QC-Module

Xin-Peng Li

September 18, 2024

Contents

1 DSE

The Dyson-Schwinger equation (DSE) in the Euclidean space takes the following form

$$S^{-1}(p) = Z_2\left(i\gamma\cdot p + m_f\left(\Lambda^2\right)\right) + \int_q^{\Lambda} g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\Gamma^a_\nu(q,p). \tag{1}$$

The dressed-quark propagator takes the form

$$S^{-1}(p) = \frac{1}{Z\left(p^2,\zeta^2\right)}\left(i\gamma\cdot p + M\left(p^2,\zeta^2\right)\right) = i\gamma\cdot pA\left(p^2,\zeta^2\right) + B\left(p^2,\zeta^2\right). \tag{2}$$

Then we apply the rainbow approximation

$$\Gamma^a_\nu = \gamma_\nu\frac{\lambda^a}{2}, \tag{3}$$

and the Qin-Chang form

$$g^2 D_{\mu\nu}(k) = \mathcal{G}\left(k^2\right)T_{\mu\nu}(k), \tag{4}$$

where

$$k^2 T_{\mu\nu}(k) = k^2\delta_{\mu\nu} - k_\mu k_\nu \tag{5}$$

and

$$\frac{1}{Z_2^2}\mathcal{G}\left(k^2\right) = \frac{8\pi^2 D}{\omega^4}e^{-\frac{k^2}{\omega^2}} + \frac{8\pi^2\gamma_m\mathcal{F}\left(k^2\right)}{\ln[\tau + \left(1+\frac{k^2}{\Lambda^2_{QCD}}\right)^2]}. \tag{6}$$

The parameters are listed below:

$$\begin{aligned} D &= 1.024GeV^2,\\ \omega &= 0.5GeV,\\ m_t &= 0.5GeV,\\ \tau &= e^2-1,\\ n_f &= 4,\\ \Lambda_{QCD} &= 0.234GeV,\\ \gamma_m &= 12/(33-2n_f). \end{aligned} \tag{7}$$

The results are shown in Fig.1.

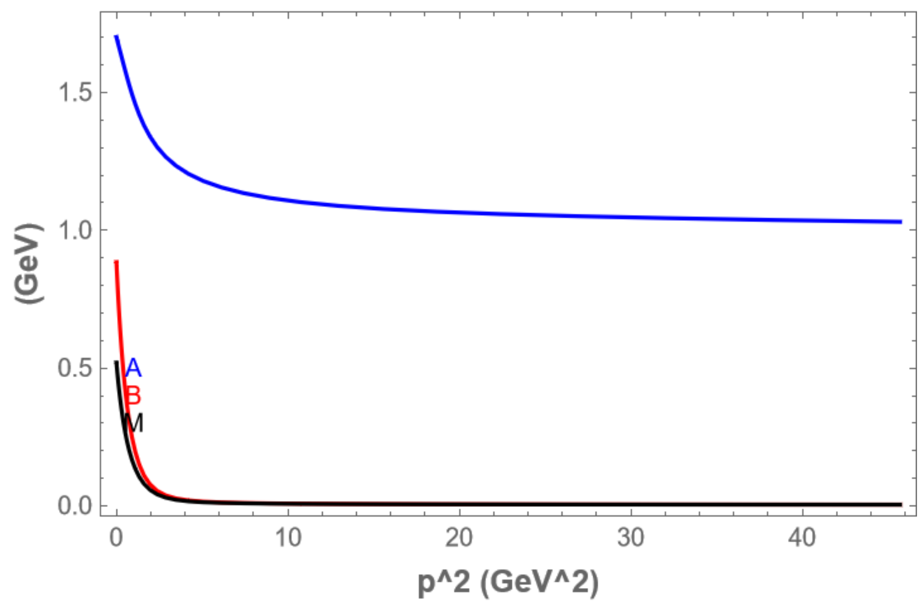

Figure 1: The function A, B and the quark mass function M.

2 BSE

The quark-antiquark bound state amplitude can be obtained by solving a homogeneous BSE. Employing the ladder truncation becomes

$$\Gamma^{f\widetilde{g}}(p,P) = -Z_2^2 \int_q^{\Lambda} \mathcal{G}\left(k^2\right) T_{\mu\nu}(k) \frac{\lambda^a}{2}\gamma_\mu S^f(q_+) \Gamma^{f\widetilde{g}}(q;P) S^g(q_-) \frac{\lambda^a}{2}\gamma_\nu. \tag{8}$$

Soving the eigenvalue equation

$$\lambda\left(P^2\right) \overrightarrow{F}_X\left(P^2\right) = \mathcal{K}_X\left(P^2\right) \overrightarrow{F}_X\left(P^2\right), \tag{9}$$

The solution at $\lambda\left(P^2 = -M^2\right) = 1$ corresponds to meson bound state.
The results are shown in Fig.2.

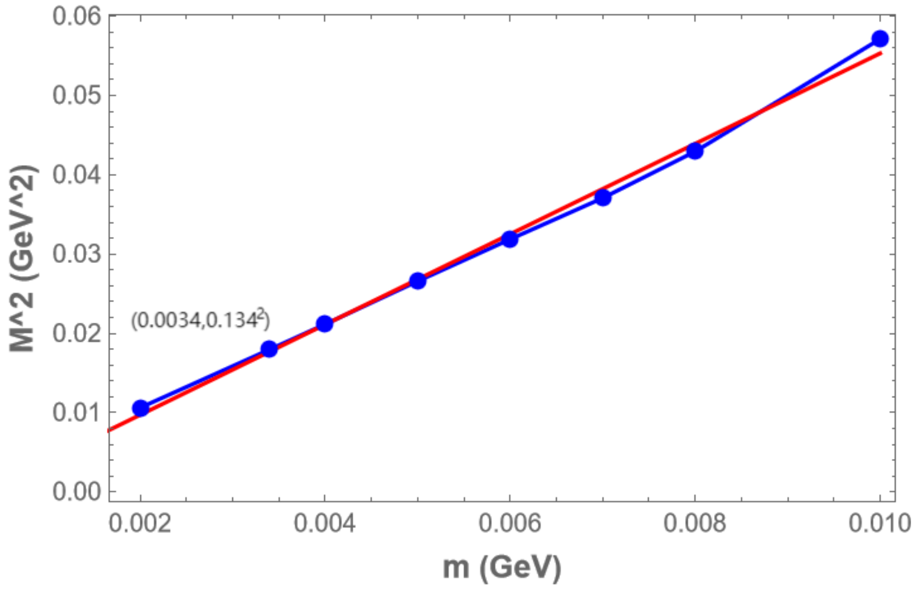

Figure 2: The blue points are directly calculated points.

This figure shows the Gell-Mann-Oakes-Renner relation.
I also calculate the pion decay constant f_π, the results are shown in the table below.

f_π
0.093665

Table 1: The pion decay constant.