Hash-Based Indexes

Introduction

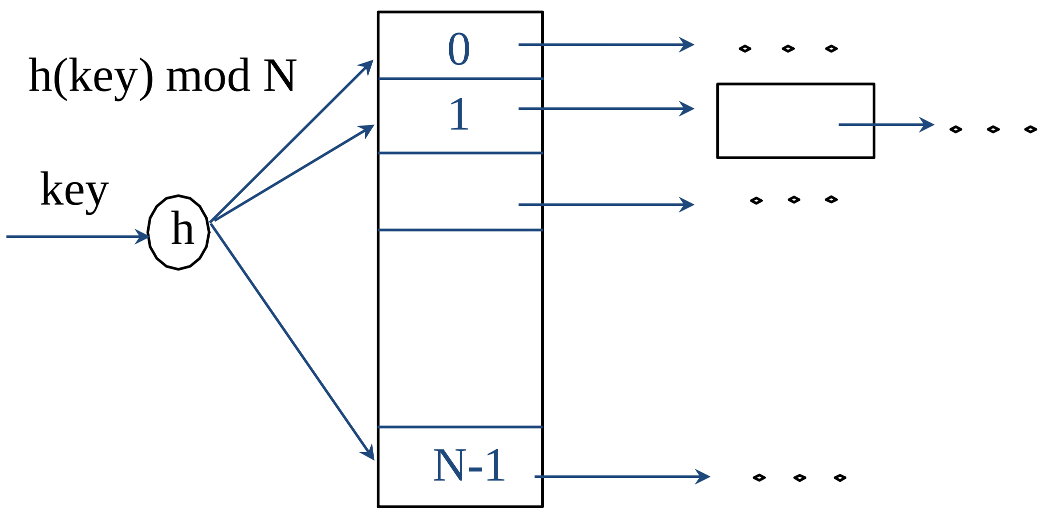
As for any index, 3 alternatives for data entries **k***:

1. Data record with key value **k**
2. <**k**, rid of data record with search key value **k**>
3. <**k**, list of rids of data records with search key **k**>

- Choice orthogonal to the *indexing technique*
- *Hash-based* indexes are the best for *equality selections*. ***Cannot*** support range searches.
- Static and dynamic hashing techniques exist; trade-offs similar to ISAM vs. B+ trees.

Static Hashing

- # primary pages (index data entry pages) fixed, allocated sequentially, never de-allocated; overflow pages if needed.
- **h**(*k*) mod M = bucket to which data entry with key *k* belongs. (M = # of buckets)



Primary bucket pages **Overflow pages**

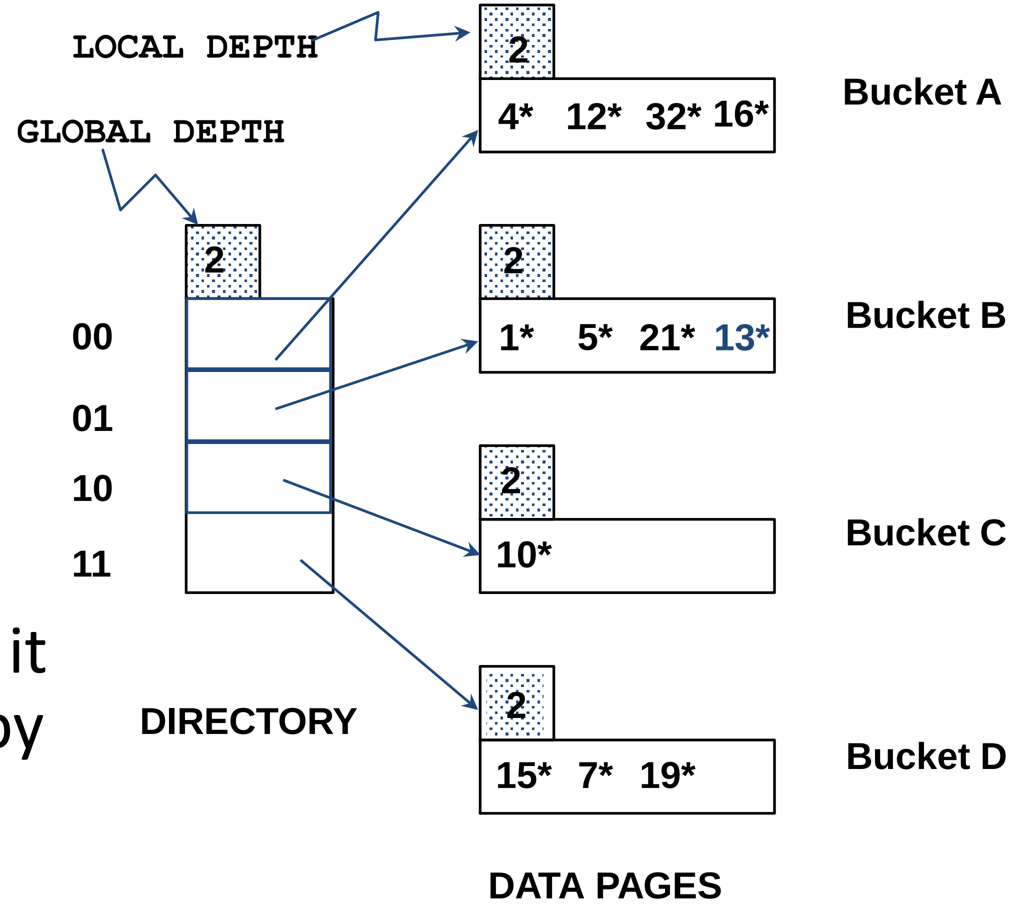
Static Hashing (Contd.)

- Buckets contain *data entries*.
- Hash function works on *search key* field of record *r.* Must distribute values over range 0 ... M-1.
  - **h**(*key*) = (a * *key* + b) usually works well.
  - **a** and **b** are constants; lots known about how to tune **h**.
- Long overflow chains can develop and degrade performance.
  - Keep 80% full initially and/or re-hashing
  - *Extendible* and *Linear Hashing*: Dynamic techniques to fix this problem.

Extendible Hashing

- Situation: Bucket (primary page) becomes full. Why not re-organize file by *doubling* # of buckets?
  - Reading and writing all pages is expensive!
  - *Idea*: Use *directory of pointers to buckets,* double # of buckets by *doubling the directory,* splitting just the bucket that overflowed!
  - Directory much smaller than file, so doubling it is much cheaper. Only one page of data entries is split. *Ensure no overflow page*!
  - Trick lies in how hash function is adjusted!

Example

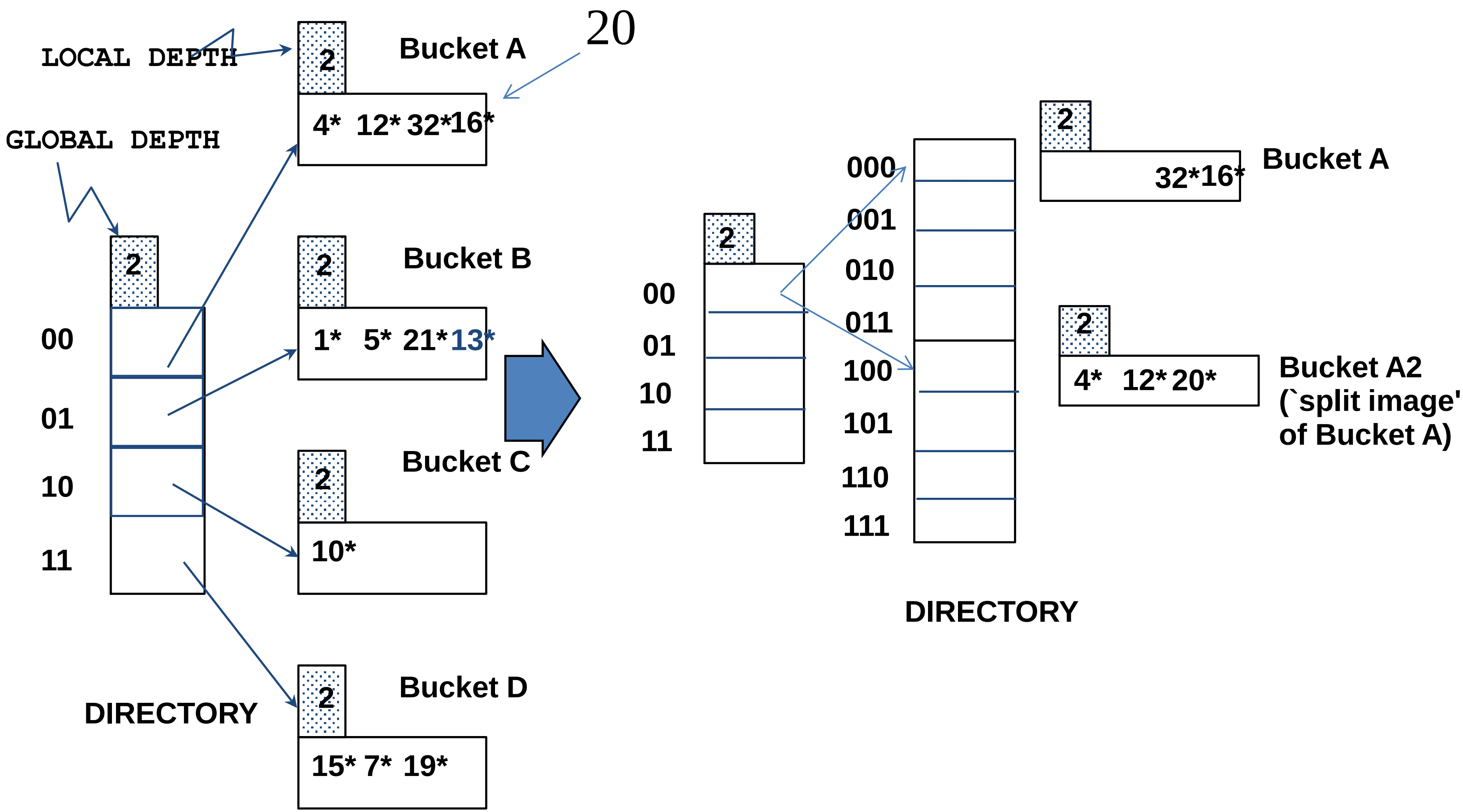


- Directory is array of size 4.
- To find bucket for *r*, take last \`*global depth*' # bits of **h**(*r*); we denote *r* by **h**(*r*).
  - If **h**(*r*) = 5 = binary 101, it is in bucket pointed to by 01.

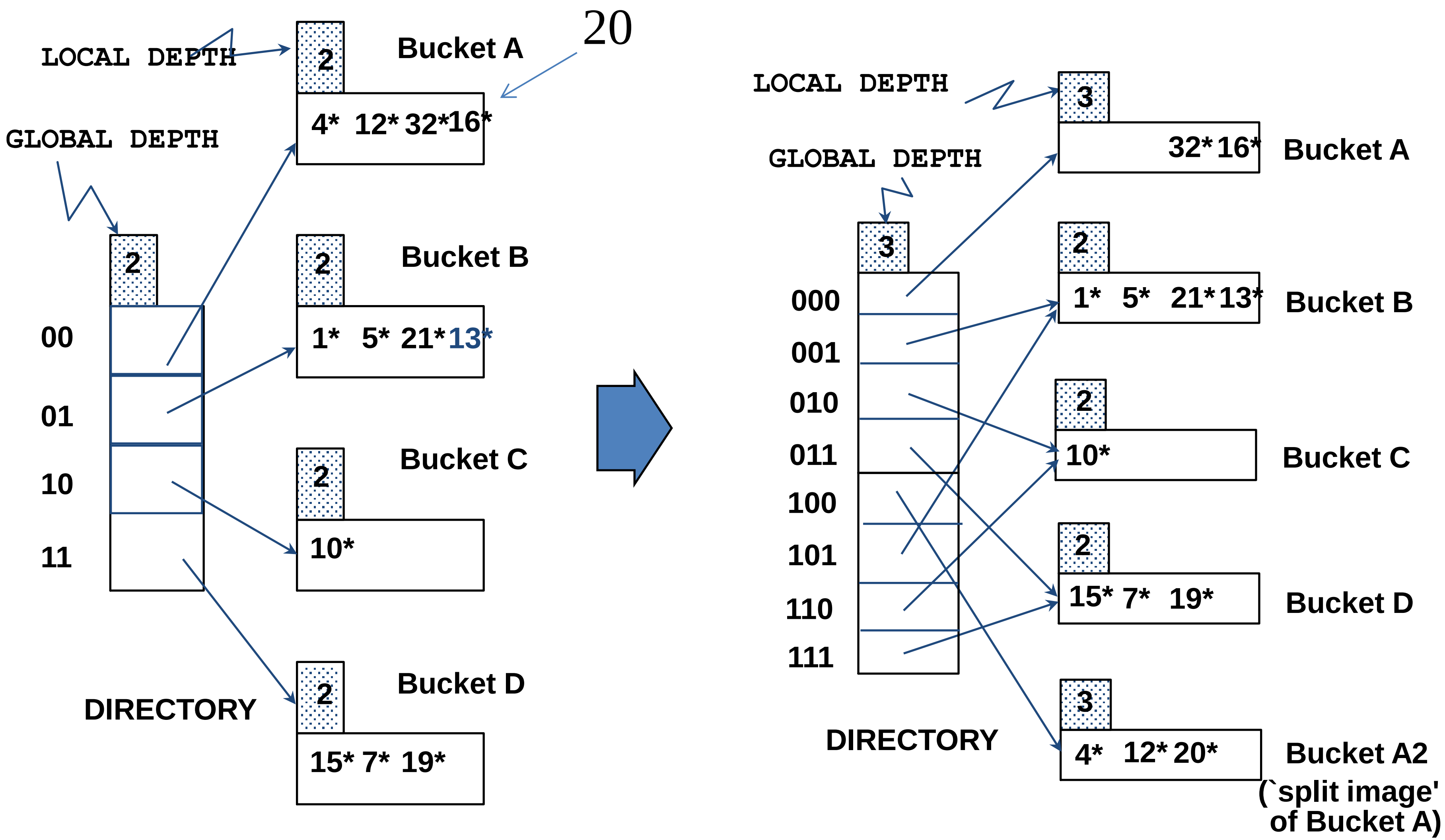
❖ **Insert**: If bucket is full, *split* it (*allocate new page, re-distribute*).

❖ *If necessary*, double the directory. (As we will see, splitting a bucket does not always require doubling; we can tell by comparing *global depth* with *local depth* for the split bucket.)

Insert **h**(r)=20 (Causes Doubling)



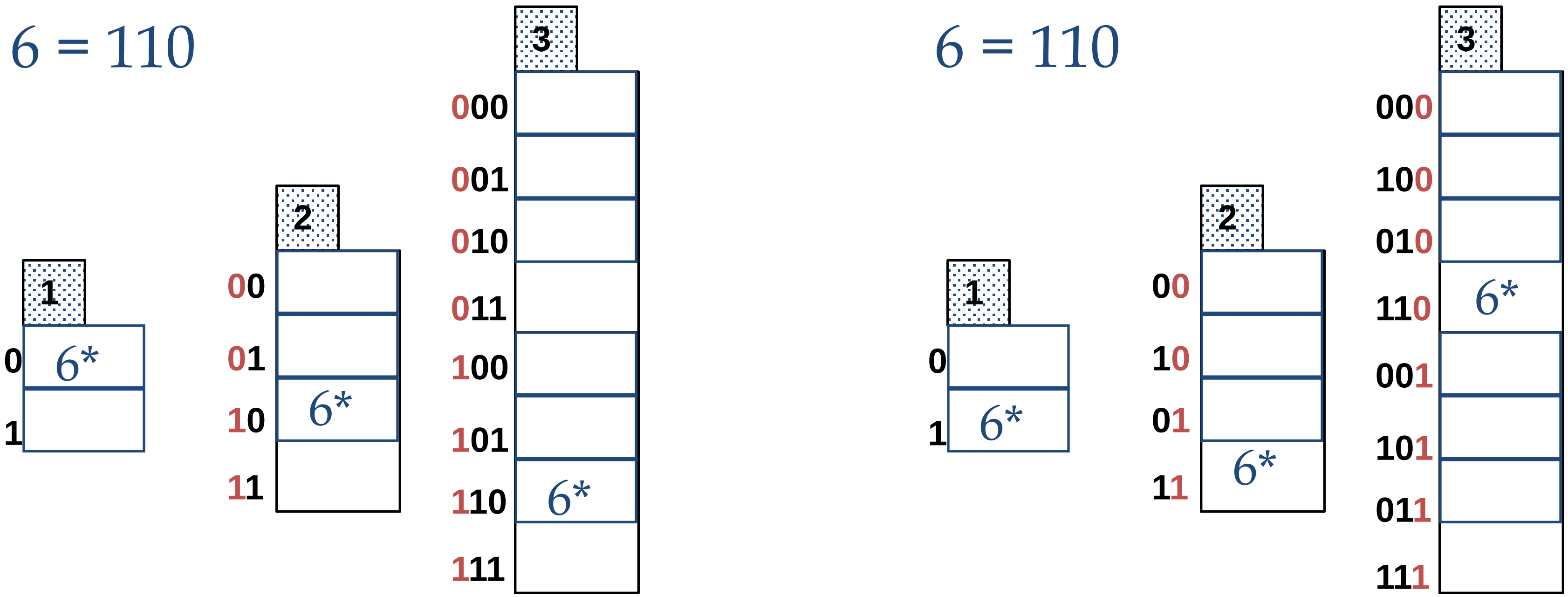
After inserting **h**(r)=20



Points to Note

- 20 = binary 10100. Last **2** bits (00) cannot tell us *r* belongs in A or A2. Last **<u>3</u>** bits can tell us the which bucket.
  - *Global depth of directory*: Max # of bits needed to tell which bucket an entry belongs to.
  - *Local depth of a bucket*: # of bits used to determine if an entry belongs to this bucket.
- When does bucket split cause directory doubling?
  - Before insert, *local depth* of bucket = *global depth*. Insert causes *local depth* to become > *global depth*; directory is doubled by *copying it over* and `fixing' pointer to split image page. (Use of least significant bits enables efficient doubling via copying of directory!)

Directory Doubling



Least Significant vs. Most Significant

Why use least significant bits in directory?

- Hard to decide where to start
- Quite biased in the most significant bids

Comments on Extendible Hashing

- If directory fits in memory, equality search answered with one disk access; else two.
  - 100MB file, 100 bytes/rec, 4K pages contains 1,000,000 records (as data entries) and 25,000 directory elements; chances are high that directory will fit in memory.
  - Directory grows in spurts, and, if the distribution *of hash values* is skewed, directory can grow large.
  - Multiple entries with same hash value cause problems!
- **Delete**: If removal of data entry makes bucket empty, can be merged with `split image'. If each directory element points to same bucket as its split image, can halve directory.

2^N, 2^{N+1}, 2^{N+2}, 2^{N+4}, 2^{N+5}

$N = 100$

00, 000, 0000, …., 000…000,

2^{101}

Linear Hashing

- This is another dynamic hashing scheme, an alternative to Extendible Hashing.
- LH handles the problem of long overflow chains without using a directory, and handles duplicates.
- <u>*Idea*</u>: Use a family of hash functions $\mathbf{h}_0$, $\mathbf{h}_1$, $\mathbf{h}_2$, ...
  - $\mathbf{h}_i(key) = \mathbf{h}(key) \bmod (2^i N)$; N = initial # buckets
  - **h** is some hash function (range is *not* 0 to N-1)
  - If $N = 2^{d0}$, for some *d0*, $\mathbf{h}_i$ consists of applying **h** and looking at the last *di* bits, where $di = d0 + i$.
  - $\mathbf{h}_{i+1}$ doubles the range of $\mathbf{h}_i$ (similar to directory doubling)

Linear Hashing (Contd.)

- Directory avoided in LH by using overflow pages, and choosing bucket to split round-robin.
  - Splitting proceeds in \`<u>rounds</u>'. Round ends when all N_R initial (for round R) buckets are split. Buckets 0 to *Next-1* have been split; *Next* to N_R yet to be split.
  - Current round number is *Level*.
  - **<u>Search:</u>** To find bucket for data entry *r,* find $\mathbf{h}_{Level}(r)$:
    - If $\mathbf{h}_{Level}(r)$ in range \`*Next* to N_R', *r* belongs here.
    - Else, r could belong to bucket $\mathbf{h}_{Level}(r)$ or bucket $\mathbf{h}_{Level}(r) + N_R$; must apply $\mathbf{h}_{Level+1}(r)$ to find out.

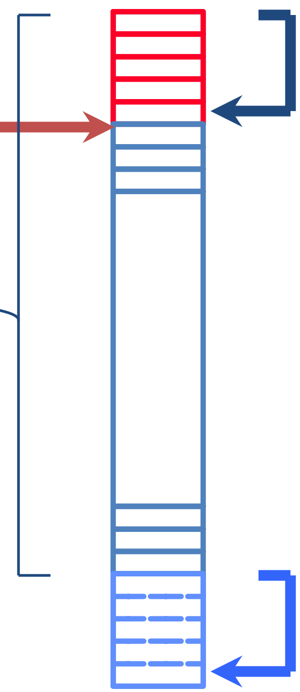
Overview of LH File

In the middle of a round.

Bucket to be split
Next

Buckets that existed at the beginning of this round: this is the range of

h_{Level}



Buckets split in this round: If h_{Level} (search key value) is in this range, must use $h_{Level+1}$ (search key value) to decide if entry is in `split image' bucket.

`split image' buckets: created (through splitting of other buckets) in this round

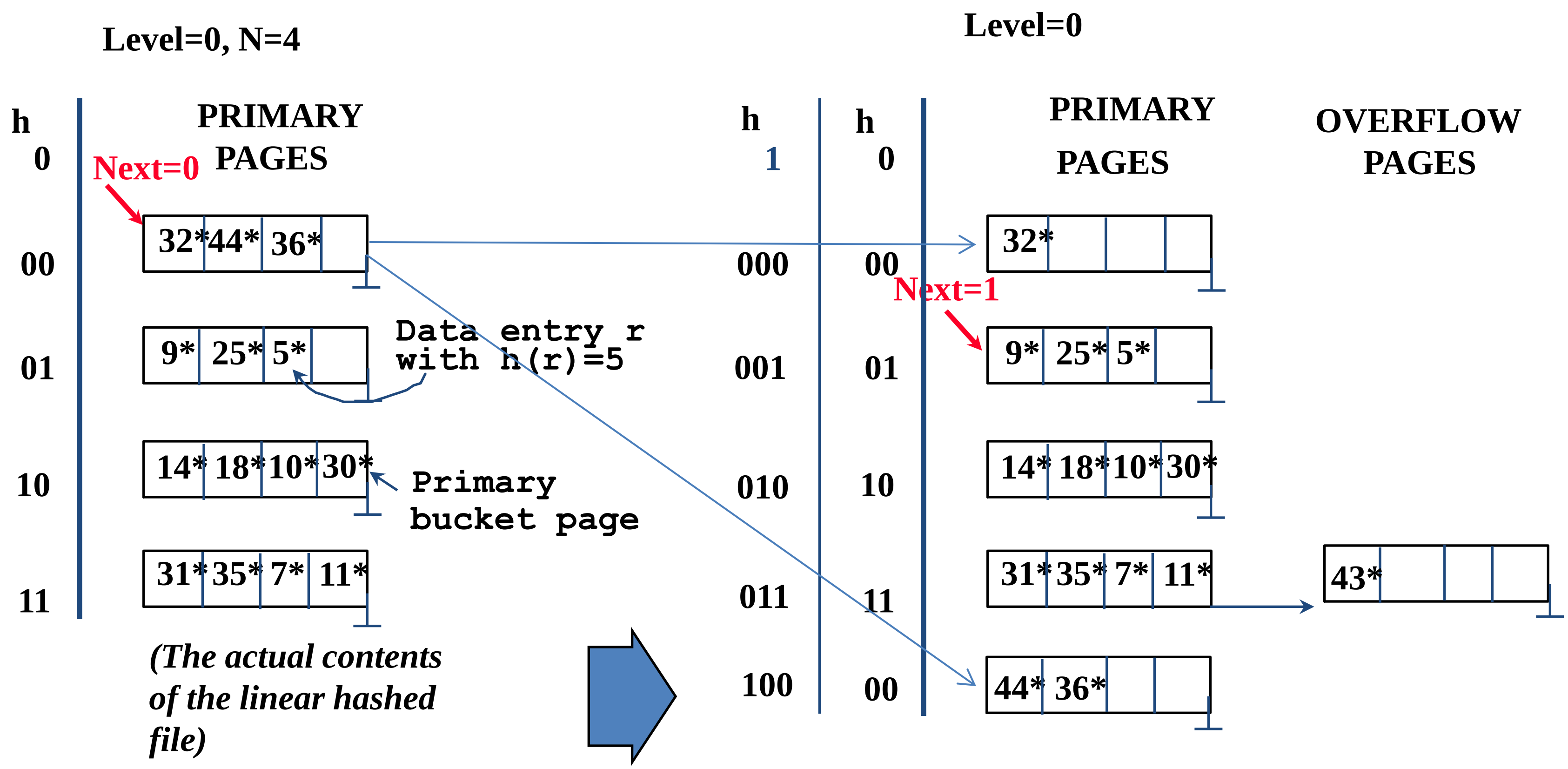
Linear Hashing (Contd.)

- **Insert**: Find bucket by applying $\mathbf{h}_{Level}$ / $\mathbf{h}_{Level+1}$:
  - If bucket to insert into is full:
    - Add overflow page and insert data entry.
    - (*Maybe*) Split *Next* bucket and increment *Next*.
- Can choose any criterion to `trigger' split.
- Since buckets are split round-robin, long overflow chains don't develop!
- Doubling of directory in Extendible Hashing is similar; switching of hash functions is *implicit* in how the # of bits examined is increased.

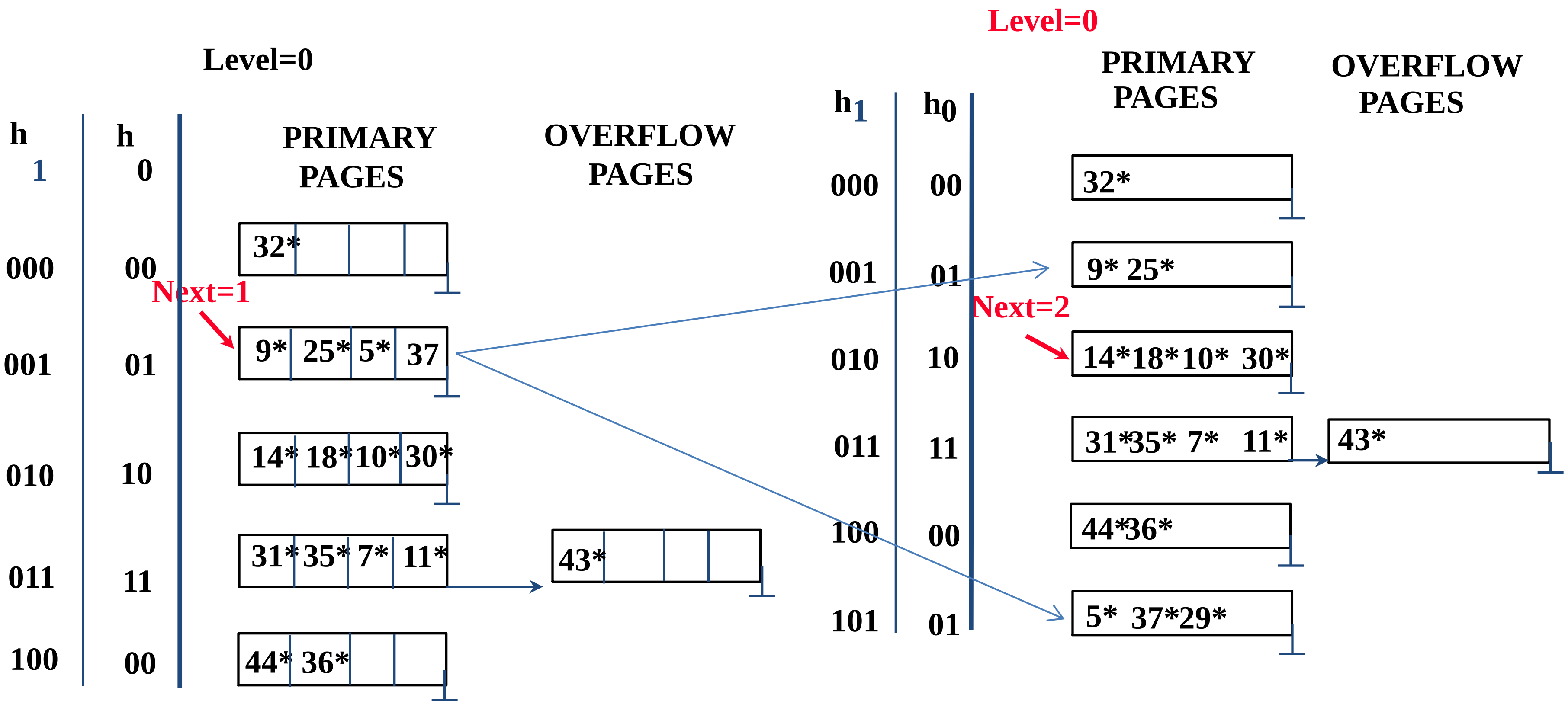
Usually, when a new overflow page is created.

Example of Linear Hashing

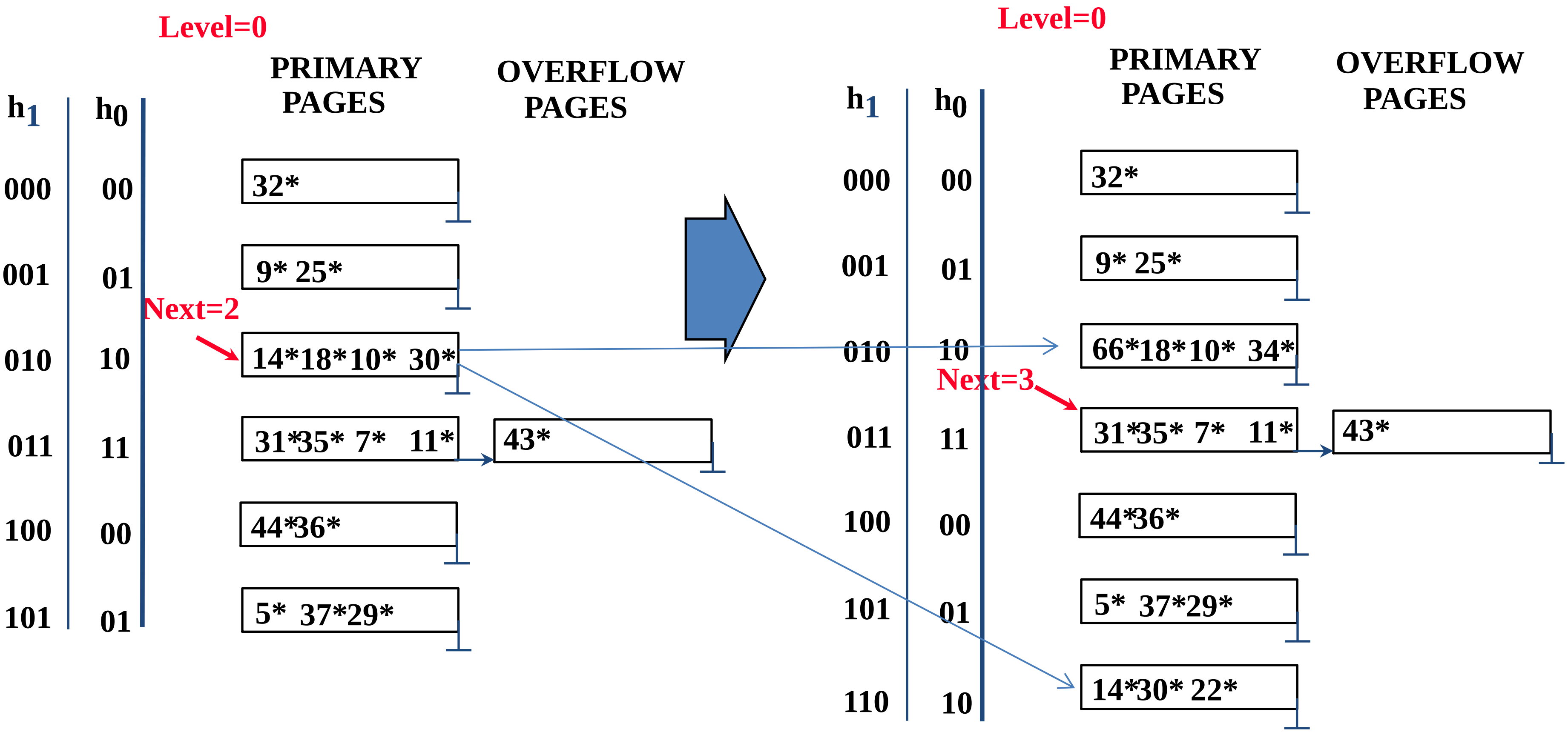
- Insert 43*



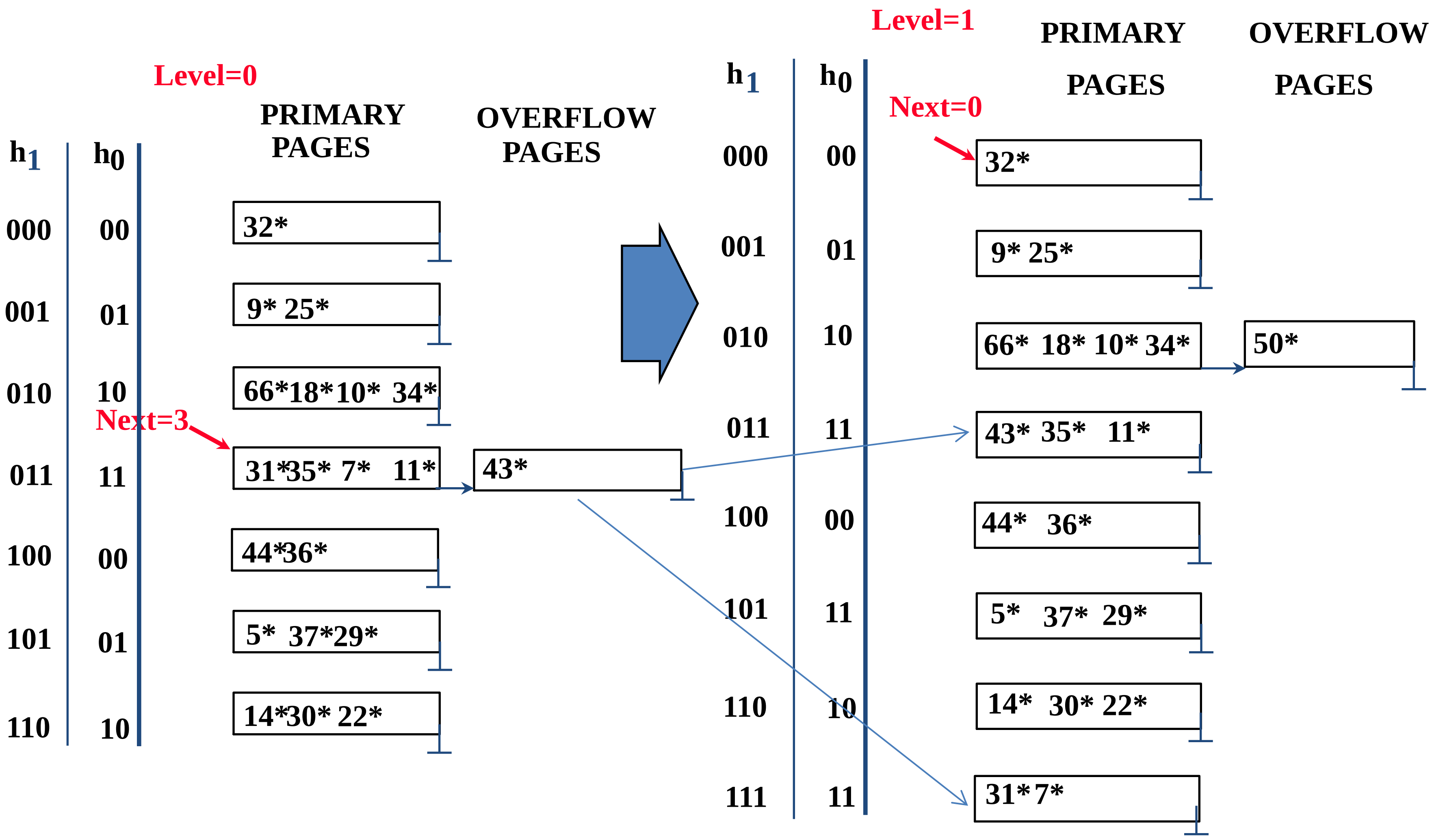
Insert 37   29


Level=0
h 1
h 0
PRIMARY PAGES
OVERFLOW PAGES
000
00
32*
Next=1
001
01
9* 25* 5* 37
010
10
14* 18* 10* 30*
011
11
31* 35* 7* 11*
43*
100
00
44* 36*
Level=0
h1
h0
PRIMARY PAGES
OVERFLOW PAGES
000
00
32*
001
01
9* 25*
Next=2
010
10
14* 18* 10* 30*
011
11
31* 35* 7* 11*
43*
100
00
44* 36*
101
01
5* 37* 29*

Insert 34 66 22



Insert 50


Level=0
h1 h0
PRIMARY PAGES
OVERFLOW PAGES
000 00 32*
001 01 9* 25*
010 10 66*18*10* 34*
Next=3
011 11 31*35* 7* 11*
43*
100 00 44*36*
101 01 5* 37*29*
110 10 14*30* 22*
Level=1
PRIMARY PAGES
OVERFLOW PAGES
h1 h0
Next=0
000 00 32*
001 01 9* 25*
010 10 66* 18* 10* 34*
50*
011 11 43* 35* 11*
100 00 44* 36*
101 11 5* 37* 29*
110 10 14* 30* 22*
111 11 31* 7*

LH Described as a Variant of EH

- The two schemes are actually quite similar:
  - Begin with an EH index where directory has *N* elements.
  - Use overflow pages, split buckets round-robin.
  - First split is at bucket 0. (Imagine directory being doubled at this point.) But elements <1,*N*+1>, <2,*N*+2>, ... are the same. So, need only create directory element *N*, which differs from 0, now.
    - When bucket 1 splits, create directory element *N*+1, etc.
- So, directory can double gradually. Also, primary bucket pages are created in order. If they are *allocated* in sequence too (so that finding i' th is easy), we actually don' t need a directory! Voila, LH.

Summary

- Hash-based indexes: best for equality searches, cannot support range searches.
- Static Hashing can lead to long overflow chains.
- Extendible Hashing avoids overflow pages by splitting a full bucket when a new data entry is to be added to it. (*Duplicates may require overflow pages.*)
  - Directory to keep track of buckets, doubles periodically.
  - Can get large with skewed data; additional I/O if this does not fit in main memory.

Summary (Contd.)

- Linear Hashing avoids directory by splitting buckets round-robin, and using overflow pages.
  - Overflow pages not likely to be long.
  - Duplicates handled easily.
  - Space utilization could be lower than Extendible Hashing, since splits not concentrated on `dense’ data areas.
    - Can tune criterion for triggering splits to trade-off slightly longer chains for better space utilization.
- For hash-based indexes, a *skewed* data distribution is one in which the *hash values* of data entries are not uniformly distributed!