COMP9024: Data Structures and Algorithms

Week Five: Trees

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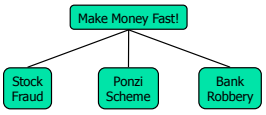
http://www.cse.unsw.edu.au/~cs9024

Outline

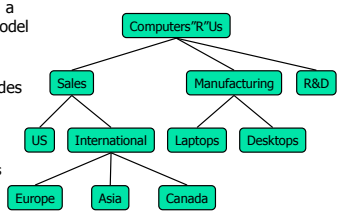
- Tree ADT
- Preorder Traversal
- Inorder Traversal
- Postorder Traversal

Trees



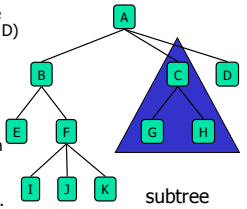
What is a Tree

- In computer science, a tree is an abstract model of a hierarchical structure
- A tree consists of nodes with a parent-child relation
- Applications:
  - Organization charts
  - File systems
  - Programming environments



Tree Terminology

- Root: node without parent (A)
- Internal node: node with at least one child (A, B, C, F)
- External node (a.k.a. leaf): node without children (E, I, J, K, G, H, D)
- Ancestors of a node: parent, grandparent, grand-grandparent, etc.
- Depth of a node: number of ancestors
- Height of a tree: maximum depth of any node (3)
- Descendant of a node: child, grandchild, grand-grandchild, etc.
- Subtree: tree consisting of a node and its descendants



subtree

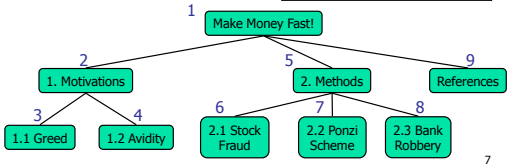
Tree ADT

- We use positions to abstract nodes
- Generic methods:
  - integer size()
  - boolean isEmpty()
  - Iterator elements()
  - Iterator positions()
- Accessor methods:
  - position root()
  - position parent(p)
  - positionIterator children(p)
- Query methods:
  - boolean isInternal(p)
  - boolean isExternal(p)
  - boolean isRoot(p)
- Update method:
  - object replace (p, o)
- Additional update methods may be defined by data structures implementing the Tree ADT

Preorder Traversal

- A traversal visits the nodes of a tree in a systematic manner
- In a preorder traversal, a node is visited before its descendants
- Application: print a structured document

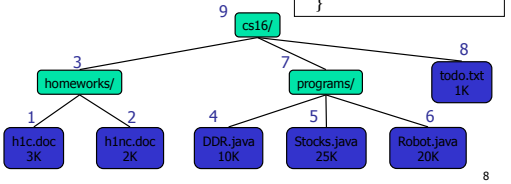
Algorithm *preOrder*(*v*)
{ *visit*(*v*);
for each child *w* of *v*
preorder(*w*);
}



Postorder Traversal

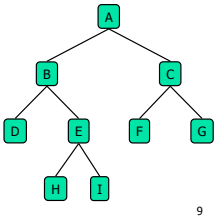
- In a postorder traversal, a node is visited after its descendants
- Application: compute space used by files in a directory and its subdirectories

Algorithm *postOrder*(*v*)
{ **for each** child *w* of *v*
postOrder(*w*);
visit(*v*);
}



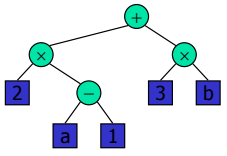
Binary Trees

- A binary tree is a tree with the following properties:
  - Each internal node has at most two children (exactly two for **proper** binary trees)
  - The children of a node are an ordered pair
- We call the children of an internal node left child and right child
- Alternative recursive definition: a binary tree is either
  - a tree consisting of a single node, or
  - a tree whose root has an ordered pair of children, each of which is a binary tree
- Applications:
  - arithmetic expressions
  - decision processes
  - searching



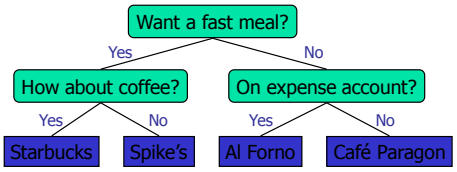
Arithmetic Expression Tree

- Binary tree associated with an arithmetic expression
  - internal nodes: operators
  - external nodes: operands
- Example: arithmetic expression tree for the expression $(2 \times (a - 1) + (3 \times b))$



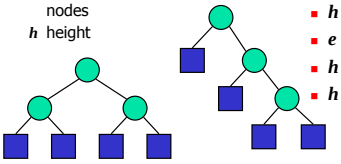
Decision Tree

- Binary tree associated with a decision process
  - internal nodes: questions with yes/no answer
  - external nodes: decisions
- Example: dining decision



Properties of Proper Binary Trees

- Notation
  - n number of nodes
  - e number of external nodes
  - i number of internal nodes
  - h height
- Properties:
  - $e = i + 1$
  - $n = 2e - 1$
  - $h \leq i$
  - $h \leq (n - 1)/2$
  - $e \leq 2^h$
  - $h \geq \log_2 e$
  - $h \geq \log_2 (n + 1) - 1$

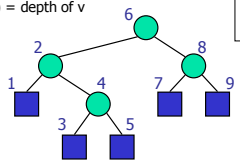


BinaryTree ADT

- The BinaryTree ADT extends the Tree ADT, i.e., it inherits all the methods of the Tree ADT
- Additional methods:
  - position left(p)
  - position right(p)
  - boolean hasLeft(p)
  - boolean hasRight(p)
- Update methods may be defined by data structures implementing the BinaryTree ADT

Inorder Traversal

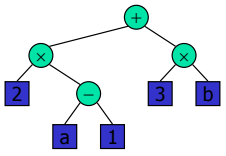
- In an inorder traversal a node is visited after its left subtree and before its right subtree
- Application: draw a binary tree
  - x(v) = inorder rank of v
  - y(v) = depth of v



Algorithm *inOrder(v)*
```
{ if ( hasLeft (v) )
    inOrder(left (v));
  visit(v);
  if ( hasRight (v) )
    ignored(right (v));
}
```


Print Arithmetic Expressions

- Specialization of an inorder traversal
  - print operand or operator when visiting node
  - print "(" before traversing left subtree
  - print ")" after traversing right subtree

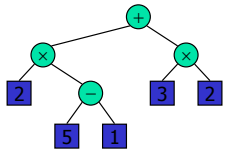


Algorithm *printExpression(v)*
```
{ if ( hasLeft(v) )
    { print("(");
      printExpression(left(v)); }
  print(v.element ());
  if ( hasRight(v) )
    { printExpression(right(v));
      print (") "); }
}
```

((2 × (a − 1)) + (3 × b))

Evaluate Arithmetic Expressions

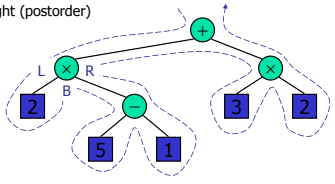
- Specialization of a postorder traversal
  - recursive method returning the value of a subtree
  - when visiting an internal node, combine the values of the subtrees



Algorithm *evalExpr(v)*
```
{ if ( isExternal (v) )
    return v.element ();
  else
    { x = evalExpr(leftChild (v));
      y = evalExpr(rightChild (v));
      ◊ = operator stored at v;
      return x ◊ y;
    }
}
```


Euler Tour Traversal

- Generic traversal of a binary tree
- Includes the special cases: the preorder, postorder and inorder traversals
- Walk around the tree and visit each node three times:
  - on the left (preorder)
  - from below (inorder)
  - on the right (postorder)



Template Method Pattern

- Generic algorithm that can be specialized by redefining certain steps
- Implemented by means of an abstract Java class
- Visit methods that can be redefined by subclasses
- Template method eulerTour
  - Recursively called on the left and right children
  - A Result object with fields leftResult, rightResult and finalResult keeps track of the output of the recursive calls to eulerTour

```
public abstract class EulerTour {
    protected BinaryTree tree;
    protected void visitExternal(Position p, Result r) { }
    protected void visitLeft(Position p, Result r) { }
    protected void visitBelow(Position p, Result r) { }
    protected void visitRight(Position p, Result r) { }
    protected Object eulerTour(Position p) {
        Result r = new Result();
        if tree.isExternal(p) { visitExternal(p, r); }
            else {
                visitLeft(p, r);
                r.leftResult = eulerTour(tree.left(p));
                visitBelow(p, r);
                r.rightResult = eulerTour(tree.right(p));
                visitRight(p, r);
                return r.finalResult;
            } ...
```

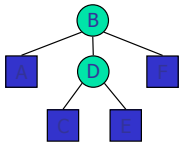
Specializations of EulerTour

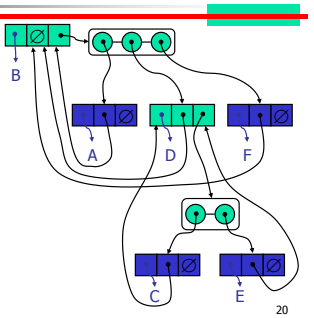
- We show how to specialize class EulerTour to evaluate an arithmetic expression
- Assumptions
  - External nodes store Integer objects
  - Internal nodes store Operator objects supporting method operation (Integer, Integer)

```
public class EvaluateExpression
                extends EulerTour {
  protected void visitExternal(Position p, Result r) {
    r.finalResult = (Integer) p.element();
  }
  protected void visitRight(Position p, Result r) {
    Operator op = (Operator) p.element();
    r.finalResult = op.operation(
                (Integer) r.leftResult,
                (Integer) r.rightResult
                );
  }
  …
}
```


Linked Structure for Trees

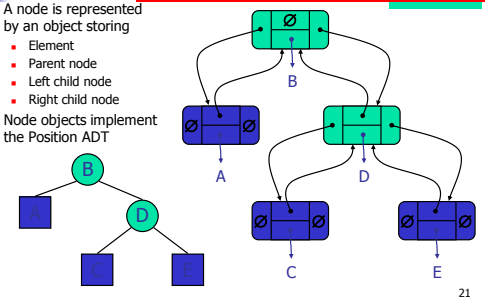
- A node is represented by an object storing
  - Element
  - Parent node
  - Sequence of children nodes
- Node objects implement the Position ADT





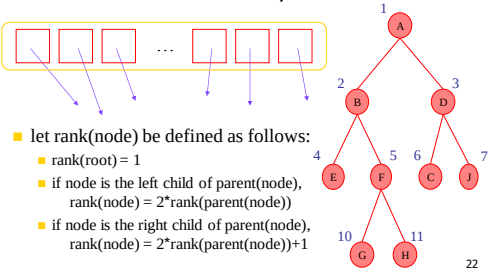
Linked Structure for Binary Trees

- A node is represented by an object storing
  - Element
  - Parent node
  - Left child node
  - Right child node
- Node objects implement the Position ADT



Array-Based Representation of Binary Trees

- nodes are stored in an array



- let rank(node) be defined as follows:
  - rank(root) = 1
  - if node is the left child of parent(node), rank(node) = 2*rank(parent(node))
  - if node is the right child of parent(node), rank(node) = 2*rank(parent(node))+1

References

1. Chapter 7, Data Structures and Algorithms by Goodrich and Tamassia.