1 Motivation for the Empirical Test

We have presented our results to many traders. The reaction has in general been positive, especially regarding the conclusions of the analysis regarding the hig-reates regimes (ie, the transition from normal to quasi-log-normal behaviour above 7 or 8%). Some traders have raised objections, however, to the log-normal behaviour for very low rates. Their reservations (sometimes very strong) were motivated by the observation that 'traders who used the log-normal model in JPY in the early 1990s (when rates where extremely low) lost a lot of money'. The implication of this statement is that the market was right (ie, correctly knew that rates were nt behaving in a log-normal manner), and that wrong the log-normal option traders were being punished for their errors.

We want to test the logical correctness of these objections. Clearly, we do not have the details about the positions that were 'punished by the market' in those days. However, we can set up a stylized experiment. First we assume that the market is right, and that two traders exist: one who hedges according to the true market model; and another who hedges using his erroneous belief about the market. We give a high degree of 'model perfection' to the experiment (no transaction costs, ability to transact in all sizes, etc), but we impose a finite, although very high, hedging frequency. We focus on the difference between the value of the option and of the replicating portfolio (what we call the 'slippages'). For reasons that we explain later, we look at the slippges not only at expiry, but also during the life of the option. We then explore whether the 'signal' (that we take to be the first two moments of the distribution of the slippages) is sufficiently strong in the presence of the noise arising from the finite re-hedging rfrequency to discrimnate between the right and the wrong model.

We then perform a second experiment: we entertain the possibility that the market migh thave been wrong. Looking again at final and intermediate slippages, we ask the question whether, inthe presence of instituional constraints, the trader would have been better off hedging according to the wrong market or the true model.

Of course, traditional hedging theory always assumes that the best strategy is to hedge according to the true process. The implicit (efficient-market) assumption is also almost invariably made that the market 'knows' the true model – more precisely, the assumption is made that the market acts as an efficient Bayesian agent, who assesses in the best possible way on the basis of the existing evidence the likelihood of different models being right. In this setting, if one is concerned with the magnitude of the slippages, there is no conceptual difference in looking at hedging all the way to the expiry time of an option, or only up to some intermediate times, because at all times the market price and the true price always coincide.

However, If the market *can* be wrong, this would no longer apply, because only at expiry would in general the true and and market prices coincide, and what happens at intermediate times becomes very important. If the market can be wrong the trader can no longer afford just to look at the slippages between the payoff of the option and of the replicating portfolio at the expiry of the option (that can be many years away). This concern for intermediate slippages arises because in realistic settings the trader will be subject to insututional constraints, such as stop losses, VaR, delta and vega limits[1] or withdrawal of capital if the mark-to-wron-market losses become too large. This situation has been well explored in the limit-of-arbitrage literature – see Shleifer, Vishni, etc. The general point made in this literature is that, in the presence of these institutional constraints, (the 'divorce between brawn and brains') the non-informed providers of capital might prevent the pseudo-arbitrageurs from carrying out their marlet-efficiency-enhancing trades by forcing the trader to cut hisposiotns before he can be proven right. When this is the case, the market can remain 'wrong'.

In the presence of these real-life instiutional constraints it is no longer obvious what the best strategy for a trader (even one who knows the true process) should be. There is a tension between following the true strategy, and remaining as close to the market as possible – and hence avoiding being stop-lossed out.

The study carried above quantifies this tension, and clarifies how the trader should behave under different situations (eg, lenght of hedging programme, at-the-moneiness of the option) and institutional constraints (eg, stop losses, risk management limits, withdrawal of capital).

Can the market be wrong? This is not the place to try to address the question of (the limits to) market efficiency. One can make, however, a couple of observations: if one maintained that the option market is always right, was the market right before or after the week of the October 1987 market crash (since equity smiles suddenly appeared after the crash)? And was the market right beofre or after the LTCM events, when suddenly non-monotonic smiles appeared in the interest-rate smile surface?

As I said, I do not intend to address this question in this note. However, the casual observations above, and the arguments provided by the limit-to arbitrage school at the very least suggest that the efficiency of the option market should not be accepted in a dogmatic fashion.

2 The Set-Up

Let N be the number of re-hedging time steps, T the expiry of a call option on a forward rate, K its strike, and Δt the length of each re-hedging interval. N and Δt are fixed, but T is not: $T \geq N\Delta t$, When $T = N\Delta t$ the option expiries at the end of the hedging simulation. For simplicity we assume zero interest rates (for borrowing and lending) and zero bid-offer spread.

[1] Traders whose mandate is to replicate the value of a portfolio of options advantgeously bought or sold, rather than taking outright directional views on the volatility market, are typically subject to tight risk management limits (VaR, equivlent delta, vega mismatch etc) to avoid that they may engage in 'mission creep'. If they hadge according to the true model, and if the riskiness of their positions is calculated using the deltas, begas and other statistics calcualted using the 'wrong' market model, then they can easily appear to be in breach of their risk limits event if they are closely hedged. This can cause a request from risk management to cut their positions befire option expiry.

A CEV process would be of the type

$$df_t = f_t^{\beta} \sigma_{CEV} dz_t \tag{1}$$

in the low-rate portion of our data set, our empirical findings would translate in the CEV process having an exponent close to 1. Rather than using a CEV process, we have preferred to use for our simulations the analytically more tractable displaced-diffusion model (Rubinstein (19xx)). As Marris (199X), Rebonato (200Y) and Svoboda (200X) have shown, this process has very strong links with the CEV process. For the two exponents that we wil explore ($\beta = 0$ and $\beta = 1$) the coincidence between the CEV and the displaced-diffusion processes is almost exact. See Rebonato (200Y)

Let the true process for the forward rate under its own terminal measure be given by:

$$\frac{df_t}{f_t + a_{true}} = \sigma_{a_{true}} dz_t \tag{2}$$

The market believes the true process to be:

$$\frac{df_t}{f_t + a_{mkt}} = \sigma_{amkt} dz_t \tag{3}$$

The trader does not know the real process, and possibly disagrees with the market, ie, he assumes that the process is

$$\frac{df_t}{f_t + a_{trad}} = \sigma_{a_{trad}} dz_t \tag{4}$$

However, he chooses the volatility, $\sigma_{a_{trad}}$, so that the price of the call at time 0 (ie, at trade time) is the same as in the market. In other words, following market practice, the trader 'calibrates' his model to market process. This is a universal market practice, and, if we want our simulations to have any degree of realism, this condition is a must.

3 The Market Is Right

For simplicity we begin by assuming that $a_{true} = a_{mkt}$, ie, the market knows the true process.

Let C_i^{true} be the true value of the call at time step i. C_i^{trad} will be the value of the call at time step i according to the trader's faulty model. If $T = N\Delta t$, then

$$C_N^{trad} = C_N^{true} = (f_T - K)^+ = Payoff(T) \tag{5}$$

The trader hedges using a delta amount of FRA struck at the same strike. He reckons the delta amount of FRA, Δ^{trad}, using his faulty model.

If the trader used the correct delta amount of FRA, Δ^{true}, then

$$E\left[\sum_i^N \Delta C_i^{true} - \sum_i^N \Delta^{true} FRA_i\right] = 0 \tag{6}$$

$$Var\left[\sum_i^N \Delta C_i^{true} - \sum_i^N \Delta^{true} FRA_i\right] > 0 \tag{7}$$

because of finite rehedging time step, Δt. Defining for compactness

$$A_N^{true,true} = \left[\sum_i^N \Delta C_i^{true} - \sum_i^N \Delta^{true} FRA_i\right]$$

we have

$$E\left[A_N^{true,true}\right] = 0 \tag{8}$$

$$Var\left[A_N^{true,true}\right] > 0 \tag{9}$$

We now look at the case when the trader does not hedge with the right amount of FRA. We will consider two different settings (both under the assumption that the market knows the true model).

In the first setting let's suppose that the trader does not have mark his book to market until expiry and can continue his faulty hedging strategy without stop losses. Note that, since the trader calibrates his faulty model to market, $C_0^{trad} = C_0^{true}$.

Consider now the quantity, $A_N^{trad,trad}$,

$$A_N^{trad,trad} = \sum_i^N \Delta C_i^{trad} - \sum_i^N \Delta^{trad} FRA_i \tag{10}$$

Even in the limit $\Delta t \longrightarrow 0$ there is no reason to expect that

$$E\left[A_N^{trad,trad}\right] = E\left[\sum_i^N \Delta C_i^{trad} - \sum_i^N \Delta^{trad} FRA_i\right] = 0 \tag{11}$$

(because the trader believes in the wrong process). We can similarly define the quantity, $A_N^{true,trad}$,

$$A_N^{true,trad} = \sum_i^N \Delta C_i^{true} - \sum_i^N \Delta^{trad} FRA_i \tag{12}$$

This quantity will give the final (time-T) slippage due to finite rehedging interval and wrong choice of model. *This is always the quantity of economic interest to track.* Some simple re-arramgements can show more cleraly how its compenents in different special settings.

Since it is always true that

$$C_N^{trad} = C_0^{trad} + \sum_i^N \Delta C_i^{trad} \tag{13}$$

$$C_N^{true} = C_0^{true} + \sum_i^N \Delta C_i^{true} \tag{14}$$

One can write

$$A_N^{trad,trad} = \sum_i^N \Delta C_i^{trad} - \sum_i^N \Delta^{trad} FRA_i + \left[C_0^{true} + \sum_i^N \Delta C_i^{true} - C_N^{true} \right] =$$

$$= C_N^{trad} - C_0^{trad} - \sum_i^N \Delta^{trad} FRA_i + \left[C_0^{true} + \sum_i^N \Delta C_i^{true} - C_N^{true} \right]$$

By calibration, $C_0^{trad} = C_0^{true}$. It follows that

$$A_N^{trad,trad} = \left[C_N^{trad} - C_N^{true} \right] + \left[\sum_i^N \Delta C_i^{true} - \sum_i^N \Delta^{trad} FRA_i \right] \qquad (15)$$

and therefore

$$A_N^{true,trad} = A_N^{trad,trad} + \left[C_N^{true} - C_N^{trad} \right] \qquad (16)$$

If $T = N\Delta t$,

$$C_N^{trad} = C_N^{true} = (f_T - K)^+ = Payoff(T) \qquad (17)$$

and

$$\sum_i^N \Delta C_i^{true} = \sum_i^N \Delta C_i^{trad}$$

(becasue the starting and final points – $C_N^{trad} = C_N^{true}$ and $C_0^{trad} = C_0^{true}$ – are the same) and therefore

$$A_N^{true,trad} = A_N^{trad,trad} \quad \text{for } T = N\Delta t \qquad (18)$$

So, if the trader does not have to mark his book to market at intermediate times, and his hedging programme ends at option expiry, the terminal cumulative slippages only depend on his trading strategy (despite the fact that the individual terms ΔC_i^{true}, ΔC_i^{trad} will, in general, be different).

This is no longer true, however, if the expiry of the option expiry is greater than the last re-hedging step: $T > N\Delta t$, because now $C_N^{trad} \neq C_N^{true}$. In this case

$$A_N^{true,trad} = \left[\sum_i^N \Delta C_i^{true} - \sum_i^N \Delta^{trad} FRA_i \right] \quad \text{for } T = N\Delta t \qquad (19)$$

$$A_N^{trad,trad} = \left[\sum_i^N \Delta C_i^{true} - \sum_i^N \Delta^{trad} FRA_i \right] + \left[C_N^{trad} - C_N^{true} \right] \quad \text{for } T > N\Delta t \qquad (20)$$

and hence

$$A_N^{true,trad} = A_N^{trad,trad} + \left[C_N^{trad} - C_N^{true} \right] \quad \text{for } T > N\Delta t \qquad (21)$$

The equation above shows that the cumulative 'slippage' seen by trader who follows the wrong hedging strategy but does not have to mark to market could well be greater (by the term $C_N^{trad} - C_N^{true}$) if he stops the hedging strategy before the option expiry because he will be exposed to the difference between the market value and his faulty value for the option.

Consider now another trader, who also hedges according to the same faulty model, but has to mark his book to market at every step, and is subject to stop losses. Now what matters is not just the quantity $A_N^{true,trad}$, but all the quantities $A_k^{true,trad}$, $k < N$, becasue the trader may have to cut his postion before the expiry of the option. Extending the same notation, we can write

$$A_k^{trad,trad} = \sum_i^k \Delta C_i^{trad} - \sum_i^k \Delta^{trad} FRA_i \tag{22}$$

We can reapeat the reasoning above: since it is always true that

$$C_k^{trad} = C_0^{trad} + \sum_i^k \Delta C_i^{trad} \tag{23}$$

$$C_k^{true} = C_0^{true} + \sum_i^k \Delta C_i^{true} \tag{24}$$

One can write

$$A_k^{trad,trad} = \sum_i^k \Delta C_i^{trad} - \sum_i^k \Delta^{trad} FRA_i + \left[C_0^{true} + \sum_i^k \Delta C_i^{true} - C_k^{true} \right] =$$

$$= C_k^{trad} - C_0^{trad} - \sum_i^k \Delta^{trad} FRA_i + \left[C_0^{true} + \sum_i^k \Delta C_i^{true} - C_k^{true} \right] \tag{25}$$

By calibration,

$$A_k^{trad,trad} = \left[C_k^{trad} - C_k^{true} \right] + \left[\sum_i^k \Delta C_i^{true} - \sum_i^k \Delta^{trad} FRA_i \right] \tag{26}$$

So, even if the option expires exactly after N time steps, the trader is now exposed to the difference during the life of the option between his wrong model price and the true market price. Therefore a trader who uses a faulty model *and is subject to mark to market* may be forced because a stop loss to cut his position earlier and more frequently than a trader who uses the same faulty model, but can run his position until time T (which may be or may not be the expiry of the option).

To get a qualtitative feel for the effect, consider for instance Fig 1 below:In this picture Series 1 diaplays for a paerticular simulation the time series of the cumulative difference between the changes in the value of the call and the changes in the hedge (the 'slippage') when the correct model is assumed for hedging; Series 2 displays the slippage when the wrong model is used both for

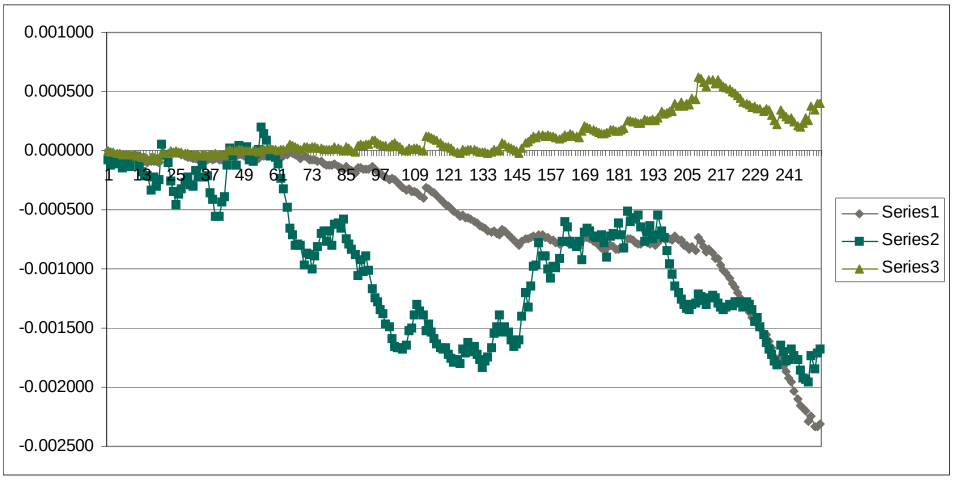

Figure 1: Fig 1

marking and for hedging; Series 3 shows the slippage when when the wrong model is used for hedging, but the book is correctly marked to market. Suppose that a stop loss had been placed at the level of, say, -0.0015. Then the trader who has to mark to market would have had to cut his position much earlier, even if by option expiry the wrong hedging produced a similar terminal slippage, for both ways of marking.

4 Results – Market is Right

The Table below displays the results for the case when the market is right.

T	K		$A_N^{true,true}$	$A_N^{true,trad}$	$A_N^{trad,trad}$	$\max\lvert A_k^{true,true}\rvert$	$\max\lvert A_k^{true,trad}\rvert$	$\max\lvert A_k^{t}$
1	4.0%	Avg	**0.0000**	**-0.0001**	**-0.0001**	**0.0004**	**0.0008**	**0.0006**
1	4.0%	StDev	*0.0004*	*0.0007*	*0.0007*	*0.0003*	*0.0003*	*0.0003*
1	5.0%	Avg	**0.0001**	**0.0000**	**0.0000**	**0.0004**	**0.0008**	**0.0007**
1	5.0%	StDev	*0.0004*	*0.0008*	*0.0008*	*0.0003*	*0.0005*	*0.0005*
2	4.0%	Avg	**0.0000**	**0.0002**	**0.0002**	**0.0002**	**0.0013**	**0.0007**
2	4.0%	StDev	*0.0002*	*0.0011*	*0.0007*	*0.0001*	*0.0004*	*0.0003*
2	6.0%	Avg	**0.0000**	**-0.0003**	**-0.0005**	**0.0002**	**0.0010**	**0.0006**
2	6.0%	Stdev	*0.0002*	*0.0009*	*0.0004*	*0.0001*	*0.0005*	*0.0001*

The details of the simulation were as follows. The number of time steps was 252 ($N = 252$), time step was daily ($\Delta t = 1/252$ yrs), the initial value of the forward rate was 4.00%, the percentrage volatility was 40.00%, no bid-offer spread, $a_{true} = a_{mkt} = 0$, $a_{trad} = 0.20$.

The following observations are in order.

- If the hedging strategy is followed, the expectation of the terminal slippage

($A_N^{true,true}$) is zero, as expexted, and the standard devaition of the slippage is 4 bp. This finite stabdard deviation is due to the finite re-hedging frequency, and provides the best possible benchmark against which the significance of other quantities can be recknoned.

- When $T = N\Delta t$, ie, when $T = 1$ yr, and one only looks at the terminal slippages ($A_N^{true,trad}$ and $A_N^{trad,trad}$), it clearly does not matter whether the intemediate changes in call price are recorded using the true proess or the trader-assumed process, because the (wrong) hedging strategy only depends on the (univerasally agreed upon) value of the forard rate, and by expiry $C_N^{trad} = C_N^{true} = (f_T - K)^+ = Payoff(T)$ (see Equation (18)).

- This is no longer true is $T \doteq N\Delta t$, becasue now $C_N^{trad} \neq C_N^{true}$.

- It is also not true at imtermediate times $k < N$, irrespective of the matirity, because $C_k^{trad} \neq C_k^{true}$.

- The quantities $\max|A_k^{true,true}|$, $\max|A_k^{true,trad}|$ and $\max|A_k^{trad,trad}|$ record the maximum cumulative slippage out to time k, in the case when

  - the trader hedges using the true (market) model and records at every time step the true market changes in call price
  - the trader hedges using the wrong (market) model and records at every time step the true market changes in call price
  - the trader hedges using the wrong (market) model and records at every time step the wrong market changes in call price, respectively. These are the quantities that would drive the stop loss of a trader.

- When $T = N\Delta t$, ie, when $T = 1$ yr, the trader incurs the smallest intermediate cumulative slippages using the true model (and, of course marking using the true model); when the wring hedging ratios are used, the average of intermiate slippages is somewhat smaller when the trader can mark-to-market daily using its own (fasle) model, than using the true call prices. The significance of the difference (judged in terms of 1 standard deviation of the results) is dubious. The same applies fo rat-the-money and out-of-the-money strikes. It is fair to say that, when the hedging and simulation end at option expiry it makes very little difference in the intemediate slippages whether one marks the call price to true market or wrong model.

- The results are very different when the option has not come to expiry bythe end of the simulation (ie, when the trader takes off his hedge before exoiry). Now the intemediate slippages are significantly smaller when the trader is allowed to mark at his wrong model, rather than at the tue market price. Specifically, $\max|A_k^{true,trad}|$ is six and four standard deivations higher than $\max|A_k^{trad,trad}|$. In terms of stop losses, allowing the tradetr to mark to his faulty model confers a substantial advantage.

5 The Market Is Wrong

Now we consider a more interesting situation: we entertain the possibility that the market may be 'wrong',

$$a_{mkt} \neq a_{true} \tag{27}$$

but that the trader is 'right', ie

$$a_{trad} = a_{true} \tag{28}$$

What strategy should the trader pursue? Clearly, if the trader is not subject to mark to market, and continues his hedging strstegy until option expiry, he should pursue his preferred (right) hedging strategy. The results will be the same as in the case analyzed above, and we will have

$$E\left[\sum_i^N \Delta C_i^{true} - \sum_i^N \Delta^{true} FRA_i\right] = E\left[\sum_i^N \Delta C_i^{trad} - \sum_i^N \Delta^{trad} FRA_i\right] = 0 \tag{29}$$

$$Var\left[\sum_i^N \Delta C_i^{true} - \sum_i^N \Delta^{true} FRA_i\right] = Var\left[\sum_i^N \Delta C_i^{trad} - \sum_i^N \Delta^{trad} FRA_i\right] > 0 \tag{30}$$

becausue $\Delta C_i^{true} = \Delta C_i^{trad}$ and $\Delta^{true} FRA_i = \Delta^{trad} FRA_i$.

However, if the trader follows the correct hedging strategy and is subject to mark-to-faulty-market during the life of the option he will experience greater P&L swings than if he were marked to the true model. If large enough, these swings may force him into a stop loss position before he can be shown right (ie, before expiry).

So, the trader who is marked to market can follow two strategies: he can hedge according to the wrong (market-endorsed) model; or he can hedge according to the true model. Let's consider the two cases in turn,

Consider

$$B_k^{true,true} = \left[\sum_i^N \Delta C_i^{true} - \sum_i^N \Delta^{true} FRA_i\right] \tag{31}$$

where the are using a different symbol (B) to remind ourselves that now the market is worng. As usual,

$$\left[C_0^{mkt} + \sum_i^k \Delta C_i^{mkt} - C_k^{mkt}\right] = 0 \tag{32}$$

and therefore

$$B_k^{true,true} = \left[\sum_i^N \Delta C_i^{true} - \sum_i^N \Delta^{true} FRA_i\right] + \left[C_0^{mkt} + \sum_i^k \Delta C_i^{mkt} - C_k^{mkt}\right] =$$

$$\left[C_k^{true} - \sum_i^N \Delta^{true} FRA_i\right] + \left[\sum_i^k \Delta C_i^{mkt} - C_k^{mkt}\right] =$$

$$= \left[\sum_i^k \Delta C_i^{mkt} - \sum_i^N \Delta^{true} FRA_i\right] + \left[C_k^{true} - C_k^{mkt}\right] \tag{33}$$

If we define

$$B_k^{mkt,true} = \left[\sum_i^N \Delta C_i^{mkt} - \sum_i^N \Delta^{true} FRA_i\right] \tag{34}$$

we now have

$$B_k^{mkt,true} = B_k^{true,true} + \left[C_k^{mkt} - C_k^{true}\right] \tag{35}$$

and, if $k = N$ and $T = N\Delta t$,

$$B_T^{mkt,true} = B_T^{true,true} \tag{36}$$

(Here we use the symbol B to reminf opurselves that the market is now 'wrong'). Again, if the trader is left alone to hedge in his own world without intemediate mark-to-market or stop losses, the cumulative slippage will, of course, be the one that applies to his hedging with the true model. Before expiry, however, he will be exposed to the differnce between his true option value and the faulty market value, $\left[C_k^{mkt} - C_k^{true}\right]$.

What if the trader hedged accoridng to the faulty model? Straightfoward rearrangements as above give

$$B_k^{mkt,mkt} = B_k^{true,mkt} + \left[C_k^{mkt} - C_k^{true}\right] \tag{37}$$

and therefore

$$B_k^{mkt,mkt} = B_k^{mkt,true} + \left[B_k^{true,mkt} - B_k^{true,true}\right] \tag{38}$$

The cumulative slippages arising from hedging using the wrong, but market-consistent) delta amount of FRA will therefore be greater or smaller than the cumulative slippages using the right amount of FRA depending on the correlation between the slippages $B_k^{mkt,true}$ and $\left[B_k^{true,mkt} - B_k^{true,true}\right]$.

The equation above can be rewritten as

$$B_k^{mkt,mkt} = \left[\sum_i^k \Delta C_i^{mkt} - \sum_i^k \Delta^{true} FRA_i\right] + \left[\sum_i^k \Delta^{true} FRA_i - \sum_i^k \Delta^{mkt} FRA_i\right] =$$

$$B_k^{mkt,true} + \left[\sum_i^k \Delta^{true} FRA_i - \sum_i^k \Delta^{mkt} FRA_i\right]$$

Whether the slippages $B_k^{mkt,mkt}$ are smaller or greater than the slippages $B_k^{mkt,true}$ will depend on the correlation between the terms of the terms $B_k^{mkt,true}$ and $\left[\sum_i^k \Delta^{true} FRA_i - \sum_i^k \Delta^{mkt} FRA_i\right]$. Intuition suggests that the sign should be negative. Suppose, for instance, that the wrong model sees the option too much in the money and the stock price goes up. Then

$$B_k^{mkt,true} = \left[\sum_i^k \Delta C_i^{mkt} - \sum_i^k \Delta^{true} FRA_i \right] > 0 \tag{39}$$

and

$$\left[\sum_i^k \Delta^{true} FRA_i - \sum_i^k \Delta^{mkt} FRA_i \right] < 0 \tag{40}$$

Similar reasoning canbe applied to the case when the stock price goes down. This suggests that negative correlation may prevail, and that, therefore,

$$B_k^{mkt,mkt} < B_k^{mkt,true} \tag{41}$$

This is a hypothesis that can be quantitatively tested with our simulation.

6 Results – Market is Wrong

The Table below displays the results for the case when the market is wrong.

T	K		$A_N^{true,true}$	$B_N^{mkt,true}$	$B_N^{mkt,mkt}$	$\max \lvert A_k^{true,true} \rvert$	$\max \lvert B_k^{mkt,true} \rvert$	$\max \lvert B_k^{m}$
1	4.0%	Avg	0.0000	-0.0001	-0.0001	0.0003	0.0005	0.0007
1	4.0%	StDev	0.0002	0.0002	0.0006	0.0002	0.0002	0.0003
1	5.0%							
1	5.0%							
2	4.0%	Avg	0.0000	-0.0002	0.0000	0.0002	0.0009	0.0006
2	4.0%	StDev	0.0002	0.0005	0.0006	0.0001	0.0002	0.0003
2	6.0%							
2	6.0%							

The following observations are in order

- First of all, we report the results of the simulation when the hedge is carried out on the basis of knowledge of the true process and also the changes in call prices are recorded according to the true (non-market) model. This gives us a best-case benchmark of precision.
- When the option comes to its own expiry by the end of the simulation ($T = N\Delta t$, ie, when $T = 1$ yr), there is little difference in the average value of the *terminal* value of the slippages, but there is a significantly greater standard deviation (uncertainty of outcome) if the trader follows the hedging strategy implied by the wrong market model, than if he follows the true hedging strategy. So, if the trader is only interested in the terminal slippage and carries out his hedging programme to option expiry, *he is better off using the true process.*
- If we still remain in the case when the option comes to its own expiry by the end of the simulation ($T = N\Delta t$, ie, when $T = 1$ yr), but we now look

at the intermediate slippages, we note the following smallest value for the intermrediate slippages.

  - again, unsurprisigly, hedging and marking according to the true (non-market) model would give the smallest value for the intermrediate slippages;
  - when the call prices are recorded according to the wring model, the slippages become statistically insignificant depending on whether the trader hedges following the true or the wrong market model;
  - there is indeed a negative correlation between the terms terms $B_k^{mkt,true}$ and $\left[\sum_i^k \Delta^{true} FRA_i - \sum_i^k \Delta^{mkt} FRA_i\right]$, as anticipated, but it is rather small, and border line significant.

- If we move to the case when the option does not expire by the end of the hedging programme (and the simulation), the resullts are radically different:

  - by end of the hedging programme there is no longer any statistically siginificant difference between hedging according to the true or the wrong (market) model;
  - at intermediate times *there now is a statistically significant advantage in hedging according to the wring (market) model, rather than according to the true model.*
  - the negative correlation between the terms terms $B_k^{mkt,true}$ and $\left[\sum_i^k \Delta^{true} FRA_i - \sum_i^k \Delta^{mkt} FRA\right.$ is now very high and very clearly significant.

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7 Conclusions

We have shown that, if we are prepared to entertain the possibility that the market can be wrong, then how the trader should hedge depends in a subtle but important way on the nature of its option portfolio and on the institutional constraints he faces. If the trader cannot be sure that he will be allowed to complete his hedging programme, I have shown that hedging according to the true model while being marked to the wrong market can be extremely dangerous. When the market is wrong, hedging in line with the way one's book is marked may be theoretically dubious, but is often the better part of valour.

Given that, if the market is wrong, very large losses can be incurred even by the 'true' trader, one must take with extreme cautions statements like the ones reported above about the Japanese option market of the 1990s. At least for a model feature like the true exponent of a CEV-like process, the signal-to-noise ratio in the slippages (which are, in a theoretical set-up, the ultimate punishment of the wrong trader) is just too weak to give an unambiguous indication of what the true model is. This applies even in the extremely stylized and favourable conditions set out in this study, even when, that is,

- the noise has been kept at extremely, and unrealistically, low levels (zero transaction costs, daily rehedging in any size),
- we have endowed the trader with *perfect* knowledge of the true model, and we have made the difference between the true and the wrong model the largest possible within the CEV family.

If ensuring one's own preservation is the traders' first goal, being able to co-ordinate one's actoin with the actions of the fellow traders becomes a much more advisable strategy than pursuing with dedication the true hedging strategy. Keynes' parable of the beauty context obviously springs to mind.

This analysis therefore shows that, in a way, yes, the market *is* always right, but more in the sense of 'right' that theuse when we say that 'the customer is always right', rather than in the sense of a perfect Bayesian information-processing machine.