Measuring Market Fear

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- Study of fear index based trading strategies

MARKET FEAR COMPONENTS

- There are a variety of market fear factors.
- We have market risk. The higher the volatility the more market uncertainty there is and the wider swings in the market can occur.
- We have liquidity risk. The bid and ask spreads widen in periods of high uncertainty.
- We have herd-behavior. In a systemic crisis, all assets move into the same direction. The more comonotone behavior we have, the more assets move together and the more systemic risk there is.
- We have counterparty risk. In heavily distressed periods, counterparty risk is omnipresent. The failure of a counterparty could lead to a domino effect. Counterparty risk can be measured through Credit Default Swaps and other credit derivatives.

MARKET FEAR COMPONENTS

- The aim is to measure the market fear factors on the basis of market option data in a single intuitive number.
- The measure will be an overall market measure and hence will be based on vanilla index options and individual stock options.
- By making use of option data and not of historical data we have a forward looking measure indicating markets expectations for the near future.
- The classical example of using of option data is the measurement of market volatility by the VIX methodology.
- We will measure volatility, herd-behavior and liquidity in a similar manner and hence will be able of exactly decomposing the overall market fear into its components.

VIX

- The VIX index is often referred to as the fear index or fear gauge. It is a key measure of market expectations of near-term volatility conveyed by SP 500 stock index option prices.
- Since its introduction in 1993, the VIX has been considered by many to be a good barometer of investor sentiment and market volatility.
- It is a weighted blend of prices for a range of options on the SP500 index.
- The formula uses as inputs the current market prices for all out-of-the-money calls and puts for the front month and second month expirations.
- The goal is to estimate the implied volatility of the SP500 index over the next 30 days.

VIX

- The VIX calculation is very related to the implementation of a Variance Swap (cfr. work by P. Carr, D. Madan, A. Neuberger and others)
- On March 26, 2004, the first-ever trading in futures on the VIX Index began on CBOE Futures Exchange (CFE).
- As of February 24, 2006, it became possible to trade VIX options contracts.
- The VIX methodology has been applied on many other indices.
- On the January 5, 2011, CBOE announced to also VIX-ify individual stocks like (APPL, IBM, GS, GOOG, …).

VIX

- The magic VIX formula is :

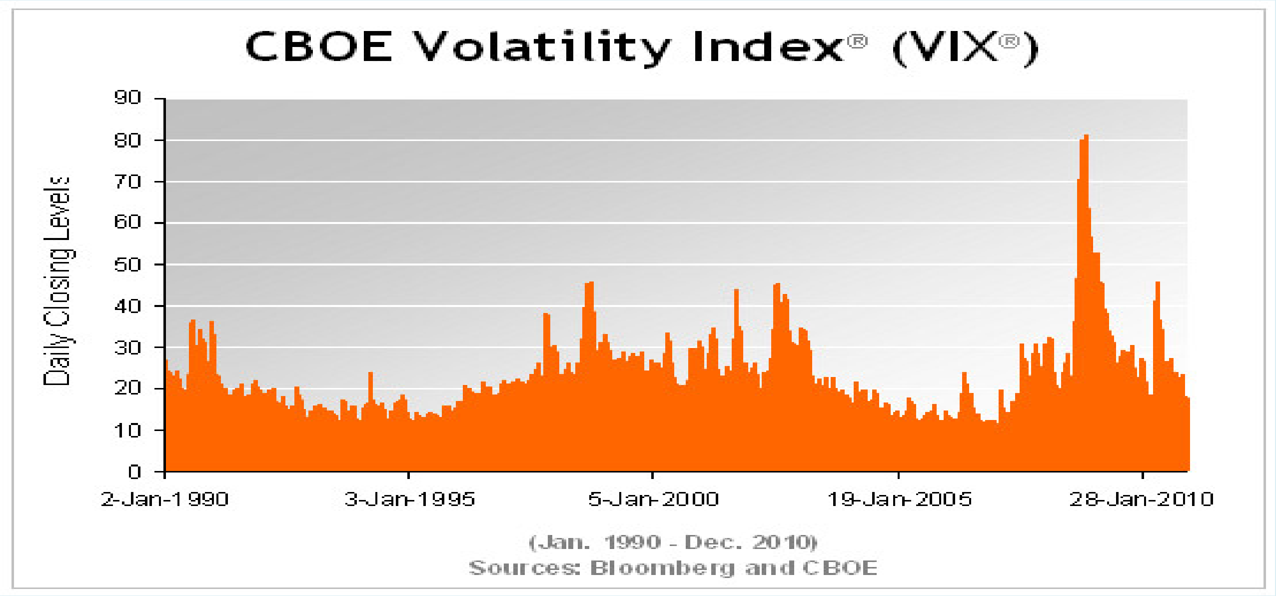
$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2$$

- VIX = σ x 100
- T is time to maturity
- F is forward index level
- K_i are strikes
- R is interest rate and
- Q(.) are mid prices

VIX

- The formula is applied to the front month (with T > 1 week) and the next month and is finally obtained by inter/extrapolation on the 30 days point:

$$\mathrm{VIX} = 100 \times \sqrt{\left\{ T_1 \sigma_1^2 \left[\frac{N_{T_2} - N_{30}}{N_{T_2} - N_{T_1}} \right] + T_2 \sigma_2^2 \left[\frac{N_{30} - N_{T_1}}{N_{T_2} - N_{T_1}} \right] \right\} \times \frac{N_{365}}{N_{30}}}$$



MEASURING LIQUIDITY

- How to measure and quantify in an isolated manner liquidity ?
- Bid-ask spread are a good indication but can be misleading.

- **Example:** Which European Call Option is the most liquid ?

EC1 on Stock1	EC2 on Stock2
Maturity = 1y	Maturity = 1y
Bid = 9 EUR	Bid = 9 EUR
Mid = 10 EUR	Mid = 10 EUR
Ask = 11 EUR	Ask = 11 EUR

A) EC1
B) EC2
C) Both
D) Can’t say

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Maturity = 1y	Maturity = 1y
r=0%; q=0%	r=0%; q=0%
S1=100	S2=20
K=100	K=10
Vol=25.13%	Vol=1.0%
Bid = 9 EUR	Bid = 9 EUR
Mid = 10 EUR	Mid = 10 EUR
Ask = 11 EUR	Ask = 11 EUR

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Probability that Stock2 after one year will trade above 19.0 EUR is 0.9999997 (5 sigma event).
And hence option will "always" payout more than 9 EUR.

MEASURING LIQUIDITY

- It is very difficult to measure liquidity in an isolate manner.
- Bid and ask spreads can move around in a non-linear manner if spot, vol, or other market parameters move, without a change in liquidity.
- The concept of implied liquidity in a unique and fundamental founded way isolates and quantifies the liquidity risk in financial markets.
- This makes comparison over times, products and asset classes possible.
- The underlying fundamental theory is based on new concepts of the two-ways price theory of conic finance.
- These investigations open the door to stochastic liquidity modeling, liquidity derivatives and liquidity trading.

CONIC FINANCE

- We will make use of the minmaxvar distortion function:

$$\Phi(u;\lambda) = 1 - \left(1 - u^{\frac{1}{1+\lambda}}\right)^{1+\lambda}$$

- We use distorted expectation to calculate (bid and ask) prices.
- The distorted expectation of a random variable with distribution function *F(x)* is defined

$$de(X;\lambda) = \mathrm{E}^{\lambda}[X] = \int_{-\infty}^{+\infty} x \mathrm{d}\Phi(G(x);\lambda).$$

- The ask price of payoff *X* is determined as

$$ask(X) = -\exp(-rT)\mathrm{E}^{\lambda}[-X].$$

- The bid price of payoff X is determined as

$$bid(X) = \exp(-rT)\mathrm{E}^{\lambda}[X].$$

CONIC FINANCE

- These formulas are derived by noting that the cash-flow of selling X at its ask price and buying *X* at its bid price is acceptable in the relevant market .

- We say that a risk *X* is acceptable if

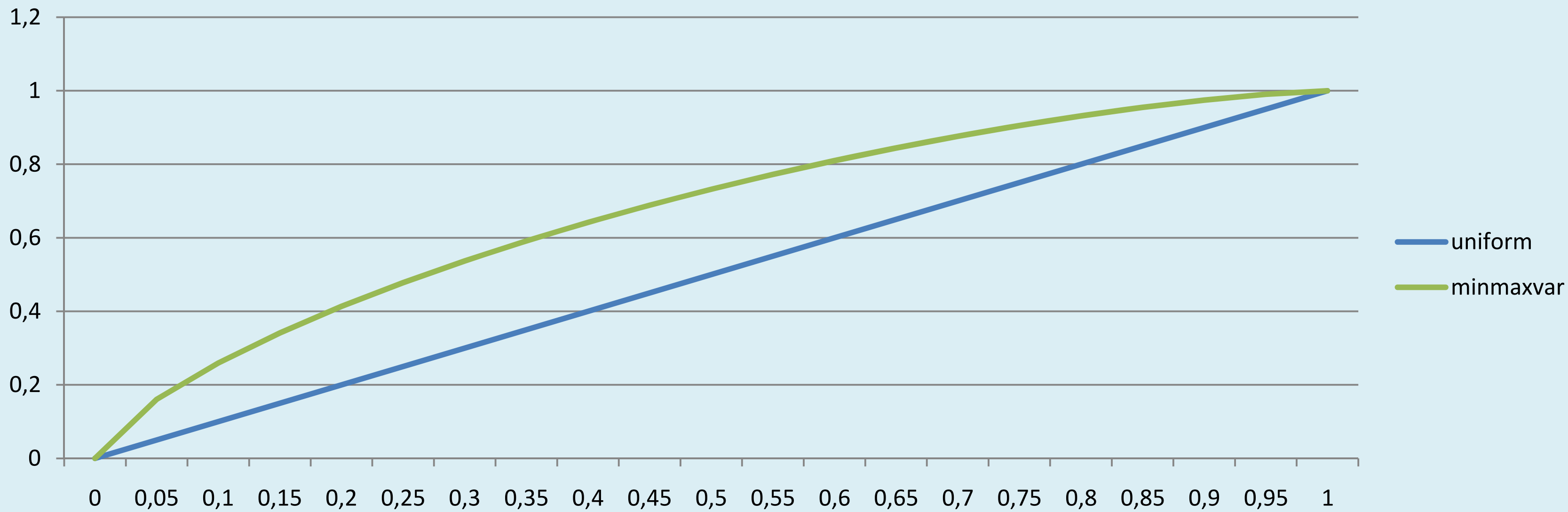
$$E_Q[X] \geq 0 \text{ for all measures } Q \text{ in a convex set } \mathcal{M}.$$

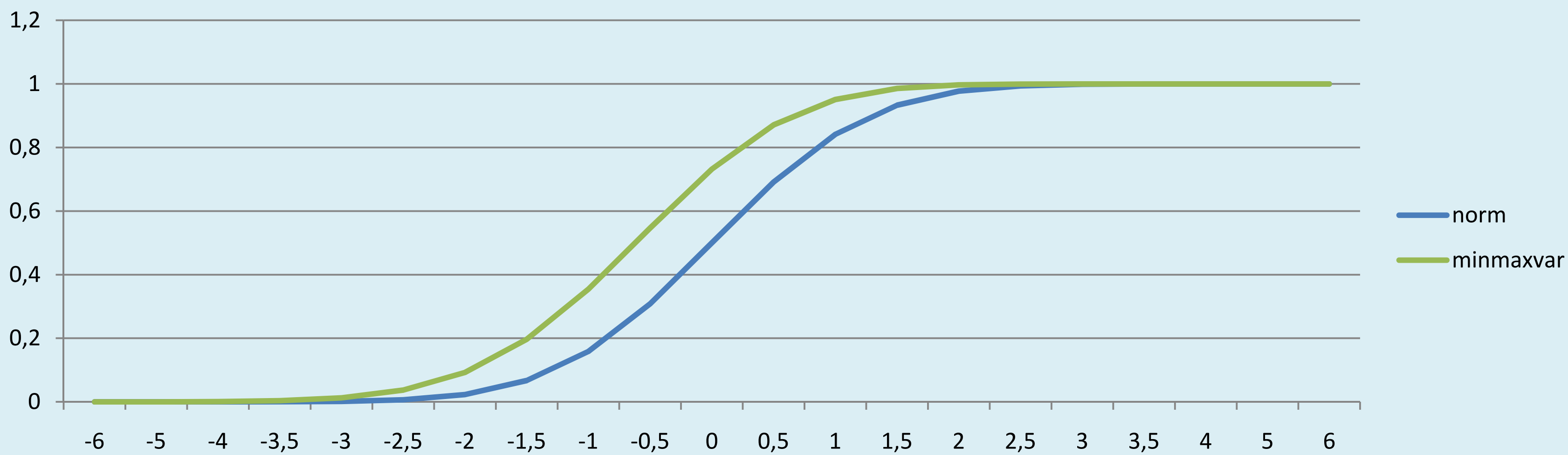
M is a set of test-measures under which cash-flows need to have positive expectation.

- Operational cones were defined by Cherney and Madan and depend solely on the distribution function *G(x)* of *X* and a distortion function. To have acceptability we need to have that the distorted expectation is positive:

$$de(X;\lambda) = \mathrm{E}^{\lambda}[X] = \int_{-\infty}^{+\infty} x \mathrm{d}\Phi(G(x);\lambda).$$

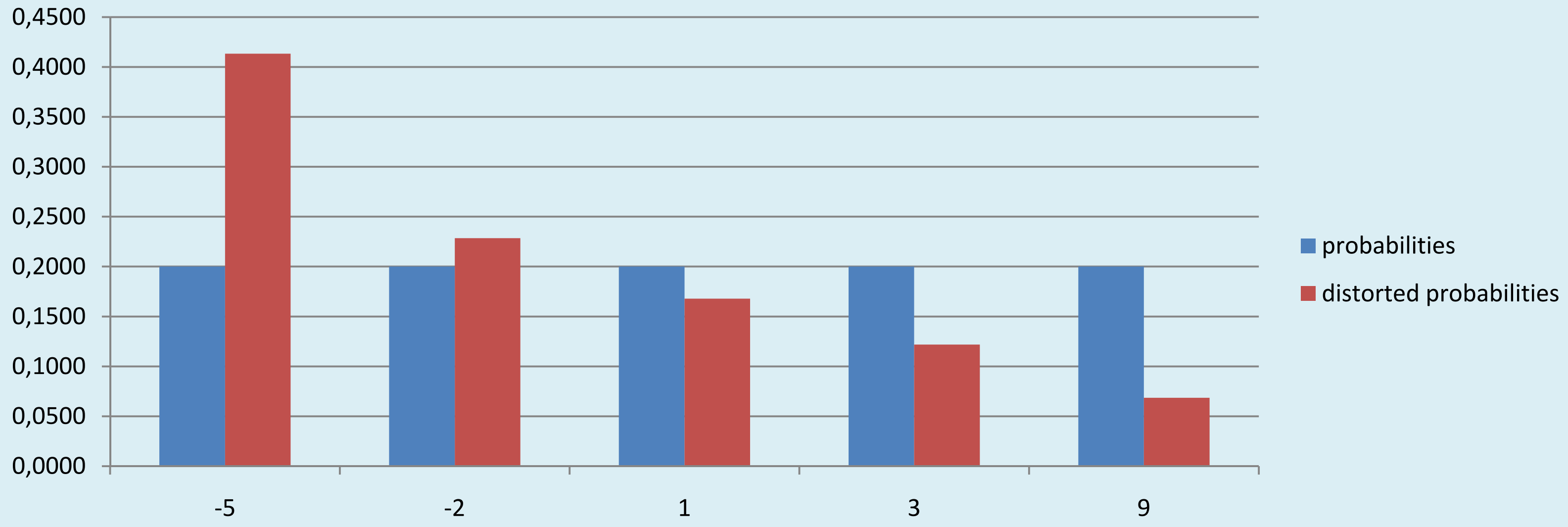
CONIC FINANCE





CONIC FINANCE

cash flow (sorted)	-5	-2	1	3	9
probabilities	0,2000	0,2000	0,2000	0,2000	0,2000
cumulative probabilities	0,2000	0,4000	0,6000	0,8000	1,0000
distorted cumul. probs	0,4133	0,6418	0,8097	0,9315	1,0000
distorted probabilities	0,4133	0,2285	0,1679	0,1218	0,0685



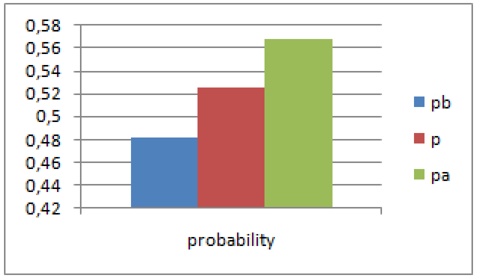
CONIC FINANCE

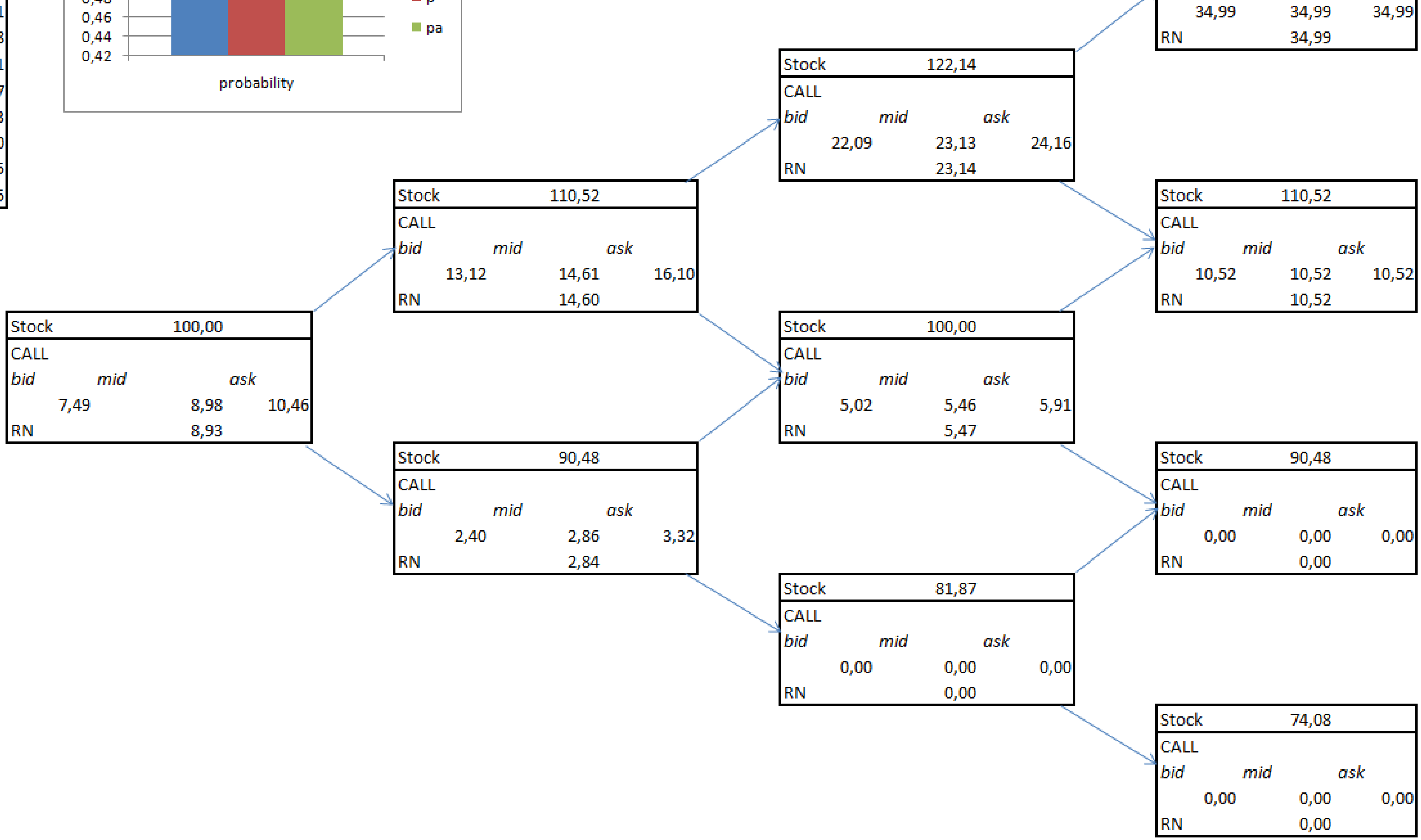
sorted neg. CF	-9	-3	-1	2	5
probabilities	0,2000	0,2000	0,2000	0,2000	0,2000
cumulative probabilities	0,2000	0,4000	0,6000	0,8000	1,0000
distorted cum probs	0,4133	0,6418	0,8097	0,9315	1,0000
distorted probabilities	0,4133	0,2285	0,1679	0,1218	0,0685

risk-neutral	(discounted) average cash flow	1,200
bid	(discounted) distored average cash flow	-1,374
ask	(discounted) negative distorted average negative cash flow	3,987
mid		1,307

CONIC FINANCE

S0	100
vol	0,1
r	0,03
T	1
lambda	0,2
alpha	1
p	0,525188
u	1,105171
d	0,904837
dt	0,333333
K	100
pa	0,567335
pb	0,482235



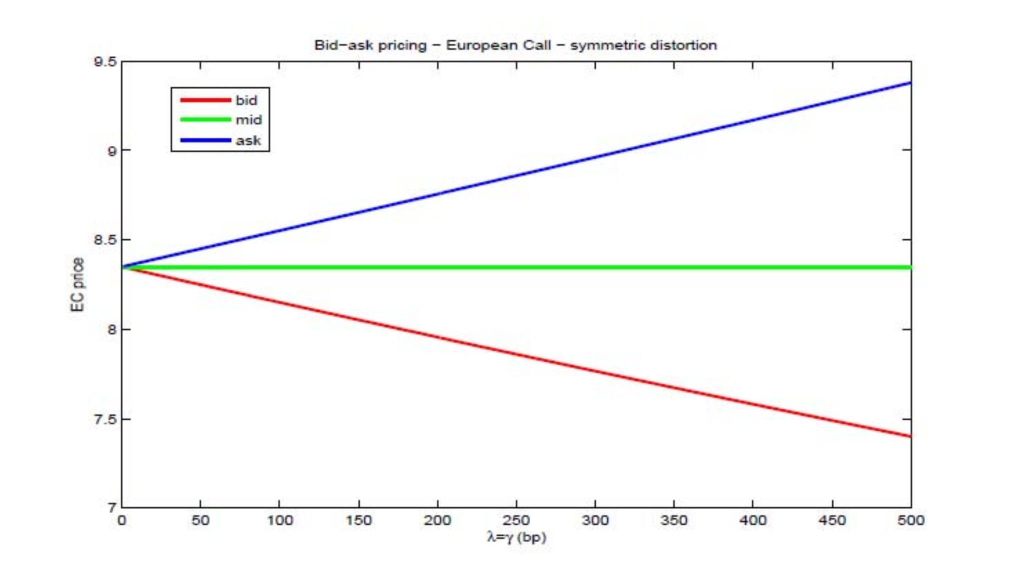


CONIC FINANCE

$$\begin{aligned} bid(X) &= \exp(-rT)\int_0^{+\infty} x \mathrm{d}\Phi(G(x);\lambda), \\ ask(X) &= \exp(-rT)\int_{-\infty}^{0} (-x)\mathrm{d}\Phi(1-G(-x);\lambda). \end{aligned}$$

- For a EC (K,T), we have

$$G(x) = 1 - \mathrm{N}\left(\frac{\log(S_0/(K+x)) + (r-q-\sigma^2/2)T}{\sigma\sqrt{T}}\right), \qquad x \geq 0$$



IMPLIED LIQUIDITY

- We will call the parameter, fitting the bid-ask around the mid price, the **implied liquidity parameter**.

- Hence for the EC(K,T) with given market bid, *b*, and ask, *a*, prices, the implied liquidity parameter is the specific $\lambda > 0$, such that:

$$a = -\exp(-rT)\mathrm{E}^{\lambda}[-(S_T - K)^+] \text{ and } b = \exp(-rT)\mathrm{E}^{\lambda}[(S_T - K)^+],$$

MEASURING LIQUIDITY

- How to measure and quantify in an isolated manner liquidity ?
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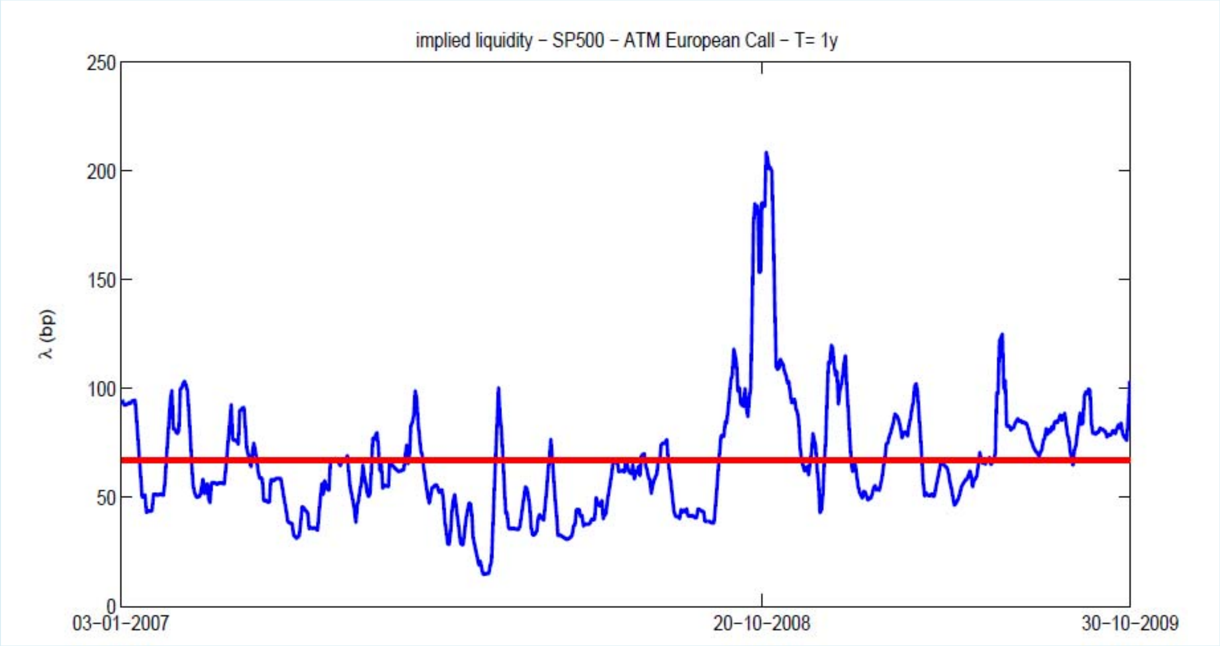
S1=100	S2=20
K=100	K=10
Vol=25.13%	Vol=1.0%
Bid = 9 EUR	Bid = 9 EUR
Mid = 10 EUR	Mid = 10 EUR
Ask = 11 EUR	Ask = 11 EUR
λ = 626 bp	*λ = 53769 bp*

A) EC1
B) EC2
C) Both
D) Can't say

IMPLIED LIQUIDITY– EVOLUTION OVER TIME

- We clearly see that liquidity is non constant over time and exhibits a mean-reverting behavior.
- The long run average of the implied liquidity of the data set and over the period of the investigation this equals 67.11 bp.
- The highest value for the implied liquidity parameter was 283.1 bp on the 20th of October 2008. Around that day (and the week-end before) several European banks were rescued by government interventions.



HERD BEHAVIOR AND COMONOTONICITY

- Comonotonicity measures herd behavior.
- A random vector $Y = (Y_1, \dots, Y_N)$ is comonotonic if

$$Y =^d (F_{Y_1}^{[-1]}(U), \dots, F_{Y_n}^{[-1]}(U)),$$

where *U* is a Uniform(0,1) random variable and

$$F_{Y_i}^{[-1]}(u) = \inf\{x \in \mathbb{R} | P(Y_i \leq x) = F_{Y_i}(x) \geq u\}.$$

- A comonotonic vector is driven by just one single factor.
- Given a vector $X = (X_1, \dots, X_N)$ we call the comonotonic counterpart of *X* the vector

$$X^c = (X_1^c, \dots, X_N^c) =^d (F_{X_1}^{[-1]}(U), \dots, F_{X_n}^{[-1]}(U))$$

HERD BEHAVIOR AND COMONOTONICITY

- Dow Jones, SP500 and any other indices are a weighted basket:

$$I(t) = \sum_{i=1}^{n} w_i S_i(t), \qquad t \geq 0$$

- We will denote by $I^c(T) = \sum_{i=1}^{n} w_i S_i^c(T)$, where $S^c(T)$ is the comonontonic counterpart of

$$S(T) = (S_1(T), \ldots, S_n(T))$$

- The comonotonic version incorporates perfect herd behavior.

- Intuitively, call options under perfect herd-behavior are more expensive, since each component moves in the same direction.

HERD BEHAVIOR AND COMONOTONICITY

- We will derive a bound for call options on the Index in terms of options in individual stocks.

$$C_{\text{index}}(K,T) = (I(T) - K)^+ \qquad C_{\text{stock}_i}(K,T) = (S_i(T) - K)^+$$

- Comonotonic theory tells us that

$$C_{\text{index}}(K,T) \leq \sum_{i=1}^{n} w_i C_{\text{stock}_i}(K_i^*, T) \leq \sum_{i=1}^{n} w_i C_{\text{stock}_i}(\tilde{K}_i^*, T)$$

where K_i^* is a specially "optimal" strike and $\tilde{K}_i^*$ is the closest lower market traded strike.

$$K_i^* = F_{S_i(T)}^{[-1]}(p^*) \text{ and } p^* = \sup\left\{ p \in [0,1] \middle| \sum_{i=1}^{n} w_i F_{S_i(T)}^{[-1]} \leq x \right\}$$

HERD BEHAVIOR AND COMONOTONICITY

- It is well know that the cdf of the stocks can be extracted out option info:

$$F_{S_i(T)}(x) = 1 + \exp(rT)\frac{\partial C_i(x+,T)}{\partial K}$$

- We hence have an upper bound for each traded vanilla index options in terms of the traded component options.

$$\frac{C_{\text{index}}(K,T)}{\sum_{i=1}^{n} w_i C_{\text{stock}_i}(\tilde{K}_i^*,T)}$$

- A similar expression exists for put options.
- We repeat this for each option in the market and come (cfr. VIX) to a VIXified 30 days herd-behavior measure.

HERD BEHAVIOR AND COMONOTONICITY

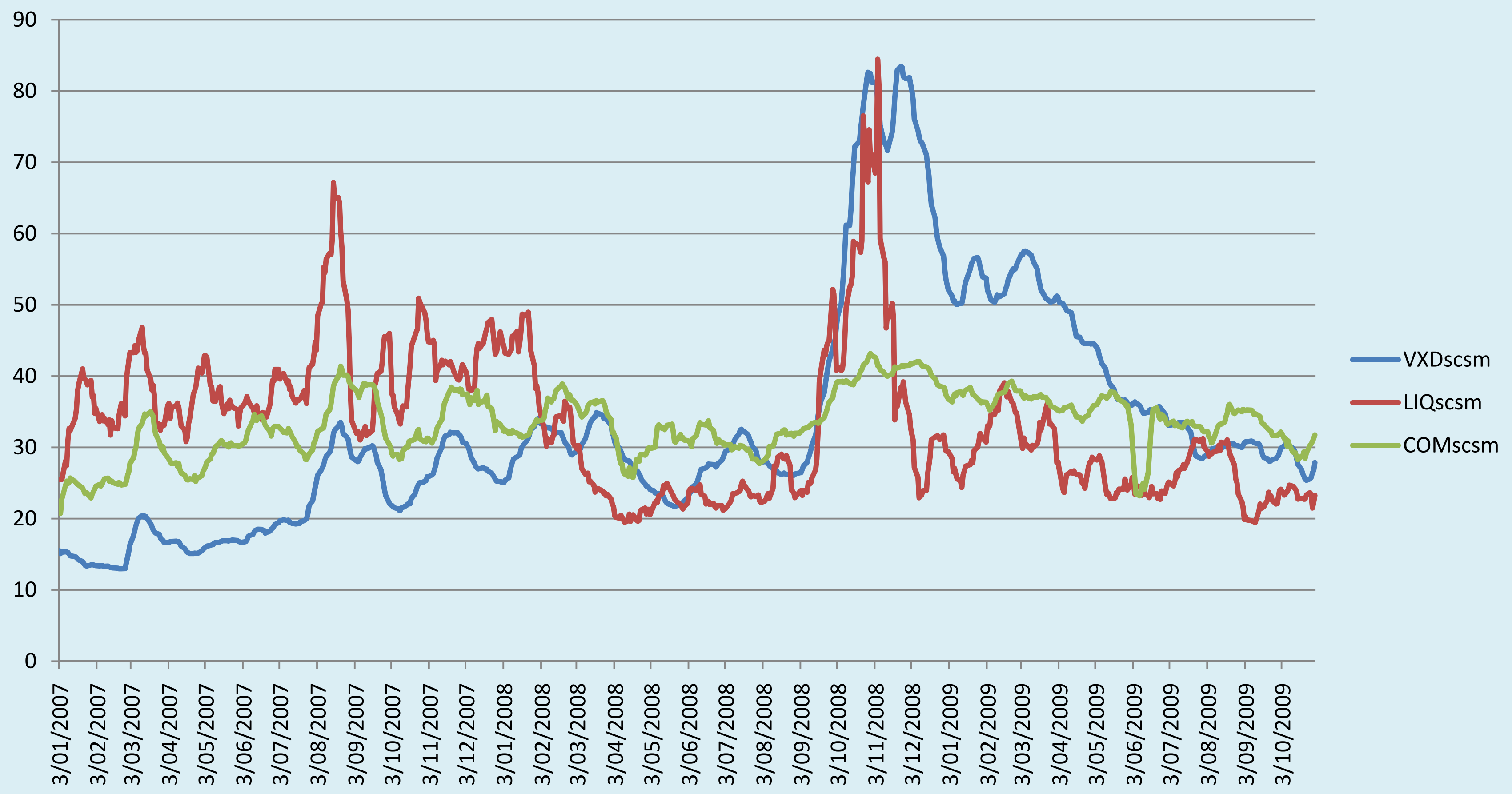
- We call the quantities

$$\frac{C_{\text{index}}(K,T)}{\sum_{i=1}^{n} w_i C_{\text{stock}_i}(\tilde{K}_i^*,T)} \qquad \frac{VIX}{VIX_{\text{comonotone}}}$$

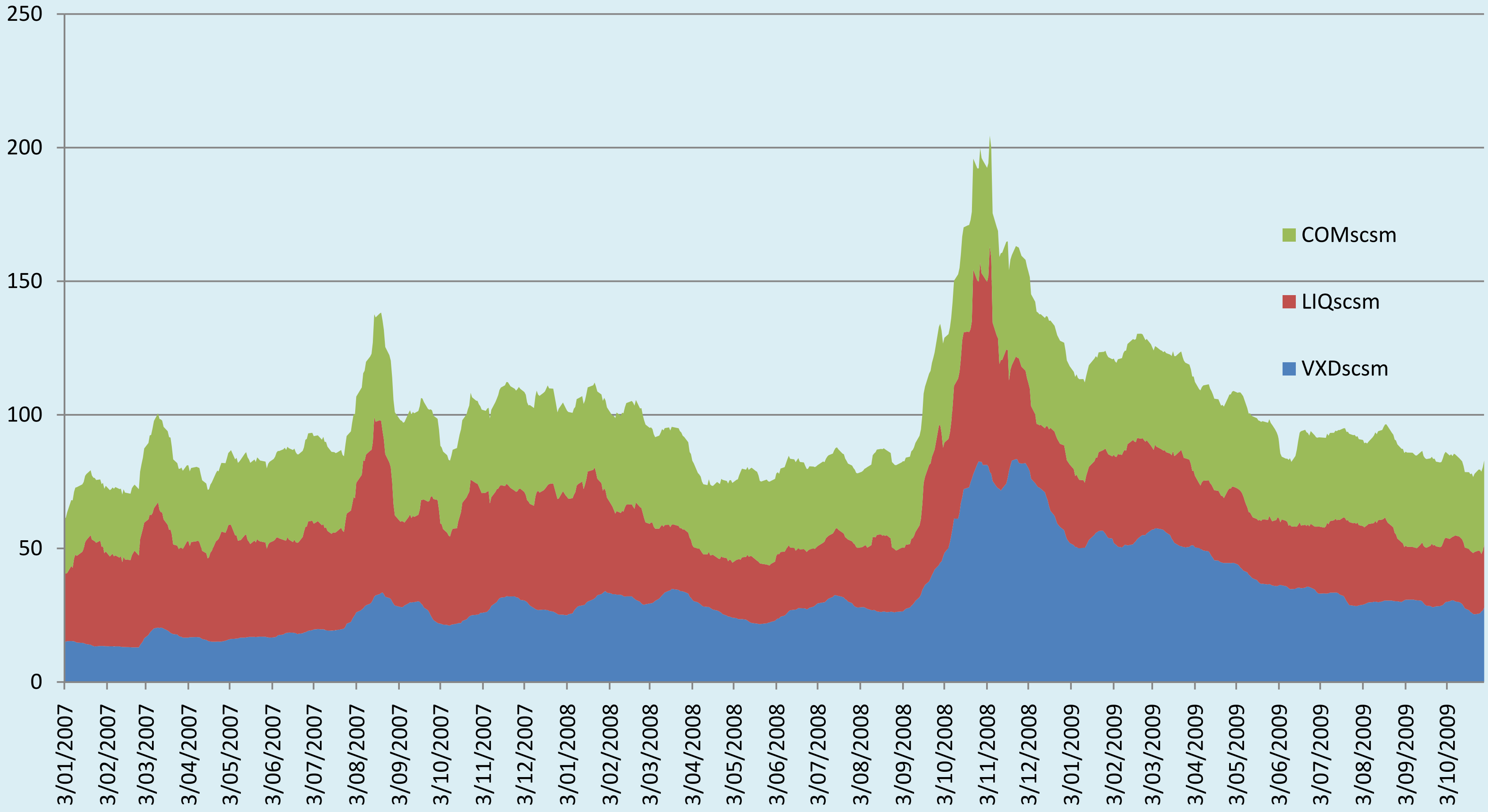
  the comonotonicity ratios.
- The closer this number is to 1, the closer we are to the comonotonic situation.
- If the ratio equals 1, we hence have perfect herd behavior.
- In conclusion, the above gives us a way to compute how much herd behavior there is on the basis of option surfaces.
- Furthermore, the gap between fully comonotonic and the current market situation can be monetized via a long-short position in options.

THE MARKET FEAR COMPONENTS

We smooth and rescale the 3 fear components :

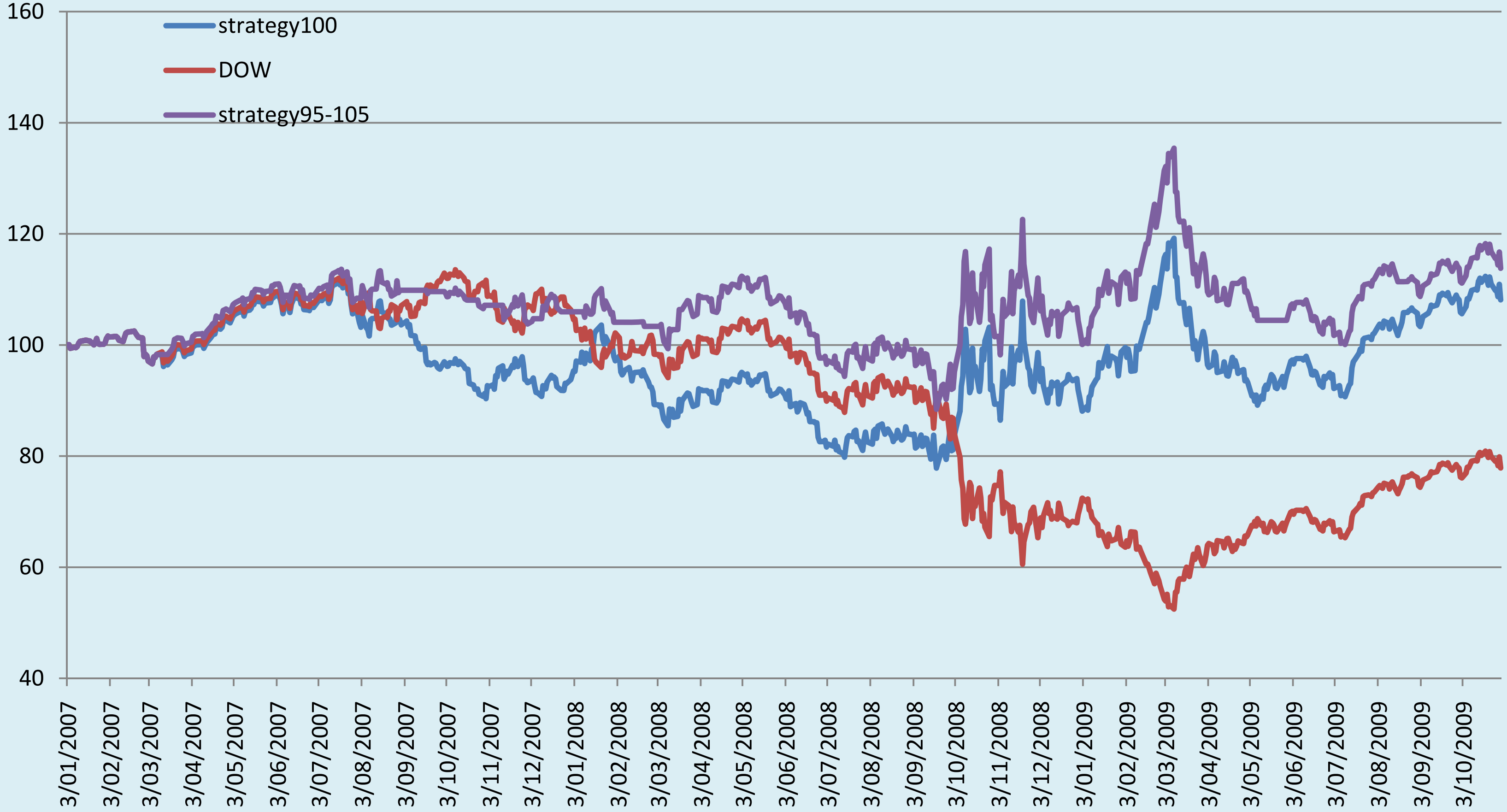


WE PROUDLY PRESENT YOU : THE FEAR INDEX



100 is base value; a value above 100 reflects a more than average stress situation; a value below 100 is a less than average stress situation

TRADING STRATEGIES



DOW : *long DJI*

Strategy100 : *short DJI if FIX >100; long DJI if FIX < 100*

Strategy95-105 : *short DJI if FIX >105; long DJI if FIX < 95*

CONCLUSION

- There are a variety of market fear factors.
- We have market risk and nervousness. The higher the volatility the more market uncertainty there is and the wider swings in the market can occur.
- We have liquidity risk. The bid and ask spread widens in periods of high uncertainty.
- We have herd-behavior. In a systemic crises, all assets move into the same direction. The more comonotonic behavior we have the more assets move together and the higher the systemic risk there is.
- The aim is to measure the market fear factors on the basis of market option data in a single intuitive number.
- We have presented the FIX as an overall market measure. The calculations are solely based on vanilla index options and individual stock options.

CONCLUSION

"'Be careful'! All you can tell me is 'be careful'?"

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