C_w -type calls and C_r -type is the following. If the OEO is allocated to convert wavelengths, then the assigned wavelengths on the two links adjacent to the 3R node are necessarily different. But if the OEO node is allocated to regenerate the signal,

³By an abuse of notation, we use the cardinality to also mean the type of call.

⁴If an OEO is assigned because of both TR and wavelength continuity, the call is considered to be a C_w -type call.

the assigned wavelengths on the two links may or may not be different since the wavelengths on the two segments are randomly picked from the available wavelengths. This important distinction will be relevant to the calculation of $\mathcal{P}_4(k|i,j)$ below.

For the two links $a-b$ and $b-c$, we have

$$\rho_e = \sum_{p:a-b \in p, b-c \notin p} \lambda_p, \quad (6)$$

$$\rho_f = \sum_{p:a-b \notin p, b-c \in p} \lambda_p, \quad (7)$$

$$\rho_c = \sum_{p:a-b \in p, b-c \in p} \lambda_p - \rho_r - \rho_w. \quad (8)$$

Due to the assumption that connections arrive at each path according to a Poisson process, and the connections' holding times are exponentially distributed, we can model the state of connections on a two-link path as a five-dimensional (5D) Markov chain (along the lines of [11]). The steady-state probability for state $(C_e, C_f, C_c, C_r, C_w)$ is

$$\pi(C_e, C_f, C_c, C_r, C_w) = \frac{\rho_e^{C_e} \rho_f^{C_f} \rho_c^{C_c} \rho_r^{C_r} \rho_w^{C_w}}{C_e! C_f! C_c! C_r! C_w!}, \quad (9)$$

where the normalization factor Δ can be calculated as

$$\Delta = \sum_{h=0}^{\min(R,W)} \sum_{g=0}^{\min(R-h,W-h)} \sum_{k=0}^{W-h-g} \sum_{i=0}^{W-k-h-g} \sum_{j=0}^{W-k-h-g} \frac{\rho_e^i \rho_f^j \rho_c^k \rho_r^h \rho_w^g}{i! j! k! h! g!} \quad (10)$$

with constraints $0 \leq C_r + C_w \leq R$, $0 \leq C_r + C_c + C_e + C_w \leq W$ and $0 \leq C_r + C_c + C_f + C_w \leq W$ for links $a-b$ and $b-c$, respectively.

3) *Distributions of ρ_r and ρ_w* : We still need ρ_r and ρ_w to compute the above equation. If the middle node b is a transparent node, then $\rho_r = \rho_w = 0$. If node b is a 3R node, the values are obtained as follows. We first find $\hat{x}_b^l(p)$ and $\bar{x}_b^l(p)$ for the paths that include links $a-b$ and $b-c$ (l is link $b-c$); denote these paths as set P . Then

$$\rho_r = \sum_{p \in P} \hat{x}_b^l(p), \quad (11)$$

$$\rho_w = \sum_{p \in P} \bar{x}_b^l(p). \quad (12)$$

The challenge in computing these loads is in part due to the several events (beyond the two-link path) that need to be considered, as suggested by the example in Fig. 1.

We introduce some additional notation in Table IV. In the table, u, v , and t are 3R nodes on a path. Node u is closer to the source, node t is closer to the destination, and node v is between u and t . Also, the phrase “there is no blocking from source to node v ” means that OEOs can be allocated so that the constraints of wavelength continuity and TR can be satisfied on the path from source to v .

The offered loads \hat{x}_i and \bar{x}_i for an arbitrary 3R node i on path p are computed as follows. Consider the example in Fig. 1. We have

$$\hat{x}_i^l(p) = \sum_{\substack{s \leq h \leq i-1 \\ D(p_{h,i}) \leq \Omega}} \sum_{\substack{i+1 \leq g \leq d \\ D(p_{h,g}) \leq \Omega}} X(h, i, g) \lambda_p, \quad (13)$$

where the previous allocated 3R node is node h ; this can be any 3R node between source s and 3R node $i-1$, with the condition that the distance from h to i is not longer than the TR. The next 3R node g with free OEOs can be any node from 3R node $i+1$ to destination d , with the condition that the distance from h to g is not longer than the TR.

On the other hand,

$$\bar{x}_i^l(p) = \sum_{\substack{s \leq h \leq i-1 \\ D(p_{h,i}) \leq \Omega}} \sum_{\substack{i+1 \leq g \leq d \\ D(p_{h,g}) > \Omega \\ D(p_{i,g}) \leq \Omega}} Z(h, i, g) \lambda_p. \quad (14)$$

4) *Distributions of \mathcal{P}_2 , \mathcal{P}_3 , and \mathcal{P}_4* : These probabilities are used in obtaining the blocking probabilities on paths without 3R nodes below. We can obtain these probabilities [Eqs. (15)–(17)] from the steady-state probabilities of the Markov chain. The probability $P(\alpha, K, C_r, C_w, C_c, C_e, C_f)$ in Eq. (17) is defined as the joint probability of the following events: there are $W - C_c - C_e - C_w$ available wavelengths on the first link, $W - C_c - C_f - C_w$ on the second link, K available (common) wavelengths on both links, and when C_r wavelengths are randomly chosen from the available wavelengths on each of the two links, α wavelengths (out of the C_r) are identical on both links. Its calculation is given in Appendix A:

$$\begin{aligned} \mathcal{P}_2(i) &= P(C_c + C_e + C_r + C_w = W - i) \\ &= \sum_{C_r=0}^{\min(R,W-i)} \sum_{C_w=0}^{\min(R-C_r,W-i-C_r)} \sum_{C_c=0}^{W-i-C_r-C_w} \sum_{C_e=0}^{W-C_r-C_c-C_w} \pi(W-i-C_c-C_r-C_w, C_f, C_c, C_r, C_w), \end{aligned} \quad (15)$$

$$\begin{aligned} \mathcal{P}_3(j|i) &= P(C_f + C_c + C_r + C_w = W - j | C_e + C_c + C_r + C_w = W - i) \\ &= \frac{\sum_{C_r=0}^{\min(R,W-i,W-j)} \sum_{C_w=0}^{\text{up1}} \sum_{C_c=0}^{\text{up2}} \pi(W-i-C_c-C_r-C_w, W-j-C_c-C_r-C_w, C_c, C_r, C_w)}{\sum_{C_r=0}^{\min(R,W-i)} \sum_{C_w=0}^{\min(R-C_r,W-i-C_r)} \sum_{C_c=0}^{W-i-C_r-C_w} \sum_{C_e=0}^{W-C_r-C_c-C_w} \pi(W-i-C_c-C_r-C_w, C_f, C_c, C_r, C_w)}, \end{aligned} \quad (16)$$

where $\text{up1} = \min(R - C_r, W - i - C_r, W - j - C_r)$ and $\text{up2} = \min(W - i - C_r - C_w, W - j - C_r - C_w)$,

$$\begin{aligned}
P_4(k|i,j) &= P(C_c + \alpha = k | C_e + C_c + C_r + C_w = W - i, C_f + C_c + C_r + C_w = W - j) \\
&= \frac{\sum_{\alpha=0}^{\min(R,k)} \sum_{C_w=0}^{\text{up3}} \sum_{C_e=\text{low1}}^{W-i-k-C_w} \sum_{K=\alpha}^{\text{up4}} P(\alpha, K, W-i-C_e-C_w-k+\alpha, C_w, k-\alpha, C_e, i-j+C_e)}{\sum_{C_r=0}^{\min(R,W-i,W-j)} \sum_{C_w=0}^{\text{up1}} \sum_{C_c=0}^{\text{up2}} \pi(W-i-C_c-C_r-C_w, W-j-C_c-C_r-C_w, C_c, C_r, C_w)}, \quad (17)
\end{aligned}$$

where $\text{up3} = \min(R - \alpha, W - i - k + \alpha, W - j - k + \alpha)$, $\text{up4} = \min(W - C_e - C_w - k + \alpha, W - C_f - C_w - k + \alpha)$, and $\text{low1} = \max(0, j - i, W - i - k + \alpha - R - C_w)$; the upper and lower bounds of C_e are found with $\alpha \leq C_r \leq R$.

5) *Blocking on a Path Without 3R Nodes*: The blocking probabilities on paths without any 3R nodes can be calculated by $B_p = F_0(s, d) = \sum_{e=0}^W T^m(0, e)$, where m is the number of hops on the path.

The values of $T^m(h, e)$ can be calculated recursively from $T^{m-1}(f, j)$ as follows by assuming the subpath with $m - 1$ hops as a link, and the last hop of the path as the second link:

$$\begin{aligned}
T^m(h, e) &= \sum_{f=h}^W \sum_{j=f}^W \sum_{k=0}^{\min(W-e, W-j)} P_1(h|f, e, k) P_4(k|j, e) \\
&\quad \times P_3(e|j) T^{m-1}(f, j). \quad (18)
\end{aligned}$$

The starting point of the recursion $T^1(h, e) = 0$ when $h \neq e$ and is equal to the probability of having e free wavelengths on a link, $P_2(e)$, when $h = e$.

6) *Blocking on a Path With 3R Nodes*: For a path p from source s to destination d , we first find its acceptance probability A_p as follows; then $B_p = 1 - A_p$.

An example path from s to d with several 3R nodes is shown in Fig. 2; the 3R nodes on the paths are numbered as 1, 2, 3, ..., and suppose node h is the last 3R node on the

path. i, k, j are three variable 3R nodes; their relative positions are shown in Fig. 2. We use the notations $E_{u,v}$, H_n , and $G_{u,v}$, which are defined in Table I.

We can get ρ_n^l from $x_n^l(p)$, $\rho_n^l = \sum_{p:l \in p} x_n^l(p)$. Then b_n^l can be calculated by the Erlang B formula, i.e.,

$$b_n^l = \frac{\frac{(\rho_n^l)^R}{R!}}{\sum_{r=0}^R \frac{(\rho_n^l)^r}{r!}}. \quad (19)$$

Based on the OEO availability at 3R node 1, we can calculate A_p as

$$\begin{aligned}
A_p &= \Pr\{H_1 = 1, G_{s,d} = 1\} + \Pr\{H_1 = 0, G_{s,d} = 1\} \\
&= \Pr\{G_{s,d} = 1 | E_{s,1} = 1, H_1 = 1\} \Pr\{E_{s,1} = 1, H_1 = 1\} \\
&\quad + \Pr\{G_{s,d} = 1 | E_{s,1} = 0, H_1 = 1\} \Pr\{E_{s,1} = 0, H_1 = 1\} \\
&\quad + \Pr\{H_1 = 0, G_{s,d} = 1\} \\
&= (1 - b_1^l) F(s, 1) \Pr\{G_{s,d} = 1 | E_{s,1} = 1, H_1 = 1\} + b_1^l \\
&\quad \times \Pr\{G_{s,d} = 1 | H_1 = 0\}. \quad (20)
\end{aligned}$$

Note that $\Pr\{G_{s,d} = 1 | E_{s,1} = 0, H_1 = 1\} = 0$, since if there is no available wavelength from the source to the first 3R node on the path, the connection is blocked, i.e., $\Pr\{G_{s,d} = 1\} = 0$.

To compute Eq. (20), we need to consider the OEO availability at 3R node 2, node 3, and so on until node h . Each time only one 3R node is under consideration. The idea is that if there is no blocking on the path, the current 3R node under consideration is node j , and the previously allocated 3R node (to the connection) is i ; then we know there is no blocking from source s to node i (some 3R nodes could be allocated in between), and there should be no blocking from i to d . It is possible that an OEO is allocated at node j to make this true (if necessary and if there are available OEOs at node j).

Thus, we need two definitions. Let $M(i, j, d)$ denote the probability that there is no blocking from i to d , given that there are available OEOs at node j , the previously allocated 3R node is node i (if there is no previously allocated 3R node, node i is source node s), and there are available wavelengths on $p_{i,j}$. Let $M'(i, j, d, k)$ denote the probability that there is no blocking from i to d , given that there are no available OEOs at node j , the previously allocated 3R node

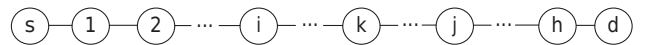


Fig. 2. Example showing a path with 3R nodes.

TABLE IV
NOTATION TO CALCULATE ρ_r AND ρ_w

Symbol	Meaning
$\hat{F}(u, v)$	= 0, if $D(P_{u,v}) > \Omega$; otherwise, $\hat{F}(u, v) = F(u, v)$
$\tilde{F}(u, v, t)$	= 0, if $D(P_{u,v}) > \Omega$ or $D(P_{v,t}) > \Omega$; otherwise, it is the probability that $E_{u,v} = 1$ and $E_{u,t} = 0$
$F'(u, v, t)$	= 0, if $D(P_{u,v}) > \Omega$ or $D(P_{v,t}) > \Omega$; otherwise, it is the probability that $E_{u,v} = 1$, $E_{v,t} = 1$, and $E_{u,t} = 0$
$X(u, v, t)$	given $D(p_{u,t}) \leq \Omega$, the probability that $G_{s,v} = 1$, the previously allocated 3R node of the path is node u , the next 3R node with available OEOs on the path is node t , and $E_{u,v} = 1$, $E_{u,t} = 0$
$X'(u, v, t)$	given $D(p_{u,t}) \leq \Omega$, the probability that $G_{s,v} = 1$, the previously allocated 3R node of the path is node u , the next 3R node with available OEOs on the path is node t , $E_{u,v} = 1$, and $E_{u,t} = 0$
$Z(u, v, t)$	given $D(p_{u,t}) > \Omega$, $D(p_{u,v}) \leq \Omega$ and $D(p_{v,t}) \leq \Omega$, the probability that $G_{s,v} = 1$, the previously allocated 3R node of the path is node u , the next 3R node with available OEOs on the path is node t , and $E_{u,v} = 1$
$Z'(u, v, t)$	given $D(p_{u,t}) > \Omega$, $D(p_{u,v}) \leq \Omega$ and $D(p_{v,t}) \leq \Omega$, the probability that $G_{s,v} = 1$, the previously allocated 3R node of the path is node u , the next 3R node with available OEOs on the path is node t , $E_{u,v} = 1$, and $E_{v,t} = 1$

is node i (if there is no previously allocated 3R node, node i is source node s), and the previous 3R node with available OEOs, but not yet allocated, is node k ; there are available wavelengths on $p_{i,k}$. (Node k is set as 0 if there is no such k , e.g., $j = i + 1$; in this case, there is no condition in which there are available wavelengths on $p_{i,k}$).

In order to compute $M(i, j, d)$ and $M'(i, j, d, k)$, we need to consider the availability of OEOs at node $j + 1$. Also, the OEOs at node j or node k , respectively, may be allocated to regenerate signal and/or convert the wavelengths to ensure there is no blocking from i to destination d . When we reach $j = h$, the next 3R node is d (i.e., $j + 1 = d$), and this procedure is finished. The details of the computation of $M(i, j, d)$ and $M'(i, j, d, k)$ are in Appendix B.

Equation (20) can be calculated with $M(i, j, d)$ and $M'(i, j, d, k)$ as

$$A_p = (1 - b_1^l)F(s, 1)M(s, 1, d) + b_1^l M'(s, 1, d, 0). \quad (21)$$

The final equations for $M(i, j, d)$ and $M'(i, j, d, k)$ are as follows:

$$M(i, j, d) = \begin{cases} (1 - b_{j+1}^l) \frac{F(i, j+1)}{F(i, j)} M(i, j+1, d) + (1 - b_{j+1}^l) \frac{F'(i, j+1)}{F'(i, j)} \\ \times M(j, j+1, d) + b_{j+1}^l M'(i, j+1, d, j), & \text{if } j \neq h, D(p_{i, j+1}) \leq \Omega, \\ (1 - b_{j+1}^l) F(j, j+1) M(j, j+1, d) + b_{j+1}^l M'(i, j+1, d, j), & \text{if } j \neq h, D(p_{i, j+1}) > \Omega, \\ \frac{F(i, d)}{F(i, j)} + \frac{F'(i, j, d)}{F'(i, j)}, & \text{if } j = h, D(p_{i, d}) \leq \Omega, \\ F(j, d), & \text{if } j = h, D(p_{i, d}) > \Omega, \\ 0, & \text{otherwise,} \end{cases} \quad (22)$$

and

$$M'(i, j, d, k) = \begin{cases} (1 - b_{j+1}^l) \frac{F(i, j+1)}{F(i, k)} M(i, j+1, d) + (1 - b_{j+1}^l) \frac{F'(i, k, j+1)}{F'(i, k)} \\ \times M(k, j+1, d) + b_{j+1}^l M'(i, j+1, d, k), & \text{if } k \neq 0, j \neq h, D(p_{i, j+1}) \leq \Omega, \\ (1 - b_{j+1}^l) F(k, j+1) M(k, j+1, d) + b_{j+1}^l M'(i, j+1, d, k), & \text{if } k \neq 0, j \neq h, D(p_{i, j+1}) > \Omega, D(p_{k, j+1}) \leq \Omega, \\ (1 - b_{j+1}^l) F(i, j+1) M(i, j+1, d) + b_{j+1}^l M'(i, j+1, d, k), & \text{if } k = 0, j \neq h, D(p_{i, j+1}) \leq \Omega, \\ \frac{F(i, d)}{F(i, k)} + \frac{F'(i, k, d)}{F'(i, k)}, & \text{if } k \neq 0, j = h, D(p_{i, d}) \leq \Omega, \\ F(k, d), & \text{if } k \neq 0, j = h, D(p_{i, d}) > \Omega, D(p_{k, d}) \leq \Omega, \\ F(i, d), & \text{if } k = 0, j = h, D(p_{i, d}) \leq \Omega, \\ 0, & \text{otherwise.} \end{cases} \quad (23)$$

The constraint $D(p_{i,k}) \leq \Omega$ in the above two equations ensures that the denominator $F(i, k)$ is greater than zero.

7) *Summary:* The procedure to find the network blocking probability for the RW policy is summarized in Algorithm 1 (for a single iteration) and the flowchart in Fig. 3.

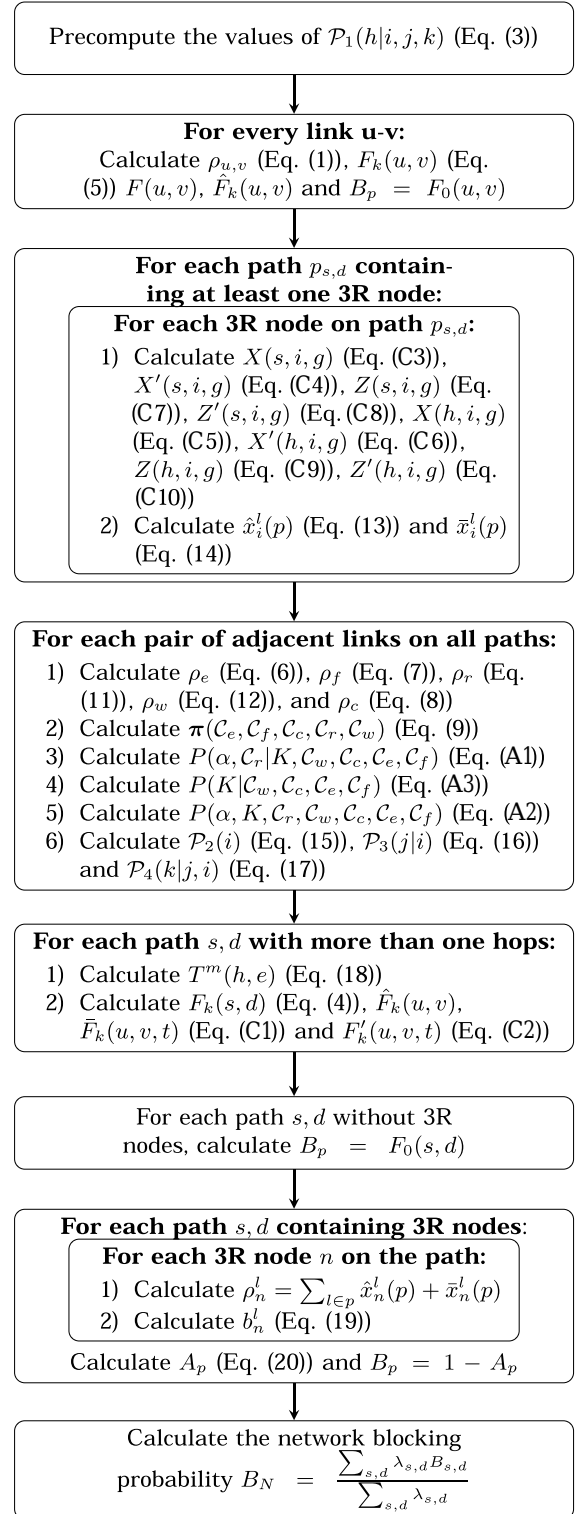


Fig. 3. One iteration of the computation for the RW policy.

Algorithm 1 Blocking Computation for the RW Policy

Step 1: Precompute the values of $\mathcal{P}_1(h|i, j, k)$ [Eq. (3)], which are independent of traffic loads.

Step 2: Consider each single link. The values of $\rho_{u,v}$, $F_k(u, v)$, $F(u, v)$, and $\hat{F}(u, v)$ will be used later when considering paths with multiple links.

for each link $l_{u,v}$ **do**

1. Calculate offered load $\rho_{u,v}$ [Eq. (1)]
2. Calculate the probability that the link has k free wavelengths, $F_k(u, v)$ [Eq. (5)]
3. Calculate the probability that the link has at least one free wavelength, $F(u, v) = \sum_{k=1}^W F_k(u, v)$
4. Calculate the probability $\hat{F}_k(u, v)$ (defined in Table IV)
5. Calculate the blocking probability of single-hop path p between nodes u, v , $B_p = F_0(u, v)$

end for

Step 3: Then we consider paths with more than one link. We need the steady-state distribution of the 5D Markov chain, and the offered loads for the five types of connections, ρ_e , ρ_f , ρ_c , ρ_w , and ρ_r . In this step, the values of $\hat{x}_i^l(p)$ and $\bar{x}_i^l(p)$ are calculated, which are used to get ρ_w and ρ_r in the next step.

for each path $p_{s,d}$ containing at least one 3R node **do**

for each 3R node i on path $p_{s,d}$, starting from the one closest to s and proceeding to the one closest to d **do**

1. Calculate the values of $X(s, i, g)$ [Eq. (C3)], $X'(s, i, g)$ [Eq. (C4)], $Z(s, i, g)$ [Eq. (C7)], $Z'(s, i, g)$ [Eq. (C8)], $X(h, i, g)$ [Eq. (C5)], $X'(h, i, g)$ [Eq. (C6)], $Z(h, i, g)$ [Eq. (C9)], and $Z'(h, i, g)$ [Eq. (C10)] (in Appendix C) (The values of b_n^l , $F(u, v)$, $\hat{F}_k(u, v)$, $\bar{F}_k(u, v, t)$, and $F'_k(u, v, t)$ needed are those found in Steps 5 and 6 of the previous iteration, as explained in Subsection III.C)
2. Calculate $\hat{x}_i^l(p)$ [Eq. (13)] and $\bar{x}_i^l(p)$ [Eq. (14)] (where l is the corresponding outgoing link of 3R node i on path p)

end for

end for

Step 4: Now we can calculate the offered loads of the five types of connections. Then we can get the steady-state distribution of the Markov chain. In turn, we can compute the values of $\mathcal{P}_2(i)$, $\mathcal{P}_3(i)$, and $\mathcal{P}_4(i)$, which are used in the next step to find the value of $T^m(h, e)$.

for each pair of adjacent links on all paths **do**

1. Calculate the values of ρ_e [Eq. (6)], ρ_f [Eq. (7)], ρ_r [Eq. (11)], ρ_w [Eq. (12)], and ρ_c [Eq. (8)]
2. Calculate the steady-state probability $\pi(C_e, C_f, C_c, C_r, C_w)$ [Eq. (9)]
3. Calculate $P(\alpha, C_r | K, C_w, C_c, C_e, C_f)$ [Eq. (A1)] based on the value of $Y(\alpha, b | m, u, v)$ [Eqs. (D1) and (D2)]
4. Calculate $P(K | C_w, C_c, C_e, C_f)$ [Eq. (A3)]
5. Calculate $P(\alpha, K, C_r, C_w, C_c, C_e, C_f)$ [Eq. (A2)]
6. Calculate $\mathcal{P}_2(i)$ [Eq. (15)], $\mathcal{P}_3(j | i)$ [Eq. (16)], and $\mathcal{P}_4(k | j, i)$ [Eq. (17)]

end for

Step 5: In this step we calculate $T^m(h, e)$, which is used to find the blocking probabilities for paths without 3R nodes.

for each path s, d ($m > 1$ hops) **do**

1. Calculate $T^m(h, e)$ [Eq. (18)]
2. Calculate $F_k(s, d)$ [Eq. (4)], $\hat{F}_k(u, v)$ (defined in Table IV), $\bar{F}_k(u, v, t)$ [Eq. (C1)], and $F'_k(u, v, t)$ [Eq. (C2)].

(Note that these values will be used in Step 3 of the next iteration.)

end for

Step 6:

for each path s, d ($m > 1$ hops) without 3R nodes **do**

Calculate the blocking probability $B_p = F_0(s, d)$

end for

Step 7: Now we consider the paths with 3R nodes. In this step, we calculate the offered loads to each 3R node's outgoing link l , ρ_n^l , and in turn, the probability that there are no available OEO converters at a 3R node b_n^l . Then we can get the acceptance probability of a path A_p based on $M(i, j, d)$ and $M'(i, j, d, k)$.

for each path s, d containing 3R nodes **do**

for each 3R node n on the path **do**

1. Calculate the offered load to 3R node n on outgoing link l ; $\rho_n^l = \sum_{l \in p} \hat{x}_n^l(p) + \bar{x}_n^l(p)$
2. Calculate the probability that there are no OEOs available at the node b_n^l [Eq. (19)] (where l is the corresponding outgoing link with node n on the path) (Note that these values will be used in Step 3 of the next iteration.)

end for

1. Calculate the acceptance probability of the path A_p [Eq. (20)] depending on the OEO availability at the first 3R node on the path. This calculation is based on the values of $M(i, j, d)$ [Eq. (22)] and $M'(i, j, d, k)$ [Eq. (23)]
2. Calculate the blocking probability $B_p = 1 - A_p$

end for

Step 8: Calculate the network blocking probability $B_N = \frac{\sum_{s,d} \lambda_{s,d} B_{s,d}}{\sum_{s,d} \lambda_{s,d}}$

E. OEO Allocation Policy II: Reach Only

We now consider the *reach only* (RO) OEO allocation policy. According to this policy, OEOs are allocated only when the signals need regeneration. For connections with candidate paths no longer than the TR, no OEOs are allocated. For connections with candidate paths longer than the TR, there is at least one 3R node on the path. The procedure to allocate OEOs to the path is as follows. The source allocates an OEO at the furthest possible node to the connection so the TR constraint is met. Similar to the previous policy, it is possible that all OEOs are being used at the furthest 3R node, so the algorithm may have to backtrack to the previous 3R node. After an OEO is allocated at a 3R node, the constraint is reset, and the algorithm starts afresh from that node; the algorithm terminates when the destination node is reached or the call is blocked. Consider the example shown in Fig. 4. (Only 3R nodes on the path are shown here.) Suppose the farthest 3R node within the TR from node s is node c ; the policy first tries to allocate an OEO at node c to the path; if there is no available OEO at node c , node b is considered, and so on. Similarly, if the currently allocated 3R node is node b , and the farthest 3R node within the TR from b is node e , then the policy first tries to allocate node e to the path; if it fails, then node d is tried. (Note that in this case, node c will not be considered, since

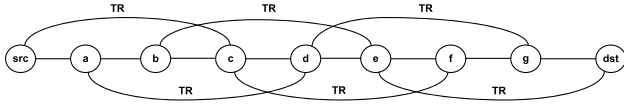


Fig. 4. Example for the RO policy.

the case in which node b is allocated means there is no available OEO at node c .) If node e (or f, g) is the currently allocated OEO node, then no more 3R nodes are allocated to the path, since $D(p_{e,d}) \leq \Omega$. Note, however, that while the allocation policy does not consider the wavelength availability, a connection is assumed to be blocked if any OEO segment has no available wavelengths. The changes in the model essentially come from the fact that wavelength conversion needs are not checked when OEOs are allocated. (However, we remind the reader that if an OEO is allocated, it may be used to provide wavelength conversion.) In this subsection, we develop the analytical model for the RO OEO allocation policy. It turns out that the previous model can be simplified for this policy. We present the necessary modifications below.

1) *Single-Hop Path*: The blocking probabilities of paths including a single link are still found by Eqs. (1) and (2).

2) *Two-Link Model*: A four-dimensional Markov chain suffices for this policy (by discarding the C_w part from the previous 5D Markov chain used in the RW model). Equations (6) and (7) are still used for calculating ρ_e and ρ_f . Instead of Eq. (8), we now use $\rho_c = \sum_{p:a-b \in p, b-c \in p} \lambda_p - \rho_r$.

Then, the steady-state probability for state (C_e, C_f, C_c, C_r) and the normalization factor Δ are similar to Eqs. (9) and (10) by removing C_w . (Again, note that this is a different Markov chain from the one used for the RW policy, with C_w being subsumed within other connection types.)

In order to calculate the values of ρ_r , only $\bar{x}_n^l(p)$, $Z(u, v, t)$, and $Z'(u, v, t)$ are used. The equations of $Z(u, v, t)$ and $Z'(u, v, t)$ are the same as the ones in Appendix B, except that the values of all X and X' are assumed to be zero. In addition, we have $\bar{x}_n^l(p) = \bar{x}_n^l(p)$.

The equations to find $\mathcal{P}_2(i)$, $\mathcal{P}_3(j|i)$, and $\mathcal{P}_4(k|j, i)$ are similar to Eqs. (15)–(17) by removing the C_w part.

For paths whose distances are shorter than the TR (note that no OEOs are allocated to them), their blocking probabilities can be calculated by $B_p = F_0(s, d) = \sum_{e=0}^W T^m(0, e)$, where m is the number of hops in the path. The values of $T^m(h, e)$ are calculated by Eq. (18).

3) *Blocking on a Path With Distance Longer Than TR*: For paths longer than TR, at least one 3R node needs to be allocated to the path. Recall that the policy allocates the possible 3R node that is furthest from the previously allocated 3R node (to the path) (or from the source if no 3R node has been allocated to the path yet). The probability that a 3R node has available OEOs is found by Eq. (19).

For a path p from source s to destination d with r 3R nodes, number the 3R nodes as $1, 2, 3, \dots, r$. Suppose $D(p_{s,h}) \leq \Omega$ while $D(p_{s,h+1}) > \Omega$. We can calculate the value of A_p as follows:

$$\begin{aligned} A_p &= \sum_{g=1}^h \Pr\{G_{s,d} = 1, \text{ and the first allocated 3R node is } g\} \\ &= \sum_{g=1}^h \Pr\{G_{s,d} = 1, H_g = 1, \text{ but there are no available OEOs at nodes from } g+1 \text{ to } h\} \\ &= \sum_{g=1}^h (1 - b_g^l) \prod_{i=g+1}^h b_i^l \Pr\{E_{s,g} = 1, G_{g,d} = 1 \mid \text{there are no available OEOs at nodes from } g+1 \text{ to } h\} \\ &= \sum_{g=1}^h (1 - b_g^l)(1 - B_{p_{s,g}}) \prod_{i=g+1}^h b_i^l \Pr\{G_{g,d} = 1 \mid \text{there are no available OEOs at nodes from } g+1 \text{ to } h\}. \end{aligned} \quad (24)$$

Denote $N(g, h, d)$ as follows: $\Pr\{G_{g,d} = 1 \mid \text{there are no available OEOs at nodes from } g+1 \text{ to } h\}$; then we can have

$$A_p = \sum_{g=1}^h (1 - b_g^l)(1 - B_{p_{s,g}}) \prod_{i=g+1}^h b_i^l N(g, h, d). \quad (25)$$

To calculate $N(g, h, d)$, we have two cases based on the length of path $p_{g,d}$, as below:

(a) $D(p_{g,d}) \leq \Omega$: there is no need to allocate any OEO converter to regenerate the signal, so

$$N(g, h, d) = 1 - B_{p_{g,d}}. \quad (26)$$

(b) $D(p_{g,d}) > \Omega$: Suppose the farthest node within the TR from node g is node t , i.e., $D(p_{g,t}) \leq \Omega$, and $D(p_{g,t+1}) > \Omega$. Then an OEO converter between node $h+1$ and t must be allocated to regenerate the signal. Denoting this node as e , we have

$$N(g, h, d) = \sum_{e=h+1}^t (1 - b_e^l)(1 - B_{p_{g,e}}) \prod_{f=e+1}^t b_f^l N(e, t, d). \quad (27)$$

F. Independence Model

In order to demonstrate the necessity of considering link load correlation (as in the two-link model described above) in certain cases, we also developed a model that is similar to the two-link but assuming independence of wavelength usages on links (called the independence model).

The only difference between the two-link model and the independence model is the method used to find $F_k(u, v)$. For a single-hop path $p_{s,d}$, $F_k(s, d)$ is still found by the Erlang B formula with capacity W and offered load $\rho_{s,d}$ using Eq. (5). We do not need the Markov chain that is used in the two-link model to calculate $F_k(u, v)$ for a multihop path. $F_k(u, v)$ for a multihop path is obtained as follows.

In a two-link path, given that there are a available wavelengths on the first link and b available wavelengths on the

second link, the probability that there are k available wavelengths on the path is denoted by

$$U(k|a, b) = \frac{\binom{a}{k} \binom{W-a}{b-k}}{\binom{W}{b}}.$$

For a two-hop path $s-i-d$, $F_k(s, d) = \sum_{a=k}^W \sum_{b=k}^W U(k|a, b) F_a(s, i) F_b(i, d)$. For a path $p_{s,d}$ with $m > 2$ hops, we find the $F_k(s, d)$ values with $m' = 2, 3, \dots, m$ hops by considering the first $m' - 1$ hops (from node s to j) as one hop, and the last hop as another hop (from node j to $j + 1$); then, $F_k(s, j + 1) = \sum_{a=k}^W \sum_{b=k}^W U(k|a, b) F_a(s, j) F_b(j, j + 1)$.

Note that the reduced load approximation is also used in the independence model.

IV. NUMERICAL RESULTS

We now validate the models by comparing analytical model results with simulation results. We also conduct a detailed performance evaluation of the two OEO allocation policies (RW and RO) in a variety of scenarios. For the simulation, unidirectional requests for connections are assumed to arrive at the network according to a Poisson process. The source and destination of each connection are randomly selected (from a uniform distribution), and fixed shortest path routing is assumed. For each data point in the graphs, we simulated several instances of at least 10,000 connection arrivals (until we observed 10 blocked connections) and obtained 95% confidence intervals. The TR value $\Omega = 3600$ km (according to the physical layer model in [13] for a QPSK 100 Gbps signal with 50 GHz channel spacing),⁵ and the number of wavelengths per fiber $W = 32$.

We present results for three network topologies, the European Optical Network (EON), NSFNET, and a ring network shown in Figs. 5–7, respectively. The numbers on the links are link distances (in hundreds of kilometers), and the blue (shaded) nodes are 3R nodes. We select the 3R nodes according to the following algorithm: First, find the shortest path for each pair of nodes, and count the number of times a node is traversed by these paths. Nodes are sorted in descending order of this number. For each topology, the first five nodes in this order are selected as 3R nodes.⁶

The results in Fig. 8 show that the two-link model accurately predicts the blocking probability as compared with simulation, unlike the “independence model” that greatly overestimates it for the ring network.

Results with different numbers of OEO converters per 3R node for the NSFNET are shown in Fig. 9 (the

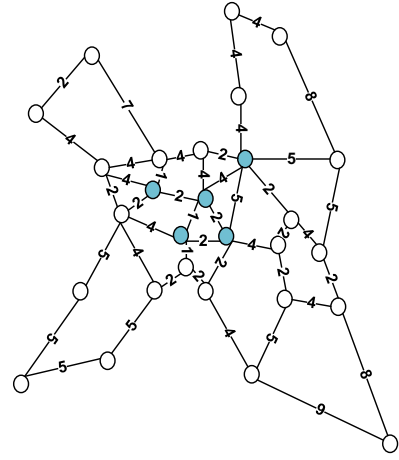


Fig. 5. 28-node EON.

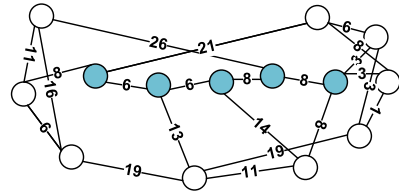


Fig. 6. 14-node NSFNET.

performance has similar trends for the other two topologies, and is therefore omitted). Once again, notice the excellent match between analytical and simulation results. Further, the blocking is reduced significantly as the number of OEO converters is increased, especially at low loads.

We have also developed an analytical model for the *wavelength only* (WO) policy that allocates regenerators only based on wavelength conversion needs. However, calls are admitted only if the QoT is satisfied. This policy is also referred to as QoT-guaranteed in the literature [10]. Results for this policy show that the blocking probability is several orders of magnitude higher than for the RW and RO policies, and they are therefore omitted from the paper.

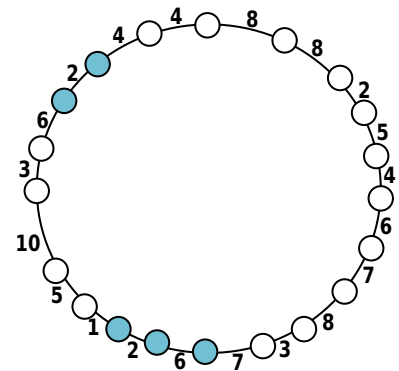


Fig. 7. 20-node ring network.

⁵Note that our models use TR as an input parameter, and any physical layer model that computes the TR could be used. We chose [13] in our paper because of its detailed and well-cited model.

⁶We emphasize that this paper's focus is not on regenerator placement. We merely present a reasonable method for selecting the 3R node locations in order to investigate the RWA algorithms' performance.

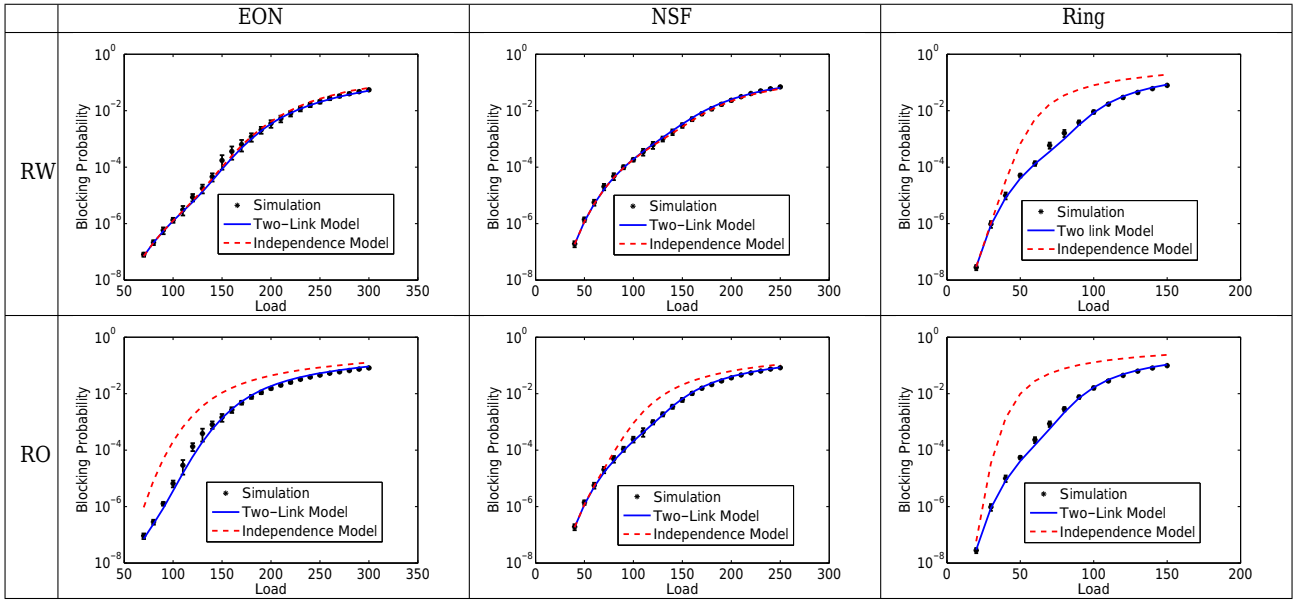


Fig. 8. Model validation and comparison of two-link and independence models; 10 OEOs per 3R node.

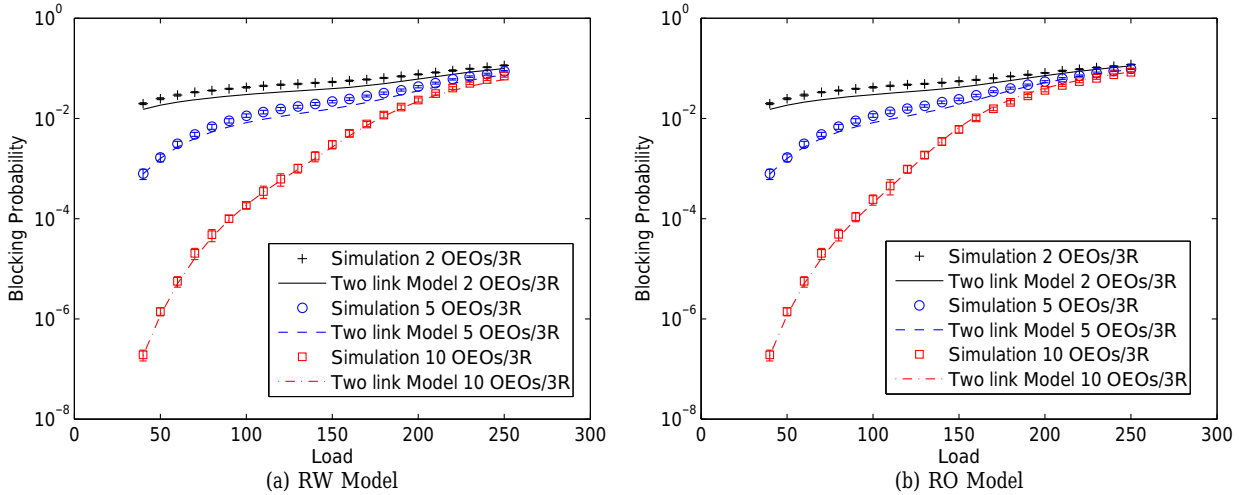


Fig. 9. Blocking versus load; NSF network. Results from the two-link model are shown as lines, while simulation results are indicated by symbols.

V. CONCLUSIONS

Transnational and transcontinental optical networks are translucent—they are mostly optically switched but have sparsely located 3R regenerators (with a limited number of OEO converters) that help to mitigate PLIs and/or perform wavelength conversion. Given their wide applicability, analyzing the performance of such networks is an important issue. In this paper, we present the first analytical models, to the best of our knowledge, for calculating the connection blocking probability in translucent optical networks. Two OEO allocation policies—RW and RO—are analyzed. Our models are rigorously constructed and maintain a balance between accuracy and computational complexity. The models are shown to predict blocking performance extremely well as demonstrated by extensive

simulation results for a variety of topologies and parameter settings. In this work, we assume fixed paths and random wavelength assignment for connections. Future extensions can consider alternate paths, first-fit wavelength assignment, grooming, and other OEO sharing models.

APPENDIX A

The probability $P(\alpha, K, C_r, C_w, C_c, C_e, C_f)$ in Eq. (17) is defined as the joint probability of the following events: There are $W - C_c - C_e - C_w$ available wavelengths on the first link, $W - C_c - C_f - C_w$ on the second link, K available (common) wavelengths on both links, and when C_r wavelengths are randomly chosen from the available wavelengths on each of the two links, α of the C_r are identical on both links. To find this probability, let us define $P(\alpha, C_r | K, C_w, C_c, C_e, C_f)$

as follows. Suppose there are $W - C_c - C_e - C_w$ available wavelengths on the first link, $W - C_c - C_f - C_w$ available wavelengths on the second link, and K available (common) wavelengths on both links; then we randomly choose C_r wavelengths from the available wavelengths on each link (C_r from the first link and, independently, C_r from the second link). Then, the probability that α wavelength pairs of C_r have the same wavelength on the two links is denoted by $P(\alpha, C_r | K, C_w, C_c, C_e, C_f)$.

In order to find $P(\alpha, C_r | K, C_w, C_c, C_e, C_f)$, we first find $Y(a, b | m, u, v)$ defined as follows. Given that there are u available wavelengths on the first link, v available wavelengths on the second link, and m available (common) wavelengths on both links, randomly choose $a + b$ wavelengths from the available wavelengths on each link. Then $Y(a, b | m, u, v)$ is the number of ways that a wavelength pairs have the same wavelengths on the two links, while b wavelength pairs have different wavelengths on the two links, for $m \leq u, v; a + b \leq u, v; a \leq m$. We show how to calculate $Y(a, b | m, u, v)$ in Appendix D.

Now, we can write

$$P(\alpha, C_r | K, C_w, C_c, C_e, C_f) = \frac{Y(\alpha, C_r - \alpha | K, W - C_c - C_e - C_w, W - C_c - C_f - C_w)}{\binom{W - C_c - C_e - C_w}{C_r} \binom{W - C_c - C_f - C_w}{C_r}}. \quad (\text{A1})$$

Using the law of total probability, we then have

$$\begin{aligned} P(\alpha, K, C_r, C_w, C_c, C_e, C_f) &= P(\alpha, C_r | K, C_w, C_c, C_e, C_f) P(K, C_w, C_c, C_e, C_f) \\ &= P(\alpha, C_r | K, C_w, C_c, C_e, C_f) P(K | C_w, C_c, C_e, C_f) \\ &\quad \times \sum_{C_r} \pi(C_e, C_f, C_c, C_r, C_w), \end{aligned} \quad (\text{A2})$$

where

$$\begin{aligned} P(K | C_w, C_c, C_e, C_f) &= \frac{\binom{W}{C_c} \binom{W - C_c}{C_e + C_w} \binom{W - C_c - C_e - C_w}{K} \binom{C_e + C_w}{W - C_c - C_f - C_w - K}}{\binom{W}{C_c} \binom{W - C_c}{C_e + C_w} \binom{W - C_c}{C_f + C_w}} \\ &= \frac{\binom{W - C_c - C_e - C_w}{K} \binom{C_e + C_w}{W - C_c - C_f - C_w - K}}{\binom{W - C_c}{W - C_c - C_f - C_w}}. \end{aligned} \quad (\text{A3})$$

APPENDIX B

In this appendix, we calculate $M(i, j, d)$ and $M'(i, j, d, k)$.

1. Calculation of $M(i, j, d)$

Recall that $M(i, j, d)$ denotes the probability that there is no blocking from i to d , given that there are available OEOs at node j , the previously allocated 3R node is node i (if there is no previously allocated 3R node, node i is source node s), and there are available wavelengths on $p_{i,j}$, i.e.,

$$M(i, j, d) = \Pr\{G_{i,d} = 1 | E_{i,j} = 1, H_j = 1, \text{ the previously allocated 3R node is node } i\}$$

We need to consider the cases $D(p_{i,j+1}) \leq \Omega$ and $D(p_{i,j+1}) > \Omega$ separately when calculating $M(i, j, d)$, since in the case in which $D(p_{i,j+1}) > \Omega$, OEOs at node j are allocated to meet the TR requirement. However, in the case in which $D(p_{i,j+1}) \leq \Omega$, the OEOs at node j are allocated only when there are no available wavelengths from node i to $j + 1$. Recall that h is the last 3R node on the path.

Case 1: $D(p_{i,j+1}) \leq \Omega$ and $j \neq h$

Based on the availability of OEOs at node $j + 1$, and the availability of wavelengths on path $p_{i,j+1}$, we have three subcases. When $E_{i,j+1} = 0$, the 3R node j is allocated to convert wavelengths:

$$\begin{aligned} M(i, j, d) &= \Pr\{H_{j+1} = 1, E_{i,j+1} = 1 | E_{i,j} = 1\} \\ &\quad \times \Pr\{G_{i,d} = 1 | E_{i,j+1} = 1, H_{j+1} = 1\} + \Pr\{H_{j+1} = 1, \\ &\quad E_{i,j+1} = 1, E_{i,j+1} = 0 | E_{i,j} = 1\} \\ &\quad \times \Pr\{G_{j,d} = 1 | E_{j,j+1} = 1, H_{j+1} = 1\} + \Pr\{H_{j+1} = 0, \\ &\quad G_{i,d} = 1 | E_{i,j} = 1, H_j = 1\} \\ &= (1 - b_{j+1}^l) \frac{F(i, j+1)}{F(i, j)} M(i, j+1, d) + (1 - b_{j+1}^l) \\ &\quad \times \frac{F'(i, j+1)}{F(i, j)} M(j, j+1, d) + b_{j+1}^l M'(i, j+1, d, j). \end{aligned}$$

Case 2: $D(p_{i,j+1}) > \Omega$ and $j \neq h$

In this case, the 3R node j is allocated to regenerate the signal, no matter the availability of wavelengths on path $p_{i,j+1}$. Based on the availability of OEOs at node $j + 1$, we have two subcases:

$$\begin{aligned} M(i, j, d) &= \Pr\{H_{j+1} = 1, E_{j,j+1} = 1 | E_{i,j} = 1\} \\ &\quad \times \Pr\{G_{j,d} = 1 | E_{j,j+1} = 1, H_{j+1} = 1\} + \Pr\{H_{j+1} = 0\} \\ &\quad \times \Pr\{G_{i,d} = 1 | E_{i,j} = 1, H_{j+1} = 0, H_j = 1\} \\ &= (1 - b_{j+1}^l) F(j, j+1) M(j, j+1, d) \\ &\quad + b_{j+1}^l M'(i, j+1, d, j). \end{aligned}$$

Case 3: $j = h$

$$M(i, j, d) = (F(i, d) / F(i, j)) + (F'(i, j, d) / F(i, j)) \text{ if } D(p_{i,d}) \leq \Omega; \\ M(i, j, d) = F(j, d) \text{ if } D(p_{i,d}) > \Omega.$$

2. Calculation of $M'(i, j, d, k)$

Recall that $M'(i, j, d, k)$ denotes the probability that there is no blocking from i to d , given that there are no available OEOs at node j , the previously allocated 3R node is node i (if there is no previously allocated 3R node, node i is source node s), and the previous 3R node with available OEOs, but not yet allocated, is node k ; there are available wavelengths on $p_{i,k}$:

$$M'(i, j, d, k) = \Pr\{G_{i,d} = 1 | E_{i,k} = 1, H_j = 0, H_k = 1, \\ \text{the previously allocated 3R node is node } i, \\ \text{3R node } k \text{ has not been allocated yet.}\}$$

Node k is set as 0 if there is no such k , e.g., $j = i + 1$; in this case, there is no condition in which there are available wavelengths on $p_{i,k}$. For the case $k = 0, j \neq h, D(p_{i,j+1}) \leq \Omega$, depending on the availability of OEOs at node $j + 1$, we have $M'(i, j, d, k) = (1 - b_{j+1}^l)F(i, j + 1)M(i, j + 1, d) + b_{j+1}^l M'(i, j + 1, d, k)$. If $k = 0, j = h$, and $D(p_{i,d}) \leq \Omega$, $M'(i, j, d, k) = F(i, d)$. Otherwise, $M'(i, j, d, 0) = 0$.

Suppose now that $k \neq 0$.

Case 1: $D(p_{i,j+1}) \leq \Omega$ and $j \neq h$

Similar to $M(i, j, d)$, based on the availability of OEOs at node $j + 1$, and the availability of wavelengths on path $p_{i,j+1}$, we have three subcases. When $E_{i,j+1} = 0$, the 3R node k is allocated to convert wavelengths:

$$M'(i, j, d, k) = \Pr\{H_{j+1} = 1, E_{i,j+1} = 1 | E_{i,k} = 1\} \\ \times \Pr\{G_{i,d} = 1 | E_{i,j+1} = 1, H_{j+1} = 1\} \\ + \Pr\{H_{j+1} = 1, E_{k,j+1} = 1, E_{i,j+1} = 0 | E_{i,k} = 1\} \\ \times \Pr\{G_{k,d} = 1 | E_{k,j+1} = 1, H_{j+1} = 1\} \\ + \Pr\{H_{j+1} = 0, G_{i,d} = 1 | E_{i,k} = 1, H_k = 1\} \\ = (1 - b_{j+1}^l) \frac{F(i, j + 1)}{F(i, k)} M(i, j + 1, d) + (1 - b_{j+1}^l) \\ \times \frac{F'(i, k, j + 1)}{F(i, k)} M(k, j + 1, d) + b_{j+1}^l M'(i, j + 1, d, k).$$

Case 2: $D(p_{i,j+1}) > \Omega, D(p_{k,j+1}) \leq \Omega$, and $j \neq h$

In this case, the 3R node k is allocated to regenerate the signal, no matter the availability of wavelengths on path $p_{i,j+1}$. Based on the availability of OEOs at node $j + 1$, we have two subcases:

$$M'(i, j, d, k) = \Pr\{H_{j+1} = 1, E_{k,j+1} = 1 | E_{i,k} = 1\} \\ \times \Pr\{G_{k,d} = 1 | H_{j+1} = 1, E_{k,j+1} = 1\} \\ + \Pr\{H_{j+1} = 0, G_{i,d} = 1 | E_{i,k} = 1, H_k = 1\} \\ = (1 - b_{j+1}^l) F(k, j + 1) M(k, j + 1, d) + b_{j+1}^l M'(i, j + 1, d, k).$$

Case 3: $j = h$

If $D(p_{i,d}) \leq \Omega$, $M'(i, j, d, k) = (F(i, d)/F(i, k)) + (F'(i, k, d)/F(i, k))$. If $D(p_{i,d}) > \Omega, D(p_{k,d}) \leq \Omega$, $M'(i, j, d, k) = F(k, d)$.

APPENDIX C

In this appendix, we calculate several variables defined in Table IV. From the definitions in Table IV, we can write

$$\bar{F}(u, v, t) = \begin{cases} 0, & \text{if } D(P_{u,v}) > \Omega \text{ or } D(P_{v,t}) > \Omega, \\ F_0(u, t) - F_0(u, v), & \text{otherwise.} \end{cases} \quad (C1)$$

$$F'(u, v, t) = \begin{cases} 0, & \text{if } D(P_{u,v}) > \Omega \text{ or } D(P_{v,t}) > \Omega, \\ F_0(u, t) - F_0(u, v) - F(u, v)F_0(v, t), & \text{otherwise.} \end{cases} \quad (C2)$$

Now, let us consider the path p . Assume the source s and destination d of the path are dummy 3R nodes, and let $b_s^l = 0$ and $b_d^l = 0$. Figure 1 shows the positions of nodes s, q, h, i, g , and d . Suppose the current 3R node under consideration is node i .

We first find $X(s, i, g)$, the probability that the next 3R node (from s) with free OEOs is node g (which means that all the 3R nodes j between i and g have no free OEOs), and there are available wavelengths on $p_{s,i}$, but there is no wavelength available on $p_{s,g}$. So we have

$$X(s, i, g) = \begin{cases} (1 - b_g^l) \prod_{j=i+1}^{g-1} b_j^l \bar{F}(s, i, g), & \text{if } D(P_{s,g}) \leq \Omega, \\ 0, & \text{otherwise.} \end{cases} \quad (C3)$$

$X'(s, i, g)$ is the same as $X(s, i, g)$ except that it requires that there are available wavelengths on $p_{i,g}$ also; so by replacing $\bar{F}(s, i, g)$ in the previous equation with $F'(s, i, g)$, we have

$$X'(s, i, g) = \begin{cases} (1 - b_g^l) \prod_{j=i+1}^{g-1} b_j^l F'(s, i, g), & \text{if } D(P_{s,g}) \leq \Omega, \\ 0, & \text{otherwise.} \end{cases} \quad (C4)$$

Now, we show the equations corresponding to the general case in which the previously allocated 3R node is some node h (as opposed to the source s). This implies that node h has free OEOs, and this happens with probability $(1 - b_h^l)$. Similar to $X(s, i, g)$, there are free OEOs at 3R node g , and no free OEOs at the 3R nodes j between i and g . Thus, we need to find the probability that there is no blocking from s to i , and there are available wavelengths on $p_{h,i}$, but there is no wavelength available on $p_{h,g}$. Assume the 3R node allocated before node h is node q , and the first 3R node after h with free OEOs is node m (m can be i). Now, note that node h , the previously allocated 3R node, may be allocated either to convert wavelengths or to regenerate signals, depending on the distance of the path $p_{q,m}$. $X'(q, h, m)$ and $Z'(q, h, m)$ reflect these two cases, respectively. We thus have

$$\begin{aligned}
X(h, i, g) &= (1 - b_h^l)(1 - b_g^l) \prod_{j=i+1}^{g-1} b_j^l \sum_{q=8}^{h-1} \left(\sum_{\substack{h+1 \leq m \leq i \\ D(p_{q,m}) \leq \Omega}} X'(q, h, m) \right. \\
&\quad \left. + \sum_{\substack{h+1 \leq m \leq i \\ D(p_{q,m}) > \Omega \\ D(p_{h,m}) \leq \Omega}} Z'(q, h, m) \right) \Pr\{E_{h,i} = 1, E_{h,g} = 0 | E_{h,m} = 1\} \\
&= (1 - b_h^l)(1 - b_g^l) \prod_{j=i+1}^{g-1} b_j^l \sum_{q=8}^{h-1} \left(\sum_{\substack{h+1 \leq m \leq i \\ D(p_{q,m}) \leq \Omega}} X'(q, h, m) \right. \\
&\quad \left. + \sum_{\substack{h+1 \leq m \leq i \\ D(p_{q,m}) > \Omega \\ D(p_{h,m}) \leq \Omega}} Z'(q, h, m) \right) \frac{\bar{F}(h, i, g)}{F(h, m)}. \quad (C5)
\end{aligned}$$

To find $X'(h, i, g)$, besides replacing $\bar{F}(h, i, g)$ with $F'(h, i, g)$, we also need to consider the case in which $m = i$, since the value of $X'(h, i, g)$ is used for the calculation of $X'(i, g, z)$ (or $Z'(i, g, z)$), which includes the probability that there are free OEOs at node i , that is $(1 - b_i^l)$. However, if $m = i$, this probability is already included in $X'(q, h, m)$ and $Z'(q, h, m)$, and we need to subtract that probability from $X'(h, i, g)$. Let $I = 1$ if $D(p_{q,i}) \leq \Omega$. We have

$$\begin{aligned}
X'(h, i, g) &= (1 - b_h^l)(1 - b_g^l) \prod_{j=i+1}^{g-1} b_j^l \sum_{q=8}^{h-1} \left(\sum_{\substack{h+1 \leq m \leq i-1 \\ D(p_{q,m}) \leq \Omega}} X'(q, h, m) \right. \\
&\quad \left. + \frac{X'(q, h, i)}{1 - b_i^l} I + \sum_{\substack{h+1 \leq m \leq i-1 \\ D(p_{q,m}) > \Omega \\ D(p_{h,m}) \leq \Omega}} Z'(q, h, m) \right. \\
&\quad \left. + \frac{Z'(q, h, i)}{1 - b_i^l} (1 - I) \right) \frac{F'(h, i, g)}{F(h, m)}. \quad (C6)
\end{aligned}$$

Then we consider Z and Z' . $Z(s, i, g)$ is the probability that the next 3R node having free OEOs is node g , and there are available wavelengths on $p_{s,i}$. So we have

$$Z(s, i, g) = \begin{cases} (1 - b_g^l) \prod_{j=i+1}^{g-1} b_j^l F(s, i), & \text{if } D(P_{s,g}) > \Omega, \\ 0, & \text{otherwise,} \end{cases} \quad (C7)$$

$Z'(s, i, g)$ is the same as $Z(s, i, g)$ except that it requires that there are available wavelengths on $p_{i,g}$ also. (Note that for Z , we do not care whether there are free wavelengths on $p_{s,g}$ as with X .) So we have

$$Z'(s, i, g) = \begin{cases} (1 - b_g^l) \prod_{j=i+1}^{g-1} b_j^l F(s, i) F(i, g), & \text{if } D(P_{s,g}) > \Omega, \\ 0, & \text{otherwise.} \end{cases} \quad (C8)$$

Now we show the equations corresponding to the general case in which the previously allocated 3R node is node h . As in finding $X(h, i, g)$, we also need the values of $X'(q, h, m)$ and $Z'(q, h, m)$, the difference being that we only need to include the probability that there are available wavelengths on $p_{h,i}$, without consideration of $p_{h,g}$:

$$\begin{aligned}
Z(h, i, g) &= (1 - b_h^l)(1 - b_g^l) \prod_{j=i+1}^{g-1} b_j^l \sum_{q=8}^{h-1} \left(\sum_{\substack{h+1 \leq m \leq i \\ D(p_{q,m}) \leq \Omega}} X'(q, h, m) \right. \\
&\quad \left. + \sum_{\substack{h+1 \leq m \leq i \\ D(p_{q,m}) > \Omega \\ D(p_{h,m}) \leq \Omega}} Z'(q, h, m) \right) \Pr\{E_{h,i} = 1 | E_{h,m} = 1\} \\
&= (1 - b_h^l)(1 - b_g^l) \prod_{j=i+1}^{g-1} b_j^l \sum_{q=8}^{h-1} \left(\sum_{\substack{h+1 \leq m \leq i \\ D(p_{q,m}) \leq \Omega}} X'(q, h, m) \right. \\
&\quad \left. + \sum_{\substack{h+1 \leq m \leq i \\ D(p_{q,m}) > \Omega \\ D(p_{h,m}) \leq \Omega}} Z'(q, h, m) \right) \frac{F(h, i)}{F(h, m)}. \quad (C9)
\end{aligned}$$

The difference between $Z'(h, i, g)$ and $Z(h, i, g)$ is that it includes the probability that there are available wavelengths on $p_{i,g}$ and we also need to consider the case in which $m = i$ as we did when finding $X'(h, i, g)$:

$$\begin{aligned}
Z'(h, i, g) &= (1 - b_h^l)(1 - b_g^l) \prod_{j=i+1}^{g-1} b_j^l \sum_{q=8}^{h-1} \left(\sum_{\substack{h+1 \leq m \leq i-1 \\ D(p_{q,m}) \leq \Omega}} X'(q, h, m) \right. \\
&\quad \left. + \frac{X'(q, h, i)}{1 - b_i^l} I + \sum_{\substack{h+1 \leq m \leq i-1 \\ D(p_{q,m}) > \Omega \\ D(p_{h,m}) \leq \Omega}} Z'(q, h, m) \right. \\
&\quad \left. + \frac{Z'(q, h, i)}{1 - b_i^l} (1 - I) \right) \frac{F(h, i) F(i, g)}{F(h, m)}. \quad (C10)
\end{aligned}$$

APPENDIX D

In this appendix, we show how to calculate $Y(a, b|m, u, v)$. $Y(a, b|m, u, v)$ can be computed by first picking the a common wavelength pairs from the m common available wavelengths, and then selecting the b different wavelengths from the remaining wavelengths. It is given by the following expression:

$$Y(a, b|m, u, v) = \binom{m}{a} Y(0, b|m - a, u - a, v - a). \quad (D1)$$

If $b = 0$, we have $Y(a, 0|m, u, v) = \binom{m}{a}$.

Then,

$$\begin{aligned}
Y(0, b|m - a, u - a, v - a) &= \binom{u - a}{b} \binom{v - a}{b} - \sum_{i=1}^{\min(b, m-a)} Y(i, b - i|m - a, u - a, v - a) \\
&= \binom{u - a}{b} \binom{v - a}{b} - \sum_{i=1}^{\min(b, m-a)} \binom{m - a}{i} \\
&\quad \times Y(0, b - i|m - a - i, u - a - i, v - a - i). \quad (D2)
\end{aligned}$$

The above is derived as follows. We want b different wavelength pairs to be selected on the two links. To get this, we find the total number of ways of selecting b wavelengths

on each link from the set of available wavelengths, respectively, (the first term), and then subtract from it the number of ways in which there are one or more common wavelength pairs on the two links (the second term of the first equation). Now, notice that the second term is equal to what we are seeking, but with a smaller number of wavelengths. We can thus compute $Y(\cdot)$ recursively, provided that we know how to compute the base case. The base case is

$$Y(0, 1|m', u', v') = \binom{m'}{1} \binom{v'-1}{1} + \binom{u'-m'}{1} \binom{v'}{1}. \quad (\text{D3})$$

Here, $\binom{m'}{1} \binom{v'-1}{1} + \binom{u'-m'}{1} \binom{v'}{1}$ means if one wavelength on the first link is selected from a set of m' wavelengths ($\binom{m'}{1}$ ways), then that wavelength cannot be selected on the second link ($\binom{v'-1}{1}$ ways). If the wavelength on the first link is not selected from the set of m' wavelengths ($\binom{u'-m'}{1}$ ways), then the wavelength selected on the second link can be any one of the available wavelengths on the second link ($\binom{v'}{1}$ ways).

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