

Lecture 2

1. Strings

Problem 3: HW2

String Append

(String-append "abc" "def")

\Rightarrow "abcdef"

(String-append "ab" "bc" "de")

\Rightarrow "abbcde"

Problem 6: HW2

You need to compare two boolean values

(boolean=? #t #f) \Rightarrow false

(

Data Type: Symbols

Set of keyboard characters

Suppose you're designing checkers app.

Red, Black

We can store this information using symbols.

Input: 'Red \Rightarrow symbol Red

'Red \neq "Red"

For Problem 2:

Grades will be in symbols

Write which outputs ' R ', ' B ', ' G '
 $(n \bmod 3) == 1 \rightarrow 'Red'$
 $(n \bmod 3) == 2 \rightarrow 'Blue'$
 $(n \bmod 3) == 0 \rightarrow 'Green'$

(: num-to-color : Integer \rightarrow Symbol)

```
(define (num-to-color n)
  (if (= (remainder n 3) 0)
      'Green
      (if (= (remainder n 3) 1)
          'Red
          'Blue))))
```

Recursion

Recursion → Divide and Conquer
Recursion → Mathematical Induction

Main Idea: You have a problem P .
Divide it into sub problems P_1 and P_2 . Suppose we can
solve P_1 and P_2 .

Combine the solution of P_1 and P_2
to solve P .

Example:

Write a program to sum first
 n natural numbers.

Input: n .

Output: $1+2+3+4+\dots+n$.

$$\text{sum}(3) = 6$$

$$\text{sum}(4) = 10.$$

$$\begin{aligned}\text{sum}(3) &= \underbrace{1+2}_{\text{sum}(2)} + 3 \\ &= \text{sum}(2) + 3.\end{aligned}$$

In general.

$$\text{sum}(n) = \text{sum}(n-1) + n.$$

(Recurrence Relation).

$$\text{sum}(4) = 4 + \text{sum}(3)$$

$$= 4 + 3 + \text{sum}(2)$$

$$= 4 + 3 + 1 + \text{sum}(1)$$

$$= 4 + 3 + 1 + 0 + \text{sum}(-1)$$

$$= 4 + 3 + 1 + 0 + (-1) + \text{sum}(-2)$$

$\text{sum}(0) = 0 \leftarrow \text{Base Case}$

$$\begin{aligned}\text{sum}(4) &= 4 + \text{sum}(3) \\ &= 4 + 3 + \text{sum}(2) \\ &= 4 + 3 + 2 + \text{sum}(1) \\ &= 4 + 3 + 2 + 1 + \text{sum}(0) \\ &= 4 + 3 + 2 + 1 + 0 \Rightarrow 10\end{aligned}$$

→ Recurrence Relation

$$\left[\begin{array}{l} \text{sum}(n) = \text{sum}(n-1) + n . \\ \text{sum}(0) = 0 \rightarrow \text{Base Case} \end{array} \right]$$

($\because \text{sum} : \text{Integer} \rightarrow \text{Integer}$)

(define (sum n)

(if (= n 0) 0

(+ (sum (- n 1)) n)))

(check-expect (sum 3) 6)

$$\begin{aligned} (\text{sum } 3) &\Rightarrow 3 + (\text{sum } 2) \\ &\Rightarrow 3 + 2 + (\text{sum } 1) \\ &\Rightarrow 3 + 2 + 1 \quad (\text{sum } 5) \\ &\Rightarrow 3 + 2 + 1 + 0 \\ &= 6. \end{aligned}$$

(sum -1) \Rightarrow Infinite Loop

Example 2:

Write a function to count 'c', 's' letters in a string.

(count-CS "abc") \Rightarrow 1

(count-CS "SCSC") \Rightarrow 4.

$S = "abc"$

Extract the last character

and check if it is 'C' or 'S'.

+ 1

Recurrence Relation

count-CS(S) = count-CS($S[1, n]$)

+ if($S[1] = 'C'$ or
 $S[1] = 'S'$)

else

0 . —

$\text{len}(S) > 1$

if $n == 0$ (string is empty)

count-CS(S) = 0.

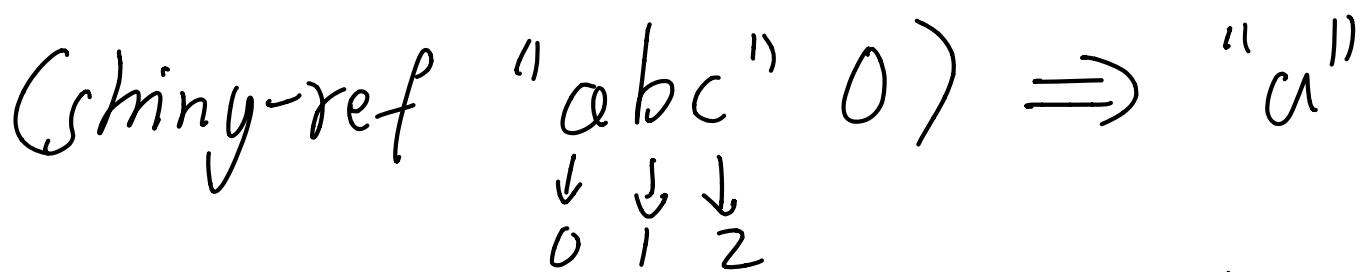
$s[0, n-1]$ → The string you get
by removing the last character.
 $s[n]$ → the last character.
 n → length of the
string.

Racket function

```
(: count-cs : String → Integer)
(define (count-cs s)
  (if (= (string-length s) 0)
      0
      (+ (count-cs (subString s 1))
```

(if (or removes the first character.
(char=? #'|c| (string-ref s 0))
(char=? #'|s| (string-ref s 0)))
1
0)))) → The first character is '|c' or '|s)

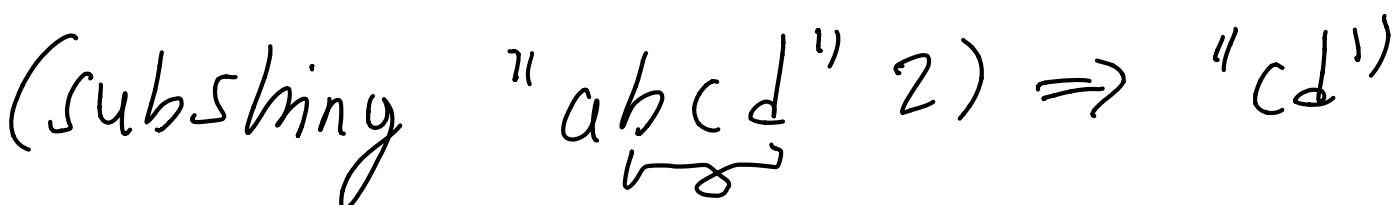
string-length
string-ref

(String-ref "abc" 0) ⇒ "a"


The string "abc" is shown with three vertical arrows pointing down to the indices 0, 1, and 2 respectively, indicating the position of each character in the string.

(String-ref "abc" 1) ⇒ "b"

(SubString "abc" 1) ⇒ "bc"

(SubString "abcd" 2) ⇒ "cd")


The string "abcd" is shown with four vertical arrows pointing down to the indices 1, 2, 3, and 4 respectively, indicating the position of each character in the string.

Example 3

$\text{factorial}(n) = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdots n$.

Recurrence for factorial

$\text{factorial}(n) = \text{factorial}(n-1) \cdot n$

$\text{factorial}(0) = 1$ → Base case.

Demonstrate error functions

($\text{: fact : Integer} \rightarrow \text{Integer}$)

(define (fact n)

(if (> 0 n)

(error "Input less than zero"))

(if (= n 0)
 1 \underline{Base case}
 (* n (fact (- n 1)))))).
 \underbrace{\hspace{1cm}}

Recurrence -

(check-error (fact -1) "Input less
than zero").

Cond

Avoid nested if statements

Example: Using Cond
Write function which takes
a country as input
and prints its population.

(define (population country)
(cond
[(string=? country "USA") 3,000,000]
[(string=? country "Mexico") 1,200,000]
[(string=? country "Canada") 2,000,000]
[else (error "I don't know
the answer")])

country ← Input
(String=? country "USA") $\Rightarrow \#t$

(population "USA") $\Rightarrow 3,04,00,000.$

General Template for Recursive functions.

C: func : Integer \rightarrow Integer
(define (func n)
(cond
[(base-case 1) result1]
[(base-case 2) result2].
[(base-case 3) result3]
[else recursive-case]))

List

[97, 100, -15, 6]

(: my-list : (Listof Integer))

(define my-list (list 97 100 -15 6))

(Listof (Listof (Listof Integer)))
(Listof Boolean)

Write important functions for list

- first

(first my-list) \Rightarrow 97

- rest

(rest my-list) \Rightarrow '(100 -15 6)

- empty?

(empty? my-list) \Rightarrow #f

Example:

Write a program to add

every element of List of
Integers.

(10 - 11 13 14)

$\Rightarrow 26$

First Recurrence

(sum-list nums)

= (first nums) + (sum-list
(rest nums))

\rightarrow Recurrence
Relation.

Sum-list ((100 -11 23 4))

= 100 + sum-list ((-11 23 4))

What is the base-case?

When the list is empty?

If list is empty, sum is 0.

(\therefore sum-list : (Listof Integer) \rightarrow Integer)

(define (sum-list input))

(cond

[(empty? input) 0]

[else (+ (first input) (sum-list
(rest input)))])

Example

You have a list of strings which are actually books you own, you have to count how many copies of "Oedipus Rex" do you have.

Count-occurrence (input)

```
=if (string=? (first input)
              "Oedipus Rex"
              )
```

* 1
0)

+ (count-occurrence ((rest input)))

If my input is empty,
output is 0.

(: count-occurrences : (Listof String)
String → Integer)

(define (count-occurrences my-books
title)

(cond
[(empty? my-books) 0])

[else (+

(count-occurrences (rest my-books))

(if (string=? (first my-books)
 title)
 1
 0))))

(check-expect (count-occurrences
(list "ABC" "DEF" "XYZ")
 "XYZ") 1)

Example

Selling Coconuts

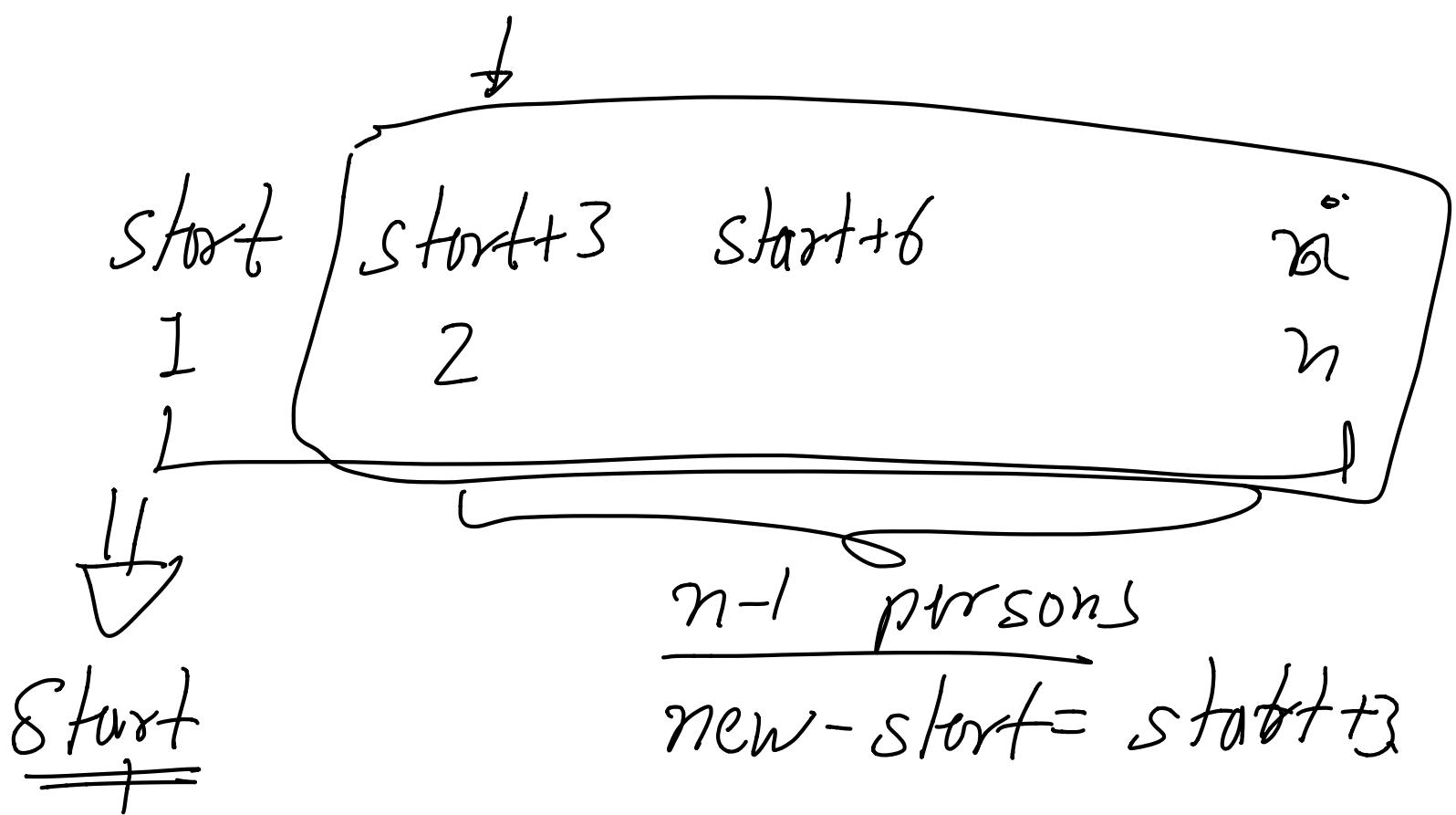
You're selling coconuts. Each time you sell a coconut, you raise the price by \$3. Initially, price of the coconut is \$1. If there n people, how much will you make.

Summation of arithmetic progression.

Let's assume we have
have an arbitrary initial
price. start

profit(n , start)
subproblem ↓

$$= \text{profit}(n-1, \text{start}+3) + \text{start}$$



If number of people is 0; then you make zero profit.

if $n=0$, $\text{profit}(n, \text{start}) = 0$.

(: profit : Integer Integer
→ Integer)

(define (profit n start))

(cond
[(= n 0) 0]

[else (+ start (profit (- n 1)
(+ start 3))))])

(: my-profit : Integer → Integer)

((define (my-profit n)

(profit n 1))).

Fibonacci

$$F_0 = 0, F_1 = 1$$

$$F_n = F_{n-1} + F_{n-2}$$

0, 1, 1, 2, 3, 5, 8, 13

Write a racket function
to compute n^{th} fibonacci.

Here we have two base cases.

(`fibo : Integer → Integer`)

(define (fib0 n)

(cond
[(= n 0) 0]
[(= n 1) 1]

[else (+ (fib0 (- n 1))
(fib0 (- n 2))))]]))