QC-Module

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October 15, 2024

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1 DSE

The Dyson-Schwinger equation (DSE) in the Euclidean space takes the following form

$$S^{-1}(p) = Z_2\left(i\gamma\cdot p + m_f\left(\Lambda^2\right)\right) + \int_q^{\Lambda} g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\Gamma^a_\nu(q,p). \tag{1}$$

The dressed-quark propagator takes the form

$$S^{-1}(p) = \frac{1}{Z\left(p^2,\zeta^2\right)}\left(i\gamma\cdot p + M\left(p^2,\zeta^2\right)\right) = i\gamma\cdot pA\left(p^2,\zeta^2\right) + B\left(p^2,\zeta^2\right). \tag{2}$$

Then we apply the rainbow approximation

$$\Gamma^a_\nu = \gamma_\nu\frac{\lambda^a}{2}, \tag{3}$$

and the Qin-Chang form

$$g^2 D_{\mu\nu}(k) = \mathcal{G}\left(k^2\right)T_{\mu\nu}(k), \tag{4}$$

where

$$k^2 T_{\mu\nu}(k) = k^2\delta_{\mu\nu} - k_\mu k_\nu \tag{5}$$

and

$$\frac{1}{Z_2^2}\mathcal{G}\left(k^2\right) = \frac{8\pi^2 D}{\omega^4}e^{-\frac{k^2}{\omega^2}} + \frac{8\pi^2\gamma_m\mathcal{F}\left(k^2\right)}{\ln[\tau + \left(1+\frac{k^2}{\Lambda^2_{QCD}}\right)^2]}. \tag{6}$$

The parameters are listed below:

$$\begin{aligned} D &= 1.024GeV^2,\\ \omega &= 0.5GeV,\\ m_t &= 0.5GeV,\\ \tau &= e^2-1,\\ n_f &= 4,\\ \Lambda_{QCD} &= 0.234GeV,\\ \gamma_m &= 12/(33-2n_f). \end{aligned} \tag{7}$$

The results are shown in Fig.1.

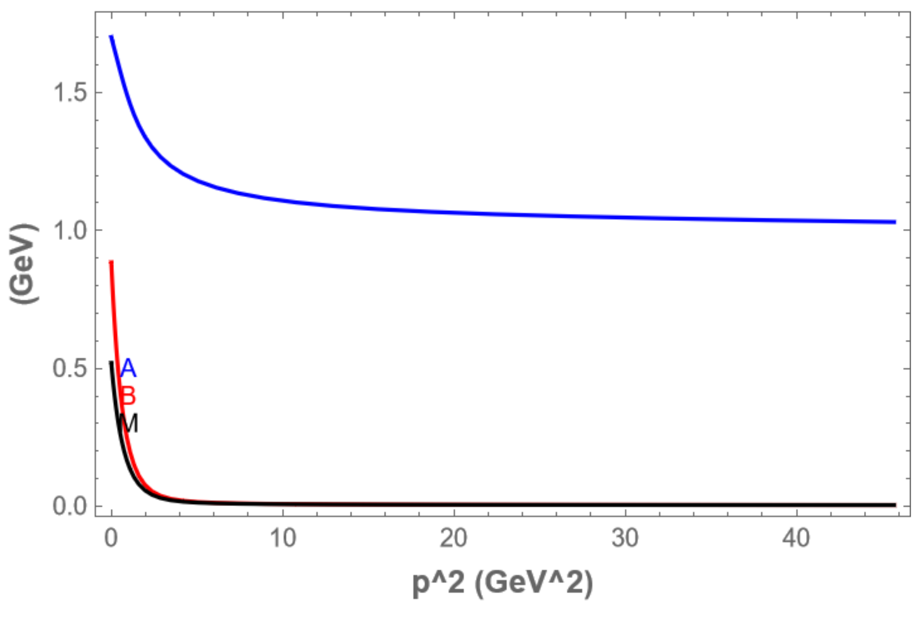

Figure 1: The function A, B and the quark mass function M.

2 BSE

The quark-antiquark bound state amplitude can be obtained by solving a homogeneous BSE. Employing the ladder truncation becomes

$$\Gamma^{f\widetilde{g}}(p,P) = -Z_2^2 \int_q^{\Lambda} \mathcal{G}\left(k^2\right) T_{\mu\nu}(k) \frac{\lambda^a}{2}\gamma_\mu S^f(q_+)\,\Gamma^{f\widetilde{g}}(q;P)\,S^g(q_-)\frac{\lambda^a}{2}\gamma_\nu. \tag{8}$$

Soving the eigenvalue equation

$$\lambda\left(P^2\right)\overrightarrow{F}_X\left(P^2\right) = \mathcal{K}_X\left(P^2\right)\overrightarrow{F}_X\left(P^2\right), \tag{9}$$

The solution at $\lambda\left(P^2=-M^2\right)=1$ corresponds to meson bound state.
The results are shown in Fig.2.

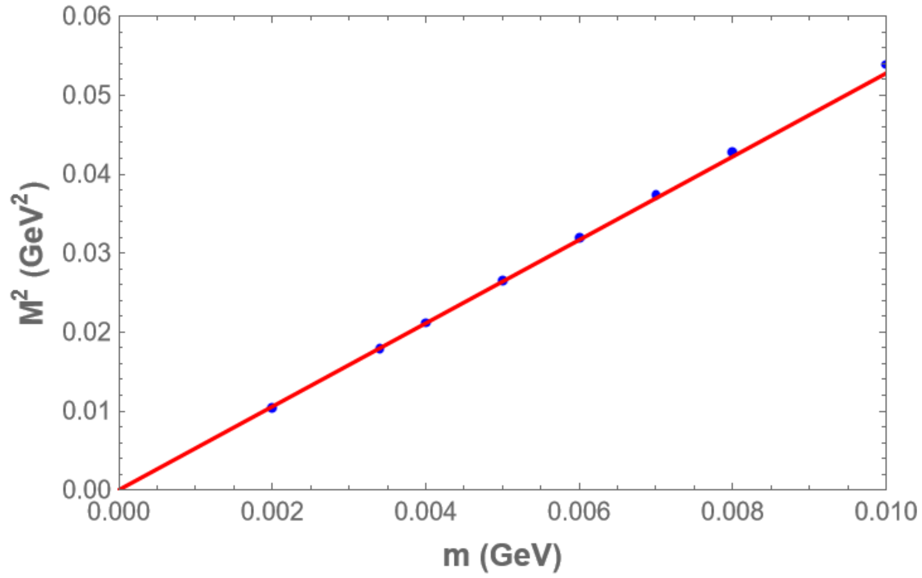

Figure 2: The red line shows the equation $M^2 = \frac{2r_P(0.0034)}{f_\pi(0.0034)}m_\mu$; the blue points are calculated by $M^2 = \frac{2r_P(m_\mu)}{f_\pi(m_\mu)}m_\mu$.

This figure shows the Gell-Mann-Oakes-Renner relation.
All the calculation results are shown below.

m	M	λ	f_π	r_P
0.002	0.102	1.0001	0.0925411	0.241155
0.0034	0.134	1.00007	0.093628	0.246978
0.004	0.1454	0.99999	0.094167	0.249134
0.005	0.163	0.99999	0.094167	0.249134
0.006	0.1785	0.99990	0.0956966	0.254830
0.007	0.1935	0.99994	0.0964696	0.257463
0.008	0.2072	0.99990	0.0972156	0.259971
0.01	0.2325	1.00000	0.0982149	0.264597

Table 1: The calculation results.