Impact of the Transmitted Signal Initial Dispersion Transient on the Accuracy of the GN-Model of Non-Linear Propagation

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Abstract. *The GN-model neglects the initial signal dispersion transient in the fiber. We show that this circumstance causes the coherent-accumulation GN-model to provide a lower-bound to system performance estimation, typically less than 0.5 dB away from actual, while the simpler incoherent GN-model typically incurs lower performance estimation errors.*

1. Introduction

The recent introduction of DSP-enabled coherent detection has made it possible to carry out electronic CD compensation at the receiver (Rx), allowing uncompensated transmission (UT). In UT links, it appears that fiber non-linearity can be modeled through relatively simple perturbative analytical models, one of which is the Gaussian-Noise (GN) model [1]-[6].

The GN-model, like other perturbative models, relies on the assumption that the transmitted signal, due to the effect of the uncompensated CD, is so dispersed that it assumes a pseudo-random nature, with its four components becoming equally-distributed, uncorrelated Gaussian processes. This assumption is crucial for deriving the model fundamental formulas. On the other hand, it clearly takes some substantial accumulated dispersion to actually turn the signal into Gaussian noise and therefore, during an initial propagation transient, the signal 'Gaussianity' assumption is not well satisfied. This paper aims at investigating what error such initial dispersion transient (IDT) can induce on the predictions of the GN-model.

To perform this study, very accurate, CPU-intensive simulations were performed, where great care was taken towards suppressing split-step induced artifacts and other simulation errors. We found that significant deviations from the amount of non-linear-interference (NLI) noise predicted by the GN-model are observed in the first spans, whereas over the long-haul they tend to become less significant. Specifically, we looked at both the *coherent* and *incoherent* NLI accumulation versions of the GN-model [5,6]. When computing system performance, the coherent GN-model (CGN-model) turned out to provide a rather tight *lower bound*. Its incoherent version (IGN-model) typically provides a more accurate estimate, although without any guarantee of being either a lower or an upper bound.

2. The Simulation Set-Up

We concentrated on a quasi-Nyquist-WDM system based on either polarization-multiplexed (PM) QPSK or PM-16QAM. At the transmitter (Tx), digital pre-filtering was applied to obtain pulses with a square-root-raised-cosine spectrum, with roll-off equal to 0.02. Then, four ideal DACs generated the electrical signals driving two nested MZ modulators, operated in their linear trans-characteristic range. The symbol rate (R_s) was set to the typical industry standard value of 32 GBaud. The channel spacing (Δf) was 33.6 GHz, that is $1.05 \cdot R_s$. Data were generated using multiple independent PRBS's, four for PM-QPSK and eight for PM-16QAM, each of length $(2^{18} - 1)$. The uncompensated test link was composed of 50 spans, with lumped EDFA amplification exactly recovering span loss. The span length was 100 km. Channel selection was performed at the Rx by properly tuning the local oscillator. After balanced photo-detection, an electrical anti-alias filter of Bessel type (5 poles) with bandwidth $R_s/2$ was inserted. Its output was sampled at 2 samples per symbol. Then, electronic CD compensation was performed, followed by polarization de-multiplexing and equalization, by means of an adaptive 2x2 equalizer, driven by a decision-directed least-mean-square algorithm. Carrier and phase recovery were not needed since lasers were assumed ideal (no phase noise). The transmission fiber was either standard-single-mode (SMF), with dispersion D=16.7 ps/(nm·km) and non-linearity coefficient γ=1.3 1/(W·

km), or non-zero dispersion-shifted fiber (NZDSF), with D=3.8 ps/(nm·km) and γ=1.5 1/(W· km). Both fibers were assumed to have loss 0.22 dB/km.

ASE noise was turned off so that disturbance was due only to NLI. The estimation of the NLI variance was performed on the center channel of a 9-channel WDM comb, as follows. The noise variance of each signal point of the constellation was evaluated on both quadratures and polarizations. The results were averaged to obtain a single variance value $\sigma^2_{\rm tot}$. The same simulation was then repeated with fiber non-linearity turned off, all other parameters identical, producing an estimate of possible residual disturbance in linearity $\sigma^2_{\rm lin}$. Then, the NLI variance was calculated as: $\sigma^2_{\rm NLI} = \sigma^2_{\rm tot} - \sigma^2_{\rm lin}$. Note that, ideally, $\sigma^2_{\rm lin}$=0, since both NLI and ASE were turned off. However, we found that $\sigma^2_{\rm lin}$ was never exactly zero, possibly reflecting some minor inter-symbol interference, so that subtracting $\sigma^2_{\rm lin}$ was necessary for accurate NLI estimation.

The non-linear simulations were based on the split-step algorithm, which is known to produce various artifacts, among which spurious FWM. We applied a logarithmic step law [7], to mitigate it. To constrain the minimum step size, we imposed both a spurious FWM suppression of 50 dB [7] and a maximum non-linear phase shift, due to the total WDM instantaneous power integrated over a 35 ps time-window, not exceeding 0.025 radians. We chose these numbers by verifying that further tightening of these accuracy constraints would not alter the simulation results. For each system set-up, $\sigma^2_{\rm NLI}$ was measured after each span, from 1 to 50 spans.

We also calculated $\sigma^2_{\rm NLI}$ by means of the GN-model, whose formula was numerically integrated using its hyperbolic integration variables form ([6], Eq. 5). We did so with both the coherent and incoherent NLI noise accumulation assumptions [5,6].

Finally, all simulations were repeated using a very large CD value of 200,000 ps/nm applied at the Tx before launch, producing pre-dispersed (PD) signals whose distribution is already Gaussian at launch.

3. Results

NLI is such that $\sigma^2_{\rm NLI} \propto P^3_{\rm ch}$, where $P_{\rm ch}$ is the launch power per channel [6]. Here we focus on the quantity $\eta = \sigma^2_{\rm NLI}/P^3_{\rm ch}$, which is theoretically independent of $P_{\rm ch}$. In Fig.1-(top), η is plotted vs. the number of spans $N_{\rm span}$ for PM-QPSK over SMF. The dashed line represents the calculated result based on the CGN model, the dashed-dotted line based on the IGN-model. The solid lines are η from simulations, without PD (red curve) and with PD (blue curve). The PD simulative results are in excellent agreement with the CGN-model. In contrast, the non-PD simulations show a rather different result: over the first span, about 6 dB less NLI noise is actually produced than either the PD simulation or the GN-model predicts. The difference then decreases steadily.

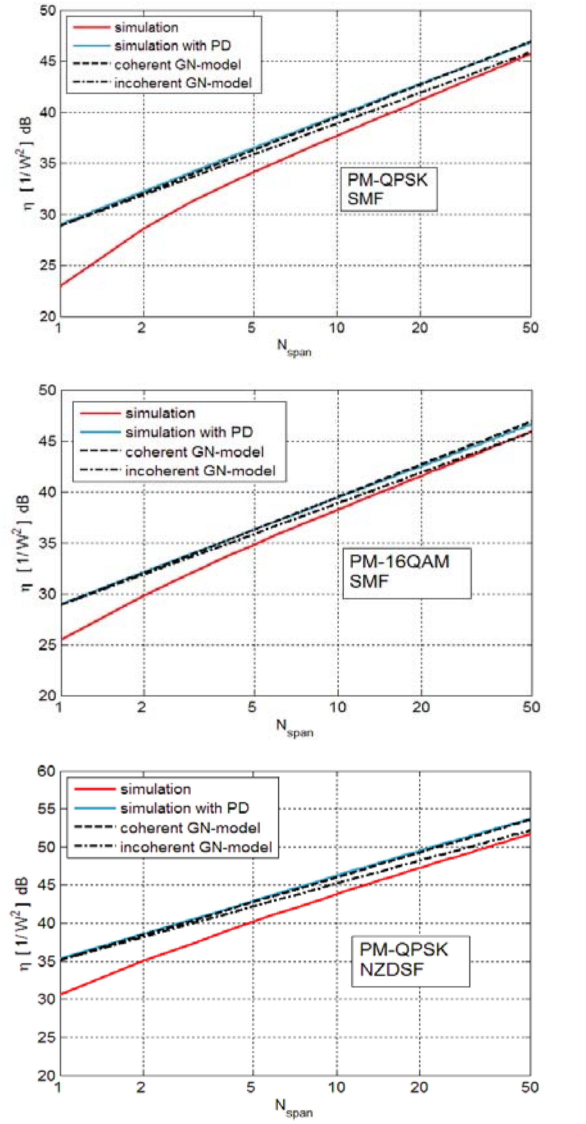

Figure 1. Normalized NLI noise variance η vs. the number of spans $N_{\rm span}$, over the center channel of 9 WDM channels, with 100 km span length. Solid lines: simulations, red no pre-distortion, blue with pre-distortion (PD). Dashed lines: CGN-model. Dash-dotted lines: IGN-model.

A similar picture emerges when considering PM-16QAM, Fig. 1-(center), although discrepancies are less pronounced. The first-span gap is 3 dB rather than 6 and, in general, the non-PD and PD simulations run closer. Again, the PD results are in very good

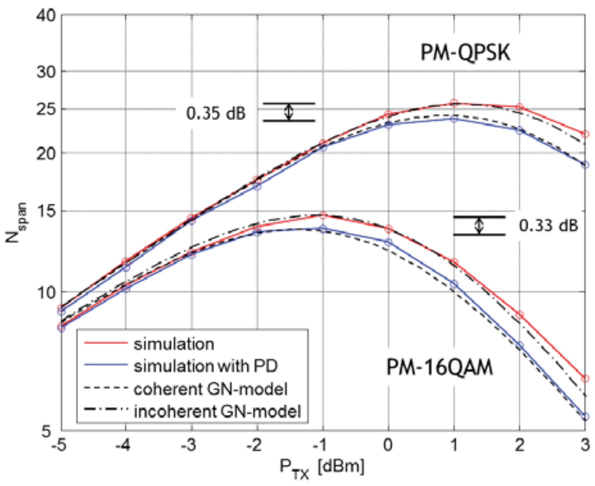

Figure 2. System reach vs. Tx power per channel P_{Tx} at BER=10^{-3}, for 9-channel PM-QPSK and PM-16QAM systems, with span length 110 and 80 km of SMF, respectively.

agreement with the CGN-model. Finally, in Fig. 1-(bottom) we show the result for PM-QPSK over NZDSF. Despite the lower fiber CD, which certainly slows down the IDT, the results are not substantially different from PM-QPSK over SMF. The gap after 1 span is smaller (4 dB vs. 6 dB). On the other hand, convergence of non-PD to PD is slightly slower.

A tentative interpretation of the overall results is that in the initial spans, and in particular in the very first, the instantaneous power variations of the conventional (non-PD) signals are smaller than those of the Gaussian-distributed PD ones. This leads to the generation of less NLI by the conventional signals than the PD ones. As the IDT progresses, and the conventional signals tend to become Gaussian-distributed too, the non-PD curve tends towards the PD curve and to what is predicted by the CGN-model. Notice that the CGN-model is always conservative with respect to the non-PD signal, in the sense that it predicts more NLI than there is. Interestingly, the IGN model typically runs closer to the non-PD simulations than the CGN model.

The impact on system performance prediction of the NLI behavior of Fig. 1 can be appreciated in Fig. 2, where the reach of an example of a 9-channel PM-QPSK or PM-16QAM system is studied, with span-length 110 and 80 km of SMF, respectively (EDFA noise figure 5 dB). The solid curves are simulations (red non-PD and blue PD signals). The PD curves are very well matched by the CGN-model (black dashed line). Expectedly, there is an error between the CGN-model and the conventional signals which, however, is limited to about 0.35 dB over the reach. Over NZDSF (not shown) it is about 0.4 dB. Such error can be theoretically shown to be roughly 1/3 (in dB) of the error on the estimation ofthe NLI power, or of η [6].

As argued before, the CGN-model error always leans towards a conservative prediction. In Fig. 2 this means a somewhat smaller reach. An interesting circumstance is that the IGN-model turns out to provide quite accurate system performance predictions for conventional (non-PD) signals, as also observed in [5]. This is just a coincidence, since the greater accuracy of the IGN-model is not due to a more faithful modeling of conventional signal propagation at a fundamental level, but rather to its different approximations that tend to cancel each other's error out. Nonetheless, this is a useful result, since the IGN-model may be quite adequate in various practical scenarios and is easier to deal with than the CGN, both analytically and numerically.

4. Comments and conclusions

Our study shows that the initial dispersion transient (IDT) does have some impact on the accuracy of the GN-model predictions. This is clearly appreciated by looking specifically at NLI noise generation (Fig. 1).

Due to the nature of the error, it turns out that the *coherent* GN-model provides a *lower bound* to system performance. Its use is therefore recommended when a conservative system assessment is mandatory. However, the performance underestimation is typically limited to less than 0.5 dB. Otherwise, the *incoherent* GN-model can be employed for its greater simplicity and ease of use, with typically better delivered accuracy, in agreement with [2].

Finally, our results suggest that, if not for the IDT, the coherent GN-model would represent non-linear fiber propagation with excellent accuracy. This suggests that efforts towards incorporating the IDT into the coherent GN-model might lead, if successful, to a very accurate, more general model.

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