Double-Spending Analysis of DAG-Based Blockchain in Wireless Blockchain Network

Abstract

Introduction

Preliminaries

In this section, we introduce the characteristics of wireless network and the consensus protocol of DAG-based blockchain respectively. Then, we describe the main procedures that a new transaction is accepted by all nodes in the wireless network whose communication protocol is CSMA/CA protocol.

Wireless Network

Wireless Local Area Networks(WLANs)[1-2] with high flexibility and convenience can provide high quality services for users in limited geographical area. Currently, as the de facto standard of WLANs, IEEE 802.11[1] has been wirdly used in wireless network. This standard include Distributed Coordination Function(DCF)[3] as Medium Access Control(MAC)[1] mechanism. The DCF, which based on Carrier Sense Multiple Access with Collision Avoidance(CSMA/CA) and binary slotted exponential backoff, can support asynchronous data transfer on a best effort basis. In this paper, we consider that any node competing for wireless channel broadcast packets by using CSMA/CA as media access protocol.

DAG-based Blockchain

DAG-based blockchain allows that appending new transaction in a forking topology. The first proposed consensus algorithm for DAG-based blockchain is Tangle[4]. In this paper, we use Tangle as consensus algorithm to analyze the consensus process of DAG-based blockchain. Compare to PoW and PoS, Tangle has higher throughput bacause it allows different branches to merge into the main chain eventually.

As shown in Fig.1, Tangle uses directed acyclic graph topology to record transaction, and the unit of Tangle should be a recorded trasaction. The basic concepts of Tangle are represented as follows:

- **Tip:** the transaction(or block) that has not been approved by any other trasnaction(or block). That is, tips are unapproved transactions in tangle graph;
- **Direct Approval:** two transactions(or blocks) is connected by a direct edge, we can say one transaction is directly approved by another transaction.
- **Indirect Approval:** two transactions are not connected by a direct edge, but there is a directed path of lenth at least two between the two transactions, then we can say the two transactions are indirectly approved.
- **Own Weight:** trasnation's own weight is propotional to the amount of work(computational power or stakes) that issuing node conssumes on proposing a trasnaction(or block).
- **Cumulative Weight:** the sum of the transaction's own weight and the overall own weight of all transactions that directly or indirectly approve this transaction.
- **Confirmation Weight Threashold:** the threshold value of trasnaction's cumulative weight, when the cumulative weight meet this value, the transaction should be confirmed.

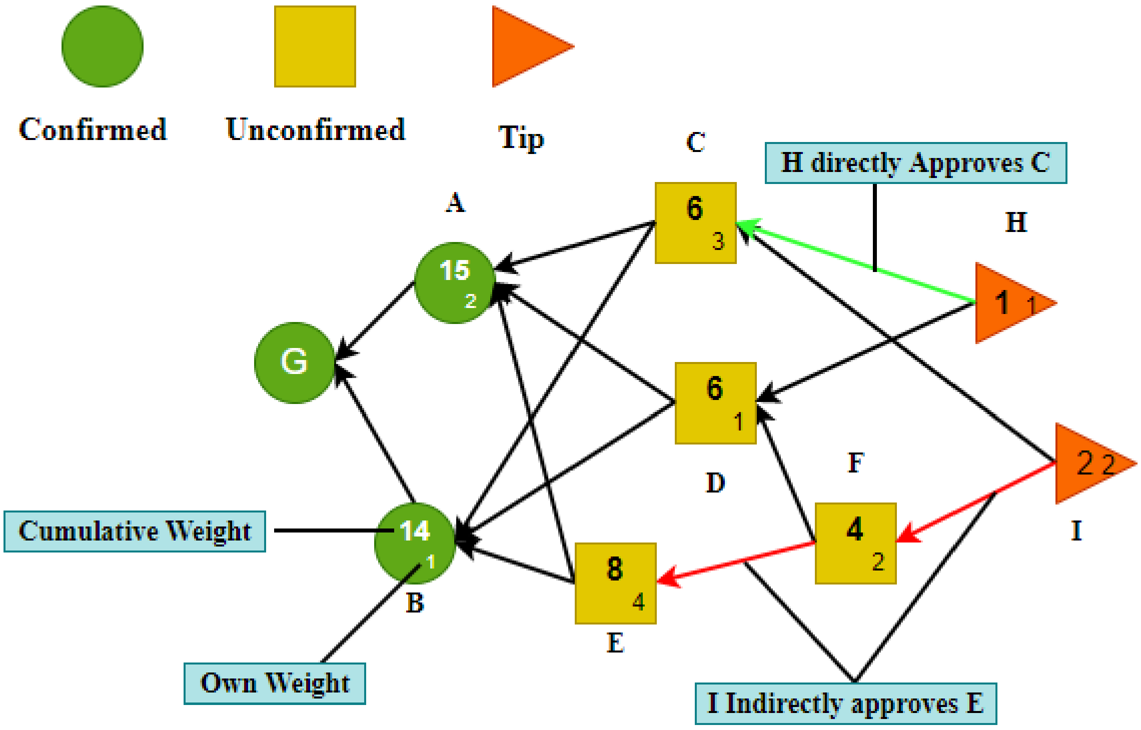


Fig. 1 Tangle

Consensus process of DAG-Based Blockchain under Wireless Network

While the broadcast procedure following CSMA/CA in wireless network, the consensus protocol should work to make sure that new issued transaction is accepted by the other nodes. For simplicity,we only consider all nodes under same local area network. Thus, the main procedures that consensus process in wireless network are as follows:

- A node finds a nonce to solve a cryptographic puzzle to meet the difficult target.
- The node issues a new transaction which will select two nonconflicting tips to approve based on local information;
- The node uses its private key to sign this new transaction. The new transaction will enter into cache waiting for broadcasting through wireless channel;
- The node competes for wireless channel following CSMA/CA while the new transaction queues in cache following first in first out(FIFO) rule;
- The node either broadcasts the transaction successfully or rebroadcasts with backoff;
- Other nodes receive the new transaction and check it to confirm legality. If the new transaction is legal, then it will become a new tip and wait for the direct or indirect approvement for confirmation.

The consensus process of an issued transaction is divided into two stages: reveal stage and weight accumulating stage.

- **Reveal Stage:** The observed transaction is appended into the DAG-based blockchain, that is all nodes can see the transaction.
- **Weight Accumulating Stage:** the cumulative weight of the observed transaction increases from its own weight to confirmation threshold gradually.

In order to simplify later analysis, we can define the second to five procedures as reveal stage, and procedure six as the weight accumulating stage of new transaction. As we can see that communication in network may cause a serious delay when nodes compete for wireless channel to broadcast the new transaction.

Double-Spending Attack in Wireless Blockchain Network

In this section, we first introduce the double-spending attack of DAG-based blockchain. And then we analyze the successfull attack probability of double-spending under wireless network with imperfect CSMA/CA protocol.

System Model

We now present some assumptions for double-spending attack analysis.

- Assume that there are n nodes running Tangle, they communicate with each other directly throught wireless channel.
- Let m be the maximum number of transactions at one broadcast.
- Assume that each node selects two tips with no-conflict by Markov Chain Monte Carlo(MCMC) tips selection algorithm[4].
- Let the own weight of each transaction be one.
- Let h be the average transmission delay to broadcast a packet through CSMA/CA protocol. In addition, h is also the reveal time to update the new transaction discussed in Tangle.
- let λ be the transaction arrival rate of each node in wireless blockchain network.
- Let $L(t)$ be the number of tips in DAG-based blockchain at time t when considering CSMA/CA protocol.
- Let $W(t)$ be the cumulative weight of an observed transaction at time t in CSMA/CA.

In CSMA/CA, all nodes will compete to send messages. We always split time into multiple slots, and let the probability of each node sending messages in a slot be $\tau = \frac{1}{n}$. If there are n nodes in wireless blockchain network, the probability of at least one node broadcasting in a slot time is

$$P_{tr} = 1 - (1-\tau)^n.$$

The probability of one node broadcasts successfully in a slot time is

$$P_s = C_n^1 \tau (1-\tau)^{n-1} = n \cdot \tau \cdot (1-\tau)^{n-1}$$

The probability of broadcast collision occuring in a slot time is

$$P_c = 1 - (1-\tau)^n - P_s.$$

Let T_s be the average time that channel is detected busy due to a successful broadcasting, and its probability is P_s. Denoting T_c is the average time that channel is collision, the probability of broadcast collision is P_c. Besides, when the channel is free that no node broadcast in a slot time, let σ be the duration time of the empty slot time, the probability of this regime is $1 - P_{tr}$. Therefore, the average transmission delay h is the expected value of the above three situations:

$$h = (1 - P_{tr}) \cdot \sigma + P_s \cdot T_s + P_c \cdot T_c.$$

Proposition Condidering wireless network that all nodes communicate with each other according to CSMA/CA protocol consists of n independent nodes. If the during time of two neighbor transactions is h, and the transaction arrival rate of each node is λ, then the number of tips in DAG-based blockchain is $L(t) = 2n\lambda h$.

Proof: Due to the faireness of CSMA/CA, the average time to compete the broadcasting on each node is nh. When the network load is stable, $L(t) = L(t - nh) = L$ for any time t, where L is a constant value. There are $n\lambda h$ new transactions between $t - nh$ and t on average. Therefore, we can write $L(t) = r + n\lambda h$, where r is the number of old tips and $n\lambda h$ is the number of tips chosen by new transactions during $t - nh$ to t (they are not tips anymore, but other nodes do not know). When a new transaction arrives at time t, two tips from $L(t)$ will be chonsen randomly by the transaction. Since $n\lambda h$ are not tips anymore, tips selection from r or $n\lambda h$ will affect the value of $L(t)$.

- If new transaction selects two tips both from $n\lambda h$, then $L(t)$ will increase by 1;
- If it selects one tip from r and $n\lambda h$, $L(t)$ will unchange;
- If it selects two tips from r, then $L(t)$ will decrease by 1.

The expected number of selected tips in r can be computed as $\frac{n\lambda h(n\lambda h-1)}{(r+n\lambda h)(r+n\lambda h-1)} \times 0 + \frac{2rn\lambda h}{(r+n\lambda h)(r+n\lambda h-1)} \times 1 + \frac{r(r-1)}{(r+n\lambda h)(r+n\lambda h-1)} \times 2 = \frac{2r}{r+n\lambda h}$.

Because of the stability of $L(t)$, we have $\frac{2r}{r+n\lambda h} = 1$. Therefore, $r = n\lambda h, L = L(t) = 2n\lambda h$.

Network Load

Because of the fairness of CSMA/CA, each node has same probability τ to compete broadcasting in wireless channel. In order to describe the queuing state in detail, we divide the network network load into two regimes.

- **Light Regime:** When the network load is light, the cache on each node may be less than m transactions, where m is the maximum number of transaction containing in a packet. Let λ_l be the the transaction arrival rate of light load regime, and we have $n\lambda_l h \leq m$. In this case, all the waited transactions in cache can be broadcasted immediately when the node successfully competes for wireless channel. And the number of tips in the DAG-based blockchain should be $L(t) = 2n\lambda_l h$. While the value of λ_l is very small, then we have $L(t) = 2n\lambda_l h \approx 1$, and the DAG-based blockchain will be convert to a single chain.
- **Heavy Regime:** When the network load is heavy, the cache on each node is always full. Let λ_h be the the transaction arrival rate of heavy load regime, The cumulative transaction on each node is $n\lambda_h h$, which satisfies $n\lambda_h h > m$. If a node compete successfully, it will broadcast at most m transactions, and m new transactions can be stored in cache accordingly. Based on the steady characteristic, the number of tips in DAG-based blockchain should be $L(t) = 2m$ at any time.

Attack Model

In this subsection, we analyze the successful attack probability from the perspective of wireless communication. In this case, attacker should win the transaction competition and broadcast the fraudulent chain successfully. In CSMA/CA, the maximum number of broadcast transactions is limited to m, thus, the maximum new transaction arrival rate is $\frac{m}{nh}$.

In Tangle[4], S.Popov proposed two approaches for double-spending attack, one is large weight attack and another is parasite chain attack. Due to the assumption that all transactions have same own weight, we only consider the parasite chain attack that an attacker privately builds a fraudulent subtangle which occasionally references the main tangle to gain higher cumulative weight.

MCMC Selection Algorithm

In order to analyze attack process, we should know the details of MCMC tips selection algorithm. Recall the assumption that all own weights are equal to 1, the cumulative weight of a tip should be 1. In Tangle, the standard of main chain is cumuleitive computational power. The greater the cumulative computational power of the link approved by tips, the more worthy the link to be approved. The idea is to place some random walkers on transactions with at least 2 cumulative weight of Tangle, and let them randomly walk towards tips. The seceltion algorithm is shown in the following way:

- Consider all transactions whose cumulative weight on the interval $[N_{cw}, 2N_{cw}]$, where N_{cw} is the cumulative weight confirmation threshold;
- Randomly select N transactions from the transactions in step 1 as walkers.
- They will perform independent discrete-time random walks "from transaction with high cumulative weight towards the tips", meaning that a path from x to y is possible if and only if y approves x;
- The two random walkers that reach the tip set first will stand on the two tips that will be approved by new transaction. To defend "lazy tips", the algorithm will discard those random walkers that reach the tips too fast.

The path probability of a walker is defined as follows: let W_x be the cumulative weight of transaction x . If transaction y approves transaction x, then the path probability should be

$$P_{xy} = \frac{\exp(-\alpha(W_x - W_y))}{\sum_{z:\text{z approves x}} \exp(-\alpha(W_x - W_z))}$$

where α is a parameter that determines the amplification degree of the difference of between W_x and W_y.

Based on the MCMC selection algorithm, random walker would like to choose the path that with higher cumulative weight transaction.
Proposition: Let x be the transaction whose weight in interval $N_{cw}, 2N_{cw}]$, where N_{cw} is the cumulative weight confirmation threshold. Let W_h and W_c be the weights of an honest trasnaction h and corresponding conflict transaction c respectively. If $W_c > W_h$, then the walker will choose the

path from x to c with high probability.

Proof: The path probabilities from x to h and from x to c can be computed respectively:

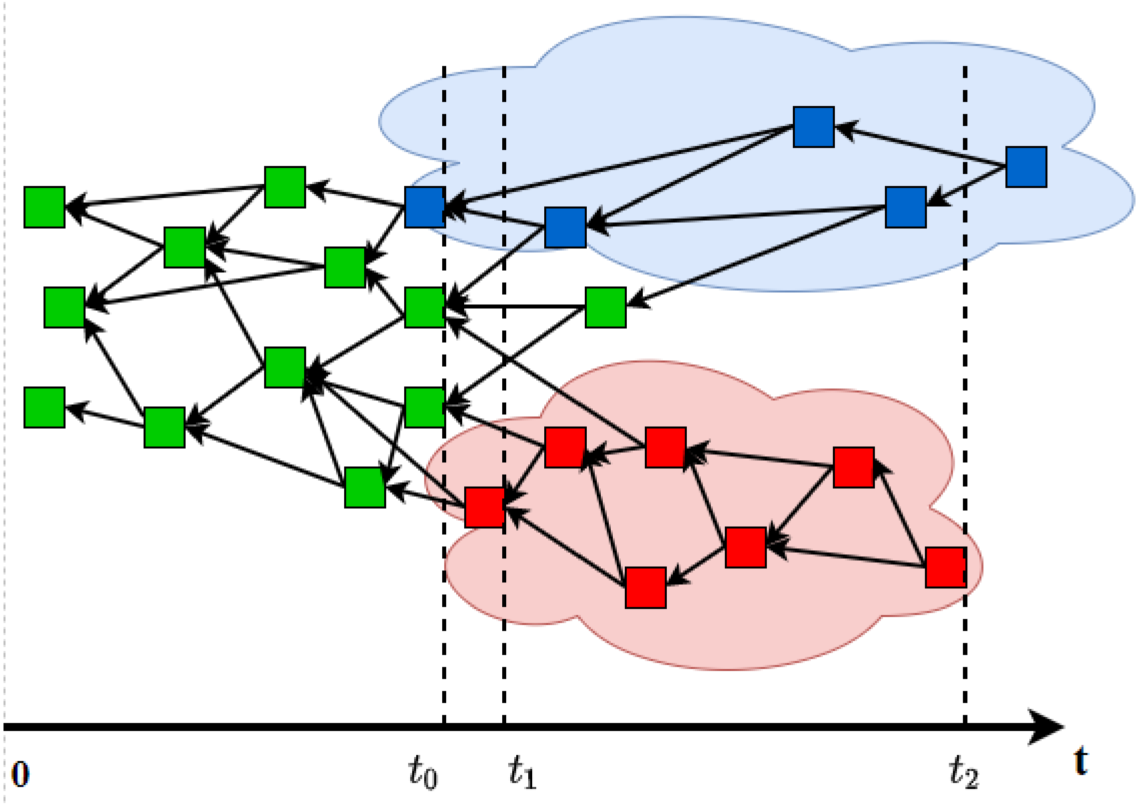
$$\begin{cases} P_{xh} = \dfrac{\exp(-\alpha(W_x - W_h))}{\sum_{z\text{:z approves x}} \exp(-\alpha(W_x - W_z))}, \\ P_{xc} = \dfrac{\exp(-\alpha(W_x - W_c))}{\sum_{z\text{:z approves x}} \exp(-\alpha(W_x - W_z))}. \end{cases}$$

When $W_c > W_h$, we have $W_x - W_c < W_x - W_h$. Thus, we can obtain that $P_{xc} \gg P_{xh}$, which means the path from x to c will be chosen by the random walker with high probability.

Attack Process Analysis

The double-spending attack process of DAG-based blockchain in wireless network is different to that in perfect network. As shown in Fig. 3, the typical way that a malicious attacker lunches double spending attack is to construct a fraudulent chain in blockchain system, the main procedures are shown as follows:

- At time t_0, attacker broadcasts an honest transaction, and honest nodes will approve it.
- At time t_1, the attacker builds a fraud chain in offchain to approve a fraudulent transaction that is conflicted with the honest transaction.
- After time t_1, the attacker will continually issue trasnactions to grow the cumulative weight of the fraudulent transaction. The time t_1 should be earlier than the end of adaption periof of the honest transaction.
- At time t_2, the honest transaction has been confirmed while its cumulative weight attaches w. In this case, the victim will send goods or services to the attacker.
- While the cumulative weight of the fraudulent transaction overweights the confirmed honest transaction after time t_2, the attacker will broadcast the fraudulent chain to the whole wireless blockchain network.
- Once the attacker contending for wireless channel to broadcast fraudulent branch updating the DAG-based blockchain, the fraud transaction will be accepted by other honest nodes based on the MCMC algorithm due to the higher cumulative weight. The confirmed honest transaction will be orphened in DAG-based blockchain, the victim cannot receive the payment even though it has provided goods or services. In this case, the attacker issues double-spending attack successfully.



Before providing goods or services to the attacker, honest nodes will choose to wait for some transactions on the honest subtangle to ensure the cumulative weight of honest transaction reaches threshold N_{cw}, which includes the own weight of the honest transaction. The attacker will publish the parasite chain if its attack was successful. Therefore, we can define two necessary conditions for double-spending attack as follows:

Definition A double-spending attack succeeds if the follwing two conditions satisfied:

- **Transaction Confirmation:** the cumulative weight of the honest transaction is greater than or equal to N_{cw}, and
- **Success in competition:** the number of tips in fraudulend subtangle is greater than that in honest subtangle.

12月22日再继续分析

The Attack Probability

We can describe the abovementioned attack process as a Markov chain. In this paper, we fit the transaction arrival process of each node using Poisson Process[5] with transaction arrival rate λ (transactions per second).

We denote the weight of the honest subtangle and fraudulent subtangle by two independent Poisson counting processes[6]. Let $H(t)$ be the weight of honest subtangle with transaction arrival rate λ at time t and $A(t)$ be the weight of fraudulent subtangle with transaction arrival rate μ at time t. Assume that attacker broadcasts an honest trasnaction at time t_0, and launches double-spending attack at time t_1. We assume that the states of these two subtangles should satisfy $H(t_1) = A(t_1) = 1$. In this case, the two subtangles have common profix before DAG-based blockchain forking occurs. The difference between $H(t)$ and $A(t)$ in a discrete-time can be represented as a random walk.

Due to the characteristics of CSMA/CA, the time interval between two new transactions should be nh, where n is the number of nodes and h is the transimission delay of a packet under wireless blockchain network. Recall that we assume there are $n-1$ honest nodes and 1 attacker in a one-hop wireless blockchain network, the arrival rates of new trasnactions on a honest node and a malicious attacker shold be

$$\begin{cases} \lambda' = \min\{\lambda, \dfrac{m}{nh}\}, \\ \mu' = \min\{\mu, \dfrac{m}{nh}\}. \end{cases}$$

where m is the maximum number of transactions of a packet that can be broadcast in wireless channel at once. In each time interval, the propabilities that a new transaction is issued by honest nodes and attacker are as follows:

$$\begin{cases} p = \dfrac{(n-1)\lambda'}{(n-1)\lambda' + \mu'}, \\ q = \dfrac{\mu'}{(n-1)\lambda' + \mu'}. \end{cases}$$

Assume that the time slot is sufficiently small that no two new transactions from the attacker and honest nodes can arrive simultaneously. The number of transactions issued by attacker in interval $[t_1, t_2]$ can be regarded as a random process obying negative binomial distribution[].

Let N_h be the number of transactions issued by honest nodes from time t_1 to time t_2, and N_a be the possible number of trasnactions issued by the attacker. Thus, potential progress function corresponds to a negative binomial distribution given by

$$P_R(p, q, N_h, N_a) = C_{N_a+N_h-1}^{N_a} p^{N_h} q^{N_a},$$

Where p, q are the propabilities that a new transaction is issued by honest nodes and attacker respectively.

The attacker can take control of the DAG-based blockchain as soon as it create a fraudulent subtangle heavier than the honest one. When discussing the successfull probability of double-spending attack, we should consider two scenarios:

- When confirming the honest transaction, the weight of the fraudulent subtangle is greater than that of the honest subtangle. In this case, attacker will publish the fraudulent subtangle, which indicates attacker launches double-spending attack successfully;
- When confirming the honest transaction, the weight of the fraudulent subtangle is smaller than or equal to that of the honest subtangle. In this case,instead of publishing the fraudulent subtangle immediately, attacker will catch up the difference between thw two subtangle. Once the weight of the fraudulent subtangle is greater than that of the honest subtangle, attacker will reveal its subtangle to ensure attack successfully.

When $N_a > N_h$, the attacker launches double-spending attack successfully at time t_2. Otherwise, the attacker requires to catch up the difference of transactions that issued by honest node and attacker until the cumulative weight of fraudulent transaction outnumbers that of honest transaction after time t_2. This process can be thought as a Gambler's Ruin problem[]. The attacker needs to catch up the difference of $N_h - N_a + 1$ transactions at least. If $p \leq q$, the attacker will eventually catch up successfully with probability 1. If $p > q$, the attacker will catch up successfully with probability a_g. Thus, the probability that an attacker constructs a fraudulent subtangle which is heavier than the honest one at any time is denoted

$$p(g) = \begin{cases} 1, & p \leq q, \\ a_g, & p > q, \end{cases}$$

where g is the number of transactions that fraudulent subtangle falls behind the honest subtangle.

Proposition: Let the probabilities with which an attacker and honest nodes issue a new transaction be q and $p = 1 - q$ respectively. Assume that the current fraudulent subtangle constructed by the attaclker is smaller than the honest subtangle made by honest nodes g transactions. Then, the probability that the fraudulent subtangle can outweigh the honest subtangle is $a_g = (\frac{q}{p})^{g+1}$.

Proof: $a_g = a_{g+1}p + a_{g-1}q, g = 0, 1, 2$

Thus, the catch up function should be given by

$$C(p,q,g) = \begin{cases} 1, & p \leq q, \\ (\dfrac{q}{p})^{g+1}, & p > q, \end{cases}$$

Proposition: Let p, q be the probabilities with which an attacker and honest nodes issue a new transaction respectively, and $p > q, p + q = 1$. When an attacker launches double-spending attak, the probability of a successful attack under $z-$ confirmation transaction validation is given by

$$\begin{aligned} P_S(p,q,z) &= P\{N_a > N_h\} \cdot 1 + P\{N_a \leq N_h\} \cdot C(p,q,N_h - N_a) \\ &= \sum_{N_a=N_h+1}^{\infty} C_{N_a+N_h-1}^{N_a} p^{N_h} q^{N_a} + \sum_{N_a=0}^{N_h} C_{N_a+N_h-1}^{N_a} p^{N_h} q^{N_a} (\frac{q}{p})^{N_h-N_a+1} \\ &= 1 - \sum_{N_a=0}^{N_h} C_{N_a+N_h-1}^{N_a} (p^{N_h} q^{N_a} - p^{N_a-1} q^{N_h+1}), \end{aligned}$$

At time t_1, the number of transactions approcving the honest trasnactionis $W(t_1) - 1$. Therefore, we can have $N_h = N_{cw} - W(t_1) + 1$ transactions from t_1 to t_2. The successful attack probability can be expressed as

$$P_S(p,q,) = 1 - \sum_{N_a=0}^{w-W(t_1)+1} C_{N_a+w-W(t_1)}^{w-W(t_1)} (p^{w-W(t_1)+1} q^{N_a} - p^{N_a-1} q^{w-W(t_1)+2}),$$

where $W(t_1)$ is the cumulative weight of the honest transaction at the end of adaption period. Propabilitis with which honest nodes and an attacker issue a new transaction are $p = \frac{(n-1)\lambda'}{(n-1)\lambda'+\mu'}, q = \frac{\mu'}{(n-1)\lambda'+\mu'}$, where $\lambda' = \min\{\lambda, \frac{m}{nh}\}, \mu' = \min\{\mu, \frac{m}{nh}\}$, and $p > q$.

We use λ, μ representing the transaction arrival rates of honest nodes and an attacker to model double-spending attack. Besides, our analysis depends on wireless communication protocol, we use m presenting the number of broadcast trasnactions in wireless blockchain network.

Successful Probabilities of Different Attack Strategies

In this subsection, we analyse the the impacts of different attack strategies for successful attack probability.

A. Advance Attack Strategy

B. Adaptive Attack Strategy

Simulation and Discussion

Conclusion

Related Work

References

[1] L. S. Committee, "ANSI/IEEE Std 802.11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications". IEEE Computer Society, 1999.

[2] B. P. Crow, J.G. Kim, "IEEE 802.11 Wireless Local Area Networks", IEEE Communications magazine, Sept. 1997.

[3] H. Wu, S. Cheng, Y. Peng, K. Long and J. Ma, "IEEE 802.11 Distributed Coordination Function (DCF): Analysis and Enhancement," 2002 IEEE International Conference on Communications. Conference Proceedings. ICC 2002 (Cat. No.02CH37333), 2002, pp. 605-609 vol.1, doi: 10.1109/ICC.2002.996924.

[4] S. Popov, "The tangle", White paper, 2018. [Online]. Available: https://www.iota.org/research/academic-papers.

[5] R. G. Gallager, "Discrete Stochastic Processes". Kluwer Academic Publishers, 1996.

[6] A. Papoulis and S. U. Pillai, “Random walks and other applications,” in Probability, Random Variables and Stochastic Processes, 4th edition., Boston, Mass.: McGraw-Hill Europe, 2002.

[7]