Extension and validation of the GN model for non-linear interference to uncompensated links using Raman amplification

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Abstract: We show the extension of the Gaussian Noise model, which describes non-linear propagation in uncompensated links of multilevel modulation formats, to systems using Raman amplification. We successfully validate the analytical results by comparison with numerical simulations of Nyquist-WDM PM-16QAM channels transmission over multi-span uncompensated links made of a single fiber type and using hybrid EDFA/Raman amplification with counter-propagating pumps. We analyze two typical high- and low-dispersion fiber types. We show that Raman amplification always induces a limited non-linear interference enhancement compared to the dominant ASE noise reduction.

OCIS codes: (060.1660) Coherent communications; (060.2330) Fiber optics communications; (060.2400) Fiber properties.

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1. Introduction

Optical communications are fast evolving towards the extensive use of multilevel modulation formats using coherent receivers based on digital signal processing (DSP) [1–3]. For these transmission techniques, it has been demonstrated that optimum setups for long-haul transmission are highly-dispersive uncompensated links [4,5]. Moreover, in these system scenarios, in order to further increase spectral efficiency with respect to *standard* wavelength-division mutiplexing (WDM), a promising option is the use of the so-called *Nyquist*-WDM (NyWDM) technique. NyWDM is a wavelength multiplexing technique with channel spacing approaching the symbol-rate and channel spectra shaped in order to minimize cross-talk [6].

In transmission of multilevel modulation formats over uncompensated links, nonlinearity generates a *Gaussian* additive disturbance, called nonlinear interference (NLI). NLI can be analytically evaluated using the Gaussian-Noise (GN) model proposed in [6] and detailed described in [7], and extensively validated by simulations [8] and experiments [9] for lumped amplified links.

With the increase of cardinality of modulation formats, for instance moving from PM-QPSK to PM-16QAM, optimization of amplifier performance is a fundamental issue in order to reach long-haul distances. To this purpose, the use of hybrid Raman/EDFA amplification (HFA) seems to be the most effective practical solution [10]. The effect of nonlinearities in links using HFA's can still be described by the GN-model. The purpose of this paper is the application of the GN-model to such scenarios and its validation through comparison with numerical simulations, extending preliminary results presented in [11].

In Sec. 2, we shortly recall the general theory of GN-model, then, in Sec. 3, we present specific expressions for Raman amplification based on counter-propagating undepleted pumps. In Sec. 4, we describe the system setups used to obtain the validation results, i.e., PM-16QAM NyWDM systems on links made up of two typical high- and low-dispersion fibers, considering several Raman pumping levels. Finally, in Sec. 5, we present simulation results showing excellent agreement with theory, demonstrating that the GN-model can be reliably applied also to links based on the use of Raman amplification.

2. Generalized theory for NLI

The general layout of the analyzed optical links is pictorially described in Fig. 1. It is a typical uncompensated multi-span link made up of N_s identical spans. Each span is composed of the transmission fiber with loss coefficient α_{dB} [dB/km], dispersion parameter D [ps/nm/km] and nonlinear coefficient γ [1/W/km]. Span length is L_s [km].

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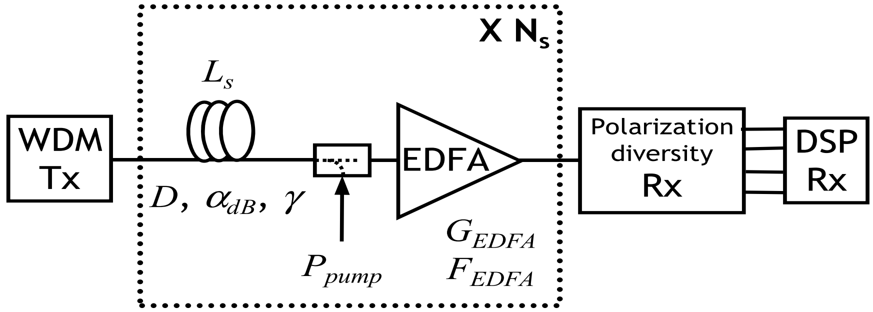

Fig. 1. Layout of the reference optical link used for the definition of the GN-model for NLI.

The fiber is possibly Raman pumped, with power P_{pump} enabling a net distributed Raman gain G_{RA} [dB] (the so-called *on-off* gain, null in case of no pumping), followed by an EDFA with gain G_{EDFA} [dB], recovering the residual loss. We assume *transparent* spans, i.e., G_{RA} + G_{EDFA} = $A_{s,dB}$, where $A_{s,dB}$ [dB] is the span loss, including possible insertion of passive components.

It has been extensively proven [12,13] that Kerr effect in propagation on uncompensated links of a WDM comb of channels based on multilevel modulation formats generates an additive *Gaussian* disturbance commonly defined as *nonlinear interference* (NLI). The characteristics of NLI at the receiver can be analytically derived using the GN-model [7]. Recalling such a theory, in general, the power spectral density (PSD) of the NLI assumes the following expression versus frequency f:

$$G_{NLI}(f)=\frac{16}{27}\gamma^2 L_{eff}^2\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} G_{WDM}(f_1)G_{WDM}(f_2)G_{WDM}(f_1+f_2-f)\rho(f_1,f_2,f)\chi(f_1,f_2,f)\,\mathrm{d}f_2\mathrm{d}f_1 \quad (1)$$

where $G_{WDM}(f)$ [W/Hz] is the power spectral density of the transmitted channel comb, L_{eff} [km] is the generalized effective length of the fiber spans, defined as:

$$L_{eff}=\int_0^{L_s} p_{ch}(z)\,\mathrm{d}z\,, \quad (2)$$

$\rho(f)$ is the generalized four-wave mixing efficiency [14] whose expression is:

$$\rho(f)=\frac{1}{L_{eff}^2}\left|\int_0^{L_s} p_{ch}(z)\exp\left\{j4\pi^2\beta_2 f^2 z\right\}\mathrm{d}z\right|^2, \quad (3)$$

where β_2 is the fiber dispersion coefficient expressed in ps^2/km, L_s [km] is the span length and $p_{ch}(z)$ is the normalized power evolution with the propagation distance z [km] defined as

$$p_{ch}(z)=\exp\left\{-2\left[\int_0^z \alpha+g(\xi)\mathrm{d}\xi\right]\right\}, \quad (4)$$

where α [1/km] is the linear fiber *field* loss coefficient and $g(z)$ [1/km] is the possible distributed amplification local *field* gain function. Equation (1) also includes the function $\chi(f_1,f_2,f)$ that models *coherent* accumulation with z of NLI and has the following closed-form [15], similar to that of the radiation pattern of a phased-array antenna:

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$$\chi(f_1,f_2,f)=\frac{\sin^2\left[2N_s\pi^2(f_1-f)(f_2-f)\beta_2L_s\right]}{\sin^2\left[2\pi^2(f_1-f)(f_2-f)\beta_2L_s\right]}. \tag{5}$$

Focusing the analysis on the transmission of N_{ch} NyWDM channels at symbol rate R_s with channel spacing $\Delta f = R_s$, we can apply a change of variables based on hyperbolic coordinates, i.e., $v^2 = f_1 \cdot f_2$, to Eq. (1), as shown in [7]. Moreover, considering the effect on the center channel, i.e., the worst-case within the transmitted comb, $G_{NLI}(f)$ variation with f is negligible, and we can approximate with excellent accuracy the Gaussian-distributed NLI disturbance as a white random process, i.e., NLI acts as an additive white Gaussian noise (AWGN). The resulting practical approximation for the NLI power spectral density at the receiver is:

$$G_{NLI}\cong\frac{256}{27}\frac{\gamma^2L_{eff}^2P_{ch}^3}{R_s^3}\int_0^{\frac{B_{opt}}{2}}\rho(\nu)\nu\frac{\sin^2\left(2N_s\pi^2\nu^2\beta_2L_s\right)}{\sin^2\left(2\pi^2\nu^2\beta_2L_s\right)}\log\left(\frac{B_{opt}}{2\nu}\right)\mathrm{d}\nu\,, \tag{6}$$

where P_{ch} [W] is the transmitted power per channel, N_s is the number of spans, R_s [sym/s] is the symbol rate and $B_{opt} = N_{ch} \cdot R_s$ [Hz] is the overall bandwidth occupied by the channel comb. Equation (6) is a general expression that holds for any NyWDM transmission independently of multilevel modulation format and can be numerically solved, provided that the fiber parameters and the evolution $g(z)$ of the possible distributed amplification are known. Note that in derivation of Eq. (6) Rayleigh backscattering has not been included being the NLI a disturbance whose intensity is always much smaller than the ASE noise intensity, thus experiencing a negligible Rayleigh backscattering enhancement [16].

It is well known that, for multilevel modulation formats using coherent receivers, the *back-to-back* bit error-rate (*BER*) has a one-to-one correspondence with the optical signal to noise ratio (*OSNR*), determined by the ASE noise, i.e, $OSNR = P_{ch}/P_{ASE}$, where P_{ASE} is the ASE noise in the reference bandwidth B_n. The monotone decreasing *sensitivity* function $BER = \Phi(OSNR)$ depends on the modulation format and on the transmitter (Tx) and receiver (Rx) structures. Since NLI can be effectively modeled as an AWGN component, *BER* at the Rx can still be calculated through the *sensitivity* function using the *nonlinear* optical signal to noise ratio ($OSNR_{NL}$) in place of *standard OSNR*, i.e., $BER = \Phi(OSNR_{NL})$. $OSNR_{NL}$ includes the effects of both ASE noise and NLI, and its expression is:

$$OSNR_{NL}=\frac{P_{ch}}{\left(G_{ASE}+G_{NLI}\right)B_n}=\frac{P_{ch}}{P_{ASE}+P_{NLI}}, \tag{7}$$

where B_n is the reference bandwidth and G_{ASE} is the overall ASE noise power spectral density:

$$G_{ASE}=N_sF_{eq}hf_0\left(A_s-1\right)\cong N_sF_{eq}hf_0A_s\,, \tag{8}$$

where f_0 is the center frequency of the channel comb, h is the Plank's constant, A_s is the span loss and F_{eq} is the equivalent noise figure of the HFA that may include Rayleigh backscattering penalty [16]. We define losses as parameters ≥ 1, i.e., given the loss A, we assume $P_{out} = P_{in}/A$.

Following the analysis described so far, in general, G_{NLI} can be numerically calculated solving Eq. (6) and *BER* for NyWDM channels transmitted over uncompensated links can be analytically estimated.

Given the target *BER*, a target $OSNR_{NL}$ can be easily derived inverting the sensitivity function Φ, and so, applying Eqs. (6), (7) and (8), the maximum reachable number of spans N_s^{max} and the corresponding optimal transmission power can be derived, as shown in Eqs. (29) and (30) of [17]. Note that N_s^{max} is always obtained balancing the overall ASE noise power (P_{ASE}) and NLI power (P_{NLI}) in B_n according to the fundamental law for optimal performances in nonlinear propagation of multilevel formats in uncompensated links, that states [18]:

$$N_s^{\max} \quad \Leftrightarrow \quad P_{ASE} = 2 \cdot P_{NLI} \;. \tag{9}$$

Applying the GN-model, N_s^{max} can be calculated and its increase using HFA's with respect to EDFA-only amplification can be expressed as:

$$10\,\log_{10}\left(\frac{N_s^{\max,HFA}}{N_s^{\max,EDFA}}\right) = \frac{2}{3}\Delta P_{ASE,dB} - \frac{1}{3}\Delta P_{NLI,dB}\;, \tag{10}$$

where $N_s^{\ maxHFA}$ and $N_s^{\ maxHFA}$ are N_s^{max} with and without Raman amplification, respectively, $\Delta P_{ASE,dB}$ is the ASE noise reduction due to F_{eq} improvement, and $\Delta P_{NLI,dB}$ is the NLI enhancement caused by distributed Raman gain. Equation (10) is a compact expression that tradeoffs positive and negative effects of Raman amplification. In particular, positive effect of noise reduction is always 2-times dominant with respect to nonlinear enhancement, in logarithmic units.

3. NLI generation in case of Raman amplification with counter-propagating pump

In this paper, we focus our analysis on the use of Raman amplification with undepleted counter-propagating pump. The assumption of undepleted pump is typically well verified when using HFA's, whereby the Raman *on-off* gain is substantially less than the span loss. If so, the distributed amplification can be analytically described by the following closed-form:

$$g(z) = \frac{1}{2} C_R P_{pump} e^{-2\alpha_p (L_s - z)}, \tag{11}$$

where C_R [1/W/km] is the fiber Raman efficiency, P_{pump} [W] is the launched un-polarized pump power and α_p [1/km] is the fiber loss at the pump wavelength. In order to clarify the meaning of Eq. (11), it is useful to remark that, typically, Raman gain is given as *on-off* gain expressed in convenient decibel units, defined as:

$$G_{RA} = 10\log_{10}\left\{\exp\left[2\int_0^{L_s} g(\xi)d\xi\right]\right\} = 10\log_{10}(\mathrm{e})\, C_R P_{pump} \frac{1-e^{-2\alpha_p L_s}}{2\alpha_p} \quad [\mathrm{dB}]. \tag{12}$$

In this case, applying Eq. (2), the generalized effective length can be analytically expressed by the following closed-form:

$$L_{eff} = \frac{\exp\left(-\frac{g(0)}{\alpha_p}\right)}{2\alpha_p}\left[-\frac{g(0)}{\alpha_p}\right]^{k_\alpha}\left\{\Gamma\left[-k_\alpha, -\frac{g(0)}{\alpha_p}\right] - \Gamma\left[-k_\alpha, -\frac{g(L_s)}{\alpha_p}\right]\right\} \tag{13}$$

where $\Gamma(s,x)$ is the upper-incomplete gamma-function [18] and $k_\alpha = \alpha_s/\alpha_p$. Moreover, the generalized FWM efficiency assumes a closed-form expression defined as:

$$\rho(\nu) = \frac{1}{L_{eff}^2}\left|\frac{\exp\left(-\frac{g(0)}{\alpha_p}\right)}{2\alpha_p}\left[-\frac{g(0)}{\alpha_p}\right]^{(k_\alpha - jB\nu^2)}\left\{\Gamma\left[-\left(k_\alpha - jB\nu^2\right), -\frac{g(0)}{\alpha_p}\right] - \Gamma\left[-\left(k_\alpha - jB\nu^2\right), -\frac{g(L_s)}{\alpha_p}\right]\right\}\right|^2 \tag{14}$$

where $B = 2\pi^2\beta_2/\alpha_p$. Equations (13) and (14) depend only on the overall Raman gain, therefore they also hold for multi-pump counter-propagating Raman setups using an *equivalent* $\alpha_{p,}$ provided that pump depletion is negligible. Equations (13) and (14) were derived in [18].

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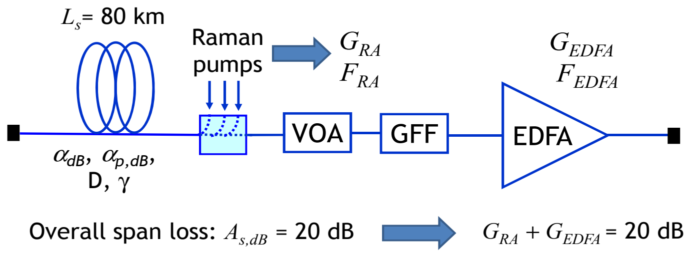

Fig. 2. Structure of the link span used in the validation analysis for both PSCF and NZDSF setups.

Table 1. Parameters of transmission fibers used for the validation analysis

	α_{dB} [dB/km]	$\alpha_{p,dB}$ [dB/km]	γ [1/W/km]	β_2 [ps^2/km]	D [ps/nm/km]
PSCF	0.185	0.28	0.8	–26.2	20.6
NZDSF	0.220	0.32	1.5	–4.8	3.8

4. Practical scenarios: description and GN-model results

In order to apply the GN-model and to perform simulative validations, we considered optical links composed of N_s identical spans of transmission fiber with length L_s = 80 km as pictorially described in Fig. 2. Each fiber span was followed by a variable optical attenuator (VOA), a coupler enabling counter-propagating Raman pumping, an ideal gain flattening filter (GFF) that compensated for Raman gain tilting and an EDFA with noise-figure F_{EDFA} = 6 dB that recovered residual loss. We took into account two fiber types: a pure silica-core fiber (PSCF) and a non-zero dispersion-shifted fiber (NZDSF). The fiber parameters, listed in Tab. 1, are meant to represent broad classes of fibers and are not related to specific commercial products. Since we are considering a three-pump Raman scheme, the pump loss coefficients are to be intended as best-fits *equivalent* loss coefficients. For both fiber scenarios VOA loss was adjusted in order to have an overall span loss $A_{s,dB}$ = 20 dB that was completely recovered by the HFA. Rayleigh backscattering was not considered in the validation process.

Raman amplification was obtained using three un-polarized counter propagating pumps at 1425, 1436 and 1459 nm with overall launched power P_{pump}. For the PSCF setups, we analyzed system scenarios based on P_{pump} ranging from 200 mW up to 1200 mW, step 200 mW. For the NZDSF links, we chose six pump levels giving the same gain values as for the PSCF. We limited P_{pump} to 1200 mW for the PSCF and to 750 mW for the NZDSF in order to have negligible pump depletion. For comparison, we also considered EDFA-only, lumped amplification (that is P_{pump} = 0 mW).

Figures 3(a) and 3(b) display the Raman *on-off* gain G_{RA} vs. the overall pump power P_{pump} for PSCF and NZDSF links, respectively. The maximum value of G_{RA} was 13.1 dB for both fibers, obtained with P_{pump} = 1200 mW for PSCF, and with P_{pump} = 750 mW for NZDSF.

Figures 3(c) and 3(d) show the values of the equivalent noise figure F_{eq} of the HFA when increasing P_{pump} for PSCF and NZDSF, respectively. For PSCF, while increasing P_{pump}, F_{eq} decreases from 6 dB for the case of EDFA only down to –4 dB for P_{pump} = 1200 mW: it corresponds to an ASE noise reduction $\Delta P_{ASE,dB}$ = 10 dB. For the NZDSF, being characterized by a larger attenuation, the resulting F_{eq} is higher, reaching a minimum value of –2.1 dB for P_{pump} = 750 mW, corresponding to $\Delta P_{ASE,dB}$ = 8.1 dB with respect to the EDFA-only.

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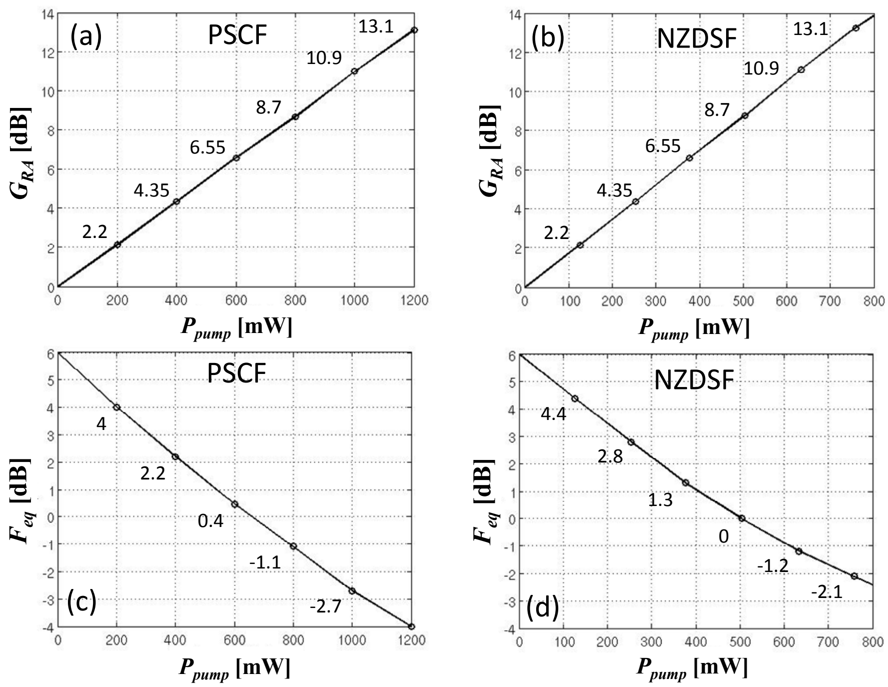

Fig. 3. On-off gain and HFA equivalent noise figure vs. P_{pump} for PSCF, (a) and (c), and NZDSF, (b) and (d).

Using Eq. (10), it can be observed that the contribution to the extension of maximum reach N_s^{max} due to only ASE noise reduction ΔP_{ASE} is up to 6.7 dB (~4.5-times) for PSCF links and up to 5.4 dB for NZDSF links (~3.5-times). In order to be able to predict the maximum reach extension using Eq. (10), the NLI enhancement ΔP_{NLI} must be evaluated as well.

First, we applied the GN-model to PSCF links in order to evaluate the NLI enhancement due to Raman amplification. We evaluated the parameter $\Delta P_{NLI,dB} = G_{NLI,HFA}/G_{NLI,EDFA}$ using Eqs. (3) and (6) for $G_{NLI,EDFA}$ and Eqs. (6), (11) and (12) for $G_{NLI,HFA}$, for the analyzed levels of Raman gain, varying the number of spans N_s from 5 to 35 ($L_{TOT} = N_s \cdot L_s$ from 400 to 2,800 km).

Results of this analysis are plotted in Fig. 4(a) as contour plot of $\Delta P_{NLI,dB}$ vs. G_{RA} and L_{TOT}. From this graph, it can be clearly observed that the iso-level curves are almost vertical with a slight tilting to the left. It means that NLI accumulation with z is slightly larger in the presence of Raman amplification with respect to EDFA only. Hence, practically we can assume that the Raman extra NLI does not depend on the number of spans and it accumulates with N_s as in the EDFA-only scenario. Such a behavior is confirmed analyzing Fig. 4(b) where the same results are plotted as $\Delta P_{NLI,dB}$ vs. G_{RA}. It can be noted that the difference between curves referring to $N_s = 5$ and $N_s = 35$ is less than 0.2 dB at $G_{RA} = 13$ dB. It means that the larger accumulation rate of NLI with the distance in case of Raman amplification is a minor effect. In particular, the maximum value of $\Delta P_{NLI,dB}$ can be estimated as 1.5 dB for $G_{RA} = 13$ dB. Considering Eq. (10), it means that the negative effect of NLI enhancement due to Raman amplification is limited to $1/3 \cdot \max\{\Delta P_{NLI,dB}\} \approx 0.5$ dB, that is a minor effect with respect to the ASE noise reduction that contributes with 6.7 dB, for the PSCF. So, from theory, we expect a maximum reach extension for the 1,200 mW HFA equivalent to 6.7 – 0.5 = 6.2 dB, i.e., a roughly four-times improvement.

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Applying the GN-model to the NZDSF links we obtained similar results, giving $1/3\cdot\max\{\Delta P_{NLI,dB}\}\approx 0.4$ dB, corresponding to a max reach extension equivalent to 5.4 - 0.4 = 5 dB, i.e., a roughly three-times improvement.

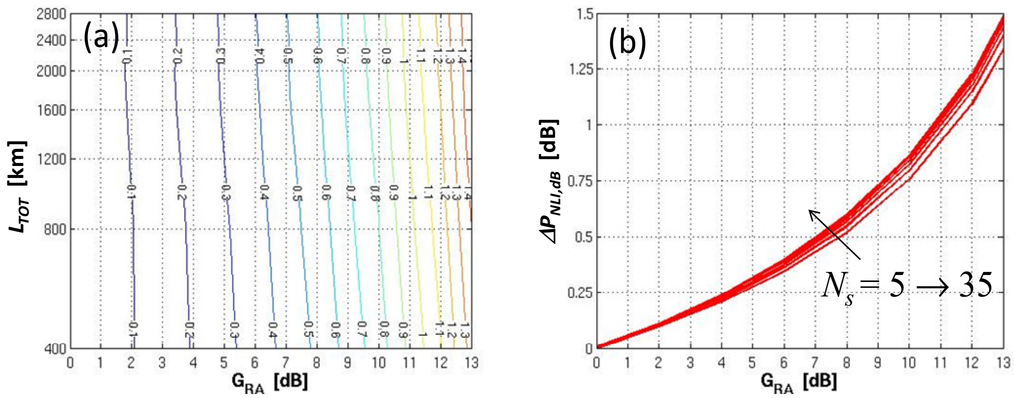

Fig. 4. (a) Contour plot of the NLI enhancement [dB] for PSCF links in the (G_{RA},L_{TOT}) plane. (b) NLI enhancement vs. G_{RA} at increasing distances for PSCF links.

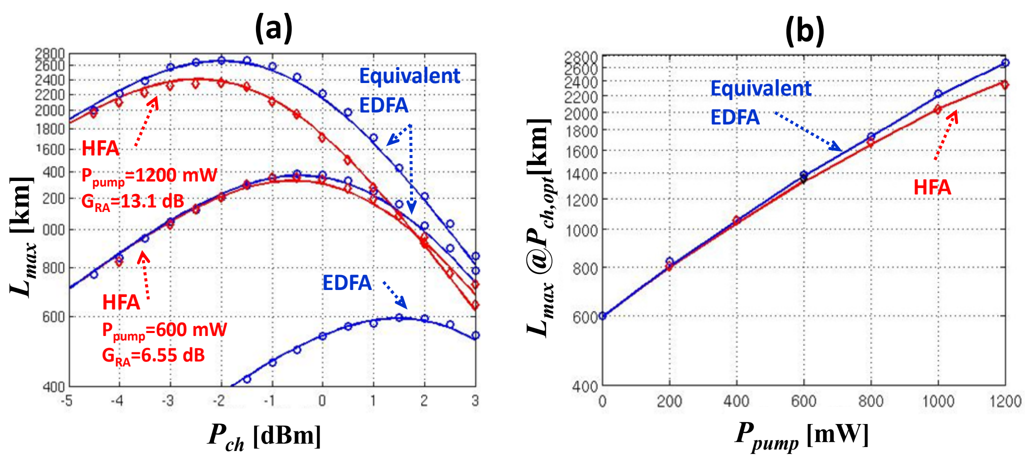

Fig. 5. PSCF setups. (a) GN-model analytical prediction of maximum reach vs. transmitted power for EDFA only setups (blue lines) and for HFA setups (red lines) and simulative results (markers). (b) Maxima vs. Pump power for HFA and equivalent "unrealistic" EDFA-only links: lines are GN-model analytical prediction while markers are simulative results.

These results can be extended to all practical links using HFA's with undepleted counter-propagating Raman pumps. Positive effect of ASE noise reduction induced by Raman amplification is always dominant with respect to negative effect given by NLI enhancement.

5. Simulative results and model validation

In order to apply and to validate the theoretical derivations, we carried out simulative studies on the links described in Sec. 4. We considered a NyWDM comb of PM-16QAM channels at the symbol rate R_s = 32 Gbaud (bit-rate R_b = 256 Gb/s) and channel spacing at the Nyquist limit, i.e., Δf = 32 GHz. We assumed the modulated signals to be formed using ideal digital-to-analog converters (DAC) at the transmitter side in order to operate as close as possible to an ideal NyWDM scenario with perfectly rectangular channel spectra. We analyzed a comb of N_{ch} = 11 channels using different pseudo-random bit sequences for each data flow and for each channel.

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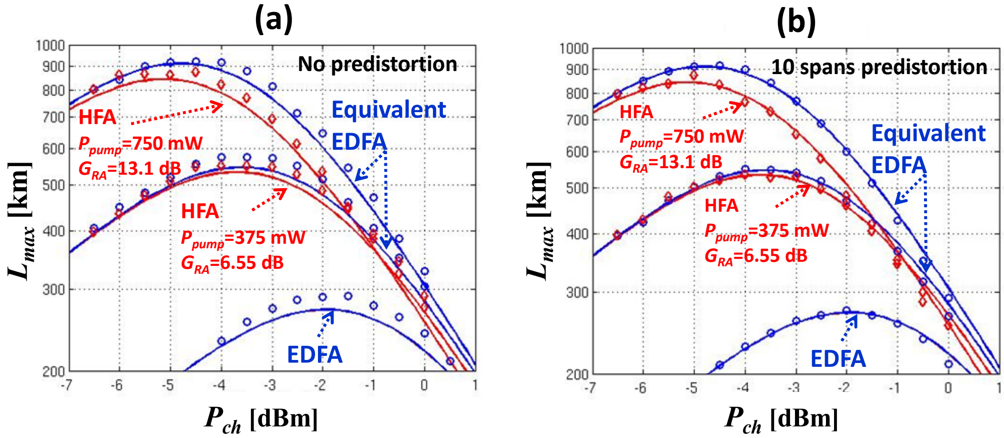

Fig. 6. NZDSF setups. GN-model analytical prediction of maximum reach vs. transmitted power for "unrealistic" EDFA-only setups (blue lines) and for HFA setups (red lines) and simulative results (markers). No predistortion (a) and transmission after predistortion equivalent to 10 fiber spans (b).

The Rx front-end was a standard polarization diversity setup based on an ideal 90-degree hybrid, followed by four balanced photodetectors and analog-to-digital converters running at 2 samples per symbol. The receiver local oscillator was assumed to be ideal, i.e., unaffected by phase noise. In the following DSP, chromatic dispersion was completely compensated for by proper FIR filters, and data were recovered by a 51-taps butterfly adaptive filter updated using the least mean square (LMS) algorithm started by a training sequence. System *BER* was evaluated through error counting over 2^{19} bits on the center channel. We characterized the *back-to-back* performance, and so we were able to accurately define the *sensitivity* function: the required OSNR in B_n = 0.1 nm in order to obtain the target $BER = 10^{-3}$ was estimated to be 23 dB.

We varied the transmitted power per channel P_{ch} from −5 up to + 3 dBm for PCSF links, and from −7 up to + 1 dBm for NZDSF links. For each P_{pump} and P_{ch} we estimated by simulation the maximum reachable distance $L_{max} = N_s^{max} \cdot L_s$ still allowing BER ≤ 10^{-3}. In this paper, L_{max} can be a fractional multiple of L_s, as N_s^{max} is obtained assuming it can be a fractional number, by theory, and interpolating evaluated *BER's* in order to estimate the non-integer N_s^{max} at *BER* target, by simulation.

In order to obtain comparative results of theory vs. simulation, we applied the GN-model and calculated the theoretical maximum reach as N_s^{max} inducing $OSNR_{NL}$ = 23 dB in B_n = 0.1 nm (see Eq. (7)) . For the only purpose of better assessing the effect of NLI enhancement due to Raman amplification, we also considered links based on *unrealistic* equivalent-EDFAs with noise-figure equal to that of the considered HFAs. These unrealistic links have the same ASE noise as the corresponding HFAs, but the same NLI as the EDFA-only case.

Figure 5(a) shows results in terms of L_{max} vs. P_{ch} for PSCF links. Blue lines refer to the GN-model analytical calculations for the EDFA-only setup and for the equivalent EDFA-only links, while red lines are analytical calculations for the 600 mW and 1,200 mW HFA's. Circles and diamonds are simulative results for EDFA and HFA links, respectively. Figure 5(b) uses the same color- and mark-coding and displays maxima (at optimal P_{ch}) of L_{max} vs. P_{pump} for all the HFA's. From Figs. 5(a) and 5(b), we can state that the results of the GN-model and the simulative ones show an excellent agreement for all the analyzed HFA's, confirming the general validity of the model, for highly dispersive fibers.

In Fig. 5(b), we observe a 4-times maximum reach extension from 600 km of EDFA-only up to 2400 km using the 1,200 mW HFA. This is in excellent agreement with theoretical results of Sec. 4, where a 6.7-dB maximum reach improvement due to ASE noise reduction

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was predicted, which is actually reduced to approximately 6 dB (4-times extension) due to the 0.5-dB decrease in maximum reach caused by NLI enhancement. These results confirm that the benefits of Raman amplification in terms of noise reduction are always dominant with respect to NLI enhancement, for highly-dispersive fibers.

We carried out similar analyses also for NZDSF links, and the results are plotted in Fig. 6(a), using the same plotting codes in terms of colors and marks, as the ones used for PSCF links in Fig. 5(a). It can be observed that the agreement between the GN-model and simulations is not as good as for the PSCF links. In particular, it appears that the theory always gives a slightly pessimistic evaluation, independently of the use of EDFA or HFA. The reason for this discrepancy, which manifests itself in the NZDSF but not in the PSCF, is the following.

The fundamental hypothesis of the GN-model is that the transmitted signal is so dispersed that it is approximately *Gaussian* distributed [7,8]. The model calculations assume this to be the case already in the first span. At R_S = 32 Gbaud, in high dispersion fibers such as the PSCF and standard single-mode fibers, dispersion acts so quickly that this assumption is well verified. However, in low-dispersion fibers, it takes longer for the signal to become *Gaussian* distributed. This causes the GN-model to overestimate the NLI produced in the first few spans of the link, so that system performance is somewhat underestimated.

In order to verify such an explanation, we simulated NZDSF links applying to the transmitted signal a linear pre-distortion equivalent to the chromatic dispersion inserted by 10 fiber spans (D_{pre} = 3040 ps/nm). The results of this analysis are plotted in Fig. 6(b) together with the curves obtained through analytical calculations. In this case, the agreement between GN-model and simulations is excellent, confirming the validity of the theory also for low dispersion fibers, provided that the signal *Gaussianity* is satisfied. In any case, the error in performance estimation of NZDSF links is small and it is always towards more conservative predictions. In addition, as confirmed by Fig. 6(a), the longer the link, the smaller the error. This is because NLI overestimation takes place only in the first few spans and therefore its relative impact tends to vanish in systems with many spans.

Regarding the balancing between noise reduction vs. NLI enhancement, theoretical predictions of Sec. 4 hold also for low-dispersive NZDSF links, i.e., in Raman amplification, noise reduction is always dominant with respect to NLI enhancement. So, in practice, it is always advantageous to Raman-pump fibers, at least up to the undepleted-pump threshold.

6. Conclusions

We presented the extension of the GN model for NLI to setups using HFA's with different levels of Raman counter-propagating pump. We applied the model to NZDSF and PSCF links transmitting 11 NyWDM PM-16QAM channels, and compared theory to simulative results.

We showed that the GN model presents an excellent agreement with simulations, provided that the signal *Gaussianity* is satisfied. This is always true in case of high-dispersion fibers, while for low-dispersion fibers signal *Gaussian* distribution is satisfied only after few spans. As a result, NLI is overestimated and performance is underestimated. However, the error is small and tends to vanish in long links.

We also theoretically showed that in case of Raman pumping, benefits of noise reductions are always dominant with respect to NLI enhancement, and, in practical counter-propagating pump scenarios, it is always advantageous to use Raman amplification, at least up to the pump depletion threshold.

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