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Optimal regenerator placement in translucent optical networks

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ABSTRACT

The distance an optical signal can travel, before its quality degrades to a level that requires *3R-regeneration*, is called the *optical reach*. In a *translucent optical network*, if an optical signal has to be communicated over a distance that exceeds the optical reach, the signal is regenerated at selected nodes of the network, so that the signal quality never degrades to an unacceptable level. Given a value of the optical reach, the goal of the *Regenerator Placement Problem* (RPP), in networks handling ad-hoc demands for lightpaths, is to find the minimum number of nodes capable of 3R regeneration necessary in the network and their positions, so that every pair of nodes (u, v) can establish a lightpath (either transparent or translucent) from u to v. In this paper we have presented two Integer Linear Program (ILP) formulations that can optimally solve the RPP problem for practical-sized networks within a reasonable amount of time. The first formulation works for networks having 35 nodes or less. The second formulation works for larger networks as well (we have reported results with up to 140 nodes). We have used a branch-and-cut approach to implement the second formulation, where we have intercepted the optimization process with control callbacks from the CPLEX callable library to introduce new constraints, as needed.

1. Introduction

In an *all-optical* network, *optical-bypass* is used to carry all traffic entirely in the optical domain so that, if there is a lightpath from a source $\mathcal{S}$ to a destination $\mathcal{D}$, no Optical–Electrical–Optical (OEO) conversion is needed at any intermediate node in the path from $\mathcal{S}$ to $\mathcal{D}$. All-optical networks are also referred to as *transparent* networks, in contrast to *opaque* networks that use all-electronic switching techniques [1–3]. The quality of transmission (QoT) of an optical signal propagating through a network degrades, due to physical layer impairments, such as *optical noise*, *chromatic and polarization mode dispersion*, *four wave mixing*, *cross-phase modulation* and *cross-talk* [1,4]. This leads to an increase in the *Bit Error Rate* (BER) of the signal and the corresponding lightpath becomes infeasible for communication, if the BER value crosses a certain threshold. As a result, in a wide-area backbone optical network, spanning a large geographical area, all end-to-end connections cannot always be established in the optical domain [1,4].

Physical layer impairment aware route and wavelength assignment (PLI-RWA) deals with designing optical networks, taking into account these degradations. When the quality of an optical signal becomes unacceptable, it is necessary to *reamplify*, *reshape* and *retime* the optical signal. These three processes cleanup and rectify the optical signals, and are often jointly called *3R regeneration* [1,5]. The notion of *translucent* networks, introduced by

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Ramamurthy et al. in [3,6,7], has features of both transparent and opaque networks. In a *translucent network*, at least one optical signal is regenerated at one or more regeneration points (typically a selected subset of the nodes of the network has the capacity to carry out 3R-regeneration [1]), so that the signal may be communicated over long distances. Henceforth, in this paper, we will call a node which has the capacity to carry out 3R regeneration as *a regenerator node*. Even though regeneration can be accomplished completely in the optical domain, regeneration in the electronic domain (i.e., using OEO conversion) is still the most economical and reliable technique [8]. When OEO conversion takes place, carrier wavelength conversion is available for free [5,9], so that, if a lightpath from $\mathcal{S}$ to $\mathcal{D}$ undergoes OEO conversion at some node $\mathcal{P}$, the carrier wavelength of the incoming lightpath to $\mathcal{P}$ may be different from the carrier wavelength of the outgoing lightpath from $\mathcal{P}$.

Physical layer impairments (PLI) can be classified into two categories – linear impairments and nonlinear impairments [5,10,11] (some researchers have categorized them somewhat differently as Class 1 and Class 2 impairments [12,13]). Linear impairments are independent of the signal power and affect each of the wavelengths (optical channels) individually, whereas nonlinear impairments affect not only each optical channel individually but they also cause disturbance and interference between them [5]. *Optical reach* [2,14] (also called *transparent reach* [1,5] and *transmission reach* [8]) is the maximum distance an optical signal can travel before 3R-regeneration is needed. Optical reach is a popular metric [2,8] to determine when 3R regeneration is needed and usually ranges from 2000 to 4000 km [4].[1]

A lightpath that involves one or more regenerators is often called a *translucent lightpath* [1]. Each translucent lightpath is a concatenation of two or more transparent (i.e., all-optical) lightpaths, where one transparent lightpath is from the source of the communication to a regenerator node, one is from a regenerator node to the destination for the communication and the remaining transparent lightpath(s), if any, is (are) from a regenerator node to another regenerator node. An example of a long haul network with distances between the nodes in km is shown in Fig. 1. If the optical reach is 2000 km, an optical signal from node A cannot reach node D without 3R-regeneration, so that a lightpath from A to D must be a translucent lightpath. It may be easily verified that a translucent lightpath from A to D using the route $A \rightarrow B \rightarrow C \rightarrow D$ involving a 3R-regeneration either at B or C is a valid solution.

Following [5,8,16], when carrying out network planning, we classify the requests for communication as follows:

(a) some requests for communication are relatively permanent and are called *permanent lightpath demands* (PLD) (or static demands);
(b) some requests have a lifetime (i.e., a start time when the lightpath is set up and an end time when the lightpath is taken down). These are called *dynamic lightpath demands* (DLD) and may be further subcategorized as follows:
  (i) the start time and the lifetime of the request may be known in advance. Such requests are called, *scheduled lightpath demands* (SLD),
  (ii) neither the start time nor the duration of such a request is known in advance. Such requests are called *ad-hoc lightpath demands* (ALD).

[1] In [15] it was stated that an optimal value of the optical reach ranges between 2500 km and 3500 km.

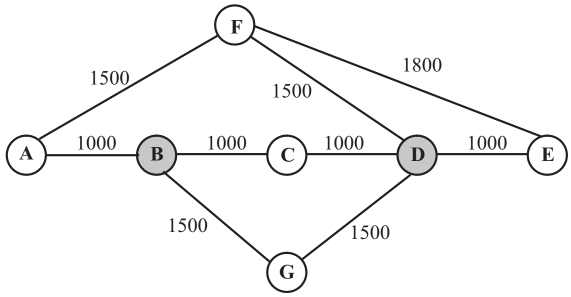

Fig. 1. Long haul optical network with distances between the nodes in km.

Route and wavelength assignment where all requests are PLDs (DLDs) are called *static*, also *offline* [5,9,10] (*dynamic*, also *on-line* [5,9,10]) PLI-RWA. In a translucent network, the PLI-RWA problem will be inevitably coupled with regeneration placement problem [5,8–10] in which the network designers are trying to plan and design, for a given network topology, a translucent network with an optimal number of regeneration sites. Static PLI-RWA in a translucent network requires users to specify beforehand all pairs $\{\mathcal{S}, \mathcal{D}\}$, such that a lightpath (translucent or transparent) will be set up from $\mathcal{S}$ to $\mathcal{D}$. In networks that handle ALD, the concepts of islands of transparency [17] and sparse regeneration [2,3,14] have been studied extensively.

In a network that handles ad-hoc lightpath demands, using sparse regeneration, there are two types of problems that have to be handled:

Problem1 (to be handled during the network design phase): Given a network topology, select a minimum number of nodes to be regenerator nodes, so that each node can communicate with any other node in the network, using either a transparent lightpath or a translucent lightpath. This is called the *Regenerator Placement Problem* (RPP) [2,6,9,14].

Problem2 (to be handled when the network is in operation): In response to a new request for communication, say from node $\mathcal{S}$ to node $\mathcal{D}$, set up, if possible, a lightpath (transparent or translucent) from $\mathcal{S}$ to node $\mathcal{D}$, taking into account all existing lightpaths in the network. This is called the *Routing with Regenerator Problem* (RRP). Given a network topology with selected nodes identified as regenerator nodes, and a number of lightpaths already in existence, the objective of RRP is to optimally route the new lightpath from

$\mathcal{S}$ to $\mathcal{D}$, using a minimum number of regenerator nodes [6,18,19].

For instance, if the network shown in Fig. 1 is using ALD, RPP has to be solved at the network design phase, in order to enable every node in the network to communicate with every other node in the network. This requires a minimum of 2 regenerator nodes and one solution is to select nodes B and D to be regenerator nodes. The RPP problem is known to be NP-complete [14,20,21] and heuristics are typically used to solve this problem [3,14,20–26] for relatively larger networks.

In this paper we present two Integer Linear Programs (ILP) to *optimally* solve the RPP for translucent networks handling ALD. The first formulation (FRM-1) is based on standard network flow programming techniques. Formulation FRM-1 can be readily solved by a commercial solver, such as the CPLEX [27], and works for relatively smaller networks (we have reported experimental results with networks having up to 35 nodes). The second formulation (FRM-2) involves an exponential number of constraints, known only implicitly. Our proposal is based on the *branch-and-cut* approach [28], a modern Operations Research (OR) technique for solving large integer linear programs, which solves this formulation with remarkable efficiency. This approach efficiently solves the RPP problem for relatively larger networks (we have solved the RPP for 140 node networks in less than 2000 s, on an average). Our experiments reveal that we only need a relatively small number of the constraints, so that the basis size is, in general, quite small and the LP relaxations can be solved very quickly.

The rest of the paper is organized as follows. In Section 2 we have presented a review of recent research on PLI-RWA, with emphasis on ad-hoc lightpath demands, along with a review on branch-and-cut. In Section 3.1 we have presented a compact node-arc formulation. In Section 3.2 we have presented our branch-and-cut formulation. In Section 4 we have presented the experimental results using the formulations FRM-1 and FRM-2 and we have concluded this paper in Section 5.

2. Literature review

2.1. Review articles on PLI-RWA

A review of algorithms for RPP and RRP appears in [2]. PLI-RWA algorithms, including the use of heuristics and meta-heuristics, and optimal algorithms, protection and resiliency issues, have been reviewed in [5]. Following [1], it is convenient to classify translucent optical networks into three categories as follows:

- Translucent networks, made up of transparent "islands" [29–33,17]. Regeneration nodes are located only on the island boundary, so that each transparent island has a limited geographic size, which is interconnected via non-transparent transponders to other islands and regeneration can take place only on the island boundary.
- Translucent networks, with sparsely placed opaque nodes, where a few "opaque" nodes are strategically placed, which convert *all* incoming signals to electronic signals. All remaining nodes are transparent OXC's which are capable of switching incoming optical signals to desired output ports.
- Translucent networks with strategically placed "translucent" (or "hybrid") nodes which can either convert an incoming signal to electronic form or retain the incoming signal in the optical domain and simply switch the optical signal to an appropriate output port.

The role of sparse regeneration in reducing the capital expenses (CAPEX), and operating expenses (OPEX) is explained in [8]. The paper discusses why translucent networks, with relatively few nodes which are selected to be regenerator nodes, represent, at the moment, the most practical scheme and classifies PLI-RWA algorithms. The objectives of algorithms for networks to handle static and ad-hoc demands for lightpaths, and the conflicting requirements of CAPEX and OPEX are reviewed in [10]. Impairment measurements using physical layer monitoring either at the impairment level or at the aggregate level appear in [9]. A comprehensive summary of a number of static and dynamic PLI-RWA and the importance of impairment-aware components (e.g., Tunable Dispersion Compensation) is also given in [9]. The use of analytical or hybrid models to obtain a composite measure of the impairments, using the optical signal to noise ratio (OSNR) or the bit error rates (BER) is reviewed in [34]. The paper also discusses traffic grooming, protection and restoration techniques for translucent networks. A comprehensive survey of the current sate of the art in PLI-RWA appears in [2].

2.2. Algorithms for static PLI-RWA

Static PLI-RWA problem has been considered in [5,9,10,35–39]. Path protection was considered in [40]. Heterogeneity in the physical layer of an optical network (e.g., different fiber types, variable amplification span distances) was considered in [41,42]. An ILP considering linear impairments and a heuristic considering both the linear and non-linear impairments appear in [35]. In [36] the routes of the lightpaths could be either pre-computed or may be determined by the algorithm and the number of regenerators at a node could have a finite bound (k) or could be unbounded. In [36], the length of the fibers connecting the nodes was not taken into account and it was shown that, for arbitrary topologies and a finite bound k, the problem is NP-hard and is not "approximable" in polynomial time. Pre-computed multiple paths, based on the notion of optical reach, were proposed in [37] in a heuristic to identify nodes which must be regenerator nodes. An ILP and a heuristic were proposed in [38] to minimize the number of nodes which must be regenerator nodes, considering some types of impairments. A comparison of the performances and tradeoffs involved, in designing and operating an optical network, when using (i) the worst case impairment estimates or (ii) the actual impairment estimates appear in [39]. Traffic grooming, to maximize the traffic that may be handled, by optimal placement of opaque nodes with traffic grooming capability, was studied in [43]. An ILP to solve the RPP considering several linear and non-linear impairments appears in [44].

2.3. Algorithms for RPP in networks handling ad-hoc lightpath demands

A review of impairment aware dynamic PLI-RWA appears in [16]. The benefits of impairment awareness in optical networks and the need for impairment aware algorithms for static and ad-hoc lightpath demands are outlined in [45]. The architecture of a system using dynamic PLI-RWA appears in [46]. The RPP problem for ALD was shown to be NP-complete [14,20,21] and is often solved using heuristics [3,14,20–26].

The concept of the *reachability graph* [21] is useful in our formulations. We construct the reachability graph $G_R = (N, E_R)$ from the graph $G = (N, E)$ representing a network, by defining E_R as follows. An edge $(\mathcal{S}, \mathcal{D})$ is in E_R, iff the shortest distance from $\mathcal{S}$ to $\mathcal{D}$ does not exceed the optical reach r.

Heuristics for RPP based on either the network topology or the expected traffic pattern were proposed in [3]. In [47] the concept of *connected dominating set* (CDS) was used. In [14] RPP was viewed as a minimum Labeled Connected Dominating Set (LCDS) problem for which a heuristic procedure was outlined. This approach removes the possibility of an invalid path that could be produced by the procedure in [47]. In [48], it was shown that the blocking probability of the sparse regenerator placemen is comparable to that of an opaque network. The paper also compares the RPP strategies in [6,49]. Constraint-based routing (CBR) algorithm to handle RPP and RRP was proposed in [11,50], taking into account the increases in non-linear impairments on existing connections, when new connections are provisioned. In [49], the problem of k-connectivity guarantying a desired minimum degree of end-to-end (optical) connectivity was studied.

Path protection scheme, based on the Shared Risk Link Group (SRLG) idea, was studied in [23], based on a non-cooperative game approach, for an approximate solution. Both static and dynamic PLI-RWA were studied in [51], considering some linear and some non-linear impairments.

RPP, using the notion of optical reach, was studied in [20] and an algorithm was proposed, based on the notion of Steiner arborescence with some constraints, including a branch-and-cut procedure that can handle up to 100 nodes. In [52], p (for some limited value of p) possible sets of lightpaths are specified, and the objective was to place a minimum total number of regenerator nodes, so that each of the sets of lightpaths may be set up. This problem has some characteristics of networks handling static requests and networks handling ad-hoc requests for lightpaths. In [21] it was proved that the RPP problem can be formulated as a Minimum Connected Dominating Set (MCDS) problem. RPP was solved in [21], using an approximation algorithm for the minimum connected dominated set problem. They have also pointed out that the algorithm presented for the solution of RPP in [36] may sometimes produce invalid solutions. In [53], RPP was done assuming (a) a uniform static matrix of demands including one bidirectional request between each pairs of network nodes and (b) dynamic traffic. The paper studied the performance of opaque networks and translucent networks and concluded that, so far as the blocking probability is concerned, opaque networks have lower blocking probabilities. This is expected since the advantage of sparse placement of nodes which must be capable of 3R regeneration is in the CAPEX. Both static and ad-hoc requests for lightpaths were considered in [54], in order to place a minimal number of regenerator nodes under various scenarios. To measure the impairments, the optical reach or the number of hops was used. Any-to-any optical node connectivity is considered in [24], to find the sites for regenerator nodes, such that the blocking probability is acceptably low. In [24], optical reach was used to measure impairments and proposed heuristics for placing regenerator nodes, which were based on the notion of (i) k-center and (ii) connected dominating sets. Mertzios et al. [26] solved the RPP, given an optical reach r and a limit k on the number of regenerators that can be placed in a regenerator node. In [55] a constraints based programming approach for multi-objective optimization was proposed, which took into account the total routing cost, the total number of regenerations, and the number of regenerator nodes. If reducing the number of sites of regenerator nodes is the first criterion for the optimization, the approach in [55] may fail in some situations. In [56] two schemes for reducing the number of sites for regeneration and one scheme for minimizing the number of regenerator nodes were proposed. Politi et al. [25] discusses how BER values may be computed in a transparent and in a translucent network. A number of heuristics that take into account linear and non-linear impairments for the RRP appear in [25].

The benefits of restricting regeneration to a limited subset of sites within the same networks, using reconfigurable optical add/drop multiplexers, were studied in [57,58]. A heuristic to find a solution to the RPP using path protection appears in [59].

The problem studied in [60] focused on the locations of a minimum number of regenerator nodes to handle a specified set of commodities $K = \{1, \ldots, |K|\}$. Each commodity denotes a node pair that communicate with each other, using two edge-disjoint paths. Since the number of constraints in the formulation *M2* in [60] (Section 6.2) was exponential, the thesis used a branch-and-cut approach. If the set of constraints K used in formulation *M2* corresponds to the set of all ordered (source, destination) pairs in the network, such that the shortest path from the source to the destination is greater than the optical reach, then the formulation in [60] is solving the RPP problem for survivable PLI-RWA – a problem closely related to the problem we have studied in this paper.

Some heuristics for solving the RPP problem was studied in [61] and in [62]. A game theoretic approach to RPP was proposed in [63]. Using optical reach to estimate impairments, an ILP for an optimal solution and a heuristic, based on game theoretic approach, were proposed in [22] for a network using path protection. Tabu Search was used in [64].

2.4. Mixed-line-rate networks handling ad-hoc lightpath demands

Recently it has become technologically possible to have mixed line rates (MLR) over different wavelength channels. A typical scenario is to have different lightpaths with bit rates of 10, 40 or 100 Gbps depending on user requirements [65].

At higher bit rates, the signal impairments are significantly worse, so that the optical reach for a communication using a higher bit rate is considerably less, compared to that for a communication using a lower bit rate. Most work on mixed line rate considered transparent networks or static traffic. In [66] grooming and RRP for mixed line rate translucent networks handling ALD were considered. They used [20] to place the 3R regenerators. In [67] RPP was done using [3]. The focus of the paper was also on grooming and RRP.

2.5. Branch-and-cut algorithm

Branch-and-bound (BB or B&B) is a well-known general algorithm for finding optimal solutions of various optimization problems involving integer variables [28,68]. It consists of a systematic enumeration of all candidate solutions, where large subsets of fruitless candidates are discarded *en masse*, by using upper and lower estimated bounds of the quantity being optimized. A branch-and-bound algorithm to minimize an objective function requires two tools [28,68]. The first one is a *splitting* procedure that, given a set $\mathbb{S}$ of candidate solutions, returns two or more smaller sets whose union covers $\mathbb{S}$. This step is called *branching*, since its recursive application defines a tree structure (the search tree), whose nodes are the subsets of $\mathbb{S}$. When the integer values are binary, the algorithm uses a divide-and-conquer strategy to partition the solution space into two *subproblems* and then recursively solves each subproblem. The other tool is a procedure that computes the upper and the lower bounds for the minimum value, within a given subset of $\mathbb{S}$. This step is called *bounding*. The key idea of the BB algorithm is that, if the lower bound for some node A, in the subtree being explored, is greater than or equal to the upper bound for the tree, then node A may be safely discarded from the search. This step is called *pruning*, and is usually implemented by maintaining a global variable that records the minimum upper bound seen among all the subregions examined so far. Any node whose lower bound is greater than this minimum upper bound can be safely discarded (the process of discarding a node often called *fathoming* in the OR literature [68]). A simple flow-chart for the Branch-and-Bound algorithm appears in [28].

A *branch-and-cut* method [28] is a branch-and-bound method with cuts (valid inequalities for the integer program) generated throughout the branch-and-bound tree. The method starts with finding an optimal solution to some linear relaxation of the integer program. If this optimal solution, call it x^*, is not a feasible solution of the integer program being solved, a "separation" problem is solved to identify, if possible, a cut which is violated by x^*. If such a valid inequality is found, it is added to the relaxation problem and the problem is resolved. Branching, as in usual branch-and-bound procedure, takes place if such a cut is not identified by the "separation" routine.

The function "CPXsetcutcallbackfunc" from CPLEX *callable library* [27] is used to set and modify the user-written callback function to add cuts. The user-written callback is called by CPLEX during Mixed Integer Linear Program (MILP) branch-and-cut for every node in the solution tree that has an LP optimal solution with an objective value below the global upper bound but which does have an integer solution. Cuts that are added at a node remain part of all the subsequent subproblems. Once cuts are added, the current subproblem is re-solved and re-evaluated. If the new LP solution still does not have an integer solution and the objective value is below the upper bound, the cut callback is called again.

3. Formulations for solving RPP optimally

To model an optical network we use an undirected graph $G=(N,E)$, where N represents the set of nodes of the network and E represents the set of edges, where each edge $(i,j)\in E$ represents a bi-directional fiber between node i and node j. Each edge $(i,j)\in E$ also has a label d_{ij}, denoting the length of the fiber from i to j.

We may view the RPP problem as a network flow problem where we treat the problem of communicating from a source $\mathcal{S}$ to a destination $\mathcal{D}$ as a problem of shipping a distinct commodity from $\mathcal{S}$ to $\mathcal{D}$. We start by considering every possible ordered source–destination pair $(\mathcal{S},\mathcal{D})$ in the network and our goal is to send one unit of the commodity corresponding to $(\mathcal{S},\mathcal{D})$ from $\mathcal{S}$ to $\mathcal{D}$. If the minimum distance from node $\mathcal{S}$ to node $\mathcal{D}$ is less than the optical reach r, no regeneration is needed for the commodity. In other words, the issue of regenerator placement is irrelevant for such commodities and, in order to reduce the size of the problem, our discussions below assume that such commodities are not included in the multi-commodity network flow problem. Furthermore, if we can find a route for a commodity from $\mathcal{S}$ to $\mathcal{D}$, then the reverse of the same route may be used for the commodity from $\mathcal{D}$ to $\mathcal{S}$. Therefore we may simplify the RPP problem further, by considering only the commodity for node pair $(\mathcal{S},\mathcal{D})$, if $\mathcal{S}<\mathcal{D}$.

For each commodity we need to consider, our formulations have to select a path and identify regenerator nodes on such a path, so that

- the path for the commodity has at least one regenerator node,
- the distance from the source of the commodity to the first regenerator node in the path $\le r$,
- the distance from the last regenerator node in the path to the destination of the commodity $\le r$,
- the distance from any regenerator node to the next regenerator node in the path $\le r$.

After the RPP has been solved and the nodes identified by RPP are equipped with regenerators, given any source–destination pair $(\mathcal{S},\mathcal{D})$, there exists at least one route from $\mathcal{S}$ to $\mathcal{D}$, so that a viable optical connection can be established from $\mathcal{S}$ to $\mathcal{D}$, using this route, which may involve 0 or more regenerator nodes.

3.1. A compact formulation to solve the RPP

In this section we present FRM-1, a MILP formulation to optimally solve the RPP problem using Multi-Commodity Network Flow (MCNF) techniques [69] based on the node-arc representation.

The idea used in this formulation is to define flow-balance constraints [69] for each commodity of interest and determine a path for each commodity in the reachability graph where each intermediate node in the path is a regenerator node. The objective of the formulation is to minimize the number of regenerator nodes and can be given as

$$\text{minimize} \quad \sum_{j \in N} \beta_j \tag{1}$$

subject to

$$\sum_{j:(i,j) \in E_R} f_{ij}^k - \sum_{j:(j,i) \in E_R} f_{ji}^k = \begin{cases} 1 & \text{if } i = S^k, \\ -1 & \text{if } i = D^k, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Eq. (2) must be satisfied $\forall k \in K, \forall i \in N$.

$$\beta_j \geq \sum_{i:(i,j) \in E_R} f_{ij}^k: \quad \forall k \in K, \forall j \in N | j \neq D^k. \tag{3}$$

$$\beta_j = \{0, 1\}: \forall j \in N. \tag{4}$$

In the above formulation, β_j is a binary variable for each node $j \in N$ in the physical topology. If the node j is chosen to be a regenerator node, $\beta_j = 1$, otherwise $\beta_j = 0$. K denotes the set of all commodities that we need to consider. f_{ij}^k represents the flow for commodity $k \in K$ on edge $(i,j) \in E_R$ so that, if the commodity k (or a part of it) uses the edge (i,j), $f_{ij}^k > 0$, otherwise $f_{ij}^k = 0$. f_{ij}^k is a continuous variable.

Eq. (2) is a standard network flow conservation equation [69]. Eq. (3) ensures that a commodity $k \in K$ can only use an edge $(i,j) \in E_R$ in the reachability graph if either the node j is its destination node ($j = D^k$), or the node j is a regenerator node (i.e., $\beta_j = 1$). Since our objective is to minimize the number of regenerator nodes, Eqs. (2), (3) and the objective function means that the formulation finds a path (or a number of paths) for each commodity, such that the total number of intermediate nodes (each representing a regenerator node), considering all the paths used by all the commodities, is minimum. Clearly this solves the RPP.

We used continuous variables f_{ij}^k for the flow variables. This reduces the number of integer variables and decreases the time needed to solve the formulation. We note the implication that we are allowing the use of multiple paths to communicate each commodity from its source to its destination. We can select any one of the paths used by commodity $k \in K$ and discard all the other paths without increasing the number of regenerator nodes in the network. This formulation can be directly given to any commercially available mathematical optimization tool like CPLEX [27] for finding an optimal solution and can generate a solution within a reasonable time with the small and medium sized networks (networks having up to 35 nodes).

3.2. RPP using the branch-and-cut algorithm

To find optimal solutions for the RPP problems for large translucent networks, in this section we propose a new formulation and a *branch-and-cut* algorithm for solving this problem using the new formulation. This formulation has, in general, an exponential number of constraints, known only implicitly. For that reason, this formulation cannot be solved directly by CPLEX. However, the formulation may be implemented by interacting with the CPLEX solver, using the control callbacks from the CPLEX callable library [27].

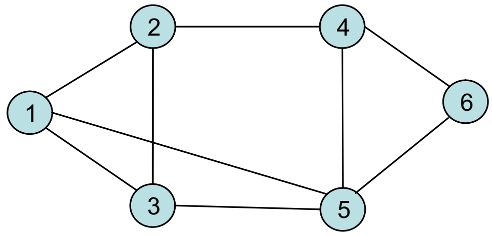

Fig. 2. Reachability graph for a 6-node network.

As we mentioned in Section 3, we need to consider only those requests for communication where the lightpath between the nodes needs at least one regenerator node. The following notion of *Disconnecting Set of Nodes (DSN)* is useful in identifying potential regenerator nodes.

Definition 1. A set $\mathbb{D} \subset N$ is a disconnecting set of nodes (DSN), if at least one of the nodes in $\mathbb{D}$ must be made a regenerator node for a feasible solution of the RPP problem.

Example. Let us consider the reachability graph for a six node network as shown in Fig. 2. As discussed in the introduction of Section 3, the commodities that need to be considered are $K^1 = (1,4)$, $K^2 = (1,6)$, $K^3 = (2,5)$, $K^4 = (2,6)$, $K^5 = (3,4)$ and $K^6 = (3,6)$. Some disconnecting sets of nodes (DSN) are

- $\{2, 3, 5\}$,
- $\{1, 3, 4\}$,
- $\{1, 2, 5\}$,
- $\{4, 5, 6\}$,
- $\{1, 3, 4, 6\}$,
- $\{4, 5\}$.

It is important to note that, for a given network, the number of disconnecting set of nodes (DSNs) is, in general, exponential.

3.2.1. An ILP formulation to solve the RPP

As before, we assume that we are given a reachability graph $G_R = (N, E_R)$. Let K be the set of commodities we need to consider, as discussed in the introduction of Section 3, where each commodity is specified by the corresponding source–destination pair. RPP can now be solved using the following binary linear integer program (BILP):

$$\text{minimize} \quad \sum_{j \in N} \beta_j \tag{5}$$

subject to

$$\sum_{j \in \mathbb{D}} \beta_j \geq 1: \quad \forall \mathbb{D} \in \Omega \tag{6}$$

$$\beta_j = \{0, 1\}: \quad j \in N \tag{7}$$

In this formulation β_j is, as before, a binary variable corresponding to node $j \in N$. If node j is selected to be a regenerator node, $\beta_j = 1$, otherwise $\beta_j = 0$. $\mathbb{D}$ is defined to be a disconnecting set of nodes (DSN). Ω is the set of all DSN's.

As before, our objective is to minimize the number of regenerator nodes. Eq. (6) ensures that at least one node in each DSN $\mathbb{D}$ must be a regenerator node. Eq. (6) must be satisfied $\forall \mathbb{D} \in \Omega$.

Lemma 1. *Any solution satisfying constraints* (6) *and* (7) *is a feasible solution for the RPP.*

Proof. Let there be a solution that satisfies all the constraints of the above BILP but is not a feasible solution for the RPP. In other words, at least one pair of nodes $(\mathcal{S}, \mathcal{D})$ cannot communicate with each other, when node i is a regenerator node, if the value of $\beta_i = 1$ in the current solution. Starting with node $\mathcal{S}$, we can identify the set R of all nodes v, such that $\mathcal{S}$ can communicate with v. Clearly $\mathcal{D} \notin R$. Let $\mathbb{D}$ be the set of all nodes i in R, which currently have $\beta_i = 0$. In order that $\mathcal{S}$ may communicate with $\mathcal{D}$, one (or more) of the nodes in $\mathbb{D}$ must be regenerator node (s). Clearly $\mathbb{D}$ is a DSN and must have been included in constraint (6). This is a contradiction. □

Since the number of elements in Ω is exponential, this formulation has an exponential number of constraints of type (6). Unless the complete set of DSN is small enough, so that the corresponding basis size can be handled by CPLEX, specifying such a formulation to CPLEX for any non-trivial network is, in general, not feasible.

We note that, the route $\mathcal{P}$ in G, from a source $\mathcal{S}$ to a destination $\mathcal{D}$, through the physical topology G, has a corresponding route $\mathcal{P}_R$ in the reachability graph G_R, such that all intermediate nodes in $\mathcal{P}_R$ are regenerator nodes. We now define a capacitated directed graph, which we will call the *extended reachability graph* G_R^X, so that, instead of considering the path $\mathcal{P}_R$ in G_R, we will consider a path $\mathcal{P}_R^X$ in G_R^X, where each edge has an appropriate capacity. Given a reachability graph G_R and the values of $\beta_i, i \in N$, $\beta_i \in \{0, 1\}$, we define the extended reachability graph $G_R^X = (V_R^X, E_R^X)$ as follows:

- for each node $u \in N$, there will be two nodes u^1, u^2 in V_R^X,
- for each edge $u, v \in E_R$, there will be two directed arcs $u^2 \rightarrow v^1$, $v^2 \rightarrow u^1$ in E_R^X, each having an infinite capacity,
- for each node $u \in N$, there will be a directed arc $u^1 \dashrightarrow u^2$ in E_R^X, with a capacity of β_u.

We will use $\rightarrow$ and $\dashrightarrow$ to distinguish between these two types of directed arcs.

As an example, Fig. 3b shows the extended reachability graph corresponding to the reachability graph shown in Fig. 3a.

If commodity k has a route $\mathcal{S}^k \rightarrow a_1 \rightarrow a_2 \rightarrow \cdots \rightarrow a_p \rightarrow \mathcal{D}^k$ in the reachability graph G_R, then commodity k has a route $\mathcal{S}^{k2} \rightarrow a_1^1 \dashrightarrow a_1^2 \rightarrow a_2^1 \dashrightarrow a_2^2 \rightarrow \cdots \rightarrow a_p^1 \dashrightarrow a_p^2 \rightarrow \mathcal{D}^{k1}$ in the extended reachability graph G_R^X. Similarly, given a path through the extended reachability graph, it is easy to find the corresponding path through the reachability graph.

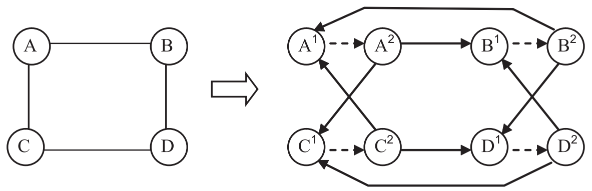

Fig. 3. Extended reachability graph created from reachability graph.

The flow of commodity k, on the path $\mathcal{S}^{k2} \rightarrow a_1^1 \dashrightarrow a_1^2 \rightarrow a_2^1 \dashrightarrow a_2^2 \rightarrow \cdots \rightarrow a_p^1 \dashrightarrow a_p^2 \rightarrow \mathcal{D}^{k1}$, is limited by *minimum* $(\beta_{a_1}, \beta_{a_2}, \ldots, \beta_{a_p})$. Since all β values are binary, the value of *minimum*$(\beta_{a_1}, \beta_{a_2}, \ldots, \beta_{a_p})$ is either 0 or 1. If this value is 0, it means that there cannot be any flow from $\mathcal{S}^{k2}$ to $\mathcal{D}^{k1}$ in G_R^X using the route $\mathcal{S}^{k2} \rightarrow a_1^1 \dashrightarrow a_1^2 \rightarrow a_2^1 \dashrightarrow a_2^2 \rightarrow \cdots \rightarrow a_p^1 \dashrightarrow a_p^2 \rightarrow \mathcal{D}^{k1}$. In other words, in G_R, at least one interior node in the route $\mathcal{S}^k \rightarrow a_1 \rightarrow a_2 \rightarrow \cdots \rightarrow a_p \rightarrow \mathcal{D}^k$ from $\mathcal{S}^k$ to $\mathcal{D}^k$ is not a regenerator node.

Lemma 2. *In a valid solution for the RPP, any cut* [69] *in* G_R^X *for any commodity must have a capacity of* 1 *or more.*

Proof. When RPP is solved, let a cut in G_R^X for commodity k have a capacity of less than 1. Since all β values are binary when RPP is solved, the capacity of this cut is 0, and hence the maximum flow for this commodity is 0. This means that there is no route from the source $\mathcal{S}^k$ to the destination $\mathcal{D}^k$, such that all interior nodes are regenerator nodes. Thus $\mathcal{S}^k$ cannot communicate with $\mathcal{D}^k$, contradicting the statement that RPP is solved. □

To lay the foundations for the branch-and-cut scheme outlined in Section 3.2.2 below, we now consider a situation where we solve a BILP, F^{new}, with the same objective function as the BILP above where we *have not* included all the DSN's in Ω, when constructing the constraints. It is quite possible that the solution to F^{new} will not correspond to a solution to the RPP, since some constraints are missing. We consider a situation where for some commodity k, node $\mathcal{S}^k$ cannot communicate with $\mathcal{D}^k$. This means that there is no path from $\mathcal{S}^k$ to $\mathcal{D}^k$, where all interior nodes are regenerator nodes.

Lemma 3. *Node* $\mathcal{S}^k$ *cannot communicate with* $\mathcal{D}^k$, *for some commodity* k, *iff the min-cut in* G_R^X *for commodity* k *has a capacity less than* 1.

Proof. The "if" part of the lemma is simply a restatement of Lemma 2 and the proof follows directly. Suppose $\mathcal{S}^k$ cannot communicate with $\mathcal{D}^k$, then there is no path from $\mathcal{S}^k$ to $\mathcal{D}^k$ with all interior nodes as regenerator nodes. Hence max-flow from $\mathcal{S}^k$ to $\mathcal{D}^k = 0$. Result follows since max-flow min-cut theorem [69]. □

Lemma 4. *The set of nodes* $\{i : (\mathcal{S}^k, i) \in E_R\}$ *is a DSN.*

Proof. The set of nodes $\{i : (\mathcal{S}^k, i) \in E_R\}$ gives the set of nodes that can be reached from $\mathcal{S}^k$ without exceeding the optical reach. Since commodity k, having source $\mathcal{S}^k$ and destination $\mathcal{D}^k$, is being considered in the RPP phase, it is not possible to go from $\mathcal{S}^k$ to $\mathcal{D}^k$ using a transparent lightpath. In other words, $\mathcal{D}^k \notin \{i : (\mathcal{S}^k, i) \in E_R\}$ and in order to reach $\mathcal{D}^k$ from $\mathcal{S}^k$, at least one of the nodes in $\{i : (\mathcal{S}^k, i) \in E_R\}$ must be designated as a regenerator node. Hence $\{i : (\mathcal{S}^k, i) \in E_R\}$ must be a DSN. □

Lemma 5. *If the max-flow for commodity k in graph* G_R^X *is less than* 1, *the min-cut in* G_R^X *for commodity k identifies a DSN.*

Proof. The min-cut in G_R^X for commodity k can only include directed edges of the type $u^1 \dashrightarrow u^2$ in G_R^X, corresponding to the nodes in G_R, since edges of the type $u^2 \to v^1$ or $v^2 \to u^1$ have a capacity of ∞. Let $\mathbb{U}$ denote the set of edges in the min-cut. If the max-flow for commodity k in graph G_R^X is 0, the capacity of every edge $u^1 \dashrightarrow u^2$ in the min-cut $\mathbb{U}$ must have a capacity of 0. In other words, the value of $\beta_u = 0$ for all such edges in $\mathbb{U}$. To allow communication from $\mathcal{S}^k$ to $\mathcal{D}^k$, we must satisfy Lemma 2 and the max-flow must become 1 or more. To achieve that, at least of one of the edges in $\mathbb{U}$ must correspond to a β value of 1, so that the set of nodes corresponding to the edges in $\mathbb{U}$ must be a DSN. □

We note that if all commodities have a min-cut of 1 or more, it means that, even though all DSN's are not included in forming the constraints, we have a valid solution for the RPP problem. This is a crucial observation that we have used below.

3.2.2. A branch-and-cut scheme to solve the RPP

Algorithm 1 gives an overview of our branch-and-cut process. The idea is that we start the process by specifying an ILP with the same objective function we used in Section 3.2.1, but with constraints corresponding to a small number of DSN's that may be generated quickly. CPLEX solves the LP corresponding to the ILP using its standard procedure, by relaxing the binary variables $\beta_i, i \in N$, so that they have continuous values, as part of the standard branch-and-bound algorithm [28]. The callback feature of CPLEX allows us to interrupt the branch-and-bound process, when some events of interest take place. Using this feature, before the branching step in a standard branch-and-bound algorithm, we take control to add additional cuts. After including these additional constraints, CPLEX resumes and solves the LP once again. We carry out this process of adding constraints (called "adding user cuts" using callback functions from CPLEX callable library [27]). This process is repeated as long as we discover new DSN's.

Symbols used in Algorithm 1:

P^0: Initial problem formulation.
z^*: The best objective value found so far.
$\vec{\beta}^*$: The values of the variables corresponding to the best objective value z^*.
List: The list of subproblems to be solved.
P^i: The problem handled in the current iteration of the loop (from Steps 3–31 of Algorithm 1).
choose_problem_from_list(*List*): This function selects, from the list of subproblems to be solved, the next subproblem to be solved. This is done using any one of a number of strategies as outlined in [28].
notDone: A boolean flag to denote whether or not the iterations in the while-loop in lines 7–26 of Algorithm 1 will be continued.
fathom_current_problem: This is a boolean flag to denote whether or not the subproblem P^i should be discarded (i.e., fathomed [28]).
z^i: The optimized value of problem P^i, after LP relaxation.
$\vec{\beta}^i$: The values of the variables corresponding to the optimized solution of problem P^i, after LP relaxation.
solve_LP_relaxation(P^i): This function solves subproblem P^i using the revised simplex method, after relaxing the integer requirements and returns the pair $(z^i, \vec{\beta}^i)$.
valid_RPP($\vec{\beta}^i$): This functions returns true, if the values in $\vec{\beta}^i$ corresponds to a valid RPP solution (i.e., every source can communicate with every destination).
branch(P^i): The function corresponding to the branching function of a branch-and-bound algorithm that partitions the solution space of P^i into two subproblems
P_1^i (P_2^i): The subproblems of P^i, generated by *branch*(P^i).

Algorithm 1. Branch-and-cut algorithm for the RPP.

Input: Initial problem formulation P^0
Output: Incumbent optimal solution $\vec{\beta}^*$
1: $List \leftarrow \{P^0\}$
2: $z^* \leftarrow \infty$
3: **while** ($List \neq \phi$) **do**
4: $P^i \leftarrow$ *choose_problem_from_list*(*List*)
5: *notDone* ← *true*
6: *fathom_current_problem* ← *false*
7: **while** (*notDone*) **do**
8: $(z^i, \vec{\beta}^i) \leftarrow$ *solve_LP_relaxation*(P^i)
9: **if** (P^i *is not feasible*) || ($z^i \geq z^*$) **then**
10: *fathom_current_problem* ← *true*
11: *notDone* ← *false*
12: **else if** (*all values in* $\vec{\beta}^i$ *are integers*) **then**
13: **if** (*valid_RPP*($\vec{\beta}^i$)) **then**
14: $(z^*, \vec{\beta}^*) \leftarrow (z^i, \vec{\beta}^i)$
15: **else if** (*violated cuts available with* $\vec{\beta}^i$) **then**
16: *add user cuts to* P^i
17: **else**
18: *fathom_current_problem* ← *true*
19: *notDone* ← *false*
20: **end if**
21: **else if** (*violated cuts available with* $\vec{\beta}^i$)**then**
22: *add user cuts to* P^i
23: **else**
24: *notDone* ← *false*
25: **end if**
26: **end while**
27: **if** (*fathom_current_problem* = *false*) **then**
28: $(P_1^i, P_2^i) \leftarrow$ *branch*(P^i)
29: $List \leftarrow List \cup \{P_1^i\} \cup \{P_2^i\}$
30: **end if**
31: **end while**
32: **return** $\vec{\beta}^*$

Our program first finds P^0, an initial problem formulation and gives it to CPLEX in Step 1, to start the branch-and-cut process. Formulation P^0 has the same objective as that in Section 3.2.1 but contains one DSN for each commodity $k \in K$ of interest, computed using Lemma 4. The algorithm keeps track of the best objective value z^* found so far and $\overrightarrow{\beta}^*$, the corresponding values of the variables. Algorithm 1 runs until it either finds an optimal RPP solution, or the list of all candidate subproblems is exhausted.

We carry out the branch-and-cut algorithm using the *while* loop (Step 3 to Step 31). In these steps, CPLEX solves all the candidate subproblems until the solution tree is empty. Following its standard procedure [27], CPLEX chooses a subproblem P^i to solve, from the list of candidate subproblems (Step 4). Steps 7–26 denote another *while* loop. Inside this loop we determine if subproblem P^i (a) should be discarded (i.e., fathomed), (b) gives a valid solution for RPP, or (c) needs to be augmented by new constraints. Details of these steps are as follows.

In Step 8 the function *solve_LP_relaxation*(P^i) finds an optimal LP value z^i, and the corresponding $\overrightarrow{\beta}^i$. In Steps 9–25, we have to consider the following cases:

1. If the LP is infeasible or its objective value is greater than or equal to the best objective value z^* found so far, *fathom_current_problem* is set to *true* by CPLEX, in order to discard the formulation P^i at the end of this iteration.
2. If all values in $\overrightarrow{\beta}^i$ are integers, we check, using Lemma 3, if the solution is a valid RPP solution (Step 13). If it is a valid RPP solution, since $z^i < z^*$, we have found a better solution than the best we found so far and we replace the best objective value (the incumbent solution) $z^*(\overrightarrow{\beta}^*)$ found so far by the current objective value (current solution) $z^i(\overrightarrow{\beta}^i)$ (Step 14).
3. If the integer solution is not a valid RPP solution, this means that, at least for one $k \in K$, S^k cannot communicate with D^k. For this case, the max-flow from S^k to D^k is less than 1. Lemma 5 indicates that the corresponding min-cut gives us a DSN that is not satisfied by the current solution. For all commodities $k \in K$, such that S^k cannot communicate with D^k, we add the DSN generated using Lemma 5 in Step 16. If no cut is found, CPLEX discards the formulation by setting *fathom_current_problem* equal to *true* and exits the *while* loop.
4. If, on the other hand, the solution to P^i is a fractional solution (i.e., at least one value in $\overrightarrow{\beta}^i$ is not an integer), using the same CPLEX callback feature for adding user cuts, our program checks if any violated cut is available for any commodity with the current values in $\overrightarrow{\beta}^i$ (Step 21).
   A simple way to identify the violated cuts is to apply Lemma 5 and find a min-cut for each commodity. If the capacity of this cut is less than 1, the DSN corresponding to the minimum cut is not satisfied by $\overrightarrow{\beta}^i$. Any polynomial time algorithm for *max-flow* [69] can find these violated cuts quite efficiently.
   If cuts are found, we add them to the formulation (Step 22) and CPLEX goes back to the beginning of the *while* loop (in Steps 7–26) and solves the LP again. If no cut is found, CPLEX exits this *while* loop.

Once out of this inner *while* loop, CPLEX checks the value of *fathom_current_problem* (Step 27). If it is true, it abandons further processing of problem P^i (in other words, problem P^i is fathomed [68]). Otherwise, CPLEX splits the current problem P^i into two subproblems P^i_1 and P^i_2 (Step 28) and adds both the subproblems to *List* (Step 29). We repeat Steps 3–31 as long as there are candidate subproblem(s) in *List*. Once *List* is empty, CPLEX returns with the incumbent solution $\overrightarrow{\beta}^*$, which is the optimal RPP solution.

It is important to note that formulations FRM-2 above and *M2* in [60] approach the RPP in very different ways, even though both involve an exponential number of constraints, and use the branch-and-cut method. It is not appropriate to compare our formulation FRM-2 and formulation *M2* in [60] by comparing the number of integer variables or the basis sizes, since [60] is finding two edge-disjoint paths, while our formulation FRM-2 finds only one path for each commodity.

Example. Let us consider the reachability graph for the six node network shown in Fig. 2. Following our discussions in Section 1, the commodities that need to be considered are $K^1=(1,4)$, $K^2=(1,6)$, $K^3=(2,5)$, $K^4=(2,6)$, $K^5=(3,4)$ and $K^6=(3,6)$).

To start the process, we find a small set of constraints, one for each commodity using Lemma 4. From the graph we can easily identify those constraints as

- $\beta_2+\beta_3+\beta_5 \geq 1$ (DSN for K^1),
- $\beta_2+\beta_3+\beta_5 \geq 1$ (DSN for K^2),
- $\beta_1+\beta_3+\beta_4 \geq 1$ (DSN for K^3),
- $\beta_1+\beta_3+\beta_4 \geq 1$ (DSN for K^4),
- $\beta_1+\beta_2+\beta_5 \geq 1$ (DSN for K^5),
- $\beta_1+\beta_2+\beta_5 \geq 1$ (DSN for K^6).

The redundant constrains from the above problem can be removed without any loss of generality:

- $\beta_2+\beta_3+\beta_5 \geq 1$ (DSN for K^1 and K^2),
- $\beta_1+\beta_3+\beta_4 \geq 1$ (DSN for K^3 and K^4),
- $\beta_1+\beta_2+\beta_5 \geq 1$ (DSN for K^5 and K^6).

One integer solution of this problem is $\beta_2=1$ and $\beta_3=1$. But this solution is not a valid RPP solution, as the max-flow for commodities K^2, K^4 and K^6 are all 0's. To get a valid RPP solution we must find some violated cuts and add them to the current problem and solve the problem once again. Using max-flow algorithm we find the following additional cuts:

- $\beta_4+\beta_5 \geq 1$ (DSN for K^2),
- $\beta_1+\beta_4+\beta_5 \geq 1$ (DSN for K^4 and K^6).

Adding these additional cuts the problem becomes

- $\beta_2+\beta_3+\beta_5 \geq 1$,
- $\beta_1+\beta_3+\beta_4 \geq 1$,
- $\beta_1+\beta_2+\beta_5 \geq 1$,
- $\beta_4+\beta_5 \geq 1$,
- $\beta_1+\beta_4+\beta_5 \geq 1$.

If we ignore the redundant (or loose) constraints, the new problem becomes

- $\beta_2+\beta_3+\beta_5 \geq 1$,
- $\beta_1+\beta_3+\beta_4 \geq 1$,
- $\beta_1+\beta_2+\beta_5 \geq 1$,
- $\beta_4+\beta_5 \geq 1$.

One optimal integer solution for this problem is $\beta_2 = 1$ and $\beta_4 = 1$, which is a valid RPP solution.

3.3. RPP for mixed-line-rate networks

The scheme for RPP described above was designed for single-line-rate networks. This can be readily extended to mixed-line-rate networks. For mixed-line-rate networks, different line rates have different optical reaches, based on factors like launch power, modulation format, dispersion map [65]. Regenerators for a particular line rate can only handle that line rate, so that different regenerators have to be installed to support different line rates [65]. We consider the following two scenarios for mixed-line-rate networks.

*Scenario*1: Each regenerator node in the network is translucent (or hybrid in the sense used in [1]), so that only selected incoming signals are regenerated at that site.

*Scenario*2: Each regenerator node in the network is opaque, so that *all* incoming signals are converted from optical to electrical domain and, if needed, reconverted to optical domain. In effect, the signals which are not dropped at this site are regenerated.

In scenario 1, if a node is a site for regeneration for a particular line rate, then that node is not involved with the regeneration of signals at any other line rate. In other words, there is no compulsion to make that a site for regeneration at other line rates. In this case, the identification of regenerator sites for a given line rate is independent of the process of identifying the regenerator sites for any other line rate. This means that the scheme outlined in Section 3.2 may be repeated for each line rate, to identify the sites for regenerating signals for that line rate.

In scenario 2, a decision to make a node a site for regenerating signals at some line rate means that the node will be an opaque node, so that all the incoming signals, regardless of the line rate will be converted to the electrical domain. To consider this case, let the network support line rates $l_1, l_2, \ldots, l_p$ have optical reaches $r_1, r_2, \ldots, r_p$ respectively. For each line speed l_i (and optical reach r_i), let $G_{R,i} = (N, E_{R,i})$ be the reachability graph. The ILP formulation is a generalization of the formulation given in Section 3.2 as follows:

$$\text{minimize} \quad \sum_{j \in N} \beta_j \tag{8}$$

subject to

$$\sum_{j \in \mathbb{D}} \beta_j \geq 1: \quad \forall \mathbb{D} \in \bigcup_i \Omega_i \tag{9}$$

$$\beta_j = \{0,1\}: \quad j \in N \tag{10}$$

Here Ω_i is the set of all DSN's for reachability graph $G_{R,i}$.

The procedure outlined in Algorithm 1 has to be modified, to take into account the fact that we have multiple reachability graphs $G_{R,i}$, for all $i, 1 \leq i \leq p$. The modifications are as follows:

- When checking whether the solution is a valid RPP (Step 13 of Algorithm 1), we have to apply Lemma 3, using *each* of the reachability graphs $G_{R,i}, 1 \leq i \leq p$, to see if S^k can communicate with D^k.
- For each graph $G_{R,i}, 1 \leq i \leq p$, if S^k cannot communicate with D^k, we have to apply Lemma 5 to generate a new DSN to be added to the constraints (Step 16 of Algorithm 1).
- If the solution to P^i is a fractional solution (i.e., at least one value in $\vec{\beta}^i$ is not an integer), we check if any violated cut is available for any commodity with the current values in $\vec{\beta}^i$ (Step 21). To do this, we apply Lemma 5 to find a min-cut for each commodity and for each reachability graph $G_{R,i}, 1 \leq i \leq p$. If the capacity of any min-cut is less than 1, the DSN corresponding to the cut is not satisfied by $\vec{\beta}^i$ and the DSN has to be added to the constraints.

4. Experimental results

For our experiments, we have considered different randomly generated topologies. For comparing the performance of our formulation FRM-2 using the branch-and-cut approach (Section 3.2), with that of our compact formulation FRM-1 (Section 3.1), we have generated a number of 15, 20, 25, 30 and 35 node networks, and have run both formulations.[2] For evaluating the performance of the formulation FRM-2 for medium and large sized networks, we have generated networks with 40, 60, 80, 100, 120 and 140 nodes. For a given size of the network, we have generated 3 categories of networks, which we have called the *low, medium* and *high* "density" networks. We have measured the density of a network by the average number of edges for each node. For networks containing 60 nodes or less, for low density (medium density, high density) networks, we have randomly chosen the degree of each node to lie in the range from 2 to 3 (3 to 5, and 4 to 7 respectively). For networks containing 80–100 nodes, we have randomly chosen the degree of each node to lie in the range 4–5, 5–7, and 6–9 for a low, a medium and a high density network respectively. For networks containing 120 nodes and more, we have randomly chosen the degree of each node to lie in the range 6–7, 7–9, and 8–11 for a low, a medium and a high density network respectively.

If there is an edge between nodes x and y, we have randomly selected the length of the fiber connecting x and y to lie in the range 800–2800 km. We have selected the optical reach r to be 3000 km.

For our first set of experiments to compare the performances of FRM-1 and FRM-2, for each category of networks with sizes from 15 nodes to 35 nodes, we have randomly

[2] For this comparison, we were limited to networks with 35 or fewer nodes, since the formulation FRM-1 takes an unacceptable amount of time to solve if the network has more than 35 nodes.

generated 10 sets of physical topologies and the results reported in this paper represent the average values of those 10 sets of physical topologies. For our second set of experiments to evaluate the performance of FRM-2, for each category of networks with sizes from 40 nodes to 140 nodes, we have randomly generated 5 sets of physical topologies and the results reported in this paper represent the average values of those 5 sets of physical topologies. We have carried out the experiments on a Sun Fire X2200 M2 Server machine [70].

We have presented two tables (Tables 1 and 2), as well as two graphs (Figs. 4 and 5) to demonstrate our experimental results. In columns 3 and 4 of both the tables, we have shown, respectively, the average number of edges ($|E|$) in the physical topology and the average number of commodities ($|K|$), for different network densities.

Table 1 illustrates, along with other data, a comparison of the average execution time (given in seconds) needed to solve the RPP problem using the formulations FRM-1 and FRM-2 (column 7 and column 8, respectively), for different network sizes and for different network densities. The results show that the formulation FRM-1 needs considerably more time than formulation FRM-2 to solve a given problem – ranging, on an average, from approximately 9 times more for 15-node, low-density network to 2150 times more for 35-node, high-density network. The dramatic improvement in the execution time of FRM-2, compared to FRM-1, particularly for larger networks, is remarkable.

We note that it always takes less time to solve problems when the average node degree is higher. The reason is that, when the node degree is higher, each node has more edges, so that, in general, for a node, there are more nodes within the optical reach of that node. Since each commodity corresponds to a source destination pair which are not within the optical reach, there are fewer commodities that the formulation has to consider. In other words, the problem becomes smaller. We compute the number of commodities in a given network, for a specified network density, as follows. Each node-pair (S, D) that is not connected by an edge in the reachability graph corresponding to the network represents the fact that the shortest distance between S and D exceeds the optical reach. Such a pair (S, D) requires at least one regenerator in the path chosen for communication from S to D (or from D to S) and constitutes a commodity for our formulation. The average number of commodities for networks of a given size, in general, varies with the average node degree. Since more node pairs are within the optical reach, when the node degree is higher, we have fewer commodities.

Table 2 illustrates, along with other data, a comparison of the average execution time (given in seconds) needed to solve the RPP problem using the formulation FRM-2 (column 7) for different network sizes and for different network densities.

The average number of regenerator nodes, determined using the heuristic for finding the minimum connected dominating set in [71] and shown in column 5 of Tables 1 and 2, is more than the optimum value shown in column 6 of Tables 1 and 2, in all the cases. This shows the need to use an optimal algorithm, rather than a heuristic, to minimize the cost of regenerator nodes.

Fig. 4 shows the comparison of average execution time using FRM-1 and FRM-2 for different sized networks with medium density.

Fig. 5 shows the average execution time needed using FRM-2 for large networks with different network densities.

5. Conclusions

In this paper we have proposed two new Integer Linear Program (ILP) formulations (FRM-1 and FRM-2) to solve the RPP problem optimally for networks handling ad-hoc demands for lightpaths. To the best of our knowledge this is the first time a formulation for an optimal solution of the RPP problem, which may easily handle large networks,

Table 1
Comparing average execution time using FRM-1 and FRM-2.

Network size	Network density	Average		Average # of regenerator nodes		Average solution time (s)	
		Node degree	Number of commodities	Heuristic	Optimal	FRM-1	FRM-2
15	Low	2.5	79.0	6.4	6.1	0.18	0.02
	Medium	3.9	57.8	3.1	2.8	0.14	0.01
	High	5.1	35.2	2.0	1.9	0.08	0.01
20	Low	2.8	155.5	9.1	8.8	1.33	0.11
	Medium	3.9	125.7	4.5	4.0	1.10	0.03
	High	4.8	94.5	2.7	2.2	0.67	0.02
25	Low	2.5	254.1	11.5	11.0	32.22	0.26
	Medium	3.8	219.1	6.5	5.3	11.77	0.07
	High	5.4	171.7	3.5	3.0	6.78	0.02
30	Low	2.5	379.5	14.0	12.8	158.29	0.47
	Medium	4.0	325.7	6.6	5.7	139.10	0.16
	High	5.4	272.2	4.5	3.8	39.68	0.05
35	Low	2.5	530.5	16.1	15.1	515.62	1.66
	Medium	3.9	473.6	7.7	6.6	427.19	0.27
	High	5.3	405.6	5.8	4.6	430.35	0.20

Table 2
Illustrating average execution time for large networks using FRM-2.

Network size	Network density	Average		Average # of regenerator nodes		Average solution time (s)
		Node degree	Number of commodities	Heuristic	Optimal	FRM-2
40	Low	2.5	709.4	18.8	17.4	3.32
	Medium	4.0	640.4	9.2	7.8	0.61
	High	5.5	565.6	6.8	5.2	0.13
60	Low	2.5	1655.0	29.0	27.4	70.27
	Medium	4.1	1558.0	13.8	11.0	9.95
	High	5.4	1398.8	8.4	6.6	1.28
80	Low	4.5	2825.0	16.6	13.4	232.58
	Medium	6.0	2595.6	11.0	8.4	35.73
	High	7.4	2407.4	8.0	6.4	4.51
100	Low	4.5	4505.2	21.4	15.8	582.72
	Medium	5.9	4256.0	13.4	11.0	77.73
	High	7.4	4003.8	11.4	7.8	11.50
120	Low	6.4	4505.2	15.0	11.8	740.17
	Medium	8.0	4256.0	11.0	8.2	96.58
	High	9.5	4003.8	9.0	6.4	15.94
140	Low	6.5	4505.2	18.4	11.8	1853.53
	Medium	8.0	4256.0	13.8	8.2	224.74
	High	9.5	4003.8	10.4	6.4	44.33

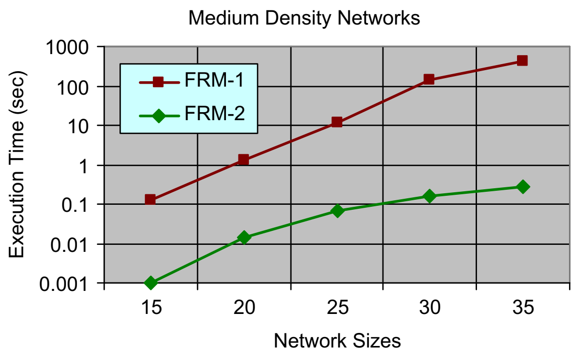

Fig. 4. Comparing average execution time for medium density networks.

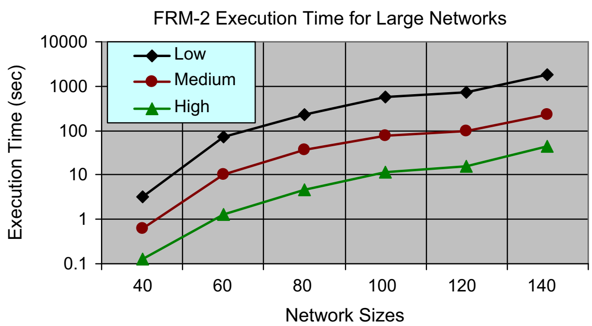

Fig. 5. Illustrating FRM-2 execution time for large networks.

has been presented. Formulation FRM-1 is compact and works only for relatively smaller networks but its advantage lies in the fact that it is based on standard network flow programming techniques that can be readily solved by any commercially available mathematical solver. The second formulation FRM-2 has an exponential number of constraints that are known only implicitly. It is impractical to generate all the constraints and solve it by a standard solver. To solve the RPP problem using FRM-2, we have proposed a branch-and-cut approach that solves the RPP problem very efficiently. This is particularly important in solving the RPP for large networks. Even though formulation FRM-2 has an exponential number of implicit constraints, experimental results reveal that, in general, we need a small number of such constraints to solve a given problem. In other words, in our experience, the basis size of the relaxed LP, when using FRM-2, remains relatively small, even after the additional constraints have been added. This ensures that the LP relaxations can be solved very quickly. We have also presented experimental results using both the formulations, clearly demonstrating the superiority of our formulation FRM-2 in terms of time complexity.

Existing optimal solutions for many optical network design problems involve Integer Linear Programs (ILP). Most of the existing research on optical network design focused on the use of conventional network flow programs. Such formulations may be readily solved, using a commercial optimization tool, such as the CPLEX. However, for most of the interesting problems on optical network design, the ILP formulations involve a large number of integer variables that grow rapidly with the size of the problem (e.g., the number of nodes in the network, the number of commodities handled) and become intractable for problems of practical size.

Our extensive literature survey on RPP problem for ALD reported in Section 2.3, reveals that only two researchers [20,60] have used branch-and-cut, where [20] has used branch-and-cut that gave optimal solutions for 536 cases out of 700 within a time period of 3600 s for networks of sizes up to 100 nodes, and [60] has studied RPP for survivable networks. In general there have been relatively few applications of modern Operations Research tools (e.g., branch-and-price, branch-and-cut, branch-price-and-cut) for solving optical network design problems. The major focus of this research is formulation FRM-2, where we used the branch-and-cut approach to solve an important problem in optical network design. Our approach can easily handle large networks (the average time to solve the RPP problem in networks having 140 nodes ranged from 44 s to 1900 s, depending on the type of topology considered). We feel that there is great scope in improving the state of the art in optimal design of optical networks by adapting modern operations research tools to solve such problems. Our FRM-2 is an example showing the potential for dramatic improvements when such approaches are used.

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