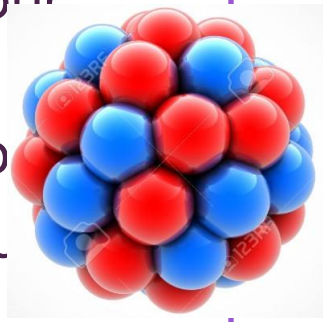
Strong Interactions and

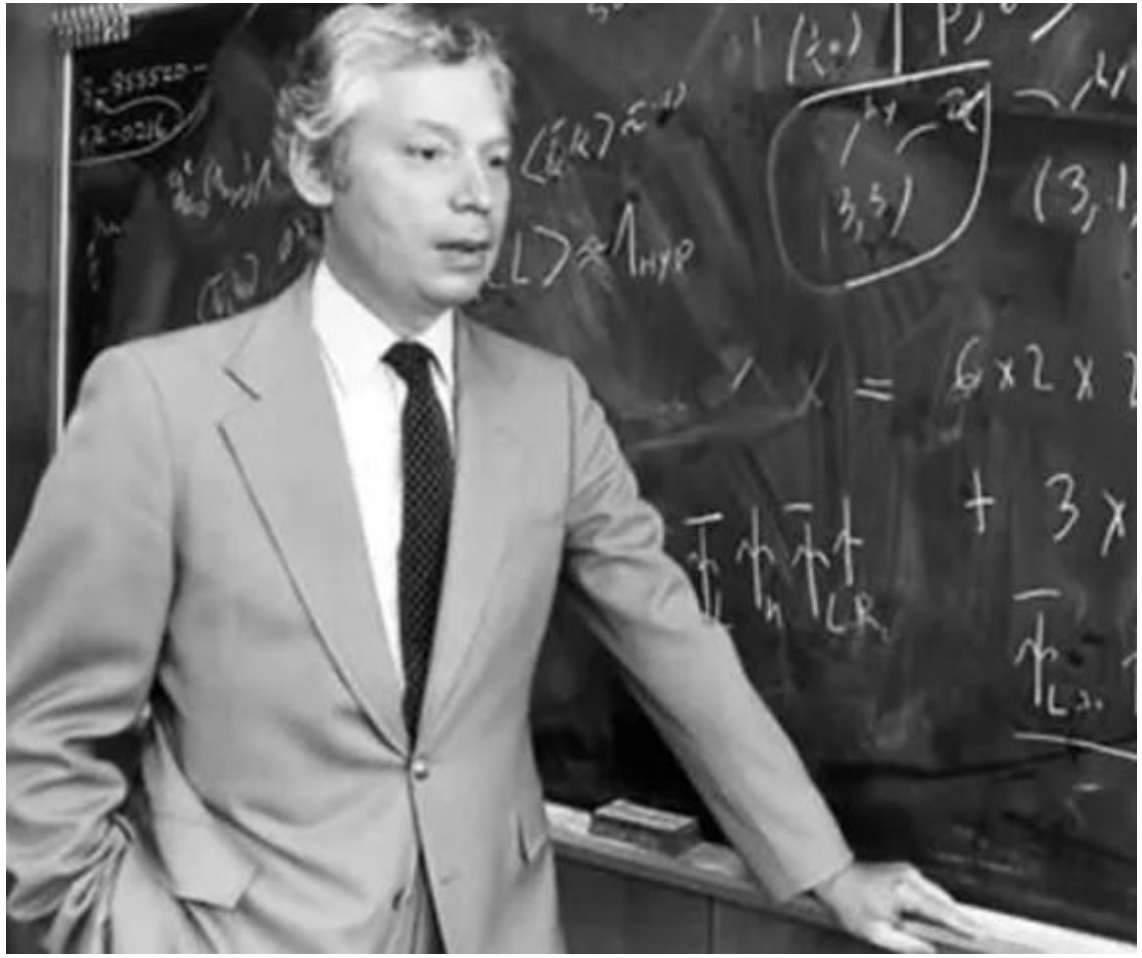
The Nature of Things

- Looking at the known Universe, one could be awed by the many complex things it contains.
- Even the Earth itself is complicated enough to generate many questions in our minds
- Basic amongst them are those which focus on our own existence and comp
- Here, too, there are many levels to be explored, running right down to the nu at the core of every atom and molecule
- Even deeper, to the neutrons and protons (nucleons) that constitute those nuclei.
- Faced with all this, physicists nevertheless assume that a few simple mathematical rules should be sufficient to provide a complete explanation of everything we can now perceive, and which might become perceptible in future.
- This may be correct



Or it might be supreme arrogance = hubris

Nature of Physics



Steven Weinberg

Born: May 3, 1933, New York, NY

Died: July 23, 2021, Austin, TX

- ✓ "The effort to understand the universe is one of the very few things that lifts human life a little above the level of farce and gives it some of the grace of tragedy."
- ✓ "Nothing in the last five hundred years has had so great an effect on the human spirit as the discoveries of modern science."
- ✓ "I know of no one who has participated actively in the advance of physics in the postwar period whose research has been significantly helped by the work of philosophers."
- ✓ "Science is the path to truth"

Basic Question: How did we

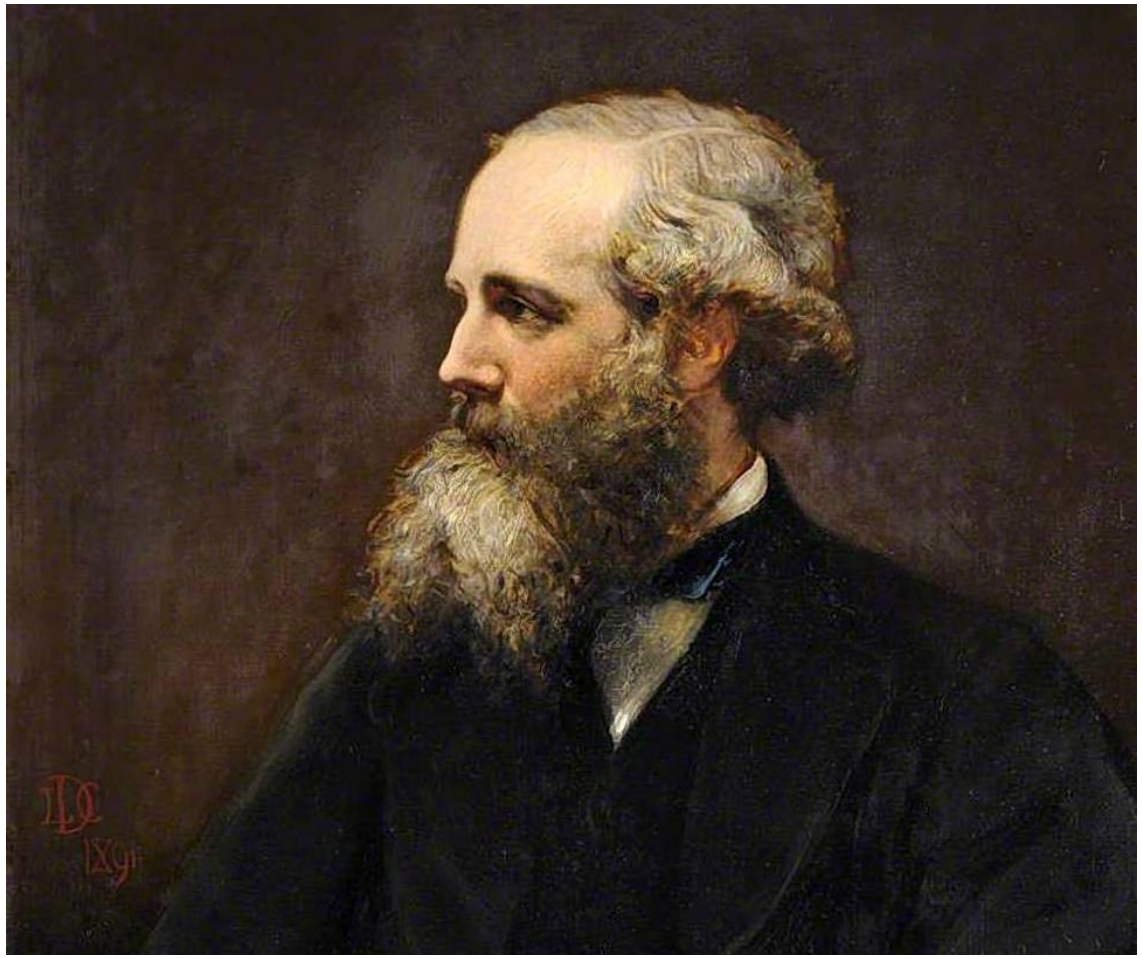
WHERE TO BEGIN

Great Unifications

- ✓ “First Great Unification" – Isaac Newton – 350 years ago
  - ✓ Universal Law of Gravitation
  - ✓ A person, fixed on earth, realised that all objects attract each other with a force of gravitational attraction, no matter where they are in the Universe.
  - ✓ Unified the understandings of the observable phenomena of gravity on Earth with the observable behaviour of celestial bodies in space

Great Unifications

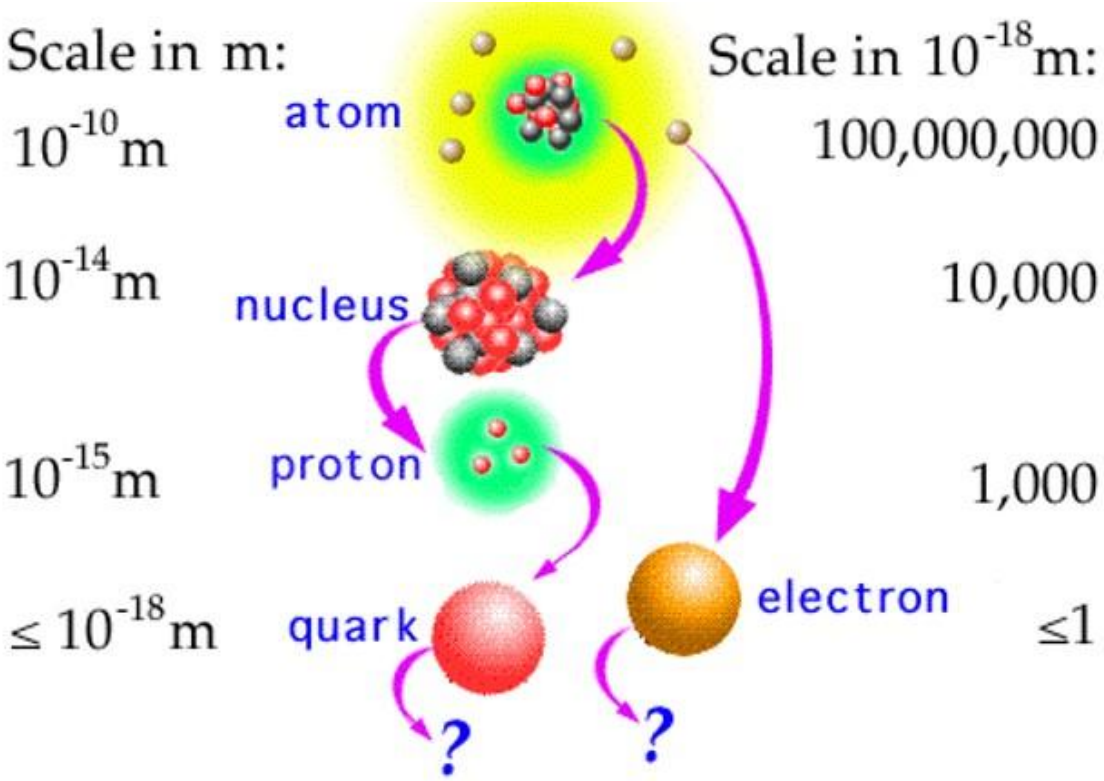
- ✓ “Second Great Unification" – 155 years ago – James Clerk Maxwell
- ✓ Formulated the classical theory of electromagnetic radiation, unifying electricity + magnetism + light as different manifestations of the same phenomenon.
- ✓ “A Dynamical Theory of the Electromagnetic Field” – 1865
  - ✓ Demonstrated that electric and magnetic fields travel through space as waves moving at the speed of light.
  - ✓ Unification of light and electrical phenomena led to Maxwell’s prediction of the existence of radio waves.



Great Unifications

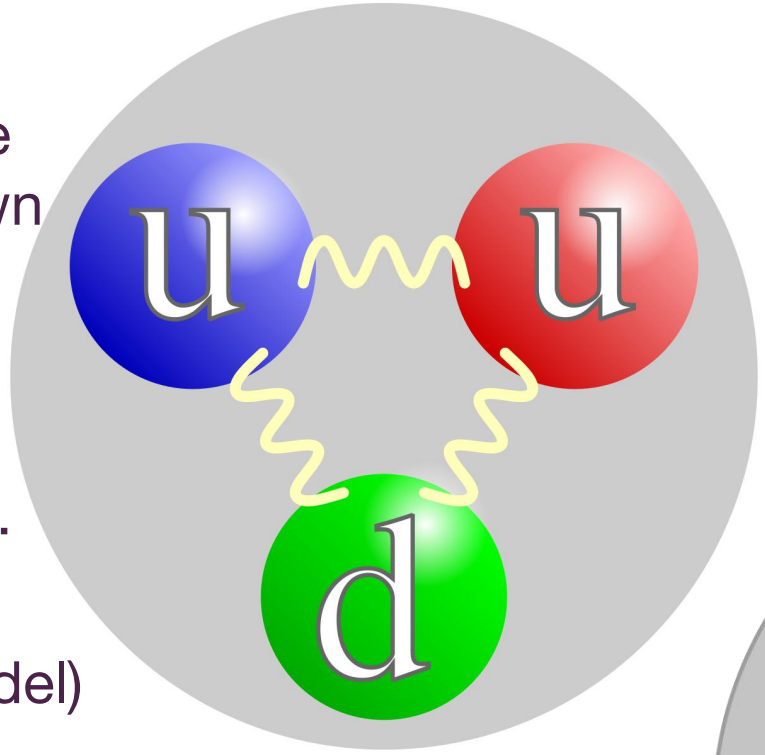
- ✓ "Third Great Unification" – Standard Model of Particle Physics
- ✓ 1967 – S. Weinberg "A Model of Leptons"

  ~ 14,800 citations
- ✓ "The most successful theory ever conceived …"
  - ✓ All particles predicted to exist have been found.
  - ✓ The masses of those particles lie within 1% of the theoretical values anticipated by the model.
  - ✓ 55 Nobel Prizes awarded for key developments and discoveries
- ✓ Description of all known fundamental physics except for gravity, and gravity has no detectable effect when particles are studied a few at a time.



Standard Model - History

- In the early 20th Century, the only particles of matter known to exist were the proton, neutron, and electron.
- Pion discovered in 1947
- For the everyday physicist …
- Basic questions:
  - what do these (quark model) pictures mean?
  - are they realistic in *any* sense?
  - why is pion mass so much smaller than proton mass?
  - what theory can explain these systems?

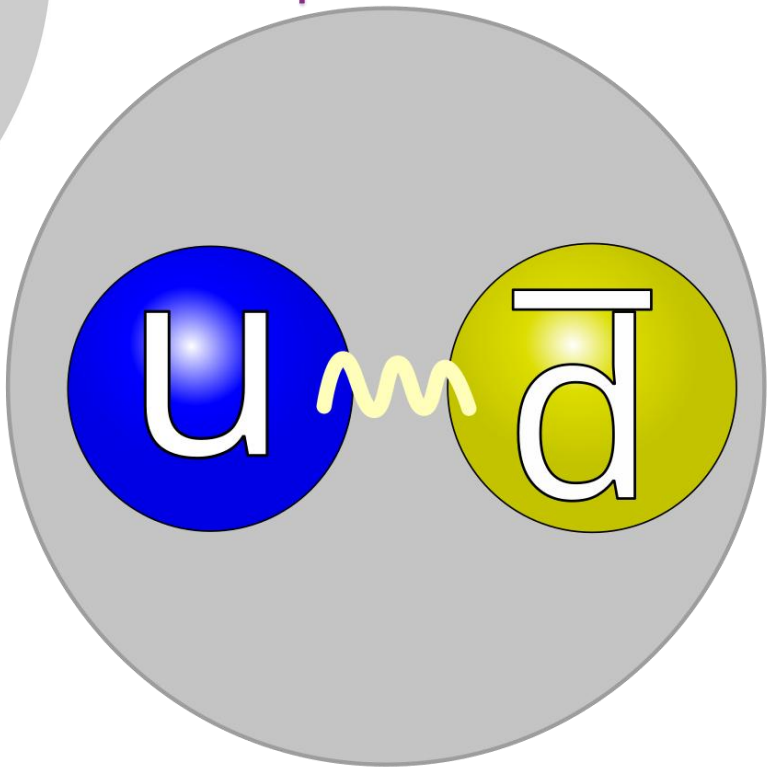


proton = baryon

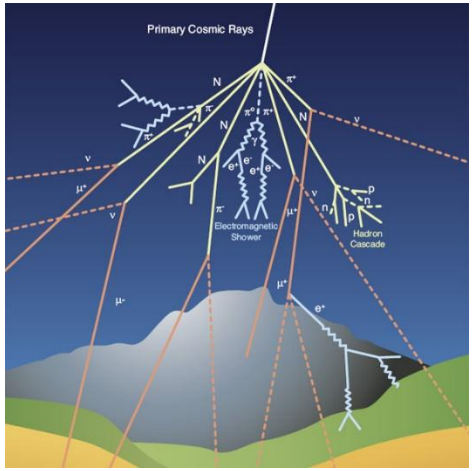
$\text{mass}_{\text{proton}} = 1\ \text{GeV}$

pion = meson

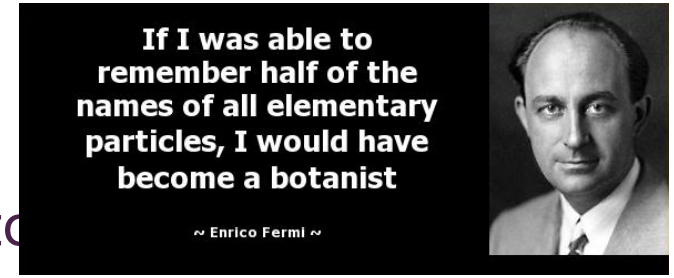
$\text{mass}_{\text{pion}} = 0.14\ \text{GeV}$



Standard Model - History



- In the early 20th Century, the only particles of matter known to exist were the proton, neutron, and electron.



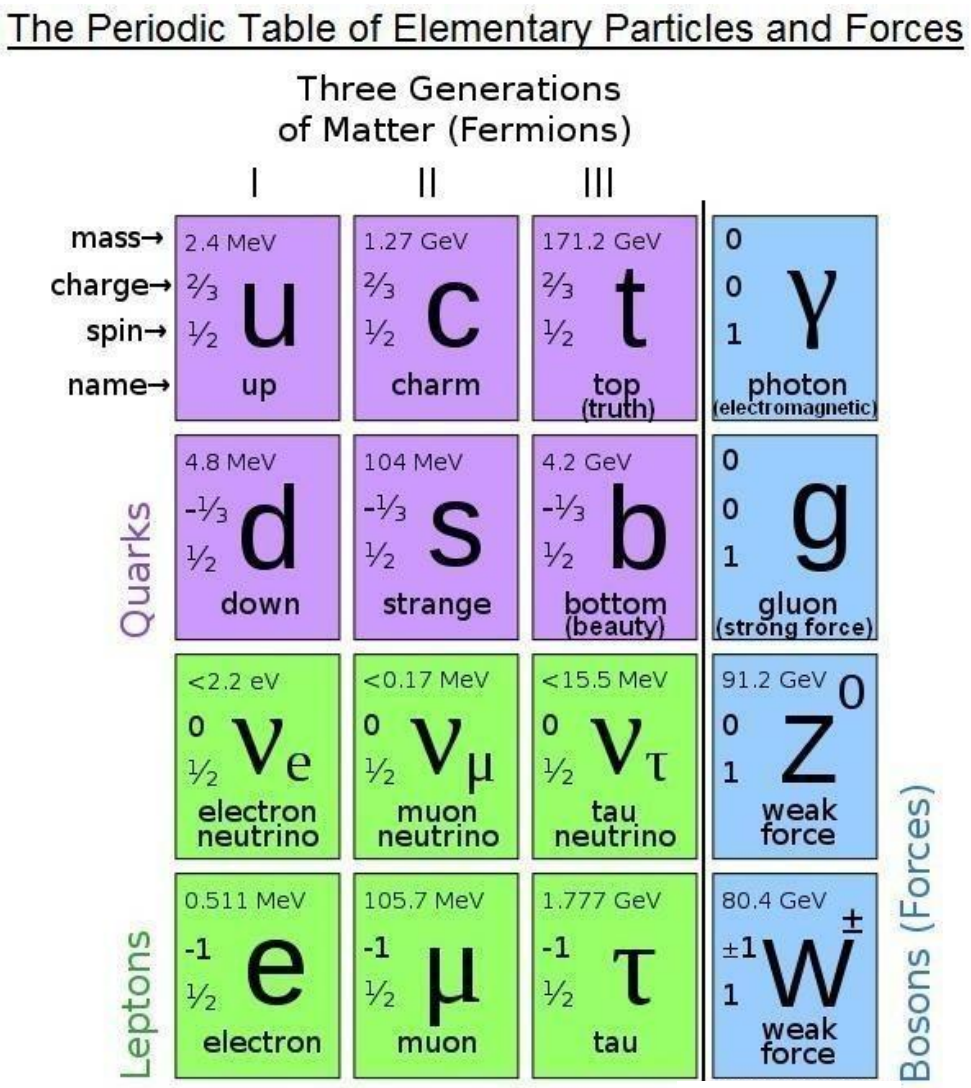
- With the advent of cosmic ray science and particle accelerato numerous additional particles were discovered:
  - muon (1937), pion (1947), kaon (1947), rho-meson – pion excitation (1961),

    Roper resonance – nucleon excitation (1963), …
- By the mid-1960s, it was apparent that not all the particles could be *fundamental*.
  - A new paradigm was necessary.
- Constituent-quark theory (1964) of Gell-Mann and Zweig, independently, was a crucial step forward.
  - Gell-Mann, Nobel Prize 1969: "*for his contributions and discoveries concerning the classification of elementary particles and their interactions*".

Nobel Prize 1938 "for his demonstrations of the existence of new radioactive elements produced by neutron irradiation, and for his related discovery of nuclear reactions brought about by slow neutrons"

Standard Model of Particle Physics

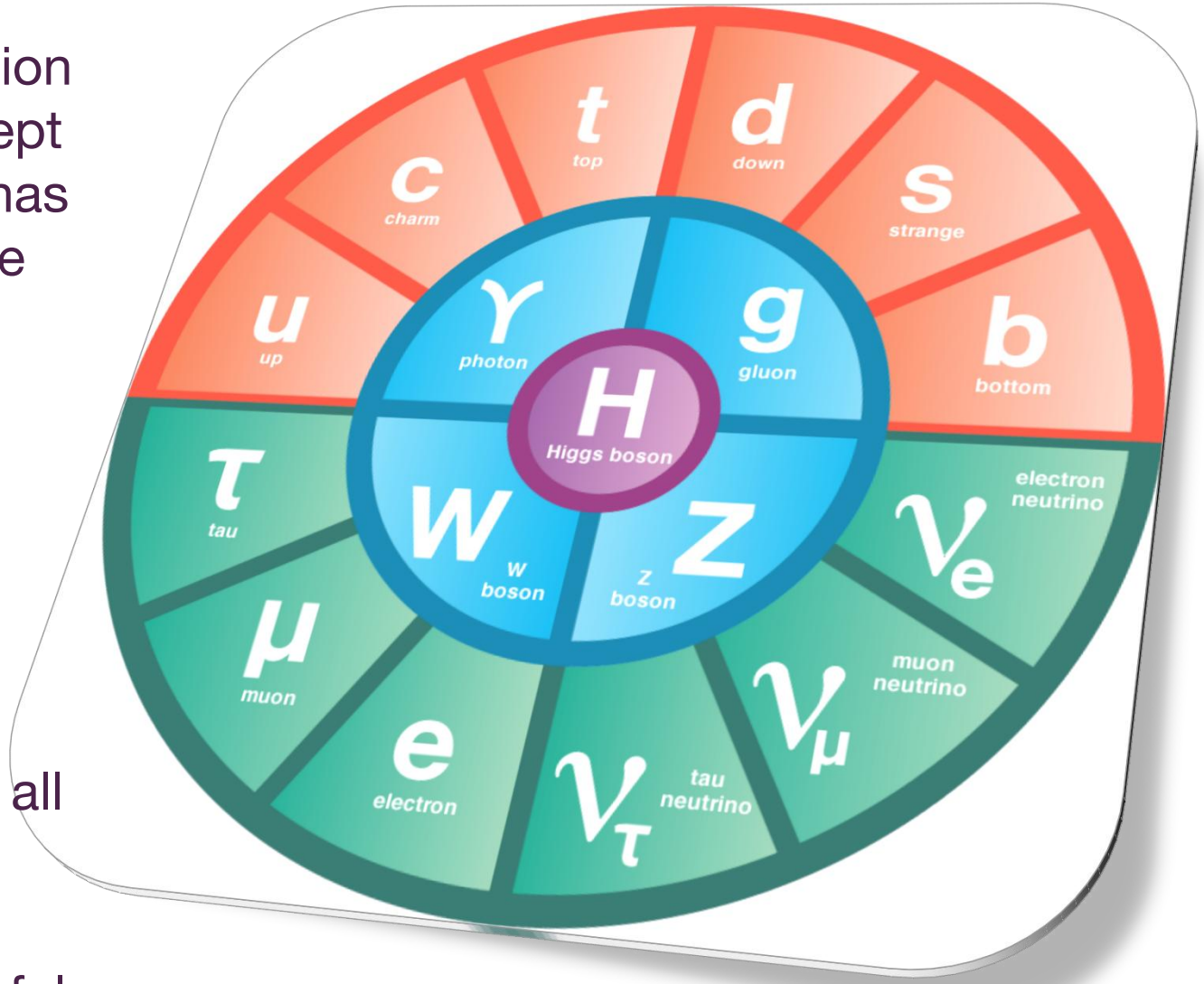
Supposed to explain and unify all the forces of Nature, *except* Gravity



13 years ago

Standard Model of Particle Physics

- Standard Model (SM) offers a description of all known fundamental physics except gravity, and gravity is something that has no discernible effect when particles are studied a few at a time.
- Since LHC's discovery of the Higgs in 2012, the Higgs Boson has been promoted to the *Centre of Things*
- Standard Model has *17 particles and 19 parameters*, most of which relate to the Higgs and all of which must be determined through comparison with experiment
- SM supposedly describes most powerful forces in Nature, yet, somewhat unsatisfactory



Standard Model of Particle Physics

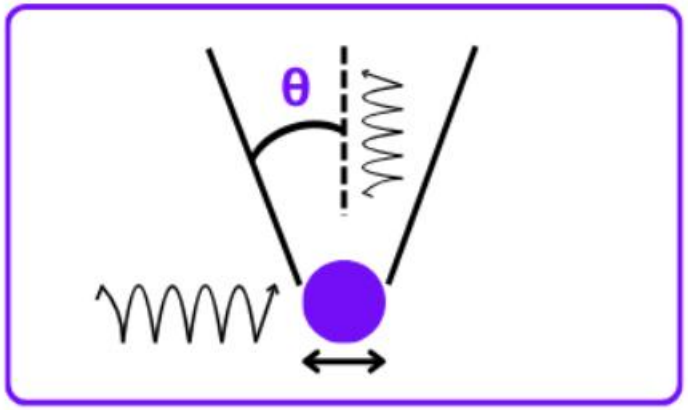
- Electroweak sector of the Standard Model (SM) is phenomenologically successful and understood at that level
  - Namely, although a Higgs boson like object has been found, there are deep theoretical problems with Higgs physics, *e.g.*:

    Hierarchy problem:
    - quantum corrections to the mass of a scalar boson are quadratically divergent ⇒ not renormalizable
    - large quantum contributions to $m^2_{\rm Higgs}$ would inevitably make this mass huge
    - avoiding such an outcome would require (incredible) fine-tuning cancellation between the quadratically divergent corrections and the Higgs bare mass.
- A means of circumventing these and related issues is being sought, *e.g.*, realising the Higgs boson as a composite particle

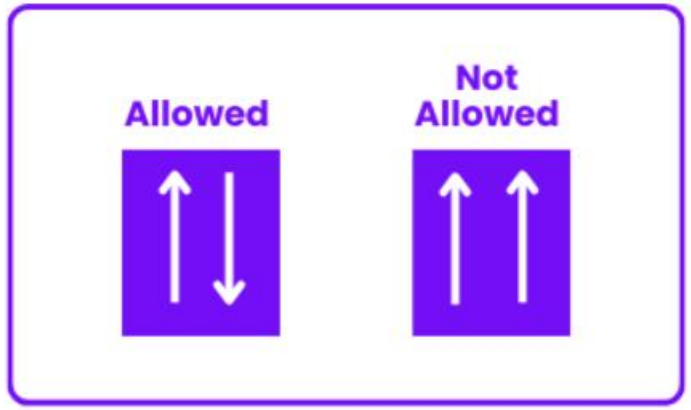
G. Cacciapaglia, C. Pica, F. Sannino, *Fundamental Composite Dynamics: A Review*, Phys. Rept. 877 (2020) 1–70.

LAWS OF QUANTUM MECHANICS

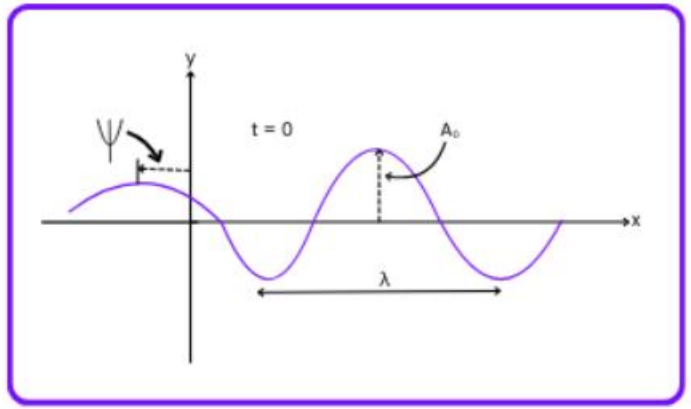
The Laws of Quantum Mechanics govern particle behavior at the atomic level, involving concepts like wave-particle duality and quantum superposition.



Heisenbergs Uncertainty Principle



Pauli Exclusion Principle



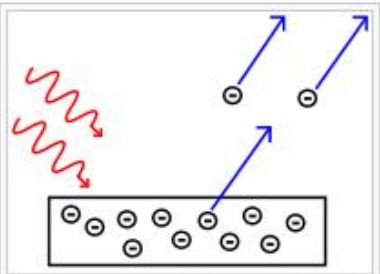
Schrödingers Equation

What is Quantum?

- Inventors of Quantum Mechanics
  - Albert Einstein
    - Nobel Prize for explaining the photoelectric effect
      - 1905 (prize 1921)
    - electromagnetic waves → photons
    - *"According to the assumption to be contemplated here, when a light ray is spreading from a point, the energy is not distributed continuously over ever-increasing spaces, but consists of a finite number of 'energy quanta' that are localized in points in space, move without dividing, and can be absorbed or generated only as a whole."*
    - 1926 … light quanta first called "photons" … Wolfers (optics) and Lewis (chemist)

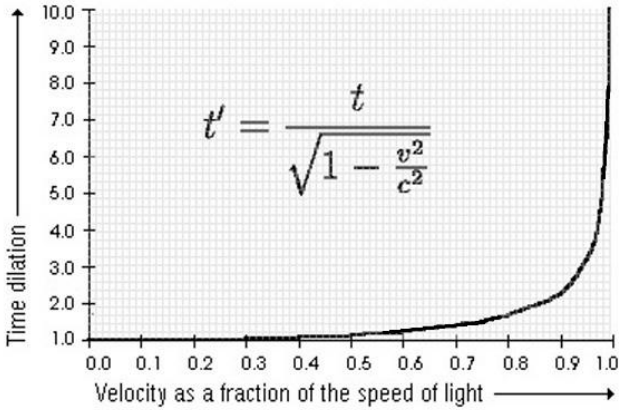
40 years later, then this huge, crucial step from Maxwell

Photoelectric effect



The emission of electrons from a metal plate caused by light quanta (photons) with energy greater than the work function of the metal.

Special Relativity
(Einstein again & again 1905)



- Einstein's original treatment (1905), it is based on two postulates:
  1. The laws of physics are invariant (*i.e.* identical) in all inertial systems (non-accelerating frames of reference).
  2. The speed of light in a vacuum is the same for all observers, regardless of the motion of the light source.
- Einstein "On the Electrodynamics of Moving Bodies".
  - Resolution of the
    - inconsistency of Newtonian mechanics with Maxwell's equations of electromagnetism
    - lack of experimental confirmation for a hypothesized luminiferous aether
  - Corrects mechanics to handle situations involving motions at a significant fraction of the speed of light
  - Special relativity is the most accurate (classical) model of motion at any speed when gravitational effects are negligible.

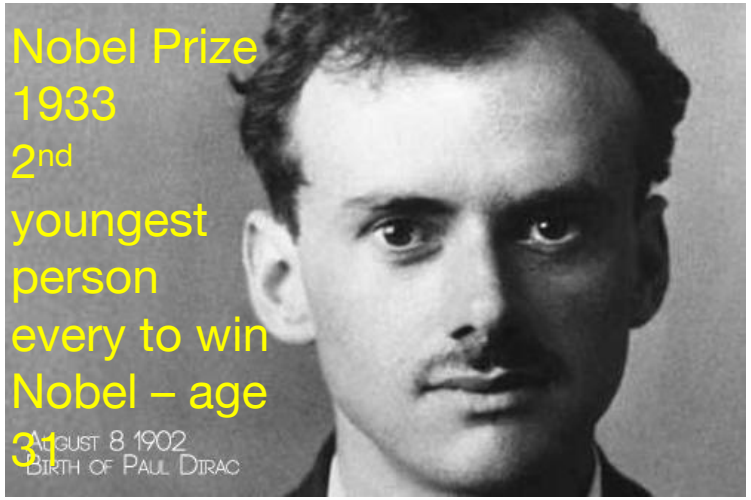
Unification of Quantum Mechanics & Special Relativity

For the discovery of new productive forms of atomic theory

- Dirac gave us the "Dirac Equation" (1928), the first and only relativistic generalisation of the Schrödinger equation for spin-1/2

The Dirac equation in the form originally proposed by Dirac is:

$$\left(\beta mc^2 + \sum_{k=1}^{3} \alpha_k p_k c\right) \psi(\mathbf{x},t) = i\hbar \frac{\partial \psi(\mathbf{x},t)}{\partial t}$$

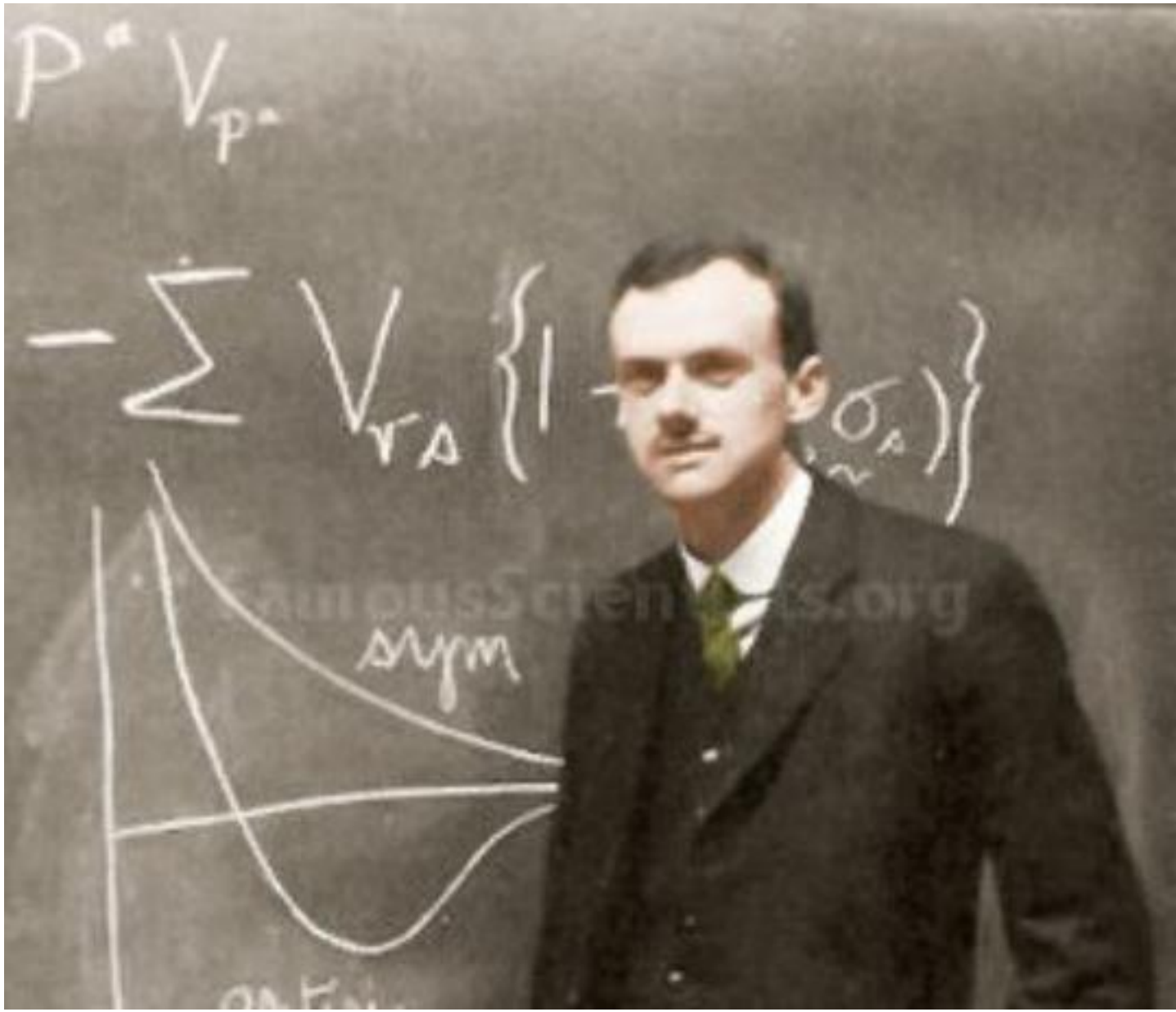


- Dirac and his equation also gave us <u>*antiparticles*</u>
  - *If we accept the view of complete symmetry between positive and negative electric charge so far as concerns the fundamental laws of Nature, we must regard it rather as an accident that the Earth (and presumably the whole solar system), contains a preponderance of negative electrons and positive protons.*
  - *It is quite possible that for some of the stars it is the other way about, these stars being built up mainly of positrons and negative protons.*
  - *In fact, there may be half the stars of each kind. The two kinds of stars would both show exactly the same spectra, and there would be no way of distinguishing them by present astronomical methods.*

Paul Dirac - man of very few words

A US journalist's interview with Dirac:

- Journalist: "Now Doctor, will you give me in a few words the low-down on all your investigations?"
- Dirac: "No."
- Journalist: "Good. Will it be alright if I put it this way – 'Professor Dirac solves all the problems of mathematical physics, but is unable to find a better way of figuring out baseball'?"
- Dirac: "Yes."
- Journalist: "Do you go to the movies?"
- Dirac: "Yes."
- Journalist: "When?"
- Dirac: "In 1920 – perhaps also in 1930."

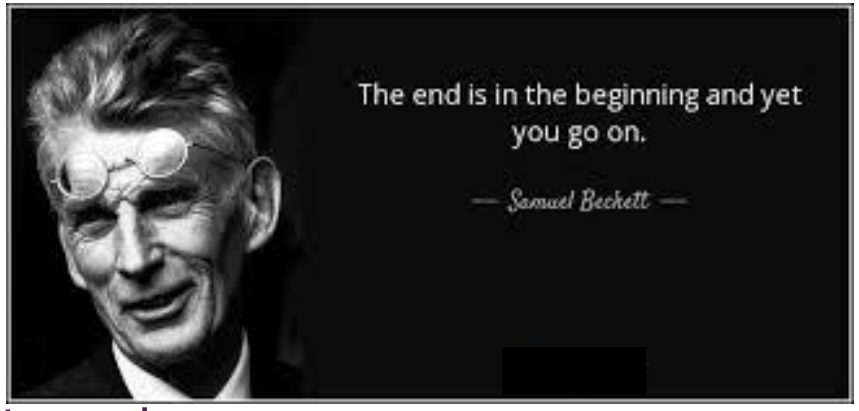


The Problem with Antimatter

- Masslessness – the absence of mass – is impossible in nonrelativistic theories. Nonrelativistically, all particles must possess a mass.
- Relativistically, masslessness is possible.
  - Einstein's Photon is a massless particle
- Antimatter = negative-energy matter

  … So, for any given system
  - It costs ZERO energy to create a virtual matter+antimatter pair
  - or 10 such pairs, or 100 pairs, …
  - or infinitely many!
- Quantum mechanics is an exactly solvable theory when the number of particles is finite

  (~ 14, with today's computers and realistic Hamiltonians)
- Quantum mechanics is *unsolvable* when there are infinitely many particles
- Relativistic quantum mechanics allows the existence of massless particles and antimatter (negative-energy matter)
  - but the existence of either/both destroys relativistic quantum mechanics!



Solving the Problem with Antimatter

- There is only one way known to combine quantum mechanics and special relativity

Poincaré-invariant Quantum Field Theory

- 1st example: *Electromagnetism*
  - Quantum electrodynamics (QED), 1946-1950
  - Feynman, Schwinger, Tomonaga
    - Nobel Prize (1965):

      "*for their fundamental work in quantum electrodynamics, with deep-ploughing consequences for the physics of elementary particles*".

Solving the Problem with Antimatter

- There is only one way known to combine quantum mechanics and special relativity

Poincaré-invariant Quantum Field Theory

- 2nd example: Weak interaction
  - Radioactive decays, parity-violating decays, electron-neutrino scattering

Glashow, Salam, Weinberg - 1963-1973

- Nobel Prize (1979):
- "*for their contributions to the theory of the unified weak and electromagnetic interaction between elementary particles, including,* inter alia*, the prediction of the weak neutral current*".

Solving the Problem with Antimatter

- 3rd example:

  Strong interaction … all of nuclear physics and anything made from nuclei = basically everything that we can see and/or touch

  - Existence and composition of the vast bulk of visible matter in the Universe:
    - proton, neutron
    - the forces that form them and bind them into nuclei
    - responsible for more than 98% of the visible matter in the Universe
  - Politzer, Gross and Wilczek – 1973-1974
    Quantum Chromodynamics – QCD
    - Nobel Prize (2004):

      "*for the discovery of asymptotic freedom in the theory of the strong interaction*".

Key to using quantum field theory is Poincaré invariant calculations of observable phenomena

- Fundamental scientific principle:
  an observer's choice of conventions cannot affect physical phenomena
- Effects that depend on such choices are artificial
  - not objective features of nature
  - *e.g.*, physics cannot depend on the choice of units
- Most critically
  - physical effects cannot depend on the observer's frame
  - valid phenomena should display Poincaré invariance for relativistic systems.

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Perspective | Published: 09 May 2022

Artificial dynamical effects in quantum field theory

Stanley J. Brodsky, Alexandre Deur & Craig D. Roberts

Nature Reviews Physics **4**, 489–495 (2022) | Cite this article

306 Accesses | **2** Citations | **4** Altmetric | Metrics

Abstract

In Newtonian mechanics, studying a system in a non-Galilean reference frame can lead to inertial pseudoforces appearing, such as the centrifugal force that seems to arise in dynamics analysed in a rotating frame. Likewise, artificial effects may arise in relativistic quantum field theory (QFT) if a system is studied in a framework that violates Poincaré invariance. In this Perspective, we highlight how such issues complicate the traditional canonical quantization of QFTs and can lead to a subjective description of natural phenomena. By contrast, the treatment of the same problem using light-front quantization is free from spurious pseudoeffects because Poincaré invariance is effectively preserved for all practical intents and purposes. We illustrate these statements using several examples: the Gerasimov–Drell–Hearn (GDH) relation, a fundamental feature of QFT; the absence of any measurable impact of Lorentz contraction in high-energy collisions; and the fictitious character of vacuum fluctuation contributions to the cosmological constant.

“Best” Introductions to Quantum Field Theory and QCD

I. Mattuck, R. D., A guide to Feynman diagrams in the many-body problem (Dover Publications, inc., New York, 1976/1992)

II. Itzykson, Claude and Zuber, Jean-Bernard, *Quantum Field Theory* (McGraw-Hill Inc., New York, 1980)

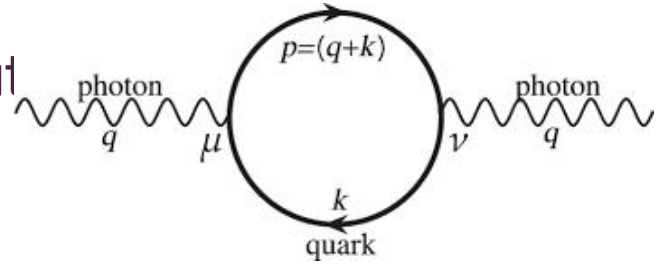
III. Pascual, P. and Tarrach, R., *QCD: Renormalization for the Practitioner*, Lecture Notes in Physics **194** (Springer-Verlag, Berlin, 1984)

IV. Roberts, C. D. and Williams, A. G., *Dyson-Schwinger equations and their application to hadronic physics,* Prog. Part. Nucl. Phys. **33** (1994) pp. 477-575. hep-ph/9403224

V. Roberts, C. D., *Strong QCD and Dyson-Schwinger Equations,* IRMA Lect. Math. and Theor. Phys. **21** (2015) pp. 356-458. arXiv:1203.5341

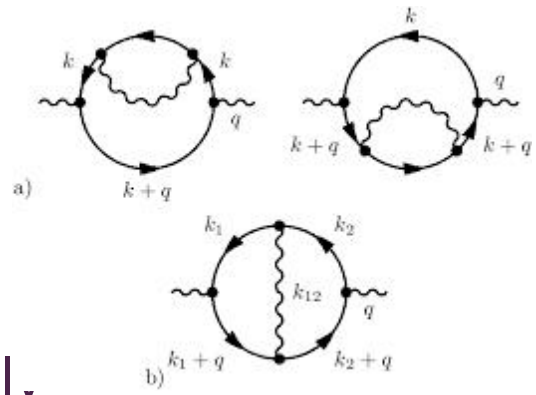
Perturbative Quantum Field Theory

- Possibly, you've studied quantum field theory (QFT) from perturbative perspective
  - Feynman diagrams
- One loop calculations in QFT are straightforward; but
  - almost always, they're divergent
  - must be regularised
  - then renormalisation scheme implemented



- Two loop calculations are complicated

  and there are many two-loop diagrams



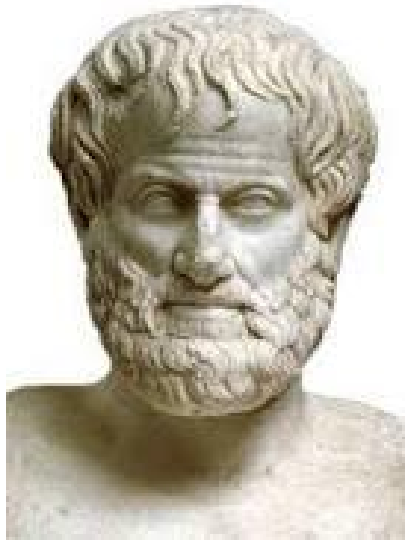
- As the number of loops increases,
  the number of Feynman diagrams
  and the complexity of the calculations grow factoriall,
- Evaluating loop integrals can be extremely challenging,

  requiring sophisticated mathematical techniques and numerical methods.
- Need to handle multiple loop integrals & their associated tensor structures adds to difficulty

Perturbative Quantum Field Theory

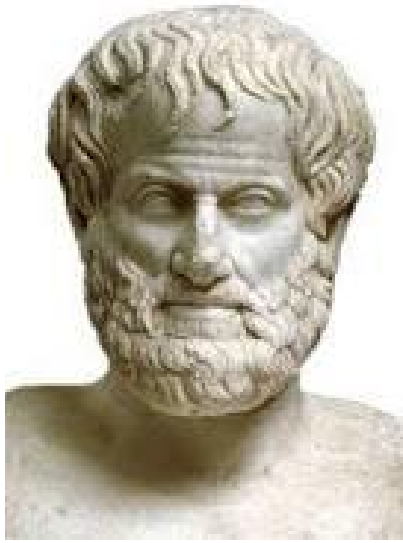
- In quantum field theory, no series of perturbatively evaluated terms is convergent
  - at any given momentum scale, k^2, there is an n such that adding more loop diagrams, *i.e.*, going to higher orders in the coupling, $\alpha^n(k^2) \to \alpha^{n+1}(k^2)$, will deliver a result that is further from the correct answer
  - the perturbative expansion delivers an "asymptotic series":
    - a useful approximation for a few terms, but at some point – some power of $\alpha(k^2)$ – it becomes less and less accurate as more and more terms are added
  - The S-matrix formal power series in all perturbative quantum field theories of interest is necessarily divergent (and hence at best an asymptotic series):
    - Freeman Dyson, *Divergence of perturbation theory in quantum electrodynamics*, Phys. Rev. 85 (1952) 631
- Furthermore, no finite sum of diagrams – no finite order in $\alpha(k^2)$ – can ever produce anything ***emergent***
- Every result is continuously deformable to free-field behaviour

Emergentism



- One might define emergent phenomena as those features of Nature which don't readily admit explanation solely in terms of known or conjectured rules.
- The concept is at least as old as Aristotle (384-322 BC), who argued that a compound item can have (emergent) properties in the whole which are not explicable merely through the independent actions of the item's constituent parts.
- His view is often represented by the statement:
  "The whole is more than the sum of its parts".
- In this sense, *emergence* has its origins in the Greek "sunergos": "together" plus "working" = origin of our concept of synergy (协同效应), *viz*. things working together more effectively than could be anticipated from their independent actions in isolation.
- This perspective is typically contrasted with that described as reductionism; namely, the view that everything in Nature can ultimately be viewed as no more complex in principle than,
  e.g. a (very good) watch, which is clearly a complex object; but

Emergentism



How does simplicity (e.g. $L_{\mathrm{StandardModel}}$) beget elegant complexity in the emergence of the most fundamental structures in Nature?

e

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  e.g. a (very good) watch, which is clearly a complex object; but

Strong Interactions in the Standard Model

Supposed to be described by quantum chromodynamics (QCD)

quark current mass generated by Higgs boson couplings into QCD

$$L = \frac{1}{4} G^a_{\mu\nu}(x) G^a_{\mu\nu}(x) + \bar{\psi}\left[\gamma \cdot \partial_x + m + ig\,\frac{\lambda^a}{2}\gamma \cdot A^a(x)\right]\psi(x)$$

No mass term for gluons

$$G^a_{\mu\nu}(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) - f^{abc} A^b_\mu(x) A^c_\nu(x)$$

H. Fritzsch, M. Gell-Mann, Proceedings of the XVI International Conference on High Energy Physics, Chicago, 1972, Vol. I, 135.

- One-line Lagrangian – expressed in terms of massless gluon (A^a_μ) and Higgs-mass quark (ψ) partons … $a = 1, \ldots, 8$ is the "colour" index
- 1973
  - ✓ In his first published article, Hugh David Politzer described the phenomenon of asymptotic freedom in QCD: the closer quarks are to each other, the weaker the strong interaction between them will be
  - ✓ Same issue of Phys. Rev. Lett., David Gross and Frank Wilczek published the same result

Strong Interactions in the Standard Model

THE DILEMMA OF ATTRIBUTION

Nobel Lecture, December 8, 2004

by

H. David Politzer

California Institute of Technology (Caltech), Pasadena, USA.

- The establishment by the mid-1970's of QCD as the correct theory of the strong interactions completed what is now known prosaically as the Standard Model.
- It offers a description of all known fundamental physics except for gravity, and gravity is something that has no discernible effect when particles are studied a few at a time.
- However, the situation is a bit like the way that the Navier-Stokes equation accounts for the flow of water. The equations are at some level obviously correct, but there are only a few, limited circumstances in which their consequences can be worked out in any detail.
- Nevertheless, many leading physicists were inclined to conclude in the late 1970's that the task of basic physics was nearly complete, and we'd soon be out of jobs.
- A famous example was the inaugural lecture of Stephen Hawking as Lucasian Professor of Mathematics, a chair first held by Isaac Barrow at Cambridge University. Hawking titled his lecture, "Is the End in Sight for Theoretical Physics?" And he argued strongly for "Yes".

Quantum Chromodynamics

$$L = \frac{1}{4} G^a_{\mu\nu}(x) G^a_{\mu\nu}(x) + \bar{\psi}\left[\gamma \cdot \partial_x + m + ig\,\frac{\lambda^a}{2}\gamma \cdot A^a(x)\right]\psi(x)$$

$$G^a_{\mu\nu}(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) - f^{abc} A^b_\mu(x) A^c_\nu(x)$$

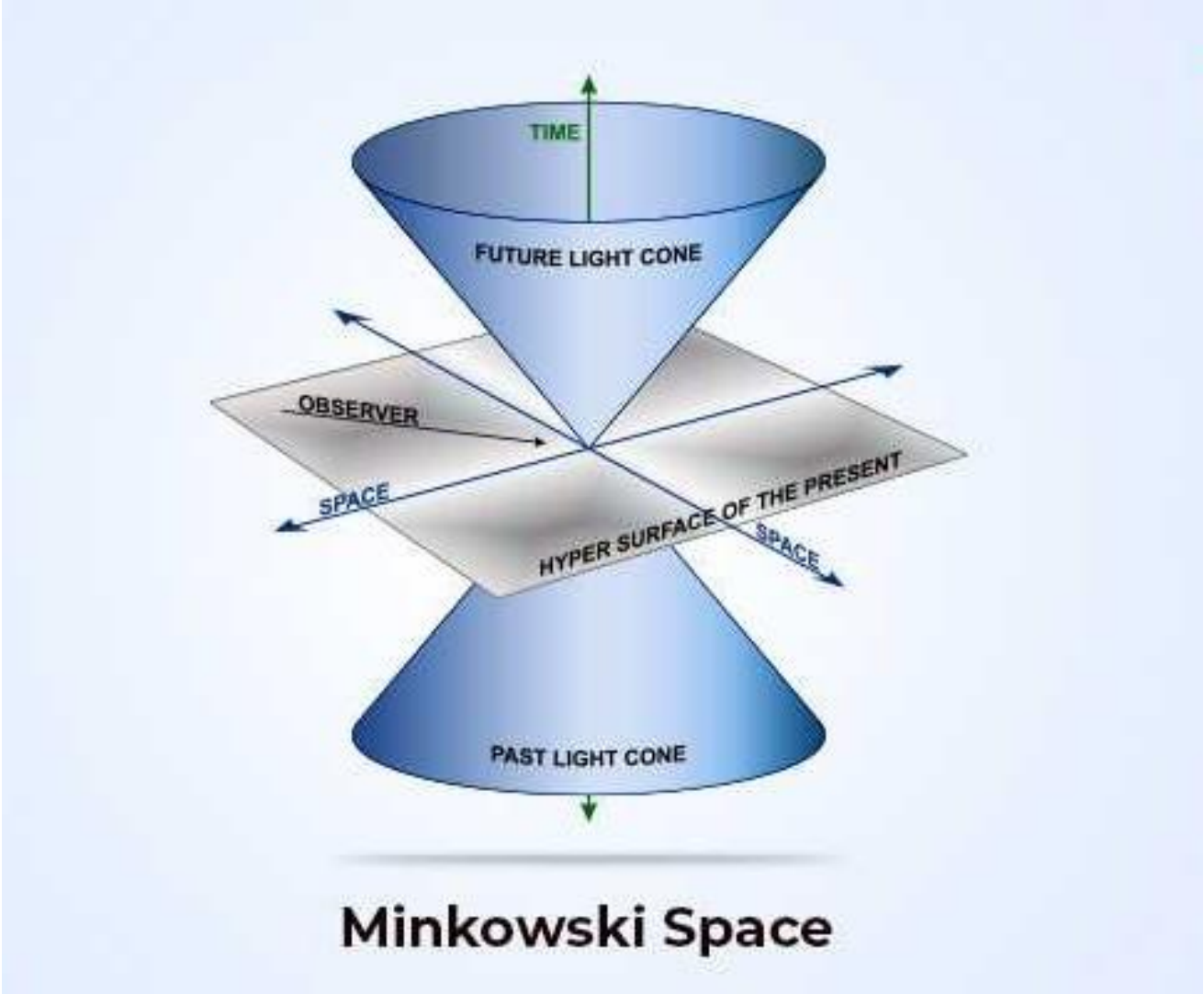
H. Fritzsch, M. Gell-Mann, Proceedings of the XVI International Conference on High Energy Physics, Chicago, 1972, Vol. I, 135

- One-line Lagrangian – expressed in terms of massless gluon (A^a_μ) and Higgs-mass quark (ψ) partons
- Which are NOT the degrees-of-freedom measured in detectors

Gluon and Quark Confinement
Essentially non-perturbative

Advantages of the Color Octet Gluon Picture, H. Fritzsch, M. Gell-Mann, H. Leutwyler, Phys. Lett. B 47 (1973) 365-368

- "The quarks come in three "colors," but all physical states and interactions are supposed to be singlets with respect to the SU(3) of color. Thus, we do not accept theories in which quarks are real, observable particles; nor do we allow any scheme in which the color non-singlet degrees of freedom can be excited."
- Emergence now becomes important: Lagrangian degrees-of-freedom are unobservable



Space and Time = Spacetime

Nature's Natural Space

- Nature is expressed in Minkowski space, where the inner product of two four-vectors is $x_\mu y^\mu = x^0 y^0 - \vec{x}\cdot\vec{y} \;\Rightarrow\; x^2$ is ***not*** positive definite for real vectors
- However, quantum field theory can only be defined in Euclidean space: $x_\mu y_\mu = \vec{x}\cdot\vec{y} + x_4 y_4 \;\Rightarrow\; x^2$ is positive definite for real vectors
- There are many reasons for adopting this perspective:
  - Lattice regularisation of theory is only possible in Euclidean space where one can use the Euclidean action to define a probability measure

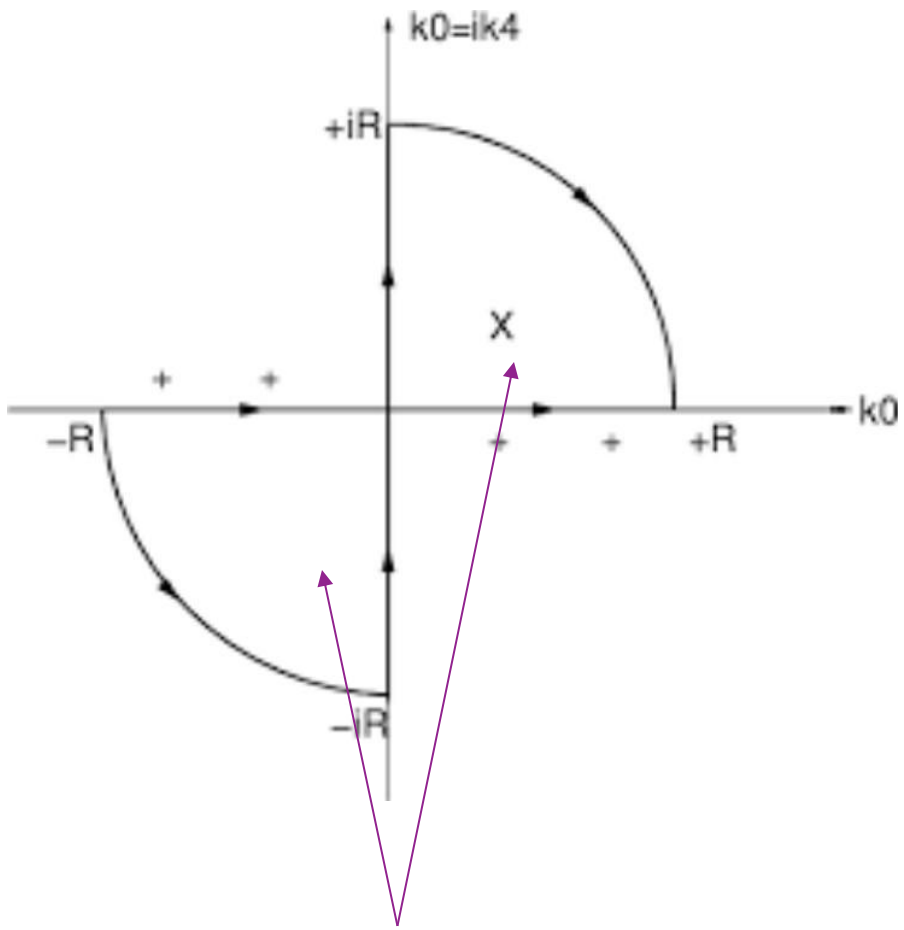
$$\int dA dq d\bar{q}\; e^{-S_{QCD}^{\text{Euclidean}}[A,q,\bar{q}]} \quad cf. \quad \int dA dq d\bar{q}\; e^{iS_{QCD}^{\text{Minkowski}}[A,q,\bar{q}]}$$

- In Euclidean space, paths with large action are exponentially suppressed
  - Summation over paths is convergent and can be achieved using computers
- Not true in Minkowski space. Instead, large actions produce wild oscillations
  - Impossible to achieve summation convergence with computers
- Moments of the probability measure are called Schwinger Functions

(Minkowski space equivalents are called Green functions.)

Nature's Natural Space

- Choice must be made because any "Wick rotation" between Minkowski space and Euclidean space is a purely formal exercise
  - validity is only guaranteed for perturbative calculations
- If QCD really does (somehow) explain emergence of hadron mass and structure, then …
  - nonlinear, nonperturbative dynamics (emergent phenomena) must be crucial
- Consequently …
  - when calculating & summing the necessarily infinite collection of processes associated with a given experimental observable
  - one *cannot* assume that any of the requirements necessary to mathematically justify a Wick rotation are satisfied



- ✓ Nonanalyticities in these quadrants will break simple form correspondence between Minkowski and Euclidean space equations
- ✓ May completely destroy all correspondences

Nature's Natural Space: Euclidean ↔ Minkowski

- A given quantum field theory, defined by probability measure is solved once all its Schwinger functions are known
- Osterwalder–Schrader axioms
  - five necessary and sufficient conditions for reconstruction of the Minkowski-space theory from the Euclidean Schwinger functions
- Guarantee equivalence between Euclidean and Minkowski formulations
- Special importance – axiom of reflection positivity $\Delta(k^2) = \int d\sigma \frac{\rho(\sigma)}{k^2 + \sigma}$
  - Schwinger function has a spectral representation
  - Spectral function is non-negative: $\forall \sigma,\ \rho(\sigma) \geq 0$
- Only Schwinger functions which satisfy reflection positivity can be connected with states that appear in Hilbert space of observables.
- Regarding QCD, all physical states are colour singlets.
  - Consequently, for QCD to be the theory of strong interactions … all its colour singlet Schwinger functions must satisfy all Osterwalder–Schrader axioms
  - Equally, all QCD's colour-nonsinglet functions must violate at least one =

Schwinger Functions Methods

- All material discussed above and in the following was obtained by supposing that QCD is defined by the Euclidean-space generating functional built using:

$$\mathcal{L}_{\rm QCD} = \sum_{f=u,d,s,\ldots} \bar{q}_f [\gamma\cdot\partial + ig\tfrac{1}{2}\lambda^a\gamma\cdot A^a + m_f] q_f + \tfrac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}$$

- Now, a new choice presents itself:
  - Lattice-regularised Schwinger functions for QCD (lQCD) – popular framework
    - growth in computer power and algorithm improvements ⇒ becoming more effective
    - artificial intelligence
  - Continuum Schwinger function methods (CSMs)
    - much has been achieved using this approach, especially during the past decade
    - particularly in connection with elucidating the origins and wide-ranging expressions of emergent hadron mass (EHM)
- *In Physics – natural philosophy – computing the answer is just a small, 1st step.*
  - *True goal = understanding the answer*
  - *One of greatest advantages of CSMs = delivering insights and understanding*

Organic Intelligence = Thinking

Origin of Mass

The Higgs boson

Elementary particles gain their mass from a fundamental field associated with the Higgs boson

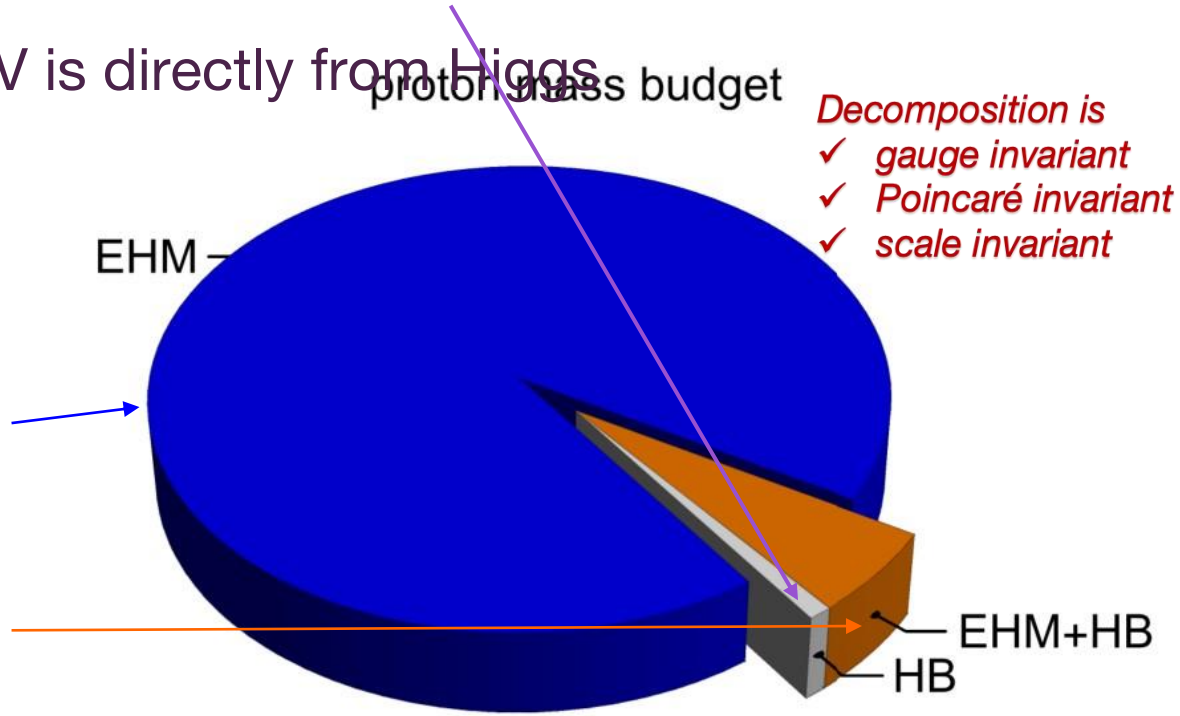
Or so the story has gone

Emergence of Hadron Mass

- Standard Model of Particle Physics has one obvious mass-generating mechanism

  = Higgs Boson … impacts are critical to evolution of Universe as we know it
- However, Higgs boson alone is responsible for just ~ 1% of the visible mass in the Universe
- Proton mass budget … only 9 MeV/939 MeV is directly from Higgs
- Evidently, Nature has another very effective mechanism for producing mass:

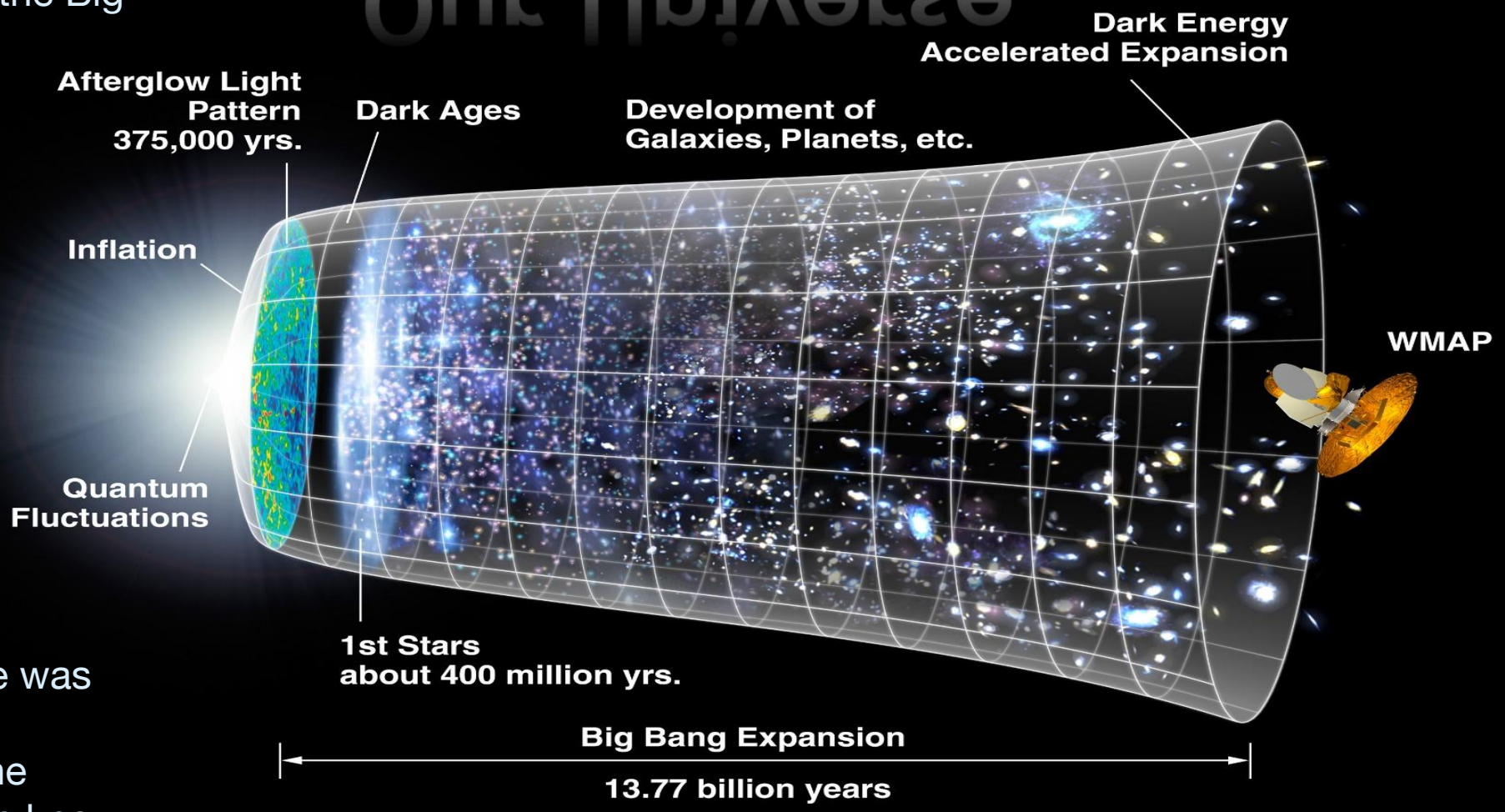
  Emergent Hadron Mass (EHM)
  - ✓ Alone, it produces 94% of the proton's mass
  - ✓ Remaining 5% is generated by constructive interference between EHM and Higgs-boson
- *What is the origin of EHM?*



Decomposition is
- ✓ *gauge invariant*
- ✓ *Poincaré invariant*
- ✓ *scale invariant*

Nuclear-scale mass emerged 0.000001 seconds after the Big Bang

Our Universe



Its appearance was crucial to the formation of the Universe as we know it

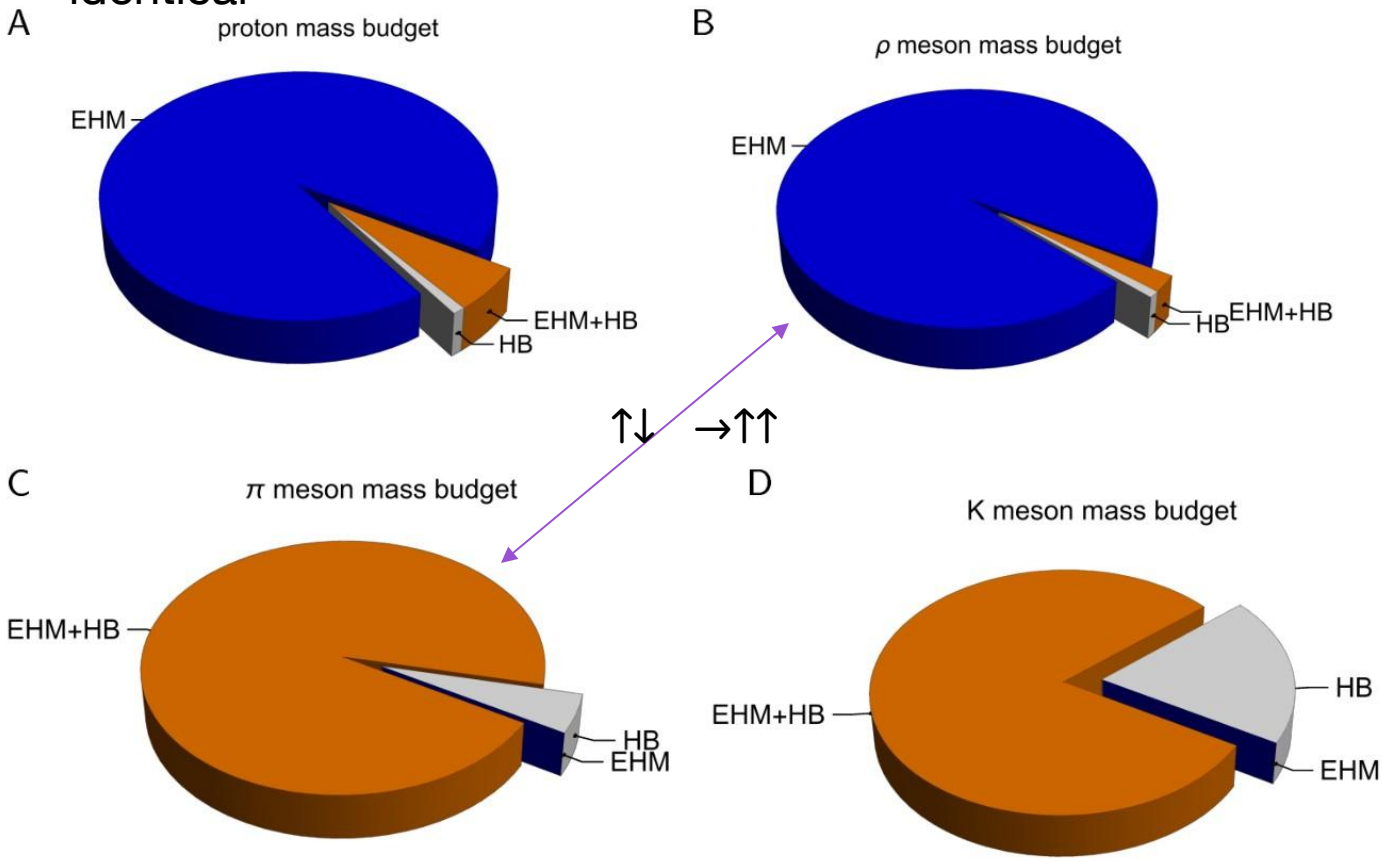
NASA/WMAP Science Team

Emergence of Hadron Mass - Basic Questions

Decompositions are
- ✓ *gauge invariant*
- ✓ *Poincaré invariant*
- ✓ *scale invariant*

- What is the origin of EHM?
- Does it lie within QCD?
- What are EHM's connections with …
  - Gluon and quark confinement?
  - Dynamical chiral symmetry breaking (DCSB) – quarks massive, yet pion massless?
  - Nambu-Goldstone modes = π & K?
- What is the role of Higgs in modulating EHM expressions in observable properties of hadrons?
  - Without Higgs mechanism of mass generation, π and K would be indistinguishable

Proton and ρ-meson mass budgets are practically identical



π- and K-meson mass budgets are completely different from those of proton and ρ

- What and wherefrom is mass?

Quantum Chromodynamics

$$L = \frac{1}{4} G^a_{\mu\nu}(x) G^a_{\mu\nu}(x) + \bar{\psi}\left[\gamma \cdot \partial_x + m + ig\,\frac{\lambda^a}{2}\gamma \cdot A^a(x)\right]\psi(x)$$

$$G^a_{\mu\nu}(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) - f^{abc} A^b_\mu(x) A^c_\nu(x)$$

- One-line Lagrangian – expressed in terms of massless gluon (A^a_μ) and Higgs-mass quark (ψ) partons

Gluon and Quark Confinement

- Which are NOT the degrees-of-freedom measured in detectors

Questions

- What are the (asymptotic) detectable degrees-of-freedom?
- How are they built from the Lagrangian degrees-of-freedom?

Hadrons

= protons, pions, etc.

- Is QCD really the theory of strong interactions?
- Is QCD really a theory … or just another EFT?

⇒ Implications far beyond Standard Model

Quantum Chromodynamics

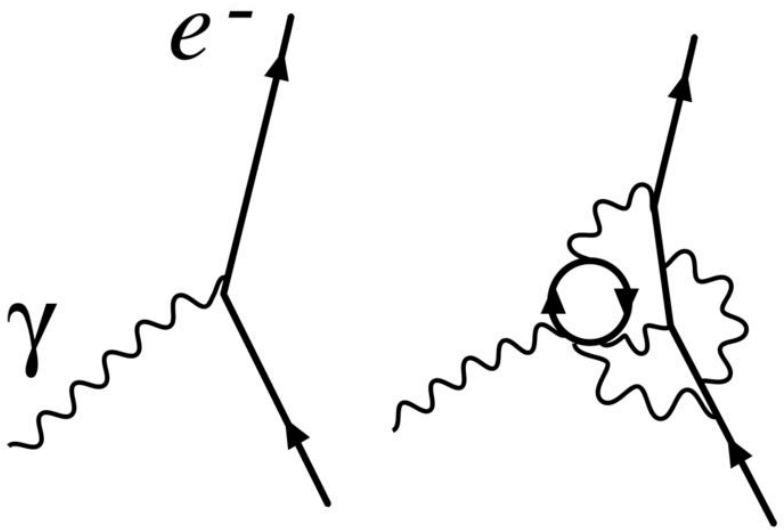
$$L = \frac{1}{4} G^a_{\mu\nu}(x) G^a_{\mu\nu}(x) + \bar{\psi}\left[\gamma \cdot \partial_x + m + ig\,\frac{\lambda^a}{2}\gamma \cdot A^a(x)\right]\psi(x)$$

$$G^a_{\mu\nu}(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) - \boxed{f^{abc} A^b_\mu(x) A^c_\nu(x)}$$

- This term is the only thing that makes QCD different from quantum electrodynamics (QED)
- It means that gluons interact with each other at leading order in perturbation theory

3-gluon vertex

4-gluon vertex



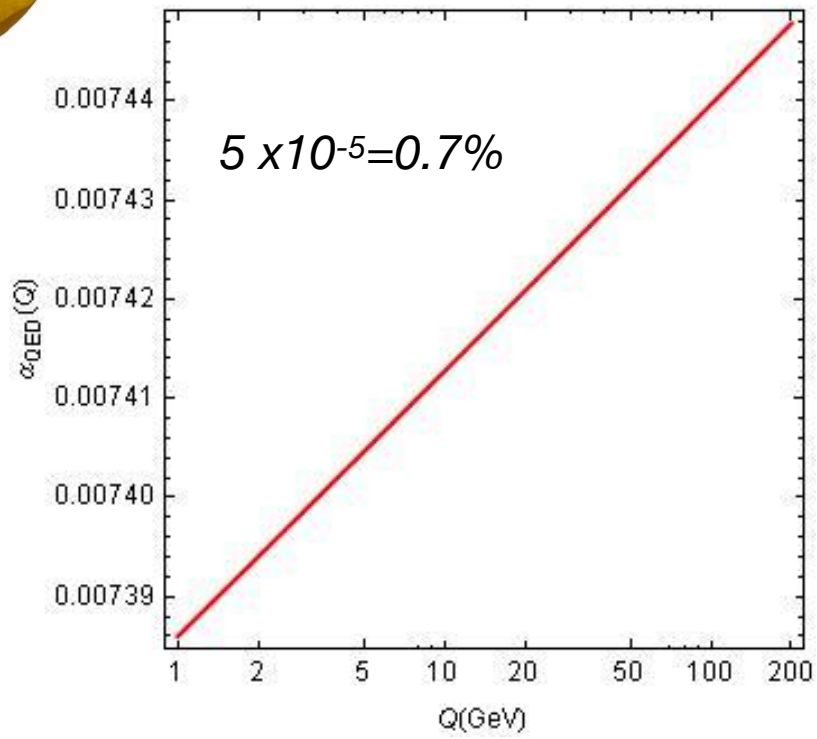
Running couplings and masses

- Quantum gauge-field theories are all typified by the feature that *Nothing is Constant*
- Distribution of charge and mass, number of particles, etc., indeed, all the things that quantum mechanics holds fixed, depend upon wavelength of tool used to measure them
  - particle number is not conserved in quantum field theory … energy can create particles
- Couplings and masses are renormalised via processes involving virtual-particles. Such effects make these quantities depend on the energy scale at which one observes them

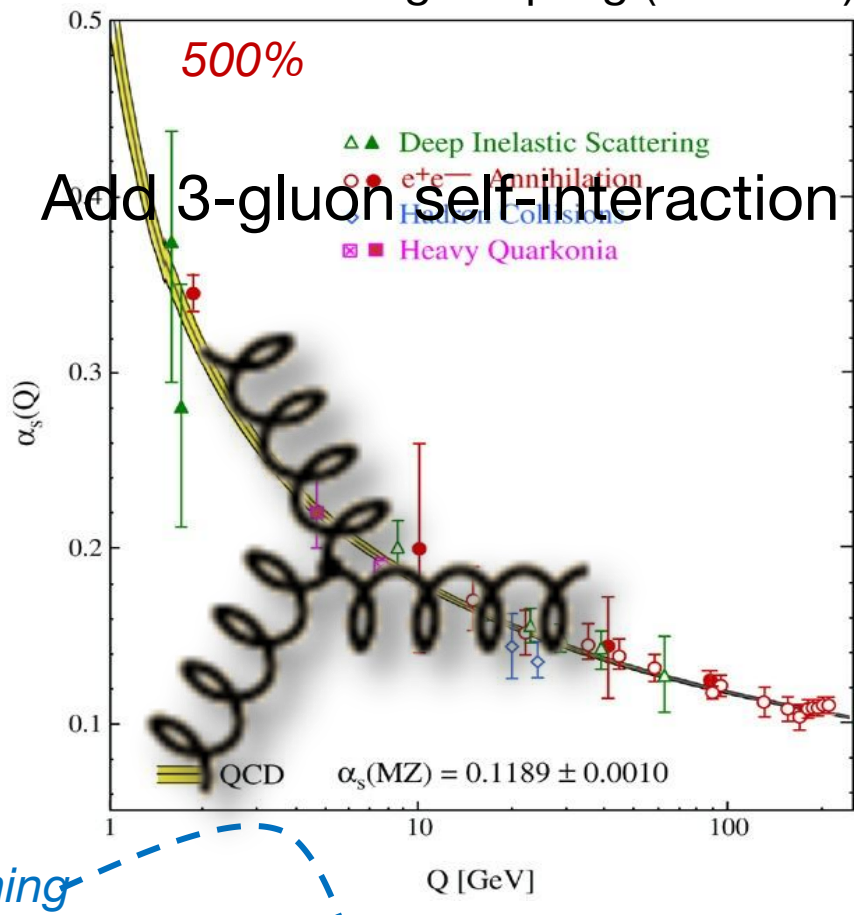
QED cf. QCD?

✓ 2004 Nobel Prize in Physics : Politzer, Gross and Wilczek

QED running coupling (textbook)



QCD running coupling (textbook)



$$a_{QED}(Q) = \frac{a}{1 - \frac{2a}{3\pi}\ln\frac{Q}{m_e}}$$

gluon antiscreening

fermion screening

$$a_{QCD}(Q) = \frac{6\pi}{(33 - 2N_f)\ln\frac{Q}{\Lambda}}$$

Millenium Madness

The Millennium Prize Problems

In order to celebrate mathematics in the new millennium, The Clay Mathematics Institute of Cambridge, Massachusetts (CMI) established seven *Prize Problems*. The Prizes were conceived to record some of the most difficult problems with which mathematicians were grappling at the turn of the second millennium; to elevate in the consciousness of the general public the fact that in mathematics, the frontier is still open and abounds in important unsolved problems; to emphasize the importance of working towards a solution of the deepest, most difficult problems; and to recognize achievement in mathematics of historical magnitude.

The prizes were announced at a meeting in Paris, held on May 24, 2000 at the Collège de France. Three lectures were presented: Timothy Gowers spoke on ***The Importance of Mathematics***; Michael Atiyah and John Tate spoke on the problems themselves.

- In 1900, the German mathematician David Hilbert opened his landmark address to the International Congress of Mathematicians in Paris, laying out 23 of the great unsolved problems of the day. “For the close of a great epoch,” Hilbert declared, “not only invites us to look back into the past but also directs our thoughts to the unknown future.”
- In May 2000, the Clay Mathematics Institute of Cambridge, Mass., emulated Hilbert, announcing (in Paris, for full effect) seven “Millennium Prize Problems,” each with a bounty of $1 million.
- In July 2000, a conference on superstring theory at the University of Michigan ended with a session called “Millennium Madness,” choosing 10 of the most perplexing problems in their field

Excerpt from the top-10

The New York Times

WORLD | U.S. | N.Y. / REGION | BUSINESS | TECHNOLOGY | SCIENCE | HEALTH | SPORTS | OPINION

10 Physics Questions to Ponder for a Millennium or Two

By George Johnson
Published: August 15, 2000

- ***Can we quantitatively understand quark and gluon confinement in quantum chromodynamics and the existence of a mass gap?***

  Quantum chromodynamics is the theory describing the strong nuclear force. Carried by gluons, it binds quarks into particles like protons and neutrons. Apparently, the tiny subparticles are permanently confined: one can't pull a quark or a gluon from a proton because the strong force gets stronger with distance and snaps them right back inside.

CLAY MATHEMATICS INSTITUTE

Millennium prize of $1,000,000 for proving that $SU_c(3)$ gauge theory is mathematically well-defined, which will necessarily prove or disprove a confinement conjecture

YANG–MILLS EXISTENCE AND MASS GAP. *Prove that for any compact simple gauge group G, a non-trivial quantum Yang–Mills theory exists on $\mathbb{R}^4$ and has a mass gap $\Delta > 0$. Existence includes establishing axiomatic properties at least as strong as those cited in* [**45, 35**].

5. Comments

An important consequence of the existence of a mass gap is clustering: Let $\vec{x} \in \mathbb{R}^3$ denote a point in space. We let H and $\vec{P}$ denote the energy and momentum, generators of time and space translation. For any positive constant $C < \Delta$ and for any local quantum field operator $\mathcal{O}(\vec{x}) = e^{-i\vec{P}\cdot\vec{x}}\mathcal{O}e^{i\vec{P}\cdot\vec{x}}$ such that $\langle\Omega, \mathcal{O}\Omega\rangle = 0$, one has

$$|\langle\Omega, \mathcal{O}(\vec{x})\mathcal{O}(\vec{y})\Omega\rangle| \leq \exp(-C|\vec{x} - \vec{y}|), \tag{2}$$

as long as $|\vec{x} - \vec{y}|$ is sufficiently large. Clustering is a locality property that, roughly speaking, may make it possible to apply mathematical results established on $\mathbb{R}^4$ to any 4-manifold, as argued at a heuristic level (for a supersymmetric extension of four-dimensional gauge theory) in [**49**]. Thus the mass gap not only has a physical significance (as explained in the introduction), but it may also be important in mathematical applications of four-dimensional quantum gauge theories to geometry. In addition the existence of a uniform gap for finite-volume approximations may play a fundamental role in the proof of existence of the infinite-volume limit.

There are many natural extensions of the Millennium problem. Among other things, one would like to prove the existence of an isolated one-particle state (an upper gap, in addition to the mass gap), to prove confinement, to

Confinement?

CLAY MATHEMATICS INSTITUTE

YANG–MILLS EXISTENCE AND MASS GAP. *Prove that for any compact simple gauge group G, a non-trivial quantum Yang–Mills theory exists on $\mathbb{R}^4$ and has a mass gap $\Delta > 0$. Existence includes establishing axiomatic properties at least as strong as those cited in* [**45, 35**].

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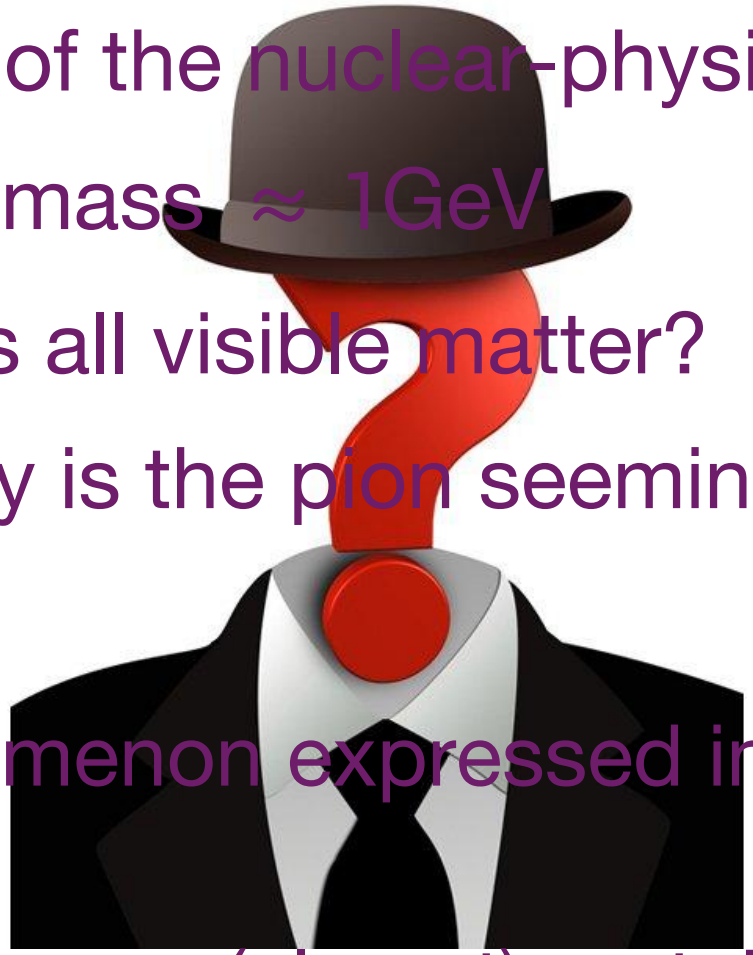
Confinement?

Basic Questions in Nature …

Basic Questions in Nature …
that insightful physics might answer in foreseeable future



- What is the origin of the nuclear-physics mass scale

  m_p = proton mass $\approx$ 1GeV

  that characterises all visible matter?
- Whatever it is, why is the pion seemingly oblivious: $m_\pi \approx \frac{1}{7} m_p$?
- How is this phenomenon expressed in measurable quantities?
  - ✓ The expressions are (almost) certainly system specific!
  - ✓ Answers should be frame and scale invariant

$$\langle p(P)|T_{\mu\nu}|p(P)\rangle = -P_\mu P_\nu$$

$$\langle p_0(P)|T_{\mu\mu}|p_0(P)\rangle = -P^2 = m_{p_0}^2 = \langle p_0(P)|\Theta_0|p_0(P)\rangle$$

Prima Facie, the QCD Lagrangian
CANNOT support a nonzero value of this expectation value.

QCD's scale anomaly

Strong Interactions in the Standard Model

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i \left(i(\gamma^\mu D_\mu)_{ij} - m\,\delta_{ij} \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

- Only apparent scale in chromodynamics is mass of the quark field
- Quark mass is generated by Higgs boson couplings into QCD
- In connection with everyday matter, that mass is less-then 0.5% of the empirical scale for strong interactions,
  viz. more-than two orders-of-magnitude smaller
- Plainly, the Higgs-generated mass is very far removed from the natural scale for strongly-interacting matter
- *Nuclear physics mass-scale* – 1 GeV – is an *emergent feature of the Standard Model*
  - No amount of staring at $L_{\rm QCD}$ can reveal that scale
- Contrast with quantum electrodynamics, *e.g*. spectrum of hydrogen levels measured in units of m_e, which appears in $L_{\rm QED}$

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i \left(i(\gamma^\mu D_\mu)_{ij} - \qquad \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

Whence Mass?

- Classical chromodynamics … non-Abelian local gauge theory
- Remove the current mass … there's no energy scale left
- *No dynamics in a scale-invariant theory*; only kinematics … the theory looks the same at all length-scales … there can be no clumps of anything … *hence bound-states are impossible*.
- *Our Universe can't exist*
- *Higgs boson doesn't solve this problem* …
  - normal matter is constituted from light-quarks
  - the mass of protons and neutrons, the kernels of all visible matter, are 100-times larger than anything the Higgs can produce
- *Where did it all begin? … becomes … Where did it all come from?*

All mass is interaction.

— Richard P. Feynman —

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

Trace Anomaly

- ✓ Dilations = compression or expansion of length (mass) scales
- ✓ Scale invariant theory = dilations don't change anything
- ✓ Current conservation:
- ✓ Divergence of dilation current vanishes in a scale invariant the

- In a scale invariant theory

  the *energy-momentum tensor must be traceless:* $\partial_\mu D_\mu = T_{\mu\mu} \equiv 0$

- Regularisation and renormalisation of (ultraviolet) divergences in Quantum Chromodynamics introduces a mass-scale

  … *dimensional transmutation*:

  Lagrangian's *constants* (couplings and masses) become dependent on a mass-scale, ζ

- $\alpha \to \alpha(\zeta)$ in QCD's (massless) Lagrangian density, L(m=0)

  $\Rightarrow \partial_\mu D_\mu = \delta L/\delta\sigma = \alpha\beta(\alpha)\, dL/d\alpha = \beta(\alpha)\, ¼G_{\mu\nu}\, G_{\mu\nu} = T_{\mu\mu} =: \Theta_0$

  QCD β function … specifies how the coupling "runs"

Trace anomaly

Quantisation of renormalisable four-dimensional theory forces nonzero value for trace of energy-momentum tensor

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

Trace Anomaly

- Knowing that a trace anomaly exists does not deliver a great deal
  … Indicates only that a mass-scale must exist
- Key Question: Can one compute and/or understand the magnitude of that scale?
- One can certainly *measure* the magnitude … consider proton:

$$\langle p(P)|T_{\mu\nu}|p(P)\rangle = -P_\mu P_\nu$$

$$\langle p(P)|T_{\mu\mu}|p(P)\rangle = -P^2 = m_p^2$$

$$= \langle p(P)|\Theta_0|p(P)\rangle$$

- In the chiral limit the entirety of the proton's mass is produced by the trace anomaly, Θ_0
  … In perturbative QCD, Θ_0 measures the strength of gluon self-interactions
  … so, from one perspective,
  m_p is (somehow) completely generated by glue.

The mass of visible matter is almost entirely produced by massless non-matter fields

On the other hand …

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

Trace Anomaly

- In the chiral limit, pion is massless Nambu-Goldstone boson:

$$\langle\pi(q)|T_{\mu\nu}|\pi(q)\rangle = -q_\mu q_\nu \quad \Rightarrow \quad \langle\pi(q)|\Theta_0|\pi(q)\rangle = 0$$

- Does this mean that the scale anomaly vanishes trivially in the pion state, *i.e.* gluons contribute nothing to the pion mass?
- Difficult way to obtain "zero"!

"Nothing can come of nothing."
William Shakespeare, "King Lear"

- Easier to imagine that "zero" owes to cancellations between different operator contributions to the expectation value of Θ_0.
- Of course, such precise cancellation should not be an accident.

  It could only arise naturally because

  of some symmetry and/or symmetry-breaking pattern.

Whence "1" and yet "0" ?

$$\langle p(P)|\Theta_0|p(P)\rangle = m_p^2\,, \qquad \langle \pi(q)|\Theta_0|\pi(q)\rangle = 0$$

- *No statement of the question*

  "How does the mass of the proton arise?"

  is complete without the additional clause

  "How does the pion remain **massless***?"*

- Natural visible-matter mass-scale must emerge simultaneously with apparent preservation of scale invariance in related systems
  - Expectation value of Θ_0 in pion is always zero, irrespective of the size of the natural mass-scale for strong interactions $= m_p$

Whence "1" and yet "0" ?

$$\langle p(P)|\Theta_0|p(P)\rangle = m_p^2\,, \qquad \langle \pi(q)|\Theta_0|\pi(q)\rangle = 0$$

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  "How does the mass of the proton arise?"

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  "How does the pion remain **massless***?"*

- Modern Physics must
  Elucidate the entire array of Empirical Consequences
  of the Mechanism responsible
  so that the Standard Model can be Tested and
  Bounded

G E N E S I S

In the Beginning…

GOOGLE

gauge boson mass

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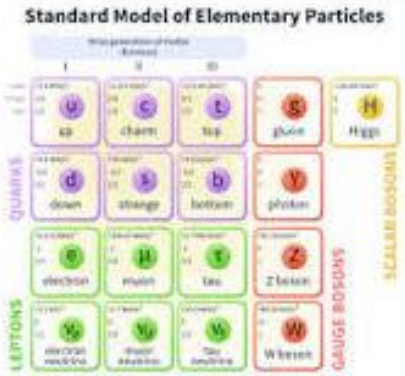
About 302,000 results (0.48 seconds)

In particle physics, a **gauge boson** is a force carrier, a bosonic particle that carries any of the fundamental interactions of nature, commonly called forces. Elementary particles, whose interactions are described by a **gauge** theory, interact with each other by the exchange of **gauge bosons**—usually as virtual particles.



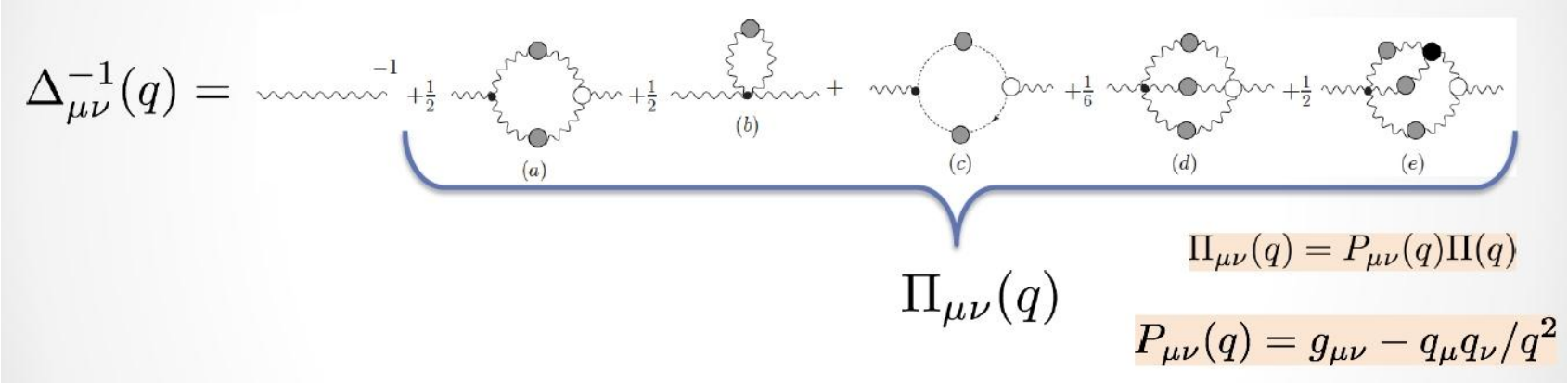
Gauge boson - Wikipedia
https://en.wikipedia.org/wiki/Gauge_boson

Do gluons have mass?

So far **gluons** appear massless, and if they **have mass** this would mean there is some sort of symmetry breaking or Higgs mechanism involved with QCD. So far there is no evidence of this, and theory **does** not make predictions of **gluon masses**. May 29, 2016

Google is *not* infallible

Gluon gap (Dyson) equation



- Known behaviour of QCD running coupling

  ⇒ long wavelength (low energy+momentum) gluons are strongly interacting
- Potentially huge nonperturbative feedback effects

Ancient History

Cornwall's estimate: $m_g = 0.5(2)\,\mathrm{GeV} \approx \frac{1}{2} m_p$

- ≈ 45 years ago

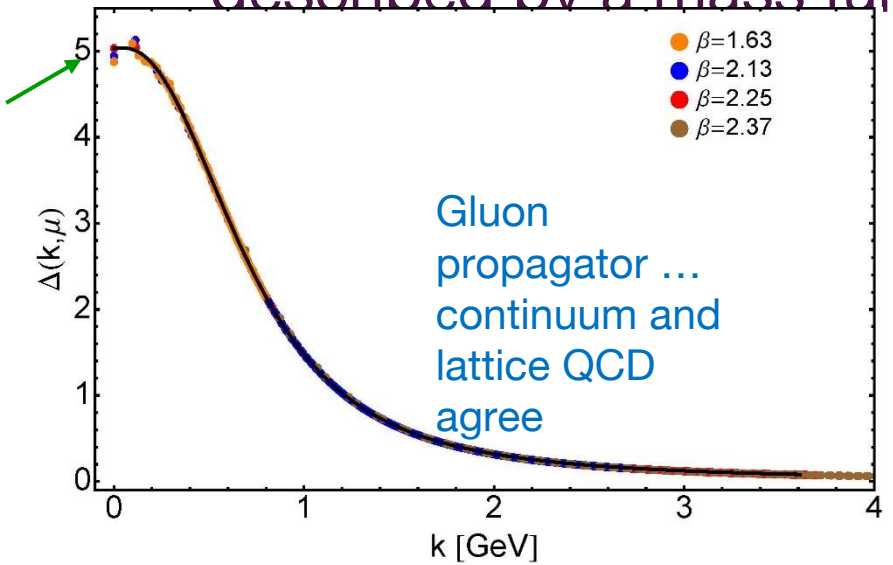
  Dynamical mass generation in continuum quantum chromodynamics,
  J.M. Cornwall, Phys. Rev. D **26** (1981) 1453 … > 1100 citations

3-gluon vertex

- Owing to strong self-interactions, gluon partons ⇒ gluon quasiparticles, described by a mass function that is large at infrared momenta

4-gluon vertex

$\frac{1}{m_g^2}$



Gluon propagator … continuum and lattice QCD agree

Truly mass from nothing
An interacting theory, written in terms of massless gluon fields, produces dressed gluon fields that are characterised by a mass function that is large at infrared momenta

- ✓ *QCD fact*
- ✓ *Continuum theory and lattice simulations agree*
- ✓ *Empirical verification?*

D. Binosi, *Emergent Hadron Mass in Strong Dynamics*, Few Body Syst. **63** (2022) 42.

M. N. Ferreira, J. Papavassiliou, *Gauge Sector Dynamics in QCD*, Particles **6** (1) (2023) 312–363

Ancient History

Today's best estimate: $m_g^{\mathrm{RGI}} = 0.43(1)\,\mathrm{GeV}$

Gauge Invariance and Mass,
J. S. Schwinger
Phys. Rev. 125 (1962) 397-398
It is argued that the gauge invariance of a vector field does not necessarily imply zero mass for an associated particle if the current vector coupling is sufficiently strong.

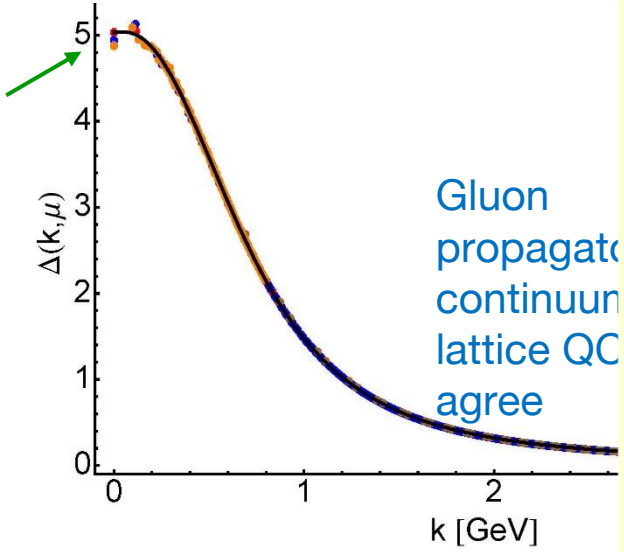
EHM means Gluons are massive via Schwinger Mechanism

3-gluon vertex

ynamics,
ons
n quasiparticles,
d momenta

4-gluon vertex

$\frac{1}{m_g^2}$



Gluon propagato
continuun
lattice QC
agree

- ✓ *QCD fact*
- ✓ *Continuum theory and lattice simulations agree*
- ✓ *Empirical verification?*

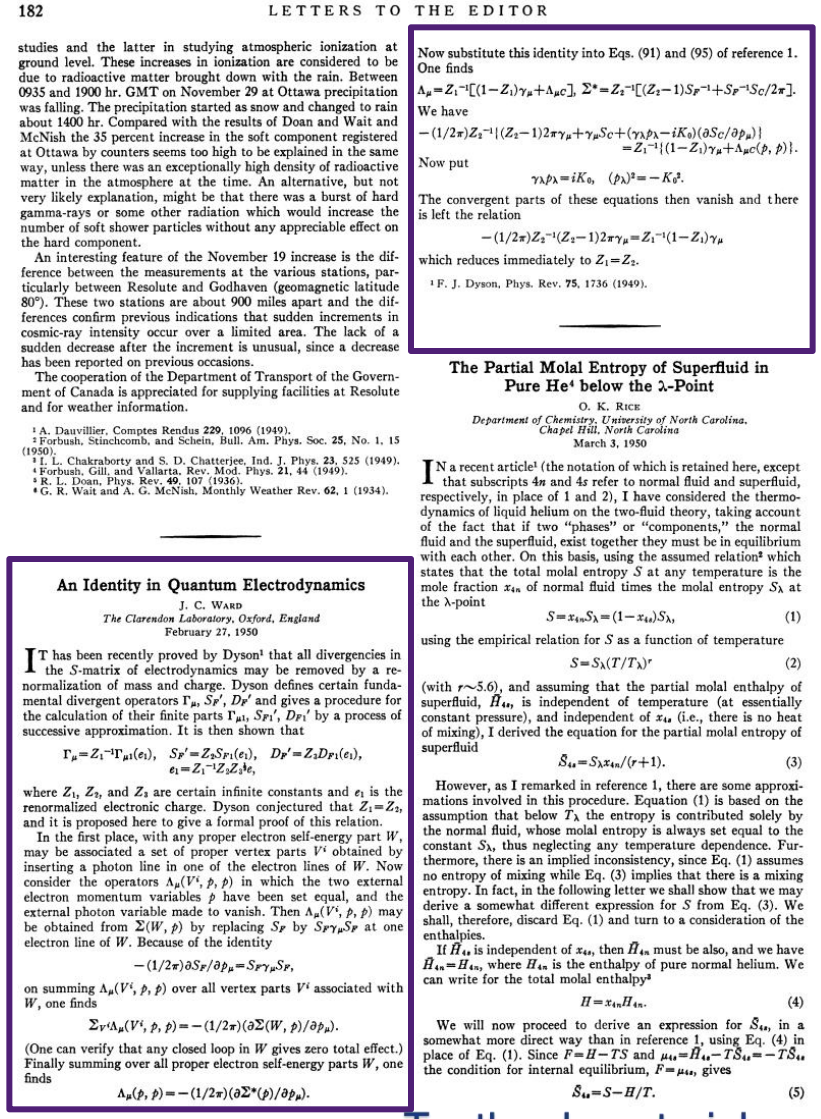
QED Running Coupling – Gell-Mann – Low effective charge

- To define QED, one introduces three renormalisation constants:

  Z_1… fermion-photon vertex & Z_2 … fermion wave function

  & Z_3 … photon wave function
- Ward identity:

  $Z_1 = Z_2$ … basically, this is necessary and sufficient to ensure current conservation
- Only one renormalisation constant remains
  - Z_3 … photon wave function; hence, photon propagator
  - So, running of coupling in QED is completely determined by renormalisation-induced flow (momentum dependence) of the photon propagator
- $\alpha_{QED}(q^2)$ … most familiar & best-known running



182 LETTERS TO THE EDITOR

studies and the latter in studying atmospheric ionization at ground level. These increases in ionization are considered to be due to radioactive matter brought down with the rain. Between 0935 and 1900 hr. GMT on November 29 at Ottawa precipitation was falling. The precipitation started as snow and changed to rain about 1400 hr. Compared with the results of Doan and Wait and McNish the 35 percent increase in the soft component registered at Ottawa by counters seems too high to be explained in the same way, unless there was an exceptionally high density of radioactive matter in the atmosphere at the time. An alternative, but not very likely explanation, might be that there was a burst of hard gamma-rays or some other radiation which would increase the number of soft shower particles without any appreciable effect on the hard component.

An interesting feature of the November 19 increase is the difference between the measurements at the various stations, particularly between Resolute and Godhaven (geomagnetic latitude 80°). These two stations are about 900 miles apart and the differences confirm previous indications that sudden increments in cosmic-ray intensity occur over a limited area. The lack of a sudden decrease after the increment is unusual, since a decrease has been reported on previous occasions.

The cooperation of the Department of Transport of the Government of Canada is appreciated for supplying facilities at Resolute and for weather information.

[1] A. Dauvillier, Comptes Rendus **229**, 1096 (1949).
[2] Forbush, Stinchcomb, and Schein, Bull. Am. Phys. Soc. **25**, No. 1, 15 (1950).
[3] I. L. Chakraborty and S. D. Chatterjee, Ind. J. Phys. **23**, 525 (1949).
[4] Forbush, Gill, and Vallarta, Rev. Mod. Phys. **21**, 44 (1949).
[5] R. L. Doan, Phys. Rev. **49**, 107 (1936).
[6] G. R. Wait and A. G. McNish, Monthly Weather Rev. **62**, 1 (1934).

Now substitute this identity into Eqs. (91) and (95) of reference 1. One finds

$$\Lambda_\mu = Z_1^{-1}[(1-Z_1)\gamma_\mu + \Lambda_\mu C],\quad \Sigma^* = Z_2^{-1}[(Z_2-1)S_F^{-1} + S_F^{-1}S_C/2\pi].$$

We have

$$-(1/2\pi)Z_2^{-1}\{(Z_2-1)2\pi\gamma_\mu + \gamma_\mu S_C + (\gamma_\lambda p_\lambda - iK_0)(\partial S_C/\partial p_\mu)\} = Z_1^{-1}\{(1-Z_1)\gamma_\mu + \Lambda_\mu C(p,p)\}.$$

Now put

$$\gamma_\lambda p_\lambda = iK_0,\quad (p_\lambda)^2 = -K_0^2.$$

The convergent parts of these equations then vanish and there is left the relation

$$-(1/2\pi)Z_2^{-1}(Z_2-1)2\pi\gamma_\mu = Z_1^{-1}(1-Z_1)\gamma_\mu$$

which reduces immediately to $Z_1 = Z_2$.

[1] F. J. Dyson, Phys. Rev. **75**, 1736 (1949).

The Partial Molal Entropy of Superfluid in Pure He^4 below the λ-Point

O. K. Rice
Department of Chemistry, University of North Carolina, Chapel Hill, North Carolina
March 3, 1950

In a recent article[1] (the notation of which is retained here, except that subscripts $4n$ and $4s$ refer to normal fluid and superfluid, respectively, in place of 1 and 2), I have considered the thermodynamics of liquid helium on the two-fluid theory, taking account of the fact that if two "phases" or "components," the normal fluid and the superfluid, exist together they must be in equilibrium with each other. On this basis, using the assumed relation[2] which states that the total molal entropy S at any temperature is the mole fraction x_{4n} of normal fluid times the molal entropy S_λ at the λ-point

$$S = x_{4n}S_\lambda = (1-x_{4s})S_\lambda, \quad (1)$$

using the empirical relation for S as a function of temperature

$$S = S_\lambda(T/T_\lambda)^r \quad (2)$$

(with $r \sim 5.6$), and assuming that the partial molal enthalpy of superfluid, $\bar{H}_{4s}$, is independent of temperature (at essentially constant pressure), and independent of x_{4s} (i.e., there is no heat of mixing), I derived the equation for the partial molal entropy of superfluid

$$\bar{S}_{4s} = S_\lambda x_{4n}/(r+1). \quad (3)$$

However, as I remarked in reference 1, there are some approximations involved in this procedure. Equation (1) is based on the assumption that below T_λ the entropy is contributed solely by the normal fluid, whose molal entropy is always set equal to the constant S_λ, thus neglecting any temperature dependence. Furthermore, there is an implied inconsistency, since Eq. (1) assumes no entropy of mixing while Eq. (3) implies that there is a mixing entropy. In fact, in the following letter we shall show that we may derive a somewhat different expression for S from Eq. (3). We shall, therefore, discard Eq. (1) and turn to a consideration of the enthalpies.

If $\bar{H}_{4s}$ is independent of x_{4s}, then $\bar{H}_{4n}$ must be also, and we have $\bar{H}_{4n} = H_{4n}$, where H_{4n} is the enthalpy of pure normal helium. We can write for the total molal enthalpy[3]

$$H = x_{4n}H_{4n}. \quad (4)$$

We will now proceed to derive an expression for $\bar{S}_{4s}$, in a somewhat more direct way than in reference 1, using Eq. (4) in place of Eq. (1). Since $F = H - TS$ and $\mu_{4s} = \bar{H}_{4s} - T\bar{S}_{4s} = -T\bar{S}_{4s}$ the condition for internal equilibrium, $F = \mu_{4s}$, gives

$$\bar{S}_{4s} = S - H/T. \quad (5)$$

An Identity in Quantum Electrodynamics

J. C. Ward
The Clarendon Laboratory, Oxford, England
February 27, 1950

It has been recently proved by Dyson[1] that all divergencies in the S-matrix of electrodynamics may be removed by a renormalization of mass and charge. Dyson defines certain fundamental divergent operators Γ_μ, S_F', D_F' and gives a procedure for the calculation of their finite parts $\Gamma_{\mu 1}$, S_{F1}', D_{F1}' by a process of successive approximation. It is then shown that

$$\Gamma_\mu = Z_1^{-1}\Gamma_{\mu 1}(e_1),\quad S_F' = Z_2 S_{F1}(e_1),\quad D_F' = Z_3 D_{F1}(e_1),\quad e_1 = Z_1^{-1}Z_2 Z_3^{\frac{1}{2}} e,$$

where Z_1, Z_2, and Z_3 are certain infinite constants and e_1 is the renormalized electronic charge. Dyson conjectured that $Z_1 = Z_2$, and it is proposed here to give a formal proof of this relation.

In the first place, with any proper electron self-energy part W, may be associated a set of proper vertex parts V^i obtained by inserting a photon line in one of the electron lines of W. Now consider the operators $\Lambda_\mu(V^i, p, p)$ in which the two external electron momentum variables p have been set equal, and the external photon variable made to vanish. Then $\Lambda_\mu(V^i, p, p)$ may be obtained from $\Sigma(W, p)$ by replacing S_F by $S_F\gamma_\mu S_F$ at one electron line of W. Because of the identity

$$-(1/2\pi)\partial S_F/\partial p_\mu = S_F\gamma_\mu S_F,$$

on summing $\Lambda_\mu(V^i, p, p)$ over all vertex parts V^i associated with W, one finds

$$\Sigma_{V^i}\Lambda_\mu(V^i, p, p) = -(1/2\pi)(\partial\Sigma(W, p)/\partial p_\mu).$$

(One can verify that any closed loop in W gives zero total effect.) Finally summing over all proper electron self-energy parts W, one finds

$$\Lambda_\mu(p, p) = -(1/2\pi)(\partial\Sigma^*(p)/\partial p_\mu).$$

Textbook material
1292 citations

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  & Z_3 … photon wave function
- Ward identity:

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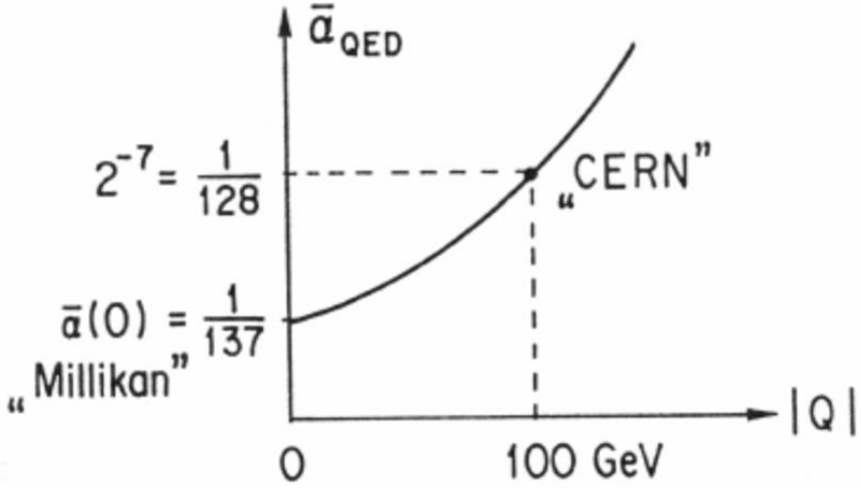
  - Z_3 … photon wave function; hence, photon propagator
  - So, running of coupling in QED is completely determined by renormalisation-induced flow (momentum dependence) of the photon propagator



- $\alpha_{QED}(q^2)$ … most familiar & best-known running

QCD Running Coupling

- QCD has 8 coupling renormalisation constants
- Slavnov-Taylor (generalised Ward) identities relate the various constants
  - ensure uniqueness of coupling and "gauge invariance" in perturbation theory
- However, with so many choices of couplings & vertices & propagators, then using standard formulation of QCD equations of motion:
  - Running coupling cannot be defined via gluon vacuum polarisation
  - No unique definition of running coupling
  - Actually, uncountable infinity of distinct nonperturbative definitions

$$g_{0YM} = \tilde{g}_0 \implies \frac{Z_{3YM}}{Z_{1YM}} = \frac{\tilde{Z}_3}{\tilde{Z}_1}$$

$$g_{0YM} = g_{0F} \implies \frac{Z_{3YM}}{Z_{1YM}} = \frac{Z_{2F}}{Z_{1F}}$$

$$g_{0YM} = g_{0S} \implies Z_S = Z_{1YM}\frac{Z_{1YM}}{Z_{3YM}}$$

ELSEVIER

Progress in Particle and Nuclear Physics

Volume 134, January 2024, 104081

Review

QCD running couplings and effective charges

Alexandre Deur [a], Stanley J. Brodsky [b], Craig D. Roberts [c d]

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https://doi.org/10.1016/j.ppnp.2023.104081

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Abstract

We discuss our present knowledge of α_s, the fundamental running coupling or effective charge of Quantum Chromodynamics (QCD). A precise understanding of the running of $\alpha_s\left(Q^2\right)$ at high momentum transfer, Q, is necessary for any perturbative QCD calculation. Equally important, the behavior of α_s at low Q^2 in the nonperturbative QCD domain is critical for understanding strong interaction phenomena, including the emergence of mass and quark confinement. The behavior of $\alpha_s\left(Q^2\right)$ at all momentum transfers also provides a connection between perturbative and nonperturbative QCD phenomena, such as hadron spectroscopy and dynamics. We first sketch the origin of the QCD coupling, the reason why its magnitude depends on the scale at which hadronic phenomena are probed, and the resulting consequences for QCD phenomenology. We then summarize latest measurements in both the perturbative and nonperturbative domains. New theory developments include the derivation of the universal nonperturbative behavior of $\alpha_s\left(Q^2\right)$ from both the Dyson–Schwinger equations and light-front holography. We also describe theory advances for the calculation of gluon and quark Schwinger functions in the nonperturbative domain and the relation of these quantities to α_s. We conclude by highlighting how the nonperturbative knowledge of α_s is now providing a parameter-free determination of hadron spectroscopy and structure, a central and long-sought goal of QCD studies.

Effective charge from lattice QCD, *Zhu-Fang Cui, Jin-Li Zhang et al., NJU-INP 014/19, arXiv:1912.08232 [hep-ph], Chin. Phys. C 44 (2020) 083102/1-10*

2649 total downloads

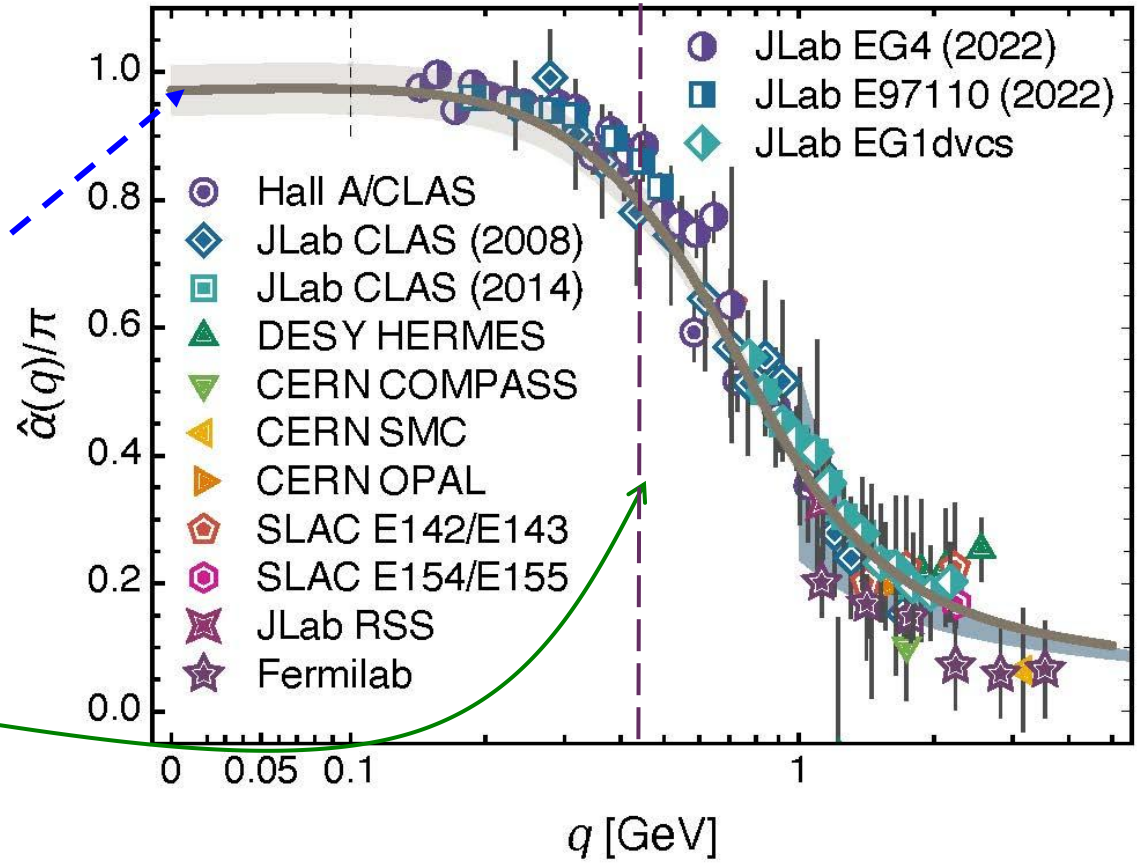
Process independent effective charge = running coupling

- Modern theory – exploiting pinch technique and background field method – enables unique QCD analogue of

  "Gell-Mann – Low"

  running charge to be rigorously defined and calculated
- Analysis of QCD's gauge sector

  yields a *parameter-free prediction*
- N.B. Qualitative change in $\hat{\alpha}_{PI}(k)$ at $k \approx \frac{1}{2} m_p$
- No Landau Pole
- Below $k \sim \hat{m}_0$, interactions become scale independent, just as they were in the Lagrangian; so, QCD becomes practically conformal again



- ✓ *Process independent strong running coupling* Daniele Binosi *et al.*, arXiv:1612.04835 [nucl-th], Phys. Rev. D 96 (2017) 054026/1-7
- ✓ *Experimental determination of the QCD effective charge $\alpha_{g1}(Q)$.* A. Deur; V. Burkert; J.-P. Chen; W. Korsch, Particles 5 (2022) 171
- ✓ *QCD Running Couplings and Effective Charges,* Alexandre Deur, Stanley J. Brodsky and Craig Roberts, e-Print: 2303.00723 [hep-ph], Prog. Part. Nucl. Phys. **134** (2024) 104081

Process-independent Effective Charge

https://1drv.ms/b/s%21Apbnb6VLY1RplPRdL0J4Wm-tlzQd-Q?e=hdZxHH

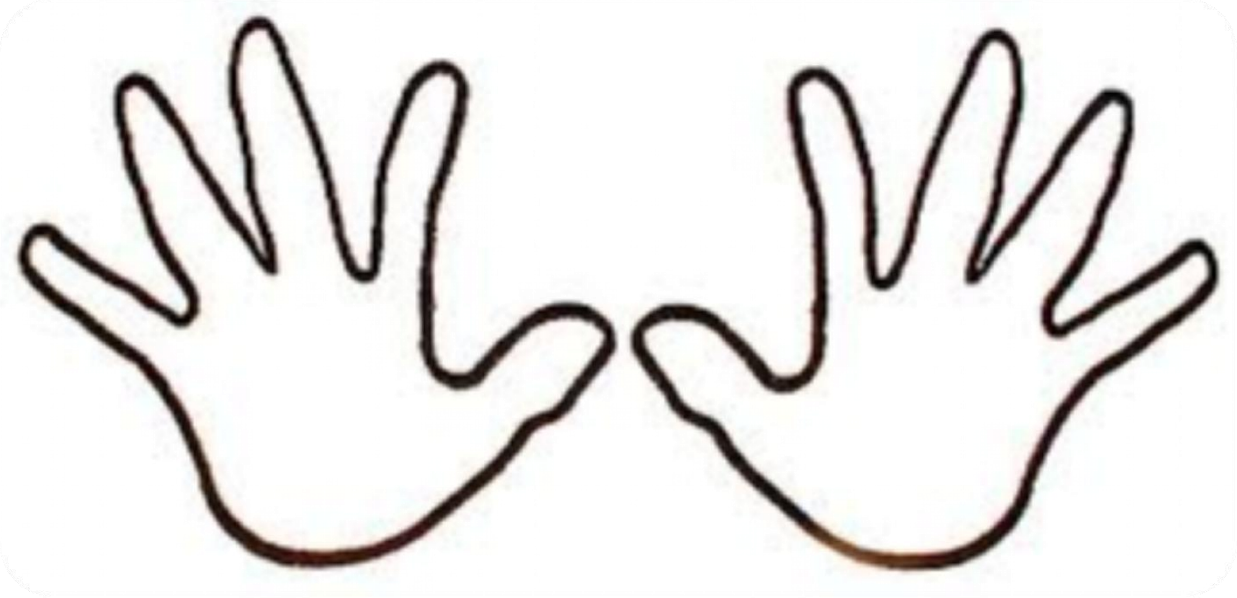
SCIENTIFIC AMERICAN

- Most-read article in the May issue of SciAm
- Translated into Chinese, French, German, Italian
- Cover story on Chinese and German editions

May 2024

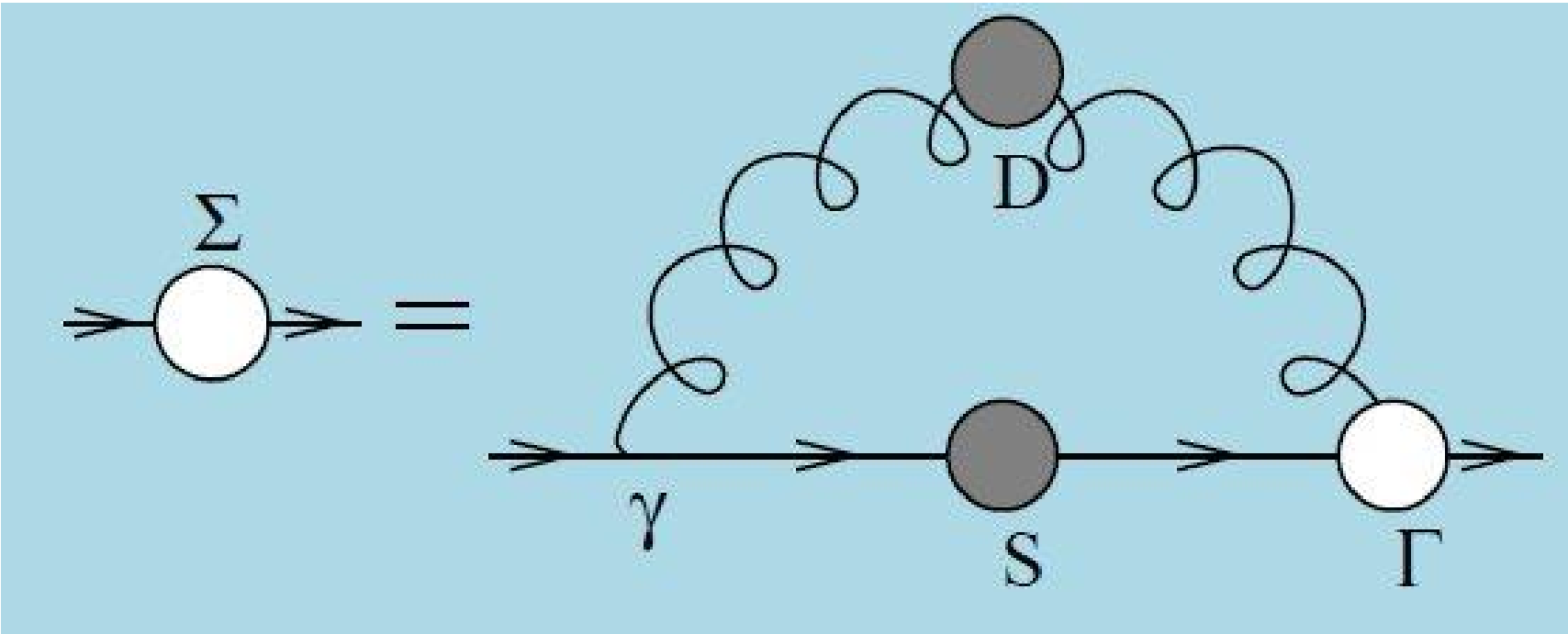
Pour la Science

- ✓ *SciAm is one of the Top-5 popular science magazines in the world.*
- ✓ *International circulation that exceeds 10,000,000 readers per month in subscriptions and millions more via social media etc.*

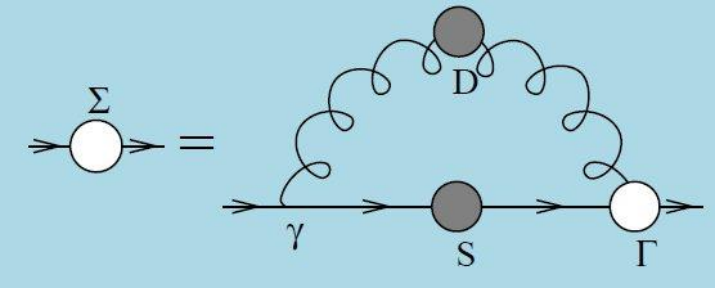


Dynamical Chiral Symmetry Breaking

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



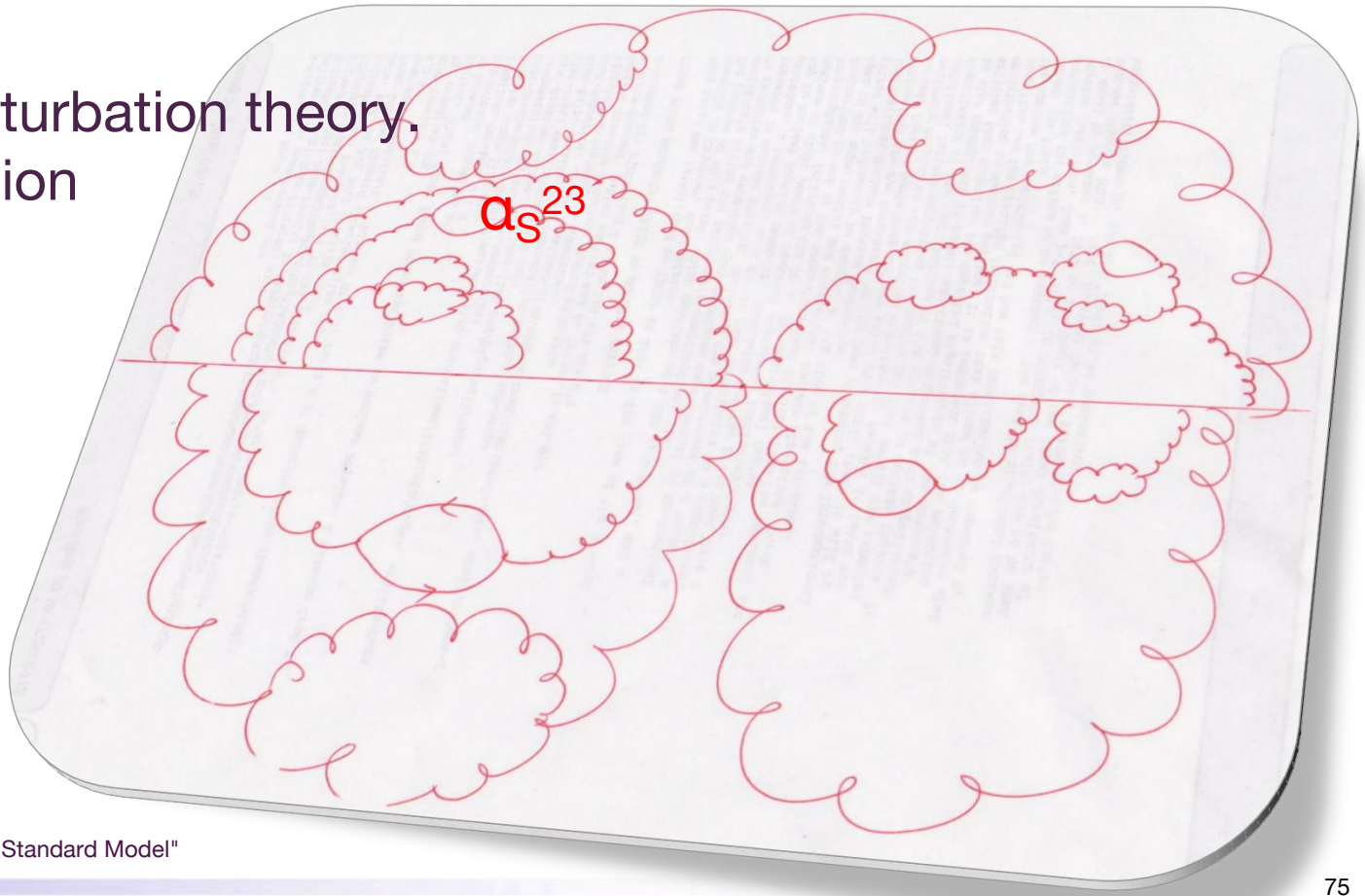
Quark Gap (Dyson) Equation



Where does the mass come from?

- Deceptively simply picture
- Corresponds to the sum of a countable infinity of diagrams.

  NB. QED has 12,672 α^5 diagrams
- Impossible to compute this in perturbation theory. The standard algebraic manipulation tools are just inadequate



Just one of the terms that are summed in a solution of the simplest, reasonable truncation of QCD's gap equation

DCSB à la Nambu

- Mathematical form of the gap equation

$$S^{-1}(p) = i\gamma\cdot p\,A(p^2) + B(p^2) = i\gamma\cdot p + m$$
$$+\int^{\Lambda}\frac{d^4\ell}{(2\pi)^4}\,g^2\,D_{\mu\nu}(p-\ell)\,\gamma_\mu\frac{\lambda^a}{2}\frac{1}{i\gamma\cdot\ell A(\ell^2)+B(\ell^2)}\Gamma^a_\nu(\ell,p)$$

NJL: $\Gamma^a_\mu(k,p)_{\rm bare} = \gamma_\mu\,\frac{\lambda^a}{2}$;

$$g^2 D_{\mu\nu}(p-\ell) \to \delta_{\mu\nu}\,\frac{1}{m_G^2}\,\theta(\Lambda^2-\ell^2)$$

- Model is not renormalisable
  $\Rightarrow$ regularisation parameter (Λ) plays a dynamical role.

- Contact-interaction gap equation

$$i\gamma\cdot p\,A(p^2) + B(p^2)$$
$$= \quad i\gamma\cdot p + m + \frac{4}{3}\,\frac{1}{m_G^2}\int\frac{d^4\ell}{(2\pi)^4}\,\theta(\Lambda^2-\ell^2)\,\gamma_\mu\,\frac{-i\gamma\cdot\ell A(\ell^2)+B(\ell^2)}{\ell^2A^2(\ell^2)+B^2(\ell^2)}\,\gamma_\mu$$

DCSB à la Nambu

- Multiply the gap equation by (-*iγ·p*); trace over Dirac indices:

$$p^2 A(p^2) = p^2 + \frac{8}{3}\frac{1}{m_G^2}\int \frac{d^4\ell}{(2\pi)^4}\,\theta(\Lambda^2-\ell^2)\,p\cdot\ell\,\frac{A(\ell^2)}{\ell^2 A^2(\ell^2)+B^2(\ell^2)}$$

  - Angular integral vanishes, therefore $A(p^2) = 1$.
  - This owes to the fact that the model is defined by a four-fermion contact-interaction in configuration space, which entails a momentum-independent interaction in momentum space.
- Simply take Dirac tr

$$B(p^2) = m + \frac{16}{3}\frac{1}{m_G^2}\int \frac{d^4\ell}{(2\pi)^4}\,\theta(\Lambda^2-\ell^2)\,\frac{B(\ell^2)}{\ell^2+B^2(\ell^2)}$$

  - Integrand is p^2-independent, therefore the only solution is $B(p^2)$ = constant = M.
- General form of the propagator for a fermion dressed by the contact interaction:

$$S(p) = 1/[\, i\,\gamma\cdot p + M\,]$$

Critical coupling for dynamical mass generation?

Contact interaction & a mass gap?

- Evaluate the integrals

$$M = m + M \frac{1}{3\pi^2} \frac{1}{m_G^2} \mathcal{C}(M^2, \Lambda^2),$$
$$\mathcal{C}(M^2, \Lambda^2) = \Lambda^2 - M^2 \ln\left[1 + \Lambda^2/M^2\right].$$

- Λ defines the model's mass-scale. Henceforth set $\Lambda = 1$, then all other dimensioned quantities are given in units of this scale, in which case the gap equation can be written

$$M = M \frac{1}{3\pi^2} \frac{1}{m_G^2} \mathcal{C}(M^2, 1)$$

- Chiral limit, *m=0*
  - Solutions?
    - One is obvious; viz., *M=0*
      This is the *perturbative result*

    … start with no mass, end up with no mass

- Chiral limit, *m=0*
  - Suppose, on the other hand that *M≠0*, and thus may be cancelled
    - This nontrivial solution can exist if-and-only-if one may satisfy

$$3\pi^2 m_G^2 = C(M^2, 1)$$

Critical coupling for dynamical mass generation!

Contact interaction & a mass gap?

- Can one satisfy $3\pi^2 m_G^2 = C(M^2,1)$?
  - $C(M^2, 1) = 1 - M^2 \ln [1 + 1/M^2]$
    - Monotonically decreasing function of M
    - Maximum value at $M = 0$; *viz.*, $C(M^2=0, 1) = 1$
- Consequently, there is a solution *iff* $3\pi^2 m_G^2 < 1$
  - Typical scale for hadron physics: $\Lambda = 1$ GeV
    - There is a M≠0 solution iff $m_G^2 < (\Lambda/(3\pi^2)) = (0.2\ \text{GeV})^2$
- Interaction strength is proportional to $1/m_G^2$
  - Hence, if interaction is strong enough, then one can start with no mass but end up with a massive, perhaps very massive fermion

Dynamical Chiral Symmetry Breaking

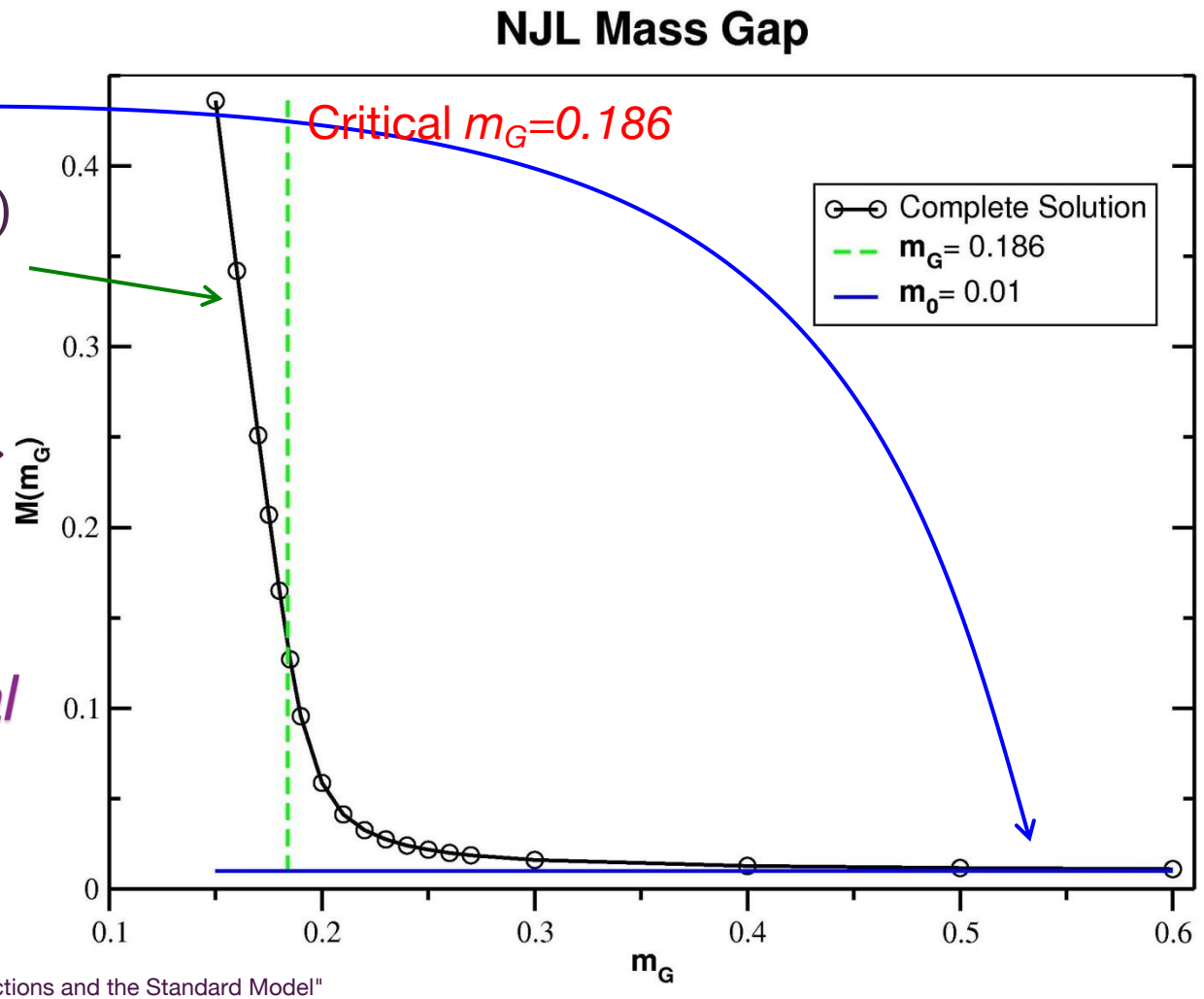
Contact Interaction & Dynamical Mass

Solution of gap equation

$$M = m + M \frac{1}{3\pi^2} \frac{1}{m_G^2} \mathcal{C}(M^2, 1)$$

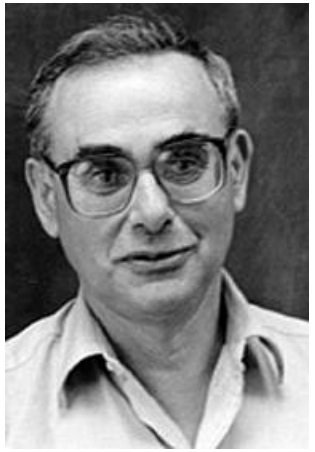
- Weak coupling corresponds to m_G large
  - … means something of $O(\Lambda)$
  - – in which case $M \approx m$
- On the other hand, strong coupling; i.e., m_G small, $M \gg m$

This is the defining characteristic of dynamical chiral symmetry breaking



Nambu-Goldstone Bosons
(Anderson-Bogoliubov modes)

Massless excitations appearing when a continuous symmetry is broken by dynamics



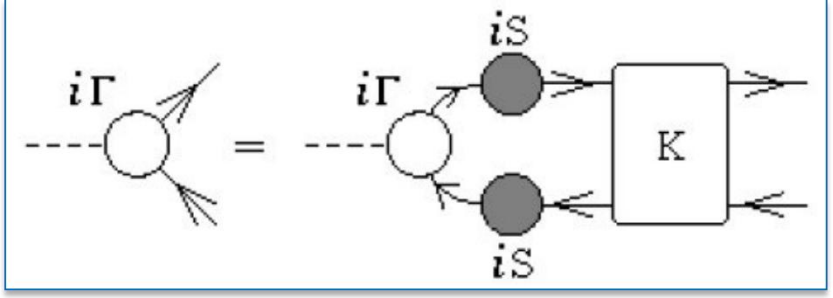
- Suppose one has a vector ⊗ vector interaction, strong enough to produce a dressed-quark mass in the absence of Higgs-coupling:
  - How do Nambu-Goldstone bosons emerge?
  - Is their emergence fine-tuned in any way?
  - How does their mass vary with current-quark mass?
  - What do their properties reveal about the origin of mass?

Pseudoscalar Mesons as Bound-States

- Poincaré covariance entails that the bound-state (Bethe-Salpeter) amplitude for an isovector pseudoscalar meson (valence-quark and valence-antiquark) takes the form

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\Big]$$

  - $\{\tau^{i=1,2,3}\}$ = Pauli matrices to represent isospin = difference between quarks *u* (isospin +1/2) and *d* (isospin -1/2)
  - P = total momentum of system
  - k = relative momentum between valence degrees-of-freedom



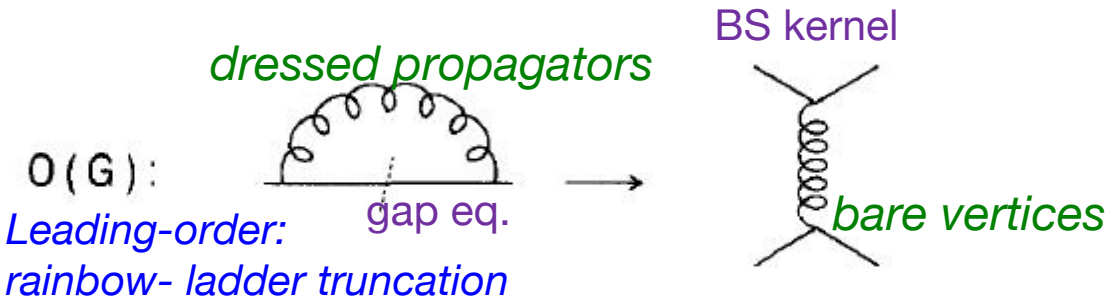
Pseudoscalar Mesons as Bound-States

- Bound-State mass and amplitude (Poincaré-covariant wave function) are determined from the homogeneous Bethe-Salpeter equation (BSE):

$$[\Gamma_\pi^j(k;P)]_{tu} = \int \frac{d^4q}{(2\pi)^4} [\chi_\pi^j(q;P)]_{sr} K_{tu}^{rs}(q,k;P)$$

  - P^2 is an eigenvalue … solution only exists for particular value(s) of $P^2 < 0$ and those values define – $\text{mass}^2_{\text{bound state}}$
  - $\chi(q;P) = S(q)\,\Gamma(q;P)\,S(q+P)$ – wave function
  - S = dressed-quark propagator
  - r,s,t,u represent colour, isospin, spinor indices
  - K = quark-antiquark scattering amplitude
- A generalisation to quantum field theory of the Lippmann-Schwinger equation = one of the most-used equations in quantum mechanics … to describe scattering and bound states

Pseudoscalar Mesons as Bound-States



- Leading-order approximation to Bethe-Salpeter kernel (rainbow-ladder truncation):

$$\mathcal{K}_{\alpha_1\alpha_1',\alpha_2\alpha_2'} = \mathcal{G}_{\mu\nu}(k)[i\gamma_\mu]_{\alpha_1\alpha_1'}[i\gamma_\nu]_{\alpha_2\alpha_2'}\,,$$

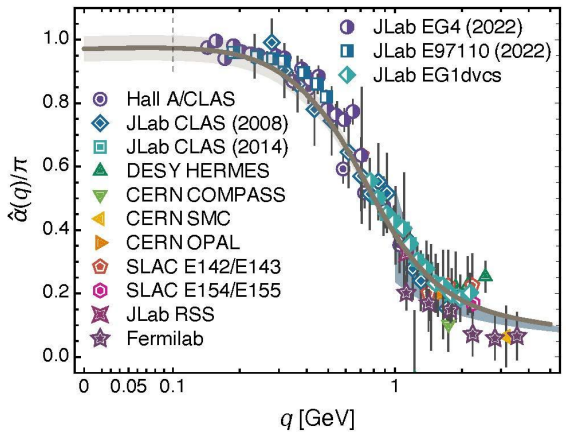
$$\mathcal{G}_{\mu\nu}(k) = \tilde{\mathcal{G}}(k^2)T_{\mu\nu}(k)\,,$$

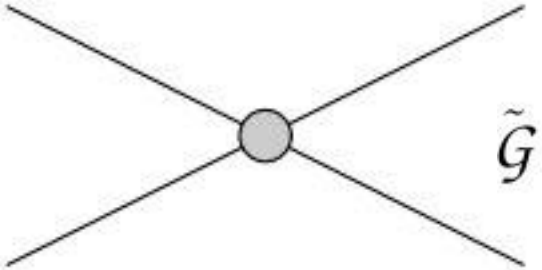
$$k^2T_{\mu\nu}(k) = k^2\delta_{\mu\nu} - k_\mu k_\nu.$$

- Recall that owing to the dynamical generation of a nonzero gluon mass-scale in QCD, $\mathcal{G}$ saturates at infrared momenta; hence, one may w

$$\tilde{\mathcal{G}}(k^2) \overset{k^2\simeq 0}{=} \frac{4\pi\alpha_{IR}}{m_G^2}$$

- In QCD
  - $m_G \approx 0.5$ GeV and $\alpha \approx \pi$





$$\tilde{\mathcal{G}}(k^2) \overset{k^2 \simeq 0}{=} \frac{4\pi\alpha_{IR}}{m_G^2}$$

Pseudoscalar Mesons as Bound-States

- This equation defines the contact interaction
- Since a contact interaction cannot support relative momentum between bound-state constituents … because no momentum exchanged between scatterers, one can simplify the tensor structure: contact-interaction RL kernel is

$$\mathcal{K}^{\rm CI}_{\alpha_1\alpha_1',\alpha_2\alpha_2'} = \frac{4\pi\alpha_{IR}}{m_G^2}[i\gamma_\mu]_{\alpha_1\alpha_1'}[i\gamma_\mu]_{\alpha_2\alpha_2'}$$

- No relative momentum, then Bethe-Salpeter amplitude simplifies

$$\Gamma_{0^-}(Q) = \gamma_5\left[iE_{0^-} + \frac{1}{M}\gamma\cdot P F_{0^-}\right]$$

  - M = dressed-mass of quarks in bound-state
  - E_{0^-} & F_{0^-} independent of relative momentum

Pseudoscalar Mesons as Bound-States

- BSE becomes (total momentum now = Q, relative momentum = t)

$$\Gamma_{0^-}(Q) = -\frac{16\pi}{3}\frac{\alpha_{\rm IR}}{m_G^2}\int\frac{d^4t}{(2\pi)^4}\gamma_\mu S_f(t+Q)\Gamma_{0^-}(Q)S_g(t)\gamma_\mu$$

- Integral is obviously quadratically divergent:
  - $\int d^4t \sim \int dt^2\, t^2$
  - Integrand $\sim 1/t^2$

  Regularisation needed.
- Same true of contact interaction gap equation:

$$S_f^{-1}(p) = i\gamma\cdot p + m_f + \frac{16\pi}{3}\frac{\alpha_{\rm IR}}{m_G^2}\int\frac{d^4q}{(2\pi)^4}\gamma_\mu S_f(q)\gamma_\mu$$

Using translationally invariant regularisation scheme, contact-interaction solution has form:

$$S(p) = 1/[i\gamma\cdot p + M]$$

- What shall we do to complete definition of theory?

Pseudoscalar Mesons as Bound-States

- With $S(p) = 1/[i\gamma\cdot p + M]$, only dynamical content of CI gap equation is determination of dressed-quark's mass (f = flavour label):

$$M_f = m_f + M_f \frac{4\alpha_{\rm IR}}{3\pi m_G^2}\left[\int_0^\infty ds\, s\, \frac{1}{s+M_f^2}\right]_{\rm reg}$$

- Follow [inSPIRE: Ebert:1996vx]

$$\frac{1}{s+M^2} = \int_0^\infty d\tau\, {\rm e}^{-\tau(s+M^2)} \to \int_{\tau_{\rm uv}^2}^{\tau_{\rm ir}^2} d\tau\, {\rm e}^{-\tau(s+M^2)} = \frac{{\rm e}^{-(s+M^2)\tau_{\rm uv}^2} - {\rm e}^{-(s+M^2)\tau_{\rm ir}^2}}{s+M^2}$$

$$M_f = m_f + M_f \frac{4\alpha_{\rm IR}}{3\pi m_G^2}\, \mathcal{C}_0^{\rm iu}(M_f^2)\,,$$

where

$$\mathcal{C}_0^{\rm iu}(\sigma) = \int_0^\infty ds\, s \int_{\tau_{\rm uv}^2}^{\tau_{\rm ir}^2} d\tau\, {\rm e}^{-\tau(s+\sigma)} = \sigma\left[\Gamma(-1,\sigma\tau_{\rm uv}^2) - \Gamma(-1,\sigma\tau_{\rm ir}^2)\right],$$

with $\Gamma(\alpha, y)$ being the incomplete gamma-function.

$\tau_{uv} = 1/\Lambda_{uv}$ & $\tau_{ir} = 1/\Lambda_{ir}$ are ultraviolet and infrared cutoffs.

τ_{ir} not necessary, but very useful

Pseudoscalar Mesons as Bound-States

- Why $\Lambda_{ir} \neq 0$, *i.e.* $\tau_{ir} < \infty$?
- Answer:

  with $\frac{1}{s+M^2} \rightarrow \frac{e^{-(s+M^2)\tau_{uv}^2} - e^{-(s+M^2)\tau_{ir}^2}}{s+M^2}$

  - then for $s + M^2 \simeq 0$, propagator becomes $= \tau_{ir}^2 - \tau_{uv}^2$
  - free-particle pole is avoided!
- Using the IR cutoff, one has a rudimentary expression of quark confinement
  - Referring to Cauchy theorem for integration in complex plane and propagator theory
  - No quark-connected singularities that can produce an imaginary part in a matrix element
  - Quarks cannot escape the bound state and propagate to a detector
- A natural choice is $\Lambda_{ir} = \Lambda_{QCD} = 0.24$ GeV
- Use this scheme for every integrand, *e.g.* those in the BSE
  - If one has $1/[s+M^2]^2$, then use $1/[s+M^2]^2 = -(d/dM^2)\, 1/[s+M^2]$

Pseudoscalar Mesons as Bound-States

- Preserving the vector and axial-vector Ward-Green-Takahashi identities is crucial when studying NG-bosons associated with axial/chiral symmetry!
  - It is nonsense to study emergent systems in a theory where the conditions for emergence are explicitly violated!
- The m = 0 axial-vector WGT identity states:

$$P_\mu \Gamma_{5\mu}(k_+, k) = S^{-1}(k_+) i\gamma_5 + i\gamma_5 S^{-1}(k)$$

  - ($k_+ = k+P$)
- $\Gamma_{5\mu}$ is the axial-vector vertex

$$\Gamma_{5\mu}(k_+, k) = \gamma_5\gamma_\mu - \frac{16\pi}{3}\frac{\alpha_{IR}}{m_G^2}\int \frac{d^4q}{(2\pi)^4}\gamma_\alpha \chi_{5\mu}(q_+, q)\gamma_\alpha$$

- (How) Is this identity satisfied using $S(p) = 1/[i\gamma\cdot p + M]$?

Pseudoscalar Mesons as Bound-States

- Insert propagator into AV WGT identity … satisfied if, and only if:

$$M = \frac{16}{3}\frac{4\pi\,\alpha_{IR}}{m_G^2}\int\frac{d^4q}{(2\pi)^4}\frac{M}{[q^2+M^2]},$$

$$0 = \int\frac{d^4q}{(2\pi)^4}\frac{\frac{1}{2}q^2+M^2}{[q^2+M^2]^2}.$$

A. Line 1 = chiral-limit quark gap equation … so the identity can only be satisfied so long as the gap equation is satisfied

B. Line 2 = entirely new:

   i. AV WGT identity is satisfied if, and only if, the model is regularised so as to ensure there are no quadratic or logarithmic divergence

Unsurprisingly, these are the just the circumstances under which a shift in integration variables is permitted, an operation required in order to prove the AV WGT identity

Pseudoscalar Mesons as Bound-States

- Lesson: the contact interaction can be defined sensibly

  if, and only if, one contains the symmetry constraints in the regularisation prescription

- For example,

  Line 2 $\Rightarrow \int_0^1 d\alpha \; [C_0^{iu}(w(\alpha)) + C_1^{iu}(w(\alpha))] = 0$

  where $C_1^{iu}(\sigma) = -\sigma \frac{d}{d\sigma} C_1^{iu}(\sigma)$

- Using this identity, one obtains NG-boson BSE in chiral limit

$$\begin{bmatrix} E_{0^-}(Q) \\ F_{0^-}(Q) \end{bmatrix} = \frac{4\alpha_{\rm IR}}{3\pi m_G^2} \begin{bmatrix} \mathcal{K}_{EE}^{0^-} & \mathcal{K}_{EF}^{0^-} \\ \mathcal{K}_{FE}^{0^-} & \mathcal{K}_{FF}^{0^-} \end{bmatrix} \begin{bmatrix} E_{0^-}(Q) \\ F_{0^-}(Q) \end{bmatrix}$$

$$\mathcal{K}_{EE} = \mathcal{C}(M^2;\tau_{\rm ir}^2,\tau_{\rm uv}^2)\,, \quad \mathcal{K}_{EF} = 0\,,$$
$$2\mathcal{K}_{FE} = \mathcal{C}_1(M^2;\tau_{\rm ir}^2,\tau_{\rm uv}^2)\,, \quad \mathcal{K}_{FF} = -2\mathcal{K}_{FE}$$

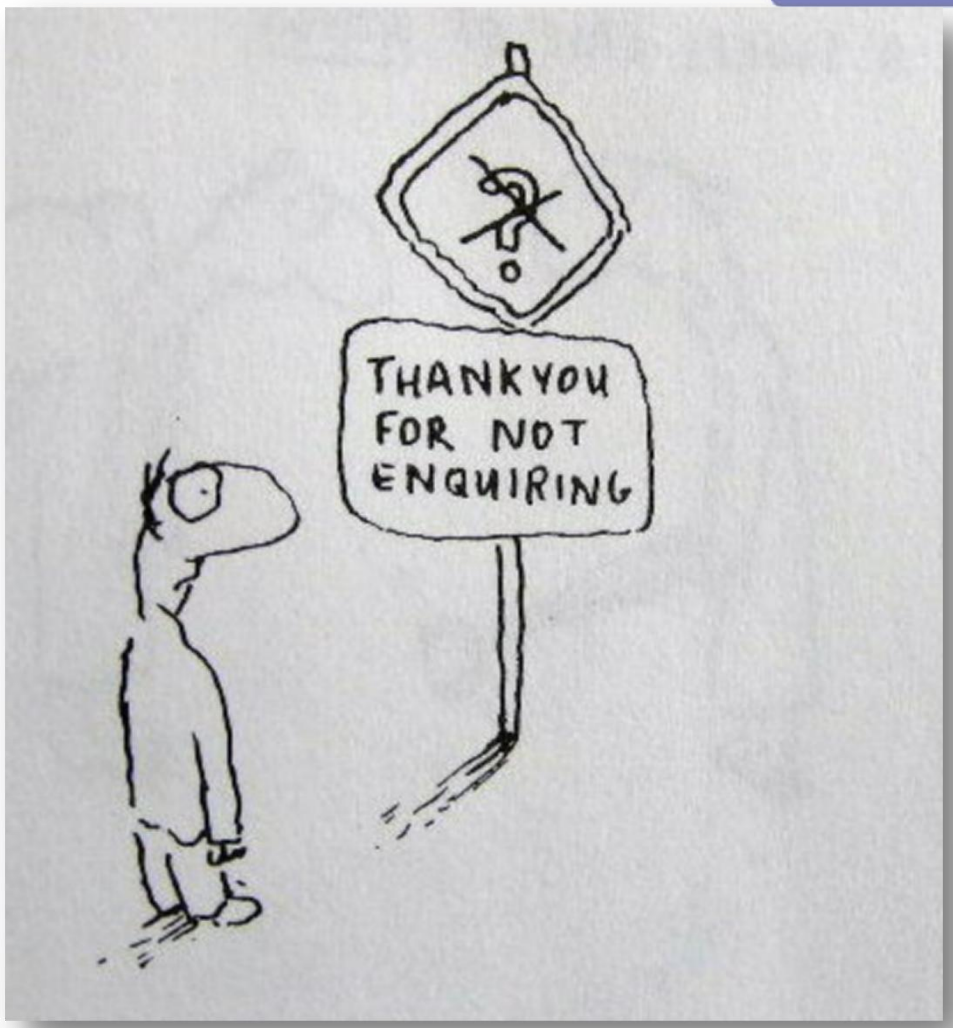
Pseudoscalar Mesons as Bound-States

- BSE … consider $Q^2 =- m_{NG}^2 = 0$ … first line

  $$E_{0^-} = \frac{4\, a_{IR}}{3\Pi m_G^2} C_0(M^2)\, E_{0^-}$$

  But this is exactly the chiral-limit gap equation!
- Plainly:
  - ✓ If ∃ M ≠ 0 solution of the chiral-limit gap equation, then the NG-boson exists because the solution of this equation is $E_{0^-} = M$ and once automatically has a nonzero wave function for he state
  - ✓ Equally if NG-boson exists, *i.e.* $E_{0^-} \neq 0$, then chiral-limit gap equation must support M ≠ 0 solution.
- NG Boson exists if, and only if, chiral symmetry is dynamically broken
  - ✓ No fine tuning involved
  - ✓ $E_{0^-} \propto M$

2-body ⇔ 1-body



Enigma of Mass

Maris, Roberts and Tandy
nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273

Pion's Goldberger -Treiman relation

- Pion's Bethe-Salpeter amplitude
  Solution of the Bethe-Salpeter equation

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\right]$$

- Dressed-quark propagator

$$S(p) = \frac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$$

$$M(p^2) = \frac{B(p^2)}{A(p^2)}$$

- Symmetry demands, via Axial-vector Ward-Takahashi identity:

$$f_\pi E_\pi(k;P=0) = B(k^2)$$

Owing to DCSB & Exact in Chiral QCD

***Miracle*: two body problem solved, almost completely, once solution of one body problem is known**

Rudimentary version of this relation is apparent in Nambu's Nobel Prize work

Model independent
Gauge independent
Scheme independent

$$f_\pi E_\pi(p^2) = B(p^2)$$

The most fundamental expression of Goldstone's Theorem and DCSB

Rudimentary version of this relation is apparent in Nambu's Nobel Prize work

Model independent
Gauge independent
Scheme independent

$$f_\pi E_\pi(p^2) = B(p^2)$$

Pion exists if, and only if, mass emerges dynamically

QCD Fact Pion (Nambu-Goldstone modes) and mass

- Higgs boson couplings → 0
- Pion exists and is massless
- Pion Bethe-Salpeter amplitude

EHM demands equivalence between one-body mass and two-body correlation strength in Nature's most fundamental Nambu-Goldstone bosons

$$f_\pi E_\pi(p^2) = B(p^2)$$

Pion wave function quark mass function

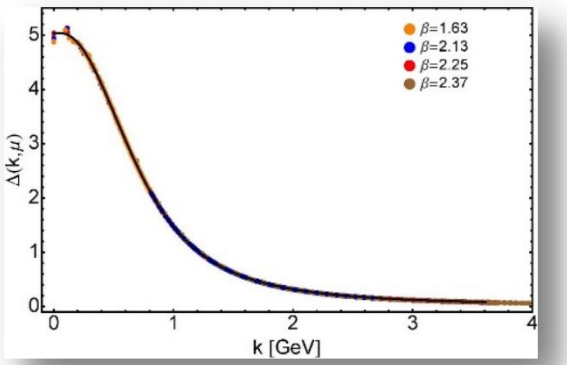
- Thus, enigmatically, properties of the nearly massless pion are the cleanest expression of EHM in the Standard Model !

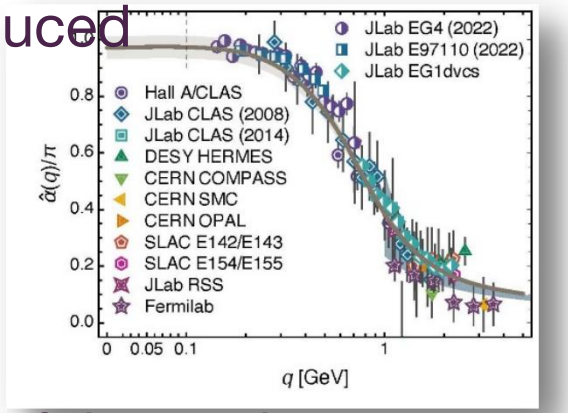
EHM Basics

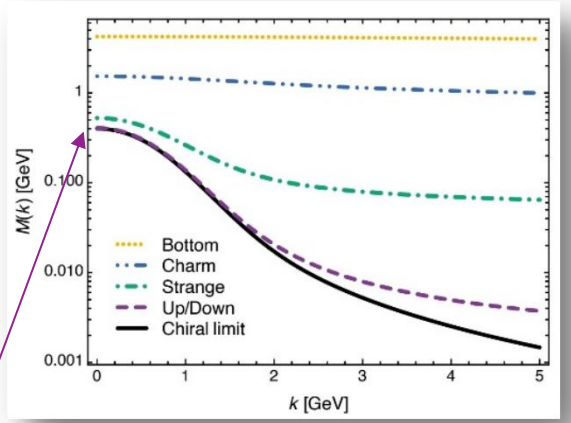
Can't build a Theory of Everything before status of QCD is decided

- Absent Higgs boson couplings, QCD Lagrangian is scale invariant
- Yet …
  - Massless gluons become massive
  - A momentum-dependent charge is produced
  - Massless quarks become massive
- EHM is expressed in
  EVERY strong interaction observable
- Challenge to Theory =
  Elucidate all observable consequences of these phenomena and highlight the paths to measuring them
- Challenge to Experiment =
  Test the theory predictions so that the boundaries of the Standard Model can finally be drawn

Three pillars of EHM







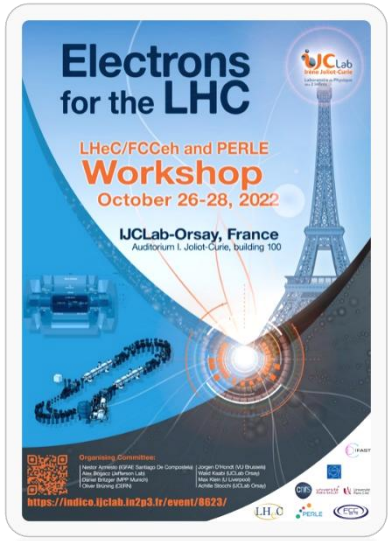
✓ $3 \times M(0)$ sets the scale of the proton mass.

✓ Meson-loops provide 20% quantum corrections

Jefferson Lab
Thomas Jefferson National Accelerator Facility

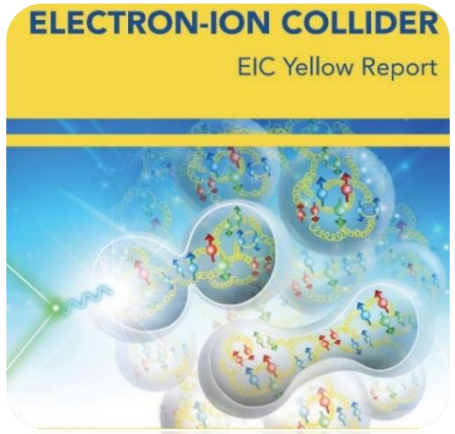
大強度陽子加速器施設
j-PARC
Japan Proton Accelerator Research Complex

CERN
AMBER
A new QCD facility at the M2 beam line of the CERN SPS



BESIII

EIC

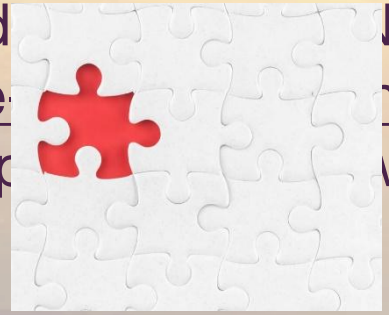


QCD at FAIR

Charting EHM using High Intensity, High Luminosity Facilities

Charting EHM

- Proton was discovered 100 years ago … It is stable; hence, an ideal target in experiments
- But just as studying the hydrogen atom ground state didn't give us QED, focusing on the ground state of only one form of hadron matter will not solve QCD
- New era is dawning … high energy + high luminosity
  ⇒ Science can move beyond the focus on the proton
- Precision studies of the structure of
  - Nature's most fundamental Nambu-Goldstone bosons (π & K) will become possible
  - Baryon excited states
    - ✓Baryons are the most fundamental three-bod ature
    - ✓If we don't understand how QCD, a Poincaré- ntum field theory, builds each of the baryons in the complete sp we don't understand Nature.



AMBER @ CERN
EIC
EicC & SCT &
CEPC
JLab12 &

- EHM is not immutable

Pseudoscalar Meson Electromagnetic Form Factors

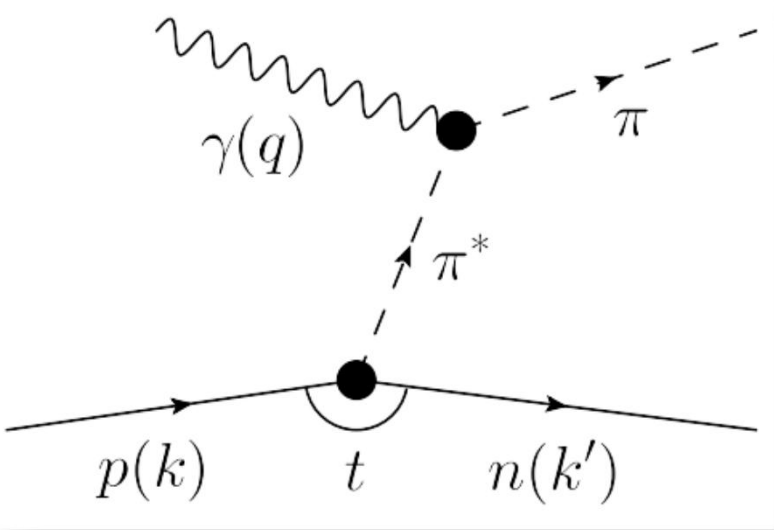
- First precise data (beyond charge radius domain) obtained at JLab and published in 2001

  J. Volmer *et al*., *Measurement of the Charged Pion Electromagnetic Form-Factor,* Phys. Rev. Lett. 86 (2001) 1713-1716

  Obtained using Sullivan Process
  = scattering electrons from pions in the proton's meson cloud

- Today, 9 precise data are available, out to $Q^2 \approx 2.5$ GeV^2
- New results from JLab anticipated within 1 year, pushing coverage to $Q^2 \approx 9$ GeV^2
- High-profile experiments planned and being developed for EIC and EicC

Why?



Hard QCD Prediction

G. P. Lepage, S. J. Brodsky, Phys. Lett. B 87 (1979) 359–365.
A. V. Efremov, A. V. Radyushkin, Phys. Lett. B 94 (1980) 245–250.
G. P. Lepage, S. J. Brodsky, *Exclusive Processes in Perturbative Quantum Chromodynamics*, Phys. Rev. D 22 (1980) 2157–2198.

- Delivered ≈ 45 years ago … positive-charge pseudoscalar meson P, constituted from valence f, $\bar{g}$ quarks:

$$\boxed{\exists\, Q_0 \gg m_p \mid Q^2 F_P(Q^2) \overset{Q^2>Q_0^2}{\approx} 16\pi\alpha_s(Q^2) f_P^2 w_{\varphi_P}^2(Q^2)\,,}$$

$$w_{\varphi_P}^2 = e_f w_{\varphi_{Pf}}^2 + e_{\bar{g}} w_{\varphi_{P\bar{g}}}^2$$

$$w_{\varphi_{Pf}} = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\, \varphi_P(x;Q^2)\,,$$

$$w_{\varphi_{P\bar{g}}} = \frac{1}{3}\int_0^1 dx\, \frac{1}{1-x}\, \varphi_P(x;Q^2)\,,$$

- $\alpha_s(Q^2)$ is QCD running coupling
- f_P is pseudoscalar meson's leptonic decay constant
- $\varphi_P(x;Q^2)$ is meson's leading parton distribution amplitude (DA) – like a quantum field theory wave function – evaluated at the hard scale of the interaction

Hard QCD Prediction

$$\exists\, Q_0 \gg m_p \mid Q^2 F_P(Q^2) \overset{Q^2>Q_0^2}{\approx} 16\pi\alpha_s(Q^2) f_P^2 w_{\varphi_P}^2(Q^2)\,,$$

- Hard QCD prediction is remarkable.
  - beyond some momentum scale, the meson form factor is precisely known
    - magnitude (normalisation) is set by meson leptonic decay constant, which is an order parameter for dynamical chiral symmetry breaking (DCSB) – a corollary of EHM
    - and scaling violations are apparent, with Q^2 dependence determined by that of the running coupling and evolution of the DA (wave function)
  - Analogous proton equation – magnitude (normalization) is not known
- On domain $m_p^2/Q^2 \simeq 0 \Rightarrow \varphi_{P=\pi,K}(x) = 6\,x(1-x)$ and right-hand side $\Rightarrow$ $16\pi\alpha_s(Q^2) f_P^2$
- Two outstanding issues:
  - what is value of Q_0 for which scaling violations become apparent?
  - and what is pointwise form of $\varphi_P(x;Q^2)$ at scales for which terrestrial experiments are

Pseudoscalar Meson Electromagnetic Form Factors

- $(p_f + p_i)F_P(Q^2) = e\langle P(p_f)|\bar{\psi}\gamma_\mu\psi|P(p_i)\rangle$

  Hadron part of $e^- M \to e^- M$ matrix element defines meson form factor
- Leading order in systematically improvable, symmetry preserving truncation of QCD equations of motion (Dyson-Schwinger equations)
- Calculate the displayed matrix element using
  - ✓ 2-point Schwinger functions (propagators) and effective charge describe above
  - ✓ Poincaré-covariant Bethe-Salpeter equation to deliver
    - ✓ meson wave function
    - ✓ photon-quark vertex

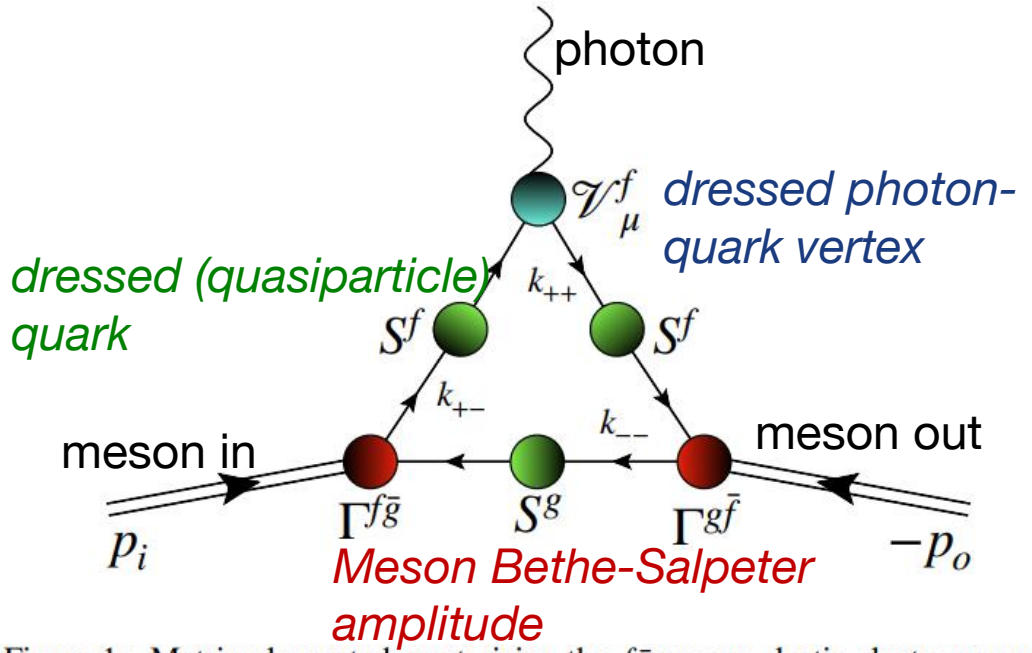
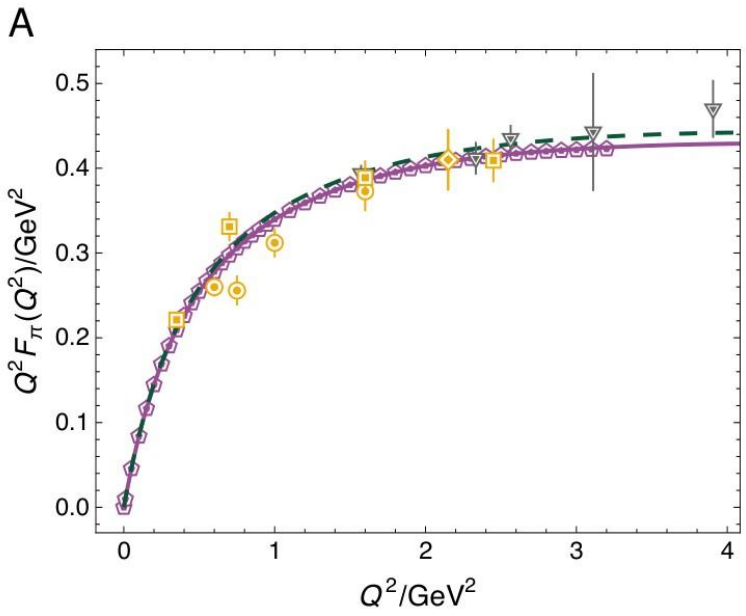


Figure 1: Matrix element characterising the $f\bar{g}$ meson elastic electromagnetic form factor in rainbow-ladder (RL) truncation. Legend. $\Gamma^{f\bar{g}}$ – Bethe-Salpeter amplitude for the $f\bar{g}$ meson; $S^{f,g}$ – dressed quark propagators; $\mathcal{V}_\mu^{f,\bar{g}}$ – dressed photon–$f,\bar{g}$-quark vertex. All quantities calculated in RL truncation.

Pion Form Factor



- Low Q^2: CSM predictions for $Q^2 F_\pi(Q^2)$ – purple curve
  - Charge radius agrees with experiment: 0.67 fm
  - Q^2dependence matches that JLab precision measurements [Horn:2007ug, Huber:2008id] (gold symbols)
- $Q^2 > 4\mathrm{GeV}^2$
  - *cf*. phenomenological – much loved, much used – vector meson dominance (VMD) result = dot-dashed red
    - VMD curve begins to deviate significantly from data and CSM prediction at upper bound of available JLab measurements
    - Best available lQCD (grey points) – pioneering work, pushing to high Q^2 – Heng-Tong Ding, *et al*., Phys. Rev. Lett. **133** (2024) 181902

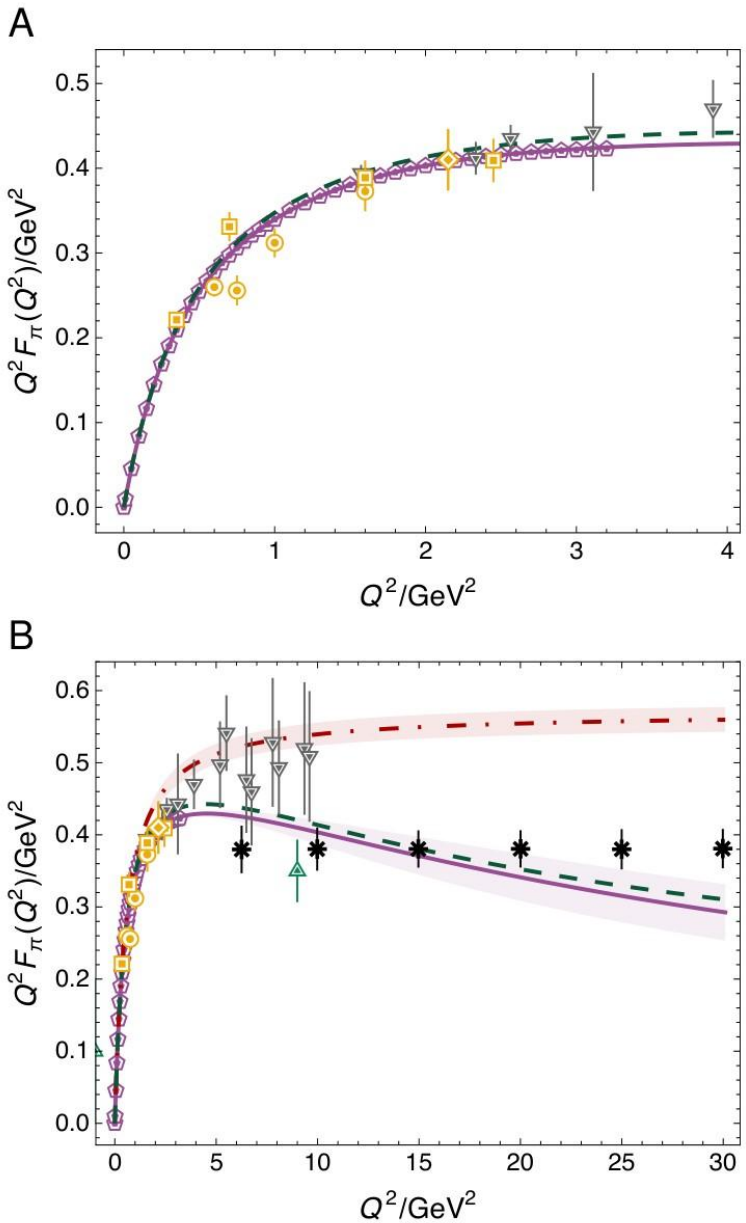
Pion Form Factor

- Salient feature of CSM predictions is existence of maximum in $Q^2 F_\pi(Q^2)$, which occurs on $Q^2 \simeq 4.6(5)\,\mathrm{GeV}^2$
- Thereafter, scaling violation apparent,

  $Q^2 F_\pi(Q^2)$ falls steadily toward zero as

  $\approx (1/\ln Q^2)^{\gamma_F}$, $\gamma_F \approx 1.1$

  consistent with hard QCD prediction
- *N.B*. Any model that expresses VMD must exhibit opposite trend, *viz*. result for $Q^2 F_\pi(Q^2)$ which rises steadily with increasing Q^2, without inflection, toward finite ultraviolet limit.
- Future: lattice-QCD reach and precision are improved to (potentially) see breakaway from VMD and scaling violation
- green point farthest reach of anticipated JLab data
  - JLab is potentially capable of seeing breakaway from



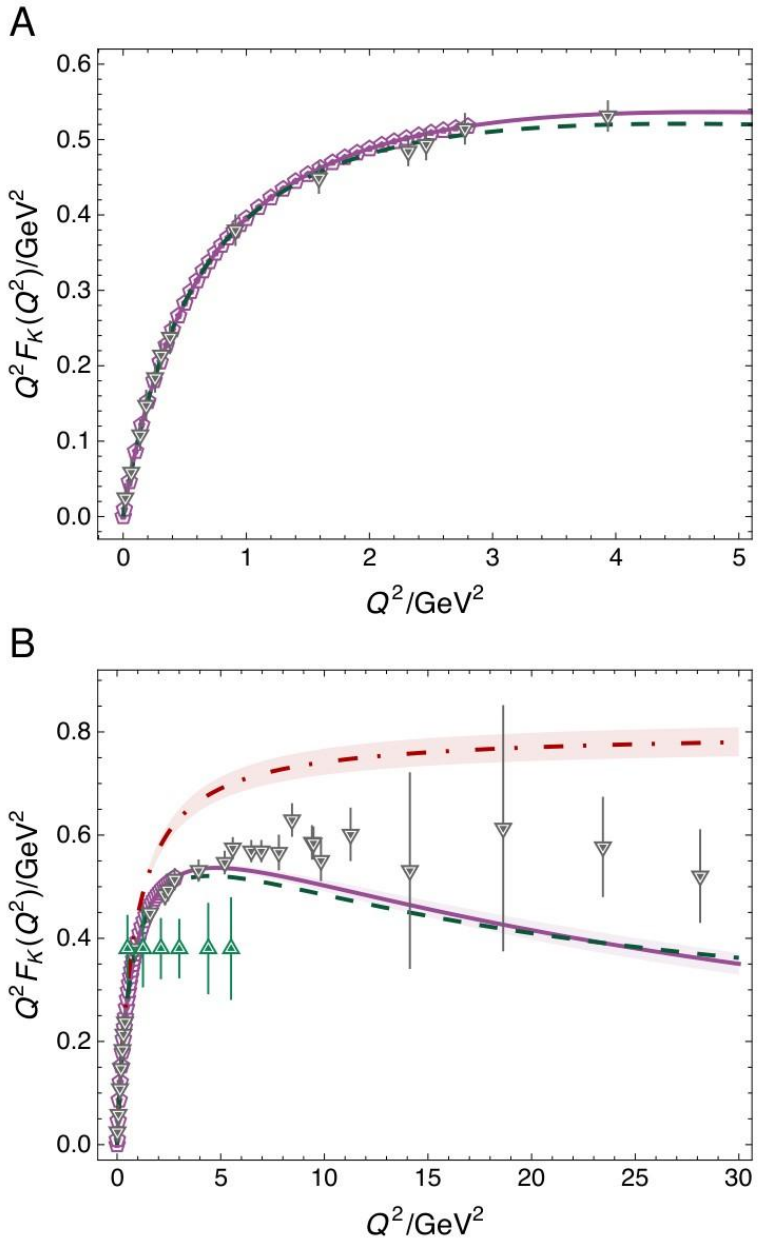
Charged kaon form factor

- CSM predictions for $Q^2 F_K(Q^2)$ – purple curve
  - Precise $F_K(Q^2)$ data are lacking, so no comparison with experiment is made
  - Predicted charge radius is commensurate with available empirical estimates: $r_K = 0.6$fm
- $Q^2 F_K(Q^2)$ exhibits maximum on $Q^2 \simeq 4.8(5)\,\mathrm{GeV}^2$

  $Q^2 F_K(Q^2)$ falls steadily toward zero as

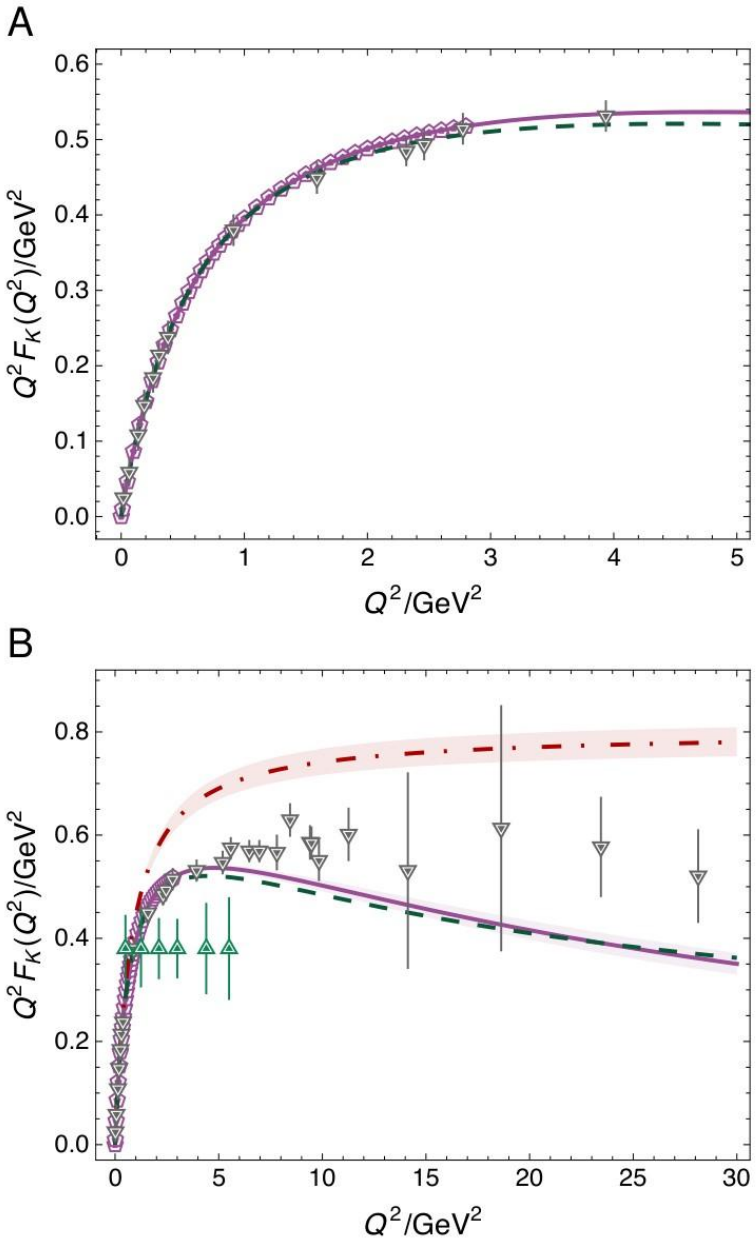
  $\approx (1/\ln Q^2)^{\gamma_F}$, $\gamma_F \approx 1.1$

  consistent with hard QCD prediction



Charged kaon form factor

- VMD result = dot-dashed red
  - Comparing $F_{\pi,K}$, notable that, with increasing mass of the meson involved, position at which VMD curve separates from CSM prediction moves to smaller Q^2
  - Considering anticipated Q^2 reach (green points), JLab experiments [E12-09-011] may be expected to confirm this breakaway
- On the other hand, discovery of scaling violation in charged kaon form factor may require a machine with EIC capabilities.
  - Analyses suggest that EicC may be better tuned to this task (EicC CDR, in preparation)
- Pioneering lattice-QCD results (Heng-Tong) are not yet precise enough to see scaling violation in $F_K(Q^2)$: on $Q^2 > 5\ \mathrm{GeV}^2$, lQCD results are consistent with $Q^2 F_K(Q^2) =$ constant



Summary $F_\pi(Q^2)$ & $F_K(Q^2)$

Onset of scaling violation in pion and kaon elastic electromagnetic form factors
Zhao-Qian Yao (姚照千), Daniele Binosi and Craig D. Roberts, NJU-INP 087/24
e-Print: 2405.04681 [hep-ph], Phys. Lett. B 855 (2024) 138823/1-7

- Predictions also for flavour separation and neutral kaon form factors
  - Possibly accessible at EIC and EicC
- CSM analysis
  - $Q^2 F_P(Q^2)$ has maximum at $Q^2 \approx 5\text{GeV}^2$
  - So …
    - proposed experiments are capable of locating the peak
    - can, potentially, provide quantitative information on value of γ_F
- Crucial steps toward ultimate verification of hard QCD prediction

  (perhaps seen in $\gamma^*\pi \to \gamma$, but this is controversial – Phys. Rev. D**93** (2016) 074017/1-9)
  - Connecting hard scattering of leptons from mesons to expressions of emergent hadron mass, via QCD effective charge and meson leptonic decay constant

Hadron Gravitational Form Factors

- *Forces inside hadrons: pressure, surface tension, mechanical radius, and all that*, M. V. Polyakov, P. Schweitzer, Int. J. Mod. Phys. A 33 (2018) 26

  "The physics related to the form factors of the energy momentum tensor spans a wide spectrum of problems, and includes

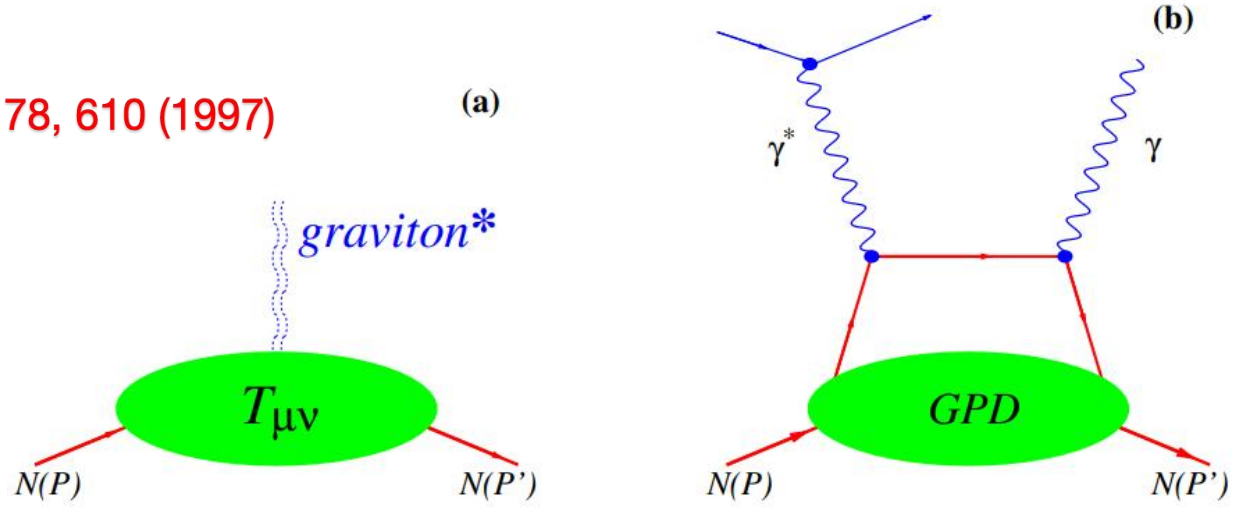
  - ✓ gravitational physics,
  - ✓ hard exclusive reactions,
  - ✓ hadronic decays of heavy quarkonia,
  - ✓ and the physics of exotic hadrons described as hadroquarkonia.

  It also provides access to the "last global unknown property:" the D-term.

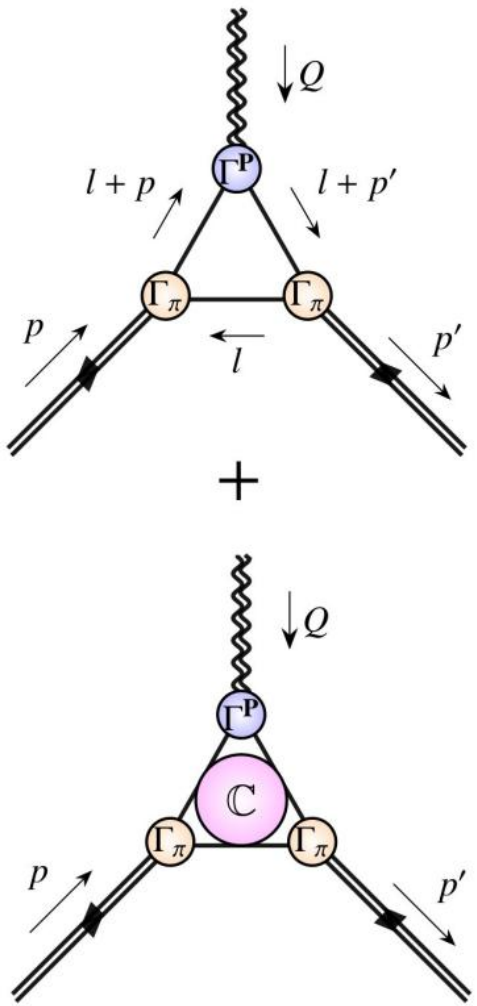
  We review the physics associated with the form factors of the energy-momentum tensor and the D-term, their interpretations in terms of mechanical properties, their applications, and the current experimental status."

Hadron Gravitational Form Factors

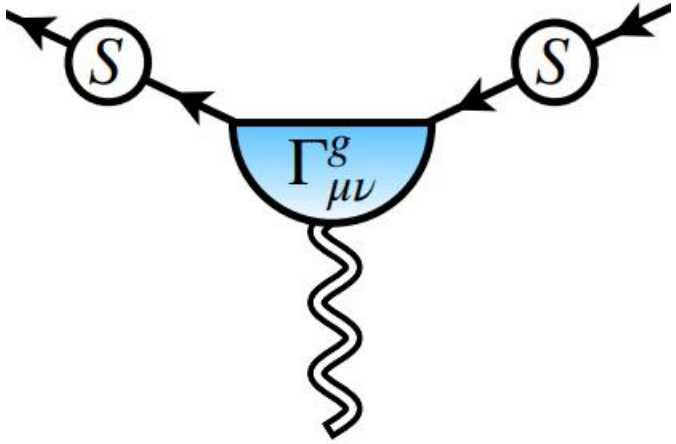
- *Forces inside hadrons: pressure, surface tension, mechanical radius, and all that*, M. V. Polyakov, P. Schweitzer, Int. J. Mod. Phys. A 33 (2018) 26
  "The most natural way to probe EMT form factors, scattering off gravitons – see left image – is also the least practical one.
  A practical opportunity to access EMT form factors emerged with the advent of generalized parton distributions (GPDs), which describe hard-exclusive reactions, such as deeply virtual Compton scattering – see right image."
- This is because the in-hadron expectation value of the QCD energy momentum tensor … $\langle H|T_{\mu\nu}|H\rangle$ X. D. Ji, Phys. Rev. Lett. 78, 610 (1997)
  appears in the operator product expansion of the in-hadron matrix element of two vector currents … $\langle H|J_\mu J_\nu|H\rangle$
- To use OPE, one of the photons, γ^*, must be far off-shell (large Q^2) = deeply virtual Compton Scattering



Pion and kaon gravitational form factors



- Precisely same framework as used for electromagnetic form factors of these mesons
  - Prediction and Unification
- Need graviton-quark vertex … made available by analysis in *Pion and kaon electromagnetic and gravitational form factors*, Yin-Zhen Xu (徐胤禛) *et al.*, e-Print: 2311.14832 [hep-ph], Eur. Phys. J. C 84 (2024) 2, 191/1-13
- In-meson expectation value of EM tensor –



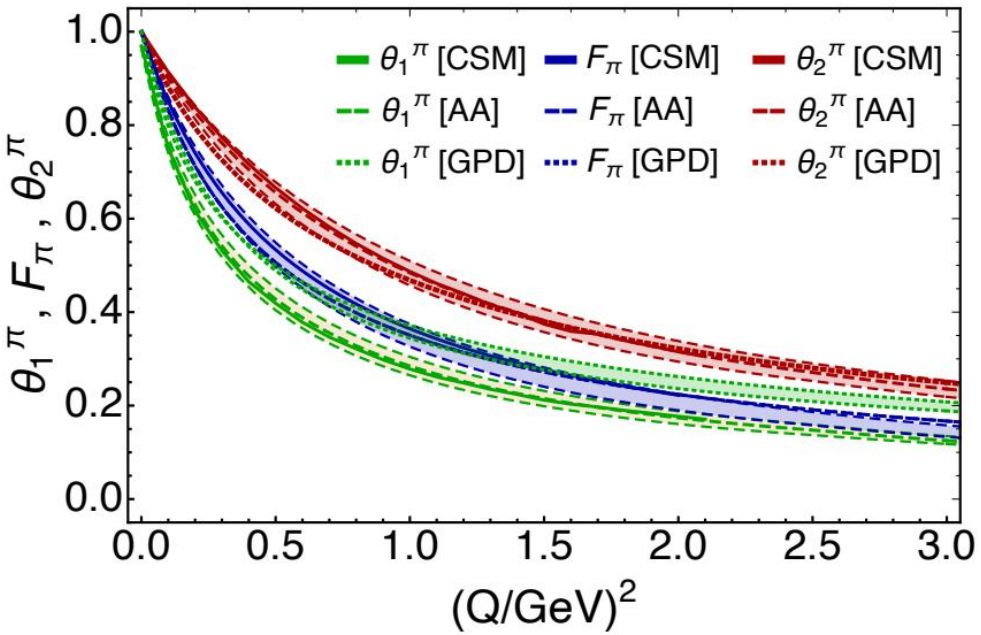
mass

$$\Lambda^g_{\mu\nu}(P,Q) = 2P_\mu P_\nu \theta_2^\pi(Q^2) + \tfrac{1}{2}[Q^2\delta_{\mu\nu} - Q_\mu Q_\nu]\theta_1^\pi(Q^2) + 2m_\pi^2 \delta_{\mu\nu}\bar{c}^\pi(Q^2)$$

pressure

Vanishes owing to energy-momentum conservation: $Q_\mu \Lambda^g_{\mu\nu}(P,Q) \equiv 0$

Meson Gravitational Form Factors



- ✓ $\theta_1(Q^2)$ is the pressure form factor
- ✓ Provides access to pressure and shear-force distributions within the meson
- ✓ Meson core pressures are more than an order of magnitude greater than those in neutron stars.

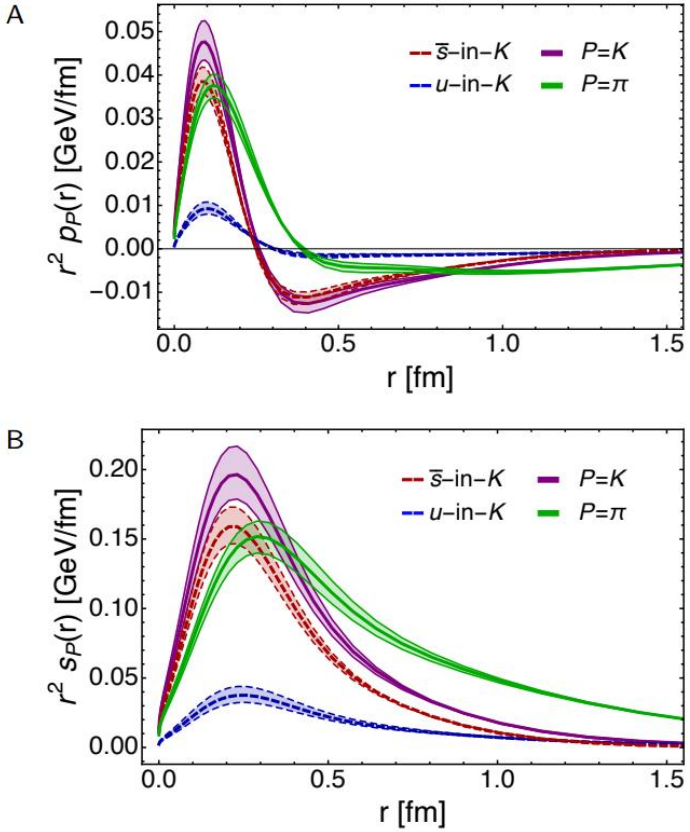


Fig. 4 Pressure (*Panel A*) and shear (*Panel B*) distributions obtained using Eqs. (28) as explained in Sec. 8. Legend. π – solid green curves; K – solid purple curves; $\bar{s}$ in K – dashed red curves; and u in K – dashed blue curves.

- ➢ Calculations reveal that each meson's mass radius is smaller than its charge radius, matching available empirical inferences
  - *Empirical Determination of the Pion Mass Distribution*, Y-Z. Xu (徐胤禛) et al., e-Print: 2302.07361 [hep-ph], Chin. Phys. Lett. *Express* **40** (2023) 041201/1-7. Express Letter.

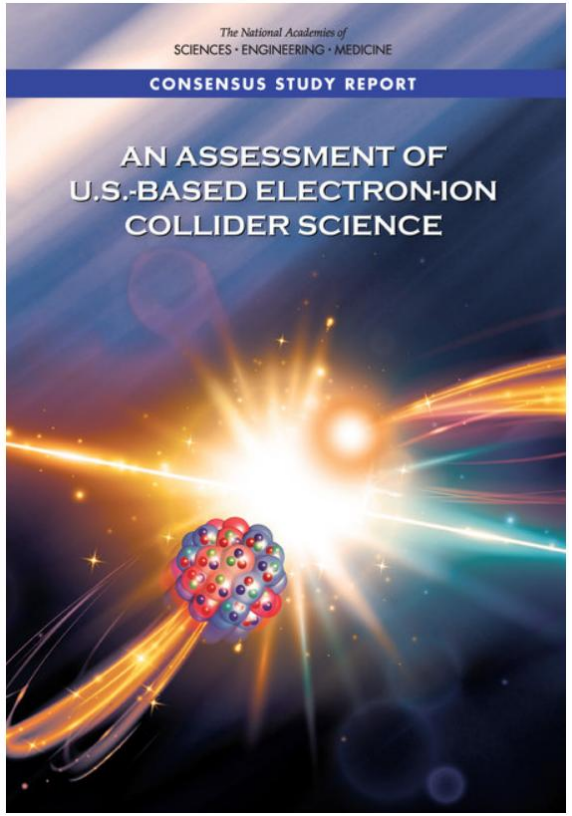
- ✓ Bound states; so, pressure integrates to zero (von Laue, 1911)
- ✓ Zero in pressure near maximum in shear force
- ✓ Kaon more compact; so, pressure higher

Electron Ion Colliders

Hearing from experts on the science that an EIC would be able to carry out, the committee finds that

Finding 1: An EIC can uniquely address three profound questions about nucleons—neutrons and protons—and how they are assembled to form the nuclei of atoms:

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- What are the emergent properties of dense systems of gluons?



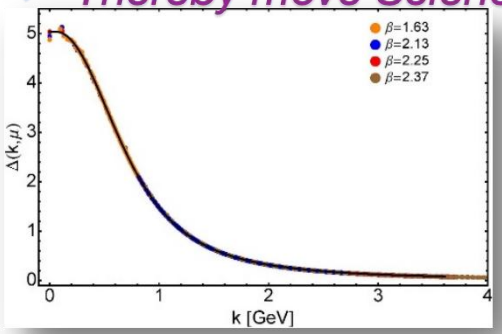
- Now available … first unified set of empirically benchmarked, parameter-free predictions for pion, kaon, proton, neutron elastic electromagnetic and gravitational form factors and their species decompositions with direct connections to three pillars of emergent hadron mass
- Also … *ab initio* predictions for pion and kaon structure functions
- Next step … nucleon structure function predictions with equal QCD fidelity

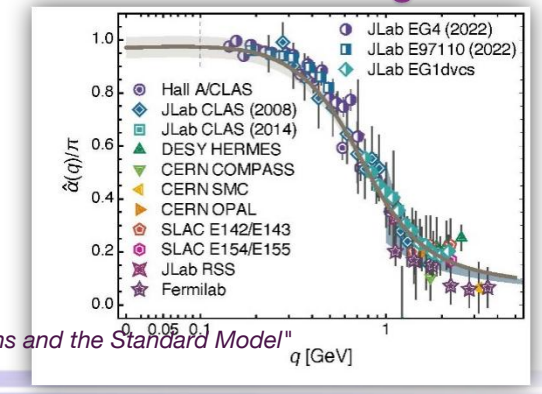
Gather all pieces of the puzzle … Reveal the source of Nature's basic mass-scale

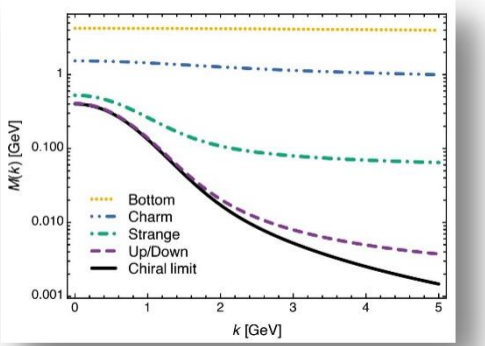
Synergy of Experiment, Phenomenology, Theory

- Continuum strong interaction theory is maturing
  - ✓ Expanding array of parameter-free predictions for the nucleon – yes
  - ✓ And all the other hadrons whose properties express the full meaning of QCD
    - Structure of Nature's most fundamental Nambu-Goldstone bosons
    - Structure of baryons, e.g.,

      Q^2-dependence of nucleon resonance transition form factors
      - ✓ Spectrum is insufficient – many models give same spectrum, but transition form factors discriminate between pictures.
- Understanding how QCD's simplicity explains the emergence of hadron mass and structure requires investment in facilities that can deliver precision data on much more than one of Nature's hadrons.
- AMBER@CERN, EIC, EicC, STCF, CEPC, JLab22, LHeC, QCD@FAIR could …
  - ✓ Deliver precise structure data on a wide range of hadrons with distinctly different quantum numbers
  - ✓ Thereby move Science into a new realm of understanding.







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国家自然科学基金委员会
National Natural Science Foundation of China

Emergent Hadron Mass

- QCD is unique amongst known fundamental theories of natural phenomena
  - The degrees-of-freedom used to express the scale-free Lagrangian are not directly observable
  - Massless gauge bosons become massive, with no “human” interference
  - Gluon mass ensures a stable, infrared completion of the theory through the appearance of a running coupling that saturates at infrared momenta, being everywhere finite
  - Massless fermions become massive, producing
    - Massive baryons and simultaneously Massless mesons
- These emergent features of QCD are expressed in every strong interaction observable
- They can also be revealed via

  EHM interference with Nature’s other known source of mass = Higgs
- We are capable of building facilities that can validate these concepts, proving QCD to be

  the 1st well defined four dimensional quantum field theory ever contemplated
- This may open doors that lead far beyond the Standard Model

Can't build a Theory of Everything before status of QCD is decided

$\mathcal{L}_{\text{Nature}} = ?$

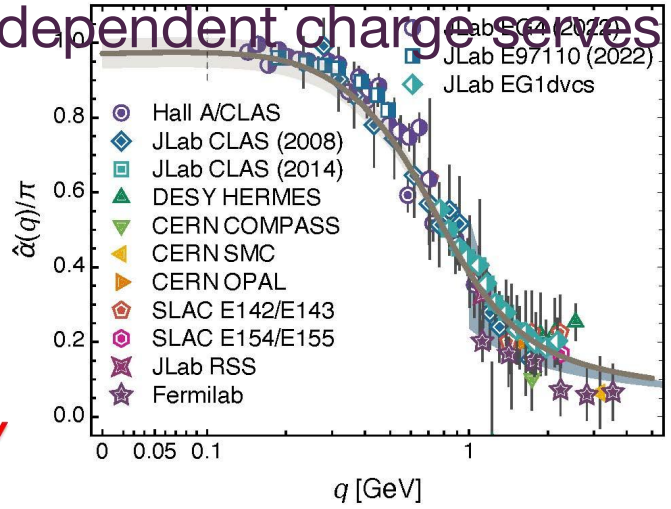
There are theories of many things,

But is there a theory of everything?

Thankyou

All-orders evolution

- **P1** – *In the context of Refs. [73, 74], there exists at least one effective charge, $\alpha_{1\ell}(k^2)$, which, when used to integrate the leading-order perturbative DGLAP equations, defines an evolution scheme for parton DFs that is all-orders exact*.
- CSM Process-Independent charge serves this purpose



- *Hadron scale, ζ_H*

  Scale at which all properties of a given hadron are carried by valence degrees-of-freedom

ELSEVIER
Physics Letters B
Volume 95, Issue 1, 8 September 1980, Pages 70-74

Renormalization group improved perturbative QCD

G. Grunberg [1]

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Abstract

The results of perturbative QCD calculations are reformulated as renormalization-scheme independent predictions; in so doing, we obtain a renormalization group improvement of perturbation theory. As an application, we show that asymptotic freedom alone does not give the correct quantitative relation between pseudoscalar charmonium decay and the scaling violations in deep inelastic scattering.

[73] G. Grunberg, Renormalization Group Improved Perturbative QCD, Phys. Lett. B 95 (1980) 70, [Erratum: Phys. Lett. B 110, 501 (1982)].
[74] G. Grunberg, Renormalization Scheme Independent QCD and QED: The Method of Effective Charges, Phys. Rev. D 29 (1984) 2315.
[75] A. Deur, S. J. Brodsky, G. F. de Teramond, The QCD Running Coupling, Prog. Part. Nucl. Phys. 90 (2016) 1–74.
[76] A. Deur, V. Burkert, J. P. Chen, W. Korsch, Experimental determination of the QCD effective charge $\alpha_{g_1}(Q)$, Particles 5 (2) (2022) 171–179.
[77] A. Deur, S. J. Brodsky, C. D. Roberts, QCD Running Couplings and Effective Charges – arXiv:2303.00723 [hep-ph] .

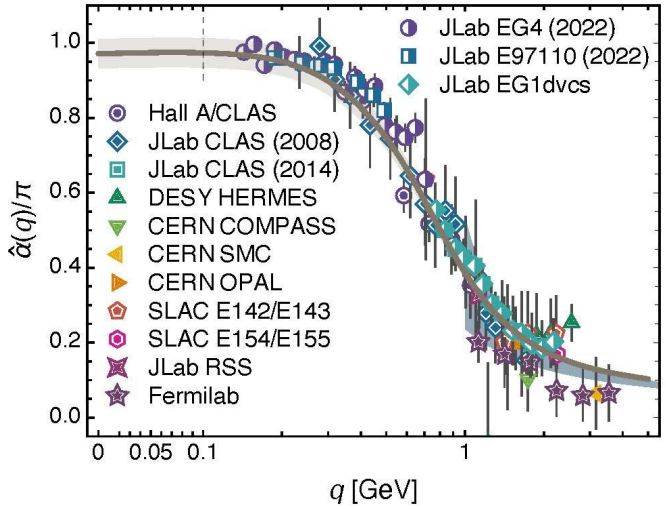
AO Evolution

- Principle
  - ✓ Hadron scale, ζ_H, is rigorously defined in connection with a Grunberg charge, in terms of which leading-order DGLAP evolution is promoted to the status of an all-orders exact procedure.
  - ✓ The AO scheme is not a model, and neither is ζ_H.
  - ✓ Instead, the AO approach, involving and defining ζ_H, is complete by definition.
- As explained by Grunberg, there is always an effective (process-dependent) effective charge for which the AO propositions are true.
  - ✓ Consequently, there is no room for doubt about the existence of ζ_H.
- Crucially, such a Grunberg effective charge is
  - ✓ consistent with the renormalization group;
  - ✓ renormalization scheme independent;
  - ✓ everywhere analytic and finite;
  - ✓ and supplies an infrared completion of any standard running coupling.

AO Evolution

- Crucially, such a Grunberg effective charge is
  - ✓ consistent with the renormalization group;
  - ✓ renormalization scheme independent;
  - ✓ everywhere analytic and finite;
  - ✓ and supplies an infrared completion of any standard running coupling.
- These features entail that all calculations of a DF at a scale within a domain of pQCD applicability are compatible with this effective charge.
- Moreover, and especially important, is the fact that one does not need to know the pointwise form of the effective charge
  - ✓ Its existence alone is sufficient to deliver the definition of ζ_H and all attendant consequences
  - ✓ *N.B.* glue and sea DFs are identically zero at ζ_H

AO Evolution



- The potential issue with a Grunberg effective charge is that reference to a specific process may make it globally us
- We know that is not the case
  - The class of process-dependent (Grunberg) charges, defined in connection with the AO scheme, may be characterized by our unique, predicted process-independent effective charge
- Thus far, the AO class of charges – in particular, our PI charge – has served in the same form for all known parton DFs
  - ✓ Namely, for a large array of structure functions and fragmentation processes.
- ✓ This is a very broad domain of applicability for one single Grunberg charge.

KeAi
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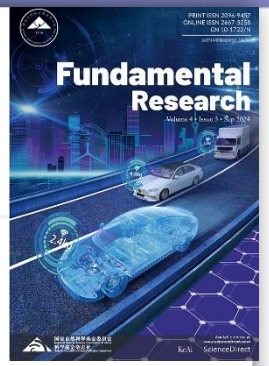
GLOBAL IMPACT

Fundamental Research
Available online 15 November 2024
In Press, Journal Pre-proof What's this?



Article

Nucleon charge and magnetisation distributions: flavour separation and zeroes

Zhao-Qian Yao [1 a b c], Daniele Binosi [1 c], Zhu-Fang Cui [1 a b], Craig D. Roberts [1 a b]

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Abstract

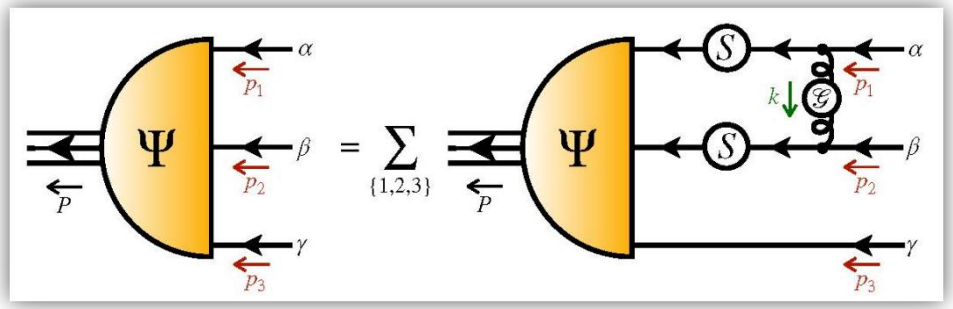
A symmetry-preserving truncation of the quantum field equations describing hadron properties is used to deliver parameter-free predictions for all nucleon elastic electromagnetic form factors and their flavour separation to large values of momentum transfer, Q^2. The proton electric form factor, G_E^p, possesses a zero, whereas that of the neutron, G_E^n, does not. The difference owes to the behaviour of the Pauli form factor of the proton's singly-represented valence d-quark. Consequently, $G_E^n > G_E^p$ on a material large-Q^2 domain. These predictions can be tested in modern experiments.

Aims & Scope

Fundamental Research is an open access, peer-reviewed, multidisciplinary journal, which is supervised by the National Natural Science Foundation of China (NSFC). Published bimonthly, it features high-calibre research covering all areas of the natural sciences and high-tech fields.
IF = 5.7

Nucleon as a Bound State

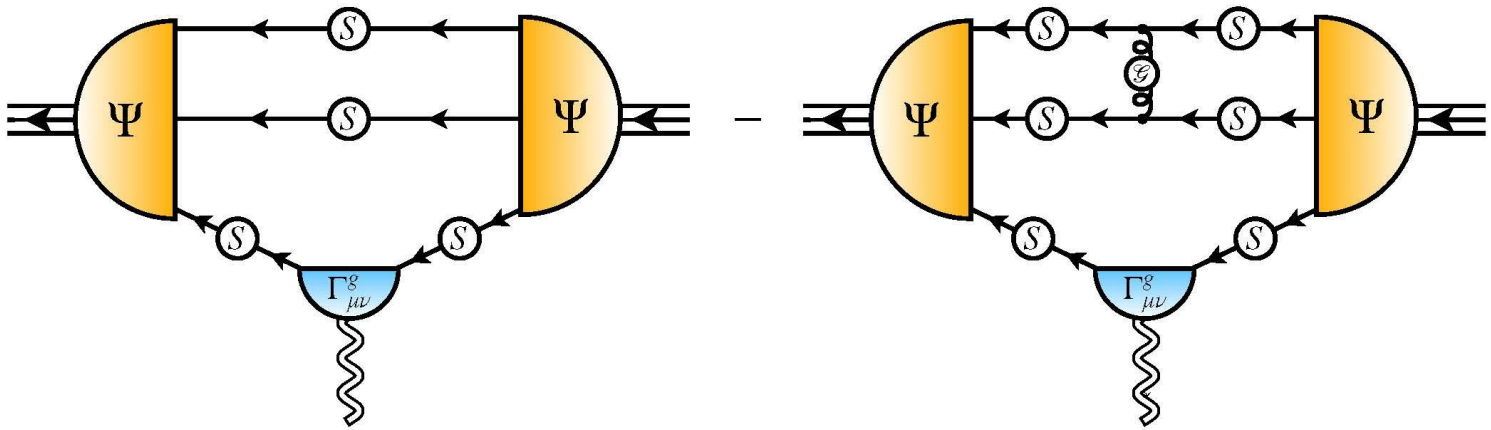
Nucleon charge and magnetisation distributions: flavour separation and zeroes, Zhao-Qian Yao (姚照千), e-Print: 2403.08088 [hep-ph], *Fund. Res. (2025) in press.*



- Nucleon bound-state problem can be addressed in any approach that provides access to the three-quark six-point Schwinger function
  - Using CSMs, one begins with the Poincaré-invariant Faddeev equation
  - Today, there are 3 people in the world who can solve this problem
    - Only one (姚照千) can handle systems with non-degenerate valence quark flavours
- The analysis to be described used precisely the same approach (approximation and interaction) as that employed for pion and kaon electromagnetic form factors
  - Thus, one arrives at unifying set of parameter-free predictions for all pion and kaon and nucleon elastic electromagnetic and gravitational form factors and their species separations to large values of momentum transfer
  - All predictions link observables directly with fundamental QCD Schwinger functions and thereby expose novel expressions of emergent phenomena in QCD

Nucleon Gravitational Form Factors

$[\Lambda^{(3)}_{n'n}]^{\delta'\delta}_{\mu\nu} =$



- Current … completing solution of Faddeev equation, then all elements are known
- Current …

$$m_N \Lambda^{Ng}_{\mu\nu}(Q) = -\Lambda_+(p_f)[K_\mu K_\nu A(Q^2) + iK_{\{\mu}\sigma_{\nu\}\rho}Q_\rho J(Q^2) + \tfrac{1}{4}(Q_\mu Q_\nu - \delta_{\mu\nu}Q^2)D(Q^2)]\Lambda_+(p_i)\,,$$

- $A(Q^2)$ mass distribution form factor

  $A(Q^2 = 0) = 1$
- $J(Q^2)$ spin distribution form factor

  $J(Q^2 = 0) = 1/2$
- $D(Q^2)$ pressure distribution form factor

  $D(Q^2 = 0) = ?$

Last unknown/unmeasured global property of nucleon

Predicted GFFs

- Solid curves … CSM predicitions
- Points lQCD results [Hackett:2023rif]
  - Agreement within (large) lattice uncertainties
- Symmetry-preserving character of CSM analysis is evident in values of
  - $A(0) = 1$
  - $J(0) = 1/2$
- With $J(0) = 1/2$, confirm that the anomalous gravitomagnetic moment of a spin-1/2 system is zero [Kobzarev:1962wt, Teryaev:1999su, Brodsky:2000ii].
- nucleon "D-term" … prediction $D(0) = -3.11(1)$
  - analogue for pion is symmetry constrained $D_\pi(0) \approx -1$

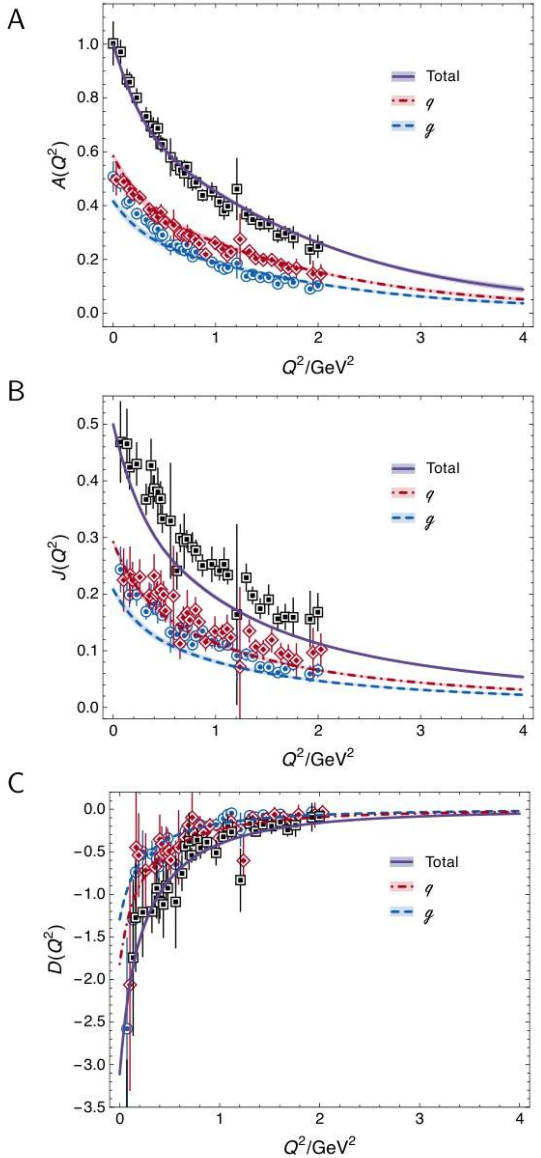
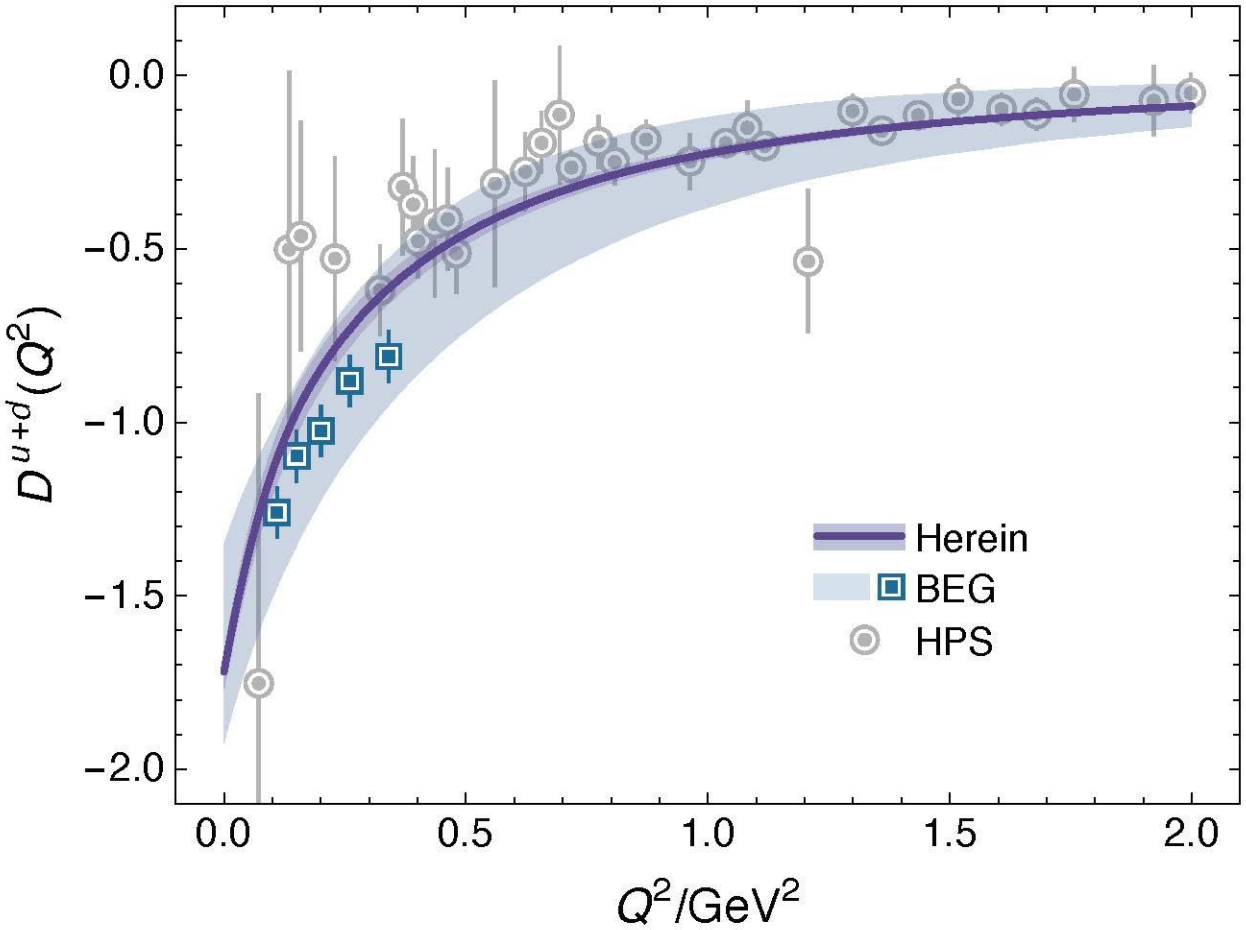


FIG. 1. Nucleon gravitational form factors. Curves – predictions herein: bracketing bands mark the extent of 1σ SPM uncertainty. In each case, the overall (species-summed) result is independent of resolving scale, ζ. The species decompositions evolve with ζ. The lQCD points in each panel are reproduced from Ref. [25]: black squares – total form factor; blue circles – glue component; red diamonds – quark.

GFF Species Decomposition (scale dependent, $\zeta_2 = 2\text{GeV}$)

- Considering light quarks alone:
  $$D^{u+d}(0;\zeta_2) = -1.73(5)$$
- Inference from available DVCS data
  $$D^{u+d}_{\text{DVCS}}(0;\zeta_2) = -1.63(29)$$
- CSM prediction is also in pointwise agreement with curve inferred from that data … see figure



BEG: Inference from existing DVCS data
V. D. Burkert, L. Elouadrhiri, F.-X. Girod, *The pressure distribution inside the proton*, Nature 557 (7705) (2018) pp. 396–399.

GFF Species Decomposition (scale dependent, $\zeta_2 = 2\text{GeV}$)

- CSM prediction – species decomposition, for each form factor:

$$\frac{F^{\text{glue}}(Q^2;\zeta)}{F^{\text{quark}}(Q^2;\zeta)} = \text{constant}(\zeta)$$

- $\text{constant}(\zeta_2) = 0.71(4)$

namely, the ratio of glue:quark contributions is the same

Q^2-independent number for each form factor

Important to test this prediction … *entails that no species decomposition contains new information: everything that can be known is already contained in the scale-invariant observable form factors*

- lQCD results … see figure
  - Consistent with prediction
  - $\text{constant}_{\text{lQCD}}(\zeta_2) = 0.82(18)$

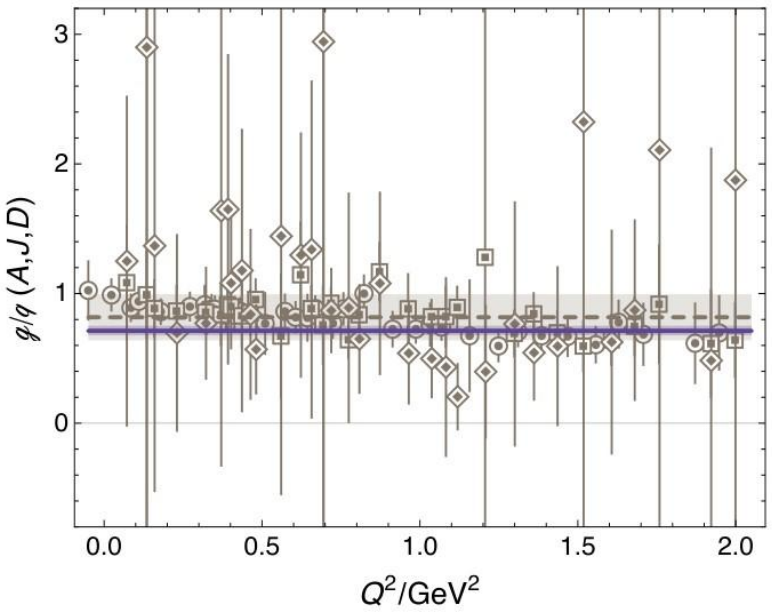


FIG. S.8. Combined A, J, D lQCD g/q results (grey points): grey line – uncertainty weighted average of all points; and grey band – 1σ around the central value: 0.82(18). Prediction herein: $g/q = 0.71(4)$ (purple line and band).

Proton Pressure Profiles

- Pressure and shear-force density profiles: determined by $D(Q^2)$ for proton and $\theta_1(Q^2)$ for pion
- Pion (green) peak values are roughly twice those in the proton
  - Expected because
    - pressures $\sim \Psi^2$
    - similar interior QCD dynamics + more compact object

      $\Rightarrow \frac{p_\pi}{r_\pi^2} \approx p_p / r_p^2 \;\;\&\;\; r_p^2 \approx 2\, r_\pi^2$
- Scale is order-of-magnitude greater than neutron star core
- $r_{\text{mass}} = 0.81(5)\, r_{\text{ch}} \;>\; r_{\text{mech}} = 0.72(2)\, r_{\text{ch}}$
- Species decomposition (ζ_2)
  - $r^q_{\text{mass}} = 0.62(4)\, r_{\text{ch}} \;>\; r^g_{\text{mass}} = 0.52(3)\, r_{\text{ch}}$
  - $r^q_{\text{mech}} = 0.55(4)\, r_{\text{ch}} \;>\; r^g_{\text{mech}} = 0.47(3)\, r_{\text{ch}}$

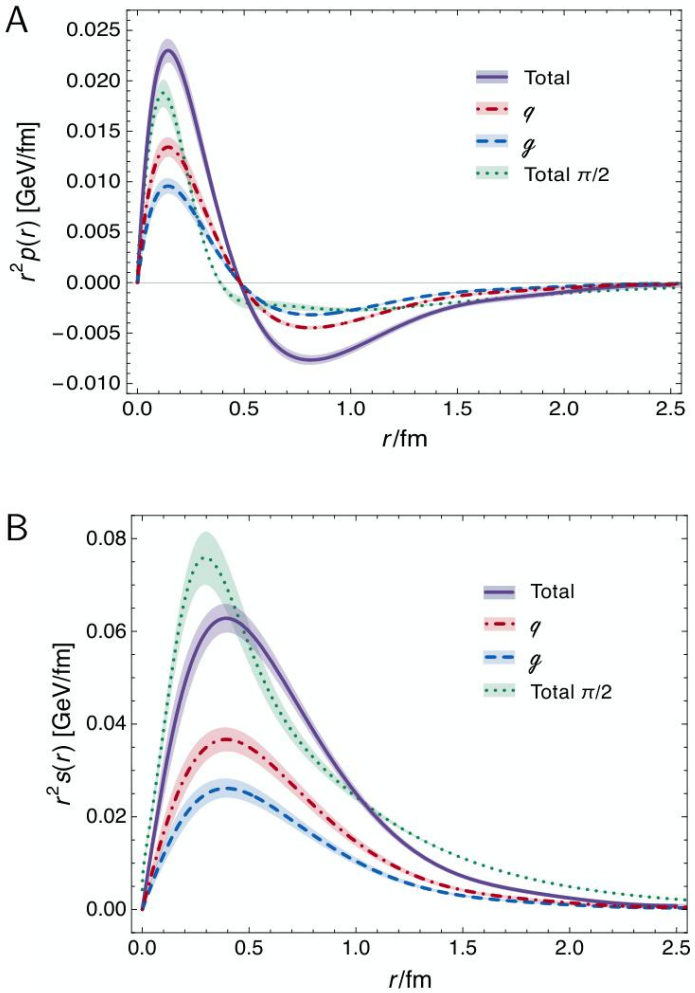


FIG. 3. Nucleon pressure, $p(r)$, and shear force, $s(r)$, distributions, along with total-quark and glue species decompositions at ζ_2. Like pion distributions are drawn in green. *N.B.* The pion distributions are divided by 2. In all cases, the like-coloured band marks the extent of 1σ SPM uncertainty.

J/ψ photoproduction: threshold to very high energy, Lin Tang (唐淋) *et al.*, e–Print: 2405.17675 [hep–ph], Phys. Lett. B **856** (2024) 138904/1–7

Gravitational Form Factors
Mass, Spin, Pressure Distributions

$$\theta(Q^2) = A(Q^2) + \frac{Q^2}{4m_N^2}[A(Q^2) - 2J(Q^2) + 3D(Q^2)]$$

- Prediction built from Zhao-Qian's results

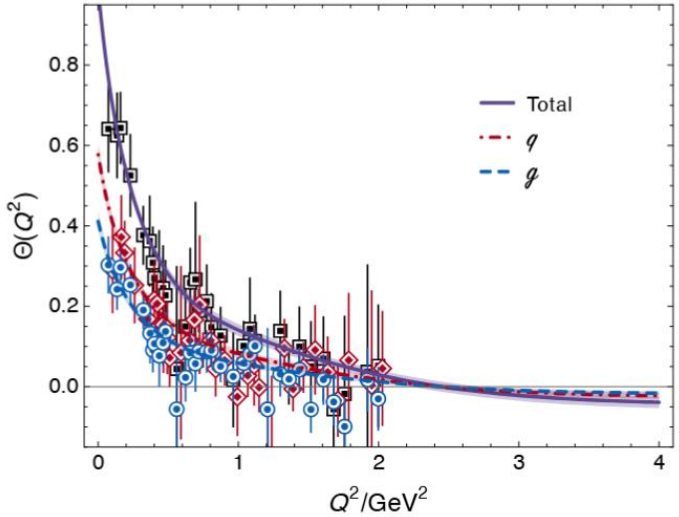


FIG. 1. Curves – CSM predictions: bracketing bands mark the extent of 1σ uncertainty in our numerical procedure for reaching large Q^2. In each case, the overall (species-summed) result is independent of resolving scale, ζ. The species decompositions evolve with ζ. The lQCD points in each panel are obtained using the results in Ref. [24]: black squares – total form factor; red diamonds – quark component; blue circles – glue.

Lattice results in left panel from [Hackett:2023rif]

i. Authors acknowledge these are first results.
ii. Statistical errors are very large.
iii. Results on only one lattice spacing & one lattice volume are available.
iv. Only one unphysical pion mass is used.
v. Renormalisation scheme is not gauge invariant and renormalisataion constants were calculated on a different lattice from the one used to compute the GFFs.
vi. No other computed observable is available on the lattice setup the authors used.
vii. Follows, *inter alia*, that the systematic errors on [Hackett:2023rif] are completely unknown and therefore uncontrolled.

Hence, no one can know whether it is better to agree or disagree with the

CSM prediction (parameter-free) in precise agreement with inference from $\pi\pi$, $K\bar{K}$ scattering data [Cao:2024zlf]

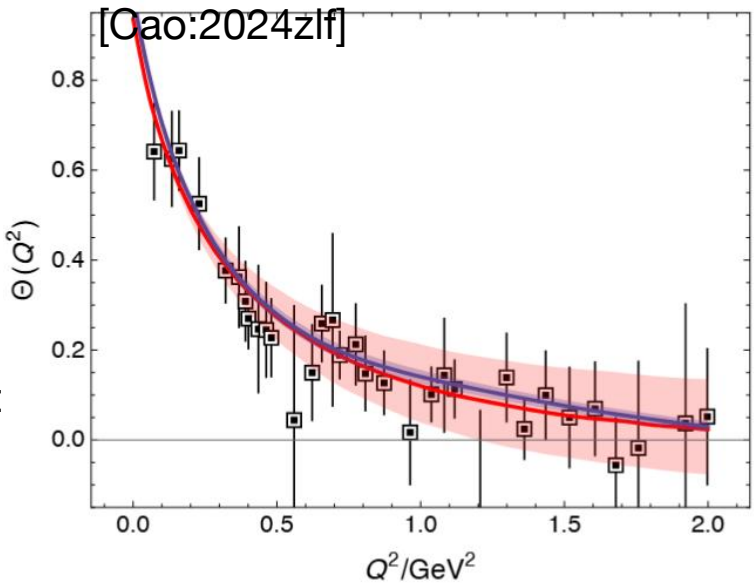


FIG. 2. $\theta(Q^2)$. Purple curve and band – CSM prediction. Red curve and band – data-based inference in Ref. [31, FKG].

Gravitational Form Factors
Mass, Spin, Pressure Distributions

$$\theta(Q^2) = A(Q^2) + \frac{Q^2}{4m_N^2}[A(Q^2) - 2J(Q^2) + 3D(Q^2)]$$

- Prediction built from Zhao-Qian's results

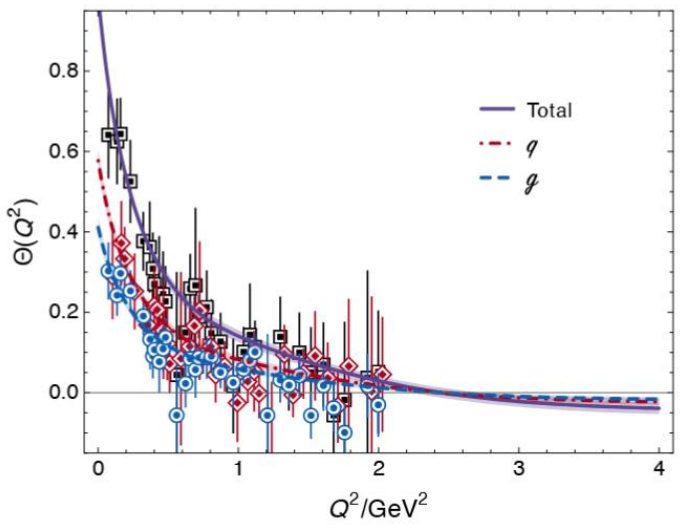


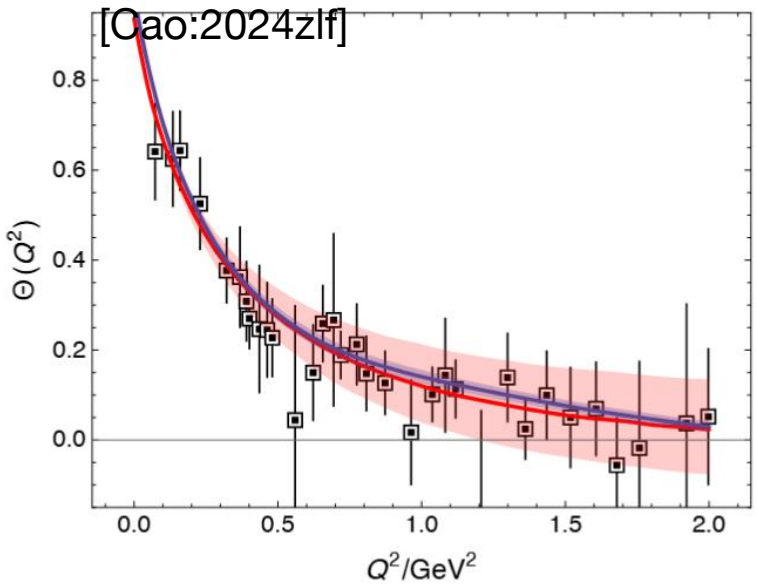
FIG. 1. Curves – CSM predictions: bracketing bands mark the extent of 1σ uncertainty in our numerical procedure for reaching large Q^2. In each case, the overall (species-summed) result is independent of resolving scale, ζ. The species decompositions evolve with ζ. The lQCD points in each panel are obtained using the results in Ref. [24]: black squares – total form factor; red diamonds – quark component; blue circles – glue.

CSM results

i. Systematic errors considered and controlled
ii. Statistical errors associated with numerical algorithms small
iii. Parameter-free unification of nucleon gravitational form factors with data-consistent predictions for pion, kaon, and nucleon electromagnetic form factors [Yao:2024drm, Yao:2024uej]

CSM predictions are fully benchmarked

CSM prediction (parameter-free) in precise agreement with inference from $\pi\pi$, $K\bar{K}$ scattering data [Cao:2024zlf]



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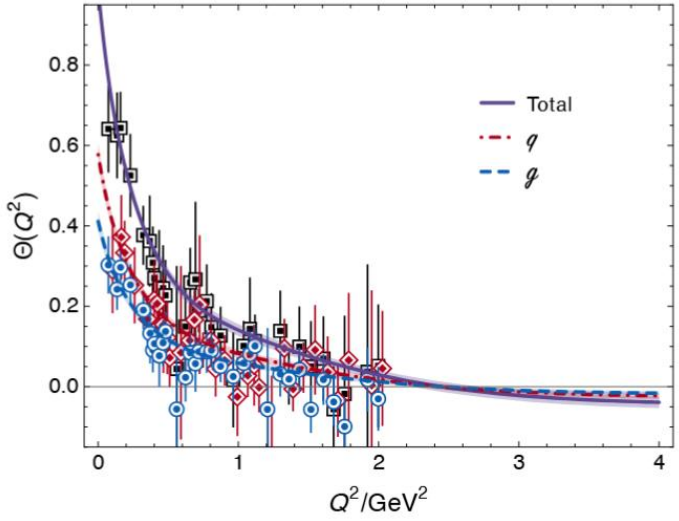

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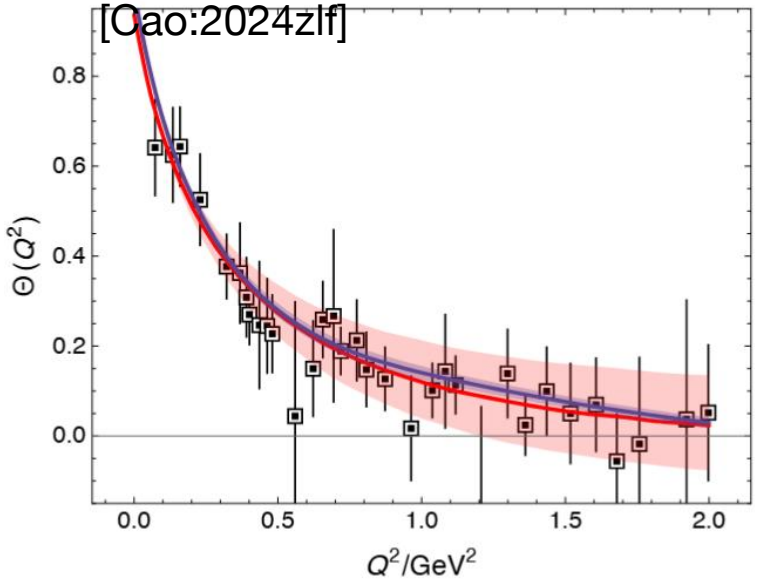
My position

- ✓ Breadths of application & success of the parameter-free framework employed suggest that predictions delivered should serve as benchmarks for future phenomenology and theory.
- ✓ No improvement on CSM study can be anticipated before lattice-QCD analyses have simultaneously delivered infinite-volume, continuum-limit, carefully renormalised, physical pion mass studies of all physical properties discussed, *viz*. pion, kaon, nucleon electromagnetic and gravitational FFs.
- ✓ CSM predictions are truly setting the standard against which future calculations/inferences should be

CSM prediction (parameter-free) in precise agreement with inference from $\pi\pi$, $K\bar{K}$ scattering data [Cao:2024zlf]



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