The Performance of Reed-Solomon Codes on a Bursty-Noise Channel

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Abstract - **The performance of a Reed-Solomon coded binary communication system on a bursty-noise channel is considered. Bursty noise is defined to be background Gaussian noise plus burst noise, where burst noise is defined to be a series of finite-duration Gaussian-noise pulses with fixed duration and Poisson occurrence times. Using the noise model, along with ideal symbol interleaving, decoded bit-error probability bounds are derived for the case where the noise bursts are long with respect to the channel symbol rate. Specific performance results are presented for the (31,15,8) Reed-Solomon Joint Tactical Distribution System (JTIDS) code with a Binary Phase Shift Keyed (BPSK) modulation scheme. Simulation results are presented and they compare well with the theoretical results.**

I. INTRODUCTION

In recent years, Reed-Solomon (RS) codes have become popular in part because they have a well understood algebraic structure which has allowed efficient decoder implementations [1]. They are also maximum distance separable (MDS) codes which mean they are the most powerful linear codes for their class. Their popularity is also due to the fact that RS codes are non-binary so they can correct bursts of binary channel errors. Error bursts can occur in military environments [2], satellite systems [3], concatenated coding schemes [4,5], and from optical storage media such as compact discs [6].

There are numerous references describing the performance of RS codes in AWGN environments, see for example [7,8]. In this paper, the theoretical performance of RS codes used with a bursty-noise channel is considered. Recently, Berman and Freedman considered the performance of RS codes with a bursty-error channel, modeled by a Markov chain, both with and without ideal interleaving [9,10]. Their analysis assumes that each RS symbol is either in a burst state or in a no-burst state and they allow the occurrence of bursts up to a specified maximum length. Approximations for the bit error rate (BER) at the RS decoder output are given.

Channels exhibiting error bursts have been modeled by Markov processes [9-14], by impulse noise [15-18], and by bursty noise [2,19-20]. The bursty noise model, as defined in this paper, allows for arbitrary rate of noise burst occurrence, arbitrary but fixed burst length, arbitrary error probability during a burst, and arbitrary error probability when no bursts occur. For example, the Tracking and Data Relay Satellite (TDRS) suffers from Radio Frequency Interference (RFI) characterized by randomly occurring noise bursts of fixed duration [3]. These results are easily generalized to noise bursts of varying duration as discussed in Section V.

The system under study, shown in Fig. 1, consists of an RS encoder and decoder, ideal code symbol interleaver and deinterleaver, binary modulator and demodulator, and a bursty-noise channel. Of interest is the decoded bit-error probability. The (n,k,t) RS encoder groups m bits per code symbol and has blocklength $n = 2^m - 1$. The RS decoder is assumed to be an errors-only, incomplete decoder. In the subsequent development, the undetected word error probability, which occurs when channel errors cause the received codeword to be mapped into a Hamming sphere about the wrong codeword, is neglected. This has been shown by McEliece and Swanson [21] to be upper bounded by $P_E \le 1/t!$ for Reed-Solomon codes. For example, the $(31,15,8)$ RS JTIDS code gives $P_E \le 3\times10^{-5}$ and the $(255,223,16)$ RS NASA code gives $P_E \le 3\times10^{-5}$, regardless of channel quality.

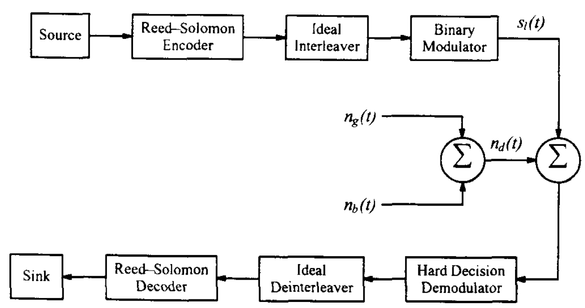

Fig. 1. System configuration.

In the following section, the bursty-noise model is described, and in Section III the uncoded system is considered. In Sections IV and V, bit-error probability bounds are derived for RS codes operating over a bursty-noise channel. In Section VI, results for a binary phase shift keyed (BPSK) modulation scheme are presented and compared with simulation results. Conclusions follow in Section VII.

II. BURSTY-NOISE MODEL DESCRIPTION

As shown in Fig. 1, the receiver input is $s_l(t) + n_d(t)$ where $s_l(t)$ is the binary signal component in the l^{th} signaling interval and $n_d(t)$ is the noise component. The noise is given by

$$n_d(t) = n_g(t) + n_b(t) \tag{1}$$

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where $n_g(t)$ is the background Gaussian-noise component and $n_b(t)$ is the burst-noise component. The combination of the background Gaussian noise and burst noise is referred to as *bursty noise.*

In order to describe the burst-noise component of the channel noise, let $a(t)$ denote a sample function from a delta-correlated Gaussian stochastic process with zero mean and double-sided power spectral density (PSD) $N_b/2$ and let $\{t_i\}$ denote a set of Poisson points with average rate ν. The burst noise component is expressed as

$$n_b(t) = a(t) \sum_{i=-\infty}^{\infty} \Pi\left(\frac{t-t_i-d/2}{d}\right) \tag{2}$$

where $\Pi(t/d)$ is defined to be a unit-amplitude pulse of width d centered at $t = 0$. When two pulses overlap, the stochastic process is doubled in amplitude in the overlapping interval. In (2), d is the time duration of each Gaussian-noise burst and t_i is the time at which the burst begins. The double-sided PSD for burst noise is

$$S_b(f) = \nu d(N_b/2), \quad -\infty < f < \infty, \tag{3}$$

and is easily derived via the autocorrelation function and the Wiener-Khinchine theorem. Since the PSD for burst noise is constant, the process is white.

The background Gaussian-noise component, $n_g(t)$, is assumed to be zero-mean and delta correlated with double-sided PSD

$$S_g(f) = N_g/2, \quad -\infty < f < \infty. \tag{4}$$

It is assumed that $n_g(t)$ and $a(t)$ are uncorrelated.

Given these descriptions for Gaussian noise and burst noise, it is clear that bursty noise is characterized by Gaussian noise which contains bursts of larger variance Gaussian noise. Only four parameters are required to completely describe bursty noise; the mean burst rate ν, the burst duration d, the single-sided PSD for the Gaussian noise N_g, and the single-sided PSD for the burst noise N_b.

We now define new bursty-noise parameters which make performance trends easier to illustrate. Since $n_g(t)$ and $a(t)$ are uncorrelated, the double-sided PSD for bursty noise is

$$S_g(f) + S_b(f) = (N_g + N_b \nu d)/2 \equiv N_t/2. \tag{5}$$

The fraction of bursty noise is defined as

$$\psi = \frac{S_b(f)}{S_g(f) + S_b(f)}. \tag{6}$$

This parameter is useful because it allows the limiting cases of bursty noise to be considered. For example, $S_b(f) = 0$ yields $\psi = 0$, which corresponds to a Gaussian-noise channel, and $S_g(f) = 0$ yields $\psi = 1$, which corresponds to a burst-noise channel. It is an easy matter to determine N_t and ψ from ν, d, N_g, and N_b. Together, ν, d, N_t, and ψ are the only parameters required to completely describe bursty noise and will be used for all subsequent developments. Since burst locations are given by a Poisson distribution, bursty noise is a stationary stochastic process.

III. UNCODED SYSTEM PERFORMANCE

The binary modulator outputs one of two signals, $s_0(t)$ or $s_1(t)$, over a time interval I of duration T_s. The bursty-noise channel can cause one of four possible events to take place as shown in Fig. 2; ξ_g when no noise burst overlaps I, ξ_p when a single noise burst partially overlaps I, ξ_b when a single noise burst completely overlaps I, and ξ_o when more than one noise burst partially or fully overlaps I. Since these events are mutually exclusive and exhaustive,

$$\begin{aligned} P(\varepsilon) = {} & P(\varepsilon|\xi_g)P(\xi_g) + P(\varepsilon|\xi_b)P(\xi_b) \\ & + P(\varepsilon|\xi_p)P(\xi_p) + P(\varepsilon|\xi_o)P(\xi_o) \end{aligned} \tag{7}$$

where ε denotes a binary symbol error. If $\nu d \ll 1$, $P(\xi_o)$ is negligibly small and if $d \gg T_s$, $P(\xi_p)$ is negligibly small so that

$$P(\varepsilon) \approx P(\varepsilon|\xi_g)P(\xi_g) + P(\varepsilon|\xi_b)P(\xi_b). \tag{8}$$

Furthermore, it is easily shown that $P(\xi_b) \approx \nu d$ and $P(\xi_g) \approx 1 - \nu d$ for $\nu d \ll 1$. These along with the conditional channel bit-error probabilities $P(\varepsilon|\xi_g)$ and $P(\varepsilon|\xi_b)$ are the only factors in (8) required to compute the uncoded error probability.

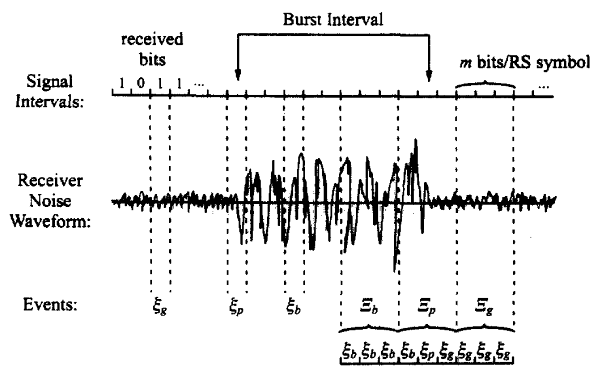


Fig. 2. Illustration of the possible noise events which can take place at the receiver

The relationship between the SNR measured at the receiver, E_b/N, and the error probability for BPSK in an AWGN environment is the well known result

$$P(\varepsilon) = Q\left(\sqrt{\frac{2E_b}{N}}\right) \tag{9}$$

where N is the single-sided noise PSD and where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\lambda^2/2}\, d\lambda. \tag{10}$$

When ξ_g occurs for a received binary symbol, $N = N_g$ by definition. Combining this with (3) through (6) yields $N = (1 - \psi)N_t$ and

$$P(\varepsilon|\xi_g) = Q\left(\sqrt{\frac{2}{(1-\psi)}\left(\frac{E_b}{N_t}\right)}\right). \tag{11}$$

When ξ_b occurs for a received binary symbol, $N = N_g + N_b$. Again combining with (3) through (6) yields

$$P(\varepsilon|\xi_b) = Q\left(\sqrt{\frac{2}{(1-\psi+\psi/\nu d)}\left(\frac{E_b}{N_t}\right)}\right). \tag{12}$$

We now turn our attention to the bursty-noise channel.

IV. IDEAL BURSTY NOISE

Before presenting the bounds in Section V, the decoded bit-error probability for a bursty-noise channel with ideal characteristics is derived. As shown in Fig. 2, which is drawn for $m = 3$, Ξ_g denotes the event that ξ_g occurs for all m binary symbols grouped to form a code symbol at the receiver. Similarly, Ξ_b denotes the event that ξ_b occurs for all m binary symbols of a code symbol and Ξ_p denotes the event that neither Ξ_g nor Ξ_b occurs for a code symbol. In this section, the burst duration is restricted to be

$$d = p\,T_{RS} = p\,m\,T_s \tag{13}$$

where p is a positive integer, T_{RS} is the time duration of an RS code symbol, and T_s is the time duration of a binary symbol. Furthermore, the noise burst locations, $\{t_i\}$, are restricted to lie on code symbol boundaries at the receiver. With these restrictions, it is not possible for Ξ_p to occur and noise bursts are always synchronized with code symbol boundaries. The restrictions on d as well as the set $\{t_i\}$ define *ideal bursty-noise.* While these artificial restrictions will not occur in a real world channel, they allow an analysis to be performed that gives rise to a useful bounding technique.

A. Performance in AWGN

Before deriving the decoded bit-error probability for ideal bursty-noise, the decoded bit-error probability on an AWGN channel is first derived. Let w_i denote the event corresponding to i binary symbol errors in a received codeword and let W_j denote the event corresponding to j code symbol errors in a received codeword. If event W_j occurs, then the received codeword must contain $j \le i \le mj$ binary symbol errors. If $i = j$, then each code symbol error must result from a single binary symbol error among its component binary symbols. Similarly, if $i = mj$ then each code symbol error must result from exactly m binary symbol errors.

Let ε denote a binary symbol error in the received codeword. Since the w_i for $0 \le i \le mn$ are mutually exclusive and exhaustive and the W_j for $0 \le j \le n$ are mutually exclusive and exhaustive, the theorem of total probability gives

$$P(\varepsilon) = \sum_{j=0}^{n} \sum_{i=0}^{nm} P(\varepsilon, w_i, W_j). \tag{14}$$

Clearly, $P(\varepsilon, w_i, W_j) = 0$ for $i < j$ or $i > mj$.

An incomplete (bounded distance) decoder detects and corrects t or fewer code symbol errors per received codeword. If more than t symbol errors occur in a received codeword and the undetected word error probability is neglected, then the errors are not changed by the decoder. Therefore, the binary symbol error probability after decoding is given by

$$P(\varepsilon_d) = \sum_{j=t+1}^{n} \sum_{i=j}^{mj} P(\varepsilon, w_i, W_j) \tag{15}$$

where ε_d denotes an error at the decoder output. Breaking down the summand using conditional probabilities gives

$$P(\varepsilon_d) = \sum_{j=t+1}^{n} \sum_{i=j}^{mj} P(\varepsilon|w_i, W_j) P(W_j|w_i) P(w_i). \tag{16}$$

Each factor of the summand is determined as described below.

The first factor in (16) represents the binary symbol error probability given that i binary symbol errors and j code symbol errors occur within a received codeword. Clearly, this must be

$$P(\varepsilon|w_i, W_j) = \frac{i}{mn}. \tag{17}$$

The third factor in (16) is only slightly more involved. Since the channel bit errors are assumed to be independent, the distribution of w_i is

$$P(w_i) = \binom{mn}{i} P^i(\varepsilon)[1-P(\varepsilon)]^{mn-i} \equiv \beta[mn, i, P(\varepsilon)] \tag{18}$$

where $\beta(\cdot,\cdot,\cdot)$ denotes the binomial distribution. The second factor in (16) is more difficult to determine and the derivation is relegated to the Appendix. The result is

$$P(W_j|w_i) = \frac{\binom{n}{j}}{\binom{mn}{i}} \sum_{q=0}^{r} (-1)^q \binom{j}{q} \binom{m(j-q)}{i} \tag{19}$$

where

$$r = \left\lfloor j - \frac{i}{m} \right\rfloor \tag{20}$$

with $\lfloor x \rfloor$ denoting the largest integer not greater than x. Substituting (17), (18), and (19) into (16) gives a new expression for the decoded bit-error probability for an RS code, using incomplete decoding, with a binary communication system operating over an AWGN channel. This result is exact subject only to the assumption of negligible undetected word error probability.

B. Performance in Ideal Bursty Noise

In this section, the RS decoded bit-error probability for an ideal bursty-noise channel is derived. Let Ξ_i denote the event that Ξ_b occurs for exactly i code symbols in a received codeword. Following a development similar to that given in Section A above, the decoded bit-error probability is

$$P(\varepsilon_d) = \sum_{i=0}^{n} P(\varepsilon_d|\Xi_i) P(\Xi_i). \tag{21}$$

Ideal symbol interleaving is assumed so that the code symbol errors at the decoder input are independent. It is easily shown, subject to $\nu d \ll 1$, that $P(\Xi_b) \approx \nu d$. Therefore, using the independence assumption, $P(\Xi_i) = \beta(n, i, \nu d)$. The first factor in (21) is described next.

Conditioned on Ξ_i, we define the burst subblock to be those i code symbols for which Ξ_b occurs and we define the Gaussian subblock to be those $n - i$ code symbols for which Ξ_g occurs, as shown in Fig. 3 for the received codeword at the decoder input. Let B represent the number of symbol errors in the burst subblock and let G represent the number of symbol errors in the Gaussian subblock. Similarly, let b represent the number of binary symbol errors in the burst subblock and let g represent the number of binary symbol errors in the Gaussian subblock. In this paper, B, G, b, and g represent random variables as well as sample values they may assume. The context will resolve the ambiguity.

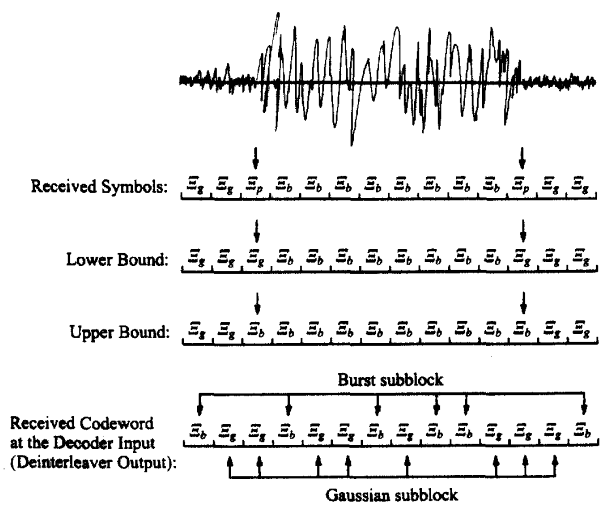


Fig. 3. Illustration of event substitutions for the lower bound and upper bound along with definitions of Gaussian and burst subblocks.

Given the event Ξ_i, it must be true that $B \le i$, $b \le m\,i$, $G \le n - i$, and $g \le m(n-i)$ errors occur. For a t-error correcting RS code, $P(\varepsilon_d|\Xi_i) = 0$ for $B + G \le t$. Therefore, the first factor in (21) can be expressed as

$$P(\varepsilon_d|\Xi_i) = \sum_{G=v}^{n-i} \sum_{B=u}^{i} P(\varepsilon|B,G,\Xi_i) \tag{22}$$

where $v = \max(0, t+1-i)$ and $u = \max(0, t+1-G)$. Recognizing that B and G are independent due to the fact that the channel noise is white, the summand in (22) can be expressed in terms of b and g by

$$P(\varepsilon|B,G,\Xi_i) = \sum_{g=G}^{mG} \sum_{b=B}^{mB} P(\varepsilon|b,g,B,G,\Xi_i)$$
$$\cdot[P(B|b,\Xi_i)P(b,\Xi_i)]\,[P(G|g,\Xi_i)P(g,\Xi_i)]\,. \tag{23}$$

Each factor of the summand in (23) relates to a factor in (16).

Conditioned on $b + g$ binary symbol errors in a received codeword, the first factor in (23) is

$$P(\varepsilon|b,g,B,G,\Xi_i) = \frac{b+g}{m\,n}\,. \tag{24}$$

The factors $P(B|b,\Xi_i)$ and $P(b,\Xi_i)$ for the burst subblock containing i code symbols and $m\,i$ binary symbols are similar to (19) and (18), respectively. By inspection

$$P(B|b,\Xi_i) = \frac{\binom{i}{B}}{\binom{m\,i}{b}} \sum_{q=0}^{s} (-1)^q \binom{B}{q}\binom{m(B-q)}{b} \tag{25}$$

where $s = \lfloor B - b/m \rfloor$, and

$$P(b,\Xi_i) = \beta[m\,i, b, P(\varepsilon|\xi_b)] \tag{26}$$

where $P(\varepsilon|\xi_b)$ is a conditional channel error probability at the receiver. Similarly, the factors $P(G|g,\Xi_i)$ and $P(g,\Xi_i)$ for the Gaussian subblock containing $n - i$ code symbols and $m(n-i)$ binary symbols are similar to (19) and (18), respectively. Again by inspection

$$P(G|g,\Xi_i) = \frac{\binom{n-i}{G}}{\binom{m(n-i)}{g}} \sum_{q=0}^{r} (-1)^q \binom{G}{q}\binom{m(G-q)}{g} \tag{27}$$

where $r = \lfloor G - g/m \rfloor$, and

$$P(g,\Xi_i) = \beta[m(n-i), g, P(\varepsilon|\xi_g)] \tag{28}$$

where $P(\varepsilon|\xi_g)$ is a conditional channel error probability at the receiver.

The equations which define the decoded bit-error probability for an RS code operating over an ideal bursty-noise channel are given by (21) through (28). All together, there are six imbedded summations which must be evaluated. Although the number of terms to be evaluated can be large for long blocklength codes, most of these terms are negligibly small.

V. PERFORMANCE BOUNDS FOR NON-IDEAL BURSTY NOISE

In the previous section, the decoded bit-error probability was derived for an ideal bursty-noise channel. In this section, decoded bit-error probability bounds are derived for a *non-ideal bursty noise* channel, where non-ideal bursty noise does not require the restrictions on d and the set $\{t_i\}$ as for ideal bursty noise. The strategy for calculating the bounds is as follows. Given the non-ideal bursty-noise channel parameters ν, d, ψ, and N_t, modified parameters are found and applied to the decoded bit-error probability formulas for ideal bursty noise of the previous section. The modified parameters are denoted ν, d_α, ψ_α, and $N_{t,\alpha}$, where $\alpha = 1$ indicates the lower bound and $\alpha = 2$ indicates the upper bound.

For each received codeword, it is possible for Ξ_g, Ξ_p, and Ξ_b to occur. Let $p = \lceil d/T_{RS} \rceil$ and note that each noise burst can cause the events Ξ_b and Ξ_p to occur. Regardless of when a burst occurs in the channel, it cannot cause more than $p + 1$ total occurrences of Ξ_b and Ξ_p. In order to establish the upper bound, the noise burst duration is artificially lengthened by substituting Ξ_b for Ξ_p at the receiver as shown in Fig. 3. Therefore, the upper-bound burst duration is defined to be

$$d_2 = (p+1)\,T_{RS}\,. \tag{29}$$

Similarly, let $q = \lfloor d/T_{RS} \rfloor$ and note that each noise burst cannot cause fewer than $q-1$ occurrences of Ξ_b. In order to establish the lower bound, the noise burst duration is artificially shortened by substituting Ξ_g for Ξ_p at the receiver. Therefore, the lower-bound burst duration is defined to be

$$d_1 = (q-1)\,T_{RS}. \tag{30}$$

For either bound, the three bursty-noise parameters ν, N_g, and N_b remain unchanged. The latter two can be written in terms of the bursty-noise parameters ψ_α and $N_{t,\alpha}$. Two cases for ψ must be considered separately, $\psi = 1$ and $0 < \psi < 1$.

For the case $\psi = 1$, the Gaussian noise component is zero regardless of the length of the bursts. Therefore, $\psi = 1$ implies that $\psi_\alpha = 1$. Combining (3) through (6) and solving for N_b gives

$$N_b = \psi \frac{N_t}{\nu d} = \psi_\alpha \frac{N_{t,\alpha}}{\nu d_\alpha} \tag{31}$$

from which it follows that

$$N_{t,\alpha} = \left(\frac{d_\alpha}{d}\right) N_t. \tag{32}$$

For the case $0 < \psi < 1$, (31) still holds. Combining (5) and (31) and solving for N_g gives

$$N_g = (1-\psi)N_t = (1-\psi_\alpha)N_{t,\alpha}. \tag{33}$$

Dividing this equation by (31) gives

$$\psi_\alpha = \left[1 + \left(\frac{1-\psi}{\psi}\right)\left(\frac{d}{d_\alpha}\right)\right]^{-1} \tag{34}$$

and from (33)

$$N_{t,\alpha} = \left(\frac{1-\psi}{1-\psi_\alpha}\right) N_t. \tag{35}$$

Consequently, using ν, d_1, ψ_1, and $N_{t,1}$ in (21) through (28) results in a lower bound for the decoded bit-error probability for the non-ideal bursty-noise channel. The upper bound follows using ν, d_2, ψ_2, and $N_{t,2}$ in (21) through (28). The bounds must be tight if $d \gg T_{RS}$ because the difference between the non-ideal bursty-noise parameters and the modified bursty-noise parameters is small. This bounding technique can also be applied to the uncoded system.

The ideal bursty-noise formulas, described in the previous section, require; (1) $\nu d \ll 1$, (2) $d \gg T_{RS}$, and (3) independent code symbol errors at the decoder input. Since the error probability bounds described in this section make use of the ideal bursty-noise formulas, these assumptions are also required for the bounds to be valid.

These results are easily extended to channels which exhibit bursts of varying duration. Suppose $f(d)$ is the conditional probability mass function (*pdf*) of the burst duration d, conditioned upon the occurrence of a burst. Summing the product of the upper bound formula and the *pdf* of d, with respect to d, will give the upper bound for the error probability at the decoder output. A similar summation will give the lower bound. However, if a significant portion of the *pdf* weight is for $d \le T_{RS}$, then the bounds may not be tight.

VI. RESULTS FOR BPSK

In this section, the theoretical decoded bit-error probability bounds are compared with simulation results. Additional theoretical performance results are presented to illustrate trends. It is assumed that the information rate and transmitter power are the same for the uncoded and coded systems. For the coded system, the conditional channel bit-error probabilities are similar to (11) and (12) except that the received symbol energy is $E_s = E_b(k/n)$ to account for the encoder redundancy. Thus, for the system with the RS encoder/decoder

$$\begin{aligned} P(\varepsilon|\xi_g) &= Q\left(\sqrt{\frac{2}{(1-\psi)}\left(\frac{E_s}{N_t}\right)}\right) \\ &= Q\left(\sqrt{\frac{2}{(1-\psi)}\left(\frac{E_b}{N_t}\right)\left(\frac{k}{n}\right)}\right) \end{aligned} \tag{36}$$

and

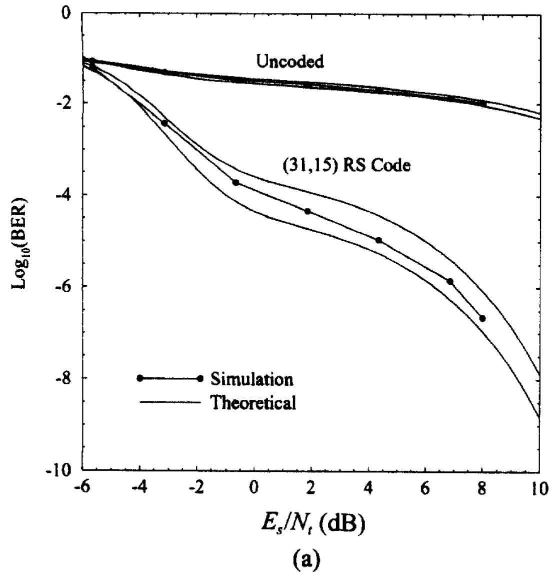
$$\begin{aligned} P(\varepsilon|\xi_b) &= Q\left(\sqrt{\frac{2}{(1-\psi+\psi/\nu d)}\left(\frac{E_s}{N_t}\right)}\right) \\ &= Q\left(\sqrt{\frac{2}{(1-\psi+\psi/\nu d)}\left(\frac{E_b}{N_t}\right)\left(\frac{k}{n}\right)}\right). \end{aligned} \tag{37}$$

A. Comparison with Simulation

To support the theoretical bounds given in the preceding section, comparisons are made with Monte Carlo simulation results. The system simulation was configured with a BPSK modulator, bursty-noise generator, coherent demodulator, code-symbol block deinterleaver, and RS decoder. The RS decoder was configured to be an errors-only, incomplete decoder and was implemented via error accumulator variables which operate on each received codeword. The block deinterleaver was used to provide the decoder with independent code symbol errors and has n columns and D rows, where D is the interleave depth.

Fig. 4(a) illustrates the uncoded and coded performance results for a (31,15,8) RS code for $\psi = 4/5$ and Fig. 4(b) illustrates the performance results for $\psi = 1$. The channel parameters chosen were $T_s = 0.5$ ms ($T_{RS} = 2.5$ ms), $d = 20$ ms, $\nu = 5\,\mathrm{s}^{-1}$, and $D = 200$. These parameters satisfy the assumptions $\nu d \ll 1$, $d \gg T_{RS}$, and independent code symbol errors input to the RS decoder required for the bounds to be valid. Clearly, the Monte Carlo bit-error probability estimates fit well within the bit-error probability bounds.

Fig. 5 shows the decoded bit-error probability as a function of the interleave depth for the same parameters used for the results in Fig. 4(a) with $E_s/N_t = 2$ dB. The decoded error probability falls within the bounds for interleave depths as low as $D = 8$. This suggests that the bounds may be useful even if ideal symbol interleaving is not assumed.



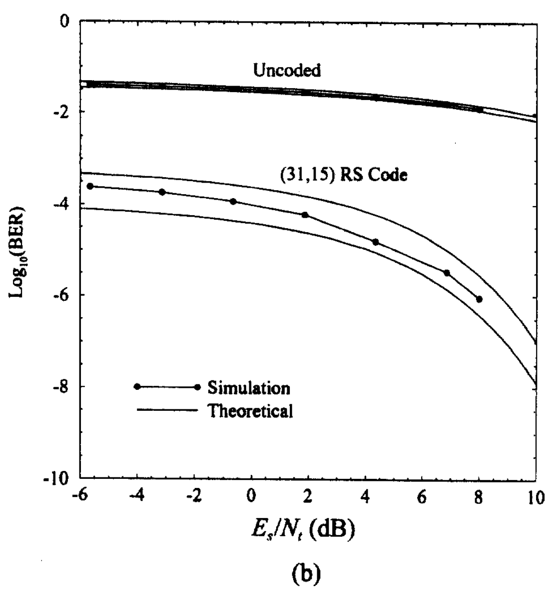


Fig. 4. Comparison of the RS decoded bit-error probability bounds with Monte-Carlo simulation results for $\nu d = 0.1$, $T_s = 0.5\,\text{ms}$, and with a) $\psi = 4/5$, and b) $\psi = 1$.

B. Further Results

As stated in Section V, the decoded bit-error probability bounds are asymptotically tight as the burst duration increases. This implies that the ideal bursty-noise error-probability formulas can be used to provide an estimate of the decoded error probability via direct substitution of the non-ideal bursty-noise channel parameters. In this section, this approach is used to estimate the decoded error probability for various bursty-noise channel parameters to illustrate trends. The uncoded error probability is approximated by (8).

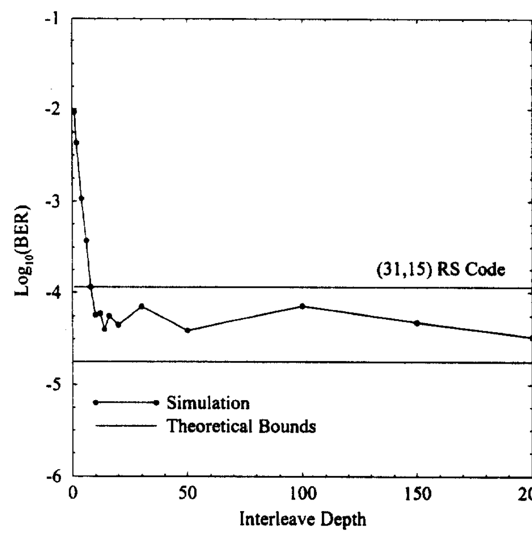


Fig. 5. RS decoded bit-error probability as a function of interleave depth for $\nu d = 0.1$, $T_s = 0.5\,\text{ms}$, and $E_s/N_t = 2\,\text{dB}$.

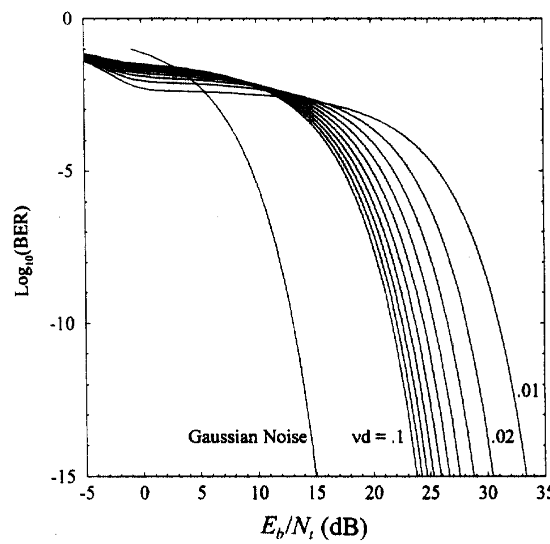


Fig. 6. Uncoded bit-error probability for $\psi = 4/5$.

Fig. 6 shows the uncoded error probability as a function of νd for $\psi = 4/5$. The curve for $\psi = 0$ (AWGN channel) is also shown for comparison. These results demonstrate that bursty noise can severely degrade the performance of a communication system. The two distinct "waterfall" segments of each error probability curve are a result of the relative values of the two terms in (8). As E_b/N_t decreases, both conditional error probabilities given by (11) and (12), approach 1/2. Since $P(\xi_b) \approx \nu d \ll 1$ by assumption, $P(\varepsilon|\xi_g)P(\xi_g)$ dominates giving rise to the first "waterfall" curve for small E_b/N_t. As E_b/N_t increases, $P(\varepsilon|\xi_b)P(\xi_b)$ dominates giving rise to the second "waterfall" curve for large E_b/N_t.

Fig. 7 shows the uncoded error probability as a function of ψ for $\nu d = 0.1$. The $\psi = 1$ curve approaches a horizontal asymptote of $\nu d/2$ as E_b/N_t decreases. This follows from (8) because $P(\varepsilon|\xi_g) = 0$ for $\psi = 1$ and $P(\varepsilon|\xi_b) \to 1/2$ as E_b/N_t decreases.

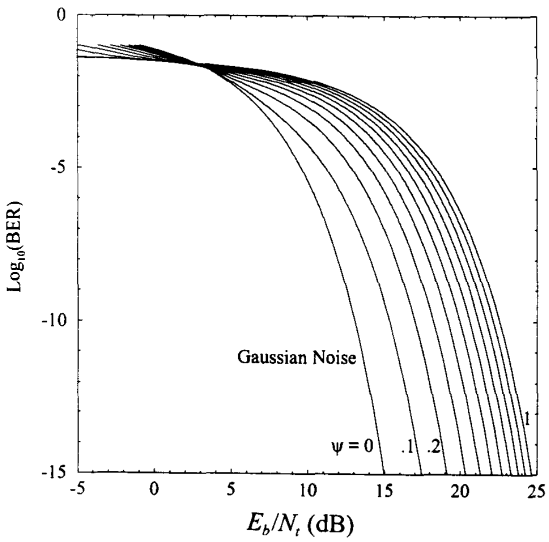


Fig. 7. Uncoded bit-error probability for $\nu d = 0.1$.

The (31,15,8) RS decoded error probability for $\psi = 4/5$, parameterized by νd, is shown in Fig. 8. These results correspond to the uncoded error probability results shown in Fig. 6. Fig. 9 shows the (31,15,8) RS decoded error probability for $\nu d = 0.1$, parameterized by ψ. These results correspond to the uncoded error probability results shown in Fig. 7. Corresponding coding gain curves, shown in Fig. 10 and 11, demonstrate that significant coding gains for a bursty-noise channel are achievable with RS codes and proper interleaving.

Note that if the noise variance at the receiver given Ξ_b is significantly greater than the noise variance given Ξ_g, then it may be possible to detect and erase RS code symbols with large probability of error. In this case, an errors-and-erasure decoder might provide more coding gain. In general, an errors-and-erasures RS decoder can correct l erasures and s errors provided $2s + l \leq 2t$.

As a final illustration, Fig. 12 shows the decoded error probability as a function of the uncoded error probability for RS codes with approximate rate 7/8 and various values of m.

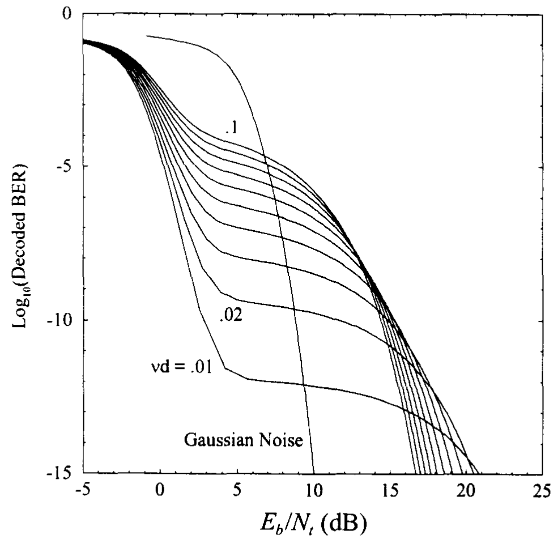


Fig. 8. Decoded bit-error probability for $\psi = 4/5$ and for a (31,15,8) Reed-Solomon code.

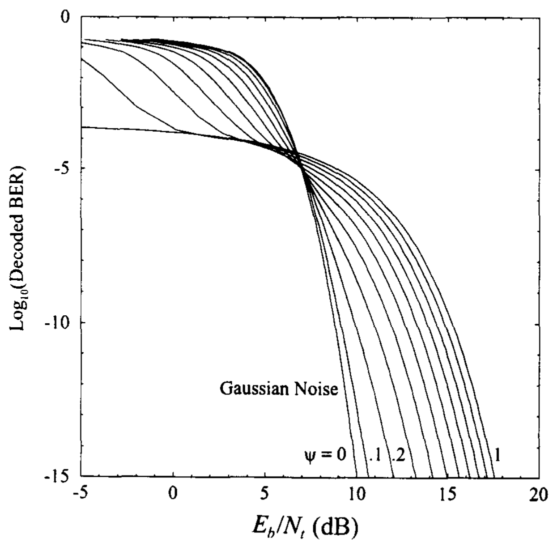


Fig. 9. Decoded bit-error probability for $\nu d = 0.1$ and for a (31,15,8) Reed-Solomon code.

The $m = 8$ curve corresponds to the (255,223,16) RS code chosen by the Consultative Committee for Space Data Systems (CCSDS) as a part of the NASA space telemetry standard. The bursty-noise parameters were taken to be $\nu d = 0.05$ and $\psi = 1$. The $m = 8$ curve is truncated due to the fact that double precision arithmetic was not sufficient to evaluate (25) due to the large values of the terms which must be alternately added and subtracted.

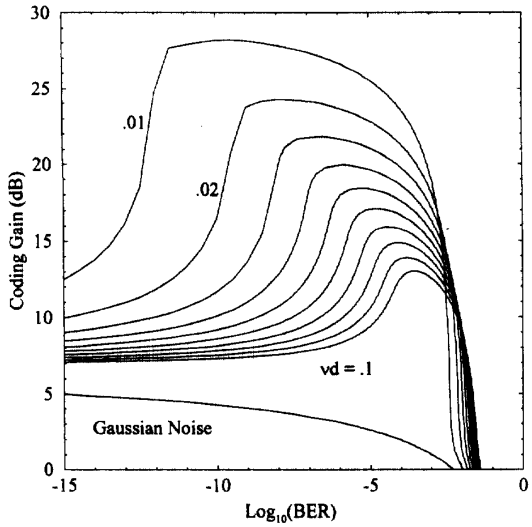


Fig. 10. Coding gains for $\psi = 4/5$ and for a (31, 15, 8) Reed-Solomon code.

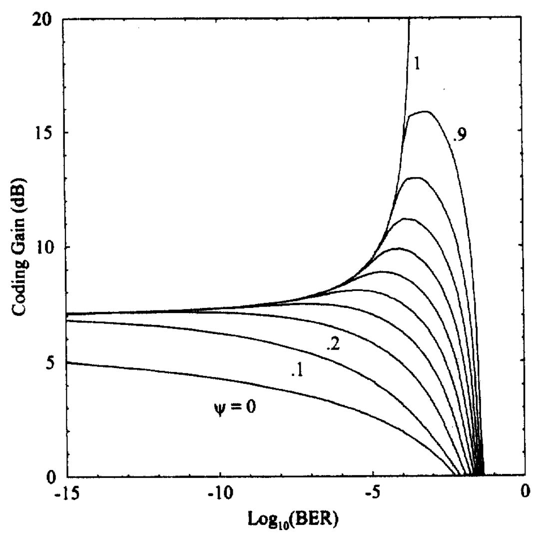


Fig. 11. Coding gains for $\nu d = 0.1$ and for a (31, 15, 8) Reed-Solomon code.

VII. SUMMARY AND CONCLUSIONS

In this paper, decoded bit-error probability bounds are derived for RS codes used with a binary communication system operating over a bursty-noise channel. As an intermediate step, a new derivation for the RS decoded bit-error probability for a binary communication system operating over AWGN is also presented and is exact subject only to negligible undetected word error probability. The decoded bit-error probability is also given for RS codes operating over an ideal bursty-noise channel. The theoretical performance bounds have been shown to agree well with simulation.

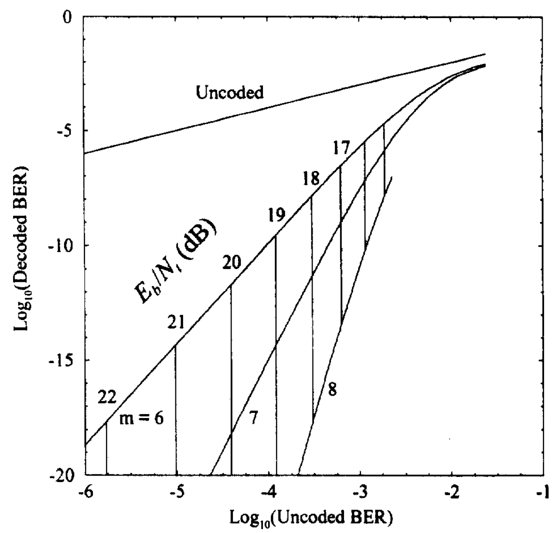


Fig. 12. Decoded bit-error probability for RS codes with approximate rate 7/8 and for $\psi = 1$ and $\nu d = 0.05$.

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APPENDIX A
DERIVATION OF (19)

Consider an (n,k,t) Reed-Solomon (RS) code with m binary symbols per code symbol and blocklength $n = 2^m - 1$. Suppose that there are i binary symbol errors contained within j code symbols in a received codeword. In this appendix, the probability that each j code symbol contains at least one binary symbol error, denoted $P(W_j|w_i)$ in Section IV, is derived.

If N_e is the number of error patterns satisfying the condition that at least one binary symbol error occurs in each of the j code symbols, then

$$P(W_j|w_i) = \frac{\binom{n}{j}}{\binom{mn}{i}} N_e . \tag{A.1}$$

The j code symbols will be referred to as the subblock and are the only code symbols referred to in the following development. The event that the k^{th} code symbol in the subblock contains no errors is denoted C_k and the event that it contains at least 1 binary error is denoted $\overline{C}_k$. If $N(x)$ is the number of error patterns which satisfy event x without regard for any other event, then

$$N_\varepsilon = N(\overline{C}_1 \overline{C}_2 \cdots \overline{C}_j). \tag{A.2}$$

Consider an arbitrary arrangement of i binary symbol errors in the subblock. Define

$$N_0 = \binom{mj}{i} \tag{A.3}$$

and define

$$N_1 = \sum_{k=1}^{j} N(C_k). \tag{A.4}$$

Since each term in the summation is identical

$$N_1 = \binom{j}{1} N(C_1) = \binom{j}{1}\binom{m(j-1)}{i}. \tag{A.5}$$

In a similar way, it can be shown that

$$N_q = \binom{j}{q}\binom{m(j-q)}{i}. \tag{A.6}$$

Using the inclusion-exclusion principle [22],

$$N_\varepsilon = \sum_{q=0}^{r} (-1)^q N_q = \sum_{q=0}^{r} (-1)^q \binom{j}{q}\binom{m(j-q)}{i} \tag{A.7}$$

where $r = \lfloor j - i/m \rfloor$. Substituting (A.7) into (A.1) gives (19).

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