Credit Default Swap Liquidity Modelling: A Survey

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Agenda

Introduction to Liquidity in Risk Measurement and Pricing

Liquidity has been one of the key drivers of the crisis.

- "In 2007 liquidity risk did not rank among the top 30 risks affecting the banking system. In 2008, liquidity risk was numberone" *(PWC & Centre for the study of financial innovation)*
- "The contraction of liquidity in certain structured products... led to severe funding liquidity strains... Banks had made assumptions about asset market liquidity that proved to be overly optimistic. The (committee) will take action aimed at strengthening banks' liquidity risk management in relation to the risks they hold" *(Basel committee on banking supervision: Liquidity risk management and supervisory challenges, 2008)*

Introduction to Liquidity in Risk Measurement and Pricing

Szego (2009) illustrates, among other factors, a negative loop involving illiquidity as fueling the crisis development. We can consider for example the following schematization:

1. (Further) liquidity reduction on asset trade;
2. (Further) price contraction due to liquidity decline;
3. (Further) decline of value of bank assets portfolio;
4. Difficulty in refinancing, difficulty in borrowing, forced to (further) sale of assets;
5. Assets left? If so, go back to 1. If not:
6. Impossibility of refinancing;
7. Bankruptcy.

Introduction to Liquidity

This simplified representation highlights three types of liquidity

- **Market/trading liquidity:** trade quickly at a low cost (O'Hara (1995)). Low transaction costs / bid-ask spreads, low price impact of trading large volumes.
- **Funding liquidity:** positions can be easily funded through different financing sources.
- Market and funding liquidity are related since timely funding of positions relies on the market liquidity risk of assets, see above loop. The recent crisis prompted discussion on new guidelines (see BIS(2008), FSA(2009)).
- A 3d kind of liquidity, implicit in the above schematization, is the **systemic liquidity risk** associated to a global financial crisis, characterized by a generalized difficulty in borrowing.
- As with Credit Risk, liquidity needs to be analyzed from both a pricing perspective (CVA) and a risk management one (Credit VaR). See references at the end.

Liquidity and CDS pricing I

In this talk we then focus on liquidity modeling in the valuation of CDS.

- The basic idea: liquidity as an additional spread, leading to a liquidity stochastic discount factor.
- Chen, Cheng and Wu (2005),
- **Buhler and Trapp** (2006) and **(2008)** (BT06 and **BT08**), and
- **Chen, Fabozzi and Sverdlove (2008) (CFS)** among others.
- All but BT08 are unrealistic as they assume the liquidity rate to be independent of the hazard rate.
- BT08 and **Predescu et al (2009)** show, although in different contexts, that liquidity and credit are correlated. We discuss their results.
- We then analyze a different approach, by **Bongaerts, De Jong and Driessen (2009) (BDD)** who use Capital Asset Pricing Model (CAPM) like arguments to deduce liquidity from CDS data.

Liquidity as a spread in reduced form models I

Zero coupon corporate bond with maturity T issued by a firm with instantaneous credit spread h_t. Rec = 0. Let the risk free rate be r_t. The price of the bond, in a reduced form model, is

$$1_{\{\tau>t\}}\mathbb{E}\left[e^{-\int_t^T (r_u+h_u)\ du}|\mathcal{F}_t\right]$$

In a reduced form framework, we may add liquidity to the picture by assuming the bond price to be

$$1_{\{\tau>t\}}\mathbb{E}\left[e^{-\int_t^T (r_u+h_u+\ell_u)\ du}|\mathcal{F}_t\right]$$

where $L(t,u) = \exp(-\int_t^u \ell_s ds)$ is an illiquidity stochastic discount factor component.

Liquidity as a spread in reduced form models II

If h is a deterministic constant, under simplifying assumptions the equilibrium CDS spread on the same entity for maturity T_b is

$$h = \frac{S_{0,b}}{\mathrm{L}_{\mathrm{GD}}} \Rightarrow S_{0,b} = h\mathrm{L}_{\mathrm{GD}} \tag{1}$$

A very rough first approach:

$$\text{Protec Seller: } h^{ask} := S^{ask}_{0,b}/\mathrm{L}_{\mathrm{GD}}, \quad \text{Pr Buyer: } h^{bid} := S^{bid}_{0,b}/\mathrm{L}_{\mathrm{GD}}.$$

$$\ell := \frac{h^{ask} - h^{bid}}{2}.$$

$$h^{mid} := \frac{h^{bid} + h^{ask}}{2} = \frac{S^{bid}_{0,b} + S^{ask}_{0,b}}{2\mathrm{L}_{\mathrm{GD}}}$$

Liquidity as a spread in reduced form models III

We notice that by definition then

$$h^{bid} = h_{mid} - \ell, \quad h^{ask} = h_{mid} + \ell,$$

and using again the flat hazard rate formula,

$$S_{0,b}^{mid} = h^{mid} \mathrm{L}_{\mathrm{GD}} = \frac{S_{0,b}^{bid} + S_{0,b}^{ask}}{2}$$

and we have consistency with a meaningful definition of quoted MID S.
[Problem: MID in spread space is not MID in PV space]
We now move to stochastic liquidity and credit spreads

CDS liquidity pricing with intensity models: CFS I

$$S_{0,b}^{CFS,MID}(0) = \frac{\mathbb{E}_0\left[\mathrm{L}_{\mathrm{GD}} \int_0^{T_b} h_u \exp(-\int_0^u (r_s + h_s + \ell_s)ds)du\right]}{\mathrm{Accrual}_{0,b} + \sum_{i=a+1}^{b} \alpha_i \mathbb{E}_0[\exp(-\int_0^{T_i}(r_s + h_s)ds)]} \quad (2)$$

CFS: CDS spread formula for MID spread quote in stochastic (credit spread volatility) intensity models, with additional ℓ term.

- Analogous formula for ASK CDS spread $S_{0,b}^{CFS,ASK}(0)$ is the same but without ℓ.
- Here ℓ, illiquidity, lowers the CDS value from ASK to MID
- For tractability they assume r, ℓ and h to be **independent**
- liquidity correction affects only the default leg of the CDS, because the premium leg is an annuity numeraire.

CDS liquidity pricing with intensity models: CFS II

- Not clear to us, the premium leg is part of a traded asset subject to mark to market.
- Also BT[06,08] make a different choice, and assume that the CDS liquidity discount appears in the payment/fix leg of the CDS and that there is a bond liquidity discount for the recovery part of the default leg.
- CFS adopt CIR models for ℓ and h (physical and pricing measures) and resort to a Maximum Likelihood Estimation method. They compute:
  - The credit parameters from a time series of CDS ask premium rates $S_{0,b}^{CFS,ASK}(0)$
  - The liquidity parameters from a time series of mid CDS premium rates $S_{0,b}^{CFS,MID}(0)$.

CDS liquidity pricing with intensity models: CFS III

- CFS find that the parameters of h are more sensitive to credit ratings and those for ℓ are more sensitive to market capitalization and number of quotes, two proxies for liquidity.
- CFS argue, through a simulation analysis, that small errors in the CDS credit premium rate can lead to substantially larger errors in the corporate bond credit spread for the same reference entity. Empirically, they use the CDS estimated hazard rate model above to reprice bonds, with (h_t and ℓ_t) and without (just h_t) CDS liquidity.
- CFS find that incorporating the CDS liquidity factor results in improved estimates of the liquidity spreads for the bonds in their sample.
- Since a small CDS liquidity premium can translate into a large liquidity discount in a bond's price (mostly due to the principal repayment at final maturity) CFS can reconcile CDS premiums and bond credit spreads by incorporating liquidity [basis].

CDS liquidity pricing with intensity models: CFS IV

- However, the relevance of this analysis for data in-crisis remains to be proven.
- Finally, it is worth noticing that in CFS work the **(il)liquidity premium is earned by the CDS protection buyer**. Indeed, adding a positive (il)liquidity discount rate to the default leg lowers the fair CDS premium rate with respect to the case with no illiquidity. This means that the protection buyer will pay a lower premium for the same protection.

CDS liquidity pricing with intensity models: BT08 I

BT08 with liquidity spread demanded by protection buyer (bid):

$$S_{0,b}^{bid}(0) = \frac{\int_0^{T_b} P(0,t)\mathbb{E}_0[(1 - Re^{-\int_0^t \ell_s^{bond} ds})h_t e^{-\int_0^t h_s ds}]dt}{\sum_{i=1}^b P(0,T_i)\alpha_i \mathbb{E}_0[e^{-\int_0^{T_i} \ell_s^{cds,bid} ds} e^{-\int_0^{T_i} h_s ds}] + \text{Accrual}},$$

$$\text{Accrual} = \int_0^{T_b} P(0,t)(t - T_{\beta(t)-1})\mathbb{E}_0[e^{-\int_0^t \ell_s^{cds,bid} ds} h_t e^{-\int_0^t h_s ds}]dt$$

- In accord with our objection above, BT[06,08] make a different choice, and assume that the CDS liquidity discount appears also in the payment/fix leg of the CDS.
- default leg has a bond liquidity discount for the recovery part.
- The ask CDS premium rate formula (protection seller) is similar with $\ell_t^{c,ask}$ instead of $\ell_t^{c,bid}$. c abbreviates “CDS”

CDS liquidity pricing with intensity models: BT08 II

- It is further assumed that risk free **interest rates are independent of the default and liquidity** intensities.
- Ask illiquidity discount rate $\ell_t^{c,ask}$ appears in the payment leg and captures the fact that part of the ask CDS premium rate may not be due to default risk but reflects an additional premium for illiquidity demanded by the protection seller.
- On the other hand $\ell_t^{c,bid}$ would capture the illiquidity premium demanded by the protection buyers. Different illiquidity ask and bid spreads reflect asymmetric transaction costs which are driven by the general observed asymmetric market imbalances.

CDS liquidity pricing with intensity models: BT08 III

- In the BT08 model the stochastic default intensity (h_t) and the illiquidity intensities (ℓ^b_t, $\ell^{c,ask}_t$, $\ell^{c,bid}_t$) are all driven by four independent latent factors X_t, Y^b_t, $Y^{c,ask}_t$, $Y^{c,bid}_t$ as follows

$$\begin{pmatrix} dh_t \\ d\ell^b_t \\ d\ell^{c,ask}_t \\ d\ell^{c,bid}_t \end{pmatrix} = \begin{pmatrix} 1 & g_b & g_{ask} & g_{bid} \\ f_b & 1 & \omega_{b,ask} & \omega_{b,bid} \\ f_{ask} & \omega_{b,ask} & 1 & \omega_{ask,bid} \\ f_{bid} & \omega_{b,bid} & \omega_{ask,bid} & 1 \end{pmatrix} \begin{pmatrix} dX_t \\ dY^b_t \\ dY^{c,ask}_t \\ dY^{c,bid}_t \end{pmatrix} \tag{3}$$

  where X_t is CIR and Y^b_t, $Y^{c,ask}_t$, $Y^{c,bid}_t$ are arithmetic Brownian motions.
- f and g shape the correlations between the default intensity and the liquidity intensities
- ω shapes the liquidity spillover effects between bonds and CDS, which are assumed to be symmetric.

CDS liquidity pricing with intensity models: BT08 IV

- Notice that the system does not guarantee $L^{c,ask}(0,t) < L^{c,bid}(0,t)$, thus not excluding $S^{bid}_{0,b}(0) > S^{ask}_{0,b}(0)$. However, in their empirical study, they find that this never occurs.
- Affine term structure model with analytical formulae for both bonds and CDS.
- Data on bonds yields and CDS premium rates on 155 European firms for the time period covering 2001 to 2007 is used to estimate the model parameters.
- Several interesting findings.

$$\begin{pmatrix} dh_t \\ d\ell^b_t \\ d\ell^{c,ask}_t \\ d\ell^{c,bid}_t \end{pmatrix} = \begin{pmatrix} 1 & ? & ? & ? \\ + & 1 & - & + \\ + & - & 1 & - \\ \text{avg}- & + & - & 1 \end{pmatrix} \begin{pmatrix} dX_t \\ dY^b_t \\ dY^{c,ask}_t \\ dY^{c,bid}_t \end{pmatrix} \tag{4}$$

CDS liquidity pricing with intensity models: BT08 V

$$\begin{pmatrix} dh_t \\ d\ell_t^b \uparrow \\ d\ell_t^{c,ask} \uparrow \\ d\ell_t^{c,bid} \downarrow \end{pmatrix} = \begin{pmatrix} 1 & ? & ? & ? \\ +(*) & 1 & - & + \\ +(**) & - & 1 & - \\ \text{avg} - (***) & + & - & 1 \end{pmatrix} \begin{pmatrix} dX_t \uparrow \\ dY_t^b \\ dY_t^{c,ask} \\ dY_t^{c,bid} \end{pmatrix} \tag{5}$$

- Credit risk has an impact on both bond and CDS liquidity. As credit risk increases, liquidity dries up for bonds (*) and for the CDS ask (protection sellers) premium rates (**)
- However the impact of increased credit risk on CDS bid liquidity spreads is mixed across different companies, but on average higher credit risk results in lower CDS bid liquidity intensity (***).

CDS liquidity pricing with intensity models: BT08 VI

$$\begin{pmatrix} dh_t \\ d\ell_t^b \uparrow \\ \downarrow d\ell_t^{c,ask} \\ \uparrow d\ell_t^{c,bid} \end{pmatrix} = \begin{pmatrix} 1 & *\text{not} & *\text{stat.} & *\text{signif.} \\ + & 1 & **- & **+ \\ + & **- & 1 & ***- \\ \text{avg}- & **+ & ***- & 1 \end{pmatrix} \begin{pmatrix} dX_t \\ \uparrow dY_t^b \\ dY_t^{c,ask} \downarrow \\ dY_t^{c,bid} \uparrow \end{pmatrix}$$

- negligible impact of bond or CDS liquidity on credit(*) , but significant spill-over effects between bond and CDS liquidities (**). Signs ** as a substitution effect between bonds and CDS: as bond liquidity dries up (illiquidity intensity $\ell_t^b \uparrow$), bond prices go down and thus taking on credit risk using bonds becomes more attractive. If trader goes long credit by CDS selling protection, she needs to drop ask price ($\ell_t^{ask} \downarrow$) given attractiveness of bonds. Lower bond prices makes shorting credit risk via bonds more costly which then drives bid quotes in CDS higher ($\ell_t^{bid} \uparrow$).

CDS liquidity pricing with intensity models: BT08 VII

- Empirical parameter and intensity estimates allow to decompose the bond spreads and MID CDS premium rates into three separate components: **the pure credit risk premium, the pure liquidity risk premium, and the credit-liquidity correlation premium.**
- CDS: credit risk = 95% of obs MID, liquidity = 4%, correl = 1%.
- BONDS: credit risk = 60%, liquidity = 35%, correlation = 5%
- Cross-sectionally all credit, illiquidity and correlation premia for bonds and CDS increase monotonically as the credit rating worsens from AAA to B and then drop for the CCC category. These findings are in contrast to Predescu et al (2009) below.
- While generally similar behavior can be observed for the credit risk premium for both investment grade (IG) and high yield (HY) firms, the same is not true for the liquidity premium.

CDS liquidity pricing with intensity models: BT08 VIII

- BT interpret the related finding (see paper for details) as the flight to quality/liquidity hypothesis: in times of stress, investors move away from assets whose liquidity would decrease as credit risk increases and instead acquire liquid assets that can be easily traded. High correlation between illiquidity and credit will thus command a high spread premium component.
- All the empirical results with respect to the difference between IG and HY should, in our view, be considered carefully since their sample is highly biased towards investment grade firms. Also, as before, no data in-crisis has been used.

Liquidity in CDS pricing with CAPM: BDD I

Inspired by Acharya and Pedersen (2005) (AP) CAPM that allows for expected liquidity and liquidity risk.

- AP ask “How does liquidity risk affect asset prices in equilibrium?”.
- Their model assumes a dynamic overlapping generations economy where risk averse agents trade securities (equities) whose liquidity changes randomly over time.
- Agents have constant absolute risk aversion utility functions and live for just one period. They trade securities at times t and $t+1$ and derive utility from consumption at time $t+1$.
- They can buy a security at a price P_t but must sell at $P_t - C_t$: liquidity cost. Liquidity risk modeled by uncertainty on C_t.
- Under further assumptions such as no short selling, AR(1) processes with i.i.d. normal innovations for the dividends and illiquidity costs, AP derive the liquidity-adjusted conditional CAPM:

Liquidity in CDS pricing with CAPM: BDD II

- $$\underbrace{\mathbb{E}_t^P\left[R_{t+1}\right]}_{Expected\ \ Asset\ \ Gross\ \ Return} = \underbrace{R^f}_{Risk\ \ Free\ \ Rate} + \underbrace{\mathbb{E}_t^P\left[c_{t+1}\right]}_{Expected\ \ Illiquidity\ \ Cost}$$
$$+ \underbrace{\pi_t}_{Risk\ \ Premium} \underbrace{\frac{Cov_t\left(R_{t+1}, R_{t+1}^M\right)}{Var_t\left(R_{t+1}^M - c_{t+1}^M\right)}}_{\beta_{Mkt,t}\text{traditional}} + \pi_t \underbrace{\frac{Cov_t\left(c_{t+1}, c_{t+1}^M\right)}{Var_t\left(R_{t+1}^M - c_{t+1}^M\right)}}_{\beta_{2,t}\text{cov assets illiquid/mkt illiquid}}$$
$$-\pi_t \underbrace{\frac{Cov_t\left(R_{t+1}, c_{t+1}^M\right)}{Var_t\left(R_{t+1}^M - c_{t+1}^M\right)}}_{\beta_{3,t}\text{cov assets return/mkt illiquid}} - \pi_t \underbrace{\frac{Cov_t\left(c_{t+1}, R_{t+1}^M\right)}{Var_t\left(R_{t+1}^M - c_{t+1}^M\right)}}_{\beta_{4,t}\text{cov assets illiquid/mkt return}}$$

where $\pi_t = \mathbb{E}_t^P\left(R_{t+1}^M - c_{t+1}^M - R^f\right)$ is the conditional market risk premium, with the expectation taken under the physical measure. The remaining notation is self-evident.

Liquidity in CDS pricing with CAPM: BDD III

- Systematic mkt and liq risks captured by 4 conditional betas.
- $R - c$ replaces R in the traditional CAPM: $\text{cov}(R_1 - C_1, R_2 - C_2) =$ $= \text{cov}(R_1, R_2) + \text{cov}(C_1, C_2) - \text{cov}(R_1, C_2) - \text{cov}(R_2, C_1)$
- The first beta ($\beta_{Mkt,t}$) is the traditional CAPM β that measures the co-variation of individual security's return with the market return.
- The second beta ($\beta_{2,t}$) measures the covariance between asset's illiquidity and the market illiquidity.
- The third beta ($\beta_{3,t}$) measures the covariance between asset's return and the market illiquidity. Negative: Investors will accept a lower return on securities that have high return in times of high market illiquidity.
- The fourth beta ($\beta_{4,t}$) measures the covariance between asset's illiquidity and the market return. Negative. Investors will accept a lower return on securities that are liquid in times of market downturns.

Liquidity in CDS pricing with CAPM: BDD IV

- To estimate the model empirically, the unconditional version of the model is derived assuming constant conditional covariances between illiquidity and returns innovations:

$$\mathbb{E}^P\left[R_t - R^f\right] = \mathbb{E}^P[c_t] + \pi\beta_{Mkt} + \pi\beta_2 - \pi\beta_3 - \pi\beta_4 \tag{6}$$

  where $\pi = \mathbb{E}^P[\pi_t]$ is the unconditional market risk premium.
- AP empirically estimate the model using daily return and volume data on NYSE and AMEX stocks over 1962-1999. The illiquidity measure is the monthly average of the daily absolute return to volume ratio proposed by Amihud (2002). Illiquid stocks have higher ratios as a small volume will have a high impact on price.
- **This is only one component of liquidity, market impact of traded volume. No broker fees, bid-ask spreads and search costs.**

Liquidity in CDS pricing with CAPM: BDD V

- Using portfolios sorted along different dimensions, AP find that the liquidity adjusted CAPM performs better than the traditional CAPM in explaining cross-sectional variations in returns, especially for the liquidity sorted portfolios. Liquidity risk and expected liquidity premiums are found to be economically significant.
- On average the 1y premium for expected liquidity, i.e. the empirical estimate for the unconditional expected illiquidity cost $E(c_t)$, is equal to 3.5%.
- The liquidity risk premium, calculated as $\pi\beta_2 - \pi\beta_3 - \pi\beta_4$, is estimated to be 1.1%. About 80% of the liquidity risk premium is due to the third component $\pi\beta_4$ which is driven by the covariation of individual illiquidity cost with the market return.
- AP work with equity data, **BDD try and adapt AP to CDS**.
- BDD extend the model to an asset pricing model for both assets in positive net supply (like equities) and derivatives in zero net supply.

Liquidity in CDS pricing with CAPM: BDD VI

- Differently from the AP framework (no short selling), in BDD some agents are exposed to non-traded risk factors and in equilibrium they hold short positions in some assets to hedge these risk factors. Hedge assets are sold short by some agents to hedge their exposures to non-traded risks. Examples of such risks are non-traded bank loans or illiquid corporate bonds held by some financial institutions such as commercial banks.
- These institutions can hedge the risks with CDS. Other agents such as hedge funds or insurance companies may not have such exposures and may sell CDS to banks to earn the spread.
- The BDD model implies that the equilibrium expected returns on the hedge assets can be decomposed in: priced exposure to the non-hedge asset returns, hedging demand effects, an expected illiquidity component, liquidity risk premia and hedge transaction costs.
- Unlike the AP model where higher illiquidity leads to lower prices and higher expected returns, the impact of the liquidity on expected returns in BDD model is more complex.

Liquidity in CDS pricing with CAPM: BDD VII

- The liquidity risk impact depends on several factors such as heterogeneity in investors' non-traded risk exposure, risk aversion, horizon and agents wealth.
- Additionally BDD model implies that, for assets in zero net supply like CDSs, sensitivity of individual liquidity to market liquidity (β_2) is not priced.
- BDD perform an empirical test of the model on CDS portfolio returns over the 2004-2008 period. The CDS sample captures 46% of the corporate bond market in terms of amount issued.
- Their results imply a statistically and economically significant expected liquidity premium $\mathbb{E}[c_t]$ priced in the expected CDS returns. On average this is 0.175% per quarter (against $3.5\%/4$ in AP) and **it is earned by the protection seller, contrary to CFS and BT above**.
- They also find that the liquidity risk premium is statistically significant, but economically very small, -0.005% (AP: $1.1\%/4$).
- Somewhat questionably, the equity and credit risk premia together account for only 0.060% per quarter.

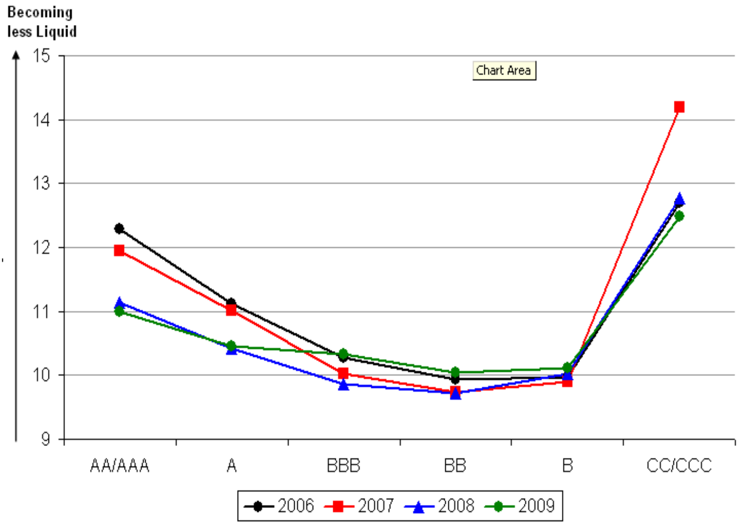
CDS Liquidity scores: Predescu et al (2009) I

- Predescu et al (2009) have built a statistical model that associates an ordinal liquidity score with each CDS reference entity.
- This provides a comparison of relative liquidity of over 2,400 reference entities in the CDS market globally, mainly concentrated in North America, Europe, and Asia.
- The model estimation and the model generated liquidity scores are based upon the Fitch CDS Pricing Service database
- The liquidity score is built using contributions from a number of active dealers quoting a reference entity: besides the actual number of contributors for each entity, one looks at
  - CDS quotes bid ask spreads
  - Inactivity measure from quotes staleness of individual dealers (depth proxy) and
  - dispersion in mid-quotes across dealers (price uncertainty proxy).

CDS Liquidity scores: Predescu et al (2009) II

- Illiquidity increases if any of the liquidity predictors increases, keeping everything else constant. Therefore less liquid names are associated with larger liquidity scores.
- The scores reveal a U-shape liquidity $\leftrightarrow$ credit with names in BB and B as most liquid and AAA + CCC/C as less liquid.
- The U-shape relation between Liquidity and Credit is interpreted as a larger order imbalance between buy and sell orders for names with a very high or a very low rating.
- In particular, one expects more buying pressure for CCC names and more selling pressure for AAA.
- Most trading will be in the middle (BBB, BB, and B). Illiquidity at the two extremes also changes over time. This is more pronounced for C entities, which were relatively less liquid in 2007.

CDS Liquidity scores: Predescu et al (2009) III



Conclusions I

- We reviewed different theoretical and empirical approaches for measuring liquidity on CDS prices.
- In the intensity models, BT08 provides the most general and realistic framework by incorporating correlation between liquidity and credit, liquidity spillover effects between bonds and CDS contracts and asymmetric liquidity effects on the Bid and Ask CDS premium rates. However the empirical testing can be improved by using a more representative sample over a longer time period including the crisis.

Conclusions II

- BDD derive an equilibrium asset pricing model with liquidity effects, tested using CDS data. Both expected liquidity and liquidity risk have a statistically significant impact on expected CDS returns. However only compensation for expected liquidity is economically significant with higher expected liquidity being associated with higher expected returns for protection sellers. Contrary to CFS where protection buyers earn liquidity premium.
- Predescu et al (2009) provide a statistical model that associates an ordinal liquidity score with each CDS over 2400 entities having contributors quotes, based on Bid Ask, inactivity and dispersion.
- Despite methodol. differences, these studies concur: CDS quotes should not be assumed to be only pure measures of credit risk.
  - CDS liquidity varies cross-sectionally and over time.
  - More importantly, CDS expected liquidity and liquidity risk premia are priced in CDS expected returns and premium rates.

Conclusions III

- Nevertheless further research is needed to test and validate the CDS liquidity premiums and the separation between credit and liquidity premia at CDS contract level.

References I

- Acerbi, C., and Scandolo, G. (2008). Liquidity risk theory and coherent measures of risk. *Quantitative Finance* 8(7), pp. 681–692.
- Acharya, V.V., and Pedersen, L.H. (2005). Asset Pricing with Liquidity Risk. *Journal of Financial Economics* 77(2), pp. 375–410.
- Amihud, Y. (2002). Illiquidity and Stock Returns: Cross-Section and Time-Series Effects. *Journal of Financial Markets* 5, pp. 31–56.
- Amihud, Y., Mendelson, H., and Pedersen, L.H. (2005). Illiquidity and Stock Returns: Liquidity and asset pricing. *Foundation and Trends in Finance* 1, pp. 269–364.
- Angelidis, T. and Benos, A. (2005). Liquidity adjusted Value-at-Risk based on the components of the bid-ask spread. *Working Paper*, available at `http://ssrn.com/abstract=661281`.

References II

- Bangia, A., Diebold, F.X., Schuermann, T. and Stroughair, J.D. (1999). Modeling Liquidity Risk With Implications for Traditional Market Risk Measurement and Management. *Working paper*, Financial Institutions Center at The Wharton School.
- Beumee, J., Brigo, D., Schiemert, D., and Stoyle, G. (2009). Charting a Course through the CDS Big Bang. Fitch Solutions research report, available at `http://www.defaultrisk.com/pp_crdrv176.htm`.
- Bielecki, T., and Rutkowski, M. (2001). Credit Risk: Modelling, Valuation and Hedging, Springer, New York, NY.
- BIS (2008). Principles for Sound Liquidity Risk Management and Supervision. Available at `http://www.bis.org/publ/bcbs144.htm`.

References III

- Bongaerts, D., De Jong, F., and Driessen, J. (2009). Derivative Pricing with Liquidity Risk: Theory and Evidence from the Credit Default Swap market. Forthcoming in *Journal of Finance*.
- Brigo, D., and Alfonsi, A. (2005). Credit Default Swaps Calibration and Derivatives Pricing with the SSRD Stochastic Intensity Model. *Finance and Stochastics* 9(1), pp. 29–42.
- Brigo, D., and Capponi, A. (2008). Bilateral counterparty risk valuation with stochastic dynamical models and application to Credit Default Swaps. Available at `papers.ssrn.com/sol3/papers.cfm?abstract_id=1318024`.

References IV

- Brigo, D., and Chourdakis, K. (2009). Counterparty Risk for Credit Default Swaps: Impact of spread volatility and default correlation. *International Journal of Theoretical and Applied Finance* 12(7), pp. 1007–1026.
- Brigo, D., and El-Bachir, N. (2008). An exact formula for default swaptions' pricing in the SSRJD stochastic intensity model. Accepted for publication in *Mathematical Finance*.
- Brigo, D., and Hanzon, B. (1998). On some filtering problems arising in mathematical finance. *Insurance: Mathematics and Economics* 22(1), pp. 53–64.
- Brigo, D., and Mercurio, F. (2001). A deterministic-shift extension of analytically tractable and time-homogeneous short rate models. *Finance and Stochastics* 5(3), pp. 369–388.

References V

- Brigo, D., and Mercurio, F. (2001). Interest Rate Models: Theory and Practice – with Smile, Inflation and Credit, Second Edition 2006, Springer Verlag.
- Brigo, D., and Nordio, C. (2010). Liquidity Adjusted Market Risk Measures with Stochastic Holding Period. Available at arXiv.org, ssrn.com and defaultrisk.com
- Brigo, D., Presescu, M., and Capponi, A. (2010). Liquidity modeling for Credit Default Swaps: an overview. To appear in: Bielecki, T., Brigo, D., and Patras, F. (Editors), Credit Risk Frontiers: The subprime crisis, Pricing and Hedging, CVA, MBS, Ratings and Liquidity, Bloomberg Press.

References VI

- Brunnermeier, M., and Pedersen, L.H. (2007). Market Liquidity and Funding Liquidity. *Review of Financial Studies* 22(6), pp. 2201–2238.
- Buhler, W., and Trapp, M. (2005). A Comparative Analysis of Liquidity in Bonds and CDS markets. *Working paper*.
- Buhler, W., and Trapp, M. (2006). Credit and liquidity risk in bond and CDS market. *Working paper*.
- Buhler, W., and Trapp, M. (2008). Time-Varying Credit Risk and Liquidity Premia in Bond and CDS Markets. *Working paper*.
- Cetin, U., Jarrow, R., and Protter, P. (2004). Liquidity risk and arbitrage pricing theory. *Finance and Stochastics* 8(3), pp. 311–341.

References VII

- Cetin, U., Jarrow, R., Protter, P., and Warachka, M. (2005). Pricing Options in an extended Black-Scholes Economy with Illiquidity: Theory and Empirical Evidence. *Review of Financial Studies* 19(2), pp. 439–529.
- Chen, R.R., Cheng, X., Fabozzi, F.J. , and Liu, B. (2008). An Explicit Multi Factor CDS pricing model with correlated factors. *Journal of Financial and Quantitative Analysis* 43(1), pp. 123–60.
- Chen, R.R., Cheng, X., and Wu, L. (2005). Dynamic Interactions Between Interest Rate, Credit, and Liquidity Risks: Theory and Evidence from the Term Structure of Credit Default Swaps. *Working Paper*, available at `http://papers.ssrn.com/sol3/papers.cfm?abstract_id=779445`.

References VIII

Chen, R.R, Fabozzi, F., and Sverdlove, R. (2008). Corporate CDS liquidity and its implications for corporate bond spreads. *Working paper*, available at http://business.rutgers.edu/files/cfs20080907.pdf.

Chen, R.R., and Scott, L. (1993). Maximum Likelihood Estimation of a Multi-Factor Equilibrium Model of the Term Structure of Interest Rates. *Journal of Fixed Income* 3(3), pp. 14–32.

Duffie, D., and Singleton, K. (1999). Modeling Term Structures of Defaultable Bonds. *Review of Financial Studies* 12(4), pp. 197–226.

Earnst, C., Stange, S., and Kaserer, C. (2009). Accounting for Non-normality in Liquidity Risk. Available at http://ssrn.com/abstract=1316769.

References IX

FSA (2009). Strengthening liquidity standards. Available at `http://www.fsa.gov.uk/pubs/policy/ps09_16.pdf`.

Garleanu, N., Pedersen, L.H., and Poteshman, A.M. (2009). Demand-based option pricing. *Review of Financial Studies* 22(10), pp. 4259–4299.

Jarrow, R., and Protter, P. (2005). Liquidity Risk and Risk Measure Computation. *Working Paper*, Cornell University.

Jarrow, R., and Subramanian, A. (1997). Mopping up Liquidity. *RISK* 10(10), pp. 170–173.

Johnson, T.C. (2008). Volume, liquidity, and liquidity risk. *Journal of Finacial Economics* 87(2), pp. 388–417.

References X

Leippold, M., and Wu, L. (2000). Quadratic term structure models. *Working paper*, available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=206329.

Longstaff, F.A., Mithal, S. and Neis, E. (2005). Corporate yield spreads: default risk or liquidity? New evidence from the credit default swap market. *Journal of Finance* 60(5), pp. 2213–2253.

Morini, M. (2009). Solving the puzzle in the interest rate market. *Working paper*, available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1506046.

O'Hara, M. (1995). Market Microstructure Theory, Blackwell Publishers, Cambridge.

References XI

- Predescu, M., Thanawalla, R., Gupton, G., Liu, W., Kocagil, A., and Reyngold, A. (2009). Measuring CDS Liquidity. Fitch Solutions presentation at the Bowles Symposium, Georgia State University, February 12 2009, available at http://www.pfp.gsu.edu/bowles/Bowles2009/Liu-LiquidityMeasure_MethodlogyResults.pdf.
- Stange, S., and Kaserer, C. (2008). Why and How to Integrate Liquidity Risk into a VaR-Framework. CEFS working paper 2008 No. 10, available at http://ssrn.com/abstract=1292289.
- Szegö, G. (2009). The Crash Sonata in D Major. *Journal of Risk Management in Financial Institutions*, 3(1), pp. 31–45.
- Tang, D.Y., and Yan, H. (2007). Liquidity and Credit Default Swap Spreads. *Working Paper*, available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=891263&rec=1&srcabs=676420.

Notes I

BT08 IG vs HY:

- While generally similar behavior can be observed for the credit risk premium for both investment grade (IG) and high yield (HY) firms, the same is not true for the liquidity premium.
- BT interpret the related finding (see paper for details) as the flight to quality/liquidity hypothesis: in times of stress, investors move away from assets whose liquidity would decrease as credit risk increases and instead acquire liquid assets that can be easily traded. High correlation between illiquidity and credit will thus command a high spread premium component.

Notes II

- During a period with high credit spreads (2001-2002, around Enron and Worldcom defaults) the bond liquidity premium for IG is very volatile and then flattens out at a higher level about mid 2003. On the other hand bond liquidity premium for HY firms reaches the highest level after Worldcom default and decreases to a lower level for the rest of the time period.
- In the CDS market the CDS liquidity premium for the IG firms is close to 0 for most of time, while for HY it is very volatile and becomes negative when credit risk is high. A negative CDS liquidity premium is consistent with more bid-initiated transactions in the market.
- The bond premium dynamics tend to comove over time with the credit risk premium dynamics. Interestingly the correlation premium is larger/smaller than the liquidity premium when credit spreads are high/low.

Notes III

- BT interpret this finding as being consistent with the flight to quality/liquidity hypothesis. In other words, in times of stress, investors will try to move away from assets whose liquidity would decrease as credit risk increases and instead acquire liquid assets that can be easily traded. High correlation between illiquidity and credit will thus command a high spread premium component.
- All the empirical results with respect to the difference between IG and HY should, in our view, be considered carefully since their sample is highly biased towards investment grade firms. Also, as before, no data in-crisis has been used.