Coherent Global Market Simulations for Counterparty Credit Risk

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References and Thanks

- Based on the paper "Coherent global market simulations for counterparty credit risk", appeared in the January issue of Quantitative Finance.
- Co-authors: Toufik Bellaj, Guillaume Gimonet and Giacomo Pietronero
- Additional references at www.albanese.co.uk
- The opinions expressed in this presentation are the author's sole responsibility.

Outline

- Our objectives

  Shortcomings of the CVA

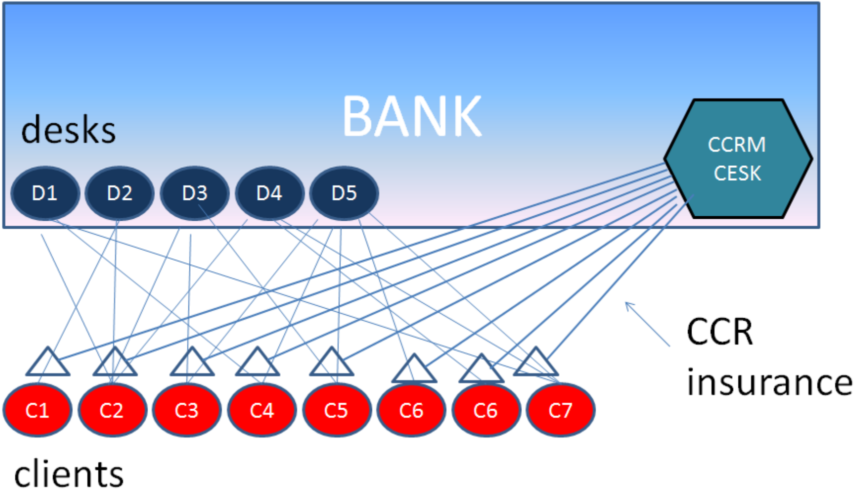
  Global valuation and single node architectures

  Applications

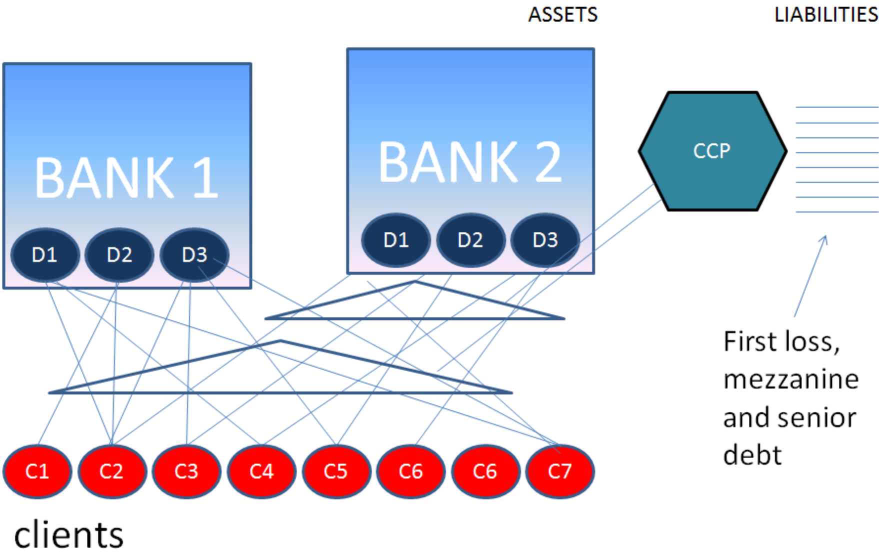
PART I. High Throughput Portfolio Analysis

- The objective is to analyse global portfolios of netting sets over long time horizons.
- We make use of global market simulations under the risk neutral measure
- We generate scenarios for market risk factors (e.g. interest rates and FX rates), credit factors (CDS spread curves and defaults) and corresponding derivatives within a consistent, fully calibrated and fully correlated framework
- Instrument coverage includes derivatives that can be valued by backward induction such as (callable) swaps, FX options and CDSs.

Application I: Internal Counterparty Credit Risk Management



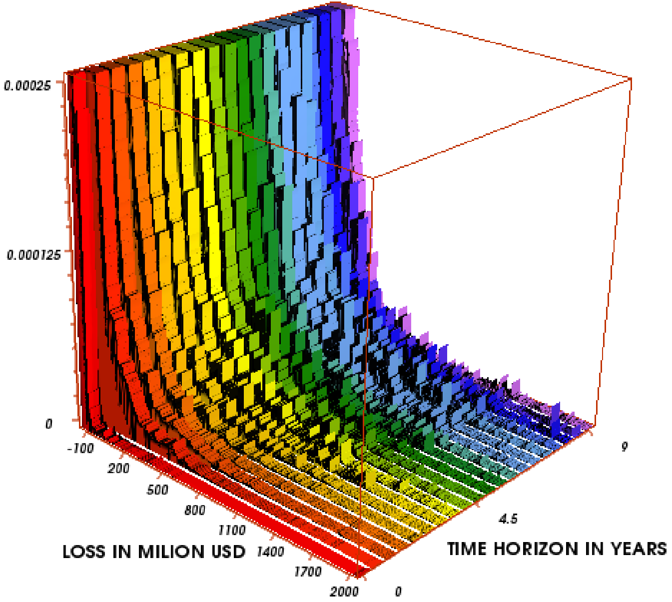
Application II: Portfolio Risk Management for Central Counterparties (CCP) and Securitization



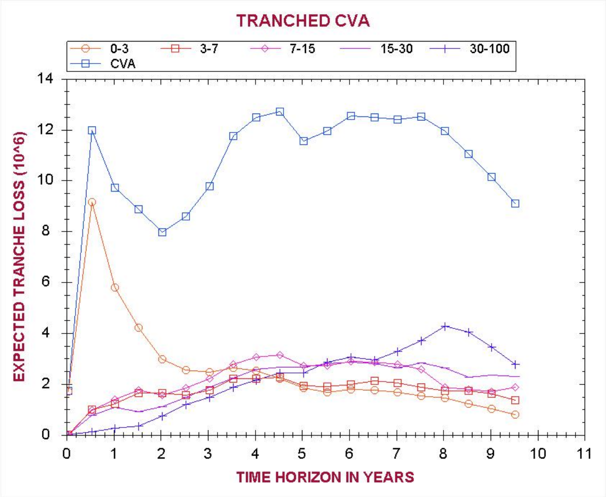
Objective: Analytics to Produce in Nearly Real Time for Global Portfolios of Netting Sets

- Point in time of loss distributions
- Cumulative loss distributions for cash waterfalls against multiple economic capital buffers (including equilibrium tranche spreads)
- Sensitivities of loss distributions and macro hedge ratios
- Risk resolutions of outliers in the tail of the loss distribution and risk concentration hedging
- Aloow the user to narrow down to arbitrary subportfolios of netting sets
- Byproducts: Standard EPE and CVA (including an exact treatment of wrong way risk)

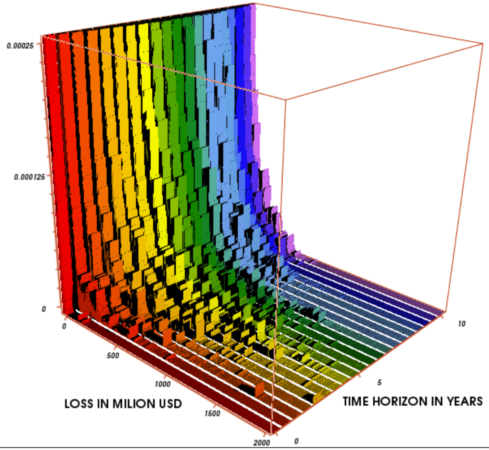
Loss distribution for a CCR portfolio of 302 netting sets.



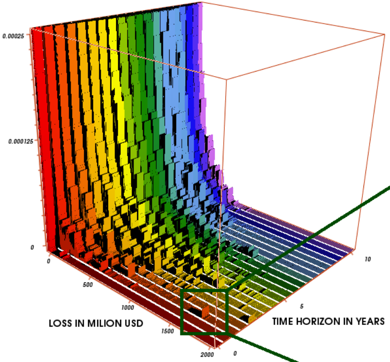
Tranched CVA for a portfolio of 302 netting sets. Tranches are given as percentages of expected exposures. (Already shown)

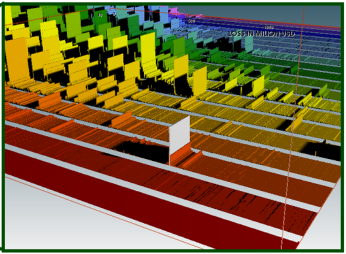


Loss distribution for the EUR denominated sub-portfolio

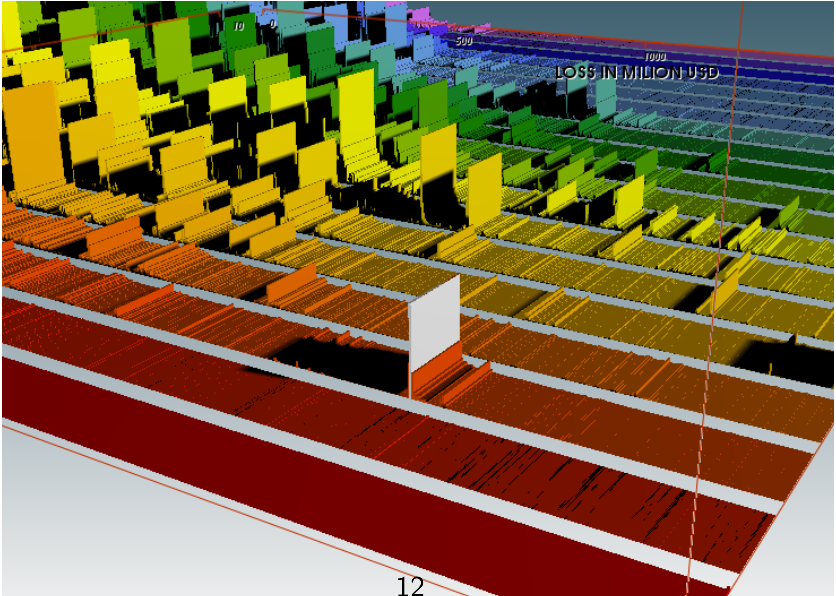


Zooming on an outlier for the EUR denominated sub-portfolio

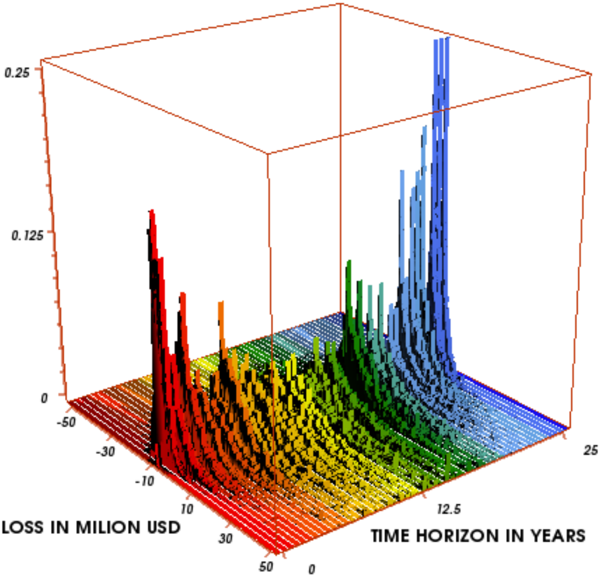




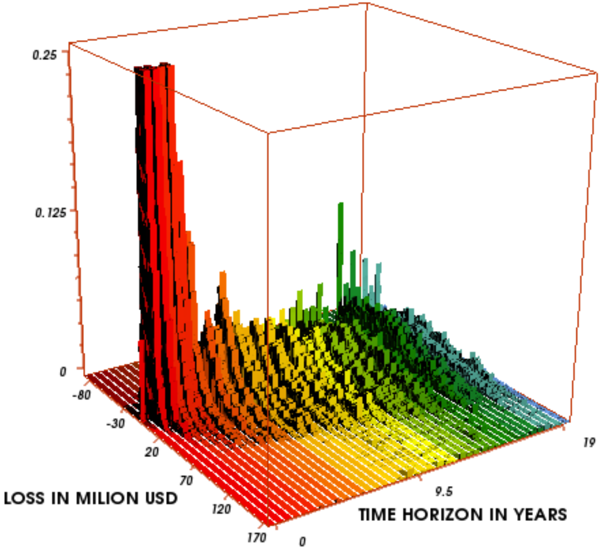
Zooming on an outlier for the EUR denominated sub-portfolio



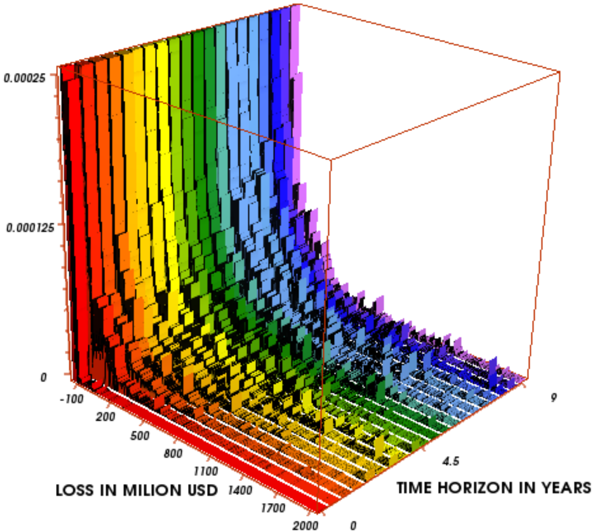
Loss distribution a EUR denominated fix-for-float swap



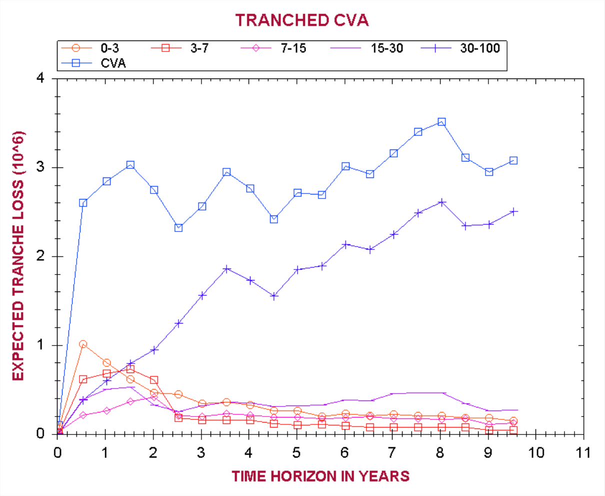
Loss distribution for a cross-currency EUR-USD swap with nominal exchange at maturity



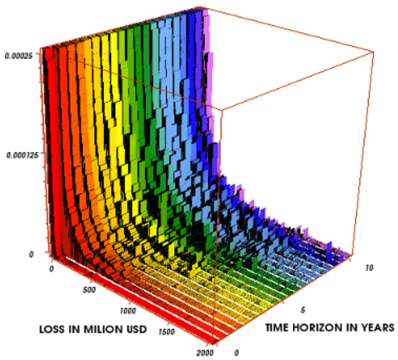
Loss distribution for a portfolio of 62 netting sets for counterparties in the financial sector.



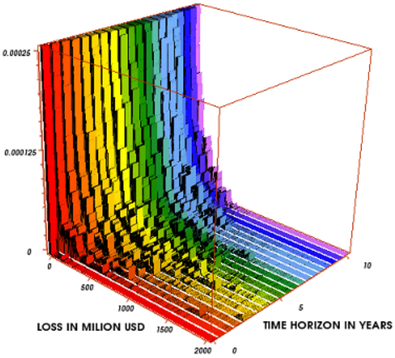
Tranched CVA for a portfolio of 62 netting sets for counterparties in the financial sector.



Comparison of loss distributions for the USD and EUR denominated subportfolios.

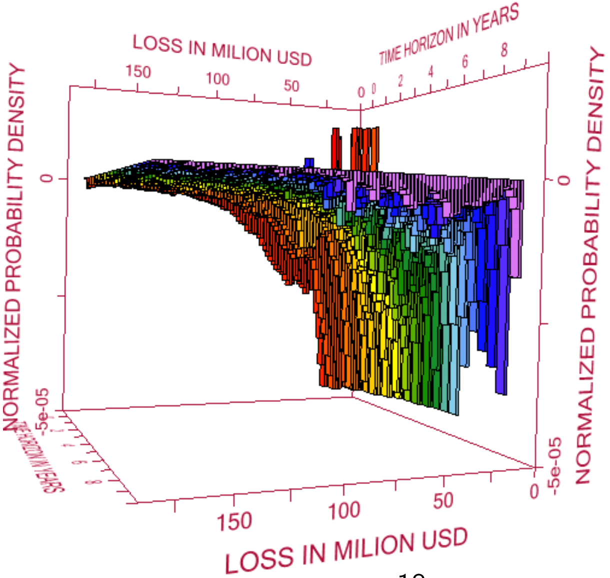


Portfolio USD

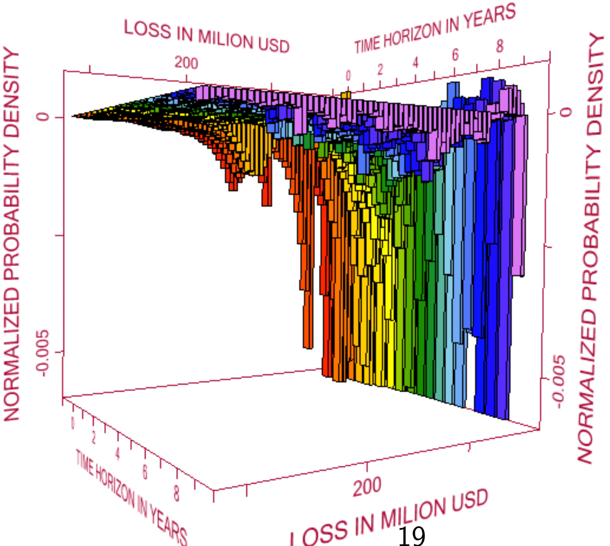


Portfolio EUR

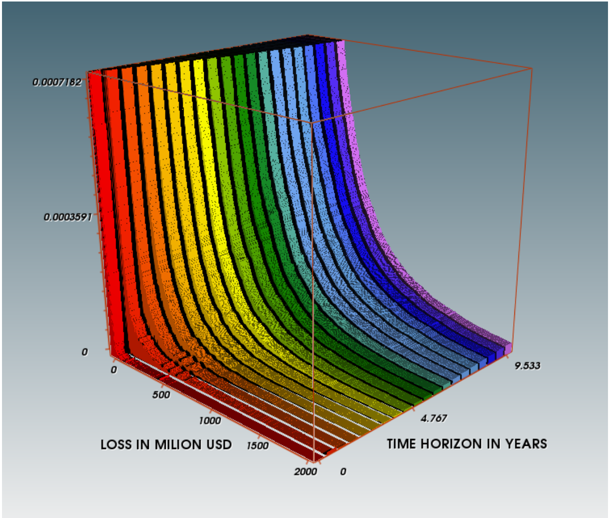
Sensitivities to a parallel shift in the USD curve for the loss distributions corresponding to the entire portfolio.



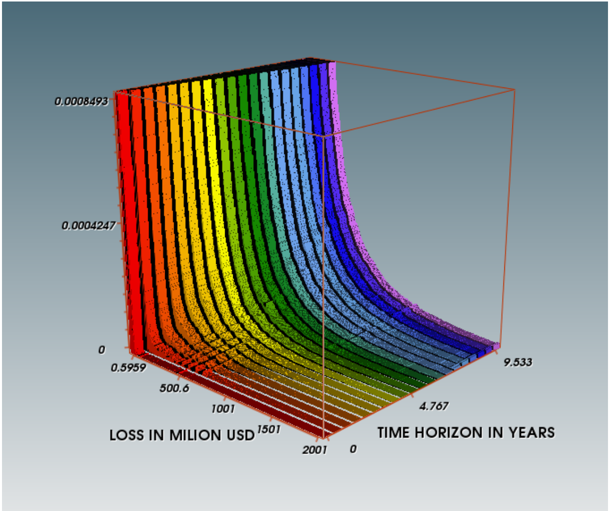
Sensitivities to a parallel shift in the USD curve for the loss distributions corresponding to the USD denominated sub-portfolio.



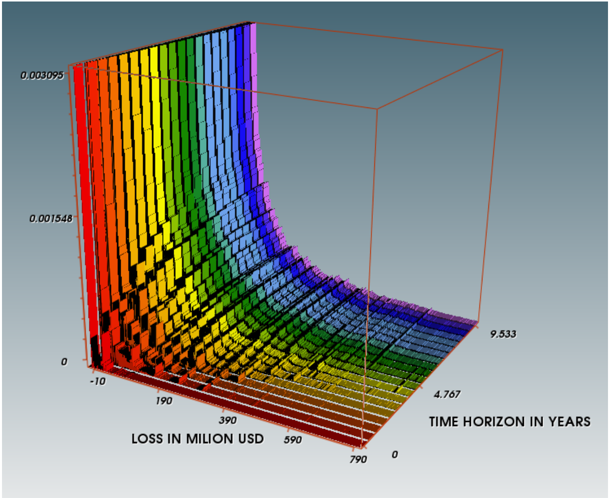
Cumulative loss distribution for the entire CCR portfolio with 302 counterparties



Cumulative loss distribution for the corporate CCR sub-portfolio



Cumulative loss distribution for the sovereign CCR sub-portfolio



Outline

- Our objectives
- Shortcomings of the CVA
- Global valuation and single node architectures
- Applications

PART II. Shortcomings of the CVA

The "Credit Valuation Adjustment" (CVA) of a derivative position is defined as the **discounted expected loss** due to counterparty default risk. More precisely, the CVA of a portfolio of netting sets is

$$\mathrm{CVA} = E_0\left[\sum_n \int_0^\infty e^{-\int_0^t r_s ds}(P_t^n)_+ d\pi_t^n\right] \tag{1}$$

where $(x)_+$ denotes $\max(x,0)$, n is an index for netting sets, P_t^n is the price process of the portfolio corresponding to the netting set n, r_t is the process for the domestic short rate and π_t^n is the process followed by the cumulative probability of default up to time t for netting set n.

Decomposition of the Credit Valuation Adjustment

The CVA can be decomposed as follows:

$$\mathrm{CVA} = \sum_n \mathrm{CVA}_n \tag{2}$$

where

$$\mathrm{CVA}_n = E_0\left[\int_0^\infty e^{-\int_0^t r_s ds}(P_t^n)_+ d\pi_t^n\right], \tag{3}$$

i.e. the CVA of a portfolio of netting sets is given by the sums of the CVAs of each individual netting set. Netting set specific CVAs also admit the following time decomposition:

$$\mathrm{CVA}_n = \int_0^\infty \gamma_n(t)dt, \tag{4}$$

where $\gamma_n(t)$ is the density of the measure

$$\gamma_n(t)dt = E_0\left[e^{-\int_0^t r_s ds}(P_t^n)_+ d\pi_t^n\right], \tag{5}$$

Shortcomings of additive risk measures

- CVA is additive across netting sets. Only the 1988 version of Basel I used additive risk measures, but that choice was then highly criticized as additive risk measures are blind to diversification.
- The 1994 Amendment for market risk and in Basel II for credit risk are based on VaR which is a measure for tail risk and is not additive.
- The current draft of the Basel III capital adequacy requirements represent a step-back as they are blind to diversification

The disincentive to diversify in Basel III

- Fix a large derivatives portfolio P.
- Suppose that Bad Bank holds all the exposures in P against a single counterparty.
- Suppose also that Good Bank holds the portfolio P but is well diversified across 2000 counterparties.
- Who pays more capital under Basel III?

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- Who pays more capital under Basel III?
- **Good Bank!**
- That's because the benefit of diversification is not recognized and the only effective way of reducing exposure is by netting.
- **In the presence ofr netting, using an additive risk measure results in an incentive to reduce the degree of diversification and concentrate risk!**

Striking an impossible balance

- By restricting on the CVA regulators are playing an impossible balancing act
- The capital requirements computed in terms of the CVA for undiversified Bad Banks is insufficient
- The capital requirements for well diversified Good Banks is excessive
- CVA was originally meant to be a price adjustment. It had to remain just that.
- The very notion of using the CVA as a risk metric for capital adequacy is just wrong!

CVA Hedging and Optimization gives rise to Systemic Risk

- If we attempt to optimize or hedge the CVA, it gets worse
- What is the most effective way of reducing the CVA of a global bank portfolio?
- Transfer first loss to investors and lever up
- This
  - creates a moral hazard situation
  - Transfers counterparty credit exposure from the body of the loss distribution to the tail, inducing systemic risk
  - To mitigate systemic risk one should give an incentive to the transfer of tail risk instead

Not only wrong, but wrongly computed

- The standard practice is to compute expected positive exposures (EPEs) with a simulation limited to market risk factors
- The CVA cannot be rigorously derived from EPEs in case counterparty credit risk is correlated with market risk (as it is)
- Approximation schemes for the CVA are controversial and uncontrollable
- To correctly compute the CVA one needs nothing short of a global market simulation including the dynamic simulation for all counterparty CDS curves, as we do

Not only wrong and wrongly computed, but implemented very inefficiently

- Typical EPE calculations take overnight run times on large grid farms
- Our system has nearly real time performance and can be used interactively on hardware that costs a small fraction of a grid farm

Our proposal to regulators

- Include credit-credit and market-credit correlations within the context of a coherent global market simulation based on solid finance principles (i.e. on the Fundamental Theorem of Finance)
- Penalize tail risk and exposure concentrations
- Decide capital adequacy requirements recognizing the existence of a hierarchy of multiple capital buffers (first loss (desk level), mezzanine (bank shareholders), senior (bank bondholders))
- Encourage banks to mitigate and transfer tail risk (not first loss risk)

Outline

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Shortcomings of the CVA

- Global valuation and single node architectures

Applications

PART II. Global valuation and single node architectures

- First comes technology
- Financial mathematics is adapted to technology
- Models are formulated by means of computational mathematics
- Regulations and business models are organized around models

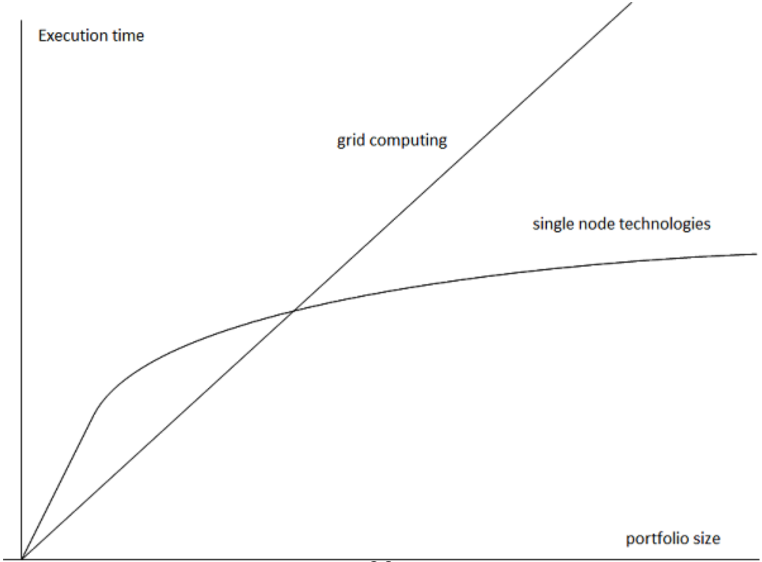
From grid computing to the CVA

- **Technology**: grid computing
- **Financial mathematics**: stochastic calculus models for individual instruments in isolation while ignoring the Fundamental Theorem at the portfolio level
- **Models**: restricted to be either solvable or with some degree of analytic tractability
- **Regulations and business models**: only additive risk measures for market risk factors such as the EEP can be computed and even the CVA needs to be fudged-up.

From single node technologies to coherent global market simulations

- **Technology**: based on single-node technology, i.e. large computing boards with dozens of CPU cores, several GPU coprocessors and TB scale memory
- **Financial mathematics**: based on algebraic operations that compute bound on current hardware
- **Models**: flexibly specified and econometrically realistic, solved numerically and calibrated globally
- **Regulations and business models**: based on real time access to coherent global market simulations, 3d visualization, risk resolution and portfolio level hedging

It's all about achieving sublinear scaling without uncontrollable approximations



Single node technologies

- The system architecture is based on a single node design whereby large portfolios are not parceled out on a grid farm but are instead loaded entirely on a single node with abundant memory (up to 1 TB), up to 4 GPU coprocessors and up to 8 CPUs.
- Having bypassed the network bottleneck at the root, we implement a number of portfolio level algorithmic simplifications to achieve **sublinear scaling**, such as
  - globally defined, high quality models for all risk factors
  - sharing of proxy models across minor currencies and credit processes
  - dictionaries of elementary building blocks for derivative valuation that is possible to compute just once and share globally at the portfolio level

The key to make it all work: The Fundamental Theorem of Finance

- In 1931, de Finetti first derived the Fundamental Theorem of Finance according to which, assuming absence of arbitrage, the relative price of all assets A_t with respect to a chosen numeraire g_t can be expressed in the form

$$\frac{A_t}{g_t} = E_t^Q \left[\frac{A_T}{g_T} \right] \tag{6}$$

for some probability measure Q, **globally defined** across all assets in the economy.

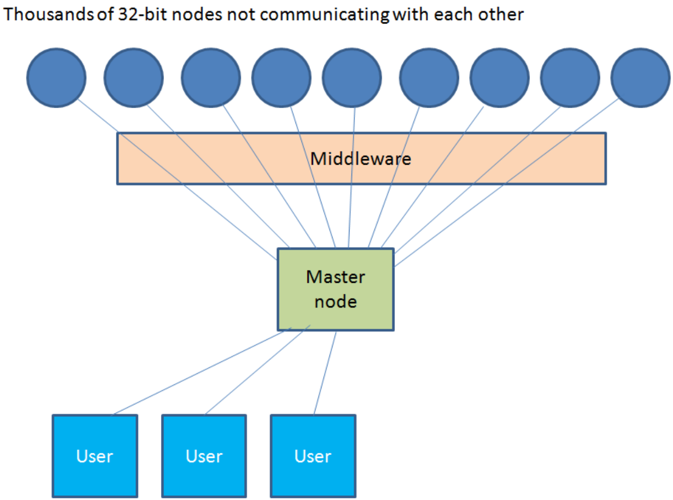
The Black-Scholes-Merton local variant

- In 1972, Black, Scholes and Merton drew attention upon a particular case of the Fundamental Theorem based on the notion of replication.
- Assuming no-arbitrage, they show that in the specific case of geometric Brownian motion, de Finetti's pricing formula can be derived by using the fact that dynamic replication of derivatives can be achieved.

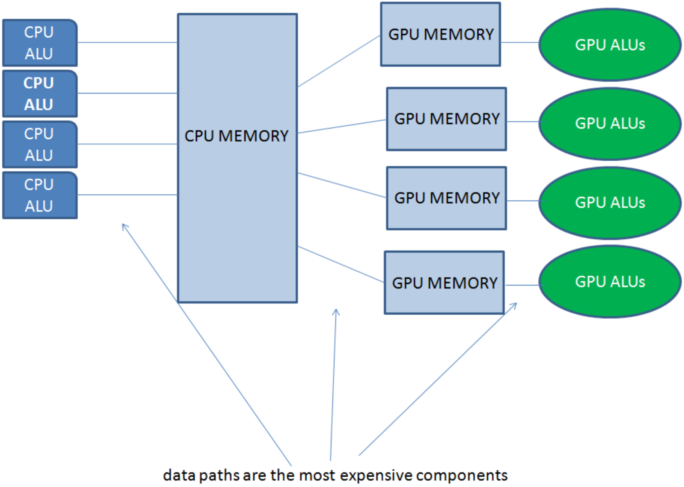
Local Valuation and Grid Computing

- The Black-Scholes-Merton result came with the theoretical promise of replicability as long as one used valuation models that admit replication to compute hedge ratios
- This gave rise to the local valuation methodology according to which one should use deal specific models, find instrument specific hedge ratios and then aggregate hedge ratios at the portfolio level
- Local valuation made it possible to achieve linear scaling on grid farms (no internode communication needed) by parceling out instrument valuation tasks to different nodes
- Sadly, local valuation lacks of any and all theoretical justification as it breaks the Fundamental Theorem at the portfolio level

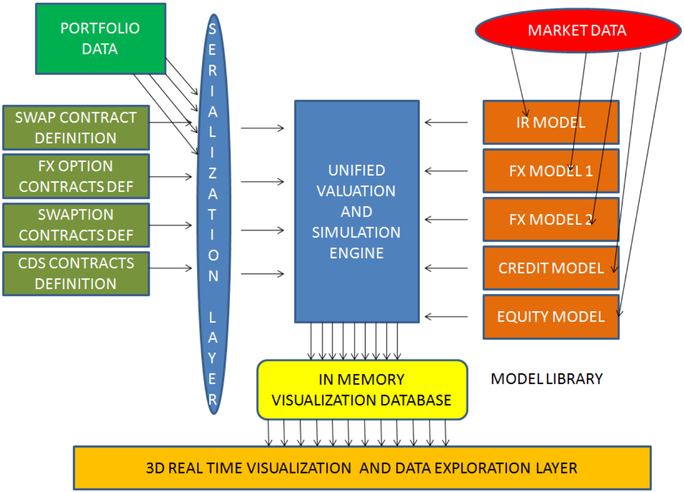
Grid of small nodes



Current boards: lots of memory, many cores and narrow data paths



Outline of a Global Valuation System Architecture

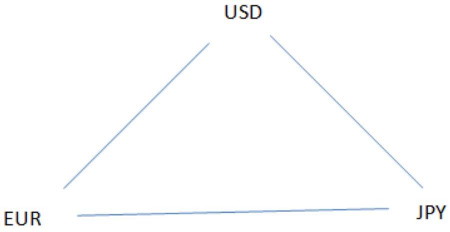


Virtues and Difficulties of Global Valuation

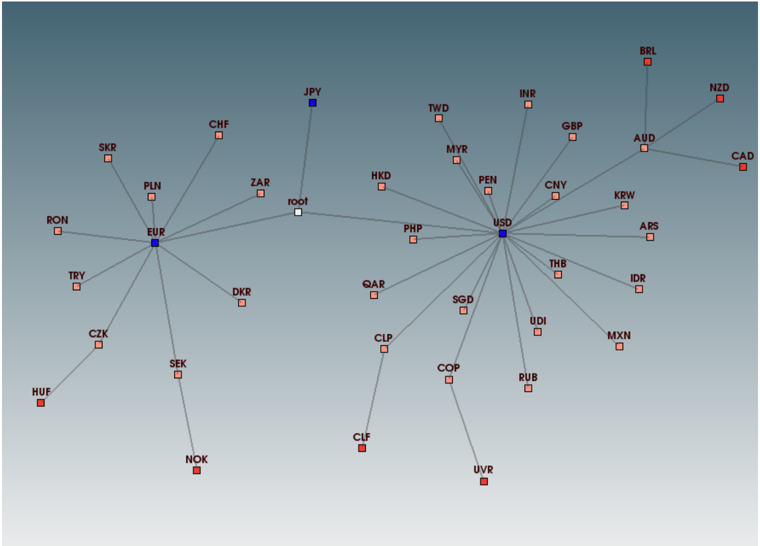
- Global valuation makes it possible to go past linear scaling and achieve strongly sub-linear scaling, allowing real time risk exploration and resolution analysis for global portfolios
- Global valuation is theoretically justified as the basis for replication and hedging as it is based on the Fundamental Theorem of Finance
- The difficulty to implement global valuation system is that they require a ground-up re-engeneering effort

Global Calibration of FX Models

Global multi-currency portfolio are an example of failure of local models for individual crosses as these ignore triangular and polygonal relations.



A Minimum Spanning Tree for Global Currency Modeling



Calibration of Global FX Models

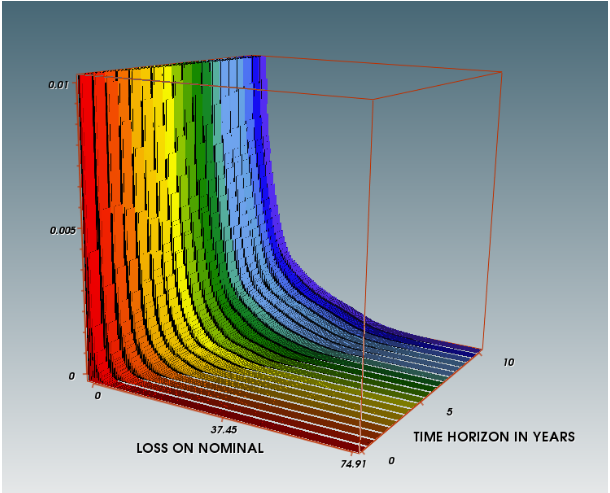
A consistent model can still use local models but as parts in a hierarchy of controllable approximations.

- The tree is devised in such a way to achieve stable historical correlations and to minimize volatilities of crosses in the tree
- The USD-EUR-JPY triangle needs to be calibrated separately by fixing the interest rate processes in the three currencies and optimizing the ROOT centered crosses
- All other crosses in the minimum spanning tree are calibrated separately
- Crosses not in the tree are implied

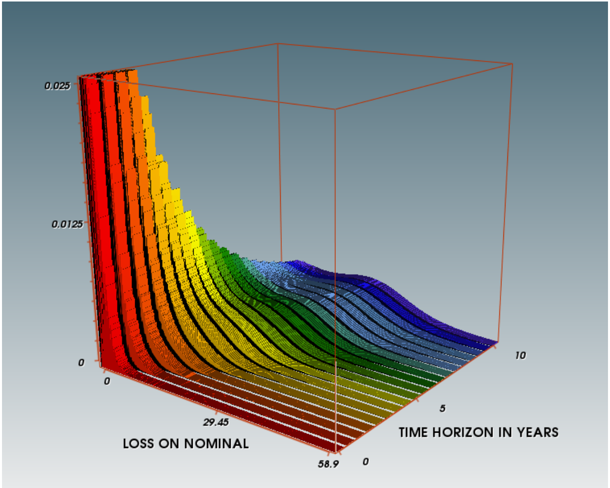
Calibration of Global Credit Models

- Global credit correlations are estimated in such a way to fit the CDX-IG, ITX-IG and the CDX-HY index tranches simultaneously
- The credit model assumes stochastic interest rates and correlates recovery rates to the interest rate cycle

Cumulative loss distribution for the CDX-IG index CDO



Cumulative loss distribution for the CDX-HY index CDO



Conclusions

- We resolve counterparty credit risk of global portfolios of netting set across the entire capital structure by means of global market simulations
- We developed single-node technologies to achieve this objective while leveraging on a rigorous application of the Fundamental Theorem of Finance
- The CVA in the current draft of Basel III is too blunt an instrument with many shortcomings
- Real time risk management and high throughput portfolio data analysis and risk visualization are likely to attract much attention in the future as the wave of technology innovation coming from Silicon Valley is understood and correctly interpreted