Portfolio Risk with Selected Revaluation

Fast and Accurate Portfolio Risk (VaR, Stessed VaR)

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NOTICE

This is a preliminary version of the presentation.
Please check for updates.

The presentation is based on the following paper:

- Portfolio Risk with Selected Revaluation (2010).
  `http://ssrn.com/abstract=1802126`

Outline

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Portfolio Risk

- Calculate the distribution of portfolio values: a function of some given distribution of market data scenarios.
- One approach: “Monte-Carlo”, i.e., re-valuate your portfolio for a given sample of market data scenarios.
- “Risk” means, we are interested in some tail behavior, e.g. behavior under large scenario shifts.
- If the portfolio consists of non-linear products, a sensitivity approximation may not be adequate.
- Re-valuations are in general computationally expensive.

Goal

- Fast calculation of the portfolio risk.
- Accurate approximation of the distribution.
- Confidence (benchmark: full-revaluation).

Portfolio Risk
Notation

- Let $T_0, T_1, T_2, \ldots$ denote a set of calculation times
- with a comparably short period length $T_{k+1} - T_k$ (e.g., one day).
- Let $X^i(T_k)$ scenarios generated from the current market data vector $M(T_k)$ and some shift Z_i
$$X^i(T_k) = G(M(T_k), Z_i).$$
- Two common situations are absolute (additive) changes and relative changes, i.e.,
$$X^i(T_k) = M(T_k) + Z_i \quad \text{or} \quad X^i(T_k) = M(T_k)(1 + Z_i),$$
where the shifts Z_i have been generated accordingly.
- Let V_j denote the j-th product in the porfolio.
- The portfolio risk is a function of the distribution of scenario valuations
$$V_j(T_k, X^i(T_k))$$

Problem: The number of valuations can be huge since the number of scenarios and the number of products are usually large.

Example: Historical Simulation

- In a historical simulation Z_i are determined from past market data changes, e.g.
  - historic absolute changes, $Z_i := M(S_{i+1}) - M(S_i)$
  - historic relative changes, $Z_i := \frac{M(S_{i+1})}{M(S_i)} - 1$

  for a given set of past dates $\{S_i\}$

Note: We assume that the scenario shifts Z_i are fixed. In a historic VaR each day one scenario drops out, a new one comes in. In a historic simulated stress VaR they are fixed.

Problem Description

- Given fixed scenario shifts Z_i.
- Calculate $V_j(T_k, X^i(T_k))$, where
- $X^i(T_k) = G(M(T_k), Z_i)$.
- for every product V_j
- in every scenario X^i
- every day T_k.

Note: We assume that the scenario shifts Z_i are fixed.

Outline

Valuation under Shift Scenarios

Approximations of the Value Function

Approximating the Value Function

Let
$$V_j^{\mathrm{A}}(T_k, M(T_k), Z_i)$$
denote some approximation of $V_j(T_k, G(M(T_k), Z_i))$.

Remarks

- We do not require the approximation to have a small approximation error.
- For example, we do not require Z_i small.[1]

Next, we give two examples for such an approximation.

[1] In applications like VaR or stressed VaR Z_i will no be small.

Simple Sensitivity Approximation

- Simple sensitivity approximation of $V_j(T_k, X^i(T_k))$ uses the valuation and sensitivities at $(T_k, M(T_k))$ to approximate the scenario at $(T_k, X^i(T_k))$.
- If $V_j^{\mathrm{A}}(T_k, M(T_k), Z_i)$ is such an approximation

$$V_j(T_k, X^i(T_k)) \approx V_j^{\mathrm{A}}(T_k, M(T_k), Z_i) \qquad \text{for } Z_i \text{ small,} \tag{1}$$

i.e. we have at least

$$|V_j^{\mathrm{A}}(T_k, M(T_k), Z_i) - V_j(T_k, M(T_k))| = O(|Z_i|). \tag{2}$$

Example:

- First order Taylor polynomial, e.g., for additive scenario changes Z_i

$$V_j^{\mathrm{A}}(T_k, M(T_k), Z_i) := V_j(T_k, M(T_k)) + \nabla V_j(T_k, M(T_k)) Z_i \tag{3}$$

Properties:

- If the sensitivity ∇V_j is numerically stable, the approximation (3) is usually suitable for small scenarios Z_i.
- but has an larger approximation error for large scenarios Z_i.

Other Approximations

- The method which will be described is not limited to sensitivity approximations.
- Other approximations can be used.
- For example an approximation by using a different valuation model and/or a different valuation algorithm.

Examples

- Analytic approximation.
- Reduced calibration accuracy (e.g., non-smile vs. smile).
- Valuation with reduced accuracy (reduced Monte-Carlo path, reduced PDE discretization).

Properties:

- If $V_j^{\mathrm{A}}(T_k, M(T_k), Z_i)$ is such an approximation we have

$$|V_j^{\mathrm{A}}(T_k, M(T_k), 0) - V_j(T_k, M(T_k))| = O(1), \tag{4}$$

Valuation under Shift Scenarios

Approximation with Scenario Error Correction

Approximation Error

- Let E_j denote the error of the approximation $V_j^{\mathrm{A}}(T_k, M(T_k), Z_i)$, i.e.,

$$E_j(T_k, M(T_k), Z_i) \;:=\; V_j(T_k, X^i(T_k)) - V_j^{\mathrm{A}}(T_k, M(T_k), Z_i).$$

Properties

- Given smoothness of E_j, the difference of two such approximation errors at T_{k_1} and T_{k_2} can be estimated as

$$\begin{aligned} &\|E_j(T_{k_1}, M(T_{k_1}), Z_i) - E_j(T_{k_2}, M(T_{k_2}), Z_i)\| \\ &= \; O\left(|T_{k_1} - T_{k_2}| + \|M(T_{k_1}) - M(T_{k_2})\|\right). \end{aligned} \tag{5}$$

Where we subsume the scenario size into the constant of $O(\ldots)$ (i.e., we assume that the scenarios are taken from a compact set). From (5) we see that the difference between the error of the two approximations is small if the time period $T_{k_1} - T_{k_2}$ and the market data movement over that time period $M(T_{k_1}) - M(T_{k_2})$ is small. Market data movements $M(T_{k_1}) - M(T_{k_2})$ are usually much smaller than scenario movements Z_i.

Approximation Error Correction

- Let us also assume that for some coarser time discretization $T_0, T_{m_1}, T_{m_2}, \ldots$ we have full valuation vectors in all scenarios.

$$V_j(T_k, X^i(T_k)) \quad \text{for } k = 0, m_1, m_2, \ldots$$

- Let l be such that $m_l \leq k < m_{l+1}$.[2]
- Then we can improve our estimate as follows

$$\begin{aligned} V_j(T_k, X^i(T_k)) &= V_j^{\mathrm{A}}(T_k, M(T_k), Z_i) + E_j(T_k, M(T_k), Z_i) \\ &= V_j^{\mathrm{A}}(T_k, M(T_k), Z_i) + E_j(T_{m_l}, M(T_{m_l}), Z_i) \\ &\qquad + O\left(|T_k - T_{m_l}|^2 + |M(T_k) - M(T_{m_l})|\right). \end{aligned}$$

- from which we may then derive the following approximation:

[2] In other words T_{m_l} is the largest time point for which we have a full-revaluation prior to T_k.

Approximation with Scenario Error Correction The approximation with error correction is given by

$$V_j^{\mathrm{A+E}}(T_k, M(T_k), Z_i) \; := \; V_j^{\mathrm{A}}(T_k, M(T_k), Z_i) + E_j(T_{m_l}, M(T_{m_l}), Z_i) \tag{6}$$

Residual Approximation Error

We have

$$\begin{aligned} &\| V_j(T_k, X^i(T_k)) \; - \; V_j^{\mathrm{A+E}}(T_k, M(T_k), Z_i) \| \\ &= \; \| E_j(T_k, M(T_k), Z_i) - E_j(T_{m_l}, M(T_{m_l}), Z_i) \| \\ &= \; O(|T_k - T_{m_l}| + |M(T_k) - M(T_{m_l})|) \, . \end{aligned}$$

Residual Approximation Error

Let us compare the approximation error

$$O(|T_k - T_{m_l}| + |M(T_k) - M(T_{m_l})|)$$

to the approximation error of the simple sensitivity approximation, which is in the case of example (3)

$$O\left(|Z_i|^2\right).$$

Obviously, the latter does not make any sense, since scenario Z_i cannot be assumed to be small, but the former does.

Valuation under Shift Scenarios

Approximation Accuracy in the Context of Numerical Errors

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