

# Successfully implementing Stochastic Intrinsic Currency Volatility Models

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# Outline of this presentation

- Overview of the intrinsic currency framework
- Overview of the stochastic intrinsic currency volatility model (“SticVol”)
- Techniques for obtaining stable calibrations to the SticVol model
- Extension to multiple less liquid currencies
  - Using maximum entropy to parameterise the correlation matrix
- Derivative pricing: FX basket options
- Using intrinsic currency volatility to trade FX correlation and covariance swaps
- Conclusion

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# Motivation for intrinsic currency modelling

- Most people in the world who travel between different countries understand the concept of “intrinsic currency values”, because
  - they can hold the notes of each currency
  - they understand how much of each currency is required to buy the different things that they want to buy



- However forex market practitioners typically don't understand intrinsic currency values!
  - Forex market practitioners usually talk in currency pairs
- The intrinsic currency framework aims to restore the natural intuition about individual currencies

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# What is the intrinsic currency framework?

- Basic definitions:

$X_i = \textit{intrinsic value}$  of 1 unit of currency  $i$  so each currency has its own value variable. Observable FX rates are given by  $X_{ij} = X_i/X_j$ . If  $X_i$  follow log-normal processes with volatility  $\sigma_i$  then the instantaneous volatility of  $X_{ij}$  is

$$\sigma_{ij} = \sigma(\sigma_i, \sigma_j) = \sqrt{\sigma_i^2 - 2\rho_{ij}\sigma_i\sigma_j + \sigma_j^2} \quad .$$

where  $\rho_{ij}$  is the correlation between  $X_i$  and  $X_j$

- The units which measure the value of the  $X_i$  are not specified because they cancel out in the ratios  $X_i/X_j$ . Not necessary to determine the units of the  $X_i$  to use the framework.

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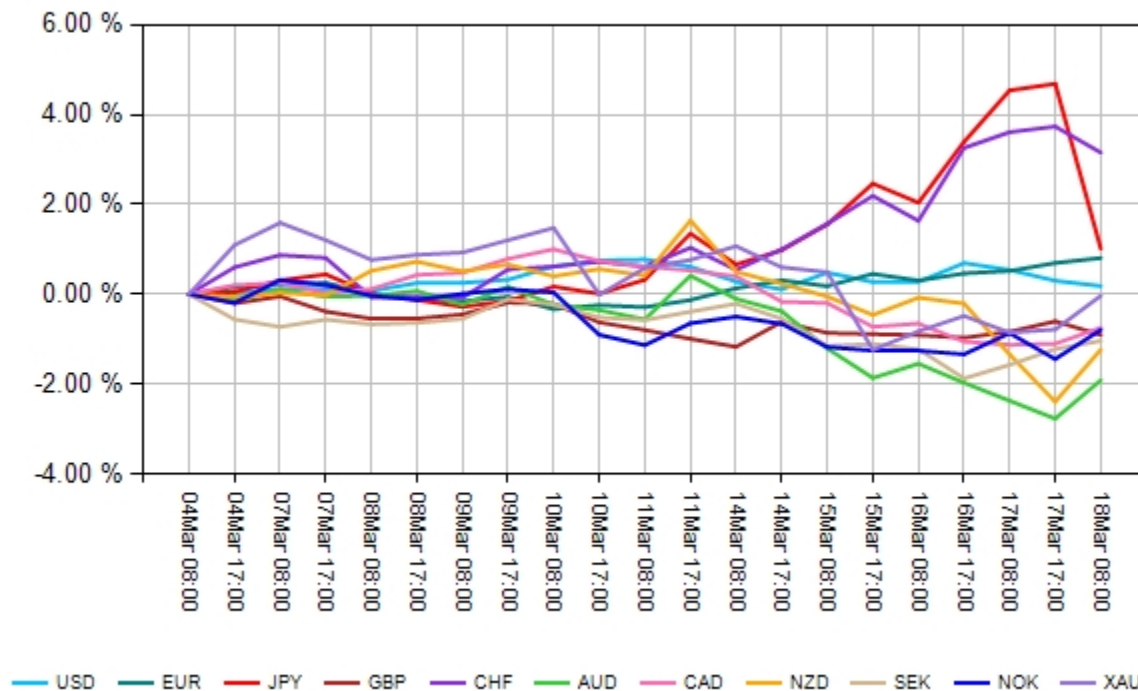
# Dealing with the extra degree of freedom

- With  $N$  currencies, there are  $N-1$  degrees of freedom which are determined by the usual FX rates. However, the intrinsic currency framework has an extra degree of freedom, because it's got a value variable for each currency.
- Use the concept of ***maximum information entropy*** to handle the extra degree of freedom in the covariance matrix between the variables of the intrinsic currency framework.
- Use ***maximum likelihood estimation*** to handle the extra degree of freedom when estimating changes in the intrinsic currency values themselves.

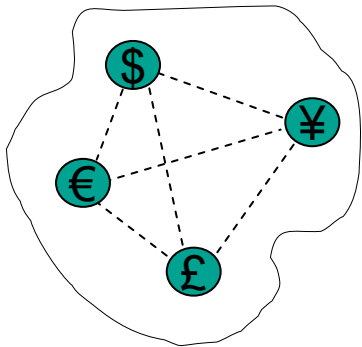
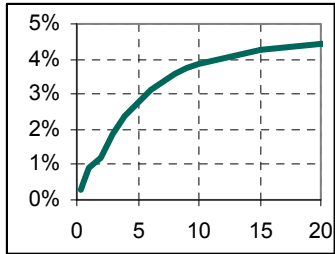
# Graphs of intrinsic currency values

- In the week following the recent Japanese Earthquake, JPY strengthened significantly, but less obvious was that CHF was highly correlated with a lot of the JPY movements:

The Movements of Intrinsic Currency Values - Majors



# Motivation for the SticVol pricing model



- With interest rate derivatives pricing, options on individual rates can be priced using the Black formula, without any regard for the behaviour of related rates along the yield curve. Libor Market Models, which deal with the whole yield curve, are more powerful but harder to formulate.
- With FX derivative pricing, the focus is usually to model FX rates individually. Simultaneously modelling all the cross rates between a set of currencies should be more powerful, if such a model can be found.
- The stochastic intrinsic currency volatility model (“**SticVol** model”) is capable of doing just that.

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# Stochastic intrinsic currency volatility model

- A SABR-style model for the FX market.
- Chose currency  $k$  as the numéraire. Then the stochastic processes for  $X_i$  and  $\sigma_i$  are given by:

$$\frac{dX_i}{X_i} = (\tilde{\lambda} - r_i + \varepsilon^2 \rho_{ik} \sigma_i \sigma_k) dt + \varepsilon \sigma_i dW_i$$

$$\frac{d\sigma_i}{\sigma_i} = \varepsilon^2 \tilde{\rho}_{ik} v_i \sigma_k dt + \varepsilon v_i dZ_i$$

where  $r_i$  is the interest rate in currency  $i$ ,  $\tilde{\lambda}$  is a variable which is the same for all  $X_i$ , and where the correlation matrices are defined by

$$\begin{pmatrix} dW \\ dZ \end{pmatrix} \begin{pmatrix} dW' & dZ' \end{pmatrix} = \begin{pmatrix} \rho & \tilde{\rho}' \\ \tilde{\rho} & r \end{pmatrix} dt ,$$

where ' denotes vector and matrix transpose.



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# Variables of the SticVol model for N currencies

- N stochastic intrinsic currency volatilities  $\sigma_i$ .
- An  $N \times N$  symmetric matrix  $\rho$  of correlations between the N intrinsic currency values  $X_i$ .
- N volatility of volatility variables  $v_i$ . It turns out that there is significant tenor dependency to the  $v_i$ , so that 1 month  $v_i$  are typically around 200%-230%, with 1 year  $v_i$  typically around 65%-85%.
- An  $N \times N$  symmetric matrix of correlations  $r$  between the N intrinsic currency volatilities  $\sigma_i$ . These are typically all positively correlated, which makes intuitive sense.
- An  $N \times N$  matrix  $\tilde{\rho}$  between all the intrinsic currency values  $X_i$  and all the intrinsic currency volatilities  $\sigma_i$ . These tend to reflect the market's risk-reversals.

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# Features of SticVol

- SABR itself cannot be used consistently with multiple FX rates, because if  $X_{ij}$ ,  $X_{jk}$  are SABR processes then  $X_{ik} = X_{ij} X_{jk}$  is not a SABR process
- With SticVol, FX rates are given by  $X_{ij} = X_i / X_j$ , so any intrinsic currency formulation automatically has the required symmetries provided:
  - it produces the correct risk-neutral processes for  $X_{ij}$ , and
  - it behaves correctly when numéraire currency changed
- When using SticVol, an approximation formula is available for the log-normal implied volatility  $\sigma_B$  as a function of strike  $K$ , as a function of the SticVol variables  $\sigma_i$ ,  $v_i$ ,  $\rho$ ,  $r$  and  $\tilde{\rho}$ .
- Smile shape of SABR and SticVol very similar

# New SABR style formula for FX volatility $\sigma_B$

$$\sigma_B = \varepsilon \sigma_{ij} \frac{z}{x(z, \sigma_i, \sigma_j)} \left( 1 + \varepsilon^2 \left( \frac{1}{4} (a_1 \sigma_{ij} + 2a_3 + 2a_4) - \frac{1}{6} a_5 + \frac{2a_2 - a_1^2}{24} \right) \tau_{ex} \right)$$

$$z = \frac{\ln\left(\frac{X_{ij}^F}{K}\right)}{\varepsilon \sigma_{ij}} \quad , \quad x(z, \sigma_i, \sigma_j) = \frac{\ln\left(\frac{\sqrt{1 - 2\varepsilon a_1 z + \varepsilon^2 (a_2 + a_5) z^2} - \frac{a_1}{\sqrt{a_2 + a_5}} + \varepsilon \sqrt{a_2 + a_5} z}{1 - \frac{a_1}{\sqrt{a_2 + a_5}}}\right)}{\varepsilon \sqrt{a_2 + a_5}}$$

$$a_1 = \frac{1}{\sigma_{ij}} D_\sigma \sigma_{ij} \quad , \quad a_3 = \tilde{\rho}_{ij} \frac{v_i \sigma_i \sigma_j}{\sigma_{ij}} \frac{\partial \sigma_{ij}}{\partial \sigma_i} + \tilde{\rho}_{jj} \frac{v_j \sigma_j^2}{\sigma_{ij}} \frac{\partial \sigma_{ij}}{\partial \sigma_j}$$

$$a_2 = \frac{v_i^2 \sigma_i^2}{\sigma_{ij}^2} \left( \frac{\partial \sigma_{ij}}{\partial \sigma_i} \right)^2 + \frac{2r_{ij} v_i v_j \sigma_i \sigma_j}{\sigma_{ij}^2} \frac{\partial \sigma_{ij}}{\partial \sigma_i} \frac{\partial \sigma_{ij}}{\partial \sigma_j} + \frac{v_j^2 \sigma_j^2}{\sigma_{ij}^2} \left( \frac{\partial \sigma_{ij}}{\partial \sigma_j} \right)^2$$

$$a_4 = \frac{\sigma_i^2 \sigma_j^2}{\sigma_{ij}^4} (1 - \rho_{ij}^2) \left( \frac{1}{2} v_i^2 - r_{ij} v_i v_j + \frac{1}{2} v_j^2 \right) \quad , \quad a_5 = D_\sigma a_1$$

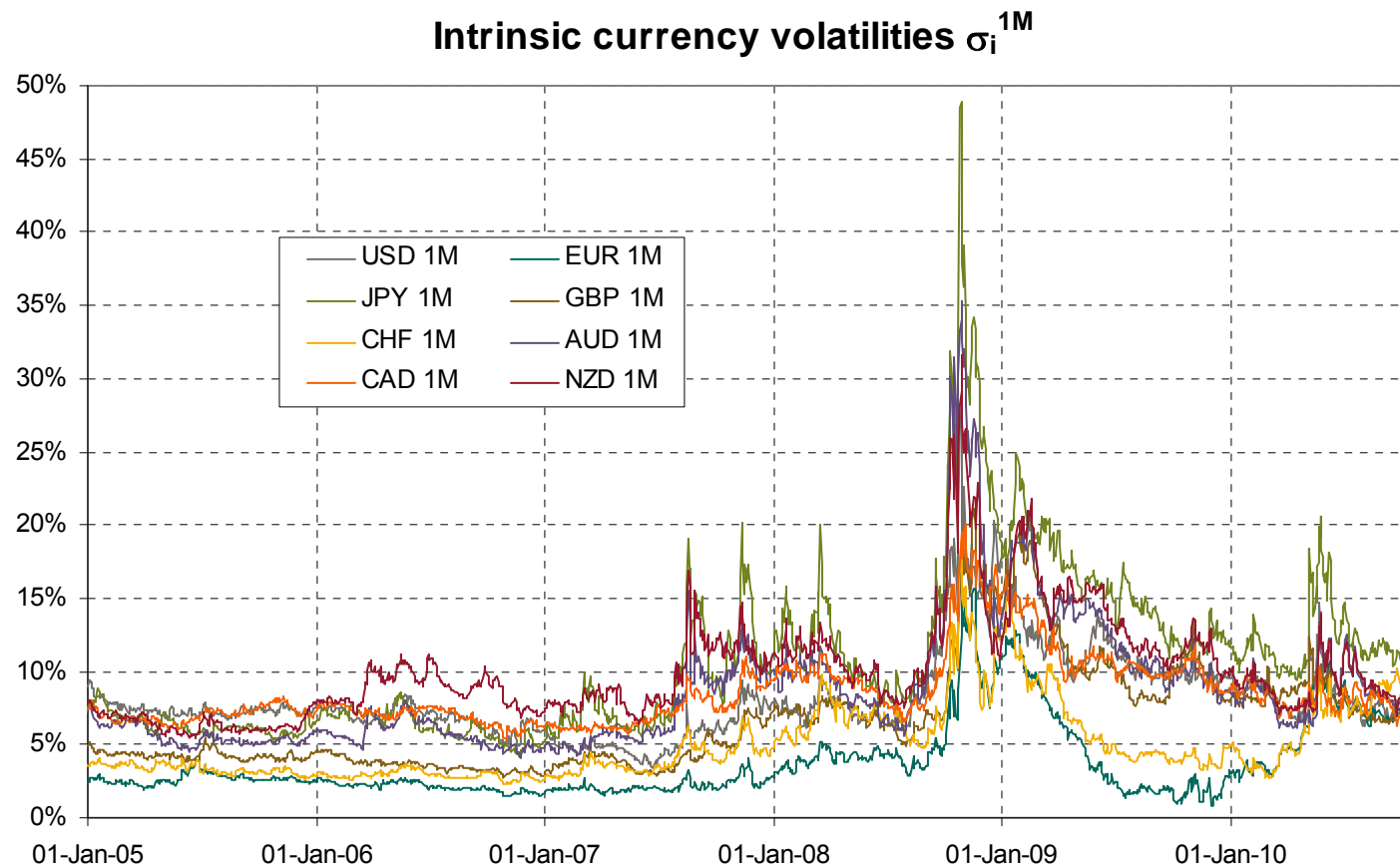
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# Advantages of intrinsic currency modelling

- SticVol demonstrates that modelling the stochastic behaviour of intrinsic currencies is a good way to build models which handle more than two currencies at the same time.
  - SticVol is just one possible multi-currency model. Other stochastic processes could be postulated.
  - The intrinsic currency approach guarantees that all natural symmetries of the forex market are respected
- For pricing models which include interest rates as well as forex rates, modelling correlations between interest rate variables and intrinsic currency values is a natural way to build models
  - A reasonable starting point might be to assume that interest rates in a particular currency are independent of intrinsic currency values in all other currencies

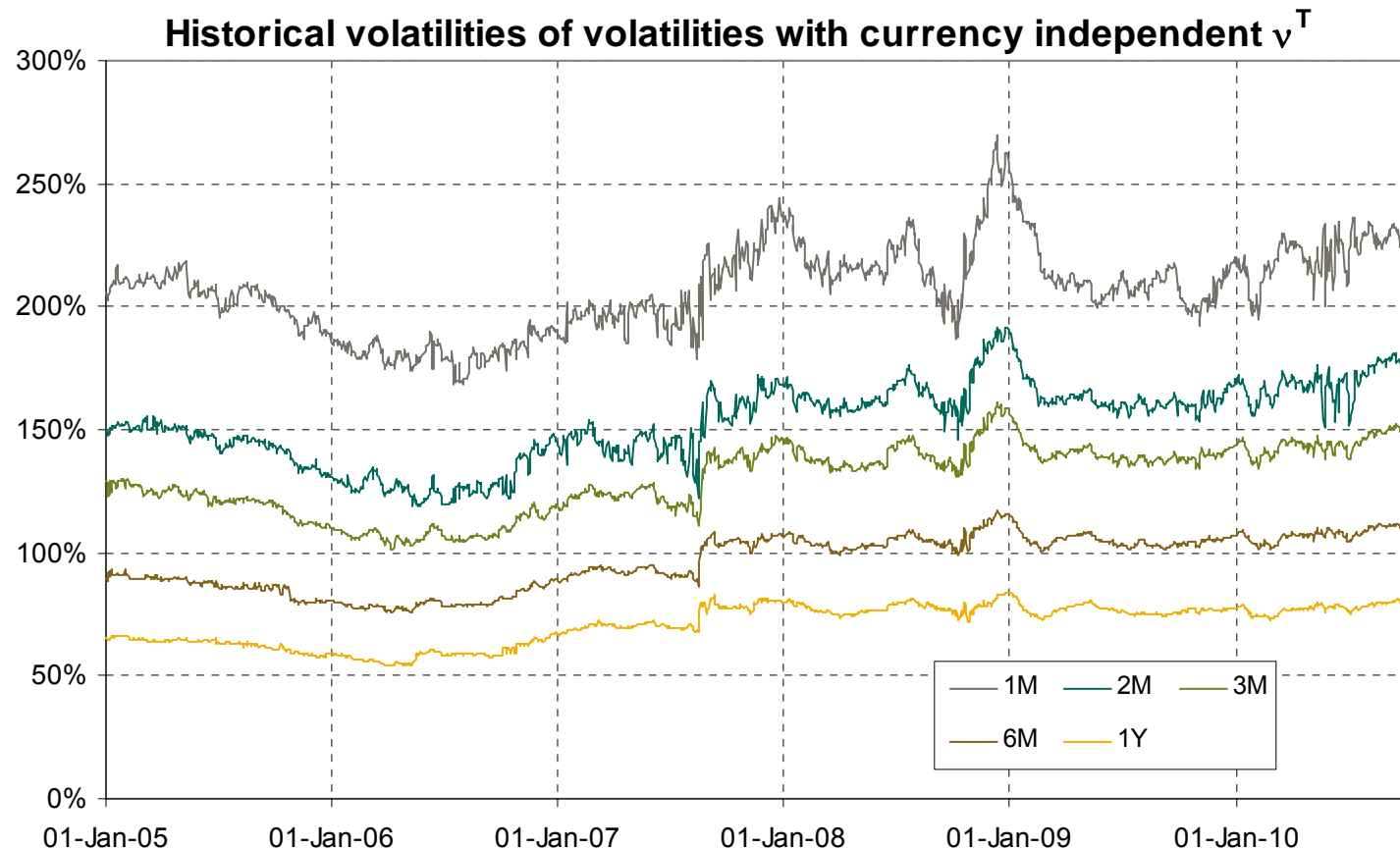
# 1-month intrinsic currency volatilities

- Up until the Greece crisis, EUR had been the safest currency, having the lowest volatility:



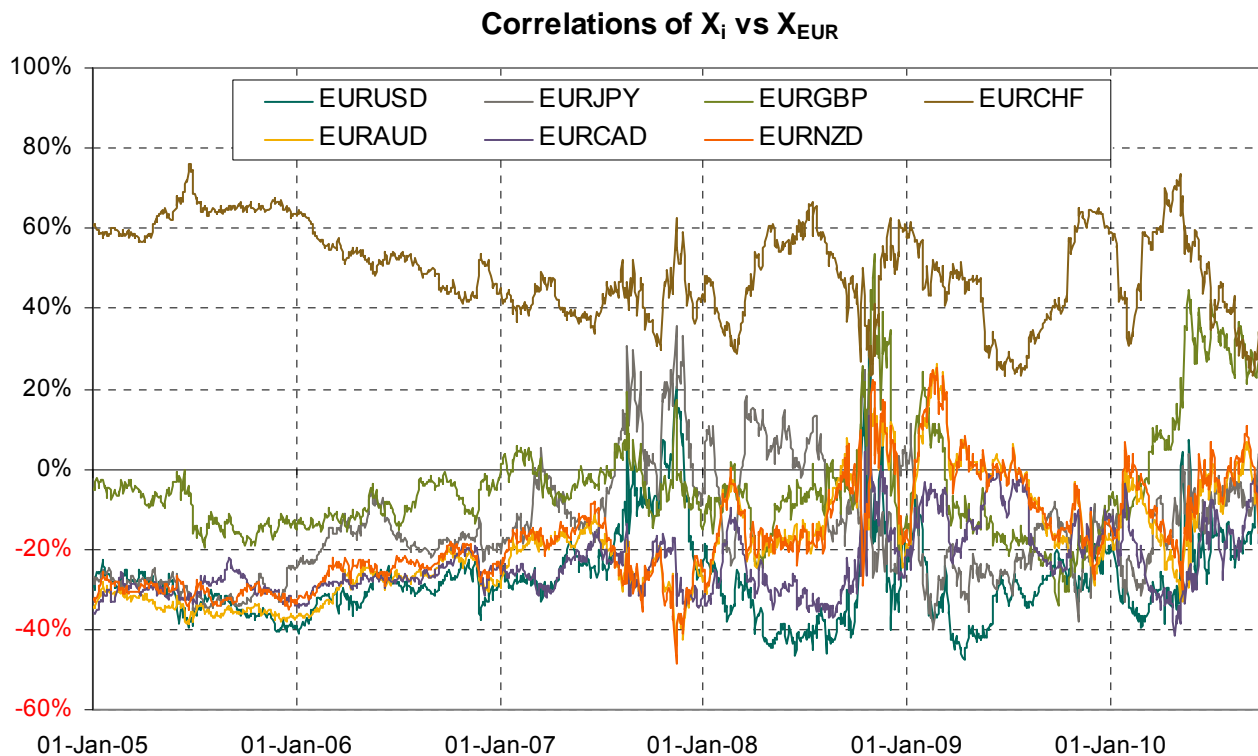
# Volatility of volatilities $v^T$

- The volatilities of volatilities  $v^T$  exhibit a strong tenor dependence:



# Correlations of $X_i$ with $X_{EUR}$

- As with the previous work, there is a significant correlation between  $X_{EUR}$  and  $X_{CHF}$ :



# Calibrating SticVol involves a lot of market data

Market data on 2-Feb-2010 (all 420 data points):

|         | At-the-money volatilities |       |       |       |       | Risk reversals |       |       |       |       | Market strangles |      |      |      |       |
|---------|---------------------------|-------|-------|-------|-------|----------------|-------|-------|-------|-------|------------------|------|------|------|-------|
|         | 1M                        | 2M    | 3M    | 6M    | 1Y    | 1M             | 2M    | 3M    | 6M    | 1Y    | 1M               | 2M   | 3M   | 6M   | 1Y    |
| EUR/USD | 10.4%                     | 10.9% | 11.2% | 11.8% | 12.2% | 1.2%           | 1.2%  | 1.3%  | 1.3%  | 1.4%  | 0.2%             | 0.2% | 0.3% | 0.4% | 0.4%  |
| USD/JPY | 12.3%                     | 12.7% | 13.0% | 13.5% | 14.0% | 1.0%           | 1.4%  | 1.8%  | 2.1%  | 2.6%  | 0.2%             | 0.3% | 0.3% | 0.3% | 0.2%  |
| GBP/USD | 10.8%                     | 11.4% | 11.7% | 12.5% | 13.0% | 1.3%           | 1.3%  | 1.5%  | 1.7%  | 1.8%  | 0.3%             | 0.3% | 0.3% | 0.4% | 0.4%  |
| USD/CHF | 9.9%                      | 10.3% | 10.6% | 11.3% | 11.8% | -0.5%          | -0.5% | -0.6% | -0.6% | -0.6% | 0.2%             | 0.3% | 0.3% | 0.4% | 0.5%  |
| AUD/USD | 14.1%                     | 14.5% | 14.9% | 15.3% | 15.6% | 2.1%           | 2.4%  | 2.7%  | 2.8%  | 3.0%  | 0.2%             | 0.3% | 0.4% | 0.5% | 0.6%  |
| USD/CAD | 11.5%                     | 12.2% | 12.6% | 13.0% | 13.2% | -1.0%          | -1.1% | -1.1% | -1.1% | -1.1% | 0.3%             | 0.3% | 0.4% | 0.5% | 0.5%  |
| NZD/USD | 14.5%                     | 15.3% | 15.6% | 16.3% | 16.7% | 1.7%           | 2.2%  | 2.6%  | 3.0%  | 3.3%  | 0.2%             | 0.3% | 0.4% | 0.5% | 0.6%  |
| EUR/JPY | 13.2%                     | 13.7% | 14.1% | 14.8% | 15.4% | 1.9%           | 2.3%  | 2.7%  | 3.2%  | 3.8%  | 0.2%             | 0.2% | 0.2% | 0.2% | 0.2%  |
| EUR/GBP | 9.2%                      | 9.8%  | 10.1% | 10.7% | 11.0% | -0.3%          | -0.5% | -0.7% | -0.8% | -0.9% | 0.2%             | 0.3% | 0.3% | 0.4% | 0.4%  |
| EUR/CHF | 3.9%                      | 4.0%  | 4.1%  | 4.2%  | 4.4%  | 0.2%           | 0.3%  | 0.5%  | 0.8%  | 1.0%  | 0.2%             | 0.3% | 0.3% | 0.3% | 0.3%  |
| EUR/AUD | 10.2%                     | 10.8% | 11.1% | 11.6% | 12.0% | -0.8%          | -1.0% | -1.3% | -1.5% | -1.6% | 0.2%             | 0.3% | 0.4% | 0.4% | 0.5%  |
| EUR/CAD | 10.3%                     | 10.7% | 11.1% | 11.5% | 11.8% | 0.3%           | 0.3%  | 0.3%  | 0.4%  | 0.4%  | 0.3%             | 0.4% | 0.4% | 0.5% | 0.5%  |
| EUR/NZD | 10.3%                     | 10.9% | 11.3% | 12.1% | 12.5% | -1.0%          | -1.4% | -1.7% | -2.0% | -2.2% | 0.3%             | 0.3% | 0.4% | 0.5% | 0.6%  |
| GBP/JPY | 14.6%                     | 15.4% | 16.0% | 16.8% | 17.4% | 2.2%           | 2.6%  | 3.0%  | 3.5%  | 4.1%  | 0.2%             | 0.2% | 0.2% | 0.2% | 0.2%  |
| CHF/JPY | 12.8%                     | 13.4% | 13.6% | 14.1% | 14.4% | 1.8%           | 2.3%  | 2.4%  | 2.8%  | 3.2%  | 0.1%             | 0.1% | 0.1% | 0.2% | 0.2%  |
| AUD/JPY | 18.8%                     | 19.5% | 20.0% | 20.5% | 21.5% | 2.4%           | 3.4%  | 4.2%  | 5.1%  | 6.6%  | 0.2%             | 0.1% | 0.1% | 0.1% | -0.2% |
| CAD/JPY | 15.7%                     | 16.2% | 16.5% | 17.1% | 17.7% | 1.9%           | 2.4%  | 2.8%  | 3.3%  | 3.8%  | 0.2%             | 0.3% | 0.3% | 0.2% | 0.2%  |
| NZD/JPY | 19.3%                     | 20.0% | 20.5% | 21.0% | 22.0% | 2.4%           | 3.3%  | 4.1%  | 5.1%  | 6.6%  | 0.2%             | 0.1% | 0.2% | 0.1% | -0.2% |
| GBP/CHF | 9.6%                      | 10.3% | 10.5% | 11.1% | 11.4% | 0.6%           | 0.7%  | 0.9%  | 1.1%  | 1.2%  | 0.2%             | 0.3% | 0.3% | 0.3% | 0.3%  |
| GBP/AUD | 12.7%                     | 13.1% | 13.3% | 13.5% | 13.8% | -0.7%          | -0.8% | -1.1% | -1.1% | -1.2% | 0.3%             | 0.3% | 0.4% | 0.5% | 0.6%  |
| GBP/CAD | 11.8%                     | 12.3% | 12.6% | 13.0% | 13.3% | 0.5%           | 0.5%  | 0.5%  | 0.6%  | 0.6%  | 0.3%             | 0.4% | 0.5% | 0.5% | 0.6%  |
| GBP/NZD | 12.8%                     | 13.3% | 13.6% | 14.2% | 14.6% | -0.2%          | -0.6% | -0.8% | -1.0% | -1.3% | 0.3%             | 0.3% | 0.4% | 0.5% | 0.6%  |
| AUD/CHF | 10.7%                     | 11.3% | 11.8% | 12.8% | 13.3% | 0.7%           | 1.0%  | 1.2%  | 1.5%  | 1.9%  | 0.3%             | 0.3% | 0.3% | 0.4% | 0.4%  |
| CAD/CHF | 10.5%                     | 11.1% | 11.6% | 12.1% | 12.5% | -0.1%          | -0.1% | -0.1% | 0.0%  | 0.1%  | 0.3%             | 0.4% | 0.5% | 0.5% | 0.6%  |
| NZD/CHF | 10.8%                     | 11.4% | 11.8% | 13.0% | 13.6% | 1.0%           | 1.3%  | 1.7%  | 2.6%  | 3.0%  | 0.3%             | 0.3% | 0.4% | 0.4% | 0.4%  |
| AUD/CAD | 10.7%                     | 11.0% | 11.2% | 11.4% | 11.5% | 1.5%           | 1.7%  | 1.9%  | 1.9%  | 2.0%  | 0.2%             | 0.2% | 0.3% | 0.4% | 0.5%  |
| AUD/NZD | 7.6%                      | 7.6%  | 7.7%  | 7.9%  | 8.1%  | -0.1%          | -0.1% | -0.1% | -0.1% | -0.1% | 0.1%             | 0.1% | 0.1% | 0.2% | 0.2%  |
| NZD/CAD | 10.8%                     | 11.5% | 11.8% | 12.2% | 12.5% | 1.0%           | 1.4%  | 1.7%  | 2.0%  | 2.3%  | 0.2%             | 0.3% | 0.4% | 0.5% | 0.5%  |



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# Calibrating the SticVol model

- To calibrate the model, minimise the target function  $f(\sigma_i, v_i, \mathbf{p}, \mathbf{r}, \tilde{\rho}) = Z_1 + Z_2 + Z_3$  where
  - $Z_1$  is the  $\chi^2$  statistic, so that minimising  $Z_1$  minimises the difference between the model and the market
$$\chi^2 = \sum_{\text{data points}} \left[ \frac{(\text{market-value}) - (\text{model-value})}{\frac{1}{2}(\text{bid-offer spread in market})} \right]^2$$
  - $Z_2$  is minus the information entropy in the correlation matrix, so minimising  $Z_2$  maximises the information entropy;
  - $Z_3$  is a term that encourages desirable properties in the calibration results, as discussed below.
- Use a conjugate gradient technique, defined in terms of variables which ensure that  $0 < \sigma_i < \sigma_{\max}$ ,  $0 < v_i < v_{\max}$ , and that the correlation matrix (comprised of  $\mathbf{p}, \mathbf{r}, \tilde{\rho}$ ) remains positive definite.

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## Imposing constraints on $\sigma_i$ and $v_i$

- Imposing constraints on  $\sigma_i$  and  $v_i$  helps a lot because conjugate gradient schemes test “silly” values as they search for the minimum.
- Allow  $\sigma_i$  to be tenor dependent (i.e.  $\sigma_i^T$ ), but for stability impose the condition that  $v_i$  is tenor dependent but currency independent (i.e.  $v^T$ ).
- $\sigma_{\max} = 80\%$  and  $v_{\max} = 500\%$  worked well in practice, and produced reasonable fits in the turbulent markets following the Lehman default.
- Use the functions  $\sigma_i^T(x_i^T)$  and  $v^T(y^T)$  where

$$\sigma_i^T(x_i^T) = \frac{\sigma_{\max}}{1 + e^{-x_i^T}} \quad , \quad v^T(y^T) = \frac{v_{\max}}{1 + e^{-y^T}} \quad .$$

# Keeping the correlation matrix positive definite

- Parameterise the Cholesky decomposition of the correlation matrix, i.e.

$$\begin{pmatrix} \rho & \tilde{\rho}' \\ \tilde{\rho} & \mathbf{r} \end{pmatrix} = \mathbf{C}\mathbf{C}' = \begin{pmatrix} 1 & 0 & 0 & \cdots \\ \cos\theta_{21} & \sin\theta_{21} & 0 & \cdots \\ \cos\theta_{31} & \sin\theta_{31}\cos\theta_{32} & \sin\theta_{31}\sin\theta_{32} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} 1 & \cos\theta_{21} & \cos\theta_{31} & \cdots \\ 0 & \sin\theta_{21} & \sin\theta_{31}\cos\theta_{32} & \cdots \\ 0 & 0 & \sin\theta_{31}\sin\theta_{32} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

so that the correlation matrix is defined in terms of angles  $\theta_{ij}$  which are unconstrained.

- Any scheme where the sums of squares of the rows of  $\mathbf{C}$  equal 1 can be used, so that the correlation matrix has 1's down the leading diagonal. Above scheme uses hypersphere angles to satisfy that condition.

# Hypersphere angle parameterisation

- If  $c_{ij}$  are the elements of the Cholesky decomposition, the hypersphere parameterisation is given by

$$c_{ij} = \begin{cases} \cos \theta_{i1} & j = 1 \\ \sin \theta_{i1} \cos \theta_{i2} & j = 2 \\ \sin \theta_{i1} \sin \theta_{i2} \cos \theta_{i3} & j = 3 \\ \dots & \\ \sin \theta_{i1} \dots \sin \theta_{i,i-2} \cos \theta_{i,i-1} & j = i - 1 \\ \sin \theta_{i1} \dots \sin \theta_{i,i-2} \sin \theta_{i,i-1} & j = i \\ 0 & j > i \end{cases}$$

- However the pyramid nature of this scheme means that for large  $i$ , all elements in row  $i$  depend on  $\theta_{i1}$ , but only the last two depend on  $\theta_{i,i-1}$ . A more balanced scheme works better.

# Alternative angle parameterisation

- Distribute the sin/cos terms more evenly:

$$\begin{aligned} i = 2 & \left\{ \begin{array}{l} c_{21} = \cos \theta_{21} \\ c_{22} = \sin \theta_{21} \end{array} \right. & i = 3 & \left\{ \begin{array}{l} c_{31} = \cos \theta_{31} \cos \theta_{32} \\ c_{32} = \sin \theta_{31} \\ c_{33} = \cos \theta_{31} \sin \theta_{32} \end{array} \right. \\ \\ i = 4 & \left\{ \begin{array}{l} c_{41} = \cos \theta_{41} \cos \theta_{42} \\ c_{42} = \sin \theta_{41} \cos \theta_{43} \\ c_{43} = \cos \theta_{41} \sin \theta_{42} \\ c_{44} = \sin \theta_{41} \sin \theta_{43} \end{array} \right. & i = 5 & \left\{ \begin{array}{l} c_{51} = \cos \theta_{51} \cos \theta_{52} \cos \theta_{54} \\ c_{52} = \sin \theta_{51} \cos \theta_{53} \\ c_{53} = \cos \theta_{51} \sin \theta_{52} \\ c_{54} = \sin \theta_{51} \sin \theta_{53} \\ c_{55} = \cos \theta_{51} \cos \theta_{52} \sin \theta_{54} \end{array} \right. \end{aligned}$$

- This scheme is more balanced because e.g. up to  $i=8$ , there are no more than 3 trigonometric functions in each  $c_{ij}$ .

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# Calculate the derivatives analytically

- With 8 currency pairs and 5 tenors, there are 165 variables to calibrate. To make the scheme computationally feasible, use the chain rule to calculate analytically the derivatives required for conjugate gradient, i.e.

$$\frac{\partial f}{\partial x_i^T} = \frac{\partial f}{\partial \sigma_i^T} \frac{d\sigma_i^T}{dx_i^T} \quad , \quad \frac{\partial f}{\partial y^T} = \frac{\partial f}{\partial v^T} \frac{dv^T}{dy^T}$$

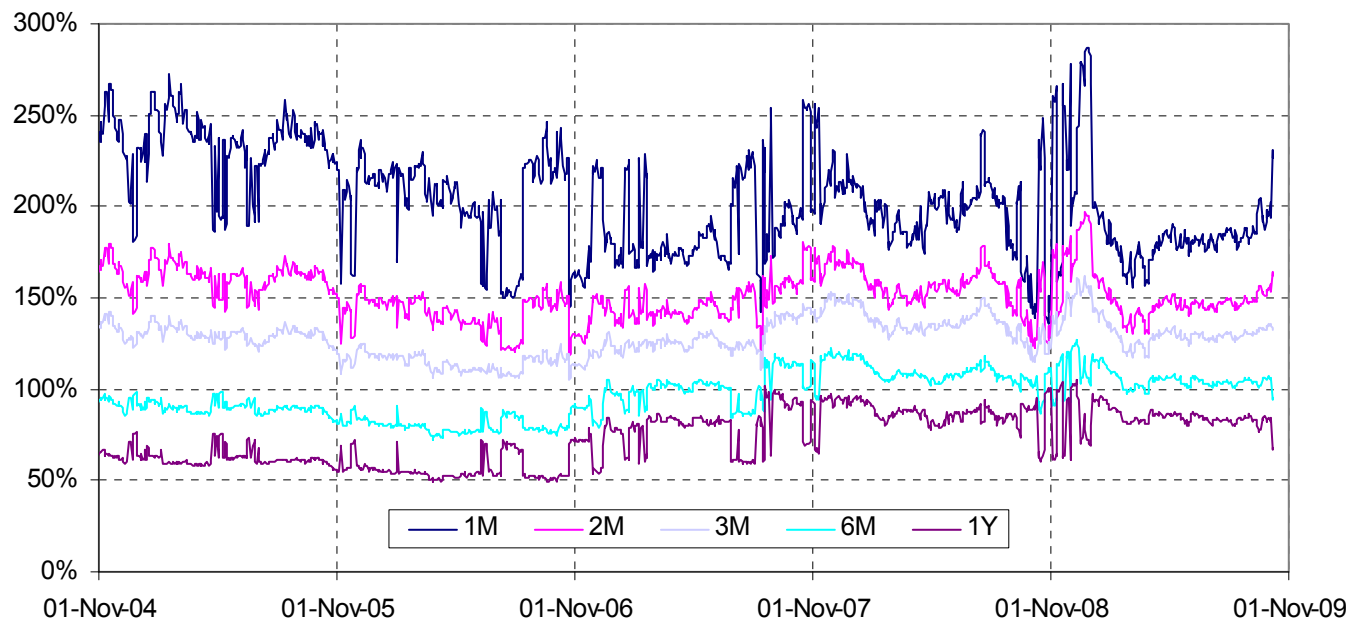
$$\frac{\partial f}{\partial \theta_{ij}} = \sum_{kl} \left( \frac{\partial f}{\partial \rho_{kl}} \frac{\partial \rho_{kl}}{\partial \theta_{ij}} + \frac{\partial f}{\partial r_{kl}} \frac{\partial r_{kl}}{\partial \theta_{ij}} + \frac{\partial f}{\partial \tilde{\rho}_{kl}} \frac{\partial \tilde{\rho}_{kl}}{\partial \theta_{ij}} \right) \quad \text{etc}$$

- Use the analytical SABR-style approximation for  $\sigma_B$ , so need to calculate all the derivatives of  $\sigma_B$  with respect to  $\sigma_i$ ,  $v_i$ ,  $\boldsymbol{\rho}$ ,  $\mathbf{r}$ ,  $\tilde{\boldsymbol{\rho}}$ .

# Define $Z_3$ to make solutions more stable

- An early calibration attempt produced the following unstable results for  $v^T$ :

Unstable  $v^T$  in early calibration attempt



# Unstable $\nu^T$ related to shape of strange errors

- Between 27-Nov-06 and 28-Nov-06, (1M, 1Y) market strangle errors switch sign  $(-, +) \rightarrow (+, -)$ :

|        | 27-Nov-06 calibration errors ( $\nu^{1M} = 170\%$ ) |       |       |       |       | 28-Nov-06 calibration errors ( $\nu^{1M} = 220\%$ ) |       |       |       |       |
|--------|---|-------|-------|-------|-------|---|-------|-------|-------|-------|
|        | 1M MS   | 2M MS | 3M MS | 6M MS | 1Y MS | 1M MS   | 2M MS | 3M MS | 6M MS | 1Y MS |
| EURUSD | -0.0%   | -0.0% | -0.0% | 0.0%  | 0.0%  | 0.0%  | 0.0%  | -0.0% | -0.0% | -0.1% |
| USDJPY | -0.0%   | -0.0% | 0.0%  | 0.0%  | 0.0%  | 0.1%  | 0.0%  | 0.0%  | -0.0% | -0.1% |
| GBPUSD | -0.0%   | -0.0% | -0.0% | 0.0%  | 0.0%  | 0.0%  | 0.0%  | -0.0% | -0.0% | -0.1% |
| USDCHF | -0.0%   | -0.0% | -0.0% | -0.0% | 0.0%  | 0.0%  | -0.0% | -0.0% | -0.1% | -0.1% |
| AUDUSD | -0.0%   | 0.0%  | 0.0%  | 0.0%  | 0.1%  | 0.1%  | 0.0%  | 0.0%  | -0.0% | -0.0% |
| USDCAD | -0.0%   | 0.0%  | 0.0%  | 0.1%  | 0.2%  | 0.1%  | 0.0%  | 0.0%  | -0.0% | -0.0% |
| NZDUSD | -0.1%   | -0.0% | 0.0%  | 0.0%  | 0.1%  | 0.0%  | 0.0%  | -0.0% | -0.0% | -0.1% |
| EURJPY | -0.0%   | -0.0% | -0.0% | -0.0% | 0.0%  | 0.0%  | 0.0%  | -0.0% | -0.0% | -0.1% |
| EURGBP | -0.0%   | -0.0% | -0.0% | 0.0%  | 0.0%  | 0.0%  | 0.0%  | 0.0%  | -0.0% | -0.0% |
| EURCHF | -0.0%   | -0.0% | -0.0% | 0.0%  | 0.1%  | 0.0%  | 0.0%  | -0.0% | -0.0% | -0.0% |
| EURAUD | -0.0%   | -0.0% | 0.0%  | 0.0%  | 0.1%  | 0.0%  | 0.0%  | 0.0%  | -0.0% | -0.0% |
| EURCAD | -0.0%   | 0.0%  | 0.0%  | 0.0%  | 0.1%  | 0.0%  | 0.0%  | 0.0%  | 0.0%  | 0.0%  |
| EURNZD | -0.0%   | 0.0%  | 0.1%  | 0.1%  | 0.2%  | 0.1%  | 0.1%  | 0.1%  | 0.0%  | 0.0%  |
| GBPJPY | -0.0%   | -0.0% | -0.0% | -0.0% | 0.0%  | 0.0%  | 0.0%  | 0.0%  | -0.0% | -0.1% |
| CHFJPY | -0.0%   | -0.0% | -0.0% | -0.0% | 0.0%  | 0.0%  | 0.0%  | -0.0% | -0.1% | -0.1% |
| AUDJPY | -0.0%   | -0.0% | -0.0% | 0.0%  | 0.0%  | 0.0%  | 0.0%  | -0.0% | -0.0% | -0.1% |
| CADJPY | -0.0%   | -0.0% | -0.0% | 0.0%  | 0.0%  | 0.1%  | 0.0%  | 0.0%  | -0.0% | -0.1% |
| NZDJPY | -0.0%   | 0.0%  | 0.0%  | 0.0%  | 0.1%  | 0.1%  | 0.0%  | 0.0%  | -0.0% | -0.1% |
| GBPCHE | -0.0%   | -0.0% | -0.0% | 0.0%  | 0.0%  | 0.0%  | 0.0%  | -0.0% | -0.0% | -0.0% |
| GBPAUD | -0.0%   | -0.0% | -0.0% | 0.0%  | 0.0%  | 0.0%  | 0.0%  | 0.0%  | -0.0% | -0.0% |
| GBPCAD | -0.0%   | -0.0% | 0.0%  | 0.0%  | 0.1%  | 0.0%  | 0.0%  | 0.0%  | 0.0%  | -0.0% |
| GBPNZD | -0.0%   | 0.0%  | 0.1%  | 0.1%  | 0.2%  | 0.1%  | 0.1%  | 0.0%  | 0.0%  | -0.0% |
| AUDCHF | -0.1%   | -0.0% | -0.0% | 0.0%  | 0.0%  | 0.0%  | -0.0% | -0.0% | -0.0% | -0.0% |
| CADCHF | -0.0%   | -0.0% | -0.0% | 0.0%  | 0.1%  | 0.0%  | 0.0%  | -0.0% | 0.0%  | -0.0% |
| NZDCHF | -0.0%   | 0.0%  | 0.0%  | 0.1%  | 0.1%  | 0.1%  | 0.0%  | 0.0%  | 0.0%  | -0.0% |
| AUDCAD | -0.0%   | -0.0% | 0.0%  | 0.0%  | 0.1%  | 0.0%  | 0.0%  | 0.0%  | -0.0% | -0.0% |
| AUDNZD | 0.0%  | 0.0%  | 0.1%  | 0.1%  | 0.2%  | 0.1%  | 0.1%  | 0.1%  | 0.1%  | -0.0% |
| NZDCAD | -0.0%   | 0.0%  | 0.0%  | 0.1%  | 0.1%  | 0.1%  | 0.0%  | 0.0%  | 0.0%  | -0.0% |



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## Define $Z_3$ to make solutions more stable

- Include in  $Z_3$  the term  $Z_{MS}$  defined by

$$Z_{MS} = \sum_T \left( \sum_{ij} \left( [MS_{\text{Market}}]_{ij}^T - [MS_{\text{Model}}]_{ij}^T \right) \right)^2$$

which penalises calibrations where all the market strangle errors for a particular tenor have the same sign.

- Also include a term which, for each currency pair, for the set of 5 ATM volatilities (1M, 2M, 3M, 6M and 1Y), the set of 5 Risk Reversals and the set of 5 Market Strangles, encourages some of the error terms in each set to be positive and some to be negative.

# Sample results for a single day: 2-Feb-2010

## Calibration errors: ModelValue – MarketValue

|        | Errors in at-the-money volatilities |       |       |       |       | Errors in the risk reversals |       |       |       |       | Errors in the market strangles |       |       |       |       |
|--------|-------------------------------------|-------|-------|-------|-------|------------------------------|-------|-------|-------|-------|--------------------------------|-------|-------|-------|-------|
|        | 1M                                  | 2M    | 3M    | 6M    | 1Y    | 1M                           | 2M    | 3M    | 6M    | 1Y    | 1M                             | 2M    | 3M    | 6M    | 1Y    |
| EURUSD | -0.0%                               | 0.0%  | -0.0% | -0.1% | -0.1% | -0.3%                        | -0.2% | -0.1% | -0.0% | -0.0% | -0.0%                          | -0.0% | -0.0% | -0.1% | -0.1% |
| USDJPY | -0.3%                               | -0.1% | 0.0%  | 0.2%  | 0.3%  | 0.4%                         | 0.4%  | 0.2%  | 0.1%  | -0.2% | 0.0%                           | 0.1%  | 0.2%  | 0.3%  | 0.4%  |
| GBPUSD | -0.0%                               | 0.1%  | 0.2%  | 0.1%  | 0.0%  | -0.2%                        | -0.0% | -0.1% | -0.0% | -0.1% | -0.1%                          | -0.0% | -0.0% | -0.1% | -0.1% |
| USDCHE | 0.2%                                | 0.1%  | 0.1%  | -0.1% | -0.2% | 0.0%                         | -0.1% | -0.1% | -0.1% | -0.1% | -0.0%                          | -0.0% | -0.0% | -0.0% | -0.0% |
| AUDUSD | -0.0%                               | -0.0% | -0.0% | 0.0%  | 0.1%  | -0.1%                        | -0.0% | -0.1% | -0.0% | -0.2% | -0.0%                          | -0.1% | -0.1% | -0.2% | -0.4% |
| USDCAD | 0.1%                                | 0.1%  | -0.0% | 0.1%  | 0.1%  | 0.3%                         | 0.1%  | 0.1%  | -0.1% | -0.2% | -0.1%                          | -0.0% | -0.1% | -0.1% | -0.1% |
| NZDUSD | 0.2%                                | -0.4% | -0.2% | -0.1% | -0.0% | 0.3%                         | 0.1%  | -0.1% | -0.2% | -0.4% | -0.0%                          | -0.1% | -0.1% | -0.2% | -0.4% |
| EURJPY | 0.2%                                | 0.0%  | -0.0% | -0.2% | -0.2% | 0.4%                         | 0.4%  | 0.2%  | -0.0% | -0.4% | 0.0%                           | 0.1%  | 0.1%  | 0.2%  | 0.2%  |
| EURGBP | 0.1%                                | -0.0% | 0.0%  | 0.0%  | 0.1%  | -0.2%                        | -0.2% | -0.0% | 0.0%  | 0.1%  | 0.0%                           | 0.0%  | -0.0% | 0.0%  | -0.0% |
| EURCHF | -0.1%                               | -0.1% | 0.1%  | 0.2%  | 0.4%  | 0.1%                         | 0.1%  | -0.0% | -0.2% | -0.3% | -0.1%                          | -0.1% | -0.1% | -0.1% | -0.1% |
| EURAUD | 0.5%                                | 0.1%  | 0.1%  | -0.1% | -0.3% | -0.3%                        | -0.3% | -0.1% | -0.0% | 0.1%  | 0.0%                           | 0.0%  | 0.0%  | -0.0% | -0.1% |
| EURCAD | 0.1%                                | 0.3%  | 0.0%  | -0.1% | -0.2% | -0.1%                        | -0.1% | -0.1% | -0.1% | -0.0% | -0.1%                          | -0.1% | -0.1% | -0.1% | -0.1% |
| EURNZD | 0.7%                                | -0.1% | 0.1%  | -0.2% | -0.1% | -0.3%                        | -0.1% | 0.1%  | 0.2%  | 0.4%  | -0.0%                          | -0.0% | -0.0% | -0.1% | -0.1% |
| GBPJPY | 0.3%                                | 0.1%  | -0.1% | -0.2% | -0.1% | 0.1%                         | 0.1%  | -0.1% | -0.3% | -0.7% | -0.0%                          | 0.0%  | 0.0%  | 0.1%  | 0.1%  |
| CHFJPY | 0.2%                                | -0.2% | -0.0% | -0.1% | 0.2%  | 0.4%                         | 0.3%  | 0.4%  | 0.3%  | 0.0%  | 0.1%                           | 0.2%  | 0.2%  | 0.2%  | 0.2%  |
| AUDJPY | 0.0%                                | -0.3% | -0.4% | -0.4% | -0.8% | 0.9%                         | 0.5%  | -0.0% | -0.7% | -2.0% | 0.0%                           | 0.1%  | 0.1%  | 0.1%  | 0.4%  |
| CADJPY | 0.1%                                | 0.3%  | 0.3%  | 0.2%  | -0.0% | 0.2%                         | 0.1%  | -0.0% | -0.2% | -0.5% | -0.0%                          | 0.0%  | 0.1%  | 0.1%  | 0.1%  |
| NZDJPY | 0.0%                                | -0.7% | -0.5% | -0.3% | -0.5% | 1.0%                         | 0.5%  | 0.0%  | -0.5% | -1.9% | 0.0%                           | 0.1%  | 0.1%  | 0.1%  | 0.4%  |
| GBPCHE | 0.0%                                | -0.3% | -0.2% | -0.3% | -0.3% | 0.1%                         | 0.1%  | -0.1% | -0.1% | -0.2% | 0.0%                           | -0.0% | -0.0% | 0.0%  | 0.0%  |
| GBPAUD | -0.3%                               | -0.1% | -0.0% | 0.4%  | 0.3%  | -0.1%                        | -0.1% | 0.1%  | 0.1%  | 0.1%  | -0.0%                          | 0.0%  | 0.0%  | -0.0% | -0.0% |
| GBPCAD | -0.2%                               | 0.1%  | -0.1% | 0.2%  | 0.0%  | -0.2%                        | -0.2% | -0.1% | -0.0% | -0.0% | -0.1%                          | -0.1% | -0.1% | -0.1% | -0.1% |
| GBPNZD | 0.1%                                | -0.2% | 0.1%  | 0.2%  | 0.3%  | -0.6%                        | -0.3% | -0.2% | -0.1% | 0.1%  | -0.0%                          | 0.0%  | 0.0%  | -0.0% | -0.1% |
| AUDCHF | 0.5%                                | -0.0% | -0.3% | -1.1% | -1.3% | 0.5%                         | 0.4%  | 0.3%  | 0.1%  | -0.3% | -0.0%                          | -0.0% | -0.0% | -0.1% | -0.0% |
| CADCHF | 0.1%                                | -0.0% | -0.5% | -0.7% | -1.0% | 0.1%                         | 0.1%  | 0.1%  | -0.0% | -0.2% | -0.1%                          | -0.1% | -0.1% | -0.1% | -0.2% |
| NZDCHF | 0.7%                                | -0.3% | -0.1% | -0.8% | -0.9% | 0.4%                         | 0.2%  | -0.0% | -0.7% | -1.1% | -0.1%                          | -0.1% | -0.1% | -0.1% | -0.0% |
| AUDCAD | -0.0%                               | 0.3%  | 0.4%  | 0.6%  | 0.7%  | -0.2%                        | -0.2% | -0.2% | -0.0% | -0.1% | 0.0%                           | 0.1%  | 0.0%  | -0.0% | -0.1% |
| AUDNZD | -0.2%                               | -0.1% | 0.3%  | 0.5%  | 0.6%  | 0.0%                         | 0.1%  | 0.0%  | -0.0% | -0.1% | 0.1%                           | 0.2%  | 0.2%  | 0.3%  | 0.2%  |
| NZDCAD | 0.3%                                | -0.0% | 0.2%  | 0.3%  | 0.3%  | 0.3%                         | 0.1%  | -0.1% | -0.1% | -0.2% | 0.0%                           | 0.0%  | -0.0% | -0.1% | -0.1% |

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# Perfect or imperfect calibration to market data?

- Perfect calibrations to market data are not possible when market data allows arbitrage
  - This happened recently in the interest rate markets, where there were arbitrages between CMS caps and CMS spread options. (NB: for mark to market, each product should be marked to its own market, but for exotics pricing a model can't be consistent with both!)
- SticVol relates implied volatility curves between different currency pairs, so the market data could contain subtle inconsistencies because the market usually doesn't look at things like that
- Forcing perfect calibrations (via e.g. tenor dependent correlation) carries the danger that the dynamics end up distorted

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# Extension to multiple less liquid currencies

- With the major currencies, ATM volatilities, risk reversals, market strangles are available for all combinations of currencies, but when adding e.g. BRL, only USD/BRL, EUR/BRL reliable
- To handles less liquid currencies, adopt a two stage process:
  - Calibrate the core model to all the major currencies and fix the corresponding  $\sigma_i^T$ ,  $v^T$ ,  $\rho$ ,  $r$ ,  $\tilde{\rho}$ .
  - Extend the model by adding the required currencies using only the market data against USD and EUR
- Calibrate the much bigger correlation matrix by using a functional form which automatically imposes maximum information entropy

# Correlation matrix imposing maximum entropy

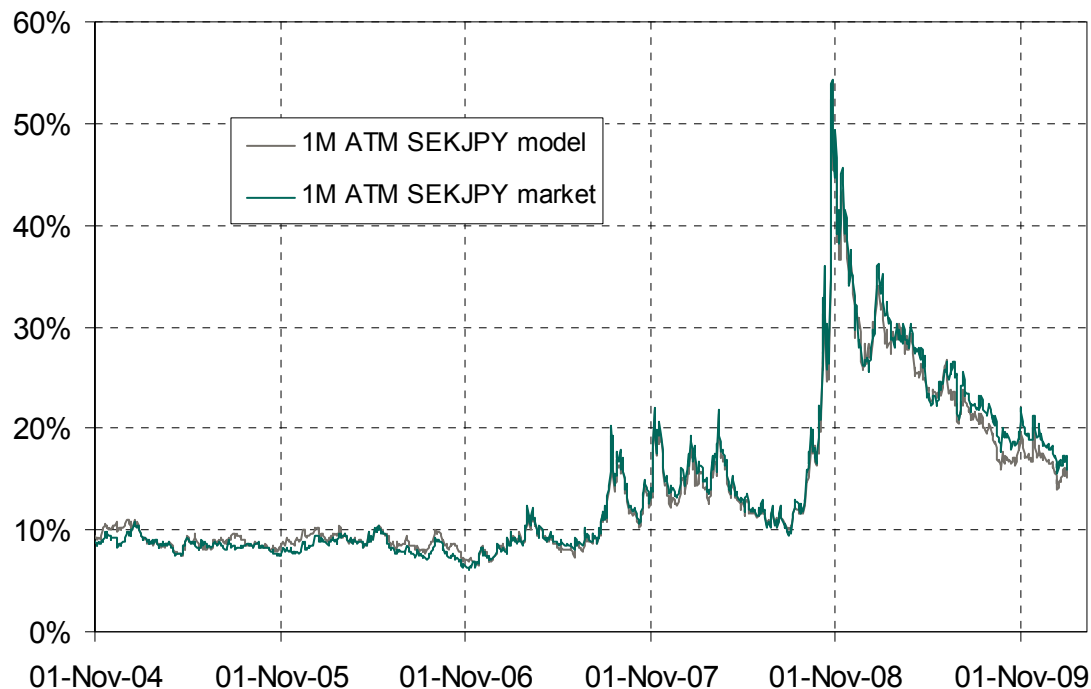
- Re-order the correlation matrix so that  $\boldsymbol{\rho}$ ,  $\mathbf{r}$ ,  $\tilde{\boldsymbol{\rho}}$  for the  $M < N$  major currencies are in the top left of  $\mathbf{C}\mathbf{C}'$ , with the USD and EUR variables in the 4x4 top left hand corner. Maximum entropy means maximum determinant of  $\mathbf{C}$ , so since  $\det(\mathbf{C}) = \prod_i C_{ii}$ ,  $\mathbf{C}$  must have the form:

$$\mathbf{C} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ C_{21} & C_{22} & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ C_{31} & C_{32} & C_{33} & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ C_{41} & C_{42} & C_{43} & C_{44} & 0 & \cdots & 0 & 0 & \cdots & 0 \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ C_{(2M)1} & C_{(2M)2} & C_{(2M)3} & C_{(2M)4} & C_{(2M)5} & \cdots & C_{(2M)(2M)} & 0 & \cdots & 0 \\ C_{(2M+1)1} & C_{(2M+1)2} & C_{(2M+1)3} & C_{(2M+1)4} & 0 & \cdots & 0 & C_{(2M+1)(2M+1)} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ C_{(2N)1} & C_{(2N)2} & C_{(2N)3} & C_{(2N)4} & 0 & \cdots & 0 & 0 & \cdots & C_{(2N)(2N)} \end{pmatrix}$$

# Example: calculation of 1M SEK/JPY ATM

- SticVol first calibrated to 28 currency pairs between USD, EUR, JPY, GBP, CHF, AUD, CAD, NZD, then extended to include SEK, NOK, BRL, calibrated against USD and EUR

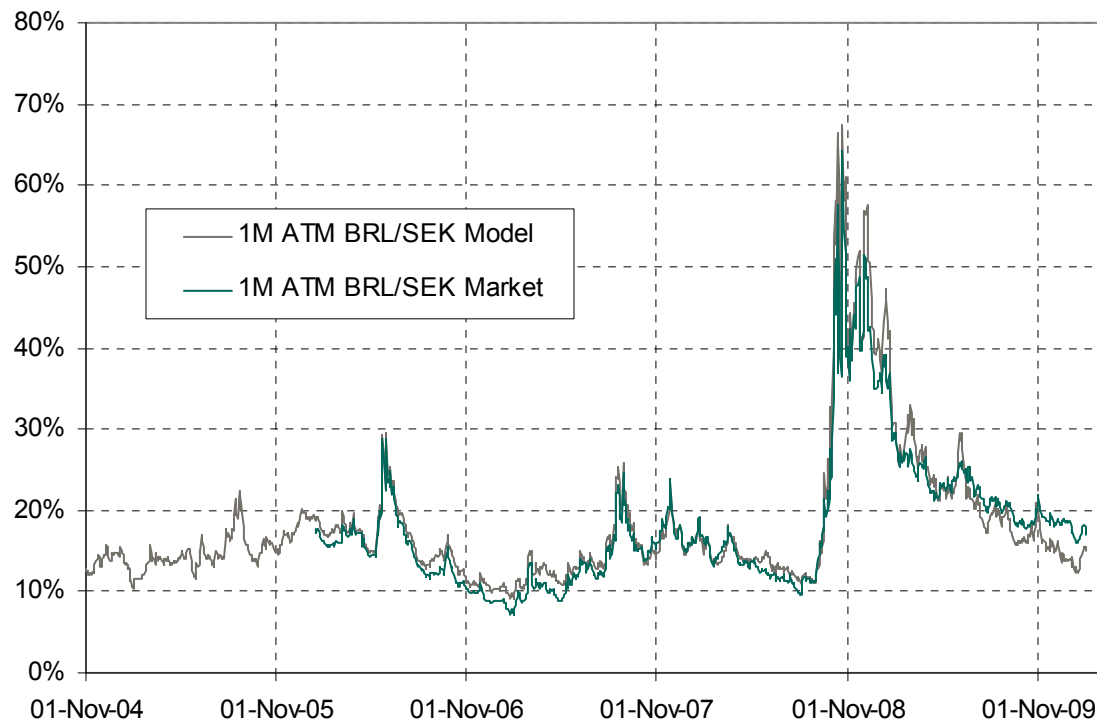
1M SEK/JPY ATM: model versus market



# Example: calculation of 1M ATM BRL/SEK

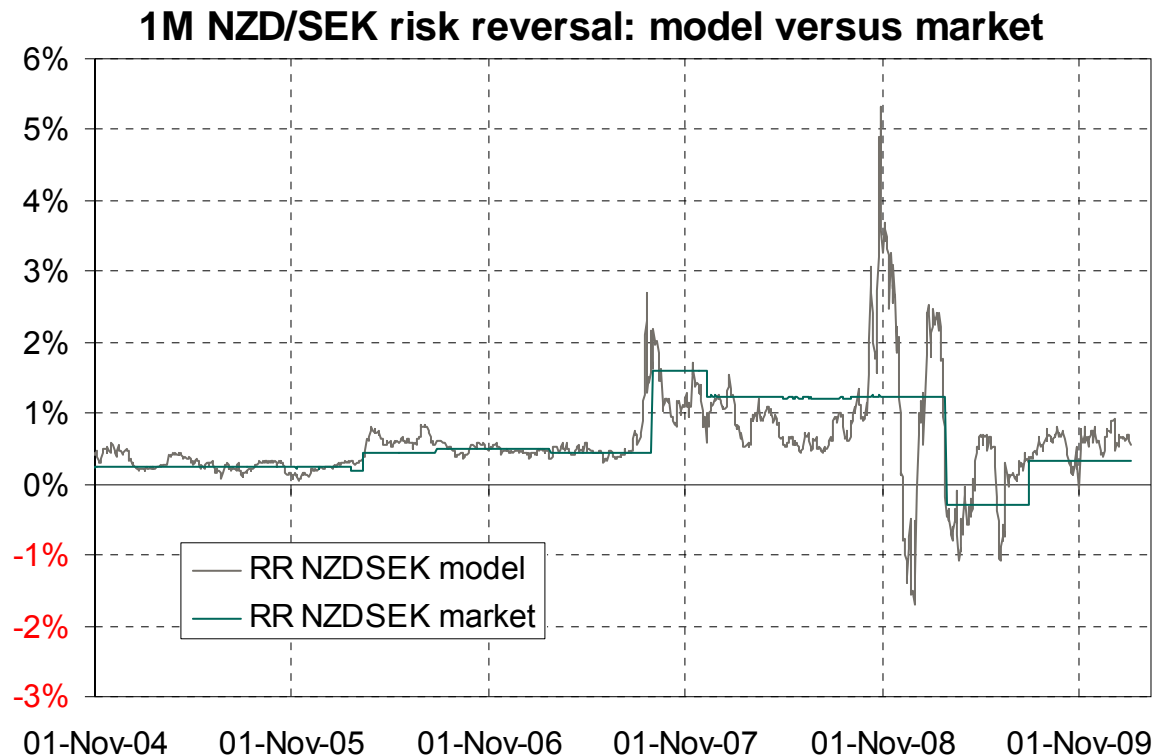
- SticVol first calibrated to 28 currency pairs between USD, EUR, JPY, GBP, CHF, AUD, CAD, NZD, then extended to include SEK, NOK, BRL, calibrated against USD and EUR

1M BRL/SEK: model versus market



# Example: calculation of 1M NZD/SEK RR

- Comparing the model to more esoteric market data points, the match is less good, however the market data here is unreliable so the model is \*probably\* more accurate





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# Derivative pricing: FX basket options

- The advantage of SticVol is that it's a model that calibrates to the implied volatility curves of ALL currency pairs under consideration.
- Difficult to find other models which can do that reliably when dealing with a lot of currencies
  - For example, with a local vol model, the covariance matrix depends on the spot rates and extreme spot rates often end up breaking the positive definiteness
- To illustrate SticVol derivative pricing, consider the option to exchange an amount  $K_{\$}$  of \$ for a basket of 7 other currencies with \$ value  $B_{\$}$

$$B_{\$} = A_{\text{€}}X_{\text{€},\$} + \frac{A_{JPY}}{X_{\$,JPY}} + A_{\text{£}}X_{\text{£},\$} + \frac{A_{CHF}}{X_{\$,CHF}} + \frac{A_{CAD}}{X_{\$,CAD}} + A_{AUD}X_{AUD,\$} + A_{NZD}X_{NZD,\$}$$

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# SticVol versus Gaussian copula models

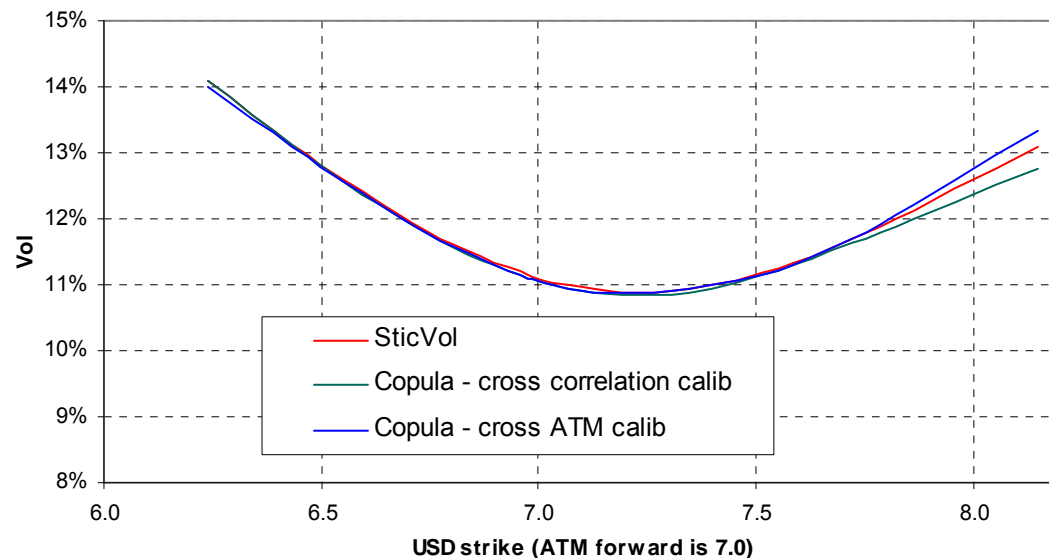
- Gaussian copula models can be calibrated robustly to multiple currency pairs, although they're incapable of a calibrating to full implied volatility curves of all currency pairs.
- For the option on  $\text{Max}(K_{\$}-B_{\$},0)$ , calibrate the Gaussian copula model to the full implied volatility curves for \$ against each basket currency, and then determine the correlations
  1. Cross ATM calibration: so that the ATM volatility of each cross is correct, or
  2. Cross correlation calibration: using the formula

$$\rho_{XY} = \frac{\sigma_{X\$}^2 + \sigma_{Y\$}^2 - \sigma_{XY}^2}{2\sigma_{X\$}\sigma_{Y\$}}$$

# Calibration: SticVol vs Gaussian Copula

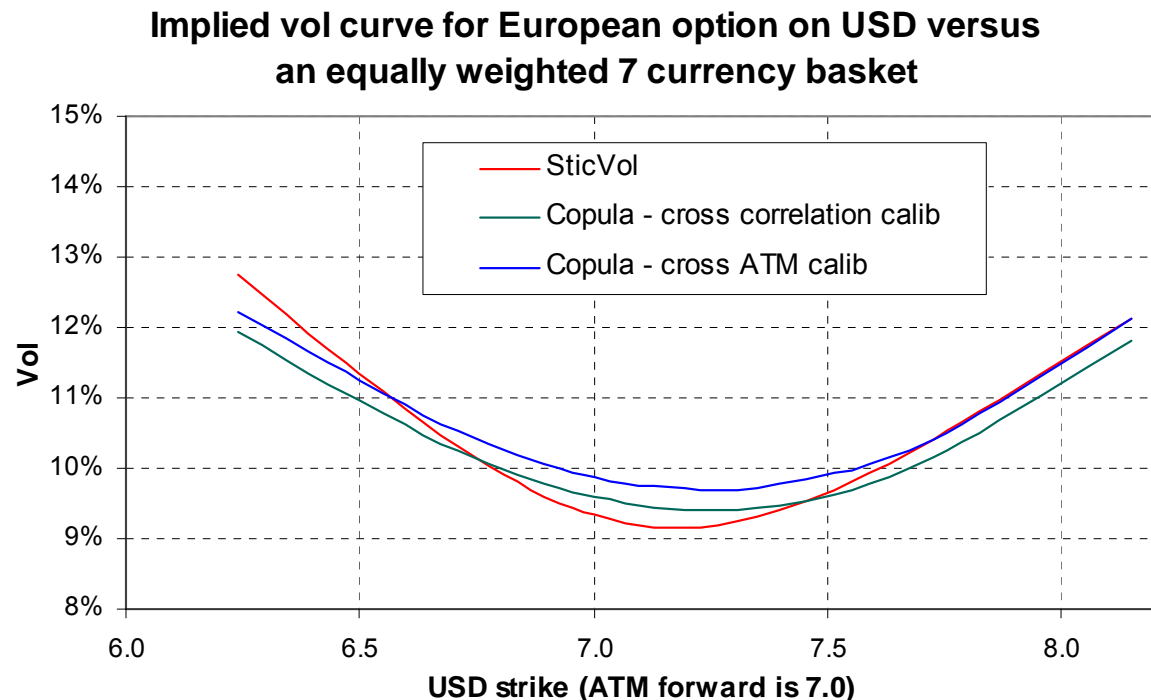
- To illustrate SticVol derivative pricing, 3-month options were considered, using Monte-Carlo simulations for pricing. For all currency pairs involving USD, the two model calibrations were very close, for example with EUR/USD:

Implied vol curve for European option on USD versus EUR (i.e. a basket of 100% EUR)



# Basket option 1: SticVol vs Gaussian Copula

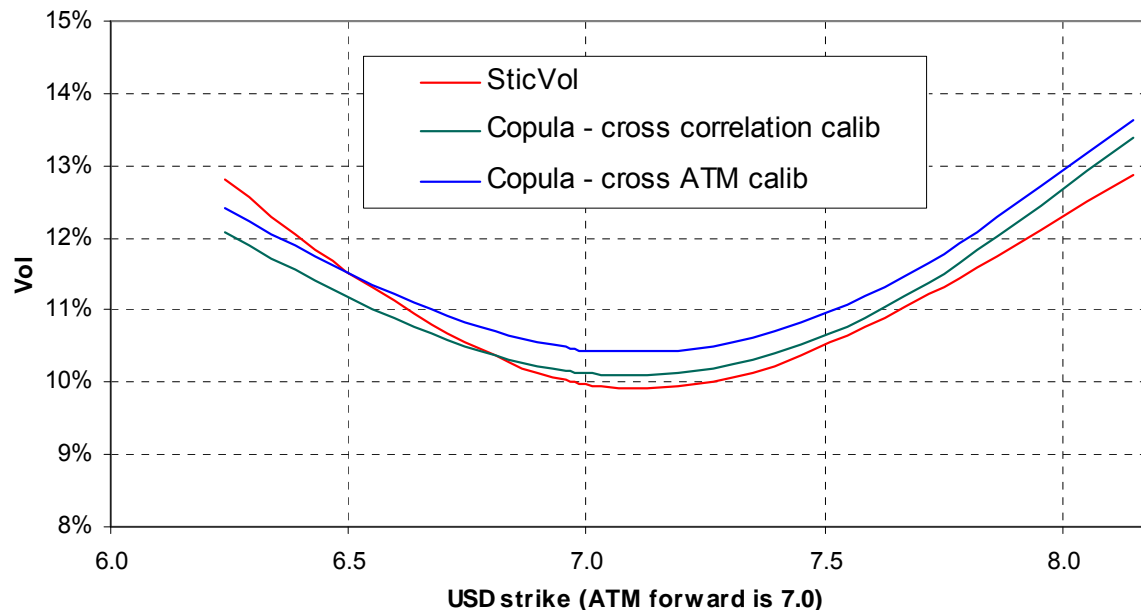
- All the non-USD currency amounts  $A_i$  were set so that at option expiry, the forward value in USD of all  $A_i$  are equal to the USD amount  $A_{\$}$ . In the volatility graph, strike  $K$  is in units of  $A_{\$}$ :



# Basket option 2: SticVol vs Gaussian Copula

- As another example, a basket of 50% JPY and 50% NZD is chosen, because NZD/JPY has a huge risk-reversal. SticVol pricing is generally lower, except for out-of-the-month USD calls

Implied vol curve for European option on USD versus a  
a basket of 50% JPY and 50% NZD

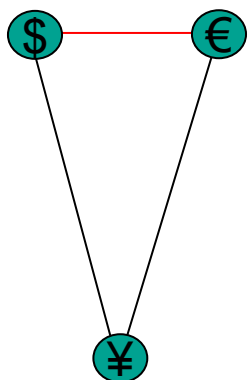


# FX correlation and covariance swaps

- These products have settlements of the form
$$\text{Payment} = \text{Notional} \times (\text{Realised} - \text{Strike}) ,$$
where ‘Realised’ is the realised correlation or covariance over the lifetime of the product, and ‘Strike’ is a fixed reference correlation or covariance which is set in advance.
- Example: long correlation position in \$/¥ vs €/¥ in \$10,000 per 0.01 of correlation. Typical vega positions in the underlying currency pairs are:

|      |         |
|------|---------|
| \$/¥ | 22,355  |
| €/¥  | 49,284  |
| €/\$ | -73,991 |

# FX correlation and covariance swaps



- Taking  $\$/¥$  vs  $€/¥$  as an example
  - A covariance swap between  $\$/¥$  and  $€/¥$  which pays the fixed covariance is mostly a long position in intrinsic  $¥$  volatility;
  - A correlation swap between  $\$/¥$  and  $€/¥$  which pays the fixed correlation is a long position in intrinsic  $¥$  volatility coupled with smaller short positions in intrinsic  $\$$  volatility and intrinsic  $€$  volatility
- Intuition: as  $¥$  vol gets bigger and bigger while  $\$, €$  vol remain constant, eventually the  $¥$  vol completely dominates. So for very large  $¥$  vol compared to  $\$, €$  vol, the correlation between  $\$/¥$  and  $€/¥$  will be close to 1. Hence as  $¥$  vol rises or falls, the covariance and correlation between  $\$/¥$  and  $€/¥$  must rise and fall too.

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# Formulas for covariance and correlation

- For two spot FX rates  $X_{ik}$  and  $X_{jk}$

$$\text{Covariance}(X_{ik}, X_{jk}) = \sigma_k^2 + \rho_{ij}\sigma_i\sigma_j - \rho_{ik}\sigma_i\sigma_k - \rho_{jk}\sigma_j\sigma_k$$

$$\text{Correlation}(X_{ik}, X_{jk}) = \frac{\sigma_k^2 + \rho_{ij}\sigma_i\sigma_j - \rho_{ik}\sigma_i\sigma_k - \rho_{jk}\sigma_j\sigma_k}{\sqrt{\sigma_i^2 - 2\rho_{ik}\sigma_i\sigma_k + \sigma_k^2} \sqrt{\sigma_j^2 - 2\rho_{jk}\sigma_j\sigma_k + \sigma_k^2}}$$

- However, intrinsic currency correlations tend to be small (because maximum possible entropy is zero correlation), so good approximations are

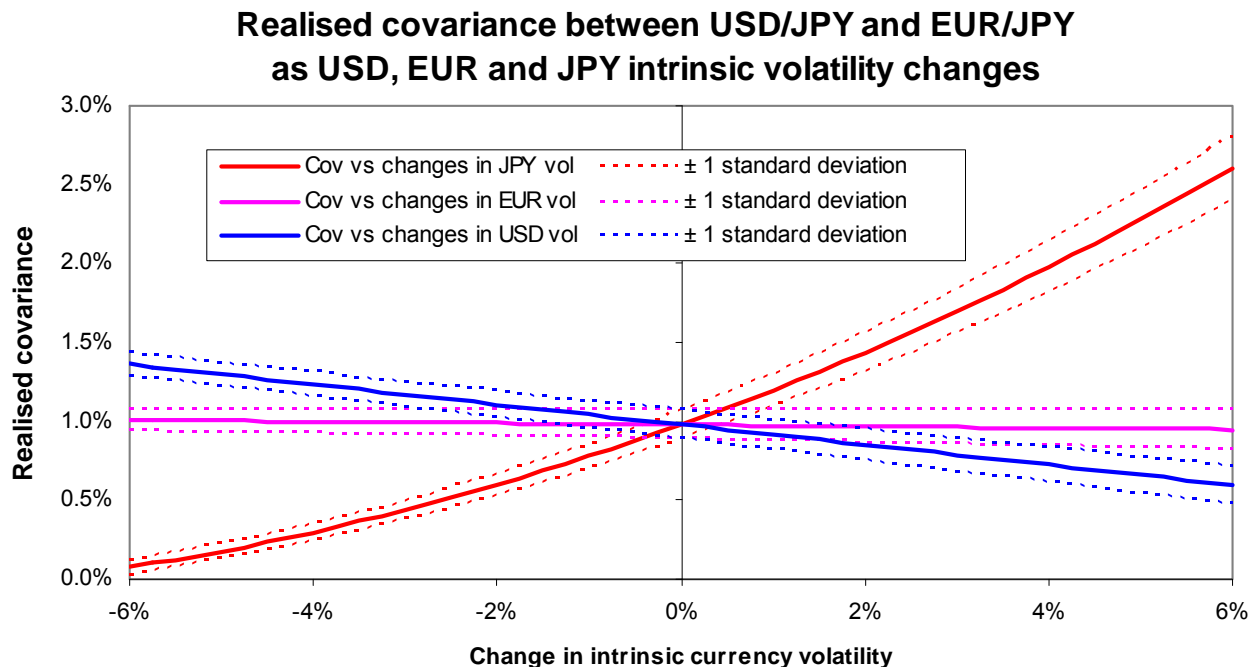
$$\text{Correlation}(X_{ik}, X_{jk}) \approx \sigma_k^2$$

$$\text{Correlation}(X_{ik}, X_{jk}) \approx \frac{\sigma_k^2}{\sqrt{\sigma_i^2 + \sigma_k^2} \sqrt{\sigma_j^2 + \sigma_k^2}}$$



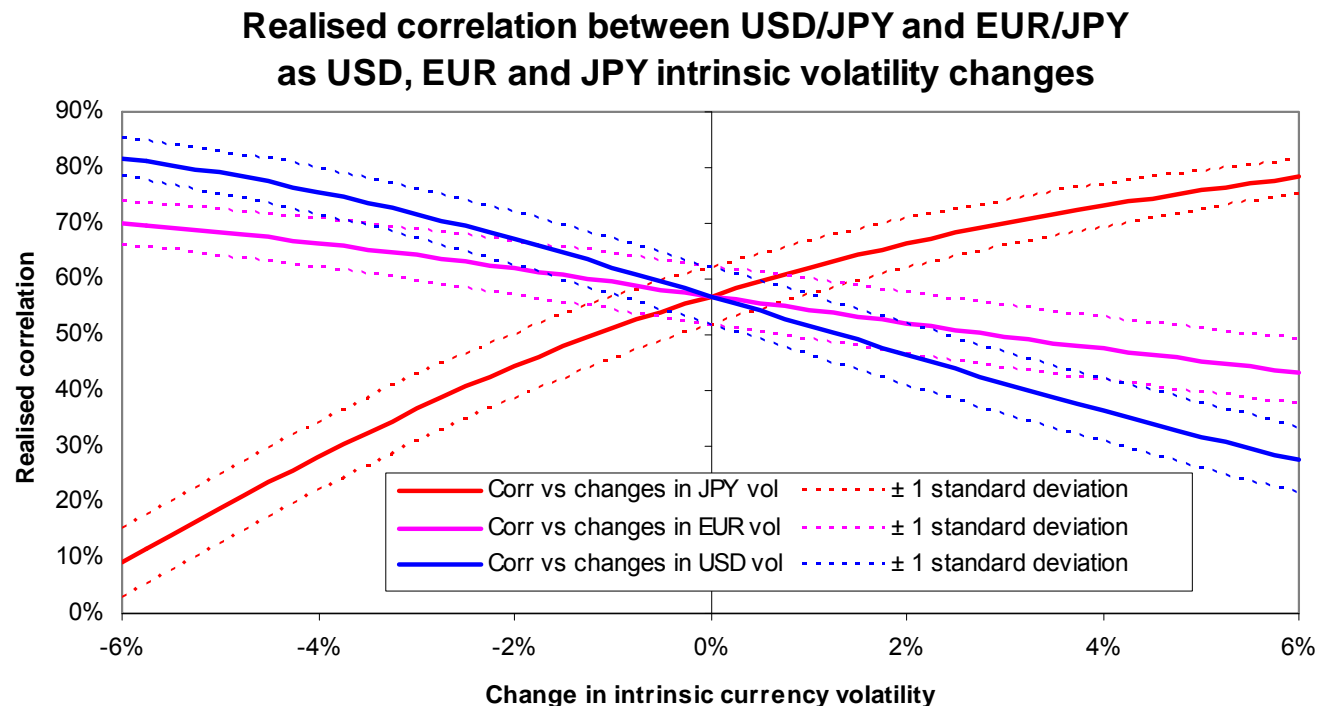
# Example \$/¥ vs €/¥ covariance swap

- Taking a 1y covariance swap as an example, as advertised this swap is mostly a long position in intrinsic ¥ volatility:



# Example \$/¥ vs €/¥ correlation swap

- Taking a 1y correlation swap as an example, as advertised this swap is a long position in intrinsic ¥ volatility coupled with smaller short positions in intrinsic \$ and intrinsic € volatilities



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# Conclusion

- It is straightforward to calibrate SticVol using conjugate gradient techniques.
  - Calculate derivatives of the target function analytically
- SticVol can be used to calculate the implied volatility curves of less liquid currency pairs.
  - With incomplete market data, imposing maximum entropy via the Cholesky decomposition is a big help in simplifying the calibration problem
- SticVol provides a robust model for pricing multi currency FX derivatives.
- The intrinsic currency concept provides a new way of looking at correlation and covariance swaps.

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