QC-Module

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Contents

1 DSE

The Dyson-Schwinger equation (DSE) in the Euclidean space takes the following form

$$S^{-1}(p) = Z_2 \left(i\gamma \cdot p + m_f\left(\Lambda^2\right)\right) + \int_q^{\Lambda} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \Gamma_\nu^a(q,p). \tag{1}$$

The dressed-quark propagator takes the form

$$S^{-1}(p) = \frac{1}{Z(p^2,\zeta^2)}\left(i\gamma \cdot p + M\left(p^2,\zeta^2\right)\right) = i\gamma \cdot p A\left(p^2,\zeta^2\right) + B\left(p^2,\zeta^2\right). \tag{2}$$

Then we apply the rainbow approximation

$$\Gamma_\nu^a = \gamma_\nu \frac{\lambda^a}{2}, \tag{3}$$

and the Qin-Chang form

$$g^2 D_{\mu\nu}(k) = \mathcal{G}\left(k^2\right) T_{\mu\nu}(k), \tag{4}$$

where

$$k^2 T_{\mu\nu}(k) = k^2 \delta_{\mu\nu} - k_\mu k_\nu \tag{5}$$

and

$$\frac{1}{Z_2^2}\mathcal{G}\left(k^2\right) = \frac{8\pi^2 D}{\omega^4}e^{-\frac{k^2}{\omega^2}} + \frac{8\pi^2\gamma_m \mathcal{F}\left(k^2\right)}{\ln[\tau + \left(1 + \frac{k^2}{\Lambda_{QCD}^2}\right)^2]}. \tag{6}$$

The parameters are listed below:

$$\begin{aligned}
D &= 1.024 GeV^2, \\
\omega &= 0.5 GeV, \\
m_t &= 0.5 GeV, \\
\tau &= e^2 - 1, \\
n_f &= 4, \\
\Lambda_{QCD} &= 0.234 GeV, \\
\gamma_m &= 12/(33 - 2n_f).
\end{aligned} \tag{7}$$

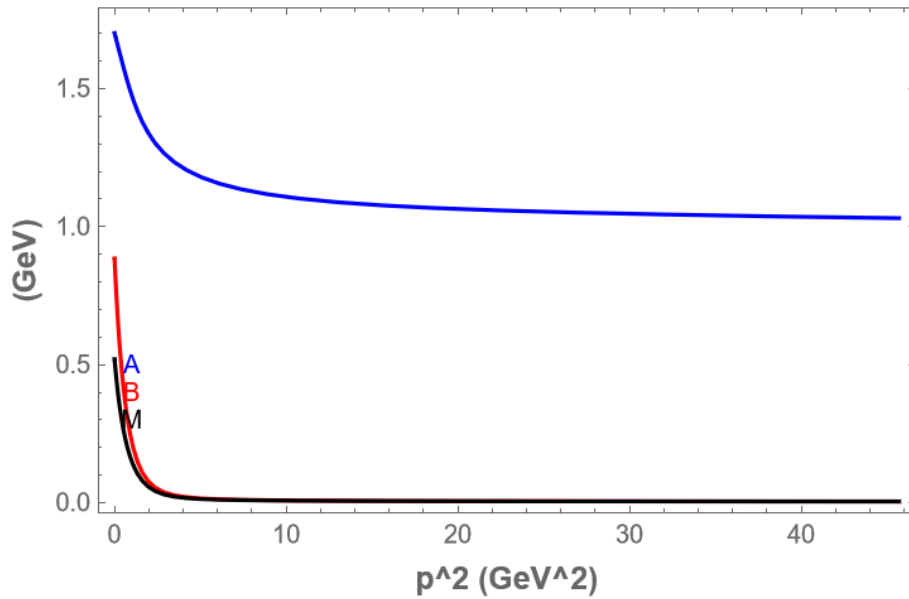The results are shown in Fig.1.

Figure 1: The function A, B and the quark mass function M.

2 BSE

The quark-antiquark bound state amplitude can be obtained by solving a homogeneous BSE. Employing the ladder truncation becomes

$$\Gamma^{f\tilde{g}}(p, P) = -Z_2^2 \int_q^\Lambda \mathcal{G}(k^2) T_{\mu\nu}(k) \frac{\lambda^a}{2} \gamma_\mu S^f(q_+) \Gamma^{f\tilde{g}}(q; P) S^g(q_-) \frac{\lambda^a}{2} \gamma_\nu. \tag{8}$$

Soving the eigenvalue equation

$$\lambda(P^2) \overrightarrow{F}_X(P^2) = \mathcal{K}_X(P^2) \overrightarrow{F}_X(P^2), \tag{9}$$

The solution at $\lambda(P^2 = -M^2) = 1$ corresponds to meson bound state.
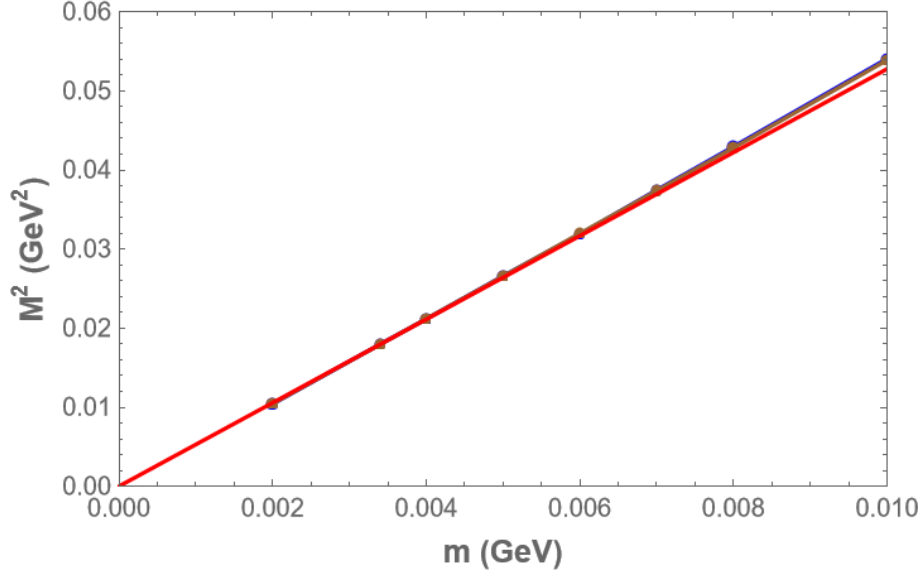The results are shown in Fig.2.

Figure 2: The red line shows the equation $M^2 = \frac{2r_P(0.0034)}{f_\pi(0.0034)} m_\mu$; the brown line shows the equation $M^2 = \frac{2r_P(m_\mu)}{f_\pi(m_\mu)} m_\mu$; the blue line shows the results directly calculated from solving BSE.

This figure shows the Gell-Mann-Oakes-Renner relation.
All the calculation results are shown below.

m	M	λ	f_π	r_P
0.002	0.102	1.0001	0.0925411	0.241155
0.0034	0.134	1.00007	0.093628	0.246978
0.004	0.1454	0.99999	0.094167	0.249134
0.005	0.163	0.99999	0.094167	0.249134
0.006	0.1785	0.99990	0.0956966	0.254830
0.007	0.1935	0.99994	0.0964696	0.257463
0.008	0.2072	0.99990	0.0972156	0.259971
0.01	0.2325	1.00000	0.0982149	0.264597

Table 1: The calculation results.