Analytical Modeling of Nonlinear Propagation in Uncompensated Optical Transmission Links

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***Abstract*—We present analytical results on the impact of nonlinear propagation in uncompensated links. We test the accuracy of our model in the context of ultradense wavelength-division-multiplexing polarization-multiplexed quadrature phase-shift keying (PM-QPSK) systems, at the Nyquist spectral efficiency limit. We show that the predicted system performance matches simulation results very accurately over a broad range of system scenarios. A simple closed-form analytical formula provides an effective tool for the quick and accurate prediction of system performance.**

***Index Terms*—Dense wavelength-division multiplexing (DWDM), Nyquist wavelength-division multiplexing (WDM), optical transmission, polarization-multiplexed quadrature amplitude modulation (PM-QAM), polarization-multiplexed quadrature phase-shift keying (PM-QPSK), uncompensated systems.**

I. Introduction

THE performance of long-haul broadband optical transmission systems is mainly limited by two distinct phenomena: amplified spontaneous emission (ASE) noise accumulation and the generation of nonlinear interference (NLI) due to the Kerr effect in the fiber. The available analytical description of ASE accumulation is simple and accurate. As for NLI, several approximated formulas describing specific Kerr-derived effects, such as cross-phase modulation (XPM) or four-wave-mixing (FWM), have been proposed over the years in the context of dispersion-managed (DM) Intensity-Modulated Direct-Detection (IMDD) systems, allowing various degrees of performance prediction. The DM nonlinear propagation scenario has however proved quite unwieldy and not amenable to compact and comprehensive analytical solutions, possibly because of fundamental reasons.

Lately, thanks to the advent of coherent detection with digital-signal-processing (DSP), uncompensated transmission (UT) has become a viable option. Recent studies have suggested that UT brings about a substantial performance improvement in long-haul dense-WDM coherent systems [1], [2] and all current transmission records have been obtained over UT links. Therefore UT appears to be the possible reference scenario for the next-generation of long-haul links.

Remarkably, UT alters the properties of signal propagation in quite a dramatic way with respect to DM. Specifically, due to dispersion, assuming polarization-multiplexed quadrature phase-shift keying (PM-QPSK) or quadrature amplitude modulation (PM-QAM), the four electric field components of each transmitted WDM channel appear to very quickly take on statistically-independent zero-mean Gaussian distributions. This was simulatively shown for PM-QPSK in [3] but can be proved rigorously for generic PM-QPSK/QAM signals. A somewhat surprising result also reported in [3] is that, after DSP, the statistical distribution of each of the received constellation points appears to be Gaussian too, with independent components, even in the absence of added ASE noise in the link. It seems, in other words, that the effect of NLI could be approximately modeled as excess additive Gaussian noise, at least for low-to-moderate NLI.

Based on such premises, we have developed a comprehensive model for the assessment of NLI in UT PM-QPSK/QAM links which eventually yields an analytical formula containing a double integral that can be solved numerically. In addition, for the special case of channel spacing equal to the baud-rate (the Nyquist limit), we have derived a simple approximate closed-form formula, which turns out to be quite accurate in expressing NLI as a function of the main link parameters.

In this letter we briefly introduce the procedure through which the model has been derived. We then present its analytical solution and its closed-form approximate expression at the Nyquist limit. Following, we show the results of a thorough validation campaign regarding PM-QPSK at the Nyquist limit, in which we checked the model predictions against a very wide range of test links, varying span length, number of channels, baud-rate, fiber dispersion, loss and nonlinearity coefficient. We concentrate on the Nyquist spacing limit for various reasons: the increasing interest towards ultrahigh spectral efficiency systems; the fact that in this limiting but fundamental case a closed-form formula can be found; the fact that the case of minimum channel spacing is challenging for some of the underlying model assumptions.

II. The Model and its Derivation

For notational convenience we assume an odd number of channels N_{ch} in the WDM comb and we concentrate on the performance of the center channel.

We assume that the effect of NLI on WDM signals in UT can be modeled as additive Gaussian noise, statistically independent of ASE noise [3]. If so, the ASE and NLI contributions add up in power. The BER then depends on the OSNR modified to include the NLI noise, defined as:

$$\mathrm{OSNR} = \frac{P_{\mathrm{Tx},ch}}{P_{\mathrm{ASE}} + P_{\mathrm{NLI}}} \tag{1}$$

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where $P_{\mathrm{Tx},ch}$ is per-channel power, $P_{\mathrm{ASE}} = G_{\mathrm{ASE}} \cdot B_n$ and $G_{\mathrm{ASE}} = [N_s(G-1)Fh\nu]$ is the dual-polarization ASE noise power spectral density (PSD), with N_s the number of spans, G the EDFA gain, equal to the span loss, F the EDFA noise figure and B_n the noise bandwidth. In this letter we assume $B_n = 12.48$ GHz (0.1 nm). Also:

$$P_{\mathrm{NLI}} = \int_{-B_n/2}^{B_n/2} G_{\mathrm{NLI}}(f) df \tag{2}$$

where $G_{\mathrm{NLI}}(f)$ is the PSD of NLI and f is frequency, relative to the center frequency of the center channel of the comb.

The fundamental quantity that needs to be assessed is then $G_{\mathrm{NLI}}(f)$. To derive it, we started out by making the key assumption that each channel of the comb and, as a consequence, the overall WDM signal, can be modeled as Gaussian periodic noise of arbitrary period T_0. The Fourier transform of the overall WDM comb can then be written as:

$$E_{\mathrm{Tx}}(f) = \sqrt{G_{\mathrm{Tx}}(f)}\sqrt{f_0} \sum_{k=-\infty}^{\infty} \xi_k \delta(f - kf_0) \tag{3}$$

where $f_0 = 1/T_0$, and the ξ_k's are complex independent Gaussian RV's of unit variance. Note that in (3) the factor $\left[\sqrt{f_0}\sum_{k=-\infty}^{\infty} \xi_k \delta(f-kf_0)\right]$ is complex periodic white Gaussian noise (PWGN) of period T_0, with unit PSD, expressed through the Karhunen–Loève formula [4]. The quantity $G_{\mathrm{Tx}}(f)$ is the WDM signal PSD which suitably shapes the PWGN.

The signal in (3) is made up of many individual spectral lines spaced $f_0 = 1/T_0$. We applied to such lines the standard formulas used to estimate FWM [5], through which we obtained the NLI field at any frequency nf_0. Note that we assumed the Manakov equation in order to account for dual-polarization propagation and we used the "undepleted pump" assumption, which corresponds to assuming operation in low-to-moderate nonlinearity.

From the NLI field, a corresponding NLI power was then found by absolute-value squaring. The squaring process creates many cross-products of *different* partial NLI terms. A large part of such cross-products is then eliminated by statistical averaging, thanks to the independence of the ξ_k's. Eventually, for each frequency nf_0, the total NLI power turns out to be expressed as a double summation of many partial NLI terms. By letting $T_0 \to \infty$ (i.e., $f_0 \to 0$) the double summation is transformed into a double integral over frequency, yielding the following final expression:

$$\begin{aligned} G_{\mathrm{NLI}}(f) &= \frac{16\gamma^2}{27} \int_{-(B_o-f)/2}^{(B_o-f)/2} \int_{-(B_o-f)/2}^{(B_o-f)/2} \frac{\sin^2\left(2N_s\pi^2|\beta_2|L_s f_1 f_2\right)}{\sin^2\left(2\pi^2|\beta_2|L_s f_1 f_2\right)} \\ &\cdot \left|\frac{1-e^{-2\alpha L_s} e^{j4\pi^2|\beta_2|L_s f_1 f_2}}{2\alpha - j4\pi^2|\beta_2| f_1 f_2}\right|^2 G_{\mathrm{Tx}}(f_1+f_2+f) \\ &\cdot G_{\mathrm{Tx}}(f_1+f) G_{\mathrm{Tx}}(f_2+f) df_1 df_2 \end{aligned} \tag{4}$$

where γ is the fiber nonlinearity coefficient; $B_o = N_{ch}\Delta f$ is the optical bandwidth of the whole WDM comb, with Δf the channel spacing; L_s is the span length; α is the fiber loss parameter and β_2 is fiber dispersion in [ps^2/km].

At the Nyquist limit, Δf equals the baud-rate R_s and the channel spectra are rectangular with bandwidth R_s. Then (4) can be effectively approximated through a closed-form expression, yielding the following formula for the NLI power P_{NLI}, to be used in (1):

$$P_{\mathrm{NLI}} \simeq \left(\frac{2}{3}\right)^3 N_s \gamma^2 L_{\mathrm{eff}} P_{\mathrm{Tx},ch}^3 \frac{\log\left(\pi^2 \mid \beta_2 \mid L_{\mathrm{eff}} N_{ch}^2 R_s^2\right)}{\pi \mid \beta_2 \mid R_s^3} B_n \tag{5}$$

where $P_{\mathrm{Tx},ch} = P_{\mathrm{Tx,tot}}/N_{ch}$, $P_{\mathrm{Tx,tot}} = \int_{-B_o/2}^{B_o/2} G_{\mathrm{Tx}}(f) df$ is the total WDM signal transmitted power and $L_{\mathrm{eff}} = [1 - \exp(-2\alpha L_s)]/(2\alpha)$ is the fiber effective length. Equation (5) is sufficiently accurate for $\left(\pi^2 \mid \beta_2 \mid L_{\mathrm{eff}} N_{ch}^2 R_s^2\right) > 50$ and $e^{-2\alpha L_s} < 1/10$. This is typically the case in practical systems and hence this approximated formula, which we call 'AF', provides a very simple and insightful means of estimating NLI in UT Nyquist-limited WDM PM-QPSK/QAM systems.

III. Model Test at the Nyquist Limit

To perform the simulative tests at baud-rate channel spacing, we used the very same PM-QPSK Tx and Rx simulation setups as described in [6]. We only scaled R_s up to 32 Gbaud (128 Gb/s) and assumed a different target BER (10^{-3}). The PRBSs degree was 16 and BERs were evaluated using direct error counting over 262144 bits (65,536 symbols). For the sake of realism, we show here results obtained using transmitted channel spectra shaped by realistic flat-top 4th order super-Gaussian (SG) filters, of bandwidth equal to R_s. The channel spectra are not exactly rectangular but this circumstance has no discernible impact on the NLI impinging over the center channel. It only causes a back-to-back (btb) penalty, due to linear crosstalk, so that the btb sensitivity was OSNR $= 16.4$ dB for BER $= 10^{-3}$ rather than the ideal 13.9 dB.

We first tested the model by addressing three fiber types and three span lengths, for a total of 9 distinct cases. Fiber dispersion [ps/nm/km], nonlinearity [1/W/km] and loss [dB/km] were, respectively: SSMF $D = 16.7$, $\gamma = 1.3$, $\alpha = 0.22$; PSCF $D = 20.1$, $\gamma = 0.9$, $\alpha = 0.18$; NZDSF $D = 3.8$, $\gamma = 1.5$, $\alpha = 0.22$. The markers in Fig. 1 show the simulated system reach results, obtained for $N_{ch} = 9$. The solid lines are obtained using the AF to estimate P_{NLI}. The results appear to be very accurate in all cases.

Next, we used SSMF and set the total WDM optical bandwidth at $B_o = 288$ GHz, for a total capacity of 1.152 Tb/s. We varied the number of channels N_{ch}, whose individual baud-rate was therefore $R_s = B_o/N_{ch}$. The test range was $N_{ch} = 3$ to 27. The AF, with (1), predicts the nontrivial result that system performance should not change when varying R_s. Fig. 2(a) confirms this prediction very well.

The AF also allows to predict that fixing the baud-rate R_s, performance should gradually get worse as N_{ch} goes up. Fig. 2(b) indicates that this prediction is accurately satisfied too.

We then set $\gamma = 1.3$, $\alpha = 0.22$ and varied dispersion, with $N_{ch} = 9$. Interestingly, the AF forecasts that performance should improve when increasing fiber dispersion. This prediction appears true and accurate as well (Fig. 2(c)).

Finally, we ran tests with ideally rectangular Tx spectra, finding the same accuracy for the AF as shown here with

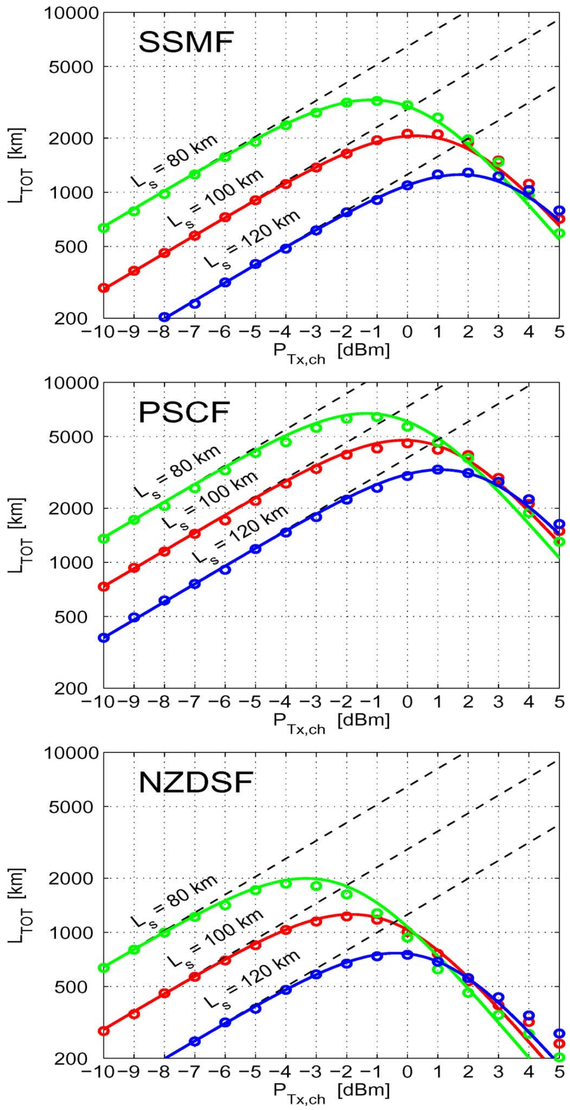

Fig. 1. System reach versus launch power per channel at BER $= 10^{-3}$ with $L_s = 100$ km, $N_{ch} = 9$, $R_s = 32$ GBaud. Markers: simulations. Solid line: analytical model of equation (5).

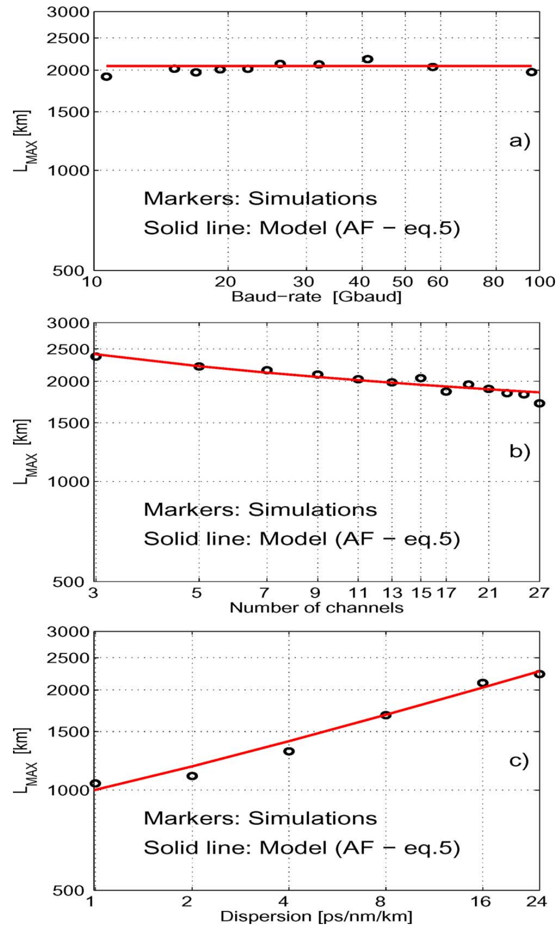

Fig. 2. Maximum reach at optimum launch power: (a) versus baud-rate, over SSMF with span length $L_s = 100$ km and variable number of WDM channels N_{ch}, at a fixed total capacity of 1.152 Tb/s ; (b) versus N_{ch}, over SSMF with $L_s = 100$ km; (c) versus dispersion with $L_s = 100$ km, loss 0.22 dB/km, nonlinearity 1.3 $(\mathrm{W} \cdot \mathrm{km})^{-1}$ and $N_{ch} = 9$.

SG shaping filters. Note that all the results shown here have been obtained with ASE noise added in-line. We have also run the same simulations using Rx noise loading and found virtually identical results, indicating a negligible impact of ASE-noise-related nonlinearity in this context.

IV. Conclusion

Our tests have shown a high accuracy of the model, across the whole parameters test range. Note that the model has no free fitting constants and that all parameter dependencies were the sole consequence of the analytical derivation. This confirms *a posteriori* that the assumptions which the model is based on must hold true to a great extent.

Thanks to the approximate, but accurate, closed-form expression of the NLI (5), the dependence of the performance of uncompensated Nyquist-limited systems on the main link parameters is made explicit. It agrees with our simulations but it is also in qualitative agreement with many experimental results that have been recently accumulating on UT systems.

Here, we have looked extensively at PM-QPSK at the Nyquist-limited spacing. Work is ongoing on the testing of the model with PM-QPSK and PM-QAM, also at larger channel spacings. Preliminary results are showing very good agreement in these more general scenarios too.

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