Time Series Analysis

Homework assignment #2

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Problems

1. Consider the $AR(1)$ model with the following choice of parameters: $\alpha = 0.1$, $\beta = 0.3$, $\sigma = 0.005$. The purpose of this Problem is to show that the MLE estimator of the parameters of the model is biased. Run three series of simulations of the models with the following numbers of observations: (i) $T = 100$, (ii) $T = 250$, and (iii) $T = 1250$.

   (a) Each run should consist of $N = 2000$ simulations. After each simulation the model parameters should be estimated using MLE.

   (b) For each run T, calculate the average values of the MLE estimators of the model parameters: $\widehat{\alpha}_T = \frac{1}{N}\sum_{j=1}^{N}\widehat{\alpha}_{j,T}$, $\widehat{\beta}_T = \frac{1}{N}\sum_{j=1}^{N}\widehat{\beta}_{j,T}$, and $\widehat{\sigma}_T = \frac{1}{N}\sum_{j=1}^{N}\widehat{\sigma}_{j,T}$.

   (c) Compare the averages with the true values of the parameters and draw the conclusions.

2. Consider the last 15 years worth of the USD_EUR exchange rate and the differentials between the official (Federal Reserve and European Central Bank) short interest rates in these currencies. You can download the time series from Bloomberg: the rates screens are FEDL01 and EUORDEPO, respectively). Economic theory says that these two time series should be

cointegrated with cointegrating vector $a = (1, -1)^{\mathsf{T}}$. Design a test to verify this theory.

3. Consider the following $AR(1)$ model with a linear drift:

$$X_t = \alpha + \delta t + \beta X_{t-1} + \varepsilon_t, \tag{1}$$

and let $x_0, x_1, \ldots, x_T$ be a set of observations. Following the method discussed in Lecture Notes #1, write down the conditional likelihood function $\mathcal{L}(\theta | x_0, x_1, \ldots, x_T)$, and derive explicit MLE estimators $\widehat{\alpha}$, $\widehat{\delta}$, $\widehat{\beta}$, and $\widehat{\sigma}$.

4. Prove the formula for the kurtosis of the $GARCH(1,1)$ model stated in Lecture Notes #2.

This assignment is due on February 23