Pricing Contingent Capital With a Capital-Ratio Trigger

Paul Glasserman
Columbia Business School

Based on joint work with Behzad Nouri

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Contingent Capital: Where Are We?

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- Swiss plan:
  - 9% CoCo requirement + 10% equity
  - Partial conversion at different levels

- Investor demand (and regulatory treatment) uncertain

Issuance To Date

- Lloyds Banking Group, Nov 2009: CoCo bonds that convert if a capital ratio is breached
- Yorkshire/Chelsea Building Society, Dec 2009
- Rabobank, Mar 2010, Jan 2011: debt with an automatic write-down
- Credit Suisse CoCos, Feb 2011 (and more to come)
- Barclays…?

All based on regulatory capital ratio triggers

Triggers

Regulatory (capital-ratio) trigger:

+ Not subject to market manipulation
+ Incorporates regulators' superior information
– Backward looking and slow to react
– May be subject to political manipulation

Market (stock price) trigger:

+ Forward-looking and quick to react
+ Incorporates market's superior information
– May be subject to market manipulation
– May be more vulnerable to false alarms

Other Dimensions

Gone concern vs. Going concern

- For valuation, just a matter of the *level* of the trigger
- Legal differences related to bankruptcy, voting rights

Stepped Conversion vs. All-In Conversion

- Stepped: Partial conversion when trigger is hit
- All-In: Full conversion first time trigger is hit

Related Work

- Flannery (2005,2009):
  - Proposed market trigger, stepped conversion
- McDonald (2010), Squam Lake Working Group (2010)
  - Dual trigger: market and systemic
- Pennacchi et al. (2010)
  - Market trigger, buyback option for equity holders
- Albul et al. (2010)
  - Asset value trigger
- Sundaresan and Wang (2010)
  - Pitfalls of market triggers

We give closed-form valuations for CC with a capital-ratio trigger and stepped conversion

Conversion: First Look

10% capital requirement just met

Assets		Liabilities	
V =	100	D =	60
		B =	30
		Q =	10

(a)

Loss of 5 in asset value absorbed by equity

Assets		Liabilities	
V =	95	D =	60
		B =	30
		Q =	5

(b)

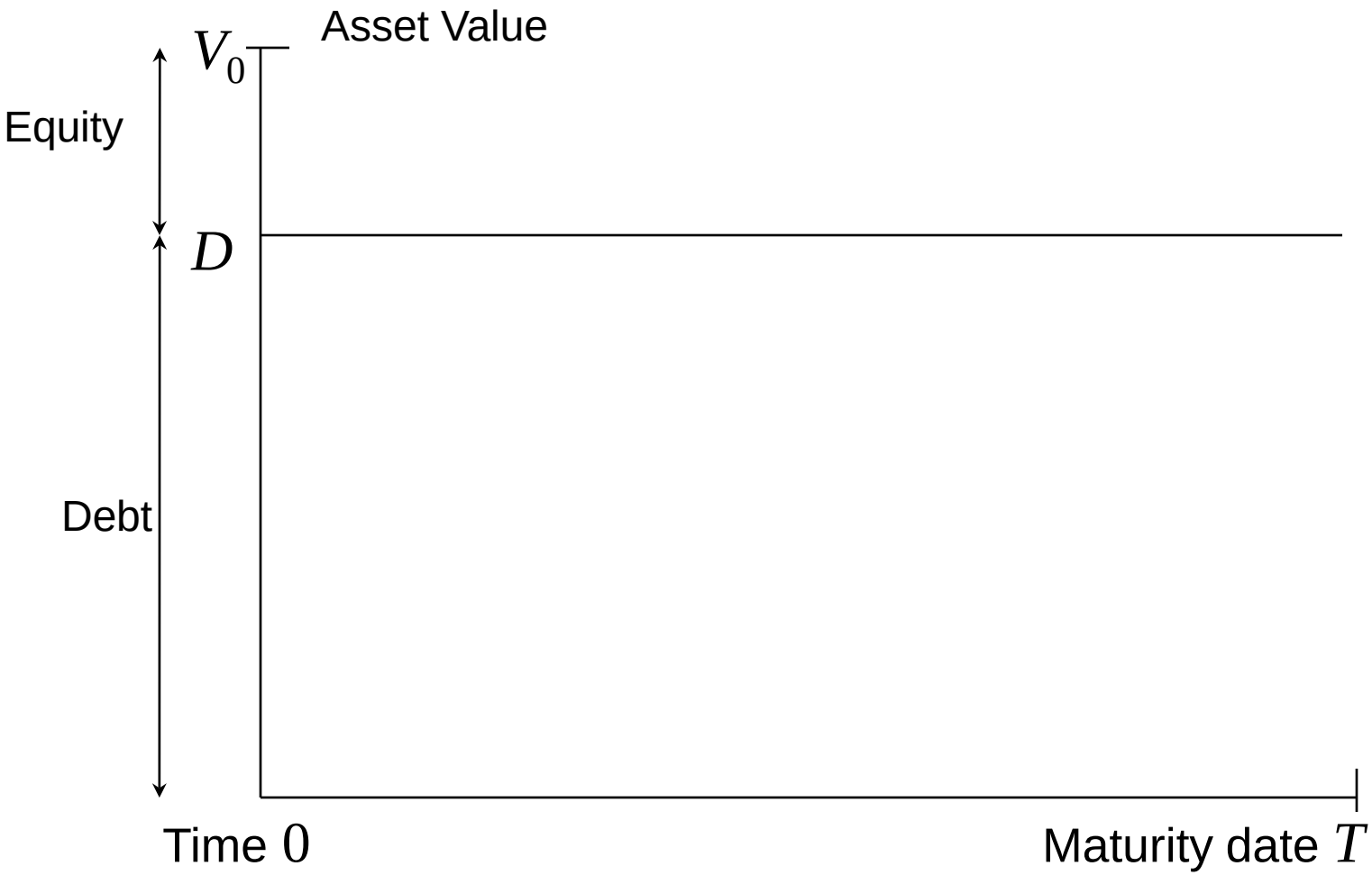
Conversion of debt restores 10% capital

Assets		Liabilities	
V =	95	D =	60
		B =	25.5
		Q =	9.5

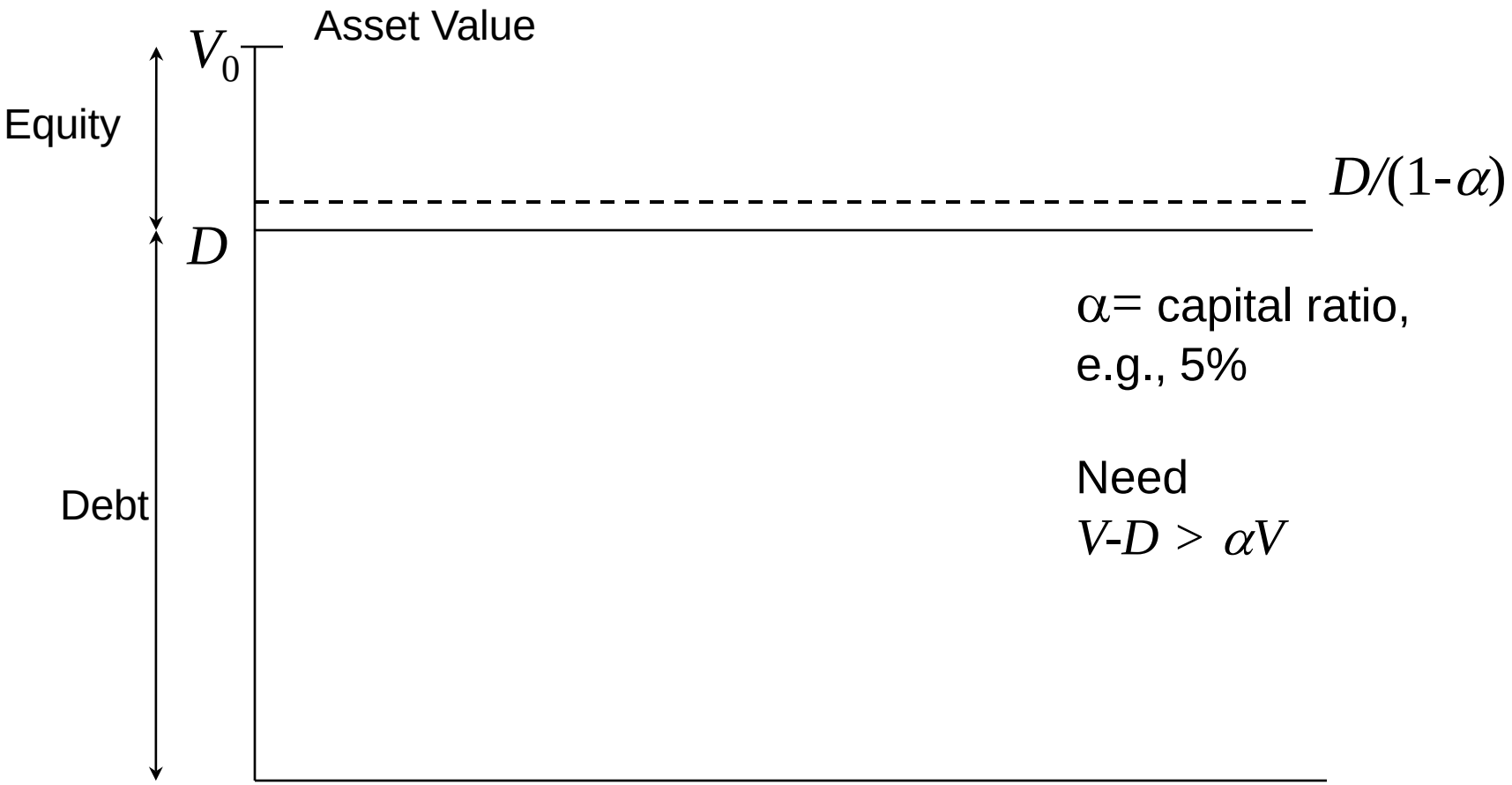
(c)

V = asset value
D = straight debt
B = convertible debt
Q = V-D-B = shareholder's equity (measure of capital)

Structural Model (Merton, Black-Cox, Leland…)



Without Contingent Capital



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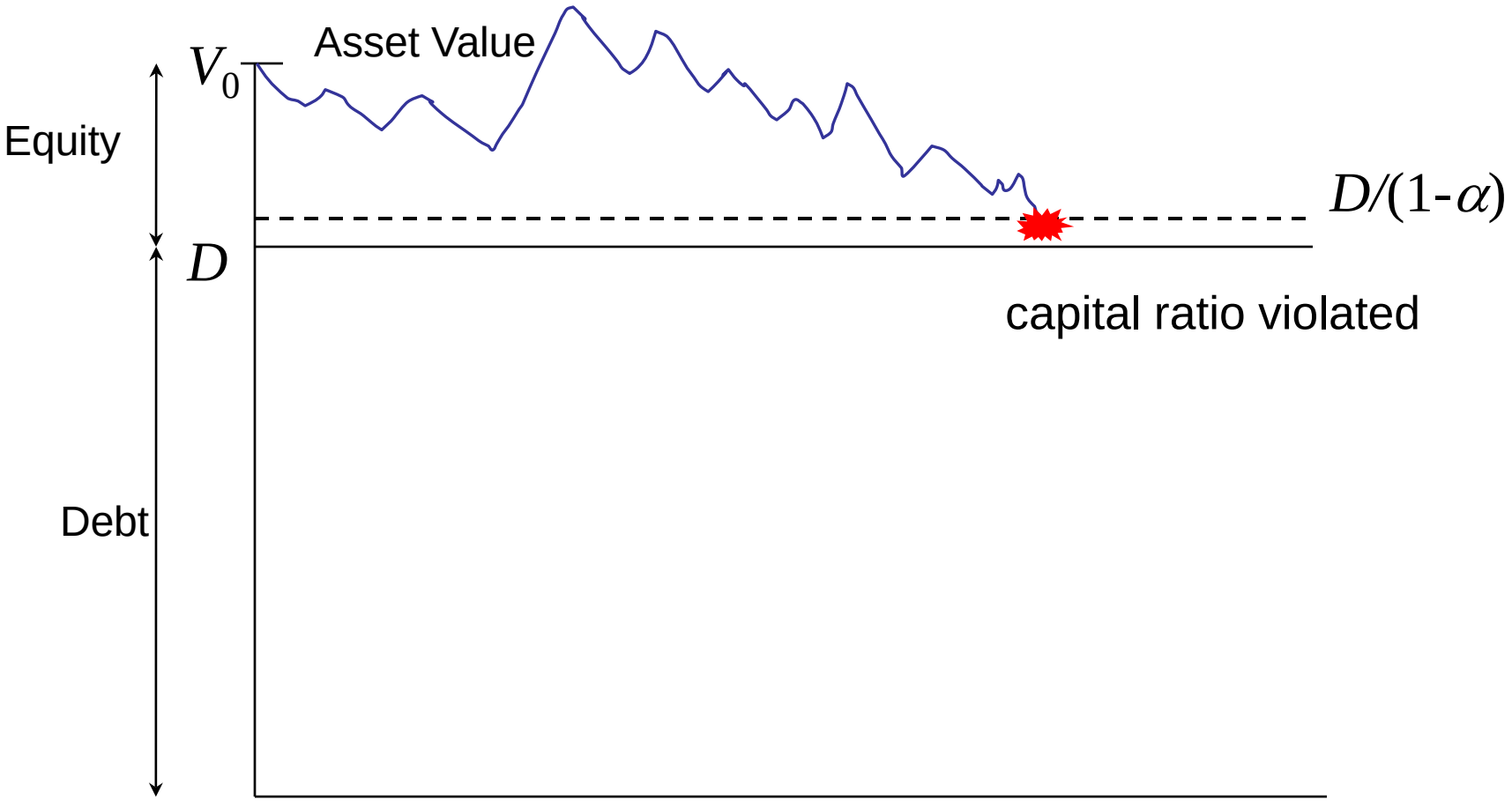

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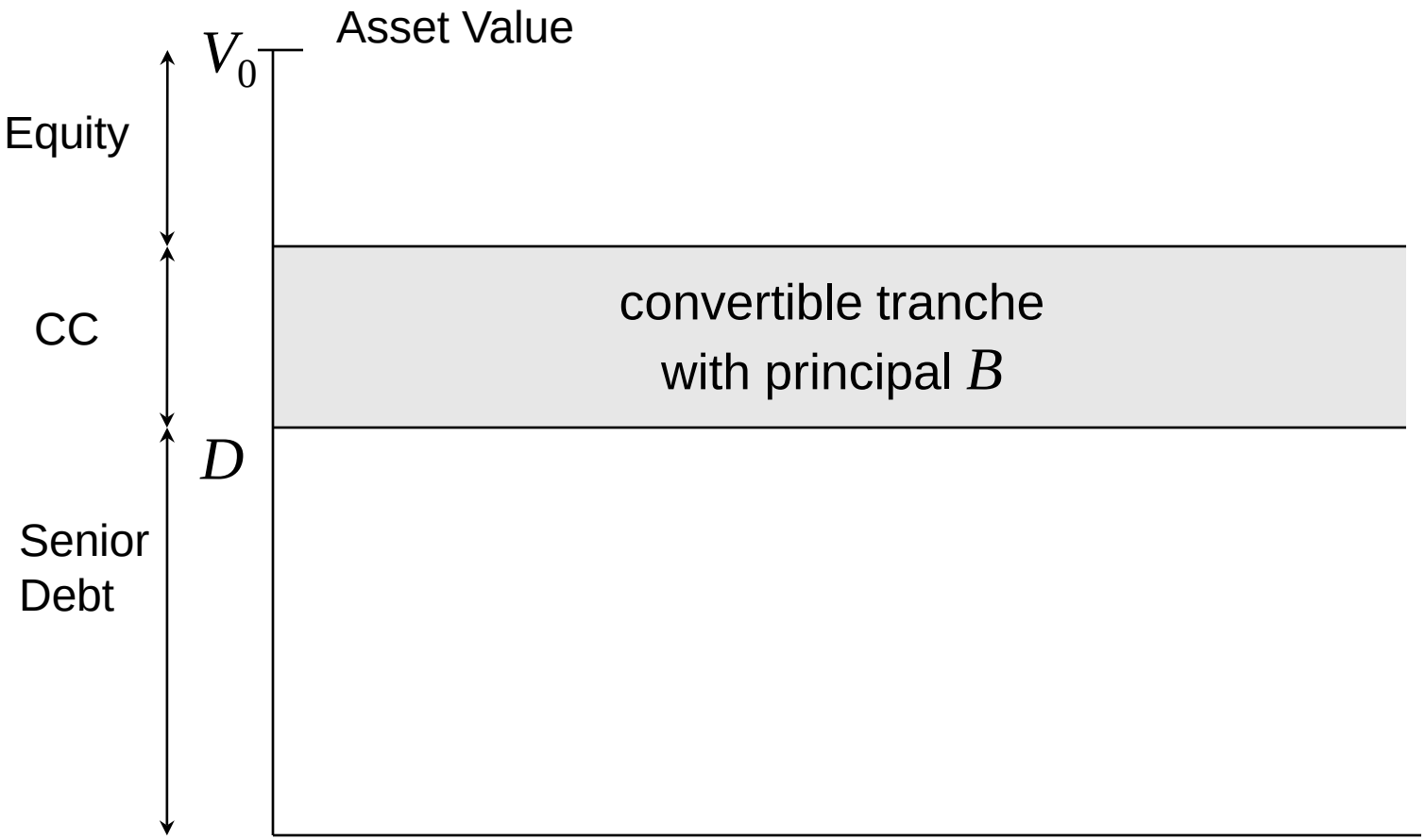
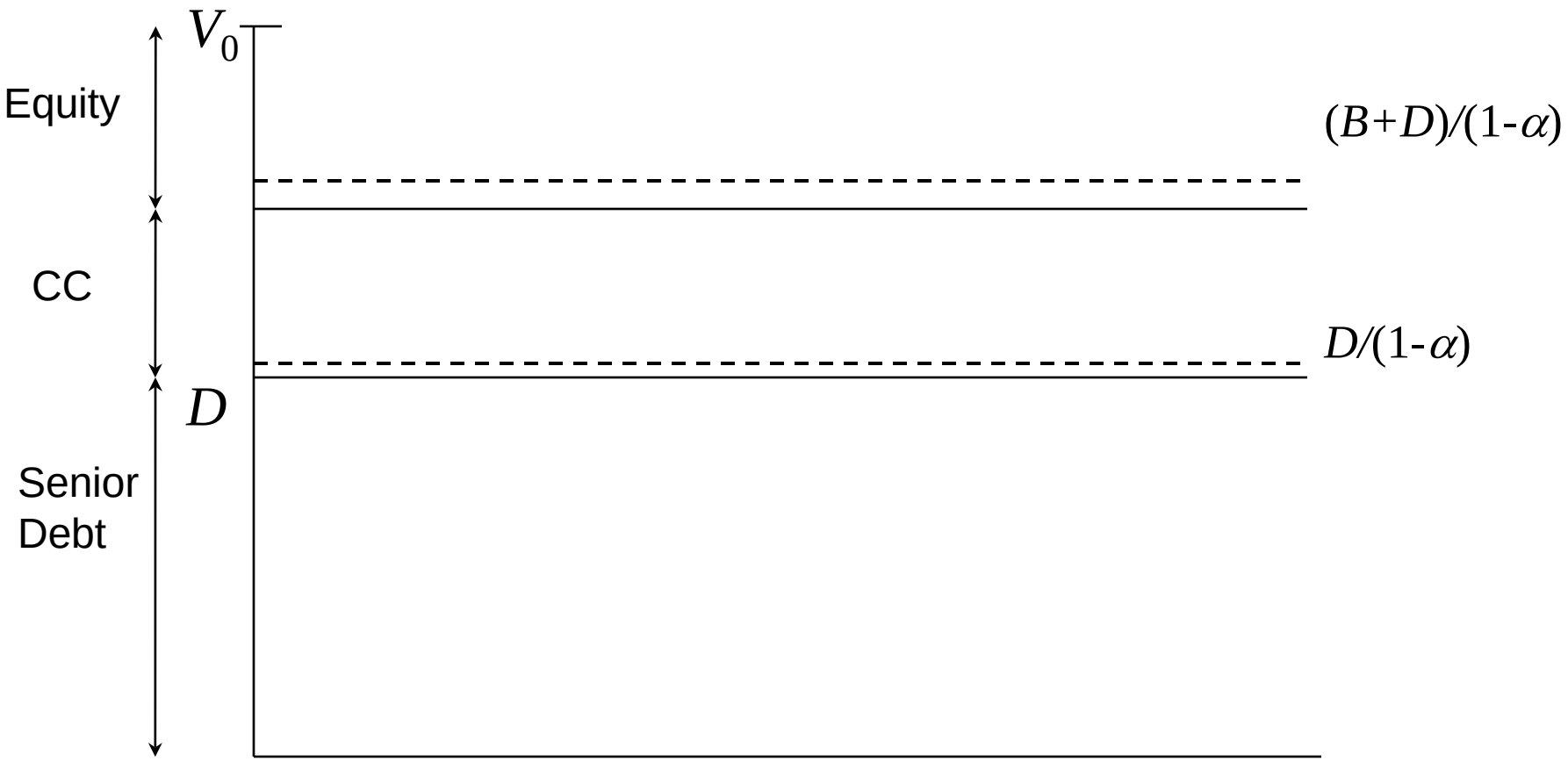
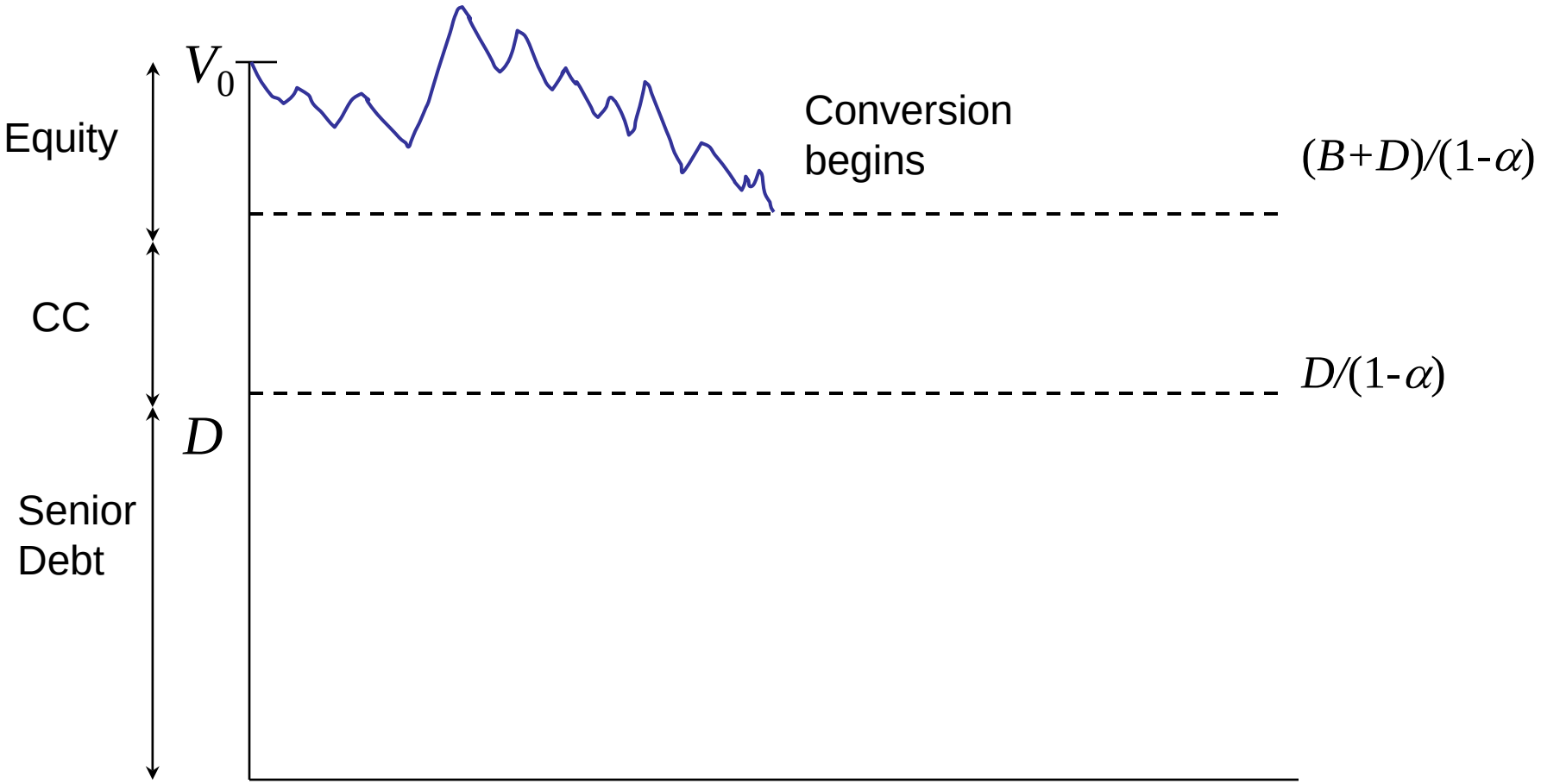


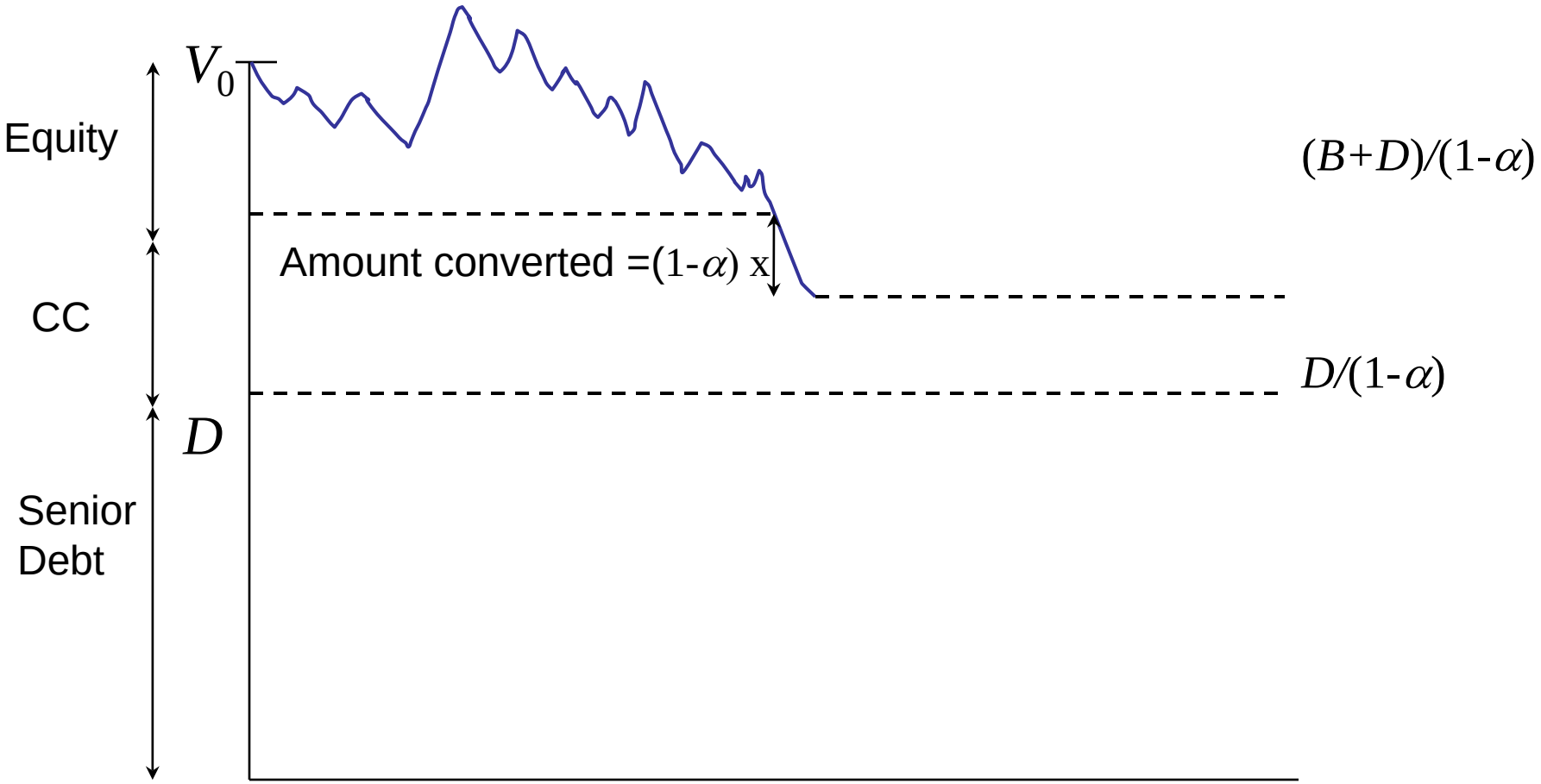
Conversion Boundaries



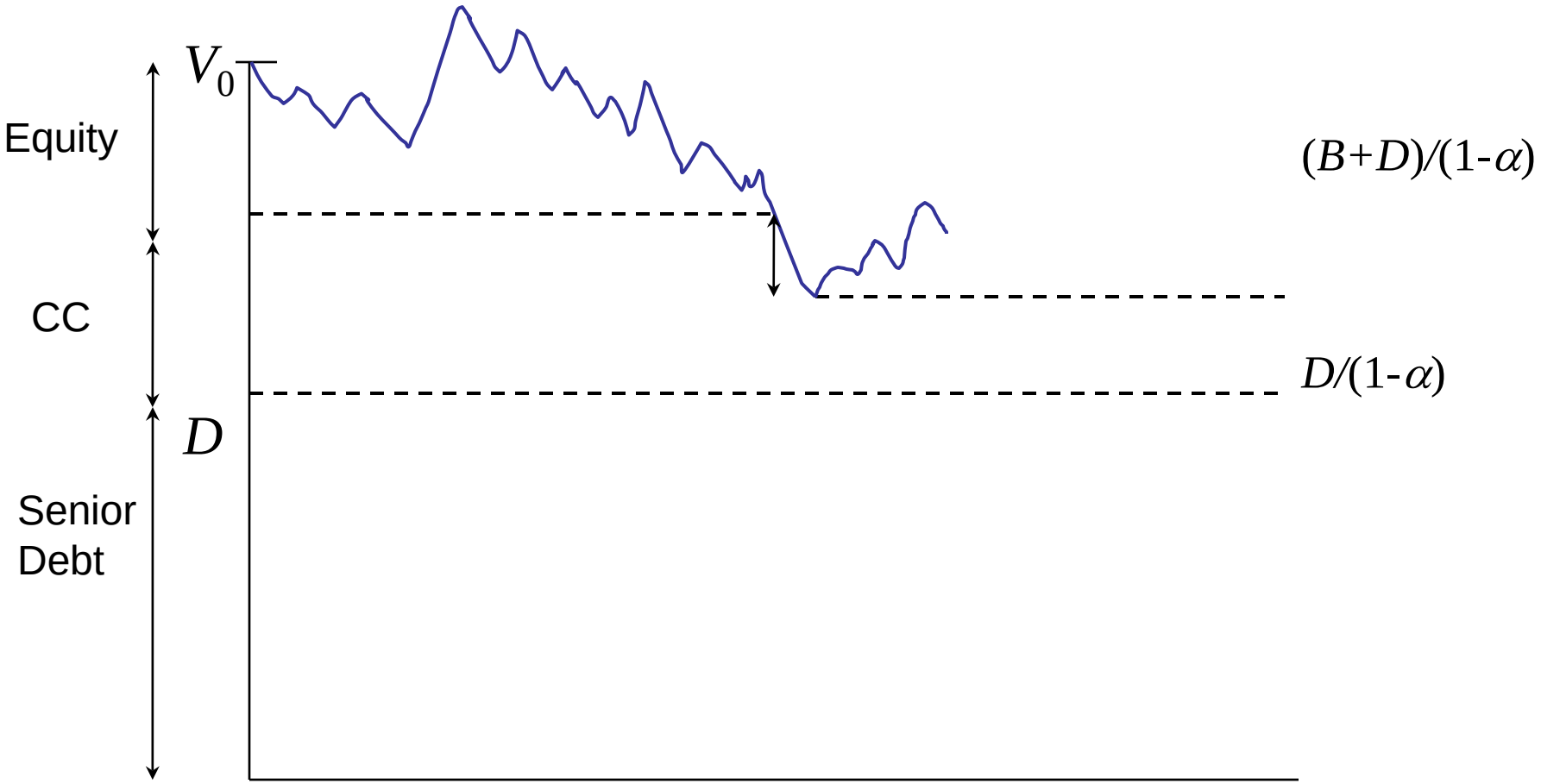
With Contingent Capital



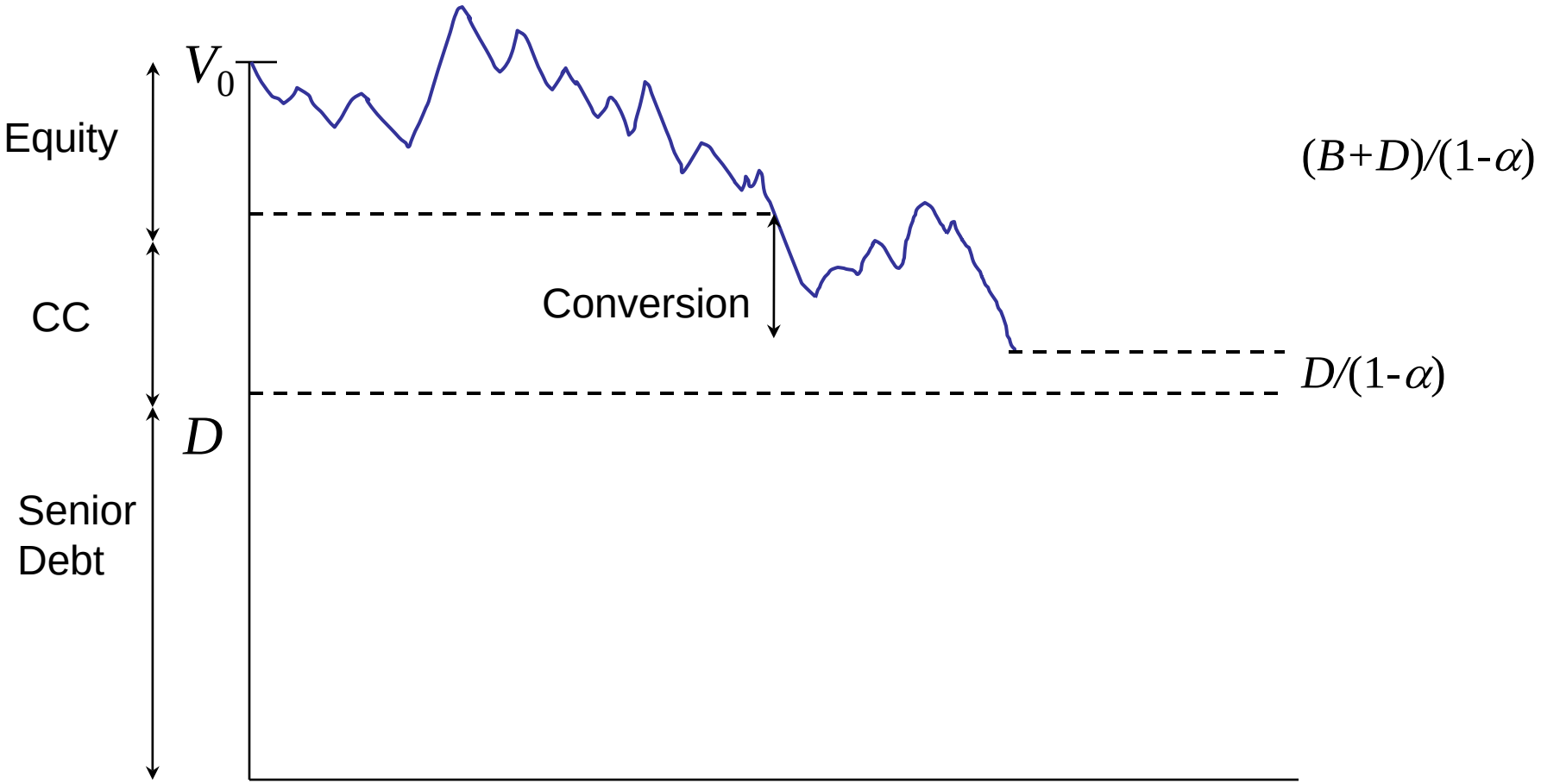
Conversion to Equity to Maintain Ratio



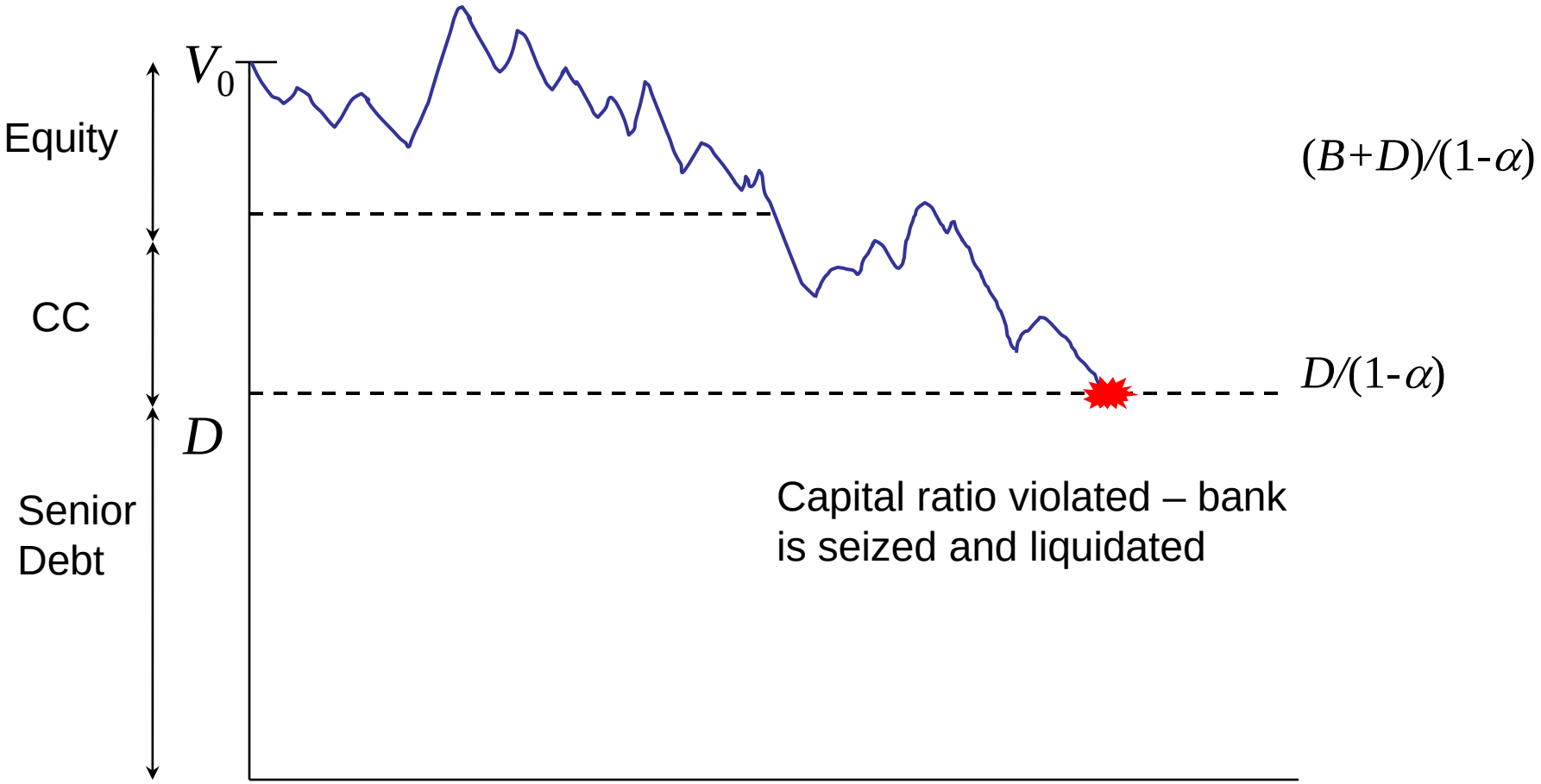
No Conversion or Unconversion



New Minimum, More Conversion



Contingent Capital Exhausted

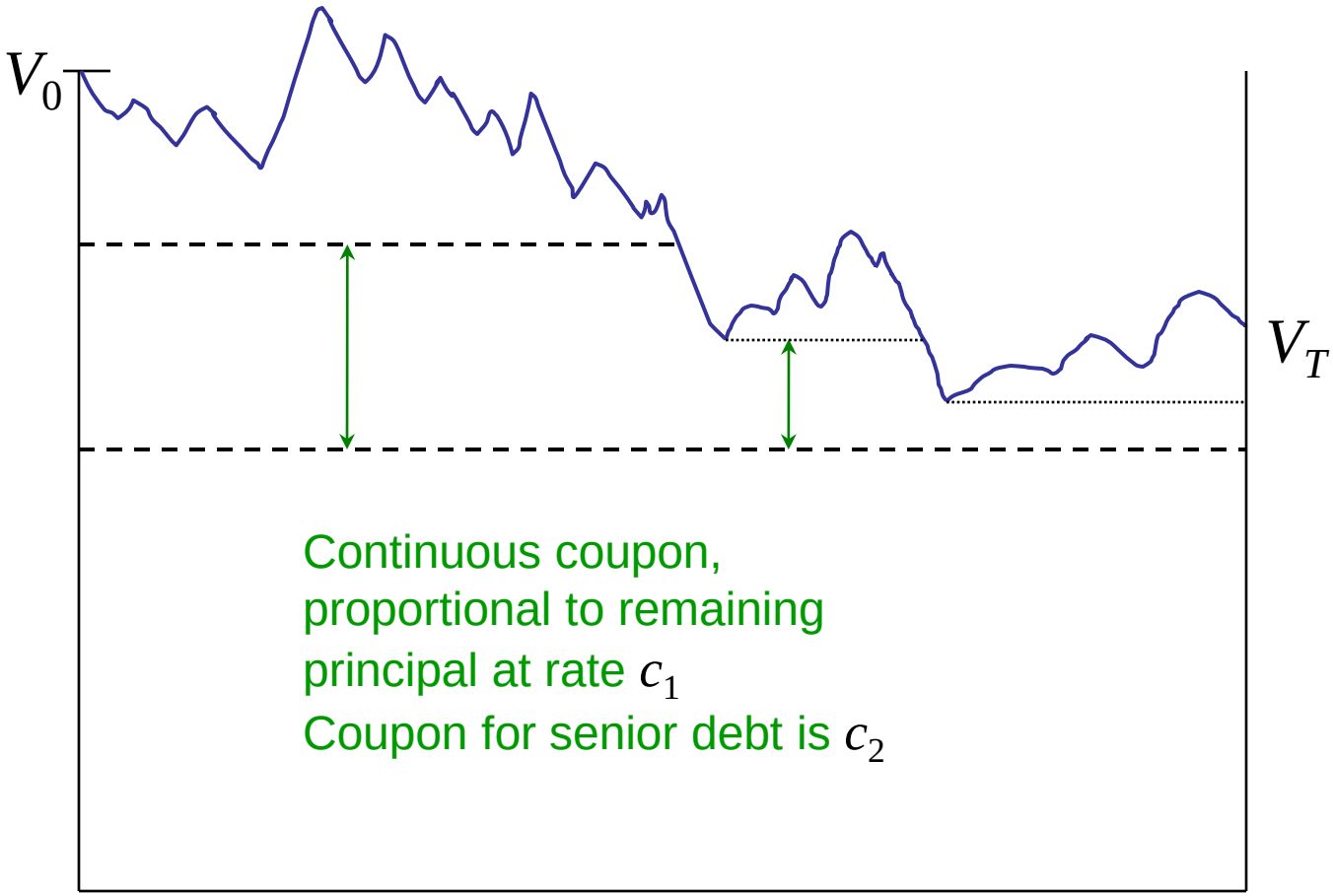


Valuation

To value the contingent capital, we need to value its pieces:

- Coupon payments on the debt
- Principal payment on the debt
- Dividends earned on equity after conversion
- Final value of earned equity at maturity

What Do Investors Get?



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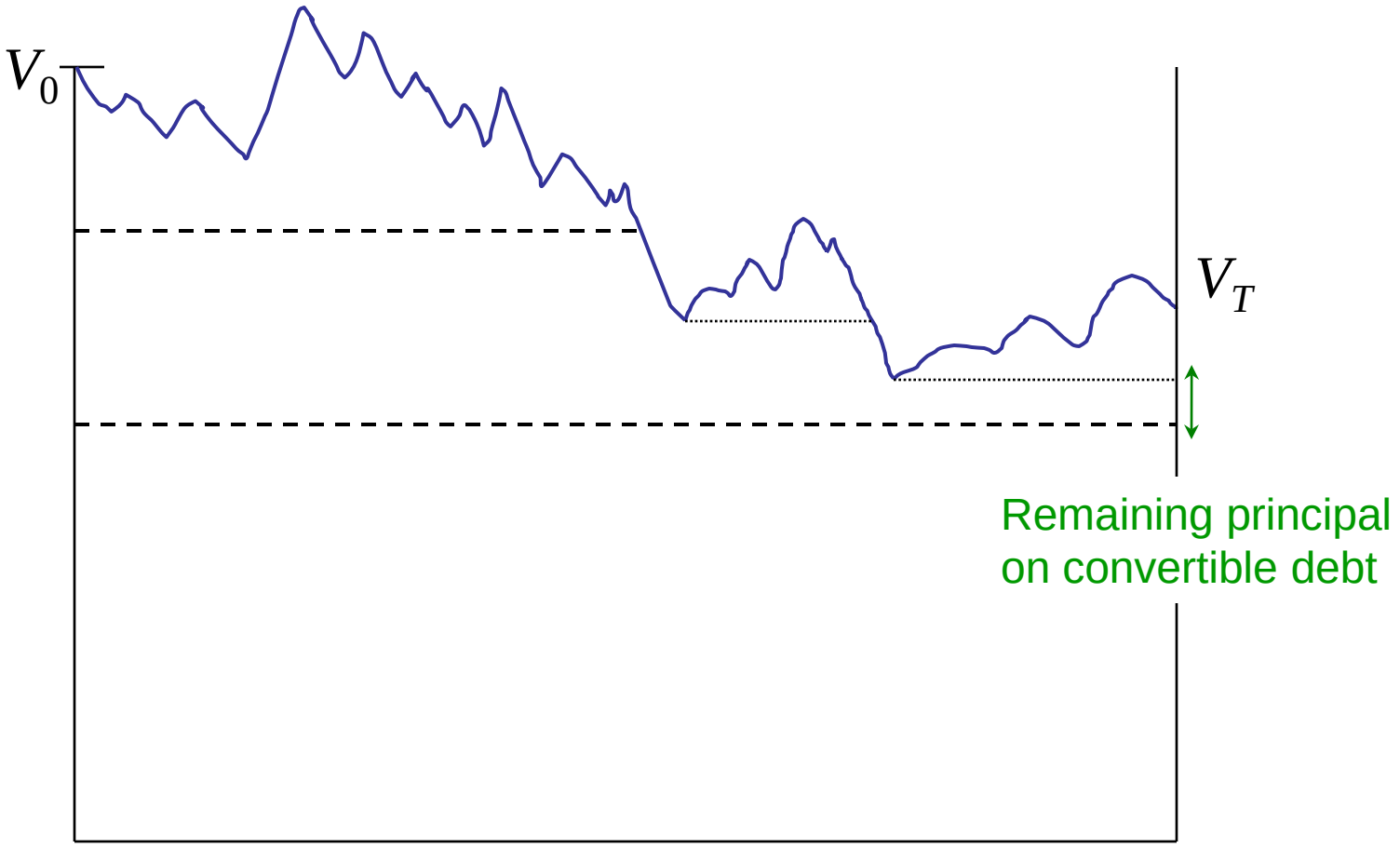

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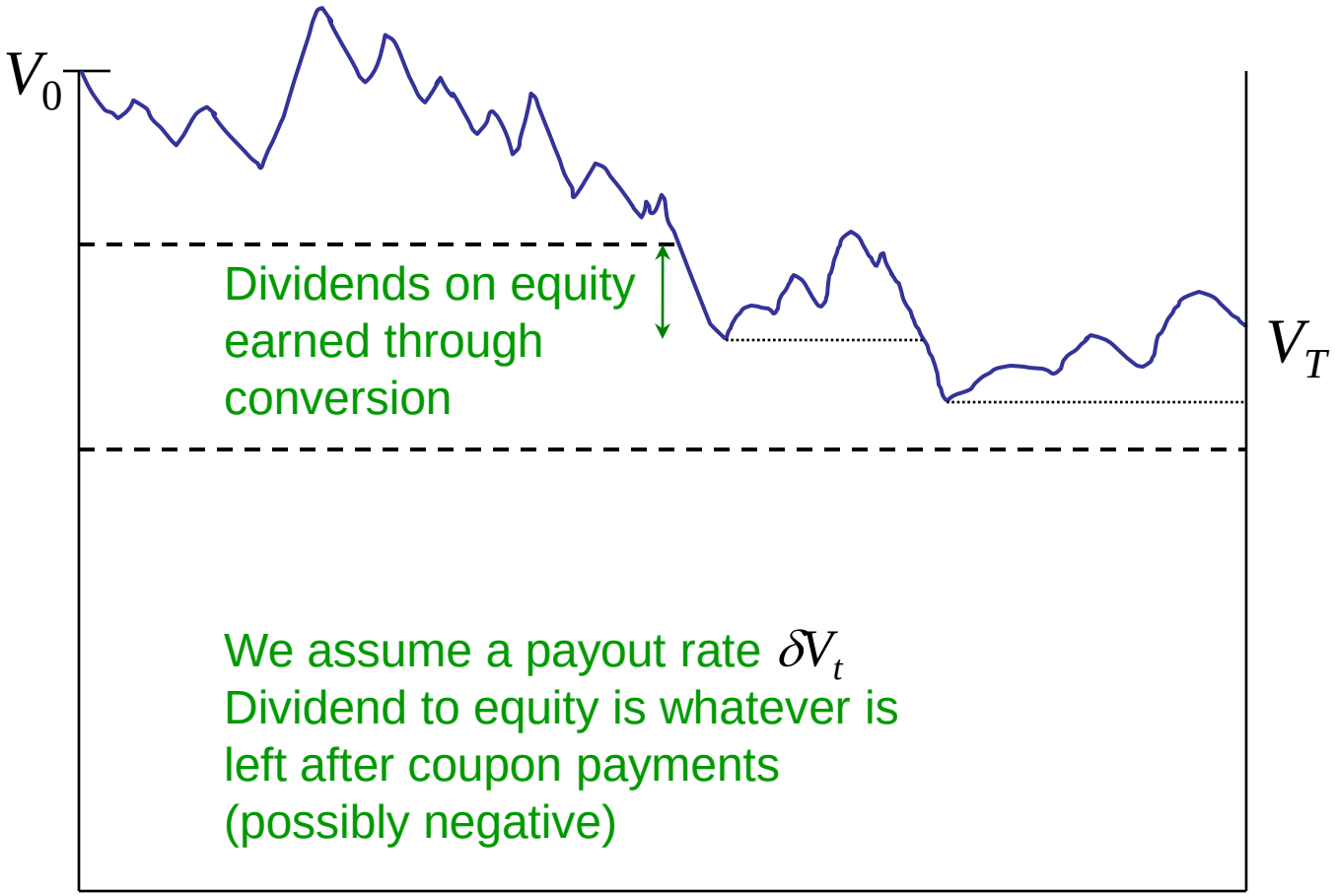

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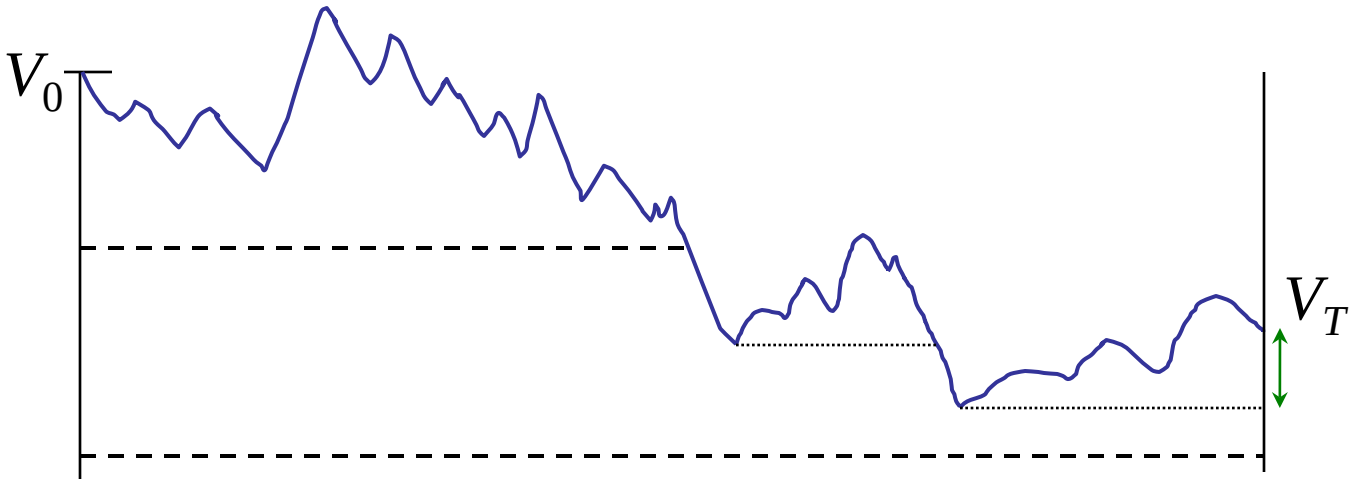


- How much debt has been converted?
- What fraction of equity is owned by converted investors?

How Much of the Debt Has Been Converted?

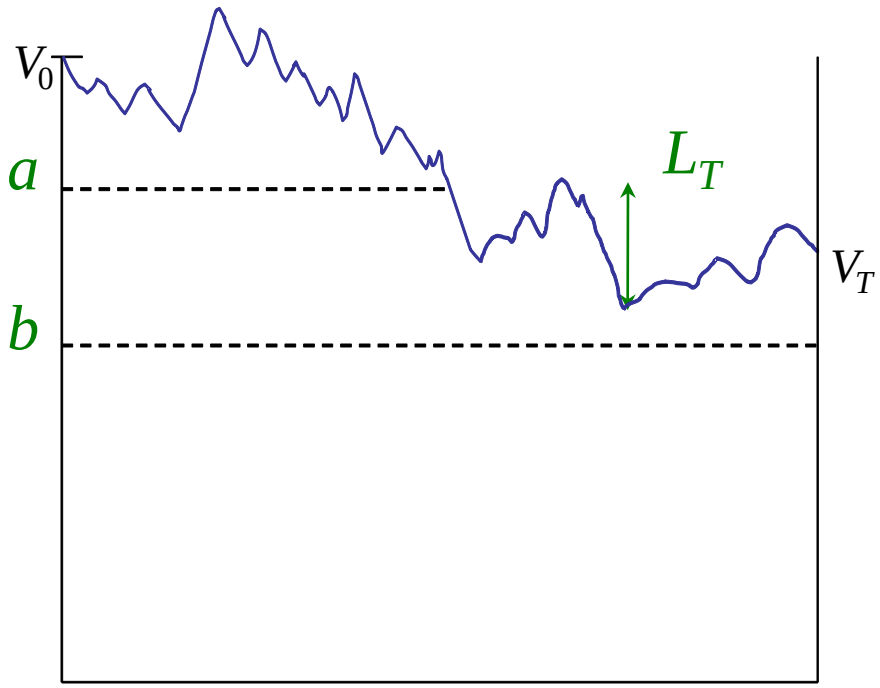
Proposition: Amount converted under minimal conversion is

$$(1\text{-}\alpha)L_t$$

where

$$L_t = \min\left\{\left(a - \min_{0\le s\le t} V_s\right)^+, a-b\right\}$$

This is essentially the reflection map control, but applied to maintain a ratio rather than a difference.



How Much of the Firm Do the Original Investors Own?

Assets		Liabilities	
V =	100	D =	60
		B =	30
		Q =	10

(a)

Assets		Liabilities	
V =	95	D =	60
		B =	30
		Q =	5

(b)

Assets		Liabilities	
V =	95	D =	60
		B =	25.5
		Q =	9.5

(c)

- With 1-for-1 conversion, original shareholders go from 100% ownership in (a) to 5/9.5 = 53% ownership in (c)
- Fraction of ownership determines allocation of dividends and terminal equity value

How Much of the Firm Do the Original Investors Own?

Total equity

$$Q_t = V_t - [B - (1-\alpha)L_t] - D$$

Define : Equity held by original shareholders

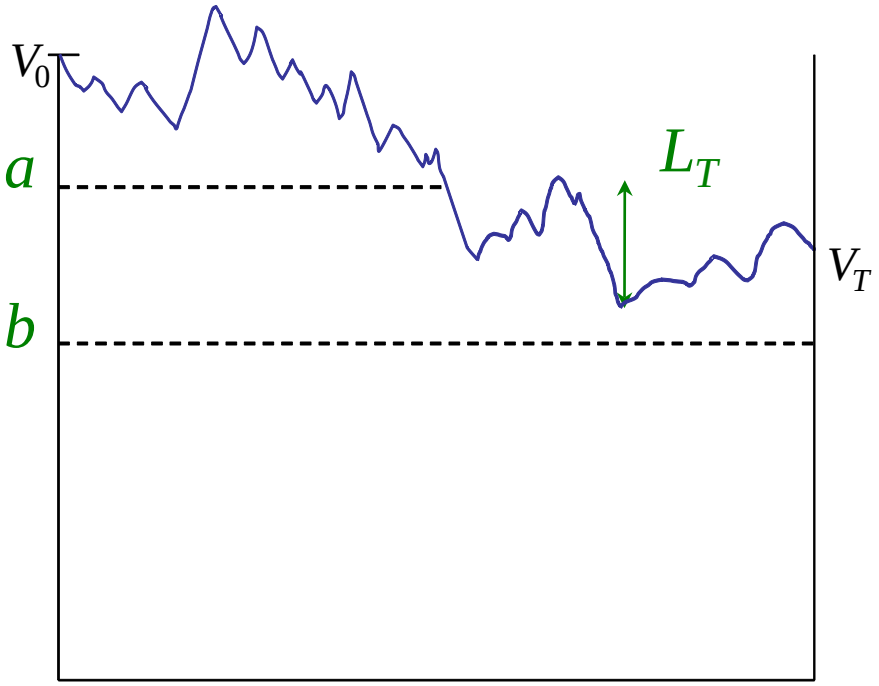
$$\frac{dQ_t^o}{Q_t^o} = \frac{dQ_t}{Q_t} - (1-\alpha)\frac{dL_t}{Q_t}, \quad Q_0^o = Q_0$$

Fraction held by original shareholders $= \pi_t = Q_t^o / Q_t$

How Much of the Firm Do the Converted Investors Own?

Theorem: If V is a continuous semimartingale, the fraction of equity owned by the original shareholders is

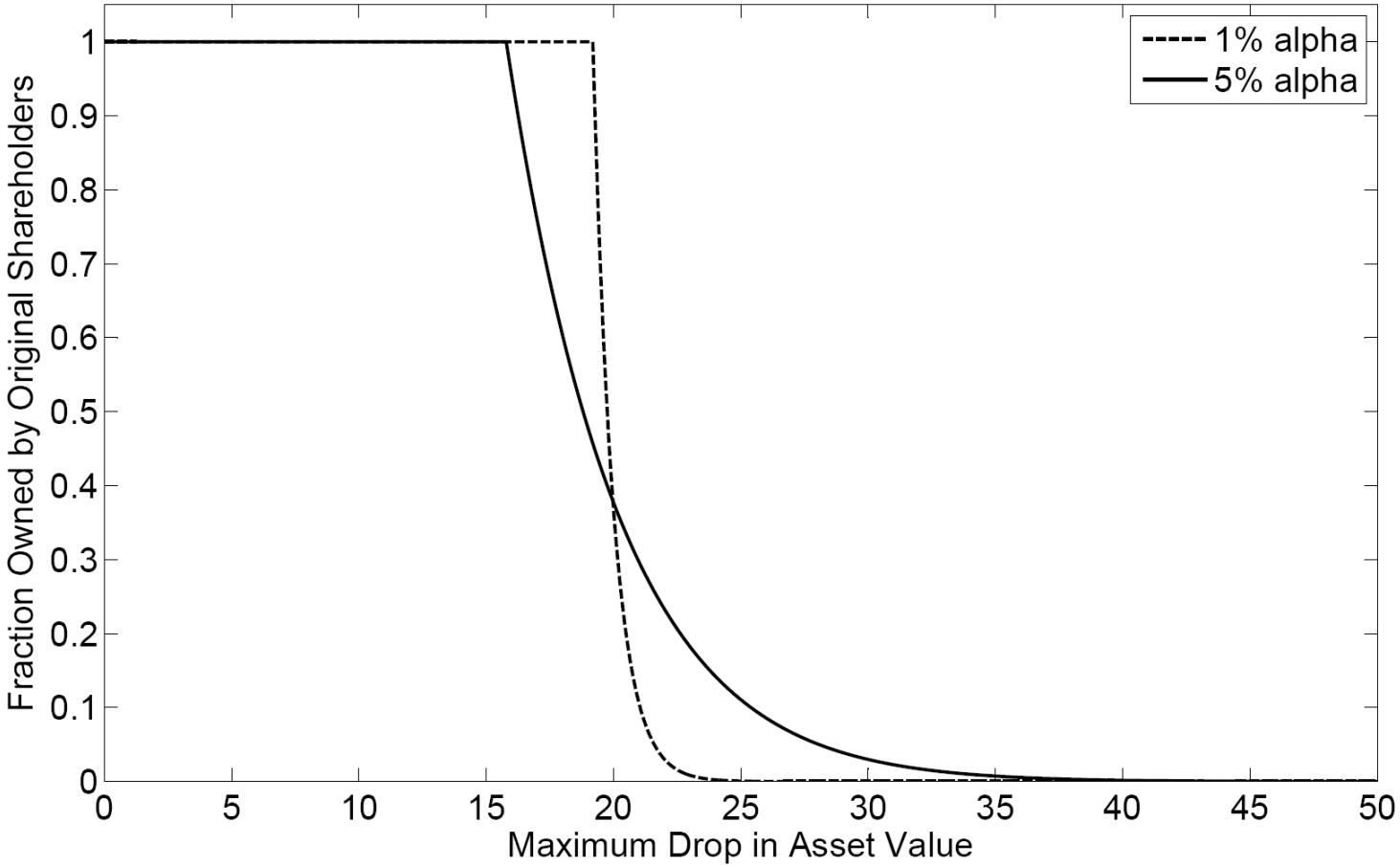
$$\pi_t = \left(\frac{a - L_t}{a} \right)^{\frac{1-\alpha}{\alpha}}$$



Key point: Terminal fractions owned by the original and new shareholders depends only on the minimum asset value

$$\frac{d\pi_t}{\pi_t} = -\frac{1-\alpha}{\alpha}\left(\frac{dL_t}{a - L_t} \right)$$

Who Benefits From Stricter Capital Requirement?



Back to Valuation

Principal payment

$$e^{-rT}\left[B-(1-\alpha)L_T\right]$$

Running coupon

$$\int_0^T e^{-rt}c_1[B-(1-\alpha)L_t]dt$$

Equity share on survival

$$e^{-rT}(1-\pi_T)(V_T-[(B-(1-\alpha)L_T)+D])1\{\tau_b>T\}$$

Equity share on early seizure

$$e^{-r\tau_b}(1-\pi_{\tau_b})R_1\alpha V_{\tau_b}1\{\tau_b<T\}$$

Running dividends on converted equity

$$\int_0^{\min(\tau_b,T)} e^{-rt}(1-\pi_t)(\delta V_t-(1-\kappa)[c_1(B-(1-\alpha)L_t)+c_2D)dt$$

All reduce to joint distribution of V and its running minimum

Master Transform

These can all be evaluated in closed-form (up to a time-average) using

$$W_t = \log(V_t / V_0), \quad m_t = \min_{0 \le s \le t} W_s$$

and

$$H(t, v, k, y) = E\left[\exp\left(vW_t + km_t\right) 1\{m_t \le y\}\right]$$

Proposition: $H(t, v, k, y) = \exp\left(-\mu v t + v^2 \sigma^2 t / 2\right) h(k, y)$

$$h(k, y) = \frac{2\theta}{2\theta + k\sigma^2} e^{ky + 2y\theta/\sigma^2} \Phi\left(\frac{y + t\theta}{\sigma\sqrt{t}}\right) - \frac{2\theta + 2k\sigma^2}{2\theta + k\sigma^2} e^{k\theta y + k^2/\sigma^2} \Phi\left(\frac{(\theta + k\sigma^2)t - y}{\sigma\sqrt{t}}\right)$$

Valuation

Each of the pieces of the payoff

$$e^{-rT}\left[B-(1-\alpha)L_T\right]$$

$$\int_0^T e^{-rt}c_1[B-(1-\alpha)L_t]dt$$

$$e^{-rT}\left(1-\pi_T\right)\left(V_T-\left[(B-(1-\alpha)L_T)+D\right]\right)1\{\tau_b > T\}$$

$$e^{-r\tau_b}(1-\pi_{\tau_b})R_1\alpha V_{\tau_b}1\{\tau_b < T\}$$

$$\int_0^{\min(\tau_b,T)} e^{-rt}(1-\pi_t)(\delta V_t-(1-\kappa)[c_1(B-(1-\alpha)L_t)+c_2D)dt$$

Is a linear combination of products of powers of V, its running minimum, and tail indicator functions of the running minimum (or time-averages thereof)

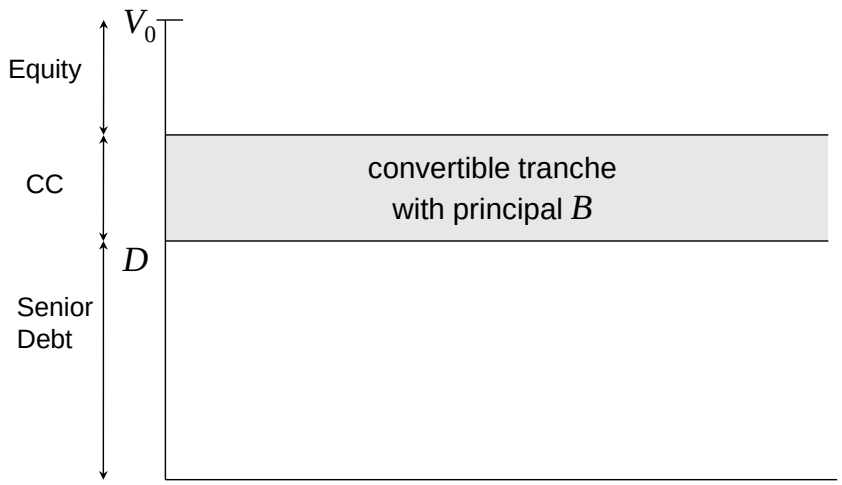
So, their expectations are linear combinations of H values (or time-averages thereof)

Closing the Model

The calculations take the coupon rates as given

For internal consistency, we now solve for the coupons that make the picture correct:

Market price of debt = face value at time zero



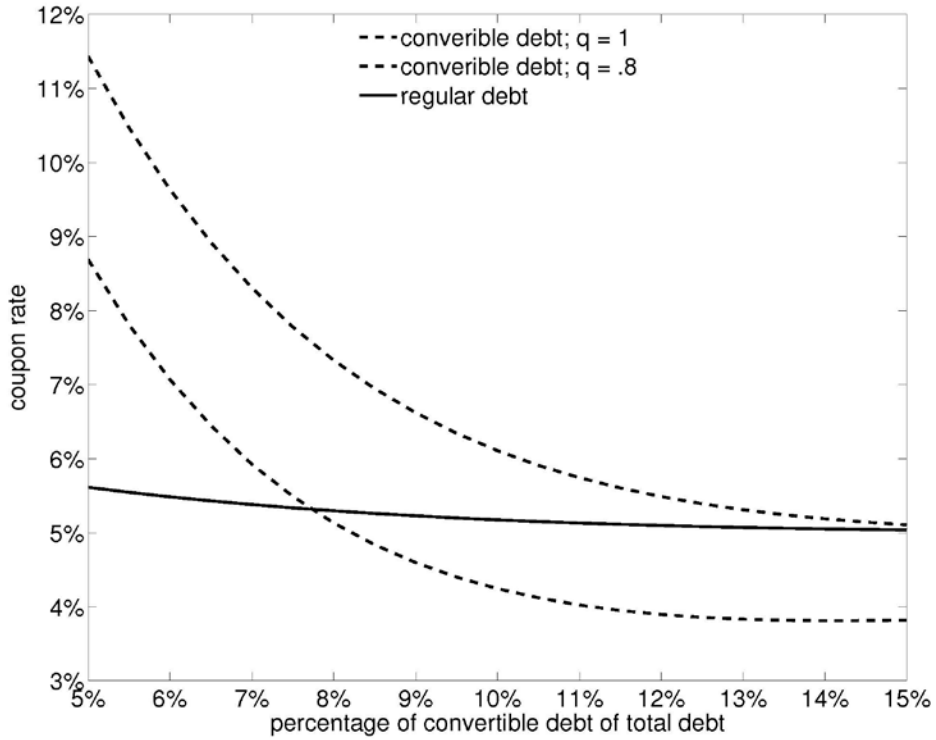
We do this in closed form

This gives the yield required by the market to compensate investors for the risk of conversion

Parameters for Numerical Examples

		I	II
Debt over assets ratio	D/V_0	90%	
Capital adequacy ratio	α	4%	
Risk free rate	r	5%	0.5%
Volatility of asset returns	σ	8%	16%
Debt maturity	T	1.5	
Fractional payout of assets	δ	3%	1.5%
Tax rate	κ	30%	
Recovery rate for equity	R_1	60%	
Recovery rate for senior debt	R_2	95%	
		~2006	~2009

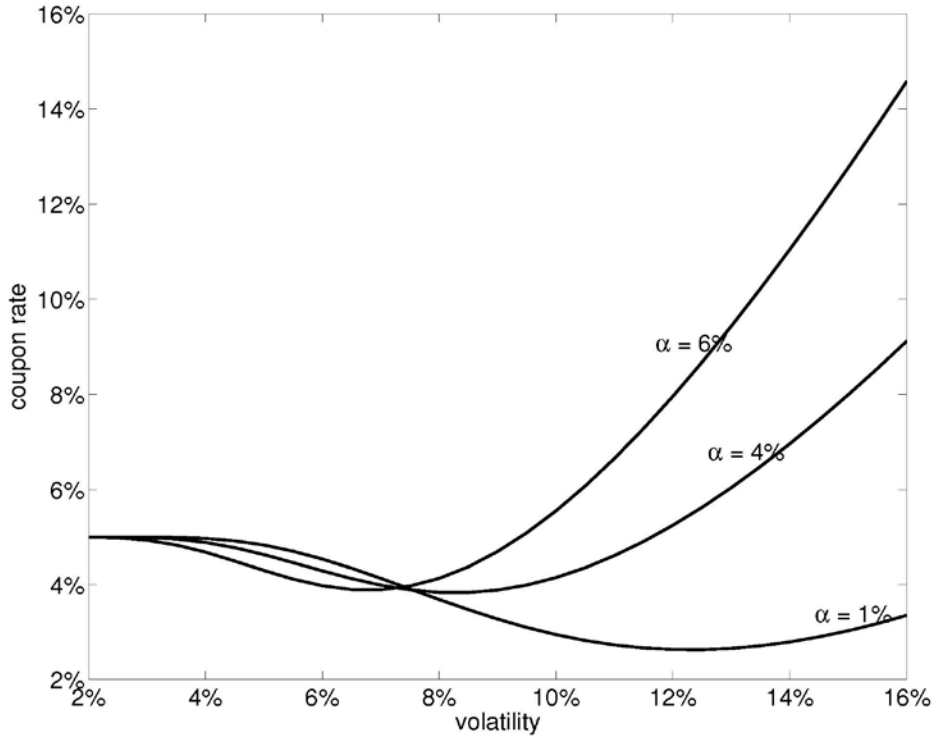
Yields At Varying Levels of CC



Impact of stepped conversion: wide tranche of CC not penalized for higher conversion probability

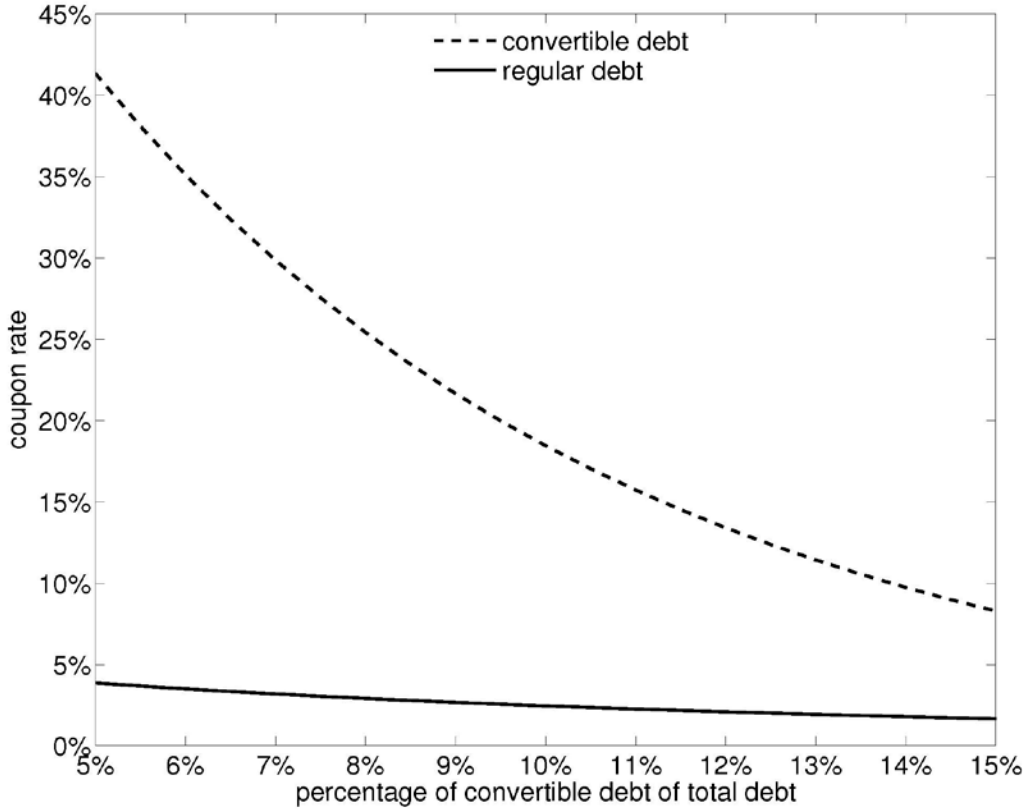
Impact of conversion ratio q

Yields At Varying Levels of Asset Volatility



Non-monotonic impact of asset volatility and capital ratio because of equity-like and debt-like features of contingent capital

Parameter Set II



Thin slice is too risky – too expensive

Summary

- We value contingent capital with a capital-ratio trigger and stepped (minimal) conversion assuming assets follow GBM
- Explicit expressions for “fair” yields
- Extension models market and book values through correlated GBMs
- With reasonable parameters we get
  - Reasonable yields
  - Some non-obvious dependence on parameters
  - Find that convertible tranche needs to be thick
- Key step is fraction of firm owned by original vs converted investors

- Currently looking at
  - Market-trigger paradoxes
  - Models with jumps
  - Modeling supervisory discretion

Thank You