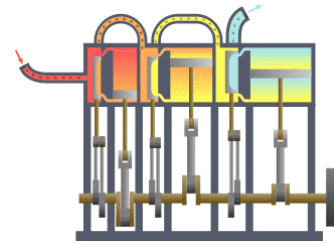


3. Thermodynamics basics

(by A. Rohrbach)

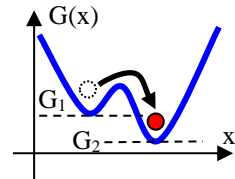
3.1. Introduction

Thermodynamics investigates systems that are controlled and characterized by physical quantities such as temperature T , heat Q , work W , different kind of energies E and entropy S . Such thermodynamic systems exchange information (and correspondingly energy).



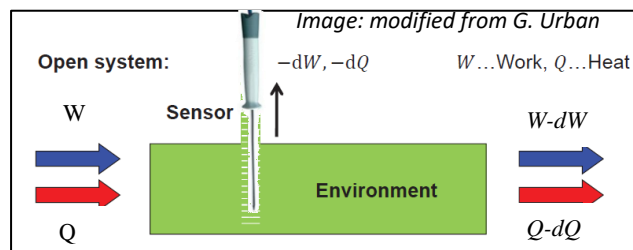
This exchange can be well investigated in thermal equilibrium, after the system has relaxed into a state of minimal energy G (e.g. from G_1 to G_2).

Thermodynamic systems can be investigated also in non-equilibrium. Then a system has not yet relaxed to an energy minimum or will even never find it, because it is constantly disturbed.



Thermodynamics has gained significant interest by the development of heat engines approximately 200 years ago,

but also plays a significant role in modern sensor technology!



The change of the physical quantities above is governed by four laws of thermodynamics

3.2. The laws of thermodynamics

In classical thermodynamics the following laws are important:

- **Zeroth Law:** *If two systems are each in thermal equilibrium with a third, they are also in thermal equilibrium with each other.* In reality this is the fourth and latest law. It provides the basis to define thermal equilibrium and it defines temperature.



- **First Law:** *The internal energy U of a closed system is constant. Or: If energy is added by heat dQ or work dW , U increases by dU just by this amount of heat or work*

$$dU = dQ + dW \quad (3.1)$$

Energy is changed from one form into another, but can never be created/destroyed.

- **Second Law:** *Heat does not flow spontaneously from a colder body to a hotter body. Or: All directional, spontaneous processes are irreversible. Or: All processes sensitive to friction (dissipated work W_{diss}) are irreversible*

To understand this, we need the definition of entropy S , which is a measure for disorder in a system: the disorder and the entropy S increase logarithmically with the count of possible arrangements (the number of states) of a number of particles (see example further down).

The product of temperature and entropy $T \cdot S$ is called entropic energy. In a closed system the change of heat is always connected to a loss in (friction) work W_{diss} . The sum of both equals the entropic energy:

$$\boxed{} \quad (3.2)$$

- **Third Law:** *It's impossible to cool a system to $T = 0$ K temperature (Nernst-theorem)*
This is not possible since temperature reduction can only be achieved by interaction with another even colder system

$$\boxed{} \quad (3.3)$$

3.2. Equilibrium states of a system

If a (closed) system reaches thermal equilibrium, neither time plays a role, nor the path the system has taken to reach equilibrium. All forces acting on the system cancel each other out.

In equilibrium the states of a system can be described by a state function (also state variable or state quantity). There are intensive state quantities like pressure p , volume V and temperature T , which do not depend on the system's size. Extensive state quantities depend on the size such as enthalpy H , internal energy U , its free energy G , or entropy S .

Ideal gas equation

For instance, the equation of state for an ideal gas (of N molecules) puts three state quantities into the following relation

$$\boxed{} \quad (3.4)$$

Or correspondingly: $\boxed{}$ or $n = \text{const}$

At constant temperature T , the pressure p increases with decreasing volume V and vice versa.

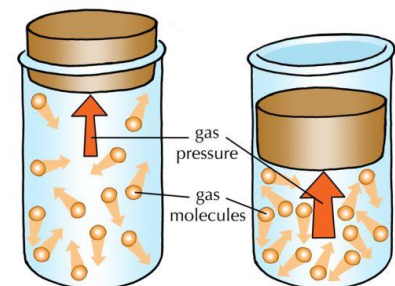


Image: mechasource.blogspot.com

Hence, an equation of state can be $p = f(V, T, n)$ or $V = f(p, T, n)$ $\boxed{}$ (3.5)

- $n = N / N_A$ is the number of moles of the gas (also amount of substance),
- $N_A = 6.0221 \cdot 10^{23} \text{ mol}^{-1}$ is the other Avogadro constant,
- $R = N_A \cdot k_B = 8.31446 \text{ J/(mol} \cdot \text{K)}$ is the ideal gas constant and
- $k_B = 1.381 \cdot 10^{-23} \text{ J/K}$ is the Boltzmann constant.

Remember the math

A (state) function $f(x, y)$ (e.g. $p(V, T, n)$) depending on several variables can be partially derived along the one or the other direction, $\frac{\partial}{\partial x} f(x, y)$ or $\frac{\partial}{\partial y} f(x, y)$, or it can be totally differentiated, e.g. to find a minimum: $\frac{\partial^2}{\partial x \partial y} f(x, y) = \frac{\partial}{\partial x} f(x, y) + \frac{\partial}{\partial y} f(x, y) = 0$.

The total differential can also be expressed as

$$df(x, y) = \boxed{} \quad (3.6)$$

Example: if $f(x, y) = xy^2 - 1$ then $\frac{\partial^2}{\partial x \partial y} f(x, y) = y^2 - 1 + 2xy = 0$ if $(x_0, y_0) = (0, \pm 1)$

Thermal energies of particles

Variations of the amount of thermal energies are called thermal fluctuations and can result in Brownian motion.

The product $k_B \cdot T$ is called **thermal energy**:

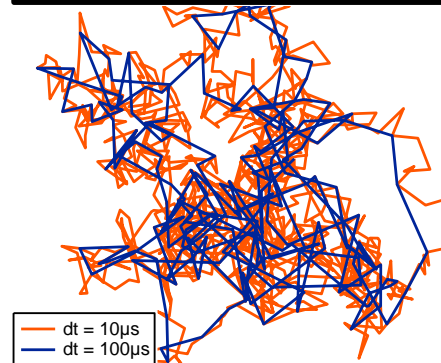
$$k_B T \approx \frac{1}{40} \text{ eV} = 25 \text{ meV} \approx 0.6 \text{ kcal/mol} = 2.5 \text{ kJ/mol}$$

$$k_B T \approx 4 \cdot 10^{-21} \text{ J} = 4 \cdot \text{pN} \cdot \text{nm} \text{ (at } T = 300 \text{ K)}$$

with $\text{pN} = 10^{-12} \text{ N (ewton)}$ and $\text{nm} = 10^{-9} \text{ m}$.

The product $R \cdot T = N_A \cdot k_B \cdot T$ is the thermal energy of 1 mol of particles ($n = N_A/N_A = 1$). E.g. an orange photon has the energy 2.5 eV.

Particle trajectory, at 2 different time samplings



Molecules in fluids and gases are bouncing against each other (stochastic collisions) leading to random paths called **Brownian motion**. Temporarily, the particles takes up a mean value of kinetic energy in direction $i = x, y, z$. Particle velocities v_i and energies W_i are the larger, the higher the temperature T :

per particle: $\langle W_i \rangle =$ (3.7)

Here the probability $p(v_i) \sim \exp(-W_i/k_B T) \sim \exp(-c^2 v_i^2)$ to find molecules with the velocity v_i is Boltzmann distributed: molecule velocities v are independent of each other, since molecules do not collide with each other. The root-mean square velocity is $v_{\text{rms}} = \sqrt{\langle v_i^2 \rangle}$.

Here the following integration rule is useful

$$\int_0^\infty v^2 \cdot p(v) dv = \int_0^\infty v^2 \cdot \exp(-c^2 \cdot v^2) dv = \frac{\sqrt{\pi}}{4c^3}.$$

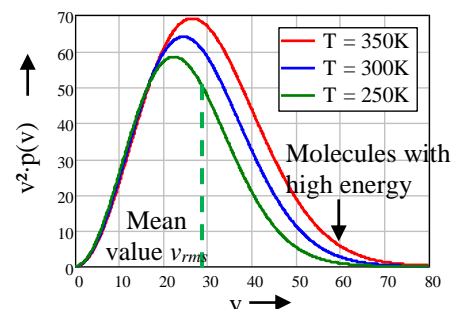


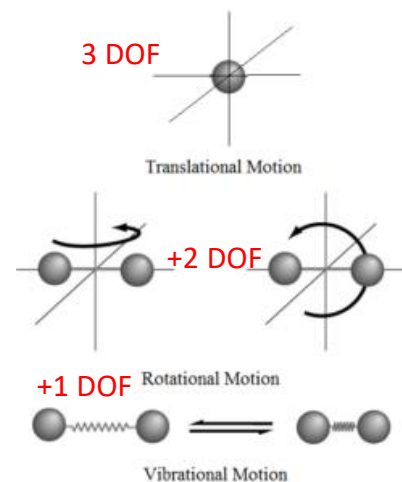
Figure 1: Three Boltzmann distributed molecule velocities

Equipartition theorem

In all his degrees of freedom (DOF) a particle has the mean energy of $\frac{1}{2} \cdot k_B T$ (i.e. for a single atom $m = 3$ DOF of translation, for a dimer plus 2 DOF of rotation and plus 1 DOF of vibration, resulting in $3.0 k_B T$ for $m = 6$ DOF).

One mol ($N_A \approx 6 \cdot 10^{23}$) of a single atomic gas (at $T = 300 \text{ K}$) has the energy $\langle W \rangle = \frac{3}{2} RT \approx 3/2 \cdot 6 \cdot 10^{23} \cdot 4 \cdot 10^{-21} \text{ J} = 3600 \text{ J}$.

For a particle the root mean square velocity $v_{\text{rms}} = \sqrt{\langle v_i^2 \rangle}$ in all 3 directions is $v_{\text{rms}} = \sqrt{3 k_B T / m}$, which can lead to a temporarily mean velocity of 640 m/s for a water molecule ($T = 25^\circ \text{C}$) and $v_{\text{rms}} = 8.6 \text{ m/s}$ for a 100 kDa protein, which is interesting to know for biotechnological sensors.



The internal energy U

is a thermodynamic potential (i.e. a state quantity) describing the total energy of a system.

For an ideal gas with $pV = nRT$, this energy is multiplied with the number m of degrees of freedom (DOF), allowing the gas to store more energy (expressed by the isochoric specific heat capacity c_v).

The internal energy depending on temperature reads

$$U(T, n) = \quad (3.8)$$

Together with the first law of thermodynamics $dU = \delta Q + \delta W$, the total differential becomes

$$dU = \quad (3.9)$$

δQ is the heat added to the system, δV is volume increase of the system.

Keeping one state fixed and let the other change

Isobaric processes ($p = \text{const}$, or $\delta p = 0$) result in $\delta Q = d \quad (3.10)$

Isochoric processes ($V = \text{const}$ or $\delta V = 0$) result in $\delta Q = d \quad (3.11)$

Isothermal processes ($\delta T = 0$, $\delta U = 0$) result in $\delta Q = d \quad (3.12)$

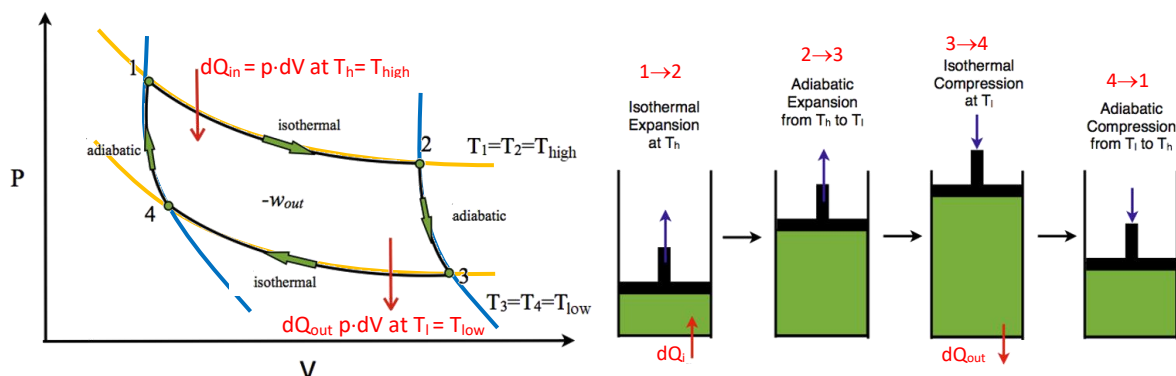
Adiabatic processes ($Q = \text{const}$ or $\delta Q = 0$) result in $dU = - \quad (3.13)$

The curves $p_1(V) = \frac{1}{RT_1} \cdot \frac{1}{V}$, $p_2(V) = \frac{1}{RT_2} \cdot \frac{1}{V}$, ... are called isotherms

The curves $p_1(V) = \frac{1}{RT_1} \cdot \frac{1}{V^q}$, $p_2(V) = \frac{1}{RT_2} \cdot \frac{1}{V^q}$, ... are called adiabates and have a strong slope because of the exponent q (adiabatic index), with $q = c_p/c_v = (m+2)/m$. We find $q = 5/3$ for an ideal (single atom) gas and $q = 7/5$ for a double atom gas.

Example: The Carnot cycle of a heat engine

Is described by a change between adiabatic and isotherm processes in pV space



The change from heat to mechanical work by isothermal extension or compression is

$$-dW_{12} = \int_{V_1}^{V_2} p_{12}(V) dV = R \cdot T_{12} \cdot \ln(V_2 / V_1) \quad \text{or} \quad -dW_{34} = \int_{V_3}^{V_4} p_{34}(V) dV = R \cdot T_{34} \cdot \ln(V_4 / V_3)$$

Adiabatic expansion or compression with $\delta Q = 0$ and $dU = -p \cdot \delta V = dW$ (eq.(3.13)) produces no work since: where $dU_{23} = U(T_{34}) - U(T_{12}) = dW_{23} = -dW_{41} = dU_{41} = -(U(T_{12}) - U(T_{34}))$

The generated work is from $W_{out} = dW_{12} + dW_{34} = R \cdot (T_{12} \ln(V_2/V_1) + T_{34} \ln(V_4/V_3))$ (3.14)

Since $\frac{V_2}{V_1} = \frac{V_3}{V_4}$ we find for the work output $W_{out} = R \cdot (T_{12} - T_{34}) \cdot \ln(V_2/V_1)$ (3.15)

The efficiency of a Carnot cycle is $\eta = \frac{W_{out}}{Q_{in}} = \frac{R \cdot (T_{12} - T_{34}) \cdot \ln(V_2/V_1)}{R \cdot T_{12} \cdot \ln(V_2/V_1)} = \frac{T_{12} - T_{34}}{T_{12}} < 1$ (3.16)

The enthalpy H

is an important thermodynamic potential opposing the entropic energy. It summarizes all energies that can be stored, important for many chemical and technological processes:

for an ideal gas: $H = U + p \cdot V$; for any sensor $H = U + pV + F \cdot x + \frac{1}{2} \kappa \cdot \theta^2 + q \cdot \Delta V_{el} \dots$ (3.17)

The enthalpy therefore considers mechanical, electrical and other potential energies as well.

Boltzmann distribution and partition function

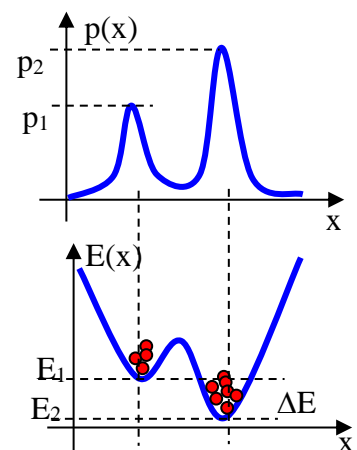
First, we need the **Boltzmann distribution**, which can be applied for processes in thermal equilibrium

$$p(E_i) = \frac{1}{Z} \exp(-E_i / k_B T) \text{ or } p(x) = \frac{1}{Z} \exp(-E(x) / k_B T) \quad (3.18)$$

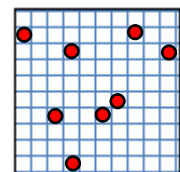
for a microstate (index i). For a given energy E_i of a particle in state i , the occupation probability is $p_i = p(E_i)$.

Now, the (canonic ensemble) **partition function Z** can be defined, which describes all possible states a system can take (changing with time):

$$Z = \quad (3.19)$$



Example: consider 8 equal particles on an array of 64 equal binding sites (see figure). The binding energy should be $E_0 = 0$, i.e. $p_0 = 1/Z$. What is the corresponding partition function Z , i.e. how many different particle arrangements (micro states) are possible? The so-called multiplicity of states N is the following combination

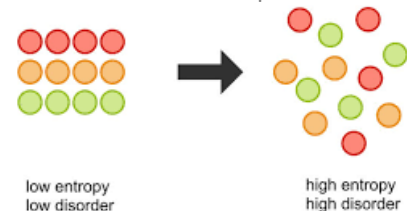


$$Z = N = \binom{64}{8} = \frac{64!}{8!(64-8)!} = 4.426 \cdot 10^9 \text{ and } p_0 = \frac{1}{Z} = 2.2596 \cdot 10^{-10}.$$

The entropy S

Is a measure of the number of possible microscopic arrangements in a thermodynamic system, i.e. it is a microscopic, statistical description. It is most relevant in physics, biology, chemistry, in information theory, and of course in sensor technology.

2 microstates from 12 particles:



The entropy S , as a function of disorder or arrangement of states, can be defined via the occupation probability:

$$S = -k_B \sum_i p_i \ln(p_i) \quad (3.20)$$

If all N particles or all N states have the same occupation probability $p_i = \frac{1}{N} = \frac{1}{Z}$, the entropy $S = -k_B \cdot Z \left(\frac{1}{Z} \cdot \ln\left(\frac{1}{Z}\right) \right)$ simplifies to

$$S = k_B \ln(Z) \quad (3.21)$$

In the example above with $Z = 4.426 \cdot 10^9$, we obtain $S = k_B \cdot \ln(Z) = 22.2 k_B$. Correspondingly, the entropic energy is $T \cdot S = 22.2 k_B T \approx 8.9 \cdot 10^{-20} \text{ J}$

The difference in entropy is described by the initial and final state (before and after a change)

$$dS = S_f - S_i = \quad (3.22)$$

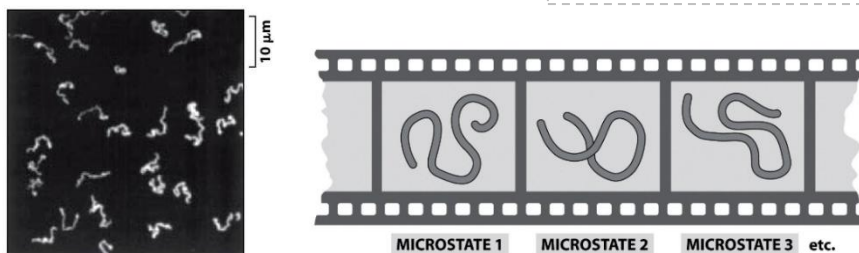


Figure 6.2b Physical Biology of the Cell (© Garland Science 2008)

Figure 2 (by R. Phillips): Microstates of DNA in solution. DNA molecules (and other polymers) jiggle around in solution exploring different conformational states. Each one of these conformations represents a different microstate. Left: Fluorescence microscopy image of λ -DNA confined to a surface. Right: Film strip showing conformations of a single DNA molecule at different instants (from Maier & Rädler, PRL 1999.)

The Gibbs free energy G

is thermodynamic potential that describes how a system is changing over time, i.e. driven by minimizing $G(H, T, S)$. It is the sum of the enthalpic and entropic energy

$$\begin{aligned} G &= H - T \cdot S \\ &= (U + p \cdot V) - T \cdot (k_B \cdot \ln Z) \end{aligned} \quad (3.23)$$

the Gibbs free energy can be minimized ($G \rightarrow 0$) when $H \approx T \cdot S$, which can be achieved for many combinations of the three state quantities T , S and H . Even when the enthalpy H does not become small, G can be minimized by a sufficiently large entropic energy $T \cdot S$. On its way to thermal equilibrium, the entropy and the number of possible states is maximized.

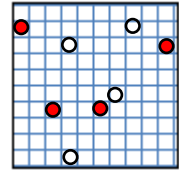
Reducing the Gibbs free energy by steps dG and the multidimensional way to equilibrium:

$$dG = d \quad (3.24)$$

Attention: If you take a movie of a process which has reached thermal equilibrium, you can play the movie backwards without finding any violation of causality. However, if you take a movie of the steady-state process (running water out of a tap), the backward movie will reveal impossible processes.

If e.g. 4 from 8 particles are removed (see example above), then the entropy change is negative (the disorder has decreased)

$$dS = S_f - S_i = k \cdot \ln \left(\frac{64!}{4!(64-4)!} / \frac{64!}{8!(64-8)!} \right) = -k \cdot 8.849$$



At constant temperature ($dT = 0$), volume ($dV = 0$) and pressure ($dp = 0$) eq. (3.24) becomes $dG = dU - T \cdot dS$, such that the free energy would increase by $dG = k_B T \cdot 8.849 > 0$, making the system slightly hotter until $dG = 0$ or $dU = T \cdot dS$.



Figure 3: the entropy of the ink-water system increases as the ink mixes with the water. Spontaneous un-mixing is never observed.

The Gibbs free energy is decreasing making the glass slightly cooler:

$$dG = dU - T \cdot dS \approx -T \cdot dS$$

Image: Pearson education Inc.

The chemical potential μ

is a thermodynamic state quantity that describes the chemical energy of a species of N particles in a system. If the number N_0 of molecules has the chemical energy μ_0 , then N molecules have the chemical potential

$$\mu(N) = \mu_0 \cdot N \quad (3.25)$$

In chemical reactions typically several species (index j) of each N_j molecules are involved, such that the change in Gibbs free energy dG can be described as

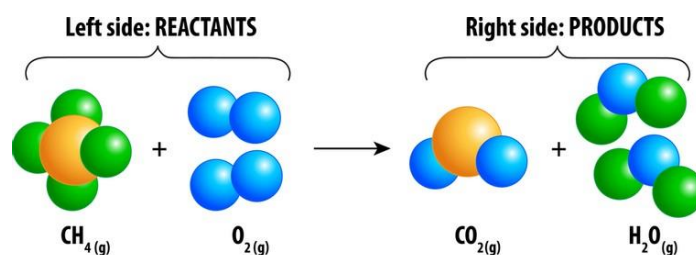
$$dG = dU + p \cdot dV - T \cdot dS + \sum_j \mu_j \cdot dN_j \quad (3.26)$$

where we assumed constant pressure ($dp = 0$) and constant temperature ($dT = 0$).

An alternative definition of the chemical potential is therefore $\mu_j = \left(\frac{dG}{dN_j} \right)_{p, T, N_{j \neq i}}$. In thermal

equilibrium, $dG = \sum_j \mu_j \cdot dN_j = 0$, one obtains for the particle numbers of each species

$$\mu_1 \cdot dN_1 + \mu_2 \cdot dN_2 + \mu_3 \cdot dN_3 + \dots = 0 \quad (3.27)$$

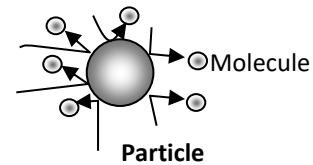


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3.3. Diffusion and fluctuation

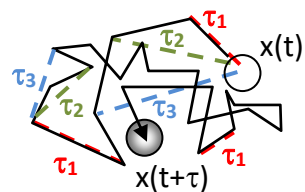
Principle:

A few micrometer small particle is hit randomly (without correlation) about 10^{20} x per sec by fluid molecules, resulting in a correlated (!) discontinuous motion of the particle.



Mathematics:

Brownian motion is a stochastic (Markow-) process, allowing to predict processes in the future from only little information obtained from past processes. With particle mass m this is called a Ornstein-Uhlenbeck process, neglecting the mass this is a Wiener-process.



Einstein relation

Einstein was able to derive a principle from the chaotic particle movements: he found that the particle's temporal mean of the square(d) displacement, $MSD(\tau) = \langle \Delta \mathbf{r}(\tau)^2 \rangle$, within a certain time interval τ , is proportional to the interval τ :

$$MSD[\mathbf{r}(t)] = MSD(\tau) = \langle \Delta \mathbf{r}(\tau)^2 \rangle = \langle (\mathbf{r}(t+\tau) - \mathbf{r}(t))^2 \rangle = 6D\tau \quad (3.28)$$

with D = diffusion constant, $[D] = \text{m}^2/\text{s}$, dimension factor $M = 1, 2, 3$

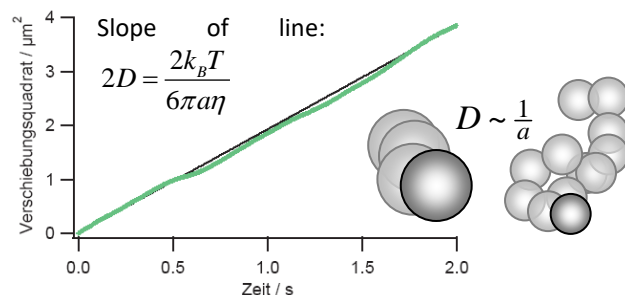
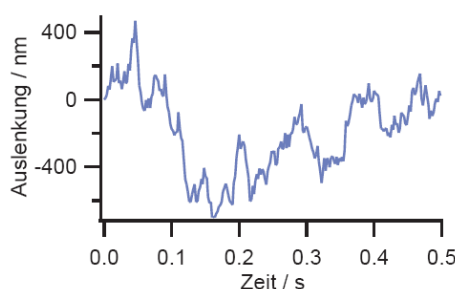
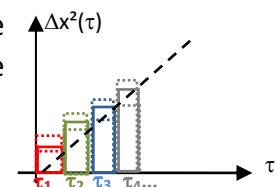


Figure 4: One dimensional particle trajectory $x(t)$ of a and resulting $MSD(\tau) = \langle |\Delta x(t, \tau)|^2 \rangle$.

The ingenious achievement from Einstein was to link the microscopic small thermal energy $k_B T$ with the microscopic friction factor $\gamma = F/v$:

The Einstein-Smoluchowski relation

(3.29)

Here, $k_B T$ is the thermal energy ($\approx 4 \cdot 10^{-21} \text{ J} = 4 \cdot \text{pN} \cdot \text{nm}$ at $T = 300 \text{ K}$) and $k_B = R/N_A$ is the Boltzmann constant, i.e the ratio between the general gas constant R and the Avogadro constant $N_A = 6.022 \cdot 10^{23}$.

$\gamma = 1/\mu$ with μ = mobility, where $\gamma = 6\pi \cdot a \cdot \eta$ for a sphere (**Stokes law**)

$\eta = 1 \text{ mPa s} = 1 \text{ cP} = 250 \text{ kT} \cdot \text{s} / \mu\text{m}^3$ ($T=20^\circ\text{C}$) viscosity of water, a = sphere radius

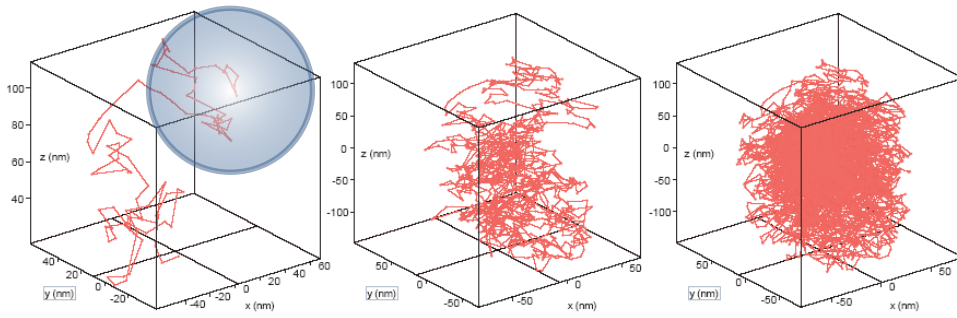
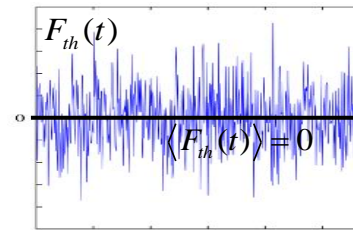


Figure 5: Measured 3D trajectories $\mathbf{r}(t)$ of an optically trapped sub-micrometer round particle undergoing Brownian motion in time steps of $50\mu\text{s}$. Shown are the first 100, 1000 and 5000 center positions. In the center image the particle has sensed the potential the particle is trapped in. On the right image the extent of the potential and the position distribution $p(\mathbf{r})$ can be determined.

Stochastic forces

Diffusing particle experience thermal random force $F_{th}(t)$ resulting from molecule collisions from different directions, with temporal mean $\langle F_{th}(t) \rangle = 0$.

$F_{th}(t)$ can be described only indirectly. Considering only white noise $N_{th}(t)$, which has a constant noise level at all frequencies, one finds for the force kicks from the molecules that $F_{th}(t)$ is uncorrelated for all time differences $\tau = t - t' \neq 0$:



$$AC[F_{th}(t)] = \langle F_{th}(t) \cdot F_{th}(t + \tau) \rangle = \quad (3.30)$$

Here, the Kronecker-Delta function is $\delta(t - t') = 1$ if $t = t'$, otherwise $\delta(t - t') = 0$, and $M = 1..3$ is the number of dimensions. $AC(\tau) = AC[f(t)]$ is the autocorrelation function of $f(t)$.

Thought through: An autocorrelation function $AC(\tau) = AC[x(t)]$ is a memory function or self-similarity function for events $x(t)$ (e.g. a change in position) within the time period $\tau = t' - t > 0$. One investigates what the quantity $x(t)$ at time t has to do with (i.e. correlates with) the quantity at a different time point $t' = t + \tau$, by forming all products $x(t) \cdot x(t + \tau)$. For good statistics, one has to average over many products $\langle x(t) \cdot x(t + \tau) \rangle$ of the same time difference τ . Hence, the trajectory $x(t + \tau)$ is shifted piecewise by τ in the overlap with $x(t)$ is calculated, which is $AC(\tau)$. If for instance the knowledge of a student simply diffuses away with time, his/her corresponding memory function $AC(\tau)$ will decrease linearly.

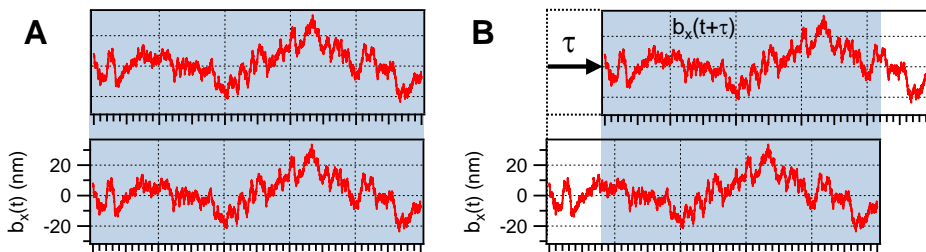


Figure 6: To determine the autocorrelation function $AC[x_i(t)]$ or $AC[b_i(t)]$ the trajectory $b_i(t)$ is shifted successively by τ . Next the temporal averages of overlaps (products) between functions $b_i(t)$ and $b_i(t + \tau)$ are calculated for each time shift τ . The blue region represents the total number of correlated data points, which for larger shifts τ .

Measuring signals on a thermal noise level

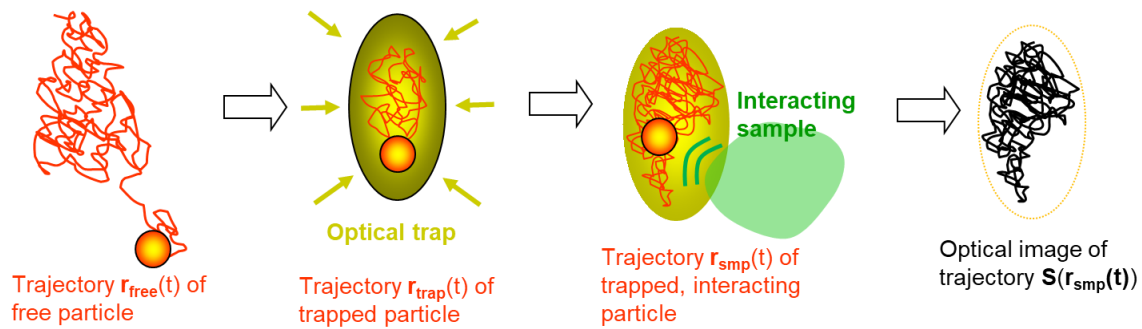


Figure 6: Sensing changes in thermal motion upon interaction with a sample. A freely diffusing particle (left) is captured in an optical trap (laser tweezers), such that Brownian motion is confined. The fluctuating particle trajectory $\mathbf{r}(t) = (x(t), y(t), z(t))$ changes in space and time (2nd figure). Then the optical trap is moved to the interaction partner (sample) and the trajectory changes again in a characteristic manner (3rd figure). A precise high-speed optical sensor (quadrant photodiodes, QPD) measures the particle trajectory (optical image) with reasonable accuracy and precision based on laser interference. The measured particle trajectory is analyzed to reconstruct the interactions with the sample.

4. Optical components of sensors

4.1. Foundations of optics and radiation

Wave - particle dualism

- Light is a spatially confined package of quantized energy $W_{ph} = h \cdot \nu = \hbar \cdot \omega$ and quantized momentum $p = \hbar \cdot \omega / c$ - called photon (with Planck constant $h = 6.626 \cdot 10^{-34} \text{Js}$, $\hbar = h/2\pi$). It belongs to the particle family of bosons (Spin ± 1) and does not obey the Pauli exclusion principle (no interaction with other particles). In Quantum Electrodynamics (QED), photons are the exchange quanta of the electromagnetic forces.
- Light is an electromagnetic wave, with the ability to interfere - also with itself. The wave carries energy in a continuous flux of photons provided the number of photons is sufficiently large (high intensity I).
- Light can be approximated as a ray, if the wave nature of light can be disregarded and the extent of the interacting object is large $d \gg \lambda$. Light is treated as a straight ray or beam undergoing reflection and deflection at interfaces

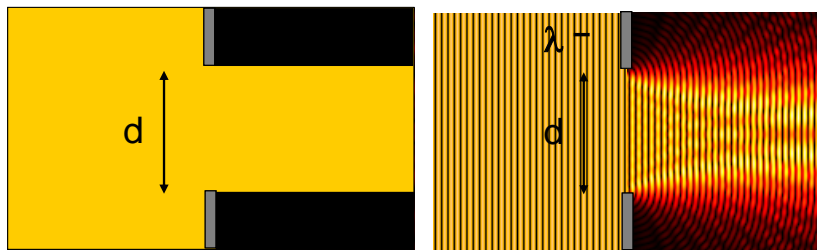


Figure 7: Transmission through an aperture.
Dimensions of ray optics $d \gg \lambda$ (left) and wave optics $d \approx 5\lambda - 50\lambda$ (right).

Where do photon and electromagnetic waves come from?

One uses the idea of a blackbody, which absorbs electromagnetic radiation of all wavelengths λ or frequencies $\omega = 2\pi \cdot c/\lambda$. In thermal equilibrium at a specific temperature T , the blackbody emits an ideal and characteristic radiation spectrum, where the spectral shape is only determined by the temperature.

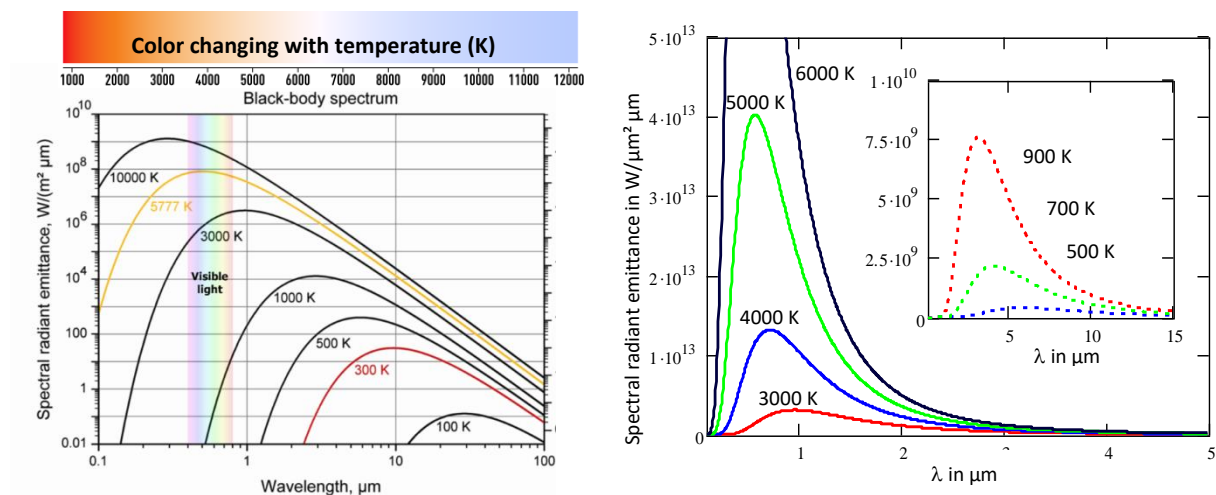


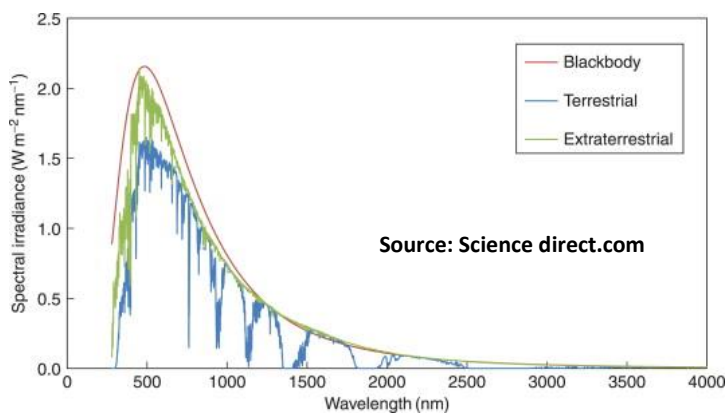
Figure 8: Blackbody radiation spectrum in logarithmic and linear scale. Left: for lower temperatures T the emission maximum shifts to higher wavelengths (Graph: Wikipedia). Right: emission power decreases with T .

The spectrum $L(\lambda, T)$ for a blackbody in thermal equilibrium at temperature T was described in 1901 by Max Planck by his radiation law:

$$L(\lambda, T) = 2\pi hc_0^2 \cdot \frac{1}{\lambda^5} \cdot \left(\exp\left(\frac{hc_0}{\lambda k_B T}\right) - 1 \right)^{-1} \quad (4.1)$$

The amplitude and thereby the emitted power (area under the curve) strongly increase the temperature by the fifth power. However, to avoid unrealistically high emission at four short wavelengths (ultraviolet catastrophe), Max Planck had to scale the energy of light with frequency $\nu = \omega/2\pi$, which is Planck's constant $h = E/\nu$.

Its total emitted power $\int L(\lambda, T) d\lambda$ is described by the emissivity, which is $\mathcal{E} = 100\%$. For all real materials the emissivity $\mathcal{E} < 1$.



For most materials are gases in the atmosphere the spectrum becomes distorted with characteristic photon absorption bands.

Figure 9: Nonideal, here terrestrial and extra-terrestrial emission spectrum relative to the black body spectrum.

Radiation at different wavelengths

The average energy of vibrating particles (atoms or molecules) is represented by temperature. The vibrating atom have an electron cloud, i.e. orbiting electric charge around the nucleus with oscillation frequency ω .

According to the laws of electrodynamics (Maxwell equations), a moving electric charge is associated with a variable electric field \mathbf{E} that produces an alternating magnetic field \mathbf{B} . In turn, when the magnetic field changes, it produces a changing electric field, and so on. Thus, a vibrating particle is a source of electromagnetic field, which propagates outwardly with the speed of light and is governed by the laws of optics, i.e. electromagnetic waves can be reflected, filtered, focused, etc.

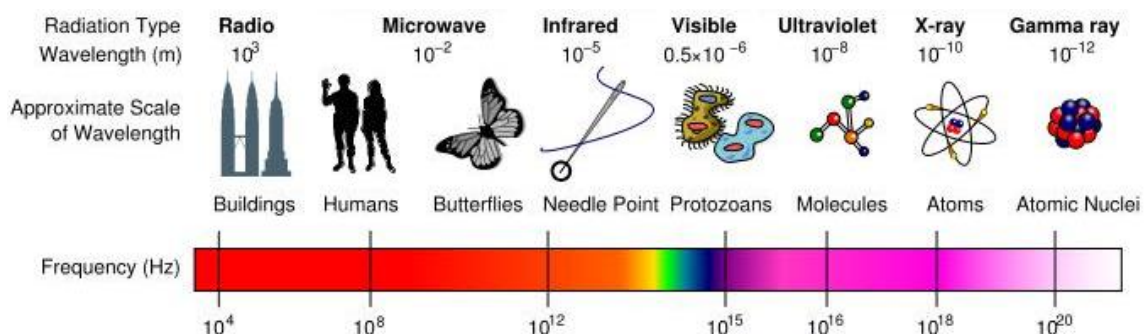


Figure 10: Radiation wavelengths, frequencies relative to objects for different radiation types (Source: Wikipedia)