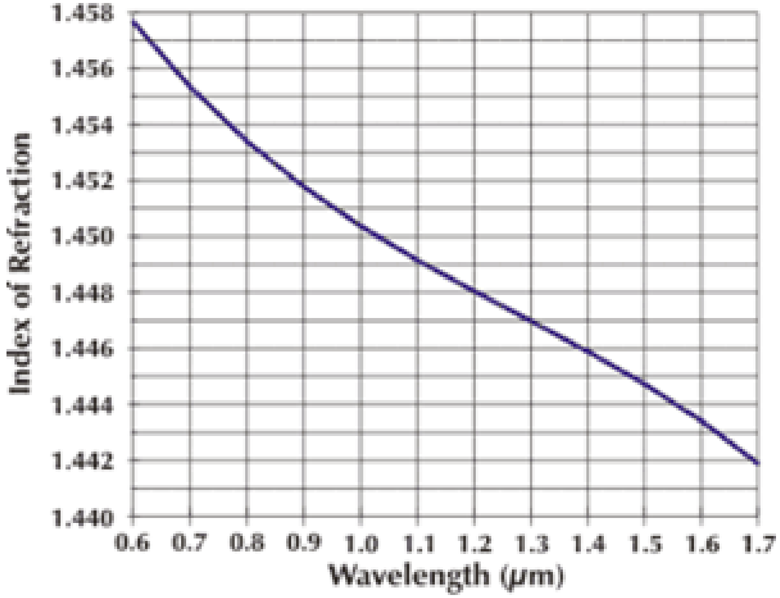
Lecture 4

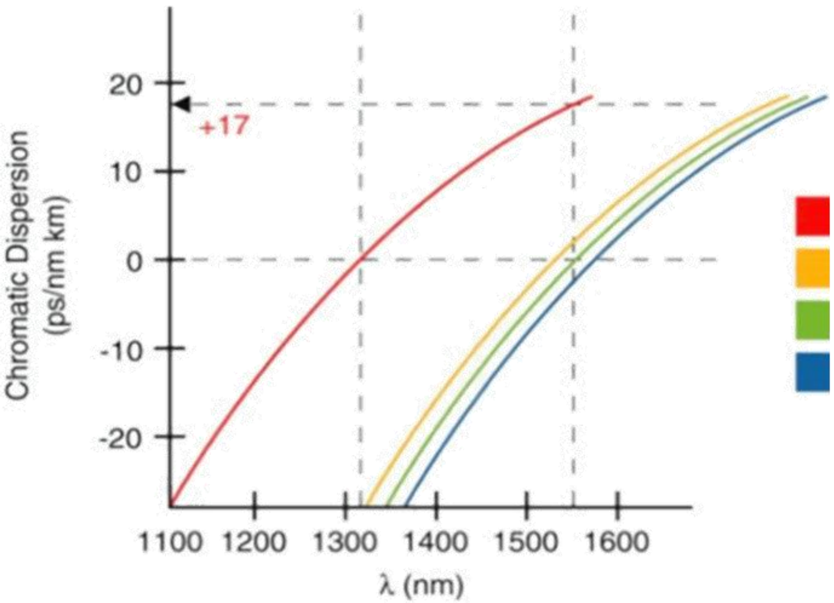
- Dispersion in single-mode fibers
  - Material dispersion
  - Waveguide dispersion
- Limitations from dispersion
  - Propagation equations
  - Gaussian pulse broadening
  - Bit-rate limitations
- Fiber losses

Dispersion, qualitatively

- Different wavelengths (frequency components) propagate differently
- A pulse has a certain spectral width and will broaden during propagation



The index of refraction as a function of wavelength



The dispersion in SMF (red) and different dispersion-shifted fibers

Group delay, group index, and GVD parameter (2.3.1)

Each spectral component of a pulse has a specific group velocity

The ***group delay*** after a distance *L* is

$$T = \frac{L}{v_g} = L\frac{d\beta}{d\omega}$$

The group velocity is related to the ***mode group index*** given by $\bar{n}_g = \bar{n} + \omega\frac{d\bar{n}}{d\omega}$

$$v_g = \frac{c}{\bar{n}_g} = \frac{c}{\bar{n} + \omega\frac{d\bar{n}}{d\omega}} = \frac{c}{\bar{n} - \lambda\frac{d\bar{n}}{d\lambda}}$$

Assuming that $\Delta\omega$ is the spectral width, the pulse broadening is governed by

$$\Delta T = \frac{dT}{d\omega}\Delta\omega = L\frac{d^2\beta}{d\omega^2}\Delta\omega \equiv L\beta_2\Delta\omega$$

where β_2 is known as the ***GVD parameter*** (unit is s^2/m or ps^2/km)

The dispersion parameter

Measuring the spectral width in units of wavelength (rather than frequency), we can write the broadening as

$$\Delta T = D\,\Delta\lambda\,L,$$

where D [ps/(nm km)] is called the ***dispersion parameter***

D is related to β_2 and the effective mode index according to

$$D = -\frac{2\pi c}{\lambda^2}\beta_2 = \left\{\beta_2 = \frac{d^2\beta}{d\omega^2}, v_g = \left(\frac{d\beta}{d\omega}\right)^{-1}\right\} = -\frac{2\pi c}{\lambda^2}\frac{d}{d\omega}\left(\frac{1}{v_g}\right) = -\frac{2\pi}{\lambda^2}\left(2\frac{d\bar{n}}{d\omega} + \omega\frac{d^2\bar{n}}{d\omega^2}\right)$$

The dispersion parameter has two contributions:

material dispersion, D_M: The index of refraction of the fiber material depends on the frequency

waveguide dispersion, D_W: The guided mode has a frequency dependence

Material dispersion (2.3.2)

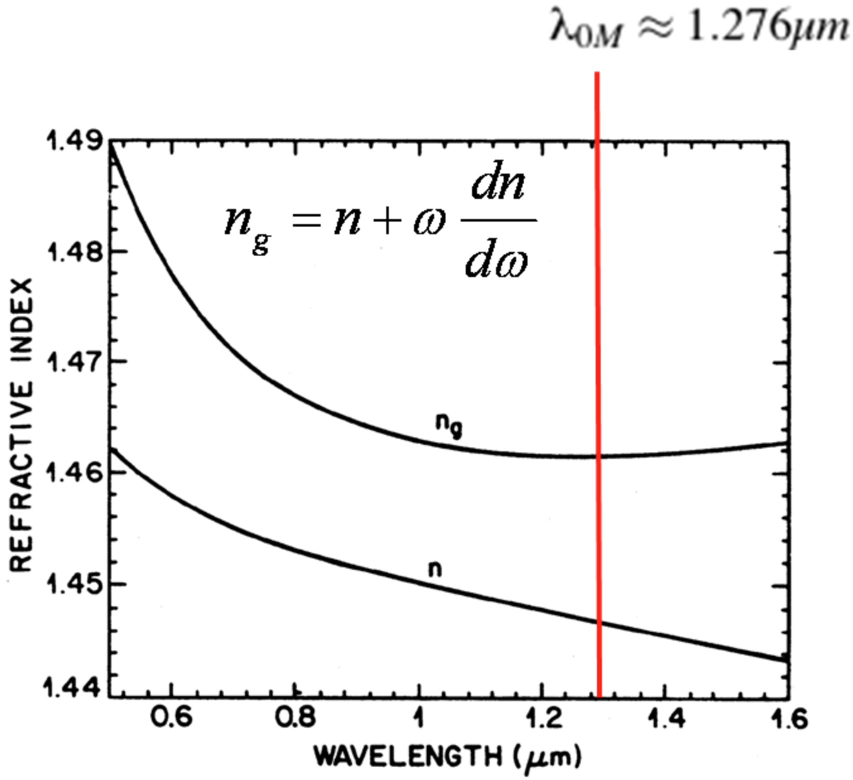
The material dispersion is related to the dependence of the cladding material's group index on the frequency

$$D_M = -\frac{2\pi}{\lambda^2}\frac{dn_{2g}}{d\omega}$$

An approximate relation for the material dispersion in silica is

$$D_M \approx 122\left(1 - \frac{\lambda_{0M}}{\lambda}\right)$$

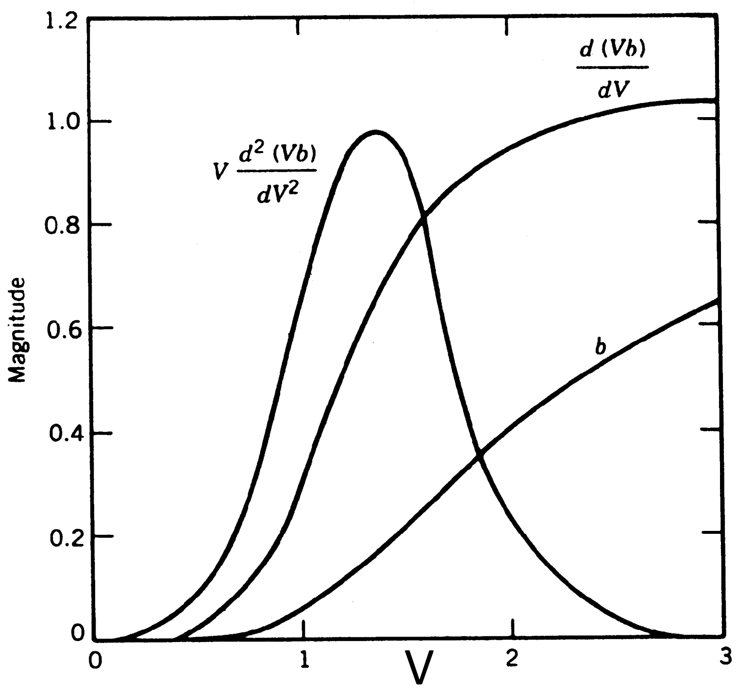
where D_M is given in ps/(nm km)



Waveguide dispersion (2.3.3)

The waveguide dispersion arises from the modes' dependence on frequency

$$D_W = -\frac{2\pi\Delta}{\lambda^2}\left[\frac{n_{2g}^2}{n_2\omega}\frac{Vd^2(Vb)}{dV^2} + \frac{dn_{2g}}{d\omega}\frac{d(Vb)}{dV}\right]$$



n_{2g}: the cladding group index

V: the normalized frequency

$$V = \frac{2\pi}{\lambda}a\sqrt{n_1^2 - n_2^2} \approx \frac{\omega}{c}an_1\sqrt{2\Delta}$$

b: the normalized waveguide index

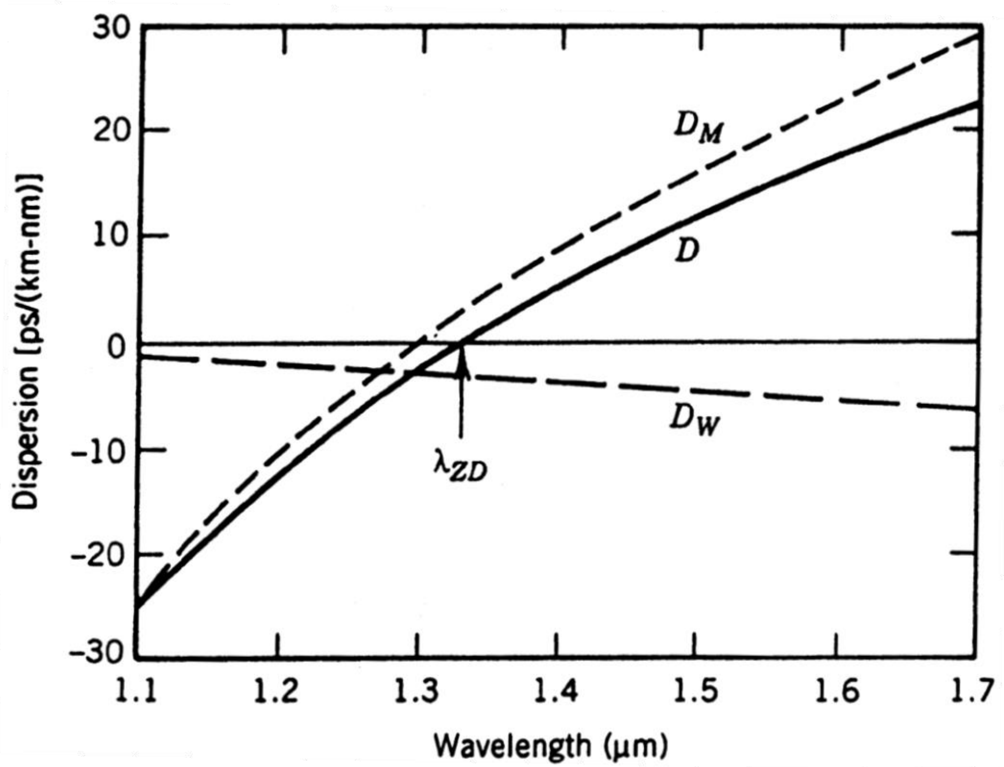
$$b = \frac{\bar{n} - n_2}{n_1 - n_2}$$

Total dispersion

The total dispersion D is the sum of the waveguide and material contributions

$$D = D_W + D_M$$

Note: D_W increases the net zero dispersion wavelength



The zero-dispersion wavelength is denoted either λ_0 or λ_{ZD}

An estimate of the dispersion-limited bit-rate is

$$|D| B \, \Delta\lambda \, L < 1$$

where B is the bit-rate, $\Delta\lambda$ the spectral width, and L the fiber length

Anomalous and normal dispersion

The dispersion can have different signs in a ***standard single-mode fiber*** (SMF)

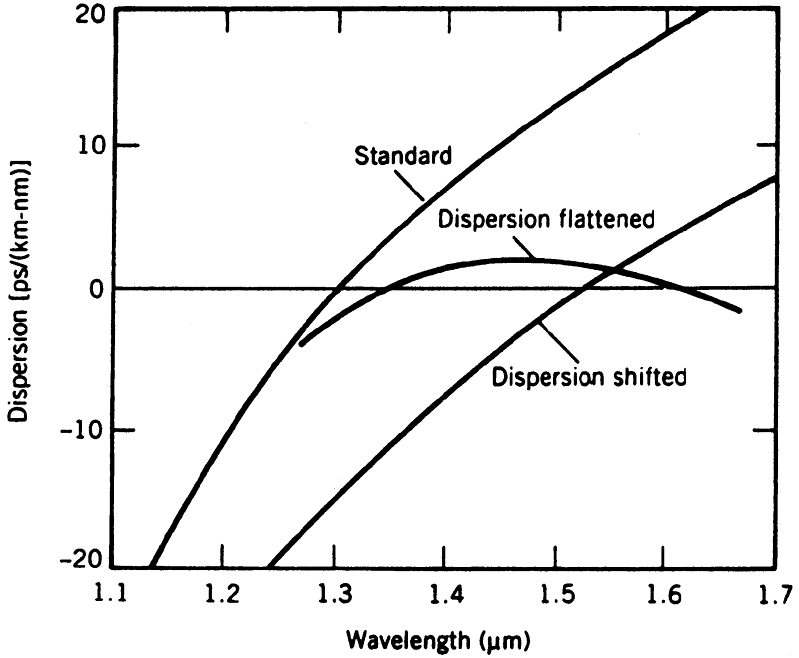
$D > 0$ for $\lambda > 1.31$ µm: ***"anomalous dispersion"***, the group velocity of higher frequencies is higher than for lower frequencies

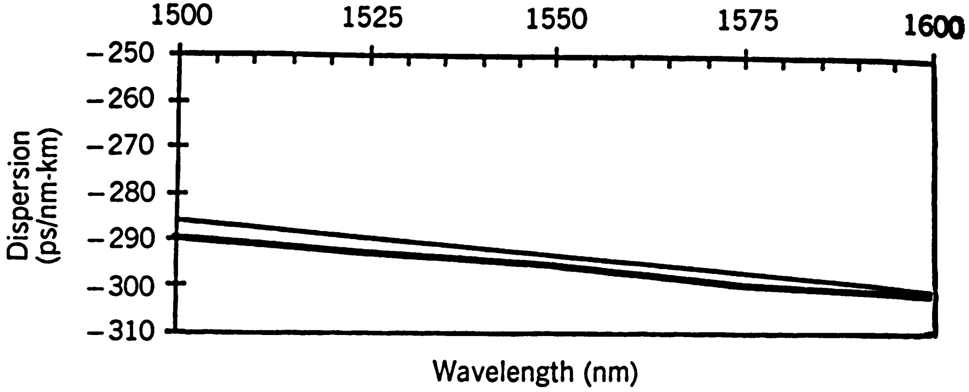
$D < 0$ for $\lambda < 1.31$ µm: ***"normal dispersion"***, the group velocity of higher frequencies is lower than for lower frequency components

Pulses are affected differently by nonlinear effects in these two cases

Different fiber types

- The fiber parameters can be tailored to shift the λ_0-wavelength from ≈1.3 µm to 1.55 µm, ***dispersion-shifted fiber*** (DSF)
- A fiber with small *D* over a wide spectral range (typically with two λ_0-wavelengths), ***dispersion-flattened fiber*** (DFF)
- A short fiber with large normal dispersion can compensate the dispersion in a long SMF, ***dispersion compensating fibers*** (DCF)

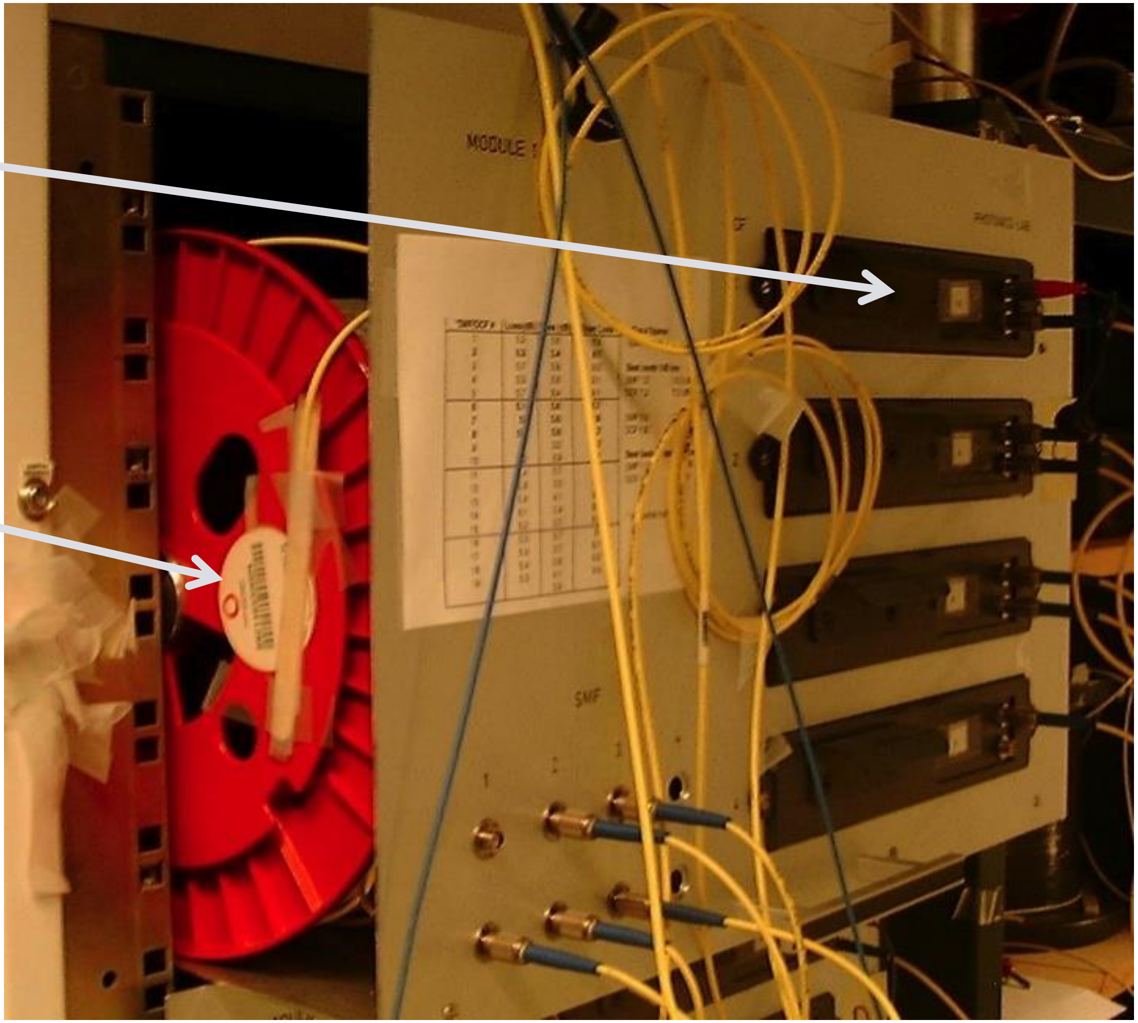




Fibers in the lab

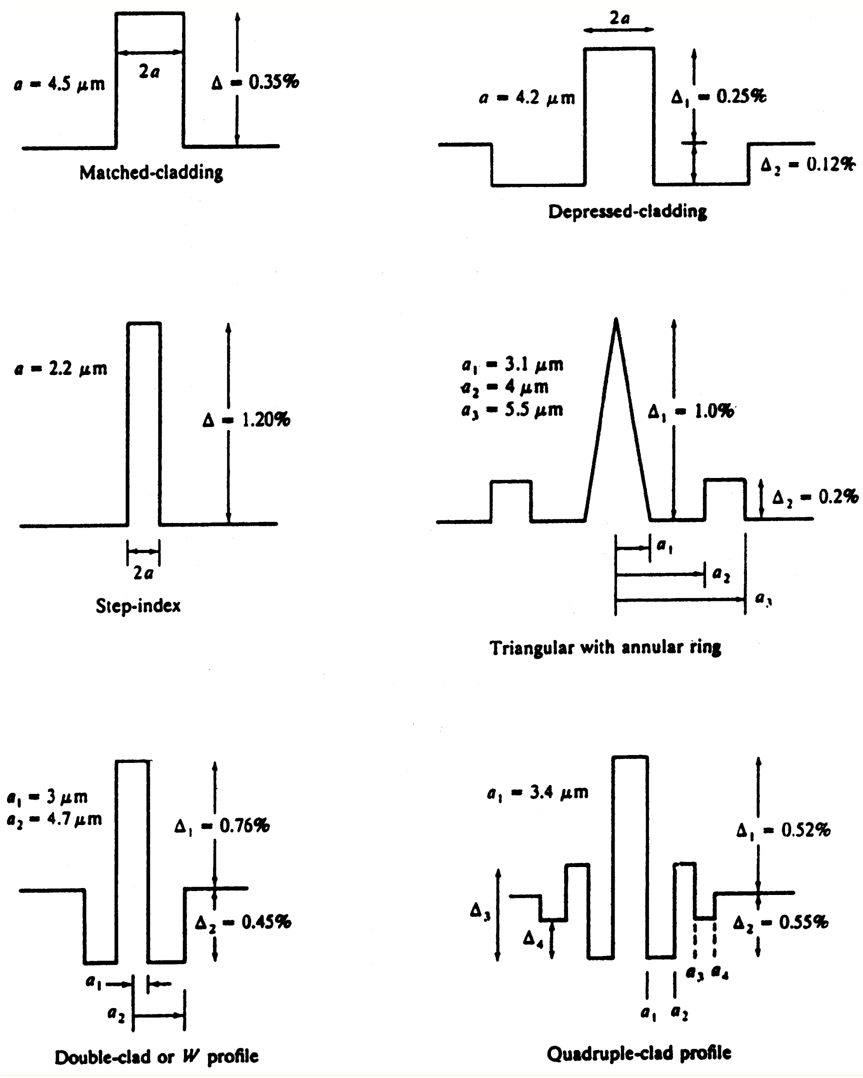
This dispersion compensating module contains 4 km of DCF...

...and it compensates the dispersion in this 25 km roll of SMF



Index profiles of different fiber types

- Standard single-mode fiber (SMF)
- Dispersion-shifted fiber (DSF)
- Dispersion-flattened fiber (DFF)

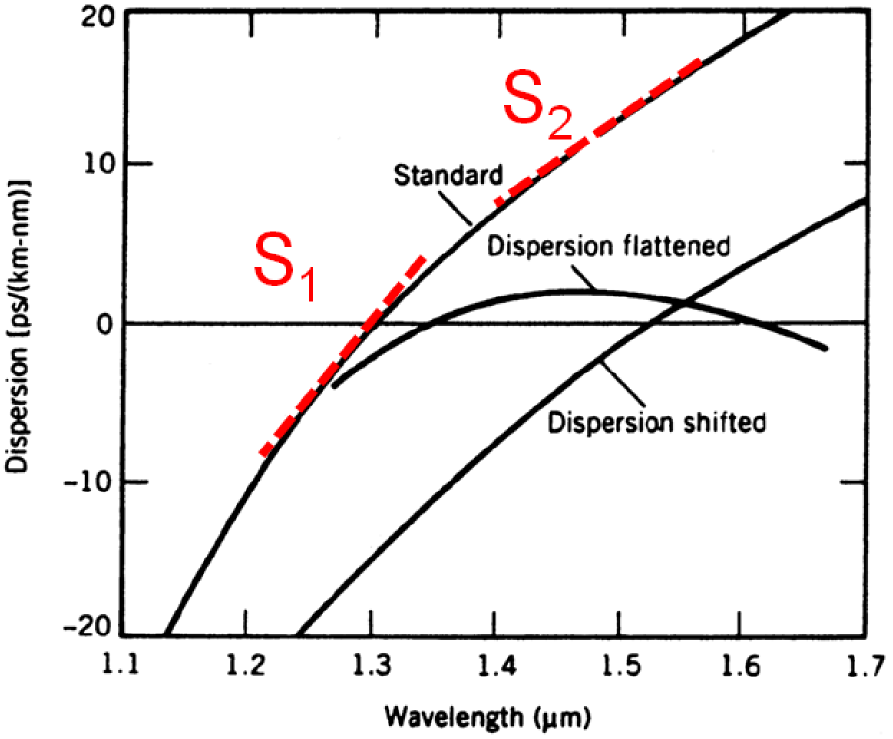


Higher order dispersion (2.3.4)

- Near the zero-dispersion wavelength $D \approx 0$
  - The variation of D with the wavelength must be accounted for

$$S \equiv \frac{dD}{d\lambda} = \left(\frac{2\pi c}{\lambda^2}\right)^2 \frac{d^3\beta}{d\omega^3}$$

  - We have used $\beta_2 = 0$
- S [ps/(nm^2 km)] is called the ***dispersion slope***
  - Typical value in SMF is 0.07 ps/(nm^2 km)

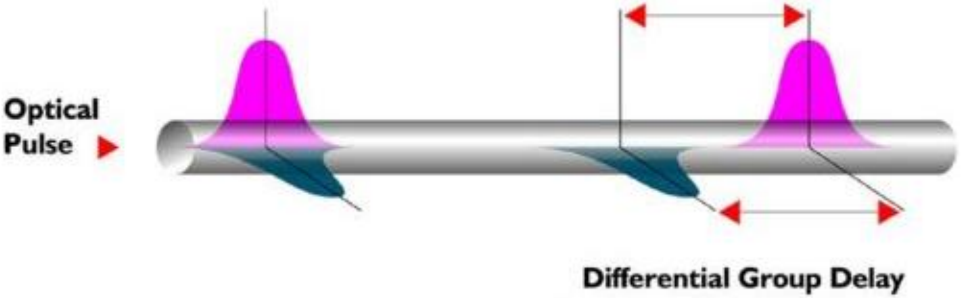


Polarization-mode dispersion (PMD) (2.3.5)

- An optical fiber is always slightly elliptical
  - Different index of refraction for *x* and *y* polarization, difference is Δ*n*
    - Changes along the fiber
    - Changes with frequency
    - Changes with time
- This leads to:
  - The state-of-polarization (SOP) of the signal in the fiber will be random
  - Different delay for different polarizations, ***differential group delay*** (DGD)

$$\Delta\tau = \left|\frac{L}{v_{gx}} - \frac{L}{v_{gy}}\right| = L\left|\beta_{1x} - \beta_{1y}\right| = L(\Delta\beta_1)$$

  - DGD acting along varying axes leads to ***polarization-mode dispersion*** (PMD)



$$\sigma_T \approx (\Delta\beta_1)\sqrt{2l_c z} = D_p\sqrt{L}$$

D_p is the PMD parameter (typical value is 0.05–1 ps/√km)

Basic propagation equation

We will now develop the theory for signal propagation in fibers

The electric field is written as

$$\mathbf{E}(\mathbf{r},t)=\mathrm{Re}\left[\hat{\mathbf{x}}\,F(x,y)A(z,t)\exp(i\beta_0 z-i\omega_0 t)\right]$$

- The field is polarized in the **x**-direction
- $F(x, y)$ describes the mode in the transverse directions
- $A(z, t)$ is the complex ***field envelope***
- β_0 is the propagation constant corresponding to ω_0

Only $A(z, t)$ changes upon propagation (described in the Fourier domain)

$$\tilde{A}(z,\omega)=\tilde{A}(0,\omega)\exp\left[i\beta(\omega)z-i\beta_0 z\right]$$

$$\left(\tilde{A}(z,\omega)=\int_{-\infty}^{\infty}A(z,t)\exp(i\omega t)\,d\omega\right)$$

Each spectral component of a pulse propagates differently

The propagation constant

- The propagation constant is in general complex

$$\beta(\omega)=[\bar{n}(\omega)+\delta n_{NL}(\omega)](\omega/c)+i\alpha(\omega)/2\approx\beta_L(\omega)+\beta_{NL}(\omega_0)+i\alpha(\omega_0)/2$$

  - α is the attenuation
  - δn_{NL} is a small nonlinear (= power dependent) change of the refractive index
- Dispersion arises from $\beta_L(\omega)$
  - The frequency dependence of β_{NL} and α is small
- We now expand $\beta_L(\omega)$ in a Taylor series around $\omega = \omega_0$ ($\Delta\omega = \omega - \omega_0$)

$$\beta_L(\omega)\approx\underbrace{\beta_0}_{1/v_p}+\underbrace{\beta_1}_{1/v_g}(\Delta\omega)+\underbrace{\frac{\beta_2}{2}}_{\text{GVD(rel. to } D)}(\Delta\omega)^2+\underbrace{\frac{\beta_3}{6}}_{\text{dispersion slope(related to } S)}(\Delta\omega)^3+\ldots,\quad \beta_m=\left.\frac{d^m\beta}{d\omega^m}\right|_{\omega=\omega_0}$$

Basic propagation equation (2.4.1)

Substitute β with the Taylor expansion in the expression for the evolution of $A(z, \omega)$, calculate ∂A/∂z, and write in time domain by using Δω↔ i ∂/∂t

$$\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} + i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} - \frac{\beta_3}{6}\frac{\partial^3 A}{\partial t^3} = i\beta_{NL} A - \frac{\alpha}{2} A$$

The nonlinearity is quantified by using $\delta n_{NL} = n_2 I$ where n_2 [m^2/W] is a measure of the strength of the nonlinearity, and I is the light intensity

$\beta_{NL} = \gamma |A|^2$, where $\gamma = 2\pi n_2/(\lambda_0 A_{\mathrm{eff}})$ is the nonlinear coefficient

A_{eff} is the effective mode area and $|A|^2$ is normalized to represent the power

γ is typically 1–20 W^{-1} km^{-1}

Basic propagation equation

- Use a coordinate system that moves with the pulse group velocity!
  - This is called ***retarded time,*** $t' = t - \beta_1 z$
  - We neglect β_3 to get

$$\frac{\partial A}{\partial z} + i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} = i\gamma|A|^2 A - \frac{\alpha}{2}A$$

- This is the ***nonlinear Schrödinger equation*** (NLSE)
  - The primes are implicit
- The loss reduces the power ⇒ reduces the impact from the nonlinearity
- The average power of the signal during propagation in the fiber is

$$P_{\text{av}}(z) = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}|A(z,t)|^2\,dt = P_{\text{av}}(0)e^{-\alpha z}$$

- Note: α is in m^{-1} while loss is often expressed in dB/km

Chirped Gaussian pulses (2.4.2)

- To study dispersion, we neglect nonlinearity and loss

$$\frac{\partial A}{\partial z} + i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} = 0$$

- The formal solution is

$$\tilde{A}(z,\omega) = \tilde{A}(0,\omega)\exp\left(i\frac{\beta_2}{2}\omega^2 z\right)$$

- Note: Dispersion acts like an all-pass filter
- We study chirped Gaussian pulses

$$A(0,t) = A_0 \exp\left[-\tfrac{1}{2}(1+iC)(t/T_0)^2\right]$$

  - A_0 is the peak amplitude
  - C is the chirp parameter
  - T_0 is the 1/e half width (power) $\quad T_{\text{FWHM}} = 2(\ln 2)^{1/2} T_0 \approx 1.665 T_0$

Chirp frequency

- For a chirped pulse, the frequency of the pulse changes with time
  - What does this mean???
- Study a CW (continuous wave)

$$\mathbf{E}(\mathbf{r},t)=\mathrm{Re}\left[\hat{\mathbf{x}}\,F(x,y)A(z,t)\exp(i\beta_0 z-i\omega_0 t)\right]$$

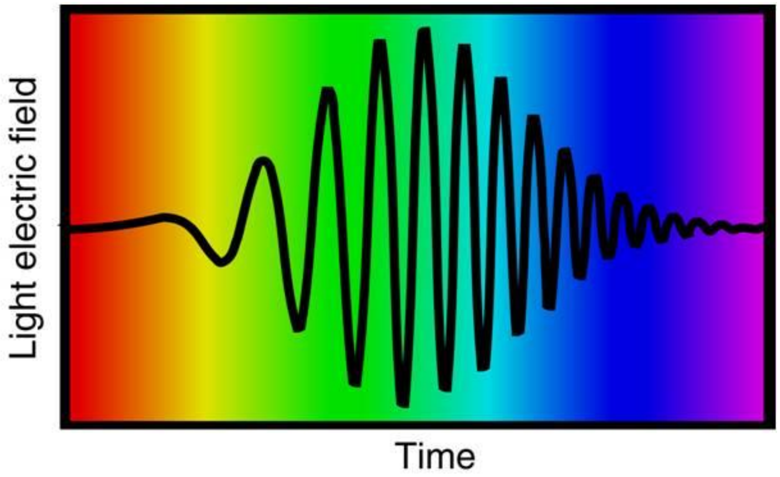
  - *A* is a constant
- Writing $A\exp(i\beta_0 z - i\omega_0 t) = A\exp(i\phi)$, we see that $\omega_0 = -\partial\phi/\partial t$
- We define the ***chirp frequency*** to be

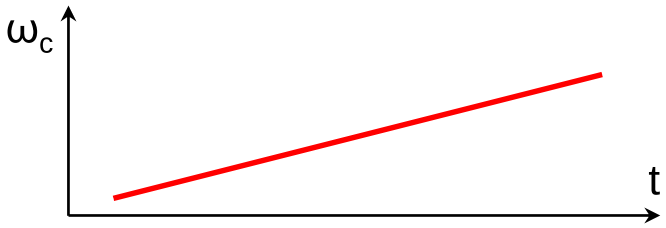
$$\boxed{\omega_c = -\partial\phi(t)/\partial t}$$

  - We allow ϕ to have a time dependence
  - We get ϕ from the complex amplitude
- In this way, the chirp frequency can depend on time
  - For the Gaussian pulse we get $\omega_c = Ct/T_0^2$

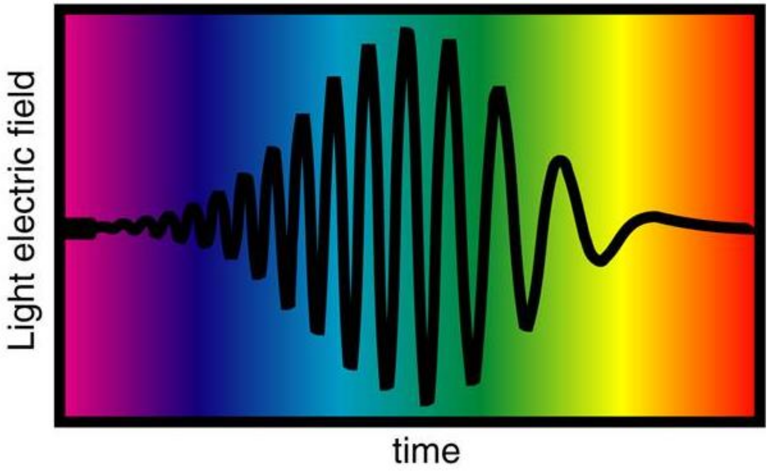
A linearly chirped pulse

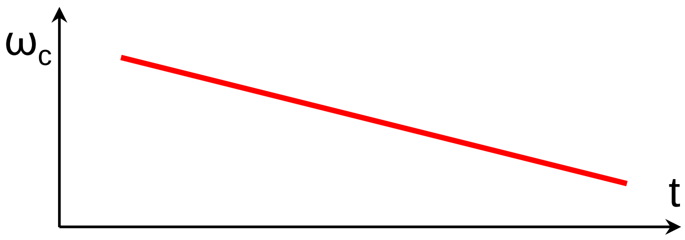
Frequency increases with time





Frequency decreases with time





Time-bandwidth product

The Fourier transform of the input Gaussian pulse is

$$\tilde{A}(0,\omega) = A_0 \left(\frac{2\pi T_0^2}{1+iC} \right)^{1/2} \exp\left[-\frac{\omega^2 T_0^2}{2(1+iC)} \right]$$

The 1/e spectral half width (intensity) is $\Delta\omega_0 = \sqrt{1+C^2} / T_0$

The product of the spectral and temporal widths is $\Delta\omega_0 T_0 = \sqrt{1+C^2}$

If C = 0 then the pulses are chirp-free and said to be ***transform-limited*** as they occupy the smallest possible spectral width

Using the full width at half maximum (FWHM), we get

$$\boxed{\Delta\nu_{\mathrm{FWHM}} T_{\mathrm{FWHM}} = \frac{2\ln 2}{\pi} \sqrt{1+C^2} \approx 0.44\sqrt{1+C^2}}$$

Chirped Gaussian pulses (2.4.2)

We introduce $\xi = z/L_D$ where the ***dispersion length*** $L_D = T_0^2/|\beta_2|$

In the time domain the dispersed pulse is

$$A(\xi,t) = \frac{A_0}{\sqrt{b_f}} \exp\left[-\frac{(1+iC_1)t^2}{2T_0^2 b_f^2} + \frac{i}{2}\arctan\left(\frac{\xi}{1+C\xi}\right)\right]$$

$$b_f(\xi) = \left[(1+sC\xi)^2 + \xi^2\right]^{1/2}$$

$$C_1(\xi) = C + s(1+C^2)\xi$$

$$s = \operatorname{sign}(\beta_2)$$

The output width (1/e-intensity point) broadens as

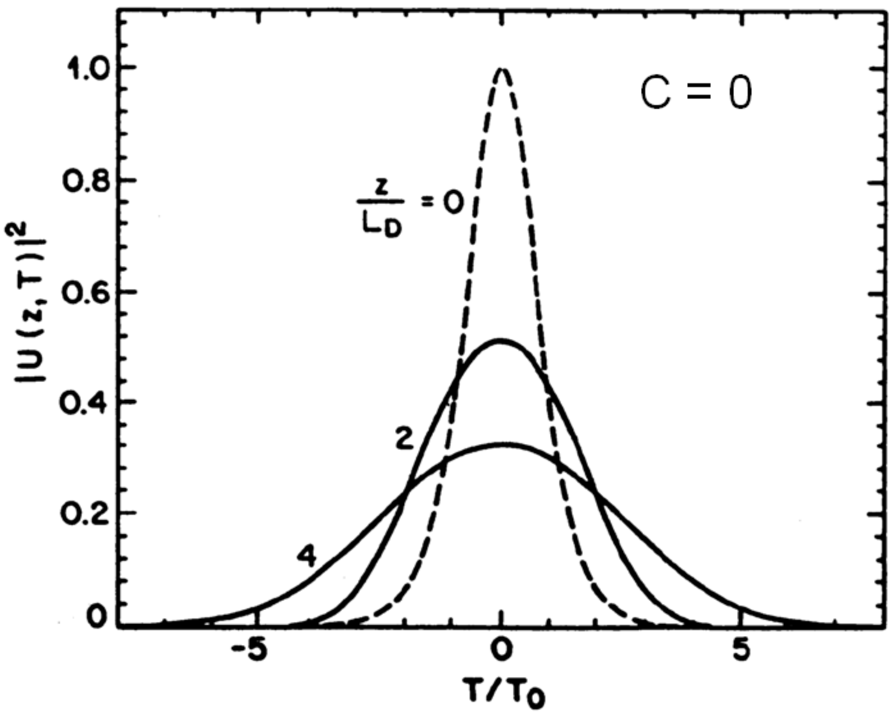
$$b_f(z) = \frac{T_1(z)}{T_0} = \left[\left(1+\frac{C\beta_2 z}{T_0^2}\right)^2 + \left(\frac{\beta_2 z}{T_0^2}\right)^2\right]^{1/2}$$

A Gaussian pulse remains Gaussian during propagation

The chirp, $C_1(\xi)$, evolves as the pulse propagates

If $(C\,\beta_2)$ is negative, the pulse will initially be compressed

Broadening of chirp-free Gaussian pulses

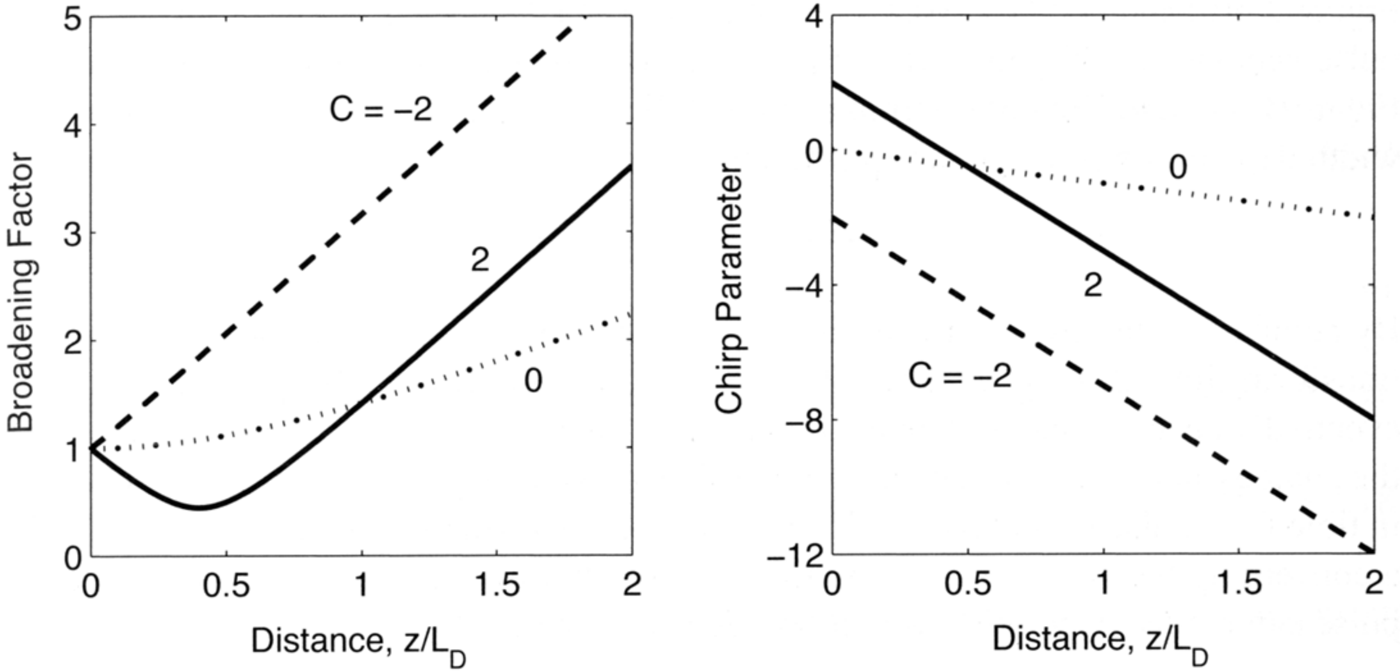


$$b_f(z) = \sqrt{1+\left(\frac{z}{L_D}\right)^2} = \sqrt{1+\left(\frac{|\beta_2|z}{T_0^2}\right)^2}$$

Short pulses broaden more quickly than longer pulses

(Compare with diffraction of beams)

Broadening of linearly chirped Gaussian pulses



For $(C\,\beta_2) < 0$, pulses initially compress and reaches a minimum at

$z = |C|/(1+C^2)L_D$ at which $C_1 = 0$ and $$T_1^{\min} = \frac{T_0}{\sqrt{1+C^2}} = \frac{1}{\Delta\omega_0}$$

Chirped pulses eventually broaden more quickly than unchirped pulses

Non-Gaussian pulses

Only Gaussian pulses remain Gaussian upon propagation

Not even Gaussian pulses remain Gaussian if β_3 cannot be ignored

In these cases, the RMS-pulse width can be used

$$\sigma_p = \sqrt{\langle t^2 \rangle - \langle t \rangle^2} \qquad \langle t^m \rangle = \frac{\int_{-\infty}^{\infty} t^m |A(z,t)|^2 dt}{\int_{-\infty}^{\infty} |A(z,t)|^2 dt}$$

σ_p can be calculated when the initial pulse is known

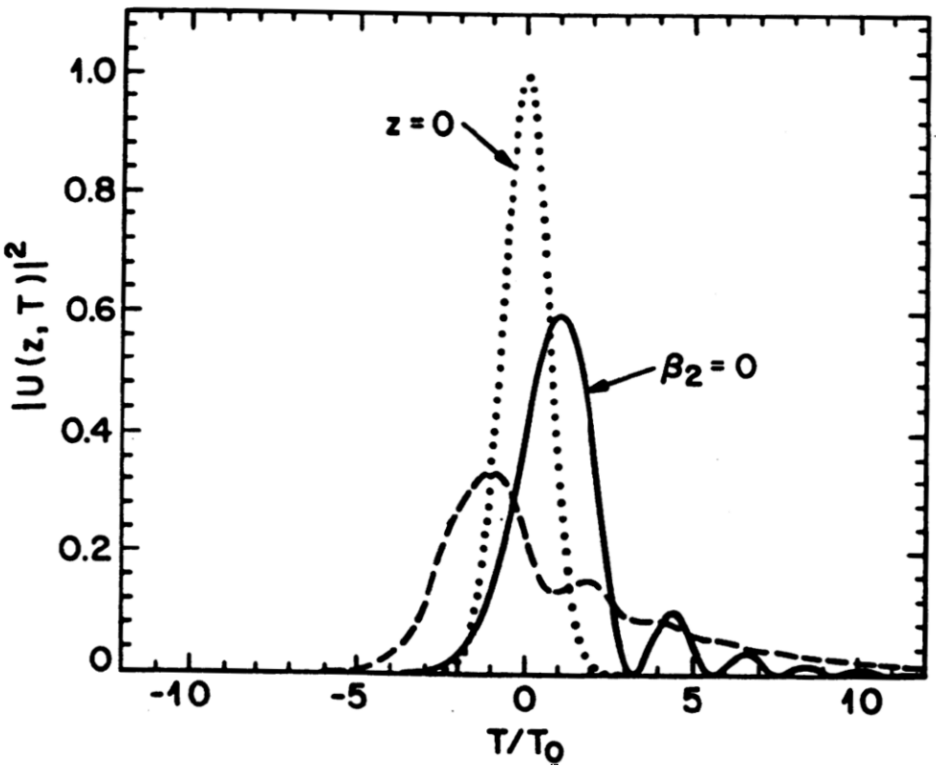
Example 1: An unchirped rectangular pulse will, in presence of only β_2, broaden as

$$\sigma_p^2 = \sigma_0^2 \left[1 + \frac{3}{2} \left(\frac{L}{L_D} \right)^2 \right]$$

Example 2: An unchirped Gaussian pulse will, in presence of only β_2, broaden as

$$\sigma_p^2 = \sigma_0^2 \left[1 + \left(\frac{L}{L_D} \right)^2 \right]$$

Chirped Gaussian pulses in the presence of β_3



Higher order dispersion gives rise to oscillations and pulse shape changes

$$\frac{\sigma^2}{\sigma_0^2} = \left(1 + \frac{C\beta_2 L}{2\sigma_0^2}\right)^2 + \left(\frac{\beta_2 L}{2\sigma_0^2}\right)^2 + \left(\frac{\beta_3 L(1+C^2)}{4\sqrt{2}\sigma_0^3}\right)^2 \qquad \sigma_0 = T_0/\sqrt{2}$$

Effect from source spectrum width

Using a light source with a broad spectrum leads to strong dispersive broadening of the signal pulses

In practice, this only needs to be considered when the source spectral width approaches the symbol rate

For a Gaussian-shaped source spectrum with RMS-width σ_ω and with Gaussian pulses, we have

$$\frac{\sigma_p^2}{\sigma_0^2} = \left(1 + \frac{C\beta_2 L}{2\sigma_0^2}\right)^2 + (1 + V_\omega^2)\left(\frac{\beta_2 L}{2\sigma_0^2}\right)^2 + (1 + C^2 + V_\omega^2)^2\left(\frac{\beta_3 L}{4\sqrt{2}\sigma_0^3}\right)^2$$

where $V_\omega = 2\sigma_\omega\sigma_0$

V_ω << 1 when the source spectral width << the signal spectral width

Limitations on bit rate, incoherent source (2.4.3)

If, as for an LED light source, $V_\omega >> 1$ we obtain approximately

$$\sigma^2 = \sigma_0^2 + (\beta_2 L \sigma_\omega)^2 \equiv \sigma_0^2 + (DL\sigma_\lambda)^2$$

A common criteria for the bit rate is that $\sigma \le T_B / 4 = 1/(4B)$

For the Gaussian pulse, this means that 95% of the pulse energy remains within the bit slot

In the limit of large broadening $\boxed{4BL|D|\sigma_\lambda \le 1}$

σ_λ is the source RMS width in wavelength units

Example: D = 17 ps/(km nm), σ_λ = 15 nm $\Rightarrow$ $(BL)_{max} \approx 1$ (Gbit/s) km

Limitations on bit rate, incoherent source

In the case of operation at $\lambda = \lambda_{ZD}$, $\beta_2 = 0$ we have

$$\sigma^2 = \sigma_0^2 + \tfrac{1}{2}(\beta_3 L \sigma_\omega^2)^2 \equiv \sigma_0^2 + \tfrac{1}{2}(S L \sigma_\lambda^2)^2$$

With the same condition on the pulse broadening, we obtain

$$\boxed{\sqrt{8} B L |S| \sigma_\lambda^2 \leq 1}$$

The dispersion slope, *S*, will determine the bit rate-distance product

Example: $D = 0$, $S = 0.08$ ps/(km nm^2), $\sigma_\lambda = 15$ nm $\Rightarrow (BL)_{max} \approx 20$ (Gbit/s) km

Limitations on bit rate, coherent source (2.4.3)

For most lasers V_ω << 1 and can be neglected and the criteria become

Neglecting β_3: $$\sigma^2 = \sigma_0^2 + (\beta_2 L / 2\sigma_0)^2 \equiv \sigma_0^2 + \sigma_D^2$$

The output pulse width is minimized for a certain input pulse width giving

$$4B\sqrt{|\beta_2| L} \leq 1$$

Example: β_2 = 20 ps^2/km → $(B^2L)_{max}$ ≈ 3000 (Gbit/s)^2 km

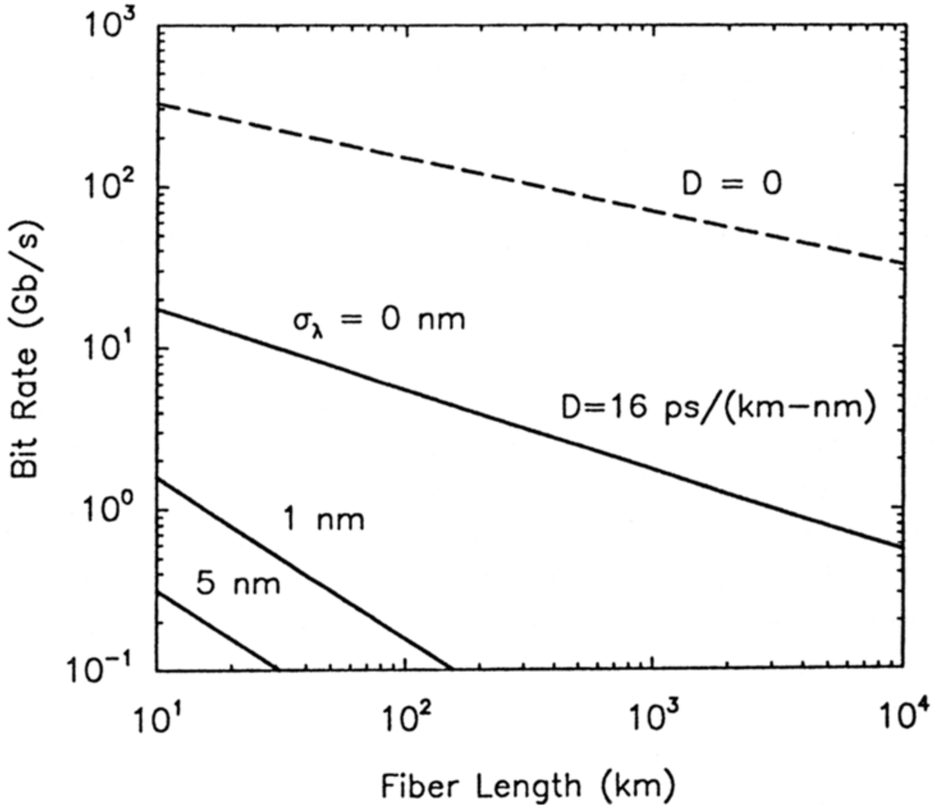
500 km @ 2.5 Gbit/s, 30 km @ 10 Gbit/s

If β_2 = 0 (close to λ_0): $$\sigma^2 = \sigma_0^2 + (\beta_3 L / 4\sigma_0^2)^2 / 2 \equiv \sigma_0^2 + \sigma_D^2$$

For an optimal input pulse width, we get

$$B(|\beta_3| L)^{1/3} \leq 0.324$$

Limitations on bit rate, summary



A coherent source improves the bit rate-distance product

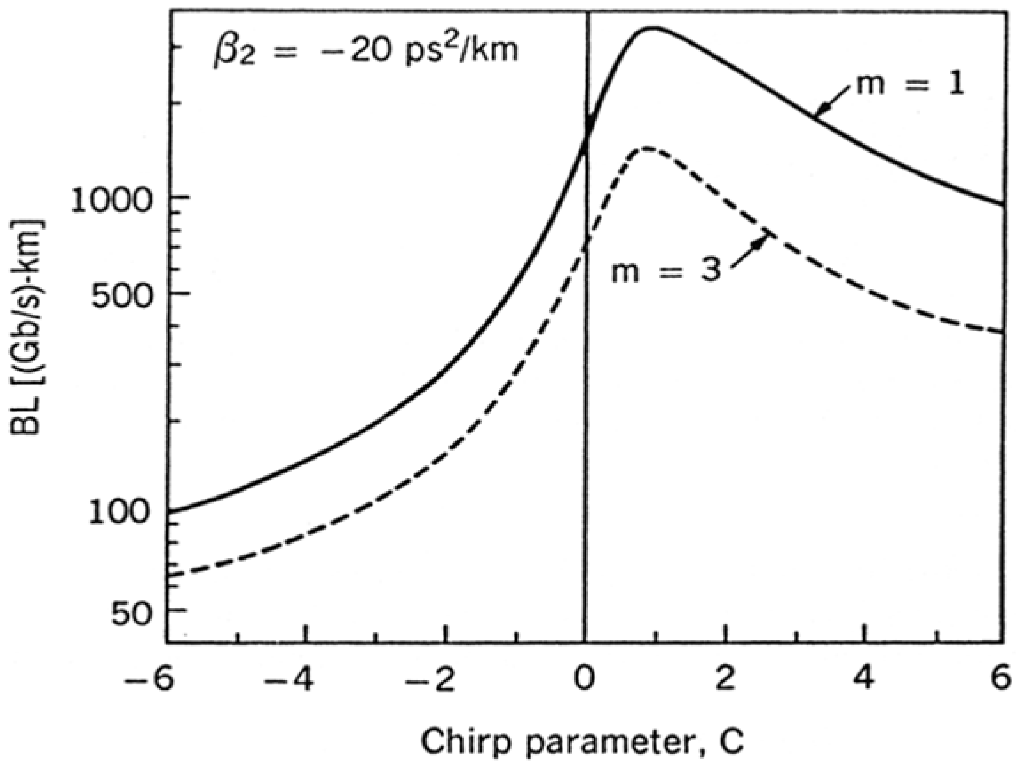
Operation near the zero-dispersion wavelength also is beneficial…

...but may lead to problems with nonlinear signal distortion

Super-Gaussian pulses

- Super-Gaussian pulses are flat-top and can be used to model NRZ…
  - …but more accurate modeling is preferred
  - If m = 1 ⇒ We recover the Gaussian
  - If m > 1 ⇒ The shape is more rectangular

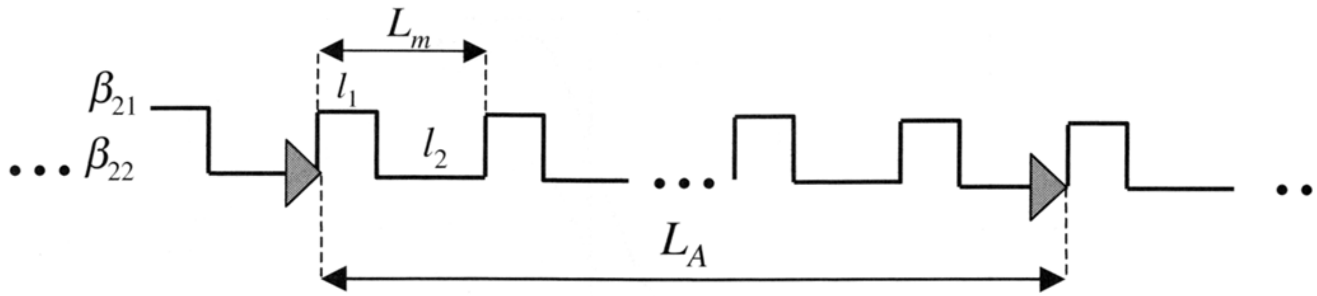
$$A(0,t) = A_0 \exp\left[-\frac{1+iC}{2}\left(\frac{t}{T_0}\right)^{2m}\right]$$



Numerical simulation shows:

- Super-Gaussian has:
  - Smaller bit rate-distance product…
  - …due to more problems with dispersion...
  - …due to the sharper edges
- A small chirp can be beneficial:
  - $C \approx 1$ is optimal

Dispersion compensation



- Dispersion is a key limiting factor for an optical transmission system
- Several ways to compensate for the dispersion exist
  - More about this in a later lecture...
- One way is to periodically insert fiber with opposite sign of *D*
  - This is called ***dispersion-compensating fiber*** (DCF)
  - Figure shows a system with both SMF and DCF
  - The GVD parameters are β_{21} and β_{22}
- Group-velocity dispersion is perfectly compensated when

  $\beta_{21}l_1 + \beta_{22}l_2 = 0$, which is equivalent to $D_1 l_1 + D_2 l_2 = 0$
- GVD and PMD can also be compensated in digital signal processing (DSP)

Fiber losses (2.5)

- Fiber have low loss but the loss grows exponentially with distance
  - Approx. 20–25 dB loss over 100 km
  - Optical receivers add noise...
  - ...and the input power may be too low to obtain sufficient SNR
- The optical power in a fiber decreases exponentially with the propagation distance as $P_{out} = P_{in} \exp(-\alpha z)$
  - α is the attenuation coefficient (unit m^{-1})
- Often, attenuation is given in dB/km and its relation to α is

$$\alpha_{\mathrm{dB}} = -\frac{1}{L} 10 \log_{10} e^{-\alpha L} = -\frac{10}{L} \frac{\log e^{-\alpha L}}{\log 10} = \frac{10}{\log 10} \alpha \approx 4.343\alpha$$

- Typical value in SMF at 1550 nm α_{dB} = 0.2 dB/km ⇒ $\alpha = 0.046\ \mathrm{km}^{-1} = 1/(21.7\ \mathrm{km})$

Attenuation mechanisms

- Material absorption
  - Intrinsic absorption: In the SiO_2 material
    - Electronic transitions (UV absorption)
    - Vibrational transitions (IR absorption)
  - Extrinsic: Due to impurity atoms
    - Metal and OH^- ions, dopants
- Rayleigh scattering
  - Occurs when waves scatter off small, randomly oriented particles
  - (Makes the sky blue!)
  - Proportional to λ^{-4}
- Waveguide imperfections
  - Core-cladding imperfections on > λ length scales (Mie scattering)
  - Micro-bending (bending curvature ~λ)
  - Macro-bending (negligible unless bending curvature < 1–5 mm)

Total attenuation

- Minimum theoretical loss is 0.15 dB/km at 1550 nm
- Main contributions: Rayleigh scattering and IR absorption
- Left figure: Theoretical curves and measured loss for typical fiber
- Right figure: Loss for sophisticated fiber with negligible loss peak