LIBOR Market Models with Stochastic Basis

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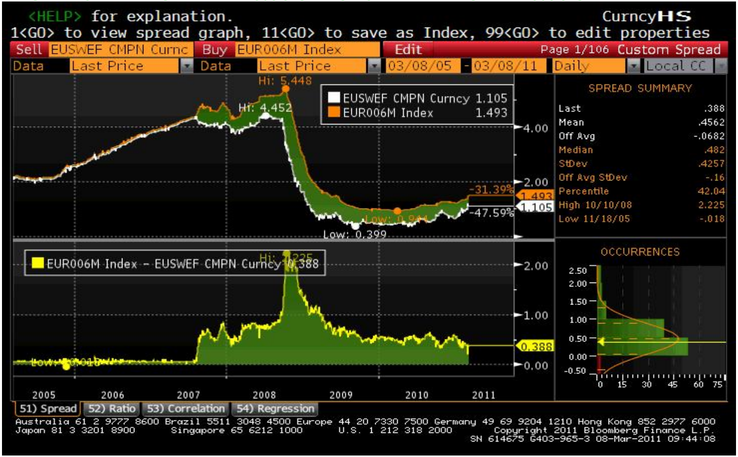
Stylized facts

Since the credit crunch of 2007, the LIBOR-OIS basis has been neither deterministic nor negligible:

EUR 6m EONIA rates vs 6m LIBOR rates



Stylized facts

Likewise, since August 2007 the basis between different-tenor LIBORs has been neither deterministic nor negligible:

EUR 5y swaps: 3m vs 6m

Market practices

- The use of different discount and forward curves (one for each tenor) has become a market practice.
- Under CSA, it is market practice to use OIS discounting:
  - Since June 2010, USD, Euro and GBP trades in SwapClear (LCH.Clearnet) have been revalued using OIS discounting.
  - Since September 2010, swaption prices in the London inter-dealer option market have been quoted on a forward basis for a number of European currencies.

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ICAP

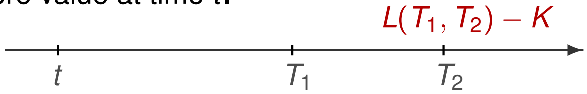
Term	1Y	2Y	3Y	4Y	5Y	6Y	7Y	8Y	9Y
1M Opt	14.0	37.5	57.0	74.5	89.5	103.0	116.0	128.0	140.0
2M Opt	19.5	51.0	77.5	102.0	123.0	142.0	160.0	177.0	193.0
3M Opt	25.0	63.5	93.0	122.0	150.0	176.0	199.0	221.0	242.0
6M Opt	40.5	92.0	136.0	178.0	215.0	250.0	283.0	315.0	344.0
9M Opt	56.5	115.0	171.0	223.0	268.0	313.0	354.0	394.0	431.0
1Y Opt	71.5	139.0	201.0	260.0	313.0	365.0	412.0	457.0	501.0
18M Opt	91.0	174.0	252.0	323.0	387.0	452.0	512.0	567.0	621.0
2Y Opt	106.5	201.0	290.0	369.0	444.0	519.0	588.0	652.0	714.0
3Y Opt	129.0	241.0	346.0	442.0	530.0	621.0	708.0	789.0	866.0
4Y Opt	144.0	267.0	385.0	494.0	594.0	697.0	796.0	890.0	978.0
5Y Opt	155.5	290.0	415.0	535.0	644.0	754.0	865.0	966.0	1067.0
7Y Opt	169.5	322.0	464.0	602.0	726.0	848.0	969.0	1085.0	1202.0
10Y Opt	185.5	358.0	520.0	674.0	820.0	959.0	1093.0	1225.0	1352.0
15Y Opt	208.5	405.0	594.0	771.0	940.0	1102.0	1258.0	1412.0	1563.0
20Y Opt	224.0	440.0	645.0	842.0	1030.0	1205.0	1381.0	1552.0	1715.0
25Y Opt	233.5	458.0	673.0	880.0	1078.0	1264.0	1446.0	1624.0	1801.0
30Y Opt	239.0	464.0	681.0	888.0	1085.0	1269.0	1451.0	1632.0	1810.0

Consistent pricing of interest rate derivatives

- The pricing of general interest rate derivatives should be consistent with the previously mentioned practice of using OIS discounting. In fact:
  - Cap - floor = swap
  - A Bermudan swaption should be more expensive than the underlying European swaptions. In addition, on the last exercise date, a Bermudan swaption becomes a European swaption.
  - A one-period ratchet is equal to a caplet.
  - Etc ...
- Therefore, we should forsake the traditional single-curve models and switch to a multi-curve framework.
- The main contributions so far are: Henrard (2007, 2009), Kijima, Tanaka and Wong (2009), Chibane and Sheldon (2009), M. (2009, 2010), Morini (2008), Bianchetti (2010), Kenyon (2010), Fujii, Shimada and Takahashi (2009, 2011), Pallavicini and Tarenghi (2010), Brace (2010), Amin (2010).

A new definition of forward LIBOR rate: The FRA rate

- Given times $t \leq T_1 < T_2$, the time-t FRA rate $\mathbf{FRA}(t; T_1, T_2)$ is defined as the fixed rate to be exchanged at time T_2 for the LIBOR rate $L(T_1, T_2)$ so that the swap has zero value at time t.



- Under the discount curve T_2-forward measure, consistently with the classic no-arbitrage pricing theory, we define:

$$\mathbf{FRA}(t; T_1, T_2) = E_D^{T_2}\left[L(T_1, T_2)|\mathcal{F}_t\right],$$

where $\mathcal{F}_t$ denotes the “information” available in the market at time t.
- In general:

$$\mathbf{FRA}(t; T_1, T_2) \neq F_D(t; T_1, T_2)$$

A new definition of forward LIBOR rate: The FRA rate

The FRA rate $\mathbf{FRA}(t; T_1, T_2)$ is the natural generalization of a forward rate to the multi-curve case

In fact:

1. The FRA rate coincides with the classically-defined forward rate in the limit case of a single interest rate curve.
2. At its reset time T_1, the FRA rate $\mathbf{FRA}(T_1; T_1, T_2)$ coincides with the LIBOR rate $L(T_1, T_2)$.
3. Its time-0 value $\mathbf{FRA}(0; T_1, T_2)$ can be stripped from market data.
4. The FRA rate is a martingale under the corresponding forward measure.
5. FRA rates are the proper building blocks for pricing interest rate derivatives in a market model set-up.

The multi-curve LIBOR Market Model (McLMM)

- In the classic (single-curve) LMM, one models the joint evolution of a set of consecutive forward LIBOR rates.
- What about our multi-curve case?
- When pricing a payoff depending on same-tenor LIBOR rates, it is convenient to model the FRA rates L_k.
- This choice is also convenient in the case of a swap-rate dependent payoff. In fact, we can write:

$$S_{a,b,c,d}(t) = \frac{\sum_{k=a+1}^{b} \tau_k P_D(t, T_k) L_k(t)}{\sum_{j=c+1}^{d} \tau_j^S P_D(t, T_j^S)} = \sum_{k=a+1}^{b} \omega_k(t) L_k(t)$$

- Modeling FRA rates is not enough. In fact, we also need to model the OIS forward rates. In fact:
  - Swap rates depend on OIS discount factors.
  - The pricing measures we consider are those defined by the OIS curve.

The McLMM

A general framework for the single-tenor case

- Let us fix a given tenor x and consider a time structure $\mathcal{T} = \{0 < T_0^x, \ldots, T_{M_x}^x\}$ compatible with x.
- Let us define forward OIS rates by

$$F_k^x(t) := F_D(t; T_{k-1}^x, T_k^x) = \frac{1}{\tau_k^x}\left[\frac{P_D(t, T_{k-1}^x)}{P_D(t, T_k^x)} - 1\right], \quad k = 1, \ldots, M_x,$$

  where τ_k^x is the year fraction for the interval $(T_{k-1}^x, T_k^x]$, and basis spreads by

$$S_k^x(t) := L_k^x(t) - F_k^x(t), \quad k = 1, \ldots, M_x.$$

- By definition, both L_k^x and F_k^x are martingales under the forward measure $Q_D^{T_k^x}$.
- Hence, their difference S_k^x is a $Q_D^{T_k^x}$-martingale as well.

A general framework for the single-tenor McLMM

- We define the joint evolution of rates F_k and spreads S_k under the spot LIBOR measure $Q_D^{\mathcal{T}}$, whose numeraire is

$$B_D^{\mathcal{T}}(t) = P_D(t, T^x_{\beta(t)-1}) / \prod_{j=0}^{\beta(t)-1} P_D(T^x_{j-1}, T^x_j)$$

  where $\beta(t) = m$ if $T^x_{m-2} < t \leq T^x_{m-1}$, $m \geq 1$, and $T^x_{-1} := 0$.
- Our single-tenor framework is based on assuming that, under $Q_D^{\mathcal{T}}$, OIS rates follow general SLV processes:

$$\mathrm{d}F_k^x(t) = \phi_k^F(t, F_k^x(t))\psi_k^F(V^F(t)) \cdot \left[\sum_{h=\beta(t)}^{k} \frac{\tau_h^x \rho_{h,k} \phi_h^F(t, F_h^x(t))\psi_h^F(V^F(t))}{1 + \tau_h^x F_h^x(t)} \mathrm{d}t + \mathrm{d}Z_k^{\mathcal{T}}(t) \right]$$

$$\mathrm{d}V^F(t) = a^F(t, V^F(t))\,\mathrm{d}t + b^F(t, V^F(t))\,\mathrm{d}W^{\mathcal{T}}(t), \quad V^F(0) = 1$$

  where ϕ_k^F, ψ_k^F, a^F and b^F are deterministic functions, $\rho_{h,k} :=\mathrm{Corr}(dZ_h^{\mathcal{T}}, dZ_k^{\mathcal{T}})$, and $\mathrm{Corr}(dW^{\mathcal{T}}, dZ_k^{\mathcal{T}}) =: \rho_k^x \neq 0$.

A general framework for the single-tenor McLMM

- We then also assume that the spreads S_k^x follow SLV processes.
- For computational convenience, we assume that spreads and their volatilities are independent of OIS rates, so that each S_k^x is a $Q_D^{\mathcal{T}}$-martingale as well.
- A non-zero correlation between rates and spreads can be introduced by setting:

$$S_k^x(t) = \rho F_k^x(t) + X_k^x(t)$$

where X_k^x is independent of F_k^x.
- Finally, the global correlation matrix that includes all cross correlations is assumed to be positive semidefinite.
- There are several different examples of dynamics that can be considered. However, the discussion that follows is rather general and requires no specification of the dynamics.

A general framework for the single-tenor McLMM

Caplet pricing

- Let us consider the x-tenor caplet paying out at time T_k^x

$$\tau_k^x [L_k^x(T_{k-1}^x) - K]^+$$

- Our assumptions on the discount curve imply that the caplet price at time t is given by

$$\begin{aligned}
&\mathbf{Cplt}(t, K; T_{k-1}^x, T_k^x) \\
&= \tau_k^x P_D(t, T_k^x) E_D^{T_k^x} \{[L_k^x(T_{k-1}^x) - K]^+ | \mathcal{F}_t\} \\
&= \tau_k^x P_D(t, T_k^x) E_D^{T_k^x} \{[F_k^x(T_{k-1}^x) + S_k^x(T_{k-1}^x) - K]^+ | \mathcal{F}_t\}
\end{aligned}$$

- Assume we explicitly know the $Q_D^{T_k^x}$-densities $f_{S_k^x(T_{k-1}^x)}$ and $f_{F_k^x(T_{k-1}^x)}$ (conditional on $\mathcal{F}_t$) of $S_k^x(T_{k-1}^x)$ and $F_k^x(T_{k-1}^x)$, respectively, and/or the associated caplet prices.

A general framework for the single-tenor McLMM

Caplet pricing

- Thanks to the independence of the random variables $F_k^x(T_{k-1}^x)$ and $S_k^x(T_{k-1}^x)$ we equivalently have:

$$\frac{\mathbf{Cplt}(t, K; T_{k-1}^x, T_k^x)}{\tau_k^x P_D(t, T_k^x)} = \int_{-\infty}^{+\infty} E_D^{T_k^x}\left\{[F_k^x(T_{k-1}^x) - (K - z)]^+ | \mathcal{F}_t\right\} f_{S_k^x(T_{k-1}^x)}(z)\,\mathrm{d}z$$
$$= \int_{-\infty}^{+\infty} E_D^{T_k^x}\left\{[S_k^x(T_{k-1}^x) - (K - z)]^+ | \mathcal{F}_t\right\} f_{F_k^x(T_{k-1}^x)}(z)\,\mathrm{d}z$$

- One may use the first or the second formula depending on the chosen dynamics for F_k^x and S_k^x.
- To calculate the caplet price one needs to derive the dynamics of F_k^x and V^F under the forward measure $Q_D^{T_k^x}$.
- Notice that the $Q_D^{T_k^x}$-dynamics of S_k^x and its volatility are the same as those under $Q_D^{\mathcal{T}}$.

A general framework for the single-tenor McLMM

Swaption pricing

- Let us consider a (payer) swaption, which gives the right to enter at time $T_a^x = T_c^S$ an interest-rate swap with payment times for the floating and fixed legs given by $T_{a+1}^x, \ldots, T_b^x$ and $T_{c+1}^S, \ldots, T_d^S$, respectively, with $T_b^x = T_d^S$ and where the fixed rate is K.
- The swaption payoff at time $T_a^x = T_c^S$ is given by

$$\left[S_{a,b,c,d}(T_a^x) - K\right]^+ \sum_{j=c+1}^{d} \tau_j^S P_D(T_c^S, T_j^S),$$

where the forward swap rate $S_{a,b,c,d}(t)$ is given by

$$S_{a,b,c,d}(t) = \frac{\sum_{k=a+1}^{b} \tau_k^x P_D(t, T_k^x) L_k^x(t)}{\sum_{j=c+1}^{d} \tau_j^S P_D(t, T_j^S)}.$$

A general framework for the single-tenor McLMM

Swaption pricing

- The swaption payoff is conveniently priced under $Q_D^{c,d}$:

$$\mathbf{PS} = \sum_{j=c+1}^{d} \tau_j^S P_D(t, T_j^S)\, E_D^{c,d}\left\{ \left[S_{a,b,c,d}(T_a^x) - K\right]^+ |\mathcal{F}_t\right\}$$

- To calculate the last expectation, we write:

$$\begin{aligned} S_{a,b,c,d}(t) &= \sum_{k=a+1}^{b} \omega_k(t) L_k^x(t) \\ &= \sum_{k=a+1}^{b} \omega_k(t) F_k^x(t) + \sum_{k=a+1}^{b} \omega_k(t) S_k^x(t) =: \bar{F}(t) + \bar{S}(t) \end{aligned}$$

- The processes $S_{a,b,c,d}$, $\bar{F}$ and $\bar{S}$ are all $Q_D^{c,d}$-martingales.
- If the chosen dynamics are sufficiently tractable, we can resort to standard approximations and calculate the swaption price in the same fashion as the caplet price.

A tractable class of multi-tenor McLMMs

- Let us consider a time structure $\mathcal{T} = \{0 < T_0, \ldots, T_M\}$ and tenors $x_1 < x_2 < \cdots < x_n$ with associated time structures $\mathcal{T}^{x_i} = \{0 < T_0^{x_i}, \ldots, T_{M_{x_i}}^{x_i}\}$.
- We assume that each x_i is a multiple of the preceding tenor x_{i-1}, and that $\mathcal{T}^{x_n} \subset \mathcal{T}^{x_{n-1}} \subset \cdots \subset \mathcal{T}^{x_1} = \mathcal{T}$.
- Forward OIS rates are defined, for each tenor $x \in \{x_1, \ldots, x_n\}$, by

$$F_k^x(t) := F_D(t; T_{k-1}^x, T_k^x) = \frac{1}{\tau_k^x}\left[\frac{P_D(t, T_{k-1}^x)}{P_D(t, T_k^x)} - 1\right], \ k = 1, \ldots, M_x,$$

  and basis spreads are defined by

$$S_k^x(t) = \mathbf{FRA}(t, T_{k-1}^x, T_k^x) - F_k^x(t) = L_k^x(t) - F_k^x(t), \ k = 1, \ldots, M_x.$$

- L_k^x, F_k^x, S_k^x are martingales under the forward measure $Q_D^{T_k^x}$.

A tractable class of multi-tenor McLMMs

- We assume that, under the spot LIBOR measure $Q_D^{\mathcal{T}}$, the OIS forward rates $F_k^{x_1}$, $k = 1, \ldots, M_1$, follow "shifted-lognormal" stochastic-volatility processes

$$\mathrm{d}F_k^{x_1}(t) = \sigma_k^{x_1}(t) V^F(t) \left[\frac{1}{\tau_k^{x_1}} + F_k^{x_1}(t) \right] \cdot \left[V^F(t) \sum_{h=\beta(t)}^{k} \rho_{h,k} \sigma_h^{x_1}(t)\, \mathrm{d}t + \mathrm{d}Z_k^{\mathcal{T}}(t) \right]$$

$$\mathrm{d}V^F(t) = a^F(t, V^F(t))\, \mathrm{d}t + b^F(t, V^F(t))\, \mathrm{d}W^{\mathcal{T}}(t)$$

where:

- For each k, $\sigma_k^{x_1}$ is a deterministic function;
- $\{Z_1^{\mathcal{T}}, \ldots, Z_{M_1}^{\mathcal{T}}\}$ is an M_1-dimensional $Q_D^{\mathcal{T}}$-Brownian motion with correlations $(\rho_{k,j})_{k,j=1,\ldots,M_1}$;
- V^F is correlated with every $Z_k^{\mathcal{T}}$, $\mathrm{d}W^{\mathcal{T}}(t)\mathrm{d}Z_k^{\mathcal{T}}(t) = \rho_k^x\, \mathrm{d}t$, and $V^F(0) = 1$.

A tractable class of multi-tenor McLMMs

- The dynamics of forward rates F_k^x, for tenors $x \in \{x_2, \ldots, x_n\}$, can be obtained by Ito's lemma, noting that F_k^x can be written in terms of shorter tenor rates $F_k^{x_1}$ as follows:

$$\prod_{h=i_{k-1}+1}^{i_k} [1 + \tau_h^{x_1} F_h^{x_1}(t)] = 1 + \tau_k^x F_k^x(t),$$

  for some indices i_{k-1} and i_k.

- We then assume, for each tenor $x \in \{x_1, \ldots, x_n\}$, the following one-factor models for the spreads:

$$S_k^x(t) = S_k^x(0)\mathcal{M}^x(t), \quad k = 1, \ldots, M_x,$$

  where, for each x, $\mathcal{M}^x$ is a (continuous and) positive $Q_D^{\mathcal{T}}$-martingale independent of rates F_k^x and of the stochastic volatility V^F. Clearly, $\mathcal{M}^x(0) = 1$.

A tractable class of multi-tenor McLMMs

- The above dynamics of F_k^x are the simplest stochastic volatility dynamics that are consistent across different tenors x.
- If 3m-rates follow shifted-lognormal processes with common stochastic volatility, the same type of dynamics (modulo the drift correction in the volatility) is also followed by 6m-rates (under the respective forward measures).
- This allows us to simultaneously price, with the same type of formula, caps and swaptions with different tenors x.
- Option prices can then be calculated as suggested before. Swaption formulas can be simplified by noting that:

$$\begin{aligned}\bar{S}(t) &= \sum_{k=a+1}^{b} \omega_k(t) S_k^x(0) \mathcal{M}^x(t) \\ &\approx \mathcal{M}^x(t) \sum_{k=a+1}^{b} \omega_k(0) S_k^x(0) = \bar{S}(0) \mathcal{M}^x(t)\end{aligned}$$

A tractable class of multi-tenor McLMMs

An explicit example of rate and spread dynamics

- We now assume constant volatilities $\sigma_k^{x_1}(t) = \sigma_k^{x_1}$ and SABR dynamics for V^F. This leads to the following dynamics for the x-tenor rate F_k^x under $Q_D^{T_k^x}$:

$$dF_k^x(t) = \sigma_k^x V^F(t)\left[\frac{1}{\tau_k^x} + F_k^x(t)\right]dZ_k^{k,x}(t)$$

$$dV^F(t) = -\epsilon[V^F(t)]^2 \sum_{h=\beta(t)}^{i_k} \sigma_h^{x_1}\rho_h^{x_1}dt + \epsilon V^F(t)\,dW^{k,x}(t),$$

with $V^F(0) = 1$, where also σ_k^x is now constant and $\epsilon \in \mathbb{R}^+$.

- We then assume that basis spreads for all tenors x are governed by the same geometric Brownian motion:

$$\mathcal{M}^x \equiv \mathcal{M}, \quad d\mathcal{M}(t) = \sigma\mathcal{M}(t)\,dZ(t)$$

where Z is a $Q_D^{T_k^x}$-Brownian motion independent of $Z_k^{k,x}$ and $W^{k,x}$ and σ is a positive constant.

A tractable class of multi-tenor McLMMs

An explicit example of rate and spread dynamics

- Caplet prices can easily be calculated as soon as we smartly approximate the drift term of V^F. We get:

$$\mathbf{Cplt}(t, K; T^x_{k-1}, T^x_k) = \int_{-\infty}^{a^x_k(t)} \mathbf{Cplt}^{\mathrm{SABR}}\Big(t, F^x_k(t) + \frac{1}{\tau^x_k}, K + \frac{1}{\tau^x_k} - S^x_k(t) e^{-\frac{1}{2}\sigma^2 T^x_{k-1} + \sigma\sqrt{T^x_{k-1}}z}; T^x_{k-1}, T^x_k\Big) \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}\,\mathrm{d}z$$
$$+ \tau^x_k P_D(t, T^x_k)(F^x_k(t) - K)\Phi\big(-a^x_k(t)\big)$$
$$+ \tau^x_k P_D(t, T^x_k) S^x_k(t)\Phi\Big(-a^x_k(t) + \sigma\sqrt{T^x_{k-1} - t}\Big)$$

where

$$a^x_k(t) := \Big(\ln\frac{K + \frac{1}{\tau^x_k}}{S^x_k(t)} + \frac{1}{2}\sigma^2(T^x_{k-1} - t)\Big)\Big/\Big(\sigma\sqrt{T^x_{k-1} - t}\Big)$$

and the SABR parameters are σ^x_k (corrected for the drift approximation), ϵ and ρ^x_k (the SABR β is here equal to 1).

A tractable class of multi-tenor McLMMs

An explicit example of rate and spread dynamics

- This caplet pricing formula can be used to price caps on any tenor x.
- In fact, cap prices on a non-standard tenor z can be derived by calibrating the market prices of standard y-tenor caps using the formula with $x = y$ and assuming a specific correlation structure $\rho_{i,j}$.
- One then obtains the output model parameters:
  - $\sigma_k^{x_1}$, $k = 1, \ldots, M_1$
  - $\rho_k^{x_1}$, $k = 1, \ldots, M_1$
  - ϵ
  - σ
- Finally, with these calibrated parameters one can price z-based caps, again using the caplet formula above, this time setting $x = z$.

A tractable class of multi-tenor McLMMs

An explicit example of rate and spread dynamics

- We finally calibrate this example of a multi-tenor McLMM to market data.
- We use EUR data as of September 15th, 2010 and calibrate 6-month caps with (semi-annual) maturities from 3 to 10 years. The strikes range from 2% to 7%.
- We minimize the sum of the squared relative differences between model and market prices.
- We assume that OIS rates are perfectly correlated with one another, that all $\rho_k^{x_1}$ are equal to the same ρ and that the drift of V^F is approximately linear in V^F.
- The average of the absolute values of these differences is 19bp.
- After calibrating the model parameters to caps with $x = 6m$, we can apply the same model to price caps based on the 3m-LIBOR ($x = 3m$), where we assume that $\sigma_{i_{k-1}}^{3m} = \sigma_{i_k}^{3m}$ for each k.

A tractable class of multi-tenor McLMMs

An explicit example of rate and spread dynamics

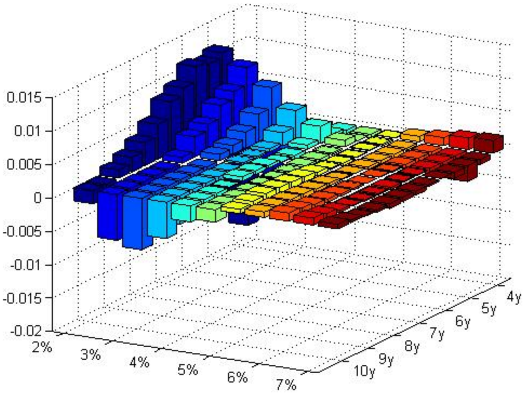


Figure: Absolute differences (in%) between market and model cap volatilities.

A tractable class of multi-tenor McLMMs

An explicit example of rate and spread dynamics

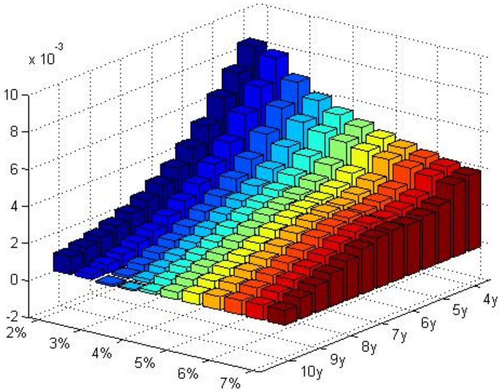


Figure: Absolute differences (in bp) between model-implied 3m-LIBOR cap volatilities and model 6m-LIBOR ones.

Conclusions

- We started by describing the changes in market interest rate quotes which have occurred since August 2007.
- We have shown how to price the main interest rate derivatives under the assumption of distinct curves for generating future LIBOR rates and for discounting.
- We have then shown how to extend the LMM to the multi-curve case, retaining the tractability of the classic single-curve LMM.
- We have finally introduced an extended LMM, where we jointly model rates and spreads with different tenors.
- References:

  - Mercurio, F. (2010a) Modern LIBOR Market Models: Using Different Curves for Projecting Rates and for Discounting. *International Journal of Theoretical and Applied Finance* 13, 1-25.
  - Mercurio, F. (2010b) A LIBOR Market Model with a Stochastic Basis. *Risk* December, 96-101.

Appendix A: The new market formula for interest rate swaps

Denote respectively by $T_a, \ldots, T_b$ and $T_c^S, \ldots, T_d^S$ the times of the floating and fixed legs of a standard interest rate swap (LIBOR set in advance and paid in arrears) and by τ_k and τ_j^S the respective year fractions.

Swap rate	**Formulas**
OLD	$\frac{\sum_{k=1}^{b} \tau_k P(0,T_k) F_k(0)}{\sum_{j=1}^{d} \tau_j^S P(0,T_j^S)} = \frac{1 - P(0,T_d^S)}{\sum_{j=1}^{d} \tau_j^S P(0,T_j^S)}$
NEW	$\frac{\sum_{k=1}^{b} \tau_k P_D(0,T_k) L_k(0)}{\sum_{j=1}^{d} \tau_j^S P_D(0,T_j^S)}$

Appendix B: The new market formulas for caps and swaptions

Type	Formulas
OLD **Cplt**	$\tau_k P(t, T_k)\,\mathrm{Bl}\left(K, F_k(t), v_k\sqrt{T_{k-1}-t}\right)$
NEW **Cplt**	$\tau_k P_D(t, T_k)\,\mathrm{Bl}\left(K, L_k(t), \bar{v}_k\sqrt{T_{k-1}-t}\right)$
OLD **PS**	$\sum_{j=c+1}^{d} \tau_j^S P(t, T_j^S)\,\mathrm{Bl}\left(K, S_{OLD}(t), \nu\sqrt{T_a-t}\right)$
NEW **PS**	$\sum_{j=c+1}^{d} \tau_j^S P_D(t, T_j^S)\,\mathrm{Bl}\left(K, S_{a,b,c,d}(t), \bar{\nu}\sqrt{T_a-t}\right)$