Believe What You See: Implicit Constraint Approach for Offline Multi-Agent Reinforcement Learning

Yiqin Yang[1][†], **Xiaoteng Ma**[1][†][‡] **Chenghao Li**[1], **Zewu Zheng**[1],
Qiyuan Zhang[2]**, Gao Huang**[1]**, Jun Yang**[1][‡]**, Qianchuan Zhao**[1]
[1]Tsinghua University, [2]Harbin Institute of Technology
{yangyiqi19, ma-xt17, lich18}@mails.tsinghua.edu.cn, zzheng17@126.com,
zhangqiyuan19@hit.edu.cn, {gaohuang, yangjun603, zhaoqc}@tsinghua.edu.cn

Abstract

Learning from datasets without interaction with environments (Offline Learning) is an essential step to apply Reinforcement Learning (RL) algorithms in real-world scenarios. However, compared with the *single-agent* counterpart, offline *multi-agent* RL introduces more agents with the larger state and action space, which is more challenging but attracts little attention. We demonstrate current offline RL algorithms are ineffective in multi-agent systems due to the accumulated extrapolation error. In this paper, we propose a novel offline RL algorithm, named *Implicit Constraint Q-learning* (ICQ), which effectively alleviates the extrapolation error by only trusting the state-action pairs given in the dataset for value estimation. Moreover, we extend ICQ to multi-agent tasks by decomposing the joint-policy under the implicit constraint. Experimental results demonstrate that the extrapolation error is successfully controlled within a reasonable range and insensitive to the number of agents. We further show that ICQ achieves the state-of-the-art performance in the challenging multi-agent offline tasks (StarCraft II). Our code is public online at https://github.com/YiqinYang/ICQ.

1 Introduction

Recently, reinforcement learning (RL), an active learning process, has achieved massive success in various domains ranging from strategy games [59] to recommendation systems [8]. However, applying RL to real-world scenarios poses practical challenges: interaction with the real world, such as autonomous driving, is usually expensive or risky. To solve these issues, offline RL is an excellent choice to deal with practical problems [3, 24, 35, 42, 15, 28, 4, 23, 54, 12], aiming at learning from a fixed dataset without interaction with environments.

The greatest obstacle of offline RL is the distribution shift issue [16], which leads to extrapolation error, a phenomenon in which unseen state-action pairs are erroneously estimated. Unlike the online setting, the inaccurate estimated values of unseen pairs cannot be corrected by interacting with the environment. Therefore, most off-policy RL algorithms fail in the offline tasks due to intractable over-generalization. Modern offline methods (e.g., Batch-Constrained deep Q-learning (BCQ) [16]) aim to enforce the learned policy to be close to the behavior policy or suppress the Q-value directly. These methods have achieved massive success in challenging single-agent offline tasks like D4RL [14].

However, many decision processes in real-world scenarios belong to multi-agent systems, such as intelligent transportation systems [2], sensor networks [37], and power grids [7]. Compared with the single-agent counterpart, the multi-agent system has a much larger action space, which grows

---

†Equal Contribution.
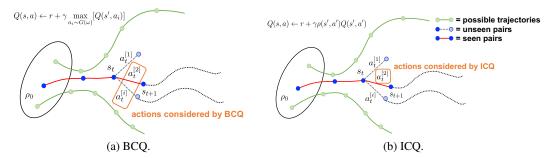‡Equal Corresponding.

Figure 1: The comparison between ICQ and BCQ for the target Q-value estimation. The spots denote states, and the connections between spots indicate actions. The red solid-lines denote seen pairs, and the gray dotted-lines are unseen pairs. (a) BCQ estimates Q-value in a defined similar action set (orange) while unseen pairs still exist in the set with low probability. (b) ICQ only adopts seen pairs (orange) in the training set for Q-value estimation.

exponentially with the increasing of the agent number. When coming into the offline scenario, *the unseen state-action pairs will grow exponentially as the number of agents increases, accumulating the extrapolation error quickly*. The current offline algorithms are unsuccessful in multi-agent tasks even though they adopt the modern value-decomposition structure [26, 48, 25]. As shown in Figure 2, our results indicate that BCQ, a state-of-the-art offline algorithm, has divergent Q-estimates in a simple multi-agent MDP environment (e.g., BCQ (4 agents)). The extrapolation error for value estimation is accumulated quickly as the number of agents increases, significantly impairing the performance.

Based on these analyses, we propose the Implicit Constraint Q-learning (ICQ) algorithm, which effectively alleviates the extrapolation error as no unseen pairs are involved in estimating Q-value. Motivated by an implicit constraint optimization problem, ICQ adopts a SARSA-like approach [49] to evaluate Q-values and then converts the policy learning into a supervised regression problem. By decomposing the joint-policy under the implicit constraint, we extend ICQ to the multi-agent tasks successfully. To the best of our knowledge, our work is the first study analyzing and addressing the extrapolation error in multi-agent reinforcement learning.

We evaluate our algorithm on the challenging multi-agent offline tasks based on StarCraft II [40], where a large number of agents cooperatively complete a task. Experimental results show that ICQ can control the extrapolation error within a reasonable range under any number of agents and learn from complex multi-agent datasets. Further, we evaluate the single-agent version of ICQ in D4RL, a standard single-agent offline benchmark. The results demonstrate the generality of ICQ for a wide range of task scenarios, from single-agent to multi-agent, from discrete to continuous control.

2   Background

Notation. The fully cooperative multi-agent tasks are usually modeled as the Dec-POMDP [31] consisting of the tuple $G = \langle S, A, P, r, \Omega, O, n, \gamma \rangle$. Let $s \in S$ denote the true state of the environment. At each time step $t \in \mathbb{Z}^+$, each agent $i \in N \equiv \{1, \ldots, n\}$ chooses an action $a^i \in A$, forming a joint action $\boldsymbol{a} \in \mathbf{A} \equiv A^n$. Let $P(s' \mid s, \boldsymbol{a}) : S \times \mathbf{A} \times S \to [0, 1]$ denote the state transition function. All agents share the same reward function $r(s, \boldsymbol{a}) : S \times \mathbf{A} \to \mathbb{R}$.

We consider a partially observable scenario in which each agent draws individual observations $o^i \in \Omega$ according to the observation function $O(s, a) : S \times \mathbf{A} \to \Omega$. Each agent has an action-observation history $\tau^i \in \mathbf{T} \equiv (\Omega \times \mathbf{A})^t$, on which it conditions a stochastic policy $\pi^i(a^i \mid \tau^i)$ parameterized by $\theta_i : \mathbf{T} \times \mathbf{A} \to [0, 1]$. The joint action-value function is defined as $Q^{\boldsymbol{\pi}}(\boldsymbol{\tau}, \boldsymbol{a}) \triangleq \mathbb{E}_{s_{0:\infty}, \boldsymbol{a}_{0:\infty}} \left[\sum_{t=0}^{\infty} \gamma^t r_t \mid s_0 = s, \boldsymbol{a}_0 = \boldsymbol{a}, \boldsymbol{\pi} \right]$, where $\boldsymbol{\pi}$ is the joint-policy with parameters $\theta = \langle \theta_1, \ldots, \theta_n \rangle$. Let $\mathcal{B}$ denote the offline dataset, which contains trajectories of the behavior policy $\boldsymbol{\mu}$.

We adopt the *centralized training and decentralized execution* (CTDE) paradigm [43]. During training, the algorithm has access to the true state s and every agent's action-observation history τ_i,

as well as the freedom to share all information between agents. However, during execution, each agent has access only to its action-observation history.

Batch-constrained deep Q-learning (BCQ) is a state-of-the-art offline RL method, which aims to avoid selecting an unfamiliar action at the next state during a value update. Specifically, BCQ optimizes π by introducing perturbation model $\xi(\tau, a, \Phi)$ and generative model $G(\tau; \varphi)$ as follows

$$\pi(\tau) = \underset{a^{[i]} + \xi(\tau, a^{[i]}, \Phi)}{\arg\max} \ Q^\pi(\tau, a^{[i]} + \xi(\tau, a^{[i]}, \Phi); \phi), \quad \text{s.t.} \quad \{a^{[i]} \sim G(\tau; \varphi)\}_{i=1}^m, \tag{1}$$

where π selects the highest valued action from a collection of m actions sampled from the generative model $G(\tau; \varphi)$, which aims to produce only previously seen actions. The perturbation model $\xi(\tau, a^{[i]}, \Phi)$ is adopted to adjust action $a^{[i]}$ in the range $[-\Phi, \Phi]$ to increase the diversity of actions.

3 Analysis of Accumulated Extrapolation Error in Multi-Agent RL

In this section, we theoretically analyze the extrapolation error propagation in offline RL, which lays the basis for Section 4. The extrapolation error mainly attributes the out-of-distribution (OOD) actions in the evaluation of Q^π [16, 21]. To quantify the effect of OOD actions, we define the state-action pairs within the dataset as *seen* pairs. Otherwise, we name them as *unseen* pairs. We demonstrate that the extrapolation error propagation from the unseen pairs to the seen pairs is related to the size of the action space, which grows exponentially with the increasing number of agents. We further design a toy example to illustrate the inefficiency of current offline methods in multi-agent tasks.

3.1 Extrapolation Error Propagation in Offline RL

Following the analysis in BCQ [16], we define the tabular estimation error[*] as $\epsilon_{\text{MDP}}(\tau, a) \triangleq Q_M^\pi(\tau, a) - Q_{\mathcal{B}}^\pi(\tau, a)$ (here we abuse τ to denote the state for analytical clarity), where the M denotes the true MDP and $\mathcal{B}$ denotes a new MDP computed from the batch by $P_{\mathcal{B}}(\tau' \mid \tau, a) = \mathcal{N}(\tau, a, \tau') / \sum_{\tilde{\tau}} \mathcal{N}(\tau, a, \tilde{\tau})$. BCQ [16] has shown that $\epsilon_{\text{MDP}}(\tau, a)$ has a Bellman-like form with the extrapolation error $\epsilon_{\text{EXT}}(\tau, a)$ as the "reward function":

$$\epsilon_{\text{MDP}}(\tau, a) \triangleq \epsilon_{\text{EXT}}(\tau, a) + \sum_{\tau'} P_M(\tau' \mid \tau, a) \gamma \sum_{a'} \pi(a' \mid s') \epsilon_{\text{MDP}}(\tau', a'),$$

$$\epsilon_{\text{EXP}}(\tau, a) = \sum_{\tau'} \left(P_M(\tau' \mid \tau, a) - P_{\mathcal{B}}(\tau' \mid \tau, a) \right) \left(r(\tau, a, \tau') + \gamma \sum_{a'} \pi(a' \mid \tau') Q_{\mathcal{B}}^\pi(\tau', a') \right). \tag{2}$$

For the seen state-action pairs, $\epsilon_{\text{EXT}}(\tau, a) = 0$ since $P_M(\tau' \mid \tau, a) - P_{\mathcal{B}}(\tau' \mid \tau, a) = 0$ in the deterministic environment. In contrast, the $\epsilon_{\text{EXT}}(\tau, a)$ of unseen pairs is uncontrollable and depends entirely on the initial values in tabular setting or the network generalization in DRL.

To further analyze how the extrapolation error in the unseen pairs impacts the estimation of actions in the dataset, we partition $\boldsymbol{\epsilon_{\text{MDP}}}$ and $\boldsymbol{\epsilon_{\text{EXT}}}$ as $\boldsymbol{\epsilon_{\text{MDP}}} = [\boldsymbol{\epsilon_s}, \boldsymbol{\epsilon_u}]^{\mathbf{T}}$ and $\boldsymbol{\epsilon_{\text{EXT}}} = [\mathbf{0}, \boldsymbol{\epsilon_b}]^{\text{T}}$ respectively according to seen and unseen state-action pairs. Let denote the transition matrix of the state-action pairs as $P_M^\pi(\tau', a' \mid \tau, a) = P_M(\tau' \mid \tau, a)\pi(a' \mid \tau')$. We decompose the transition matrix as $P_M^\pi = \left[P_{\text{s,s}}^\pi, P_{\text{s,u}}^\pi; P_{\text{u,s}}^\pi, P_{\text{u,u}}^\pi \right]$ according to state-action pairs' property (e.g., $P_{\text{s,u}}^\pi(\tau_u', a_u' \mid \tau_s, a_s) = P_M(\tau_u' \mid \tau_s, a_s)\pi(a_u' \mid \tau_u')$ denotes the transition probability from seen to unseen pairs). Then the extrapolation error propagation can be described by the following linear system:

$$\begin{bmatrix} \boldsymbol{\epsilon_s} \\ \boldsymbol{\epsilon_u} \end{bmatrix} = \gamma \begin{bmatrix} P_{\text{s,s}}^\pi & P_{\text{s,u}}^\pi \\ P_{\text{u,s}}^\pi & P_{\text{u,u}}^\pi \end{bmatrix} \begin{bmatrix} \boldsymbol{\epsilon_s} \\ \boldsymbol{\epsilon_u} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\epsilon_b} \end{bmatrix}. \tag{3}$$

Based on the above definitions, we have the following conclusion.

Theorem 1. *Given a deterministic MDP, the propagation of $\boldsymbol{\epsilon_b}$ to $\boldsymbol{\epsilon_s}$ is proportional to $\|P_{\text{s,u}}^\pi\|_\infty$:*

$$\|\boldsymbol{\epsilon_s}\|_\infty \leq \frac{\gamma \left\| P_{\text{s,u}}^\pi \right\|_\infty}{(1 - \gamma) \left(1 - \gamma \left\| P_{\text{s,s}}^\pi \right\|_\infty \right)} \|\boldsymbol{\epsilon_b}\|_\infty. \tag{4}$$

---

[*]Note that we adopt a different definition of extrapolation error with BCQ. The $\epsilon_{\text{MDP}}(\tau, a)$ is regraded as the extrapolation error in BCQ, while the generalization error of unseen pairs $\epsilon_{\text{EXT}}(\tau, a)$ is considered in this work.
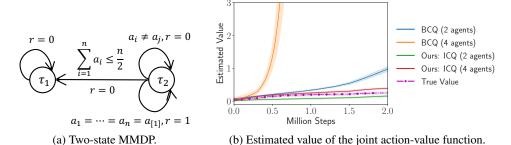
(a) Two-state MMDP.   (b) Estimated value of the joint action-value function.

Figure 2: (a) An MMDP where Q-estimates of BCQ will diverge as the number of agents increases. (b) The learning curve of the joint action-value function while running several agents in the given MMDP. The true values are similar in this task with different agent numbers, calculated by averaging the Monte-Carlo estimation under different agents. The Q-estimates of BCQ (4 agents) diverge while our algorithm (ICQ) has accurate Q-estimates. Please refer to Appendix C.2 for the complete results.

The above theorem indicates the effect of extrapolation error on seen state-action pairs is directly proportional to $\|P_{s,u}^{\pi}\|_{\infty}$. In the practice, $\|P_{s,u}^{\pi}\|_{\infty}$ is related to the size of action space and the dataset. If the action space is enormous, such as a multi-agent task with a number of agents, we need a larger amount of data to reduce $\|P_{s,u}^{\pi}\|_{\infty}$. However, the dataset size in offline learning tasks is generally limited. Moreover, when using the networks to approximate the value function, ϵ_b does not remain constant as $Q_{\mathcal{B}}(\tau_u, a_u)$ could be arbitrary during training, making the Q-values extreme large even for the seen pairs. For these reasons, we have to enforce the $P_{s,u}^{\pi} \to 0$ by avoiding using OOD actions. For example, BCQ utilizes an auxiliary generative model to constrain the target actions within a familiar action set (see Section 2 for a detailed description). However, the error propagation heavily depends on the accuracy of the generative model and is intolerable with the agent number increasing. We will demonstrate this effect in the following toy example.

3.2 Toy Example

We design a toy two states Multi-Agent Markov Decision Process (MMDP) to illustrate the accumulated extrapolation error in multi-agent tasks (see Figure 2a). All agents start at state τ_2 and explore rewards for 100 environment steps by taking actions $a_{[1]} = 0$ or $a_{[2]} = 1$. The optimal policy is that all agents select $a_{[1]}$. The MMDP task has sparse rewards. The reward is 1 when following the optimal policy, otherwise, the reward is 0. The state τ_2 will transfer to τ_1 if the joint policy satisfies $\sum_{i=1}^{n} a_i \leq \frac{n}{2}$ at τ_2, while the state τ_1 will never return to τ_2.

We run BCQ and our method ICQ on a limited dataset, which only contain 32 trajectories generated by QMIX. Obviously, the number of unseen state-action pairs exponentially grows as the number of agents increases. We control the amount of valuable trajectories ($r = 1$) in different datasets equal for fair comparisons. The multi-agent version of BCQ shares the same value-decomposition structure as ICQ (see Appendix D.2).

As shown in Figure 2b, the joint action-value function learned by BCQ gradually diverges as the number of agents increases while ICQ maintains a reasonable Q-value. The experimental result is consistent with Theorem 1, and we provide an additional analysis for the toy example in Appendix B.2. In summary, we show theoretically and empirically that the extrapolation error is accumulated quickly as the number of agents increases and makes the Q-estimates easier to diverge.

4 Implicit Constraint Approach for Offline Multi-Agent RL

In this section, we give an effective method to solve the accumulated extrapolation error in offline Multi-Agent RL based on the analysis of Section 3. From the implementation perspective, we find that a practical approach towards offline RL is to estimate target Q-value without sampled actions from the policy in training. We propose Implicit Constraint Q-learning (ICQ), which only trusts the seen state-action pairs in datasets for value estimation. Further, we extend ICQ to multi-agent tasks with a value decomposition framework and utilize a λ-return method to balance the variance and bias.

4.1 The Implicit Constraint Q-learning (ICQ) Approach

Based on the analysis of Section 3, we find that the extrapolation error can be effectively alleviated by enforcing the actions within the dataset when calculating the target values, which is the most significant difference between offline and off-policy RL. For a formal comparison of off-policy and offline algorithms, we first introduce the standard Bellman operator $\mathcal{T}^\pi$ as follows:

$$(\mathcal{T}^\pi Q)(\tau, a) \triangleq Q(\tau, a) + \mathbb{E}_{\tau'}[r + \gamma \mathbb{E}_{a' \sim \pi}[Q(\tau', a')] - Q(\tau, a)]. \tag{5}$$

Many off-policy evaluation methods, such as the Tree Backup [10] and Expected SARSA [41], are designed based on this operator. However, when coming into the offline setting, the standard Bellman operator suffers from the OOD issue as the actions sampled from current policy π are adopted for target Q-value estimation. A natural way to avoid the OOD issue is adopting the importance sampling measure [30]:

$$(\mathcal{T}^\pi Q)(\tau, a) = Q(\tau, a) + \mathbb{E}_{\tau'}[r + \gamma \mathbb{E}_{a' \sim \mu}[\rho(\tau', a')Q(\tau', a')] - Q(\tau, a)], \tag{6}$$

where $\rho(\tau', a') \triangleq \frac{\pi(a'|\tau')}{\mu(a'|\tau')}$ denotes the importance sampling weight. If we can calculate $\rho(\tau', a')$ *with action a' sampled from μ rather than π*, the unseen pairs will be avoided for target Q-value estimation. In this case, the extrapolation error is theoretically avoided since $P_{s,u}^\pi \to 0$. The estimated Q-value based on the above operation would be stable even in complex tasks with enormous action space. However, in most real-world scenarios, it is hard to obtain the exact behavior policy to calculate $\rho(\tau', a')$, e.g., using expert demonstrations. Fortunately, we find that the solution of following implicit constraint optimization problem is efficient to compute the desired importance sampling weight.

4.1.1 Implicit Constraint Q-learning

In offline tasks, the policies similar to the behavior policy are preferred while maximizing the accumulated reward $Q^\pi(\tau, a)$, i.e., $D_{\mathrm{KL}}(\pi \| \mu)[\tau] \le \epsilon$. The policy optimization with the behavior regularized constraint can be described in the following problem:

$$\pi_{k+1} = \arg\max_\pi \mathbb{E}_{a \sim \pi(\cdot|\tau)}[Q^{\pi_k}(\tau, a)], \quad \text{s.t.} \quad D_{\mathrm{KL}}(\pi \| \mu)[\tau] \le \epsilon. \tag{7}$$

This problem has well studied in many previous works [36, 1, 58]. Note that the objective is a linear function of the decision variables π and all constraints are convex functions. Thus we can obtain the optimal policy π^* related to μ through the KKT condition [9], for which the proof is in Appendix B.4:

$$\pi_{k+1}^*(a \mid \tau) = \frac{1}{Z(\tau)} \mu(a \mid \tau) \exp\left(\frac{Q^{\pi_k}(\tau, a)}{\alpha}\right), \tag{8}$$

where $\alpha > 0$ is the Lagrangian coefficient and $Z(\tau) = \sum_{\tilde{a}} \mu(\tilde{a} \mid \tau) \exp\left(\frac{1}{\alpha} Q^{\pi_k}(\tau, \tilde{a})\right)$ is the normalizing partition function. Next, we calculate the ratio between π and μ by relocating μ to the left-hand side:

$$\rho(\tau, a) = \frac{\pi_{k+1}^*(a \mid \tau)}{\mu(a \mid \tau)} = \frac{1}{Z(\tau)} \exp\left(\frac{Q^{\pi_k}(\tau, a)}{\alpha}\right). \tag{9}$$

Motivated on Equation 9, we define the Implicit Constraint Q-learning operator as

$$\mathcal{T}_{\mathrm{ICQ}} Q(\tau, a) = r + \gamma \mathbb{E}_{a' \sim \mu}\left[\frac{1}{Z(\tau')} \exp\left(\frac{Q(\tau', a')}{\alpha}\right) Q(\tau', a')\right]. \tag{10}$$

Thus we obtain a SARAR-like algorithm which not uses any unseen pairs.

Comparison with previous methods. While BCQ learns an action generator to filter unseen pairs in Q-value estimation, it cannot work in enormous action space due to the error of the generator (see Figure 1). Instead, in the value update of ICQ, we do not use the sampled actions to compute the target values, thus we alleviate extrapolation error effectively. There are some previous works, such as AWAC [29] and AWR [35], addressing the offline problem with similar constrained problem in Equation 7. However, these methods only impose the constraint on the policy loss and adopt the standard Bellman operator to evaluate Q-function, which involves the unseen actions or converges to the value of behavior policy μ. Differently, we re-weight the target $Q(\tau', a')$ with the importance sampling weight derived from the optimization problem, which makes the estimated value closer to the optimal value function.

4.1.2 Theoretical Analysis

The ICQ operator in Equation 10 results in a SARSA-like algorithm, which be re-written as:

$$\mathcal{T}_{\text{ICQ}}Q(\tau,a) = r + \gamma \sum_{a' \in \mathcal{B}} \left[\frac{1}{Z(\tau')} \mu(a' \mid \tau') \exp\left(\frac{1}{\alpha} Q(\tau',a') \right) Q(\tau',a') \right]. \quad (11)$$

This update rule can be viewed as a regularized softmax operator [46, 34] in the offline setting. When $\alpha \to \infty$, $\mathcal{T}_{\text{ICQ}}$ approaches $\mathcal{T}^{\mu}$. When $\alpha \to 0$, $\mathcal{T}_{\text{ICQ}}$ becomes the batch-constrained Bellman optimal operator $\mathcal{T}_{\text{BCQ}}$ [16], which constrains the possible actions with respect to the batch:

$$\mathcal{T}_{\text{BCQ}}Q(\tau,a) = r + \gamma \max_{a' \in \mathcal{B}} Q(\tau',a'). \quad (12)$$

$\mathcal{T}_{\text{BCQ}}$ has been shown to converge to the optimal action-value function Q^* of the batch, which means $\lim_{k \to \infty} \mathcal{T}_{\text{BCQ}}^k Q_0 = Q^*$ for arbitrary Q_0. Based on this result, we show that iteratively applying $\mathcal{T}_{\text{ICQ}}$ will result in a Q-function not far away from Q^*:

Theorem 2. *Let $\mathcal{T}_{\text{ICQ}}^k Q_0$ denote that the operator $\mathcal{T}_{\text{ICQ}}$ are iteratively applied over an initial action-value function Q_0 for k times. Then, we have $\forall (\tau,a)$, $\limsup_{k \to \infty} \mathcal{T}_{\text{ICQ}}^k Q_0(\tau,a) \leq Q^*(\tau,a)$,*

$$\liminf_{k \to \infty} \mathcal{T}_{\text{ICQ}}^k Q_0(\tau,a) \geq Q^*(\tau,a) - \frac{\gamma(|A_\tau| - 1)}{(1-\gamma)} \max \left\{ \frac{1}{(\alpha^{-1}+1)C+1}, \frac{2Q_{\max}}{1 + C\exp(\alpha^{-1})} \right\}, \tag{13}$$

where $|A_\tau|$ is the number of seen actions for state τ, $C \triangleq \inf_{\tau \in S} \inf_{2 \leq i \leq |A_\tau|} \frac{\mu(a_{[1]}|\tau)}{\mu(a_{[i]}|\tau)}$ and $\mu(a_{[1]} \mid \tau)$ denotes the probability of choosing the expert action according to behavioral policy μ. Moreover, the upper bound of $\mathcal{T}_{\text{BCQ}}^k Q_0$ - $\mathcal{T}_{\text{ICQ}}^k Q_0$ decays exponentially fast as a function of α.

While $\mathcal{T}_{\text{ICQ}}$ is not a contraction [5] (similar with the softmax operator), the Q-values are still within a reasonable range. Further, $\mathcal{T}_{\text{ICQ}}$ converges to $\mathcal{T}_{\text{BCQ}}$ with an exponential rate in terms of α. Our result also quantifies the difficulty in offline RL problems. Based on the definition of $\mu(a_{[i]}|\tau)$, C shows the proportion of the expert experience in the dataset. A larger C corresponds to more expert experience, which induces a smaller distance between $\mathcal{T}_{\text{ICQ}}^k Q_0(\tau,a)$ and $Q^*(\tau,a)$. In contrast, with a small C, the expert experience is few and the conservatism in learning is necessary.

4.1.3 Algorithm

Based on the derived operator $\mathcal{T}_{\text{ICQ}}$ in Equation 9, we can learn $Q(\tau,a;\phi)$ by minimizing

$$\mathcal{J}_Q(\phi) = \mathbb{E}_{\tau,a,\tau',a' \sim \mathcal{B}} \left[r + \gamma \frac{1}{Z(\tau')} \exp\left(\frac{Q(\tau',a';\phi')}{\alpha} \right) Q(\tau',a';\phi') - Q(\tau,a;\phi) \right]^2, \quad (14)$$

where the Q-network and the target Q-network are parameterized by ϕ and ϕ' respectively.

As for the policy training, we project the non-parametric optimal policy π_{k+1}^* in Equation 8 into the parameterized policy space θ by minimizing the following KL distance, which is implemented on the data distribution of the batch:

$$\mathcal{J}_\pi(\theta) = \mathbb{E}_{\tau \sim \mathcal{B}} \left[D_{\text{KL}} \left(\pi_{k+1}^* \| \pi_\theta \right) [\tau] \right] = \mathbb{E}_{\tau \sim \mathcal{B}} \left[-\sum_a \pi_{k+1}^*(a \mid \tau) \log \frac{\pi_\theta(a \mid \tau)}{\pi_{k+1}^*(a \mid \tau)} \right]$$

$$\overset{(a)}{=} \mathbb{E}_{\tau \sim \mathcal{B}} \left[\sum_a \frac{\pi_{k+1}^*(a \mid \tau)}{\mu(a \mid \tau)} \mu(a \mid \tau) (-\log \pi_\theta(a \mid \tau)) \right] \quad (15)$$

$$\overset{(b)}{=} \mathbb{E}_{\tau,a \sim \mathcal{B}} \left[-\frac{1}{Z(\tau)} \log(\pi(a \mid \tau;\theta)) \exp\left(\frac{Q(\tau,a)}{\alpha} \right) \right],$$

where (a) ignores $\mathbb{E}_{\tau \sim \mathcal{B}} \left[\sum_a \pi_{k+1}^*(a \mid \tau) \log \pi_{k+1}^*(a \mid \tau) \right]$ that is not related to θ, and (b) applies the importance sampling weight derived in Equation 9 under forward KL constraint. Note that tuning the α parameter in Equation 15 between 0 and ∞ interpolates between Q-learning and behavioral cloning. See Appendix A for the complete workflow of the ICQ algorithm. We provide two implementation options to compute the normalizing partition function $Z(\tau)$, which is discussed in detail in Appendix D.1.

4.2 Extending ICQ to Multi-Agent Tasks

In the previous section, we propose an implicit constraint Q-learning framework by re-weighting target Q-value $Q(\tau', a')$ in the critic loss, which is efficient to alleviate the extrapolation error. We next extend ICQ to multi-agent tasks. For notational clarity, we name the **M**ulti-**A**gent version of ICQ as ICQ-MA.

4.2.1 Decomposed Multi-Agent Joint-Policy under Implicit Constraint

Under the CTDE framework, we have to train individual policies for decentralized execution. Besides, it is also challenging to compute $\mathbb{E}_{\mu}[\rho(\tau', a')Q^{\pi}(\tau', a')]$ in multi-agent policy evaluation as its computational complexity is $O(|A|^n)$. To address the above issues, we first define the joint-policy as $\pi(a \mid \tau) \triangleq \Pi_{i \in N}\pi^i(a^i \mid \tau^i)$, and then introduce a mild value-decomposition assumption:

$$Q^{\pi}(\tau, a) = \sum_i w^i(\tau)Q^i(\tau^i, a^i) + b(\tau), \quad (16)$$

where $w^i(\tau) \geq 0$ and $b(\tau)$ are generated by the Mixer Network whose inputs are global observation-action history (see
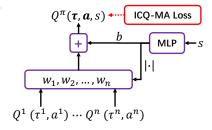

Figure 3: Mixer Network.

Figure 3). Based on the above assumptions, we propose the decomposed multi-agent joint-policy under implicit constraint in the following theorem:

Theorem 3. *Assuming the joint action-value function is linearly decomposed, we can decompose the multi-agent joint-policy under implicit constraint as follows*

$$\pi = \arg\max_{\pi^1, ..., \pi^n} \sum_i \mathbb{E}_{\tau^i, a^i \sim \mathcal{B}} \left[\frac{1}{Z^i(\tau^i)} \log(\pi^i(a^i \mid \tau^i)) \exp\left(\frac{w^i(\tau)Q^i(\tau^i, a^i)}{\alpha} \right) \right], \quad (17)$$

where $Z^i(\tau^i) = \sum_{\tilde{a}^i} \mu^i(\tilde{a}^i \mid \tau^i) \exp\left(\frac{1}{\alpha} w^i(\tau)Q^i(\tau^i, \tilde{a}^i) \right)$ is the normalizing partition function.

The decomposed multi-agent joint-policy has a concise form. We can train individual policies π^i by minimizing

$$\mathcal{J}_{\pi}(\theta) = \sum_i \mathbb{E}_{\tau^i, a^i \sim \mathcal{B}} \left[-\frac{1}{Z^i(\tau^i)} \log(\pi^i(a^i \mid \tau^i; \theta_i)) \exp\left(\frac{w^i(\tau)Q^i(\tau^i, a^i)}{\alpha} \right) \right]. \quad (18)$$

Besides, $w^i(\tau)$ achieves the trade-off between the roles of agents. If some agents have important roles, the value of corresponding $w^i(\tau)$ is relatively large. Also, if $w^i(\tau) \to 0$, π^i is approximately considered as the behavior cloning policy. As for the policy evaluation, we train $Q(\tau, a; \phi, \psi)$ by minimizing

$$\mathcal{J}_Q(\phi, \psi) = \mathbb{E}_{\mathcal{B}} \left[\sum_{t \geq 0} (\gamma\lambda)^t \left(r_t + \gamma \frac{1}{Z(\tau_{t+1})} \exp\left(\frac{Q(\tau_{t+1}, a_{t+1})}{\alpha} \right) Q(\tau_{t+1}, a_{t+1}) - Q(\tau_t, a_t) \right) \right]^2, \quad (19)$$

where $Q(\tau_{t+1}, a_{t+1}) = \sum_i w^i(\tau_{t+1}; \psi')Q^i(\tau_{t+1}^i, a_{t+1}^i; \phi_i') - b(\tau_{t+1}; \psi')$.

4.2.2 Multi-Agent Value Estimation with λ-return

As the offline dataset contains complete behavior trajectories, it is natural to accelerate the convergence of ICQ with the n-step method. Here we adopt $Q(\lambda)$ [27] to improve the estimation of ICQ, which weights the future temporal difference signal with a decay sequence λ^t. Further, the constraint in Equation 7 implicitly meets the convergence condition of $Q(\lambda)$. Therefore, we extend the ICQ operator in Equation 10 to n-step estimation, which is similar to $Q(\lambda)$:

$$(\mathcal{T}_{\text{ICQ}}^{\lambda}Q)(\tau, a) \triangleq Q(\tau, a) + \mathbb{E}_{\mu} \left[\sum_{t \geq 0} (\gamma\lambda)^t \left(r_t + \gamma\rho(\tau_{t+1}, a_{t+1})Q(\tau_{t+1}, a_{t+1}) - Q(\tau_t, a_t) \right) \right], \quad (20)$$

where $\rho(\tau_t, a_t) = \frac{1}{Z(\tau_t)} \exp(\frac{1}{\alpha}Q(\tau_t, a_t))$ and hyper-parameter $0 \leq \lambda \leq 1$ provides the balance between bias and variance.
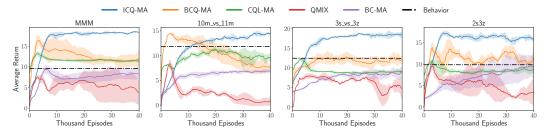
7

Figure 4: Performance comparison in offline StarCraft II tasks.

Table 1: Performance of ICQ with five offline RL baselines on the single-agent offline tasks with the normalized score metric proposed by D4RL benchmark [14], averaged over three random seeds with standard deviation. Scores roughly range from 0 to 100, where 0 corresponds to a random policy performance and 100 indicates an expert. The results for BC, BCQ, CQL, AWR and BRAC-p are taken from [14, 22].

Dataset type	Environment	ICQ (ours)	BC	BCQ	CQL	AWR	BRAC-p
fixed	antmaze-umaze	**85.0 ± 2.7**	65.0	78.9	74.0	56.0	50.0
play	antmaze-medium	**80.0 ± 1.3**	0.0	0.0	61.2	0.0	0.0
play	antmaze-large	**51.0 ± 4.8**	0.0	6.7	15.8	0.0	0.0
diverse	antmaze-umaze	65.0±3.3	55.0	55.0	84.0	**70.3**	40.0
diverse	antmaze-medium	**65.0 ± 3.9**	0.0	0.0	53.7	0.0	0.0
diverse	antmaze-large	**44.0 ± 4.2**	0.0	2.2	14.9	0.0	0.0
expert	adroit-door	**103.9 ± 3.6**	101.2	99.0	-	102.9	-0.3
expert	adroit-relocate	**109.5 ± 11.1**	101.3	41.6	-	91.5	-0.3
expert	adroit-pen	**123.8 ± 22.1**	85.1	114.9	-	111.0	-3.5
expert	adroit-hammer	**128.3 ± 2.5**	125.6	107.2	-	39.0	0.3
human	adroit-door	6.4±2.4	0.5	-0.0	**9.1**	0.4	-0.3
human	adroit-relocate	**1.5 ± 0.7**	-0.0	-0.1	0.35	-0.0	-0.3
human	adroit-pen	**91.3 ± 10.3**	34.4	68.9	55.8	12.3	8.1
human	adroit-hammer	2.0±0.9	1.5	0.5	**2.1**	1.2	0.3
medium	walker2d	71.8±10.7	66.6	53.1	**79.2**	17.4	77.5
medium	hopper	55.6±5.7	49.0	54.5	**58.0**	35.9	32.7
medium	halfcheetah	42.5±1.3	36.1	40.7	**44.4**	37.4	43.8
med-expert	walker2d	**98.9 ± 5.2**	66.8	57.5	98.7	53.8	76.9
med-expert	hopper	109.0±13.6	**111.9**	110.9	111.0	27.1	1.9
med-expert	halfcheetah	**110.3 ± 1.1**	35.8	64.7	104.8	52.7	44.2

5 Related Work

As ICQ-MA seems to be the first work addressing the accumulated extrapolation error issue in offline MARL, we briefly review the prior single-agent offline RL works here, which can be divided into three categories: dynamic programming, model-based, and safe policy improvement methods.

Dynamic Programming. Policy constraint methods in dynamic programming [20, 3, 58, 51, 17] are most closely related to our work. They attempt to enforce π to be close to μ under KL-divergence, Wasserstein distance [53], or MMD [47], and then only use actions sampled from π in dynamic programming. For example, BCQ [16] constrains the mismatch between the state-action visitation of the policy and the state-action pairs contained in the batch by using a state-conditioned generative model to produce only previously seen actions. AWR [35] and ABM [42] attempt to estimate the value function of the behavior policy via Monte-Carlo or TD(λ). Unlike these methods, our algorithm, ICQ, estimates the Q-function of the current policy using actions sampled from μ, enabling much more efficient learning. Another series of methods [52, 32, 33] aim to estimate uncertainty to determine the trustworthiness of a Q-value prediction. However, the high-fidelity requirements for uncertainty estimates limit the performance of algorithms.
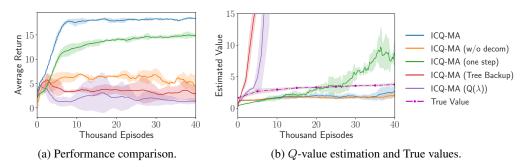
| (a) Performance comparison. | (b) Q-value estimation and True values. |

Figure 5: Module ablation study on MMM map.

Model-based and Safe Policy Improvement. Model-based methods [18, 50, 13, 56, 19] attempt to learn the model from offline data, with minimal modification to the algorithm. Nevertheless, modeling MDPs with very high-dimensional image observations and long horizons is a major open problem, which leads to limited algorithm performance [24]. Besides, safe policy improvement methods [23, 44, 6, 11] require a separately estimated model to μ to deal with unseen actions. However, accurately estimating μ is especially hard if the data come from multiple sources [29].

6 Experiments

In this section, we evaluate ICQ-MA and ICQ on multi-agent (StarCraft II) and single-agent (D4RL) offline benchmarks and compare them with state-of-the-art methods. Then, we conduct ablation studies on ICQ-MA. We aim to better understand each component's effect and further analyze the main driver for the performance improvement.

6.1 Multi-Agent Offline Tasks on StarCraft II

We first construct the multi-agent offline datasets based on ten maps in StarCraft II (see Table 2 in Appendix E). The datasets are made by collecting DOP [55] training data. All maps share the same reward function, and each map includes 3000 trajectories. We are interested in non-expert data or multi-source data. Therefore, we artificially divide behavior policies into three levels based on the average episode return (see Table 3 in Appendix E). Then, we evenly mix data of three levels.

We compare our method against QMIX [39], multi-agent version of BCQ (BCQ-MA), CQL (CQL-MA), and behavior cloning (BC-MA). To maintain consistency, BCQ-MA, CQL-MA, and BC-MA share the same linear value decomposition structure with ICQ-MA. Details for baseline implementations are in Appendix D.2. Each algorithm runs with five seeds, where the performance is evaluated ten times every 50 episodes. Details for hyper-parameters are in Appendix E.1.

We investigate ICQ-MA's performance compared to common baselines in different scenarios. Results in Figure 4 show that ICQ-MA significantly outperforms all baselines and achieves state-of-the-art performance in all maps. QMIX, BCQ-MA, and CQL-MA have poor performances due to the accumulated extrapolation error. Interestingly, since BC does not depend on the policy evaluation, it is not subject to extrapolation error. Thus BC-MA has a sound performance as StarCraft II is near deterministic. We implement BCQ and CQL according to their official code[*].

6.2 Single-Agent Offline Tasks on D4RL

To compare with current offline methods, we evaluate ICQ in the single offline tasks (e.g., D4RL), including gym domains, Adroit tasks [38] and AntMaze. Specifically, adroit tasks require controlling a 24-DoF robotic hand to imitate human behavior. AntMaze requires composing parts of sub-optimal trajectories to form more optimal policies for reaching goals on a MuJoco Ant robot. Experimental result in Table 1 shows that ICQ achieves the state-of-the-art performance in many tasks compared with the current offline methods.

---

[*]BCQ: https://github.com/sfujim/BCQ,   CQL: https://github.com/aviralkumar2907/CQL.

6.3 Ablation Study

We conduct ablation studies of ICQ-MA in the MMM map of StarCraft II to study the effect of different modules, value estimation, important hyper-parameters, and data quality.

Module and Value Estimation Analysis. From Figure 5, we find that if we adopt other Q-value estimation methods in implicit constraint policies (e.g., $Q(\lambda)$ [27] or Tree Backup), the corresponding algorithms (ICQ-MA ($Q(\lambda)$) or ICQ-MA (Tree Backup)) have poor performances and incorrect estimated values. Suppose we train ICQ-MA without decomposed implicit constraint module (e.g., ICQ-MA (w/o decom)). In that case, the algorithm's performance is poor, although the estimated value is smaller than the true value, confirming the necessity of decomposed policy. Besides, the performance of one-step estimation (ICQ-MA (one step)) indicates n-step estimation is not the critical factor for improving ICQ-MA, while one-step estimation will introduce more bias.

The Parameter α. The Lagrangian coefficient α of implicit constraint operator directly affects the intensity of constraint, which is a critical parameter for the performance. A smaller α leads to a relaxing constraint and tends to maximize reward. If $\alpha \to 0$, ICQ-MA is simplified to Q-learning [57] while $\alpha \to \infty$ results in that ICQ-MA is equivalent to behavior cloning. Indeed, there is an intermediate value that performs best that can best provide the trade-off as in Appendix C.4.

Data Quality. It is also worth studying the performance of ICQ-MA and BC-MA with varying data quality. Specifically, we make the datasets from behavior policies of different levels (e.g., Good, Medium, and Poor). As shown in Figure 9 in Appendix C.4, ICQ-MA is not sensitive to the data quality, while the performance of BC-MA drops drastically with the data quality deteriorates. Results confirm that ICQ-MA is robust to the data quality while BC-MA strongly relies on the data quality.

Computational Complexity. With the same training steps in SMAC, BCQ-MA consumes 70% time of ICQ-MA. Although ICQ-MA takes a little long time compared with BCQ-MA, it achieves excellent performance in benchmarks. The computing infrastructure for running experiments is a server with an AMD EPYC 7702 64-Core Processor CPU.

7 Conclusion

In this work, we demonstrate a critical problem in multi-agent off-policy reinforcement learning with finite data, where it introduces accumulated extrapolation error in the number of agents. We empirically show the current offline algorithms are ineffective in the multi-agent offline setting. Therefore, we propose the Implicit Constraint Q-learning (ICQ) method, which effectively alleviates extrapolation error by only trusting the state-action pairs in datasets. To the best of our knowledge, the multi-agent version of ICQ is the first multi-agent offline algorithm capable of learning from complex multi-agent datasets. Due to the importance of offline tasks and multi-agent systems, we sincerely hope our algorithms can be a solid foothold for applying RL to practical applications.

Acknowledgments and Disclosure of Funding

References

[1] Abbas Abdolmaleki, Jost Tobias Springenberg, Yuval Tassa, Remi Munos, Nicolas Heess, and Martin Riedmiller. Maximum a posteriori policy optimisation. In *International Conference on Learning Representations*, 2018.

[2] Jeffrey L Adler and Victor J Blue. A cooperative multi-agent transportation management and route guidance system. *Transportation Research Part C: Emerging Technologies*, 10(5-6):433–454, 2002.

[3] Rishabh Agarwal, Dale Schuurmans, and Mohammad Norouzi. An optimistic perspective on offline reinforcement learning. In *International Conference on Machine Learning*, pages 104–114. PMLR, 2020.

[4] Anurag Ajay, Aviral Kumar, Pulkit Agrawal, Sergey Levine, and Ofir Nachum. OPAL: Offline primitive discovery for accelerating offline reinforcement learning. In *International Conference on Learning Representations*, 2020.

[5] Kavosh Asadi and Michael L Littman. An alternative softmax operator for reinforcement learning. In *International Conference on Machine Learning*, pages 243–252. PMLR, 2017.

[6] Felix Berkenkamp, Matteo Turchetta, Angela P Schoellig, and Andreas Krause. Safe model-based reinforcement learning with stability guarantees. *arXiv preprint arXiv:1705.08551*, 2017.

[7] Duncan S Callaway and Ian A Hiskens. Achieving controllability of electric loads. *Proceedings of the IEEE*, 99(1):184–199, 2010.

[8] Yuanjiang Cao, Xiaocong Chen, Lina Yao, Xianzhi Wang, and Wei Emma Zhang. Adversarial attacks and detection on reinforcement learning-based interactive recommender systems. In *Proceedings of the 43rd International ACM SIGIR Conference on Research and Development in Information Retrieval*, pages 1669–1672, 2020.

[9] Axel Dreves, Francisco Facchinei, Christian Kanzow, and Simone Sagratella. On the solution of the KKT conditions of generalized nash equilibrium problems. *SIAM Journal on Optimization*, 21(3):1082–1108, 2011.

[10] Damien Ernst, Pierre Geurts, and Louis Wehenkel. Tree-based batch mode reinforcement learning. *Journal of Machine Learning Research*, 6:503–556, 2005.

[11] Benjamin Eysenbach, Shixiang Gu, Julian Ibarz, and Sergey Levine. Leave no trace: Learning to reset for safe and autonomous reinforcement learning. *arXiv preprint arXiv:1711.06782*, 2017.

[12] Rasool Fakoor, Pratik Chaudhari, and Alexander J Smola. P3O: Policy-on policy-off policy optimization. In *Uncertainty in Artificial Intelligence*, pages 1017–1027. PMLR, 2020.

[13] Mehrdad Farajtabar, Yinlam Chow, and Mohammad Ghavamzadeh. More robust doubly robust off-policy evaluation. In *International Conference on Machine Learning*, pages 1447–1456. PMLR, 2018.

[14] Justin Fu, Aviral Kumar, Ofir Nachum, George Tucker, and Sergey Levine. D4RL: Datasets for deep data-driven reinforcement learning. *arXiv preprint arXiv:2004.07219*, 2020.

[15] Scott Fujimoto, Edoardo Conti, Mohammad Ghavamzadeh, and Joelle Pineau. Benchmarking batch deep reinforcement learning algorithms. *arXiv preprint arXiv:1910.01708*, 2019.

[16] Scott Fujimoto, David Meger, and Doina Precup. Off-policy deep reinforcement learning without exploration. In *International Conference on Machine Learning*, pages 2052–2062. PMLR, 2019.

[17] Natasha Jaques, Asma Ghandeharioun, Judy Hanwen Shen, Craig Ferguson, Agata Lapedriza, Noah Jones, Shixiang Gu, and Rosalind Picard. Way off-policy batch deep reinforcement learning of implicit human preferences in dialog. *arXiv preprint arXiv:1907.00456*, 2019.

[18] Nan Jiang and Lihong Li. Doubly robust off-policy value evaluation for reinforcement learning. In *International Conference on Machine Learning*, pages 652–661. PMLR, 2016.

[19] Gregory Kahn, Adam Villaflor, Pieter Abbeel, and Sergey Levine. Composable action-conditioned predictors: Flexible off-policy learning for robot navigation. In *Conference on Robot Learning*, pages 806–816. PMLR, 2018.

[20] Dmitry Kalashnikov, Alex Irpan, Peter Pastor, Julian Ibarz, Alexander Herzog, Eric Jang, Deirdre Quillen, Ethan Holly, Mrinal Kalakrishnan, Vincent Vanhoucke, et al. Scalable deep reinforcement learning for vision-based robotic manipulation. In *Conference on Robot Learning*, pages 651–673. PMLR, 2018.

[21] Aviral Kumar, Justin Fu, Matthew Soh, George Tucker, and Sergey Levine. Stabilizing off-policy q-learning via bootstrapping error reduction. In *Advances in Neural Information Processing Systems*, pages 11784–11794, 2019.

[22] Aviral Kumar, Aurick Zhou, George Tucker, and Sergey Levine. Conservative q-learning for offline reinforcement learning. *arXiv preprint arXiv:2006.04779*, 2020.

[23] Romain Laroche, Paul Trichelair, and Remi Tachet Des Combes. Safe policy improvement with baseline bootstrapping. In *International Conference on Machine Learning*, pages 3652–3661. PMLR, 2019.

[24] Sergey Levine, Aviral Kumar, George Tucker, and Justin Fu. Offline reinforcement learning: Tutorial, review, and perspectives on open problems. *arXiv preprint arXiv:2005.01643*, 2020.

[25] Chenghao Li, Chengjie Wu, Tonghan Wang, Jun Yang, Qianchuan Zhao, and Chongjie Zhang. Celebrating diversity in shared multi-agent reinforcement learning. *arXiv preprint arXiv:2106.02195*, 2021.

[26] Xiaoteng Ma, Yiqin Yang, Chenghao Li, Yiwen Lu, Qianchuan Zhao, and Jun Yang. Modeling the interaction between agents in cooperative multi-agent reinforcement learning. In *Proceedings of the 20th International Conference on Autonomous Agents and MultiAgent Systems*, pages 853–861, 2021.

[27] Rémi Munos. Q(λ) with off-policy corrections. In *Algorithmic Learning Theory: International Conference, ALT 2016, Bari, Italy, October 19-21, 2016, Proceedings*, volume 9925, page 305. Springer, 2016.

[28] Ofir Nachum, Yinlam Chow, Bo Dai, and Lihong Li. DualDICE: Behavior-agnostic estimation of discounted stationary distribution corrections. *Advances in Neural Information Processing Systems*, 32:2318–2328, 2019.

[29] Ashvin Nair, Murtaza Dalal, Abhishek Gupta, and Sergey Levine. Accelerating online reinforcement learning with offline datasets. *arXiv preprint arXiv:2006.09359*, 2020.

[30] Radford M Neal. Annealed importance sampling. *Statistics and computing*, 11(2):125–139, 2001.

[31] Frans A Oliehoek, Christopher Amato, et al. *A concise introduction to decentralized POMDPs*, volume 1. Springer, 2016.

[32] Ian Osband, John Aslanides, and Albin Cassirer. Randomized prior functions for deep reinforcement learning. In *Proceedings of the 32nd International Conference on Neural Information Processing Systems*, pages 8626–8638, 2018.

[33] Brendan O'Donoghue, Ian Osband, Remi Munos, and Volodymyr Mnih. The uncertainty bellman equation and exploration. In *International Conference on Machine Learning*, pages 3836–3845, 2018.

[34] Ling Pan, Qingpeng Cai, and Longbo Huang. Softmax deep double deterministic policy gradients. In *Advances in Neural Information Processing Systems*, volume 33, pages 11767–11777, 2020.

[35] Xue Bin Peng, Aviral Kumar, Grace Zhang, and Sergey Levine. Advantage-weighted regression: Simple and scalable off-policy reinforcement learning. *arXiv preprint arXiv:1910.00177*, 2019.

[36] Jan Peters, Katharina Mulling, and Yasemin Altun. Relative entropy policy search. In *AAAI Conference on Artificial Intelligence*, volume 24, 2010.

[37] Michael Rabbat and Robert Nowak. Distributed optimization in sensor networks. In *Proceedings of the 3rd international symposium on Information processing in sensor networks*, pages 20–27, 2004.

[38] Aravind Rajeswaran, Vikash Kumar, Abhishek Gupta, Giulia Vezzani, John Schulman, Emanuel Todorov, and Sergey Levine. Learning complex dexterous manipulation with deep reinforcement learning and demonstrations. *arXiv preprint arXiv:1709.10087*, 2017.

[39] Tabish Rashid, Mikayel Samvelyan, Christian Schroeder, Gregory Farquhar, Jakob Foerster, and Shimon Whiteson. QMIX: Monotonic value function factorisation for deep multi-agent reinforcement learning. In *International Conference on Machine Learning*, pages 4295–4304, 2018.

[40] Mikayel Samvelyan, Tabish Rashid, Christian Schroeder de Witt, Gregory Farquhar, Nantas Nardelli, Tim GJ Rudner, Chia-Man Hung, Philip HS Torr, Jakob Foerster, and Shimon Whiteson. The StarCraft multi-agent challenge. In *International Conference on Autonomous Agents and Multiagent Systems*, pages 2186–2188, 2019.

[41] Juan C Santamaria, Richard S Sutton, and Ashwin Ram. Experiments with reinforcement learning in problems with continuous state and action spaces. *Adaptive behavior*, 6(2):163–217, 1997.

[42] Noah Siegel, Jost Tobias Springenberg, Felix Berkenkamp, Abbas Abdolmaleki, Michael Neunert, Thomas Lampe, Roland Hafner, Nicolas Heess, and Martin Riedmiller. Keep doing what worked: Behavior modelling priors for offline reinforcement learning. In *International Conference on Learning Representations*, 2019.

[43] Kyunghwan Son, Daewoo Kim, Wan Ju Kang, David Earl Hostallero, and Yung Yi. QTRAN: Learning to factorize with transformation for cooperative multi-agent reinforcement learning. In *International Conference on Machine Learning*, pages 5887–5896, 2019.

[44] Aaron Sonabend-W, Junwei Lu, Leo A Celi, Tianxi Cai, and Peter Szolovits. Expert-supervised reinforcement learning for offline policy learning and evaluation. *arXiv preprint arXiv:2006.13189*, 2020.

[45] H Francis Song, Abbas Abdolmaleki, Jost Tobias Springenberg, Aidan Clark, Hubert Soyer, Jack W Rae, Seb Noury, Arun Ahuja, Siqi Liu, Dhruva Tirumala, et al. V-mpo: On-policy maximum a posteriori policy optimization for discrete and continuous control. In *International Conference on Learning Representations*, 2019.

[46] Zhao Song, Ron Parr, and Lawrence Carin. Revisiting the softmax bellman operator: New benefits and new perspective. In *International Conference on Machine Learning*, pages 5916–5925. PMLR, 2019.

[47] Bharath K Sriperumbudur, Kenji Fukumizu, Arthur Gretton, Bernhard Schölkopf, Gert RG Lanckriet, et al. On the empirical estimation of integral probability metrics. *Electronic Journal of Statistics*, 6:1550–1599, 2012.

[48] Peter Sunehag, Guy Lever, Audrunas Gruslys, Wojciech Marian Czarnecki, Vinicius Zambaldi, Max Jaderberg, Marc Lanctot, Nicolas Sonnerat, Joel Z Leibo, Karl Tuyls, et al. Value-decomposition networks for cooperative multi-agent learning based on team reward. In *International Conference on Autonomous Agents and MultiAgent Systems*, pages 2085–2087, 2018.

[49] Richard S. Sutton and Andrew G. Barto. *Reinforcement Learning: An Introduction*. MIT press, 2018.

[50] Philip Thomas and Emma Brunskill. Data-efficient off-policy policy evaluation for reinforcement learning. In *International Conference on Machine Learning*, pages 2139–2148. PMLR, 2016.

[51] Emanuel Todorov. Linearly-solvable markov decision problems. In *Advances in neural information processing systems*, pages 1369–1376, 2007.

[52] Ahmed Touati, Harsh Satija, Joshua Romoff, Joelle Pineau, and Pascal Vincent. Randomized value functions via multiplicative normalizing flows. In *Uncertainty in Artificial Intelligence*, pages 422–432. PMLR, 2020.

[53] SS Vallender. Calculation of the wasserstein distance between probability distributions on the line. *Theory of Probability & Its Applications*, 18(4):784–786, 1974.

[54] Quan Vuong, Yiming Zhang, and Keith W Ross. Supervised policy update for deep reinforcement learning. In *International Conference on Learning Representations*, 2018.

[55] Yihan Wang, Beining Han, Tonghan Wang, Heng Dong, and Chongjie Zhang. Off-policy multi-agent decomposed policy gradients. *arXiv preprint arXiv:2007.12322*, 2020.

[56] Yu-Xiang Wang, Alekh Agarwal, and Miroslav Dudık. Optimal and adaptive off-policy evaluation in contextual bandits. In *International Conference on Machine Learning*, pages 3589–3597. PMLR, 2017.

[57] Christopher JCH Watkins and Peter Dayan. Q-learning. *Machine learning*, 8(3-4):279–292, 1992.

[58] Yifan Wu, George Tucker, and Ofir Nachum. Behavior regularized offline reinforcement learning. *arXiv preprint arXiv:1911.11361*, 2019.

[59] Deheng Ye, Zhao Liu, Mingfei Sun, Bei Shi, Peilin Zhao, Hao Wu, Hongsheng Yu, Shaojie Yang, Xipeng Wu, Qingwei Guo, et al. Mastering complex control in MOBA games with deep reinforcement learning. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 34, pages 6672–6679, 2020.

A Algorithms

The single-agent version of ICQ is shown in Algorithm 1. Its multi-agent version counterpart (ICQ-MA) is shown in Algorithm 2.

Algorithm 1: Implicit Constraint Q-Learning in Single-Agent Tasks.

Input: Offline buffer $\mathcal{B}$, target network update rate d.

Initialize critic network $Q^\pi(\cdot; \phi)$ and actor network $\pi(\cdot; \theta)$ with random parameters.
Initialize target networks: $\phi' = \phi$, $\theta' = \theta$.
for $t = 1$ **to** T **do**
$\quad$ Sample trajectories from $\mathcal{B}$.
$\quad$ Train policy according to $\mathcal{J}_\pi(\theta) = \mathbb{E}_{\tau \sim \mathcal{B}} \left[-\frac{1}{Z(\tau)} \log(\pi(a \mid \tau; \theta)) \exp\left(\frac{Q^\pi(\tau, a)}{\alpha} \right) \right]$.
$\quad$ Train critic according to

$$\mathcal{J}_Q(\phi) = \mathbb{E}_{\tau \sim \mathcal{B}} \left[r + \gamma \frac{1}{Z(\tau')} \exp\left(\frac{Q(\tau', a'; \phi')}{\alpha} \right) Q(\tau', a'; \phi') - Q(\tau, a; \phi) \right]^2.$$

$\quad$ **if** $t \bmod d = 0$ **then**
$\quad\quad$ | Update target networks: $\phi' = \phi$, $\theta' = \theta$.
$\quad$ **end**
end

Algorithm 2: Implicit Constraint Q-Learning in Multi-Agent Tasks.

Input: Offline buffer $\mathcal{B}$, target network update rate d.

Initialize critic networks $Q^i(\cdot; \phi_i)$, actor networks $\pi^i(\cdot; \theta_i)$ and Mixer network $M(\cdot; \psi)$ with random parameters.
Initialize target networks: $\phi' = \phi$, $\theta' = \theta$, $\psi' = \psi$.
for $t = 1$ **to** T **do**
$\quad$ Sample trajectories from $\mathcal{B}$.
$\quad$ Train individual policy according to
$\quad\quad$ $\mathcal{J}_{\boldsymbol{\pi}}(\theta) = \sum_i \mathbb{E}_{\tau^i, a^i \sim \mathcal{B}} \left[-\frac{1}{Z^i(\tau^i)} \log(\pi^i(a^i \mid \tau^i; \theta_i)) \exp\left(\frac{w^i(\boldsymbol{\tau}) Q^i(\tau^i, a^i)}{\alpha} \right) \right]$.
$\quad$ Train critic according to $\mathcal{J}_Q(\phi, \psi) =$

$$\mathbb{E}_{\mathcal{B}} \left[\sum_{t \geq 0} (\gamma\lambda)^t \left[r_t + \gamma \frac{\exp\left(\frac{1}{\alpha} Q(\boldsymbol{\tau}_{t+1}, \boldsymbol{a}_{t+1}; \phi', \psi') \right)}{Z(\boldsymbol{\tau}_{t+1}; \phi', \psi')} Q(\boldsymbol{\tau}_{t+1}, \boldsymbol{a}_{t+1}; \phi', \psi') - Q(\boldsymbol{\tau}_t, \boldsymbol{a}_t; \phi, \psi) \right] \right]^2.$$

$\quad$ **if** $t \bmod d = 0$ **then**
$\quad\quad$ | Update target networks: $\phi' = \phi$, $\theta' = \theta$, $\psi' = \psi$.
$\quad$ **end**
end

B Detailed Proof

B.1 Proof of Theorem 1

Theorem 1. *Given a deterministic MDP, the propagation of $\epsilon_{\mathbf{b}}$ to $\epsilon_{\mathbf{s}}$ is proportional to $\|P_{\mathrm{s,u}}^{\pi}\|_{\infty}$:*

$$\|\epsilon_{\mathbf{s}}\|_{\infty} \leq \frac{\gamma \left\|P_{\mathrm{s,u}}^{\pi}\right\|_{\infty}}{(1-\gamma)\left(1-\gamma\left\|P_{\mathrm{s,s}}^{\pi}\right\|_{\infty}\right)} \|\epsilon_{\mathbf{b}}\|_{\infty} . \tag{21}$$

Proof. Based on the Remark 1 in BCQ [16], the exact form of $\epsilon_{\mathrm{MDP}}(\tau, a)$ is:

$$\epsilon_{\mathrm{MDP}}(\tau, a) = Q_M^{\pi}(\tau, a) - Q_{\mathcal{B}}^{\pi}(\tau, a)$$

$$= \sum_{\tau'} (P_M(\tau' \mid \tau, a) - P_{\mathcal{B}}(\tau' \mid \tau, a)) \left(r(\tau, a, \tau') + \gamma \sum_{a'} \pi(a' \mid \tau') Q_{\mathcal{B}}^{\pi}(\tau', a') \right)$$

$$+ P_M(\tau' \mid \tau, a) \gamma \sum_{a'} \pi(a' \mid \tau') \epsilon_{\mathrm{MDP}}(\tau', a'), \tag{22}$$

where $P_{\mathcal{B}} = \frac{\mathcal{N}(\tau, a, \tau')}{\sum_{\tilde{\tau}} \mathcal{N}(\tau, a, \tilde{\tau})}$ and $\mathcal{N}$ is the number of times the tuple (τ, a, τ') is observed in $\mathcal{B}$. If $\sum_{\tilde{\tau}} \mathcal{N}(\tau, a, \tilde{\tau}) = 0$, then $P_{\mathcal{B}}(\tau_{\mathrm{init}} \mid \tau, a) = 1$. Since the considered MDP is deterministic, we have $P_M(\tau' \mid \tau, a) - P_{\mathcal{B}}(\tau' \mid \tau, a) = 0$ for $P_{\mathrm{s,s}}^{\pi}$ and $P_{\mathrm{s,u}}^{\pi}$. For notational simplicity, the error generated by $P_M(\tau' \mid \tau, a) - P_{\mathcal{B}}(\tau' \mid \tau, a)$ in $P_{\mathrm{u,s}}^{\pi}$ and $P_{\mathrm{u,u}}^{\pi}$ is attributed to $\epsilon_{\mathbf{b}}$ as they have the same dimension. Then, based on the extrapolation error decomposition assumption, we rewrite Equation 22 in the matrix form:

$$\begin{bmatrix} \epsilon_{\mathbf{s}} \\ \epsilon_{\mathbf{u}} \end{bmatrix} = \gamma \begin{bmatrix} P_{\mathrm{s,s}}^{\pi} & P_{\mathrm{s,u}}^{\pi} \\ P_{\mathrm{u,s}}^{\pi} & P_{\mathrm{u,u}}^{\pi} \end{bmatrix} \begin{bmatrix} \epsilon_{\mathbf{s}} \\ \epsilon_{\mathbf{u}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \epsilon_{\mathbf{b}} \end{bmatrix}. \tag{23}$$

The result indicates that the error is the solution of a linear program with $[0, \epsilon_{\mathbf{b}}]^T$ as the reward function. Thus, we solve this linear program and arrive at

$$\begin{bmatrix} \epsilon_{\mathbf{s}} \\ \epsilon_{\mathbf{u}} \end{bmatrix} = (I - \gamma P^{\pi})^{-1} \begin{bmatrix} 0 \\ \epsilon_{\mathbf{b}} \end{bmatrix} = \begin{bmatrix} I - \gamma P_{\mathrm{s,s}}^{\pi} & -\gamma P_{\mathrm{s,u}}^{\pi} \\ -\gamma P_{\mathrm{u,s}}^{\pi} & I - \gamma P_{\mathrm{u,u}}^{\pi} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \epsilon_{\mathbf{b}} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \epsilon_{\mathbf{b}} \end{bmatrix}. \tag{24}$$

With the block matrix inverse formula, we have

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \begin{bmatrix} A^{-1} + A^{-1} B \left(D - CA^{-1}B\right)^{-1} CA^{-1} & -A^{-1} B \left(D - CA^{-1}B\right)^{-1} \\ -\left(D - CA^{-1}B\right)^{-1} CA^{-1} & \left(D - CA^{-1}B\right)^{-1} \end{bmatrix}. \tag{25}$$

Since $\left(D - CA^{-1}B\right)^{-1}$ is just the lower right block of $(I - \gamma P^{\pi})^{-1}$, we have

$$\left\|\left(D - CA^{-1}B\right)^{-1}\right\|_{\infty} \leq \left\|(I - \gamma P^{\pi})^{-1}\right\|_{\infty} \leq \frac{1}{1-\gamma}. \tag{26}$$

Thus, we obtain

$$\left\|-A^{-1} B \left(D - CA^{-1}B\right)^{-1}\right\|_{\infty} \leq \|A^{-1}\|_{\infty} \|-B\|_{\infty} \left\|\left(D - CA^{-1}B\right)^{-1}\right\|_{\infty}$$

$$\leq \frac{1}{1-\gamma} \|A^{-1}\|_{\infty} \|-B\|_{\infty}$$

$$= \frac{1}{1-\gamma} \left\|\left(I - \gamma P_{\mathrm{s,s}}^{\pi}\right)^{-1}\right\|_{\infty} \|\gamma P_{\mathrm{s,u}}^{\pi}\|_{\infty} \tag{27}$$

$$\leq \frac{\gamma \left\|P_{\mathrm{s,u}}^{\pi}\right\|_{\infty}}{(1-\gamma)\left(1-\gamma\left\|P_{\mathrm{s,s}}^{\pi}\right\|_{\infty}\right)}.$$

Plugging the result into Equation 25, we finish our proof at

$$\|\epsilon_{\mathbf{s}}\|_{\infty} \leq \left\|-A^{-1} B \left(D - CA^{-1}B\right)^{-1}\right\|_{\infty} \|\epsilon_{\mathbf{b}}\|_{\infty} \leq \frac{\gamma \left\|P_{\mathrm{s,u}}^{\pi}\right\|_{\infty}}{(1-\gamma)\left(1-\gamma\left\|P_{\mathrm{s,s}}^{\pi}\right\|_{\infty}\right)} \|\epsilon_{\mathbf{b}}\|_{\infty} . \tag{28}$$

$\square$

B.2 Proof of Theorem 2

The proof of our Theorem 2 is based on the Theorem 3 in [46]. The main difference is that we consider a behavior policy to regularize the softmax operation. All the actions considered in the analysis are batch-constrained, thus $\mu(a \mid \tau) > 0, \forall \tau, a$ in the proof.

Lemma 1. *By assuming $f_\alpha^T(Q(\tau,))Q(\tau,)$ as target value of the Implicit Constraint Q-learning operator, we have $\forall Q$,* $\max_{a \sim \mathcal{B}} Q(\tau, a) - f_\alpha^T(Q(\tau,))Q(\tau,) \leq (|A_\tau| - 1) \max\{\frac{1}{(\frac{1}{\alpha}+1)C+1}, \frac{2Q_{\max}}{1+C\exp(\frac{1}{\alpha})}\}$, *where $Q_{\max} = \frac{R_{\max}}{1-\gamma}$ represents the maximum Q-value in Q-iteration with $\mathcal{T}_{\mathrm{ICQ}}$.*

Proof. The target value operation of Implicit Constraint Q-learning is defined as:

$$f_\alpha(\tau \mid \mu) = \frac{\left[\mu_1 \exp(\frac{1}{\alpha}\tau_1), \mu_2 \exp(\frac{1}{\alpha}\tau_2), ..., \mu_{|A_\tau|} \exp(\frac{1}{\alpha}\tau_{|A_\tau|})\right]^T}{\sum_{i=1}^{|A_\tau|} \mu_i \exp(\frac{1}{\alpha}\tau_i)}, \tag{29}$$

We first sort the sequence $\{Q(\tau, a_i)\}$ such that $Q(\tau, a_{[1]}) \geq \cdots \geq Q(\tau, a_{[|A_\tau|]})$. Then, $\forall Q$ and $\forall \tau$, we have that the distance between optimal Q-value and Implicit Constraint Q-value is:

$$\begin{aligned}
&\max_{a \sim \mathcal{B}} Q(\tau, a) - f_\alpha^T(Q(\tau, \cdot) \mid \mu(\cdot|\tau))Q(\tau,) \\
&= Q(\tau, a_{[1]}) - \frac{\sum_{i=1}^{|A_\tau|} \mu(a_{[i]} \mid \tau) \exp\left[\frac{1}{\alpha}Q\left(\tau, a_{[i]}\right)\right] Q\left(\tau, a_{[i]}\right)}{\sum_{i=1}^{|A_\tau|} \mu(a_{[i]} \mid \tau) \exp\left[\frac{1}{\alpha}Q\left(\tau, a_{[i]}\right)\right]} \\
&= \frac{\sum_{i=1}^{|A_\tau|} \mu(a_{[i]} \mid \tau) \exp\left[\frac{1}{\alpha}Q\left(\tau, a_{[i]}\right)\right] \left[Q\left(\tau, a_{[1]}\right) - Q\left(\tau, a_{[i]}\right)\right]}{\sum_{i=1}^{|A_\tau|} \mu(a_{[i]} \mid \tau) \exp\left[\frac{1}{\alpha}Q\left(\tau, a_{[i]}\right)\right]}.
\end{aligned} \tag{30}$$

Let $\delta_i(\tau) = Q\left(\tau, a_{[1]}\right) - Q\left(\tau, a_{[i]}\right)$. The distance in the Equation 30 can be rewritten as:

$$\begin{aligned}
&\frac{\sum_{i=1}^{|A_\tau|} \mu(a_{[i]} \mid \tau) \exp\left[\frac{1}{\alpha}Q\left(\tau, a_{[i]}\right)\right] \left[Q\left(\tau, a_{[1]}\right) - Q\left(\tau, a_{[i]}\right)\right]}{\sum_{i=1}^{|A_\tau|} \mu(a_{[i]} \mid \tau) \exp\left[\frac{1}{\alpha}Q\left(\tau, a_{[i]}\right)\right]} \\
&= \frac{\sum_{i=1}^{|A_\tau|} \mu(a_{[i]} \mid \tau) \exp\left[-\frac{1}{\alpha}\delta_i(\tau)\right] \delta_i(\tau)}{\sum_{i=1}^{|A_\tau|} \mu(a_{[i]} \mid \tau) \exp\left[-\frac{1}{\alpha}\delta_i(\tau)\right]} \\
&= \frac{\sum_{i=2}^{|A_\tau|} \mu(a_{[i]} \mid \tau) \exp\left[-\frac{1}{\alpha}\delta_i(\tau)\right] \delta_i(\tau)}{\mu(a_{[1]} \mid \tau) + \sum_{i=2}^{|A_\tau|} \mu(a_{[i]} \mid \tau) \exp\left[-\frac{1}{\alpha}\delta_i(\tau)\right]}
\end{aligned} \tag{31}$$

First note that for any two non-negative sequences $\{x_i\}$ and $\{y_i\}$,

$$\frac{\sum_i x_i}{1 + \sum_i y_i} \leq \sum_i \frac{x_i}{1 + y_i}. \tag{32}$$

We have the following conclusion by applying the Equation 32 to Equation 31:

$$\begin{aligned}
\frac{\sum_{i=2}^{|A_\tau|} \mu(a_{[i]} \mid \tau) \exp\left[-\frac{1}{\alpha}\delta_i(\tau)\right] \delta_i(\tau)}{\mu(a_{[1]} \mid \tau) + \sum_{i=2}^{|A_\tau|} \mu(a_{[i]} \mid \tau) \exp\left[-\frac{1}{\alpha}\delta_i(\tau)\right]} &\leq \sum_{i=2}^{|A_\tau|} \frac{\mu(a_{[i]} \mid \tau) \exp\left[-\frac{1}{\alpha}\delta_i(\tau)\right] \delta_i(\tau)}{\mu(a_{[1]} \mid \tau) + \mu(a_{[i]} \mid \tau) \exp\left[-\frac{1}{\alpha}\delta_i(\tau)\right]} \\
&= \sum_{i=2}^{|A_\tau|} \frac{\mu(a_{[i]} \mid \tau)\delta_i(\tau)}{\mu(a_{[i]} \mid \tau) + \mu(a_{[1]} \mid \tau) \exp\left[\frac{1}{\alpha}\delta_i(\tau)\right]} \\
&= \sum_{i=2}^{|A_\tau|} \frac{\delta_i(\tau)}{1 + \frac{\mu(a_{[1]}|\tau)}{\mu(a_{[i]}|\tau)} \exp\left[\frac{1}{\alpha}\delta_i(\tau)\right]} \\
&\leq \sum_{i=2}^{|A_\tau|} \frac{\delta_i(\tau)}{1 + C \exp\left[\frac{1}{\alpha}\delta_i(\tau)\right]},
\end{aligned} \tag{33}$$

17

where $C = \inf_{\tau \in S} \inf_{2 \le i \le |A_\tau|} \frac{\mu(a_{[1]}|\tau)}{\mu(a_{[i]}|\tau)}$.

If $\delta_i(\tau) > 1$, we have

$$\frac{\delta_i(\tau)}{1 + C \exp\left[\frac{1}{\alpha}\delta_i(\tau)\right]} \le \frac{\delta_i(\tau)}{1 + C \exp\left(\frac{1}{\alpha}\right)} \le \frac{2Q_{\max}}{1 + C \exp(\frac{1}{\alpha})}. \tag{34}$$

else $0 \le \delta_i(\tau) \le 1$:

$$\frac{\delta_i(\tau)}{1 + C \exp\left[\frac{1}{\alpha}\delta_i(\tau)\right]} = \frac{1}{\frac{1+C}{\delta_i(\tau)} + \frac{1}{\alpha}C + 0.5\frac{1}{\alpha^2}\delta_i(\tau)C + \cdots} \le \frac{1}{(\frac{1}{\alpha} + 1)C + 1}. \tag{35}$$

By combining these two cases with Equation 33, we complete the proof. □

Theorem 2. *Let $\mathcal{T}_{\text{ICQ}}^k Q_0$ denote that the operator $\mathcal{T}_{\text{ICQ}}$ are iteratively applied over an initial state-action value function Q_0 for k times. Then, we have $\forall (\tau, a)$, $\limsup_{k \to \infty} \mathcal{T}_{\text{ICQ}}^k Q_0(\tau, a) \le Q^*(\tau, a)$,*

$$\liminf_{k \to \infty} \mathcal{T}_{\text{ICQ}}^k Q_0(\tau, a) \ge Q^*(\tau, a) - \frac{\gamma(|A_\tau| - 1)}{(1 - \gamma)} \max\left\{\frac{1}{(\frac{1}{\alpha} + 1)C + 1}, \frac{2Q_{\max}}{1 + C \exp(\frac{1}{\alpha})}\right\}, \tag{36}$$

where $|A_\tau|$ is the size of seen actions for state τ, $C \triangleq \inf_{\tau \in S} \inf_{2 \le i \le |A_\tau|} \frac{\mu(a_{[1]}|\tau)}{\mu(a_{[i]}|\tau)}$ and $\mu(a_{[1]} \mid \tau)$ denotes the probability of choosing the expert action according to behavioral policy μ. Moreover, the upper bound of $\mathcal{T}_{\text{BCQ}}^k Q_0$ - $\mathcal{T}_{\text{ICQ}}^k Q_0$ decays exponentially fast in terms of α.

Proof. We first conjecture that

$$\mathcal{T}_{\text{BCQ}}^k Q_0(\tau, a) - \mathcal{T}_{\text{ICQ}}^k Q_0(\tau, a) \le \sum_{j=1}^{k} \gamma^j \zeta, \tag{37}$$

where $\zeta = \sup_Q \max_\tau \left[\max_{a \sim \mathcal{B}} Q(\tau, a) - f_\alpha^T(Q(\tau,)) Q(\tau,)\right]$ denotes the supremum of the difference between the BCQ and ICQ operators, over all Q-functions that occur during Q-iteration, and state τ. Equation 37 is proven using induction as follows:

- When $i = 1$, we start from the definitions for $\mathcal{T}_{\text{BCQ}}$ and $\mathcal{T}_{\text{ICQ}}$, and proceed as

$$\mathcal{T}_{\text{BCQ}} Q_0(\tau, a) - \mathcal{T}_{\text{ICQ}} Q_0(\tau, a) = \gamma \sum_{\tau'} P(\tau' \mid \tau, a) \left[\max_{a' \sim \mathcal{B}} Q_0(\tau', a') - f_\alpha^T(Q_0(\tau',)) Q_0(\tau',)\right]$$
$$\le \gamma \sum_{\tau'} P(\tau' \mid \tau, a) \zeta = \gamma \zeta. \tag{38}$$

- Suppose the conjecture holds when $i = l$, i.e., $\mathcal{T}_{\text{BCQ}}^l Q_0(\tau, a) - \mathcal{T}_{\text{ICQ}}^l Q_0(\tau, a) \le \sum_{j=1}^{l} \gamma^j \zeta$, then

$$\mathcal{T}_{\text{BCQ}}^{l+1} Q_0(\tau, a) - \mathcal{T}_{\text{ICQ}}^{l+1} Q_0(\tau, a) = \mathcal{T}_{\text{BCQ}} \mathcal{T}_{\text{BCQ}}^l Q_0(\tau, a) - \mathcal{T}_{\text{ICQ}}^{l+1} Q_0(\tau, a)$$
$$\le \mathcal{T}_{\text{BCQ}} \left[\mathcal{T}_{\text{ICQ}}^l Q_0(\tau, a) + \sum_{j=1}^{l} \gamma^j \zeta\right] - \mathcal{T}_{\text{ICQ}}^{l+1} Q_0(\tau, a)$$
$$= \sum_{j=1}^{l} \gamma^{j+1} \zeta + (\mathcal{T}_{\text{BCQ}} - \mathcal{T}_{\text{ICQ}}) \mathcal{T}_{\text{ICQ}}^l Q_0(\tau, a)$$
$$\le \sum_{j=1}^{l} \gamma^{j+1} \zeta + \gamma \zeta = \sum_{j=1}^{l+1} \gamma^j \zeta. \tag{39}$$

By using the fact that $\lim_{k\to\infty}\mathcal{T}_{\text{BCQ}}^k Q_0(\tau,a)$ and applying Lemma 1 to bound ζ, we have $\forall(\tau,a),\ \limsup_{k\to\infty}\mathcal{T}_{\text{ICQ}}^k Q_0(\tau,a) \leq Q^*(\tau,a)$ and $\liminf_{k\to\infty}\mathcal{T}_{\text{ICQ}}^k Q_0(\tau,a) \geq Q^*(\tau,a) - \frac{\gamma(|A_\tau|-1)}{(1-\gamma)}\max\{\frac{1}{(\frac{1}{\alpha}+1)C+1}, \frac{2Q_{\max}}{1+C\exp(\frac{1}{\alpha})}\}$. Based on the Equation 33, we can bound Equation 37 as:

$$\mathcal{T}_{\text{BCQ}}^k Q_0(\tau,a) - \mathcal{T}_{\text{ICQ}}^k Q_0(\tau,a) \leq \frac{\gamma(1-\gamma^k)}{1-\gamma}\sum_{i=2}^{|A_\tau|}\frac{\delta_i(\tau)}{1+C\exp\left[\frac{1}{\alpha}\delta_i(\tau)\right]}. \tag{40}$$

From the definition of $\delta_i(\tau)$, we have $\delta_{|A_\tau|}(\tau) \geq \delta_{|A_\tau|-1}(\tau) \geq \cdots \geq \delta_2(\tau) \geq 0$. Furthermore, there must exist an index $i^* \leq |A_\tau|$ such that $\delta_i > 0, \forall i^* \leq i \leq |A_\tau|$. Therefore, we can proceed from Equation 40 as

$$\frac{\gamma(1-\gamma^k)}{1-\gamma}\sum_{i=2}^{|A_\tau|}\frac{\delta_i(\tau)}{1+C\exp\left[\frac{1}{\alpha}\delta_i(\tau)\right]} = \frac{\gamma(1-\gamma^k)}{1-\gamma}\sum_{i=i^*}^{|A_\tau|}\frac{\delta_i(\tau)}{1+C\exp\left[\frac{1}{\alpha}\delta_i(\tau)\right]}$$

$$\leq \frac{\gamma(1-\gamma^k)}{1-\gamma}\sum_{i=i^*}^{|A_\tau|}\frac{\delta_i(\tau)}{C\exp\left[\frac{1}{\alpha}\delta_i(\tau)\right]} \leq \frac{\gamma(1-\gamma^k)}{1-\gamma}\sum_{i=i^*}^{|A_\tau|}\frac{\delta_i(\tau)}{C\exp\left[\frac{1}{\alpha}\delta_{i^*}(\tau)\right]} \tag{41}$$

$$= \frac{\gamma(1-\gamma^k)}{1-\gamma}\exp\left[-\frac{1}{\alpha}\delta_{i^*}(\tau)\right]\sum_{i=i^*}^{|A_\tau|}\frac{\delta_i(\tau)}{C},$$

which implies an exponential convergence rate in terms of α. $\qquad\square$

B.3 Proof of Remark 3.2

We analyze the MMDP experimental result in Section 3.2 from the perspective of the concentrability coefficient $C(\Pi)$, which illustrates the degree to which states and actions are out of distribution. In the MMDP case, we theoretically prove $C(\Pi^i)$ satisfies: $C(\Pi^1) < C(\Pi^2) < \cdots < C(\Pi^n)$, where Π^i denotes the set of joint policies including i agents. As illustrated in the above conclusion, the increase in the number of agents makes the distribution shift issue more severe in the MMDP case.

Remark 1. *Let $\varrho(s)$ denote the marginal distribution over S, ρ_0 indicate the initial state distribution, and Π^i represent the set of joint policies including i agents. Assume there exist coefficients $c(k)$ satisfying $\rho_0 P^{\pi_1}P^{\pi_2}...P^{\pi_k}(s) \leq c(k)\varrho(s)$. We define the concentrability coefficient $C(\Pi) \triangleq (1-\gamma)^2\sum_{k=1}^{\infty}k\gamma^{k-1}c(k)$, which illustrates the degree to which states and actions are out of distribution. Due to the limited datasets, the number of seen state-action pairs m is fixed. Then, $C(\Pi^i)$ is monotonically increasing with the number of agents*

$$C(\Pi^1) < C(\Pi^2) < \cdots < C(\Pi^n) \tag{42}$$

Proof. We first note that $c(k) \geq \frac{\rho_0 P^{\pi_1}P^{\pi_2}...P^{\pi_k}(s)}{\varrho(s)}$ and $c(k)$ determines the value of $C(\Pi^i)$. To compare $C(\Pi^i)$, we just need to compare $c(k)$ at iteration k. For clarity of analysis, we assume each state-action pair is visited only once, and individual policy is random $\pi^i(A^{(i)}|s) = \frac{1}{2}$. In the MMDP case, the transition matrix $P^{\boldsymbol{\pi}}$ is stable for the number of agents:

$$P^{\pi_k^1} = P^{\boldsymbol{\pi}_k} = P^{\pi_k^1\pi_k^2...\pi_k^n} = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}. \tag{43}$$

For this reason, $\rho_0 P^{\boldsymbol{\pi}_1}P^{\boldsymbol{\pi}_2}...P^{\boldsymbol{\pi}_k}(s)$ does not change with the number of agents. As $\varrho(s) = \sum_a \varrho(s,a) = \sum_a \frac{\sum_{s,a\in\mathcal{D}}\mathbf{1}[s=s,a=a]}{\sum_{s',a'\in\mathcal{D}}\mathbf{1}[s=s',a=a']}$, we can calculate $\varrho(s)$ by counting state-action pairs in $\mathcal{D}$ as follows

$$\varrho(s) = \frac{m}{2^{n+1}}. \tag{44}$$

The gradient of $\varrho(s)$ is:

$$\varrho(s)' = \left(\frac{m}{2^{n+1}}\right)' = \frac{-m\cdot 2^{n+1}\ln 2}{(2^{n+1})^2} < 0. \tag{45}$$

Therefore, $c(k)$ is monotonically increasing with the number of agents and $C(\Pi^1) < C(\Pi^2) < \cdots < C(\Pi^n)$. $\qquad\square$

B.4 Proof of Remark 2

Remark 2. *For the optimization problem*

$$\pi_{k+1} = \arg\max_{\pi} \mathbb{E}_{a \sim \pi(\cdot|\tau)}[Q^{\pi_k}(\tau, a)] \qquad \text{s.t.} \quad D_{\mathrm{KL}}(\pi\|\mu)[\tau] \le \epsilon, \quad \sum_a \pi(a|\tau) = 1, \qquad (46)$$

the optimal policy is $\pi_{k+1}^*(a \mid \tau) = \frac{\mu(a|\tau)\exp\left(\frac{1}{\alpha}Q^{\pi_k}(\tau,a)\right)}{\sum_{\tilde{a}} \mu(\tilde{a}|\tau)\exp\left(\frac{1}{\alpha}Q^{\pi_k}(\tau,\tilde{a})\right)}.$

Proof. First, note the objective is a linear function of the decision variables π. All constraints are convex functions. Thus Equation 46 is a convex optimization problem. The Lagrangian equation is

$$\mathcal{L}(\pi, \alpha) = \mathbb{E}_{a \sim \pi}[Q^{\pi_k}(\tau, a)] + \alpha\left(\epsilon - D_{\mathrm{KL}}(\pi\|\mu)[\tau]\right) + \lambda\left(1 - \sum_a \pi(a \mid \tau)\right), \qquad (47)$$

where $\alpha > 0$ denotes the Lagrangian coefficient. Differentiate π to get the following formula

$$\frac{\partial \mathcal{L}}{\partial \pi} = Q^{\pi_k}(\tau, a) - \alpha\left(1 + \log\left(\frac{\pi(a \mid \tau)}{\mu(a \mid \tau)}\right)\right) - \lambda. \qquad (48)$$

Setting $\frac{\partial \mathcal{L}}{\partial \pi}$ to zero, then:

$$
\begin{aligned}
Q^{\pi_k}(\tau, a) - \alpha\left(1 + \log\left(\frac{\pi(a \mid \tau)}{\mu(a \mid \tau)}\right)\right) - \lambda &= 0 \\
Q^{\pi_k}(\tau, a) &= \alpha\left(1 + \log\left(\frac{\pi(a \mid \tau)}{\mu(a \mid \tau)}\right)\right) + \lambda \\
\frac{Q^{\pi_k}(\tau, a)}{\alpha} - 1 - \frac{\lambda}{\alpha} &= \log\left(\frac{\pi(a \mid \tau)}{\mu(a \mid \tau)}\right) \\
\frac{\pi(a \mid \tau)}{\mu(a \mid \tau)} &= \exp\left(\frac{Q^{\pi_k}(\tau, a)}{\alpha} - 1 - \frac{\lambda}{\alpha}\right) \\
\pi(a \mid \tau) &= \mu(a \mid \tau)\exp\left(\frac{Q^{\pi_k}(\tau, a)}{\alpha} - 1 - \frac{\lambda}{\alpha}\right)
\end{aligned}
\qquad (49)
$$

Due to the second constraint in Equation 46, the policy is a probability distribution. Therefore, we adopt Z to normalize the result by moving the constant $\mu(a \mid \tau)\exp(-1 - \frac{\lambda}{\alpha})$ to Z:

$$\pi_{k+1}^*(a \mid \tau) = \frac{1}{Z(\tau)}\mu(a \mid \tau)\exp\left(\frac{Q^{\pi_k}(\tau, a)}{\alpha}\right), \qquad (50)$$

where $Z(\tau) = \sum_{\tilde{a}} \mu(\tilde{a} \mid \tau)\exp\left(\frac{1}{\alpha}Q^{\pi_k}(\tau, \tilde{a})\right)$ is the normalizing partition function. $\square$

B.5 Proof of Theorem 3

Theorem 3. *Assuming the joint action-value function is linearly decomposed, we can decompose the multi-agent joint-policy under implicit constraint as follows*

$$\boldsymbol{\pi} = \arg\max_{\pi^1,\dots,\pi^n} \sum_i \mathbb{E}_{\tau^i, a^i \sim \mathcal{B}}\left[\frac{1}{Z^i(\tau^i)}\log(\pi^i(a^i \mid \tau^i))\exp\left(\frac{w^i(\boldsymbol{\tau})Q^i(\tau^i, a^i)}{\alpha}\right)\right], \qquad (51)$$

where $Z^i(\tau^i) = \sum_{\tilde{a}^i} \mu^i(\tilde{a}^i \mid \tau^i)\exp\left(\frac{1}{\alpha}w^i(\boldsymbol{\tau})Q^i(\tau^i, \tilde{a}^i)\right)$ *is the normalizing partition function.*

Proof. Let $\mathcal{J}_{\boldsymbol{\pi}}$ denote the joint-policy loss. According to the assumption, $\mathcal{J}_{\boldsymbol{\pi}}$ is written:

$$
\begin{aligned}
\mathcal{J}_{\boldsymbol{\pi}} &= \mathbb{E}_{\boldsymbol{\tau}, \boldsymbol{a} \sim \mathcal{B}}\left[-\frac{1}{Z(\boldsymbol{\tau})}\log(\boldsymbol{\pi}(\boldsymbol{a}|\boldsymbol{\tau}))\exp\left(\frac{1}{\alpha}Q^{\boldsymbol{\pi}}(\boldsymbol{\tau}, \boldsymbol{a})\right)\right] \\
&= \mathbb{E}_{\boldsymbol{\tau}, a^1,\dots,a^n \sim \mathcal{B}}\left[-\frac{1}{Z(\boldsymbol{\tau})}\left(\sum_i \log(\pi^i(a^i \mid \tau^i))\right)\exp\left(\frac{1}{\alpha}\left(\sum_i w^i(\boldsymbol{\tau})Q^i(\tau^i, a^i) + b(\boldsymbol{\tau})\right)\right)\right].
\end{aligned}
\qquad (52)
$$

The loss function $\mathcal{J}_{\boldsymbol{\pi}}$ is equivalent to the following form by relocating the sum operator:

$$
\begin{aligned}
\mathcal{J}_{\boldsymbol{\pi}} &= \sum_i \mathbb{E}_{\boldsymbol{\tau},a^1,\ldots,a^n \sim \mathcal{B}} \left[-\frac{1}{Z(\boldsymbol{\tau})} \log(\pi^i(a^i \mid \tau^i)) \exp\left(\frac{\sum_i w^i(\boldsymbol{\tau})Q^i(\tau^i,a^i) + b(\boldsymbol{\tau})}{\alpha} \right) \right] \\
&= \sum_i \mathbb{E}_{\boldsymbol{\tau},a^1,\ldots,a^n \sim \mathcal{B}} [-\frac{1}{Z(\boldsymbol{\tau})} \log(\pi^i(a^i \mid \tau^i)) \exp\left(\frac{w^i(\boldsymbol{\tau})Q^i(\tau^i,a^i)}{\alpha} \right) \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \exp\left(\frac{\sum_{j\neq i} w^j(\boldsymbol{\tau})Q^j(\tau^j,a^j) + b(\boldsymbol{\tau})}{\alpha} \right)] \\
&= \sum_i \mathbb{E}_{\boldsymbol{\tau},a^i \sim \mathcal{B}} \mathbb{E}_{a^{j\neq i} \sim \mathcal{B}} [-\frac{1}{Z(\boldsymbol{\tau})} \log(\pi^i(a^i \mid \tau^i)) \exp\left(\frac{w^i(\boldsymbol{\tau})Q^i(\tau^i,a^i)}{\alpha} \right) \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \exp\left(\frac{\sum_{j\neq i} w^j(\boldsymbol{\tau})Q^j(\tau^j,a^j) + b(\boldsymbol{\tau})}{\alpha} \right)] \\
&= \sum_i \mathbb{E}_{\boldsymbol{\tau},a^i \sim \mathcal{B}} \left[-\frac{1}{Z^i(\tau^i)} \log(\pi^i(a^i \mid \tau^i)) \exp\left(\frac{w^i(\boldsymbol{\tau})Q^i(\tau^i,a^i)}{\alpha} \right) \right],
\end{aligned}
\tag{53}
$$

$$
\begin{aligned}
Z^i(\tau^i) &= \frac{\sum_{\tilde{a}^i} \sum_{\tilde{a}^{j\neq i}} \boldsymbol{\mu}(\bar{\boldsymbol{a}} \mid \boldsymbol{\tau}) \exp\left(\frac{1}{\alpha} w^i(\boldsymbol{\tau})Q^i(\tau^i,\tilde{a}^i)\right) \exp\left(\frac{1}{\alpha}\left(\sum_{j\neq i} w^j(\boldsymbol{\tau})Q^j(\tau^j,\tilde{a}^j) + b(\boldsymbol{\tau})\right)\right)}{\mathbb{E}_{\tilde{a}^{j\neq i} \sim \mathcal{B}}\left[\exp\left(\frac{1}{\alpha}\left(\sum_{j\neq i} w^j(\boldsymbol{\tau})Q^j(\tau^j,\tilde{a}^j) + b(\boldsymbol{\tau}) \right) \right) \right]} \\
&= \frac{\sum_{\tilde{a}^i} \sum_{\tilde{a}^{j\neq i}} \mu^i(\tilde{a}^i \mid \tau^i)\mu^{j\neq i}(\tilde{a}^j \mid \tau^j) \exp\left(\frac{1}{\alpha} w^i(\boldsymbol{\tau})Q^i(\tau^i,\tilde{a}^i)\right)}{\sum_{\tilde{a}^{j\neq i}} \mu^{j\neq i}(\tilde{a}^j \mid \tau^j) \exp\left(\frac{1}{\alpha}\left(\sum_{j\neq i} w^j(\boldsymbol{\tau})Q^j(\tau^j,\tilde{a}^j) + b(\boldsymbol{\tau}) \right) \right)} \cdot \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad \exp\left(\frac{1}{\alpha}\left(\sum_{j\neq i} w^j(\boldsymbol{\tau})Q^j(\tau^j,\tilde{a}^j) + b(\boldsymbol{\tau}) \right) \right) \\
&= \sum_{\tilde{a}^i} \mu^i(\tilde{a}^i \mid \tau^i) \exp\left(\frac{1}{\alpha} w^i(\boldsymbol{\tau})Q^i(\tau^i,\tilde{a}^i) \right).
\end{aligned}
\tag{54}
$$

$\square$

C   Additional Results

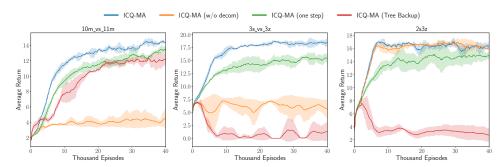### C.1   Additional Ablation Results in StarCraft II

Figure 6: Module ablation study in additional StarCraft II environments.

C.2   Additional Results in MMDP

Due to the space limits, we put the complete results in MMDP in Figure 7. BCQ gradually diverges as the number of agents increases, while ICQ has accurate estimates.
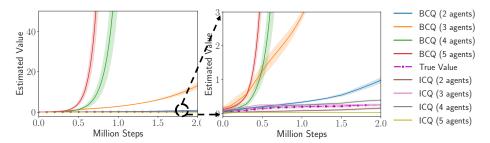
21

Figure 7: Additional results in MMDP.

C.3 Additional Results in D4RL

(a) Adroit-expert.

(b) Adroit-human.
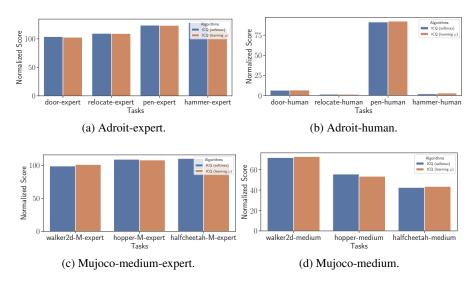
(c) Mujoco-medium-expert.

(d) Mujoco-medium.

Figure 8: The performance on D4RL tasks with different implementation of ICQ.
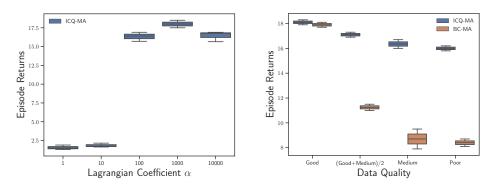
C.4 Ablation Study

Figure 9: Ablation study on MMM map.

D Experimental Details

D.1 Implementation details of ICQ

We provide the two implementation options of our methods regards whether learning μ to calculate ρ.

Learning an auxiliary behavior model $\hat{\mu}$. We first consider to learn the behavior policy $\hat{\mu}$ using conditional variational auto-encoder as BCQ. Next, we will sample actions n times ($n = 100$ in our experiment) from $\hat{\mu}$ to calculate $Z(\tau)$ on each value update:

$$\rho(\tau, a) = \frac{\exp(\frac{Q(\tau,a)}{\alpha})}{Z(\tau)} \approx \frac{\exp(\frac{Q(\tau,a)}{\alpha})}{\mathbb{E}_{\tilde{a}\sim\hat{\mu}} \exp(\frac{Q(\tau,\tilde{a})}{\alpha})}. \tag{55}$$

If $\hat{\mu} \approx \mu$, this method is favored as it provides an accurate approximation. However, since it may introduce unseen pairs sampled from the learned behavior model, it is against the principle of our analysis. Nevertheless, we believe it is still a better choice compared with BCQ. If there is any unseen pair $\tau, \tilde{a}$ with large extrapolation error sampled from $\hat{\mu}$, e.g, $Q_{\mathcal{B}}(\tau, \tilde{a}) \gg Q_M(\tau, \tilde{a})$, we will have $\hat{\rho}(\tau, a) < \rho(\tau, a)$, which means the unsafe estimation is truncated and the resulting target Q-value tends to be conservative.

Approximate with softmax operation over a mini-batch. We have the following measure to approximately calculate ρ without μ, which reduces the computational complexity:

$$\rho(\tau, a) = \frac{\exp(\frac{Q(\tau,a)}{\alpha})}{Z(\tau)} \approx \frac{\exp(\frac{Q(\tau,a)}{\alpha})}{\sum_{(\tau',a')\sim\text{mini-batch}} \exp(\frac{Q(\tau',a')}{\alpha})}, \tag{56}$$

where $Z^i(\tau^i)$ is approximated by softmax operation over mini-batch samples. The benefit of the softmax operation is that it does not include any unseen pairs, which is consistent with our theoretical analysis. However, the price is that the softmax operation ignores the difference of states over a mini-batch, which introduces an additional bias. However, we find it does not harm the performance a lot in practice. There are also some previous works using softmax to deal with the partition function, such as AWAC [29]) and VMPO [45], which has been confirmed to promote performance improvement.

Considering the concise form of the softmax operation, we prefer the the second version in the multi-agent tasks. We conduct ablation studies of these two measures on D4RL to demonstrate their superior performance (see Figure 8).

D.2 Baselines Details

BCQ-MA is trained by minimizing the following loss:

$$\mathcal{J}_Q^{\text{BCQ}}(\phi, \psi) = \mathbb{E}_{\boldsymbol{\tau}\sim\mathcal{B},\boldsymbol{a}\sim\boldsymbol{\mu}} \left[\left(r(\boldsymbol{\tau}, \boldsymbol{a}) + \gamma \max_{\tilde{\boldsymbol{a}}^{[j]}} Q^{\boldsymbol{\pi}}(\boldsymbol{\tau}', \tilde{\boldsymbol{a}}^{[j]}; \phi', \psi') - Q^{\boldsymbol{\pi}}(\boldsymbol{\tau}, \boldsymbol{a}; \phi, \psi) \right)^2 \right], \tag{57}$$

$$\tilde{\boldsymbol{a}}^{[j]} = \boldsymbol{a}^{[j]} + \xi(\boldsymbol{\tau}, \boldsymbol{a}^{[j]})$$

where $Q^{\boldsymbol{\pi}}(\boldsymbol{\tau}, \boldsymbol{a}) = w^i(\boldsymbol{\tau})Q^i(\tau^i, a^i) + b(\boldsymbol{\tau})$ and $\xi(\boldsymbol{\tau}, \boldsymbol{a}^{[j]})$ denotes the perturbation model, which is decomposed as $\xi^i(\tau^i, a^{i,[j]})$. If $\frac{a^{i,[j]}\sim G^i(\tau^i;\psi^i)}{\max\{a^{i,[j]}\sim G^i(\tau^i;\psi^i)\}_{j=1}^m} \leq \zeta$ in agent i, $a^{i,[j]}$ is considered an unfamiliar action and $\xi^i(\tau^i, a^{i,[j]})$ will mask $a^{i,[j]}$ in maximizing Q^i-value operation.

CQL-MA is trained by minimizing the following loss:

$$\mathcal{J}_Q^{\text{CQL}}(\phi, \psi) = \alpha^{\text{CQL}} \mathbb{E}_{\boldsymbol{\tau}\sim\mathcal{B}} \left[\sum_i \log \sum_{a^i} \exp(w^i(\boldsymbol{\tau})Q^i(\tau^i, a^i) + b(\boldsymbol{\tau})) - \mathbb{E}_{\boldsymbol{a}\sim\boldsymbol{\mu}(\boldsymbol{a}|\boldsymbol{\tau})}[Q^{\boldsymbol{\pi}}(\boldsymbol{\tau}, \boldsymbol{a})] \right]$$

$$+ \frac{1}{2} \mathbb{E}_{\boldsymbol{\tau}\sim\mathcal{B},\boldsymbol{a}\sim\boldsymbol{\mu}(\boldsymbol{a}|\boldsymbol{\tau})} \left[\left(y^{\text{CQL}} - Q^{\boldsymbol{\pi}}(\boldsymbol{\tau}, \boldsymbol{a}) \right)^2 \right]$$

$$\mathcal{J}_{\boldsymbol{\pi}}^{\text{CQL}}(\theta) = \sum_i \mathbb{E}_{\tau^i,a^i\sim\mathcal{B}} \left[-\log(\pi^i(a^i \mid \tau^i; \theta_i))Q^i(\tau^i, a^i) \right],$$

$$\tag{58}$$

where we adopt the decomposed policy gradient to train $\boldsymbol{\pi}$, and y^{CQL} is calculated based on n-step off-policy estimation (e.g., Tree Backup algorithm). Besides, $w^i(\boldsymbol{\tau}) = w^i(\boldsymbol{\tau}; \psi)$, $b(\boldsymbol{\tau}) = b(\boldsymbol{\tau}; \psi)$ and $Q^{\boldsymbol{\pi}}(\boldsymbol{\tau}, \boldsymbol{a}) = Q^{\boldsymbol{\pi}}(\boldsymbol{\tau}, \boldsymbol{a}; \phi, \psi)$.

BC-MA only optimize $\boldsymbol{\pi}$ by minimizing the following loss:

$$\mathcal{J}_{\boldsymbol{\pi}}^{\text{BC}}(\theta) = \sum_i \mathbb{E}_{\tau^i, a^i \sim \mathcal{B}} [-\log(\pi^i(a^i \mid \tau^i; \theta_i))]. \tag{59}$$

E   Multi-Agent Offline Dataset Based on StarCraft II

We divide maps in StarCraft II into three classifications based on difficulty (see Table 2). We divide behavior policies into three classifications based on the episode returns (see Table 3).

Table 2: Classification of maps in the dataset.

Difficulties	Maps
Easy	MMM, 2s_vs_3z, 3s_vs_3z, 3s5z, 2s3z, so_many_baneling
Hard	10m_vs_11m, 2c_vs_64zg
Super Hard	MMM2, 27m_vs_30m

Table 3: Classification of behavior policies in the dataset.

Level	Episode Returns
Good	$15 \sim 20$
Medium	$10 \sim 15$
Poor	$0 \sim 10$

E.1   Hyper-parameters

Hyper-parameters in multi-agent tasks are respectively presented in Table 4. Please refer to our official code for the hyper-parameter in single-agent tasks.

Table 4: Multi-agent hyper-parameters sheet

Hyper-parameter	Value
Shared	
Policy network learning rate	5×10^{-4}
Value network learning rate	10^{-4}
Optimizer	Adam
Discount factor γ	0.99
Parameters update rate d	600
Gradient clipping	20
Mixer network dimension	32
RNN hidden dimension	64
Activation function	ReLU
Batch size	16
Replay buffer size	1.2×10^4
Others	
Lagrangian coefficient α	1000 or 100
λ	0.8
α^{CQL}	2.0
ζ	0.3