Pulse/Spectrum Shaping

by

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Introduction:

Bandwidth (BW) is one of the most crucial resources in the communication industry. Fiber optic communication is band limited and typically occurs over the C-band. As modulation rates for optical signals increase each channel occupies more BW in the optical band, meaning less channels will be available at higher modulation rates. In order to squeeze more channels together over a band, pulse shaping can be done. Pulse shaping is a process by which an optical signal is shaped such that its BW is reduced in the frequency domain. Various techniques for pulse shaping are used, with the objective of limiting the bandwidth of each channel while eliminating Intersymbol Interference (ISI) and crosstalk.

The Rectangular Pulse:

The rectangular pulse is the most basic information unit in a digital transmission scheme. It has a defined amplitude and period. A sequence of such pulses constitutes the transmission of information. The information is encoded in the amplitude of the pulse. The simplest case is when a binary 0 is encoded as the absence of a pulse (A = 0) and a binary 1 is encoded as the presence of a pulse (A =

constant). Since each pulse spans the period T, the maximum pulse rate is 1/T Hz. The pulse amplitude can be more sophisticated and take on multiple discrete levels (including negative values), so that each pulse can represent more than one bit [7]

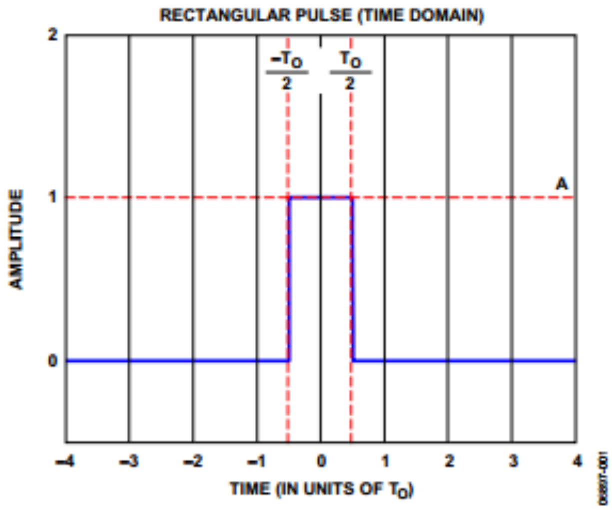

Figure 1. A Single Rectangular Pulse ($T = T_O$, $A = 1$)

Figure. A rectangular pulse, with period T0, and amplitude A.

In these cases where multiple amplitudes and/or multiple simultaneous pulses transmit a single unit of data, each unit of data (or symbol) represent multiple bits. The group of bits that a single unit of data represents is defined as symbol. The unit symbols per second, is defined as the baud rate. The data transmission bits rate is the baud rate multiplied by the number of bits represented by each symbol [7]. This means that a lower transmission rate can be used to transmit symbols as opposed to directly transmitting bits, which is the primary reason that the more sophisticated data transmission systems [7].

Spectrum of a Rectangular Pulse:

The Fourier Transform (spectrum) of the rectangular pulse is a sinc function, which is sin(x)/x. This result is the same for any amplitude or frequency of the rectangular pulse. Although the amplitude of the rectangular pulse does proportionally affect the magnitude of the frequency response peaks.

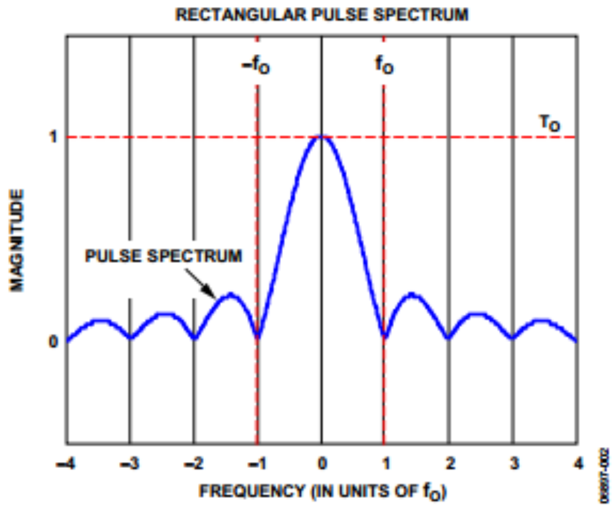

Figure 2. Spectrum of a Single Rectangular Pulse of Duration T_o

Figure. The frequency spectrum of a rectangular pulse, note the null points at integer multiples of f0. The sinc function is characterized by these peaks extending from negative infinity to positive infinity, but an important observation is that the null points (where the spectral magnitude is zero) always occurs at integer multiples of f0 (1/T), which is the symbol rate. These null points are therefore solely determined by the pulse period. If two sinc functions which have the same period were to add together, shifted by an integer multiple of the period, sampling only at the null points would yield an undistorted signal. This result will be used to eliminate Intersymbol Interference (ISI) during pulse shaping [7].

Pulse Shaping

In general optical signals can be shaped in the electrical domain before modulation, or optically after modulation. For QPSK systems, pulse shaping is done pre-modulation in digital signal processing (DSP). There are two methods used to minimize the spacing between consecutive channels; orthogonal frequency-division multiplexing (OFDM) and Nyquist wavelength division multiplexing (N-WDM). The latter is more commonly implemented in optical systems since it is easier to realize [1] and is the main focus of this report.

Nyquist Wavelength Division Multiplexing:

The ideology behind N-WDM is simple. All of the important data in a modulated optical signal is contained in the main lobe of its spectra, which has width equal to the modulation frequency. If each channel can be band-pass filtered such that the BW of the filter is the modulation frequency, then any redundant frequencies of the signal spectra in the channel can be eliminated and only the main lobe of the signal will remain. Theoretically this would allow all of the channels to be as tightly squeezed together as possible since they would be separated by the modulation rate; there would be no excess or wasted BW between consecutive channels (see Figure 1). One would need an ideal sinc-filter to implement this, which is impossible to do in practice. There are a few filters that can be used instead of the sinc filter; the raised-cosine filter is commonly used.

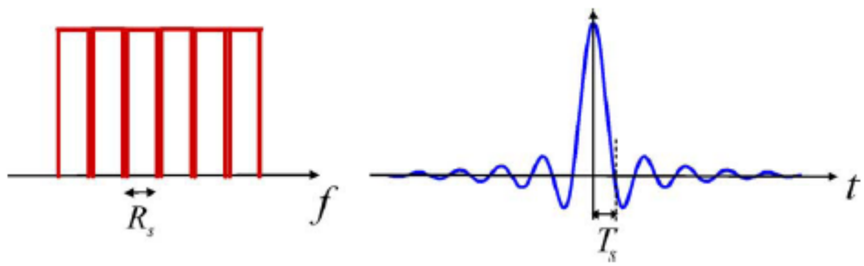

Figure 1: Ideal Nyquist wavelength division multiplexing in the frequency and time domain. Note the infinite response in the time domain [1]

Raised Cosine Filter:

The raised cosine filter is the most commonly used filter for pulse shaping in communication systems, especially multichannel communications that have channel modulation frequencies close together. The main benefits of this filter are that it can eliminate ISI, since it satisfies Nyquist ISI criterion (see Figure 2). It has a configurable excess bandwidth, which depends on the factor β. The transfer function of the filter is given by:

$$H(f) = \begin{cases} T, & |f| \leq \frac{1-\beta}{2T} \\ \frac{T}{2}\left[1 + \cos\left(\frac{\pi T}{\beta}\left[|f| - \frac{1-\beta}{2T}\right]\right)\right], & \frac{1-\beta}{2T} < |f| \leq \frac{1+\beta}{2T} \\ 0, & \text{otherwise} \end{cases}$$

$$0 \leq$$

where β is the roll-off factor and T is the symbol period. The roll-off factor corresponds to a measure of the excess bandwidth of the filter. The Nyquist bandwidth of the filter is given by 1/2T.

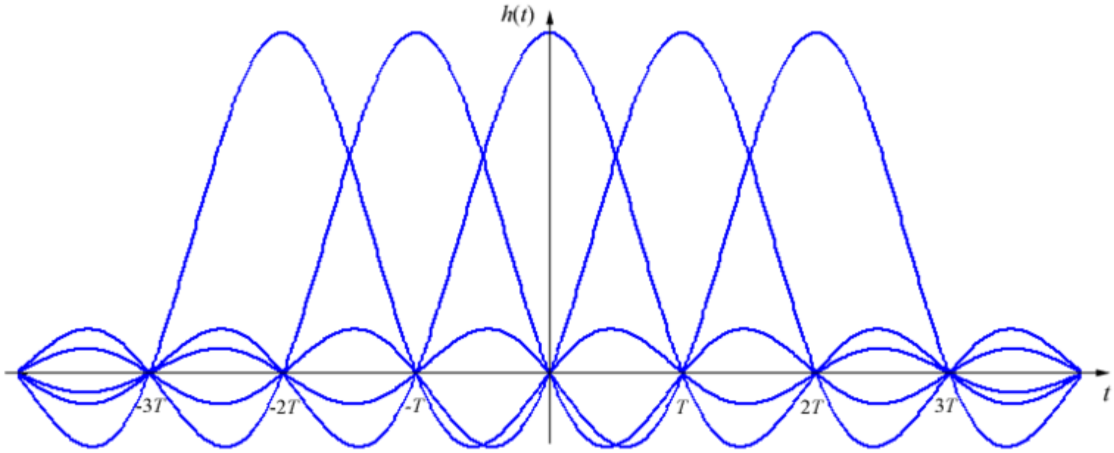

Figure 2: Illustration of the ISI Nyquist criteria. If sampling occurs and integer multiples of T, then the signal from previous and successive signals are zero. [2]

Figures 3 and 4 show the raised-cosine filter in the frequency and time domain respectively. As β approaches 0, the raised-cosine filter becomes a sinc filter. One can see that the larger β becomes, the more BW the filtered signal will have, and thus it is advantages to try to decrease β.

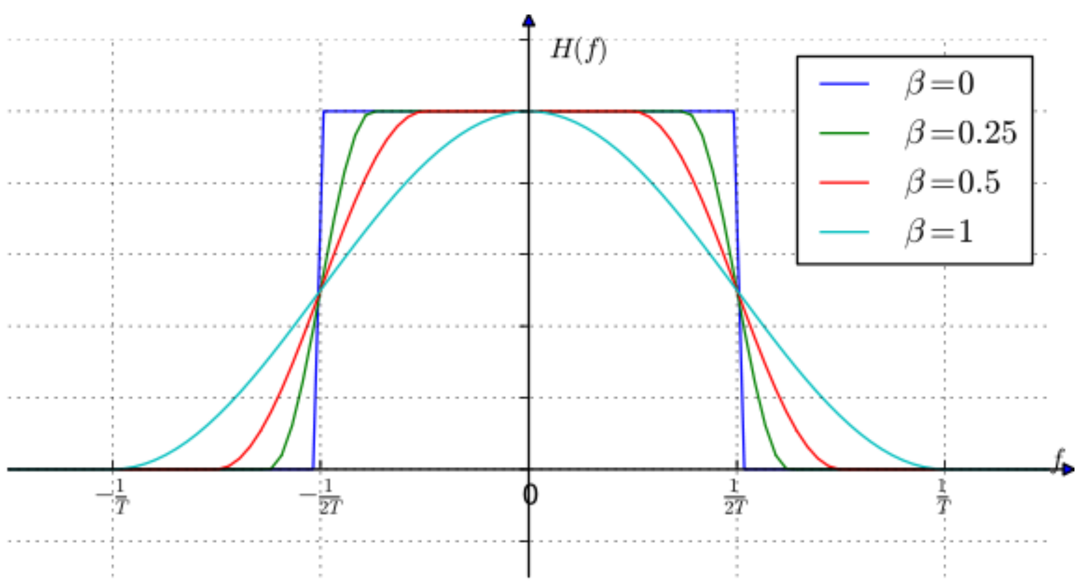

Figure 3: Frequency spectra of a raised cosine filter for various roll-off factor values

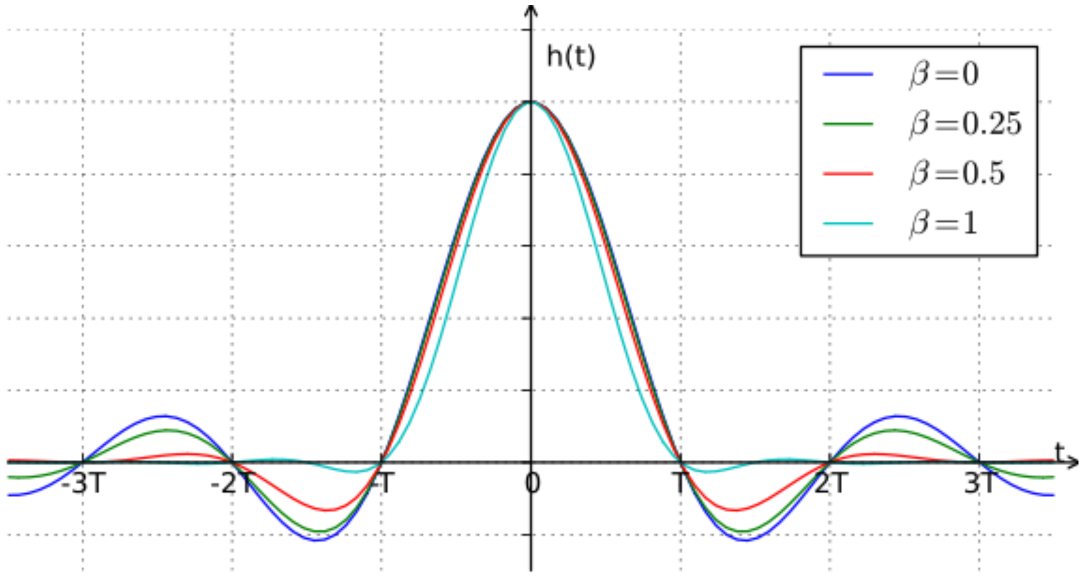

Figure 4: Impulse response of a raised cosine filter for various roll-off factor values

There are three important points in the frequency spectrum associated with raised cosine filters. The first is the Nyquist frequency, which occurs at f0/2 (1/2T). This is the minimum possible bandwidth that can be used to transmit data without loss of information. As shown in Fig.3 this bandwidth corresponds to a perfect (β=0) filter. Note that the response crosses through the half amplitude at this point regardless of β value. The second important point is the stop band frequency (fstop), defined as the frequency at which the response first reaches zero magnitude.

$$fstop=(1+\beta)(f0/2)$$

The third important point is the pass band frequency (fpass), defined as the frequency at which the response first begins to depart from its peak magnitude. The frequency response is perfectly flat from DC to fpass.

$$fpass=(1-\beta)(f0/2)$$

Also note that sometimes it is desirable to implement half a raised cosine filter, at each the receiver and transmitter ends. The two square root raised cosine filters' product form the true raised cosine response.

Intersymbol Interference and Crosstalk

The consequence of pulse shaping is that it distorts the time domain signal (from a rectangular pulse to a sinc or raised cosine wave). These ripples in time domain results from the convolution (time domain filtering) of the rectangular pulse with the raised cosine filter impulse response, and they are unavoidable due to limiting the bandwidth to a finite value. As shown previously, however, the filter zero crossings (null points) coincide with the midpoints of the adjacent pulses. As long as the receiver samples at these null points (mid pulse intervals), the ripples from adjacent pulses are crossing through zero, and therefore do not cause any interference [7]. Thus ISI is avoided. Theoretically the choice of β does not affect the fact that these ripples always have zero crossings at null points, and thus one assumes that it is always desirable to have $\beta=0$, to get the minimum bandwidth and accept the maximum ripple amplitude. The reality is that receivers are usually not accurate enough to sample exactly at these null points, and thus some ripples distortion will be introduced into the sampled signal [7]. Thus, in reality, there is always some ISI, and there is always a tradeoff between ISI reduction in time domain and bandwidth efficiency in spectral domain.

Since the raised-cosine filter is not perfectly rectangular in the frequency domain, there is a limit to how close the channels can be placed together before crosstalk effects begin to significantly distort signals. Figure 5 shows an illustration of crosstalk between consecutive channels placed too close in carrier frequency. Cross talk between channels may also occur due to the finite impulse response of the implemented filter.

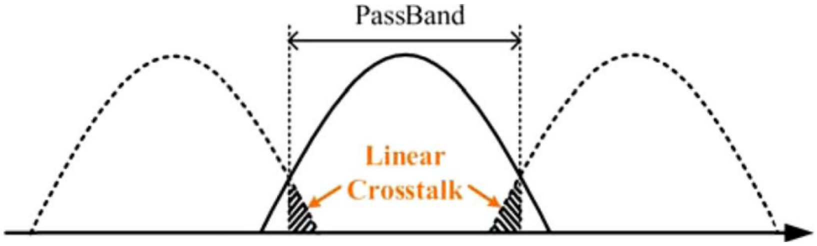

Figure 5: Cross talk between consecutive pass-band filtered channels [5]

Finite Impulse Response:

In theory the raised cosine filter provides a good solution for reducing ISI and multi channel cross-talk; however, there are some issues that arise when it is implemented in practice. The main issue that one encounters is the fact that these filters shape pulses such that they have infinite length in the time-domain. Since this is not a physical possibility, the pulse shape needs to be truncated. This will result in non-zero sidelobes in the frequency domain of the shaped pulses. The less taps that are taken for the filter, the less ideal it becomes and thus the less tight the channels can be placed together. Figure 6 shows the frequency spectra of a raised-cosine function for a various amount of taps. The number of taps used is limited by computational power.

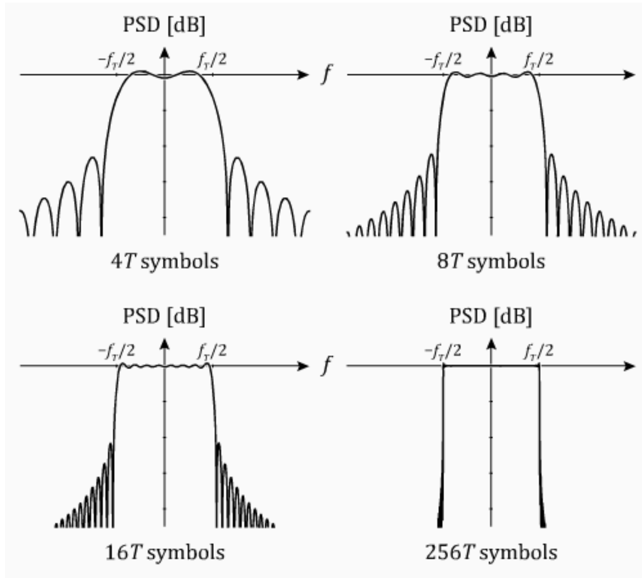

Figure 6: FIR spectra of a Nyquist filter for various tap numbers [6].

FIR filters also pose some advantages compared to IIR filters, for several reasons. Firstly, FIR filters are easily designed with linear phase response, which is important for applications that desire a constant group delay. Secondly, FIR filters do not suffer from limit cycles (small oscillations that persists at the output of the filter even after removing the inputs). Thirdly, FIR filters are intrinsically stable because they do not have feedback, whereas the IIR has feedback and places limits on the choices of the taps due to stability. Lastly, if the filter is implemented in hardware rather than software, FIR can be made from polyphase architecture, which significantly reduces the amount of hardware required. The amount of hardware required is proportional to the number of taps in the filter, good FIR filters require a lot of taps and thus a lot of hardware. Polyphase design offer to reduce the amount of hardware required per tap, and is thus attractive [7].

Regardless of the type of filter, in reality one can only achieve an approximation of the ideal raised cosine filter. The degree to which one can approach an ideal filter is dependent on the number of filter taps (N) and the amount of oversampling (M). Generally N is chosen to be an integer multiple of M to guarantee the impulse response of the filter to span an integer number of pulses,such that

$$N=D * M$$

where the ideal D is found experimentally [7]. In general, larger D can approximate the ideal filter better, whereas smaller D means simpler filter implementation.

Implementation:

Pulse shaping has been commonly implemented via digital rather than analog filters. This means that the filter is subjected to the Nyquist criterion. The filter sample rate must be twice that of the input bandwidth in order to avoid aliasing [7]. The required bandwidth can approach f0 for cases where the filter's β=1, this implies that digital pulse shaping filters must oversample the symbol rates by a factor of at least two. The actual filtering is often done in time domain [7]. The digital filter coefficients (taps) define the impulse response of the filter which produces a desired frequency response.

Several groups are using various raised-cosine filters to determine the optimal performance of their optical system. For example, a group at the University of Tokyo measured the big-error rates of N-WDM PM-QPSK signals with channel spacing equal to the symbol rate (10GHz). They used a Nyquist filter with 600 taps to shape the optical pulses, which achieved nearly rectangular like channel spectra. Figure 7 shows a block diagram of the DSP filtering used and the expected output signal frequency.

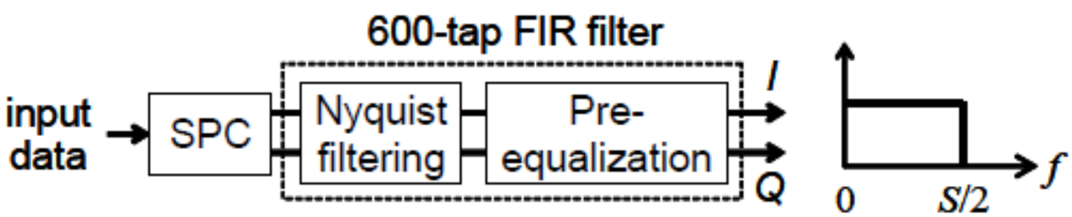

Figure 7: Experimental Setup [1].

Figure 8 shows the constellation diagram for various channel spacing. One can see that for channel spacing less than the symbol rate (10 GHz), there is a large increase in cross-talk and the signals become heavily distorted (see Figure 8d). They were able to entirely supress cross talk between channels for 10 GHz channel spacing and the signals were unaffected by ISI when filtered [1].

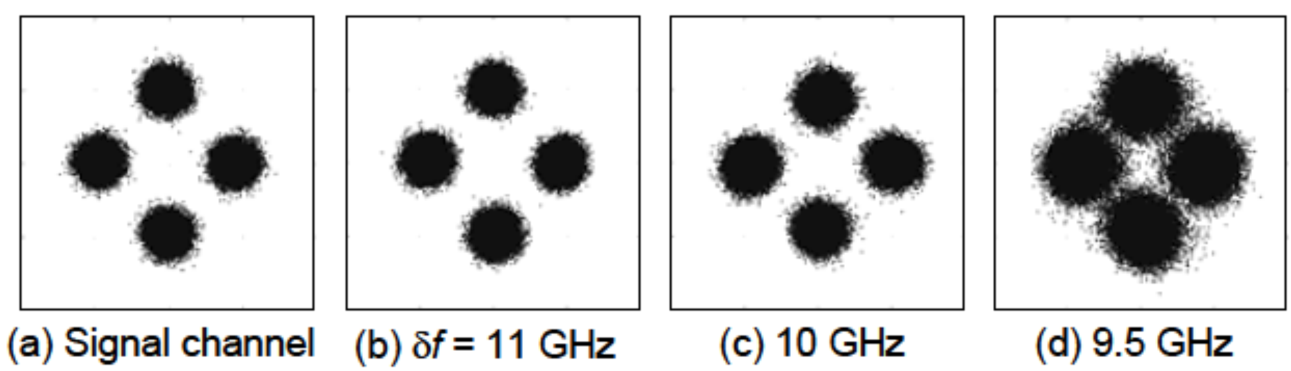

Figure 8: Experimental constellation map of a 10G modulated signal for various channel spacing denoted by δf [1].

References:

[1] - Igarashi K, et al. 'Bit-error Rate Performance of Nyquist Wavelength-Divisions Multiplexed Quadrature Phase-Shift Keying Optical Signals'. *Optical Fiber Communication Conference and Exposition (OFC/NFOEC)* (2010)

[2] - http://en.wikipedia.org/wiki/File:Raised-cosine-ISI.png

[3] - http://en.wikipedia.org/wiki/File:Raised-cosine_filter.svg

[4] - http://en.wikipedia.org/wiki/File:Raised-cosine-impulse.svg

[5] – Li J, et al. 'Approaching Nyquist Limit in WDM Systems by Low-Complexity Receiver-Side Duobinary Shaping' *Journal of Lightwave Technology, Vol 30, No 1* (2012)

[6] –http://www.marcuswinter.de/archives/1037

[7] - http://www.analog.com/static/imported-files/application_notes/AN-922.pdf