

Assignment 7

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Problem 1:

$$\begin{array}{l} \sum_{k=1}^n (2k-1) = n^2 \\ \text{when } n=1 \\ \sum_{k=1}^1 (2k-1) = 2-1 = 1 \\ n^2 = 1 \\ \text{the equation is true} \end{array} \quad \begin{array}{l} | \\ | \text{ when } n = a+1 \\ | \\ | \sum_{k=1}^{a+1} (2k-1) = \sum_{k=1}^a (2k-1) + 2(a+1) - 1 \\ | \\ | = a^2 + 2a + 2 - 1 \\ | \\ | = a^2 + 2a + 1 \\ | \\ | = (a+1)^2 \\ | \\ | \therefore \text{the equation is always true} \end{array}$$

Problem 2:

Assume that two propositional formula A & B has same number (respectively nA & nB) of left parentheses as right parentheses. Then $(A \vee B)$ has $nA + nB + 1$ pairs of parentheses. By the same method, we can prove that with combinations of other propositional formulas by \wedge and \neg are also have same number of left parentheses as right parentheses.

Problem 3:

\therefore for all nonnegative x , $0 \leq f(x) \leq x$
 \therefore in the recursion, $x_n < x_{n-1}$
 $\therefore f(0) = 0$
 \downarrow
 $f(x_n) = 0, x_n \geq 0$
 \downarrow
 $f(x_{n-1}) = x_{n-1} > x_n$
 \therefore every x would gradually decrease during the recursion,
and eventually reaches zero.