# Unit 6: Counterparty Risk Pricing: Unilateral and Bilateral CVA across asset classes with Netting and Collateral

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# Agenda

# Some common questions 1

**Q** What is counterparty risk in general?

**A** *The risk taken on by an entity entering an OTC contract with a counterparty having a relevant default probability. As such, the counterparty might not respect its payment obligations.*

> *The counterparty credit risk is defined as the risk that the counterparty to a transaction could default before the final settlement of the transaction's cash flows. An economic loss would occur if the transactions or portfolio of transactions with the counterparty has a positive economic value at the time of default.*
>
> [Basel II, Annex IV, 2/A]

# Some common questions

**Q** What is the difference between Credit VaR and CVA?

**A** *Credit VaR is a Value at Risk type measure, it measures a potential loss due to counterparty default. CVA stands for Credit Valuation Adjustment and is a price adjustment. CVA is obtained by pricing the counterparty risk component of a deal, similarly to how one would price a credit derivative.*

**Q** Are the methodologies for Credit VaR and CVA similar?

**A** *There are analogies but CVA needs to be more precise in general. Also, Credit VaR should use statistics under the physical measure whereas CVA should use statistics under the pricing measure*

**Q** What are the regulatory bodies involved?

**A** *There are many, for Credit VaR type measures it is mostly Basel II and now III, whereas for CVA we have IAS and ISDA.*

**Q** What is the focus of this presentation?

**A** *We will focus on CVA.*

# Some common questions

**Q** When is valuation of counterparty risk CVA symmetric?

**A** *When we include the possibility that also the entity computing the counterparty risk adjustment may default, besides the counterparty itself.*

**Q** When is valuation of counterparty risk CVA asymmetric?

**A** *When the entity computing the counterparty risk adjustment considers itself default-free, and only the counterparty may default.*

**Q** Which one is computed usually for valuation adjustments?

**A** *Pre-crisis it used to be the asymmetric one; At the moment there is quite a debate*

# Some common questions

Basel II on bilateral counterparty risk:

> *Unlike a firm's exposure to credit risk through a loan, where the exposure to credit risk is unilateral and only the lending bank faces the risk of loss, the counterparty credit risk creates a bilateral risk of loss: the market value of the transaction can be positive or negative to either counterparty to the transaction.* [Basel II, Annex IV, 2/A]

# Some common questions

**Q** What impacts counterparty risk?

**A** *The OTC contract's underlying volatility, the correlation between the underlying and default of the counterparty, and the counterparty credit spreads volatility.*

**Q** Is it model dependent?

**A** *It is.*

**Q** What about *wrong way risk*?

**A** *The amplified risk when the reference underlying and the counterparty are strongly correlated in the wrong direction.*

# Some common questions

**Q** What is collateral?

**A** *It is a guarantee (liquid and secure asset, cash) that is deposited in a collateral account in favour of the investor party facing the exposure. If the depositing counterparty defaults, thus not being able to fulfill payments associated to the above mentioned exposure, Collateral can be used by the investor to offset its loss.*

**Q** What is netting?

**A** *This is the agreement to net all positions towards a counterparty in the event of the counterparty default. This way positions with negative PV can be offset by positions with positive PV and counterparty risk is reduced. This has to do with the option on a sum being smaller than the sum of the options*

**Q** Is Counterparty risk CVA model dependent? **A** *It is.*

**Q** What about *wrong way risk*?

**A** *The amplified risk when the reference underlying and the counterparty are strongly correlated in the wrong direction.*

# Existing approaches for the Asymmetric Case

**Capital Adequacy based approach**

- Obtain estimates of expected exposures for the portfolio NPV at different maturities
- Buy default protection on the counterparty through Credit Default Swaps on those maturities with notionals following the expected exposures.

**Problems**

- May ignore risk premiums in the underlying portfolio's factors
- Ignores correlation structure between counterparty default and portfolio's risk factors
- Models wrong way risk inaccurately through rough coefficients

# General Notation

- We will call "investor" the party interested in the counterparty adjustment. This is denoted by "0"
- We will call "counterparty" the party with whom the investor is trading, and whose default may affect negatively the investor. This is denoted by "2" or "C".
- "1" will be used for the underlying name/risk factor(s) of the contract
- The counterparty's default time is denoted with $\tau_C$ and the recovery rate for unsecured claims with $\text{REC}_C$ (we often use $\text{LGD}_C := 1 - \text{REC}_C$).
- $\Pi_0(t, T)$ is the discounted payout without default risk seen by '0'(sum of all future cash flows between t and $T$, discounted back at $t$). $\Pi_2(t, T) = -\Pi_0(t, T)$ is the same quantity but seen from the point of view of '2'. When we omit the index 0 or 2 we mean '0'.

# General Notation

- We define $NPV_0(t,T) = \mathbb{E}_t[\Pi(t,T)]$. When $T$ is clear from the context we omit it and write $\mathrm{NPV}(t)$.
- $$\Pi(s,t) + D(s,t)\Pi(t,u) = \Pi(s,u)$$
- $$\mathbb{E}_0[D(0,u)NPV(u,T)] = \mathbb{E}_0[D(0,u)\mathbb{E}_u[\Pi(u,T)]] =$$ $$= \mathbb{E}_0[D(0,u)\Pi(u,T)] = NPV(0,T) - \mathbb{E}_0[\Pi(0,u)]$$ $$= NPV(0,T) - NPV(0,u)$$

# The mechanics of Evaluating asymmetric counterparty risk

payoff under counterparty default risk

- counterparty defaults after final maturity
  - original payoff of the instrument
- counterparty defaults before final maturity
  - all cash flows before default
    $\oplus$ recovery of the residual NPV at default if positive
    $\ominus$ Total residual NPV at default if negative

# General Formulation under Asymmetry

$$\Pi_0^D(t,T) = \mathbf{1}_{\tau_2 > T}\Pi_0(t,T)$$
$$+\mathbf{1}_{t<\tau_2\leq T}\left[\Pi_0(t,\tau_2) + D(t,\tau_2)\left(REC_2\left(\mathrm{NPV}_0(\tau_2)\right)^+ - \left(-\mathrm{NPV}_0(\tau_2)\right)^+\right)\right]$$

This last expression is the general payoff seen from the point of view of '0' ($\Pi_0$, $\mathrm{NPV}_0$) under unilateral counterparty default risk. Indeed,

1. if there is no early default, this expression reduces to first term on the right hand side, which is the payoff of a default-free claim.
2. In case of early default of the counterparty, the payments due before default occurs are received (second term)
3. and then if the residual net present value is positive only the recovery value of the counterparty $\mathrm{REC}_2$ is received (third term),
4. whereas if it is negative it is paid in full by the investor (fourth term).

# General Formulation under Asymmetry

If one simplifies the cash flows and takes the risk neutral expectation, one obtains the fundamental formula for the valuation of counterparty risk when the investor 0 is default free:

$$\mathbb{E}_t\left\{\Pi_0^D(t,T)\right\} =$$
$$\mathbf{1}_{\{\tau_2>t\}}\mathbb{E}_t\left\{\Pi_0(t,T)\right\} - \mathbb{E}_t\left\{\mathrm{LGD}_2\mathbf{1}_{\{t<\tau_2\le T\}}\mathrm{D}(t,\tau_2)\left[\mathrm{NPV}_0(\tau_2)\right]^+\right\} \quad (*)$$

- First term : Value without counterparty risk.
- Second term : Unilateral Counterparty Valuation Adjustment
- $\mathrm{NPV}(\tau_C) = \mathbb{E}_{\tau_C}\left[\Pi(\tau_C,T)\right]$ is the value of the transaction on the counterparty default date. LGD = 1 - REC_counterparty.

$$\mathrm{UCVA}_0 = \mathbb{E}_t\left\{\mathrm{LGD}_2\mathbf{1}_{\{t<\tau_2\le T\}}\mathrm{D}(t,\tau_2)\left[\mathrm{NPV}_0(\tau_2)\right]^+\right\}$$

# Proof of the formula

In the proof we omit indices: $\tau = \tau_2$, REC=REC$_2$, LGD=LGD$_2$, NPV=NPV$_0$, $\Pi = \Pi_0$. The proof is obtained easily putting together the following steps. Since

$$1_{\{\tau>t\}}\Pi(t,T) = 1_{\{\tau>T\}}\Pi(t,T) + 1_{\{t<\tau\leq T\}}\Pi(t,T)$$

we can rewrite the terms inside the expectation in the right hand side of the simplified formula (*) as

$$\begin{aligned}
& \mathbf{1}_{\{\tau>t\}}\Pi(t,T) - \left\{LGD\mathbf{1}_{\{t<\tau\leq T\}}D(t,\tau)\left[\mathrm{NPV}(\tau)\right]^+\right\} \\
& = 1_{\{\tau>T\}}\Pi(t,T) + 1_{\{t<\tau\leq T\}}\Pi(t,T) \\
+ \quad & \{(\mathrm{Rec}-1)[1_{\{t<\tau\leq T\}}D(t,\tau)(\mathrm{NPV}(\tau))^+]\} \\
= \quad & 1_{\{\tau>T\}}\Pi(t,T) + 1_{\{t<\tau\leq T\}}\Pi(t,T) \\
+ \quad & \mathrm{Rec}\ \ 1_{\{t<\tau\leq T\}}D(t,\tau)(\mathrm{NPV}(\tau))^+ - 1_{\{t<\tau\leq T\}}D(t,\tau)(\mathrm{NPV}(\tau))^+
\end{aligned}$$

Conditional on the information at $\tau$ the second and the fourth terms are equal to

# Proof (cont'd)

$$
\begin{aligned}
& E_\tau[1_{\{t<\tau\le T\}}\Pi(t,T) - 1_{\{t<\tau\le T\}}D(t,\tau)(\mathrm{NPV}(\tau))^+] \\
= \ & E_\tau[1_{\{t<\tau\le T\}}[\Pi(t,\tau) + D(t,\tau)\Pi(\tau,T) - D(t,\tau)(E_\tau[\Pi(\tau,T)])^+]] \\
= \ & 1_{\{t<\tau\le T\}}[\Pi(t,\tau) + D(t,\tau)E_\tau[\Pi(\tau,T)] - D(t,\tau)(E_\tau[\Pi(\tau,T)])^+] \\
= \ & 1_{\{t<\tau\le T\}}[\Pi(t,\tau) - D(t,\tau)(E_\tau[\Pi(\tau,T)])^-] \\
= \ & 1_{\{t<\tau\le T\}}[\Pi(t,\tau) - D(t,\tau)(E_\tau[-\Pi(\tau,T)])^+] \\
= \ & 1_{\{t<\tau\le T\}}[\Pi(t,\tau) - D(t,\tau)(-\mathrm{NPV}(\tau))^+]
\end{aligned}
$$

since

$$1_{\{t<\tau\le T\}}\Pi(t,T) = 1_{\{t<\tau\le T\}}\{\Pi(t,\tau) + D(t,\tau)\Pi(\tau,T)\}$$

and $f = f^+ - f^- = f^+ - (-f)^+$.

# Proof (cont'd)

Then we can see that after conditioning the whole expression of the original long payoff on the information at time $\tau$ and substituting the second and the fourth terms just derived above, the expected value with respect to $\mathcal{F}_t$ coincides exactly with the one in our simplified formula (*) by the properties of iterated expectations by which $\mathbb{E}_t[X] = \mathbb{E}_t[\mathbb{E}_\tau[X]]$.

# What we can observe

- Including counterparty risk in the valuation of an otherwise default-free derivative $\Longrightarrow$ credit (hybrid) derivative.
- The inclusion of counterparty risk adds a level of optionality to the payoff.
  In particular, model independent products become model dependent also in the underlying market.
  $\Longrightarrow$ **Counterparty Risk analysis incorporates an opinion about the underlying market dynamics and volatility.**

# The point of view of the counterparty "2"

The deal from the point of view of '2', while staying in a world where only '2" may default.

$$\Pi_2^D(t,T) = \mathbf{1}_{\tau_2 > T}\Pi_2(t,T)$$
$$+\mathbf{1}_{t<\tau_2\le T}\left[\Pi_2(t,\tau_2) + D(t,\tau_2)\left((\mathrm{NPV}_2(\tau_2))^+ - REC_2\left(-\mathrm{NPV}_2(\tau_2)\right)^+\right)\right]$$

This last expression is the general payoff seen from the point of view of '2' ($\Pi_2$, $\mathrm{NPV}_2$) under unilateral counterparty default risk. Indeed,

1. if there is no early default, this expression reduces to first term on the right hand side, which is the payoff of a default-free claim.
2. In case of early default of the counterparty '2", the payments due before default occurs go through (second term)
3. and then if the residual net present value is positive to the defaulted '2', it is received in full from '0' (third term),
4. whereas if it is negative, only the recovery fraction $\mathrm{REC}_2$ it is paid to '0' (fourth term).

# The point of view of the counterparty “2”

The above formula simplifies to

$$\mathbb{E}_t\left\{\Pi_2^D(t,T)\right\} =$$
$$\mathbf{1}_{\tau_2>t}\mathbb{E}_t\left\{\Pi_2(t,T)\right\} + \mathbb{E}_t\left\{\mathrm{LGD}_2\mathbf{1}_{t<\tau_2\le T}D(t,\tau_2)\left[-\mathrm{NPV}_2(\tau_2)\right]^+\right\}$$

and the adjustment term with respect to the risk free price $\mathbb{E}_t\left\{\Pi_2(t,T)\right\}$ is called

UNILATERAL DEBIT VALUATION ADJUSTMENT

$$\mathrm{UDVA}_2(t) = \mathbb{E}_t\left\{\mathrm{LGD}_2\mathbf{1}_{\{t<\tau_2\le T\}}D(t,\tau_2)\left[-\mathrm{NPV}_2(\tau_2)\right]^+\right\}$$

We note that $\mathrm{UDVA}_2 = \mathrm{UCVA}_0$.
Notice also that in this universe $\mathrm{UDVA}_0 = \mathrm{UCVA}_2 = 0$.

# Including the investor default or not?

Often the investor, when computing a counterparty risk adjustment, considers itself to be default-free. This can be either a unrealistic assumption or an approximation for the case when the counterparty has a much higher default probability than the investor.

If this assumption is made when no party is actually default-free, the valuation adjustment is asymmetric: if “2” were to consider itself as default free and “0” as counterparty, and if “2” computed the counterparty risk adjustment, this would not be the opposite of the one computed by “0” in the straight case.

Also, the total NPV including counterparty risk is similarly asymmetric, in that the total value of the position to “0” is not the opposite of the total value of the position to “2”.

# Including the investor default or not?

We get back symmetry if we allow for default of the investor in computing counterparty risk. This also results in an adjustment that is cheaper to the counterparty “2”.

The counterparty “2” may then be willing to ask the investor “0” to include the investor default event into the model, when the Counterparty risk adjustment is computed by the investor

# The case of symmetric counterparty risk

Suppose now that we allow for both parties to default. Counterparty risk adjustment allowing for default of "0"?
"0" : the investor; "2": the counterparty;
("1": the underlying name/risk factor of the contract).
$\tau_0, \tau_2$: default times of "0" and "2". $T$: final maturity
We consider the following events, forming a partition

## Four events ordering the default times

$$\begin{aligned} A &= \{\tau_0 \le \tau_2 \le T\} \qquad E = \{T \le \tau_0 \le \tau_2\} \\ B &= \{\tau_0 \le T \le \tau_2\} \qquad F = \{T \le \tau_2 \le \tau_0\} \\ C &= \{\tau_2 \le \tau_0 \le T\} \\ D &= \{\tau_2 \le T \le \tau_0\} \end{aligned}$$

# The case of symmetric counterparty risk

$$\Pi_0^D(t,T) = \mathbf{1}_{E\cup F}\Pi_0(t,T)$$
$$+\mathbf{1}_{C\cup D}\left[\Pi_0(t,\tau_2) + D(t,\tau_2)\left(REC_2\left(\mathrm{NPV}_0(\tau_2)\right)^+ - \left(-\mathrm{NPV}_0(\tau_2)\right)^+\right)\right]$$
$$+\mathbf{1}_{A\cup B}\left[\Pi_0(t,\tau_0) + D(t,\tau_0)\left(\left(\mathrm{NPV}_0(\tau_0)\right)^+ - REC_0\left(-\mathrm{NPV}_0(\tau_0)\right)^+\right)\right]$$

1. If no early default ⇒ payoff of a default-free claim (1st term).
2. In case of early default of the counterparty, the payments due before default occurs are received (second term),
3. and then if the residual net present value is positive only the recovery value of the counterparty $\mathrm{REC}_2$ is received (third term),
4. whereas if negative, it is paid in full by the investor (4th term).
5. In case of early default of the investor, the payments due before default occurs are received (fifth term),
6. and then if the residual net present value is positive it is paid in full by the counterparty to the investor (sixth term),
7. whereas if it is negative only the recovery value of the investor $\mathrm{REC}_0$ is paid to the counterparty (seventh term).

# The case of symmetric counterparty risk

$$\mathbb{E}_t\left\{\Pi_0^D(t,T)\right\} = \mathbb{E}_t\left\{\Pi_0(t,T)\right\} + \mathrm{DVA}_0(t) - \mathrm{CVA}_0(t)$$
$$\mathrm{DVA}_0(t) = \mathbb{E}_t\left\{\mathrm{LGD}_0 \cdot \mathbf{1}(A \cup B) \cdot D(t,\tau_0) \cdot [-\mathrm{NPV}_0(\tau_0)]^+\right\}$$
$$\mathrm{CVA}_0(t) = \mathbb{E}_t\left\{\mathrm{LGD}_2 \cdot \mathbf{1}(C \cup D) \cdot D(t,\tau_2) \cdot [\mathrm{NPV}_0(\tau_2)]^+\right\}$$

- Obtained simplifying the previous formula and taking expectation.
- 2nd term : adj due to scenarios $\tau_0 < \tau_2$. This is positive to the investor 0 and is called "Debit Valuation Adjustment" (DVA)
- 3d term : Counterparty risk adj due to scenarios $\tau_2 < \tau_0$
- Bilateral Valuation Adjustment as seen from 0: $\mathrm{BVA}_0 = \mathrm{DVA}_0 - \mathrm{CVA}_0$.
- If computed from the opposite point of view of "2" having counterparty "0", $\mathrm{BVA}_2 = -\mathrm{BVA}_0$. Symmetry.

# The case of symmetric counterparty risk

## Strange consequences of the formula new mid term, i.e. DVA

- credit quality of investor WORSENS $\Rightarrow$ books POSITIVE MARK TO MKT
- credit quality of investor IMPROVES $\Rightarrow$ books NEGATIVE MARK TO MKT
- Example: From a press release of a large bank.
- ... Revenues also included ...:
- A net 2.6 USD billion positive CVA on derivative positions, excluding monolines, mainly due to the widening of [the bank]s CDS spreads

# The case of symmetric counterparty risk

## When allowing for the investor to default: symmetry

- DVA: One more term with respect to the asymmetric case.
- depending on credit spreads and correlations, the adjustment to be subtracted can now be either positive or negative. In the asymmetric case it can only be positive.
- Ignoring the symmetry is clearly more expensive for the counterparty and cheaper for the investor.
- Some counterparties therefore may request the investor to include its own default into the valuation
- We assume the asymmetric case in most of the numerical presentations
- WE TAKE THE POINT OF VIEW OF ‘0” from now on, so we omit the subscript ‘0’. We denote the counterparty as ‘2” or ‘C”.

# Closeout: Substitution (ISDA?) VS Risk Free

When we computed the bilateral adjustment formula from

$$\begin{aligned}
&\Pi_0^D(t,T) = \mathbf{1}_{E\cup F}\Pi_0(t,T) \\
&+\mathbf{1}_{C\cup D}\left[\Pi_0(t,\tau_2) + D(t,\tau_2)\left(REC_2\,(\mathrm{NPV}_0(\tau_2))^+ - (-\mathrm{NPV}_0(\tau_2))^+\right)\right] \\
&+\mathbf{1}_{A\cup B}\left[\Pi_0(t,\tau_0) + D(t,\tau_0)\left((-\mathrm{NPV}_2(\tau_0))^+ - REC_0\,(\mathrm{NPV}_2(\tau_0))^+\right)\right]
\end{aligned}$$

(where we now substituted $\mathrm{NPV}_0 = -\mathrm{NPV}_2$ in the last two terms) we used the risk free NPV upon the first default, to close the deal. But what if upon default of the first entity, the deal needs to be valued by taking into account the credit quality of the surviving party? What if we make the substitutions

$$\mathrm{NPV}_0(\tau_2) \to \mathrm{NPV}_0(\tau_2) + \mathrm{UDVA}_0(\tau_2)$$

$$\mathrm{NPV}_2(\tau_0) \to \mathrm{NPV}_2(\tau_0) + \mathrm{UDVA}_2(\tau_0)?$$

# Closeout: Substitution (ISDA?) VS Risk Free

This seems to be supported by ISDA.

ISDA (2009) Close-out Amount Protocol.

”In determining a Close-out Amount, the Determining Party may consider any relevant information, including, without limitation, one or more of the following types of information: (i) quotations (either firm or indicative) for replacement transactions supplied by one or more third parties that may take into account the creditworthiness of the Determining Party at the time the quotation is provided”

# Closeout: Substitution (ISDA?) VS Risk Free

The final formula with subsitution closeout is quite complicated:

$$
\begin{aligned}
&\Pi_0^D(t,T) = \mathbf{1}_{E\cup F}\Pi_0(t,T)\\
&+\mathbf{1}_{C\cup D}\Big[\Pi_0(t,\tau_2) + D(t,\tau_2)\\
&\cdot\left(REC_2\left(\mathrm{NPV}_0(\tau_2)+\mathrm{UDVA}_0(\tau_2)\right)^+ - \left(-\mathrm{NPV}_0(\tau_2)-\mathrm{UDVA}_0(\tau_2)\right)^+\right)\Big]\\
&+\mathbf{1}_{A\cup B}\Big[\Pi_0(t,\tau_0) + D(t,\tau_0)\\
&\cdot\left(\left(-\mathrm{NPV}_2(\tau_0)-\mathrm{UDVA}_2(\tau_0)\right)^+ - REC_0\left(\mathrm{NPV}_2(\tau_0)+\mathrm{UDVA}_2(\tau_0)\right)^+\right)\Big]
\end{aligned}
$$

# Closeout: Substitution (ISDA?) VS Risk Free

**B. and Morini (2010)**

We analyze the Risk Free closeout formula in Comparison with the Substitution Closeout formula using a Zero coupon bond as a contract and in two cases:

1. Default of '0' and '2" are independent
2. Default of '0' and '2" are co-monotonic

Suppose '0' (the lender) holds the bond,
and '2' (the borrower) will pay the notional 1 at maturity $T$.

The risk free price of the bond at time 0 to '0' is denoted by $P(0, T)$.

# Closeout: Substitution (ISDA?) VS Risk Free

If we assume deterministic interest rates, the above formulas reduce to

$$P^{D,Subs}(0,T) = P(0,T)[\mathbb{Q}(\tau_2 > T) + REC_2 \mathbb{Q}(\tau_2 \le T)]$$

$$\begin{aligned} P^{D,Free}(0,T) &= P(0,T)[\mathbb{Q}(\tau_2 > T) + \mathbb{Q}(\tau_0 < \tau_2 < T) \\ &\quad + REC_2 \mathbb{Q}(\tau_2 \le \min(\tau_0, T))] \end{aligned}$$

$$= P(0,T)[\mathbb{Q}(\tau_2 > T) + REC_2 \mathbb{Q}(\tau_2 \le T) + LGD_2 \mathbb{Q}(\tau_0 < \tau_2 < T)]$$

## Credit Risk of the Lender

We see an important drawback of the risk free closeout in this case: The adjusted price of the bond DEPENDS ON THE CREDIT RISK OF THE LENDER '0' IF WE USE THE RISK FREE CLOSEOUT. This is counterintuitive and undesirable. From this point of view the Substitution Closeout is superior.

# Closeout: Substitution (ISDA?) VS Risk Free

## Co-Monotonic Case

If we assume the default of '0' and '2' to be co-monotonic, and the spread of the lender '0" to be larger, we have that the lender '0" defaults first in ALL SCENARIOS (e.g. '2' is a subsidiary of '0', or a company whose well being is completely driven by '0': '2' is a trye factory whose only client is car producer '0"). In this case the two formulas become

$$P^{D,Subs}(0,T) = P(0,T)[\mathbb{Q}(\tau_2 > T) + REC_2\mathbb{Q}(\tau_2 \leq T)]$$

$$P^{D,Free}(0,T) = P(0,T)[\mathbb{Q}(\tau_2 > T) + \mathbb{Q}(\tau_2 < T)] = P(0,T)$$

Risk free closeout gives the correct price. Either '0" does not default, and then '2" does not default either, or if '0" defaults, at that precise time 2 is solvent, and 0 recovers the whole payment. Credit risk of '2" should not impact the deal. This happens with the Risk Free closeout but not with the substitution closeout.

# Closeout: Substitution (ISDA?) VS Risk Free

## Contagion. Lenders and Borrowers

$$P^{D,Subs}(t,T) = P(t,T)[\mathbb{Q}_t(\tau_2 > T) + REC_2\mathbb{Q}_t(\tau_2 \leq T)]$$
$$P^{D,Free}(t,T) = P^{D,Subs}(t,T) + P(t,T)LGD_2\mathbb{Q}_t(\tau_0 < \tau_2 < T)$$

We focus on two cases:

- $\tau_0$ and $\tau_2$ are independent. Take $t < T$.

$$\mathbb{Q}_{t-\Delta t}(\tau_0 < \tau_2 < T) \mapsto \quad \{\tau_0 = t\} \quad \mapsto \mathbb{Q}_{t+\Delta t}(\tau_2 < T)$$

  and this effect can be quite sizeable.

- $\tau_0$ and $\tau_2$ are comonotonic. Take an example where $\tau_0 = t < T$ implies $\tau_2 = u < T$ with $u > t$. Then

$$\mathbb{Q}_{t-\Delta t}(\tau_2 > T) \mapsto \quad \{\tau_0 = t, \tau_2 = u\} \quad \mapsto 0$$
$$\mathbb{Q}_{t-\Delta t}(\tau_2 \leq T) \mapsto \quad \{\tau_0 = t, \tau_2 = u\} \quad \mapsto 1$$
$$\mathbb{Q}_{t-\Delta t}(\tau_0 < \tau_2 < T) \mapsto \quad \{\tau_0 = t, \tau_2 = u\} \quad \mapsto 1$$

# Closeout: Substitution (ISDA?) VS Risk Free

Let us put the pieces together:

- $\tau_0$ and $\tau_2$ are independent. Take $t < T$.

$$P^{D,Subs}(t - \Delta t, T) \mapsto \quad \{\tau_0 = t\} \mapsto \quad \text{no change}$$

$$P^{D,Free}(t - \Delta t, T) \mapsto \quad \{\tau_0 = t\} \mapsto \quad \text{add } \mathbb{Q}_{t-\Delta t}(\tau_0 > \tau_2, \tau_2 < T)$$

  and this effect can be quite sizeable.

- $\tau_0$ and $\tau_2$ are comonotonic. Take an example where $\tau_0 = t < T$ implies $\tau_2 = u < T$ with $u > t$. Then

$$P^{D,Subs}(t - \Delta t, T) \mapsto \quad \{\tau_0 = t\} \mapsto \quad \text{subtract } X$$

$$X = LGD_2 P(t, T)\mathbb{Q}_{t-\Delta t}(\tau_2 > T)$$

$$P^{D,Free}(t - \Delta t, T) \mapsto \quad \{\tau_0 = t\} \mapsto \quad \text{no change}$$

# Closeout: Substitution (ISDA?) VS Risk Free

### The independence case: Contagion with Risk Free closeout

The Risk Free closeout shows that *upon default of the lender*, the mark to market to the lender itself jumps up, or equivalently **the mark to market to the borrower jumps down**. The effect can be quite dramatic.
*The substitution closeout instead shows no such contagion*, as the mark to market does not change upon default of the lender.

### The co-monotonic case: Contagion with Substitution closeout

*The Risk Free closeout behaves nicely in the co-monotonic case*, and there is no change upon default of the lender.
Instead the Substitution closeout shows that *upon default of the lender* the mark to market to the lender jumps down, or equivalently **the mark to market to the borrower jumps up**.

# Closeout: Substitution (ISDA?) VS Risk Free

Impact of an early default of the Lender

| Dependence→ Closeout↓ | independence | co-monotonicity |
|---|---|---|
| Risk Free | Negatively affects Borrower | No contagion |
| Substitution | No contagion | Further Negatively affects Lender |

# A useful derivative: Contingent CDS (CCDS)

### Definition

Similar to a CDS but when the reference credit defaults at $\tau$, the protection seller pays protection on a notional that is not fixed but given by the NPV of a reference Portfolio $\Pi$ at that time if positive.
This amount is:
$(\mathbb{E}_{\tau_C}\Pi(\tau_C, T))^+$, minus a recovery $\mathrm{R}_{\mathrm{EC}}$ fraction of it.

### CCDS default leg payoff = asymmetric counterparty risk adj

The payoff of the default leg of a Contingent CDS is exactly

$$(1 - \mathrm{R}_{\mathrm{EC}})\mathbf{1}_{\{(t<\tau_C<T)\}}D(t, \tau_C)(\mathbb{E}_{\tau_C}\Pi(\tau_C, T))^+$$

# General Remarks on CCDS

*"[...]Rudimentary and idiosyncratic versions of these so-called CCDS have existed for five years, but they have been rarely traded due to high costs, low liquidity and limited scope. [...] Counterparty risk has become a particular concern in the markets for interest rate, currency, and commodity swaps - because these trades are not always backed by collateral.[...] Many of these institutions - such as hedge funds and companies that do not issue debt - are beyond the scope of cheaper and more liquid hedging tools such as normal CDS. The new CCDS was developed to target these institutions (Financial Times, April 10, 2008)."*

***Being the two payoffs equivalent, the counterparty risk adjustment valuation will hold as well for the default leg of a CCDS.***

# Methodology

1. Assumption: The *investor* enters a transaction with a *counterparty* and the investor considers itself default free.
   Note : All the payoffs seen from the point of view of the *investor*.
2. We model and calibrate the default time of the *counterparty* using a stochastic intensity default model.
3. We model the transaction underlying and estimate the deal NPV at default.
4. We allow for the counterparty default time and the contract underlying to be correlated.

# Counterparty default model: CIR++ stochastic intensity

**The model for the counterparty instantaneous credit spread:**

$$\lambda(t) = y(t) + \psi(t;\beta)$$

**Remarks:**

1. $y(t)$ is a CIR process with possible jumps
   $dy = \kappa(\mu - y)dt + \nu\sqrt{y}dW_y + dJ$
2. $\psi(t;\beta)$ is the shift that matches a given CDS curve
3. In CDS calibration we assume deterministic interest rates.
4. Calibration : Fitting model survival probabilities to survival probabilities stripped from counterparty CDS quotes

# Approximation: Default Bucketing

## General Formulation

1. Model (underlying) to estimate the NPV of the transaction.
2. Simulations are run allowing for correlation between the credit and underlying models, to determine the counterparty default time and the underlying deal NPV respectively.

## Approximated Formulation under default bucketing

$$\mathbb{E}_0 \Pi^D(0,T) := \mathbb{E}_0 \Pi(0,T)$$

$$-\mathrm{L}_{\mathrm{GD}} \sum_{j=1}^{b} \mathbb{E}_0[1\{\tau \in (T_{j-1}, T_j]\}\ D(0,T_j)(\mathbb{E}_{T_j}\Pi(T_j,T))^+]$$

1. In this formulation defaults are bucketed but we still need a joint model for $\tau$ and the underlying $\Pi$ including their correlation.
2. Option model for $\Pi$ is implicitly needed in $\tau$ scenarios.

# Approximation: Default Bucketing and Independence

## Approximated Formulation under independence (and 0 correlation)

$$\mathbb{E}_0 \Pi^D(0,T) := \mathbb{E}_0 \Pi(0,T)$$
$$- \mathrm{L}_{\mathrm{GD}} \sum_{j=1}^{b} \boxed{\mathbb{Q}\{\tau \in (T_{j-1}, T_j]\}} \, \mathbb{E}_0[D(0,T_j)(\mathbb{E}_{T_j}\Pi(T_j,T))^+]$$

1. In this formulation defaults are bucketed and only survival probabilities are needed (no default model).
2. Option model is STILL needed for the underlying of Π.

# 3 cases: Interest Rates, Credit, Commodities

We now examine 3 specific cases of underlying contracts:

- Interest Rate Swaps and Derivatives Portfolios
- Commodities swaps (Oil)
- Credit: CDS on a reference credit

# Interest Rates Swap Case

## Formulation for IRS under independence (no correlation)

$$\mathrm{IRS}^D(t,K) = \mathrm{IRS}(t.K)$$
$$-\mathrm{L{\small GD}} \sum_{i=a+1}^{b-1} \mathbb{Q}\{\tau \in (T_{i-1}, T_i]\}\, \mathrm{S{\small WAPTION}}_{i,b}(t; K, S_{i,b}(t), \sigma_{i,b})$$

## Modeling Approach with corr.

Gaussian 2-factor G2++ short-rate $r(t)$ model:
$r(t) = x(t) + z(t) + \varphi(t;\alpha), r(0) = r_0$

$dx(t) = -ax(t)dt + \sigma dW_x$
$dz(t) = -bz(t)dt + \eta dW_z$

$dW_x dW_z = \rho_{x,z} dt$

$\alpha = [r_0, a, b, \sigma, \eta, \rho_{1,2}]$

$dW_x dW_y = \rho_{x,y} dt, dW_z dW_y = \rho_{z,y} dt$

## Calibration

- The function $\varphi(\cdot;\alpha)$ is deterministic and is used to calibrate the initial curve observed in the market.
- We use swaptions and zero curve data to calibrate the model.
- The $r$ factors $x$ and $z$ and the intensity are taken to be correlated.

# Interest Rates Swap Case

## Total Correlation Counterparty default / rates

$$\bar{\rho} = \mathrm{Corr}(dr_t, d\lambda_t) = \frac{\sigma \rho_{x,y} + \eta \rho_{z,y}}{\sqrt{\sigma^2 + \eta^2 + 2\sigma\eta\rho_{x,z}}\sqrt{1 + \frac{2\beta\gamma^2}{\nu^2 y_t}}}.$$

where $\beta$ is the intensity of arrival of $\lambda$ jumps and $\gamma$ is the mean of the exponentially distributed jump sizes.

## Without jumps ($\beta = 0$)

$$\bar{\rho} = \mathrm{Corr}(dr_t, d\lambda_t) = \frac{\sigma \rho_{x,y} + \eta \rho_{z,y}}{\sqrt{\sigma^2 + \eta^2 + 2\sigma\eta\rho_{x,z}}}.$$

# IRS: Case Study

**1) Single Interest Rate Swaps (IRS)**

At-the-money fix-receiver forward interest-rate-swap (IRS) paying on the EUR market.

The IRS's fixed legs pay annually a 30E/360 strike rate, while the floating legs pay LIBOR twice per year.

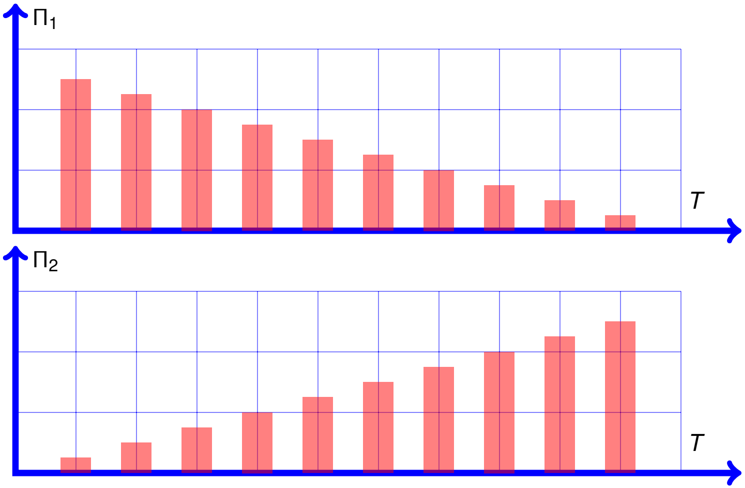
**2) Netted portfolios of IRS.**

- Portfolios of at-the-money IRS either with different starting dates or with different maturities.

1. ($\Pi$1) annually spaced dates $\{T_i : i = 0 \ldots N\}$, $T_0$ two business days from trade date; portfolio of swaps maturing at each $T_i$, with $i > 0$, all starting at $T_0$.
2. ($\Pi$2) portfolio of swaps starting at each $T_i$ all maturing at $T_N$.

**Can also do exotics (Ratchets, CMS spreads, Bermudan)**

# IRS Case Study: Payment schedules



# IRS Results

Counterparty risk price for netted receiver IRS portfolios Π1 and Π2 and simple IRS (maturity 10Y). Every IRS, constituting the portfolios, has unit notional and is at equilibrium. Prices are in bps.

| $\lambda$ | correlation $\bar{\rho}$ | Π1 | Π2 | IRS |
|---|---|---|---|---|
| 3% | -1 | -140 | -294 | -36 |
| | 0 | -84 | -190 | -22 |
| | 1 | -47 | -115 | -13 |
| 5% | -1 | -181 | -377 | -46 |
| | 0 | -132 | -290 | -34 |
| | 1 | -99 | -227 | -26 |
| 7% | -1 | -218 | -447 | -54 |
| | 0 | -173 | -369 | -44 |
| | 1 | -143 | -316 | -37 |

# Compare with ”Basel 2” deduced adjustments

Basel 2, under the ”Internal Model Method”, models wrong way risk by means of a 1.4 multiplying factor to be applied to the zero correlation case, even if banks have the option to compute their own estimate of the multiplier, which can never go below 1.2 anyway.

Is this confirmed by our model?

$$(140 - 84)/84 \approx 66\% > 40\%$$

$$(54 - 44)/44 \approx 23\% < 40\%$$

So this really depends on the portfolio and on the situation.

# A bilateral example and correlation risk

Finally, in the bilateral case for Receiver IRS, 10y maturity, high risk counterparty and mid risk investor, we notice that depending on the correlations

$$\bar{\rho}_0 = \mathrm{Corr}(dr_t, d\lambda_t^0), \quad \bar{\rho}_2 = \mathrm{Corr}(dr_t, d\lambda_t^2), \quad \rho_{0,2}^{Copula} = 0$$

the DVA - CVA or Bilateral CVA does change sign, and in particular for portfolios Π1 and IRS the sign of the adjustment follows the sign of the correlations.

| $\bar{\rho}_2$ | $\bar{\rho}_0$ | **Π1** | **Π2** | **10×IRS** |
|---|---|---|---|---|
| **-60%** | **0%** | -117(7) | -382(12) | -237(16) |
| **-40%** | **0%** | -74(6) | -297(11) | -138(15) |
| **-20%** | **0%** | -32(6) | -210(10) | -40(14) |
| **0%** | **0%** | -1(5) | -148(9) | 31(13) |
| **20%** | **0%** | 24(5) | -96(9) | 87(12) |
| **40%** | **0%** | 44(4) | -50(8) | 131(11) |
| **60%** | **0%** | 57(4) | -22(7) | 159(11) |

# Payer vs Receiver

- Counterparty Risk (CR) has a relevant impact on interest-rate payoffs prices and, in turn, correlation between interest-rates and default (intensity) has a relevant impact on the CR adjustment.
- The (positive) CR adjustment to be subtracted from the default free price **decreases with correlation for receiver payoffs**. Natural: If default intensities increase, with high positive correlation their correlated interest rates will increase more than with low correlation, and thus a receiver swaption embedded in the adjustment decreases more, reducing the adjustment.
- The adjustment for payer payoffs increases with correlation.

# Further Stylized Facts

- As the default probability implied by the counterparty CDS increases, the size of the adjustment increases as well, but the impact of correlation on it decreases.
- Financially reasonable: Given large default probabilities for the counterparty, fine details on the dynamics such as the correlation with interest rates become less relevant
- **The conclusion is that we should take into account interest-rate/ default correlation in valuing CR interest-rate payoffs.**
- In the bilateral case correlation risk can cause the adjustment to change sign

# Exotics

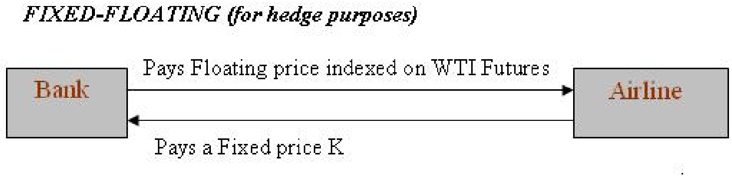
For examples on exotics, including Bermudan Swaptions and CMS spread Options, see

## Papers with Exotics and Bilateral Risk


- Brigo, D., and Pallavicini, A. (2007). Counterparty Risk under Correlation between Default and Interest Rates. In: Miller, J., Edelman, D., and Appleby, J. (Editors), Numerical Methods for Finance, Chapman Hall.
- Brigo, D., Pallavicini, A., and Papatheodorou, V. (2009). Bilateral counterparty risk valuation for interest-rate products: impact of volatilities and correlations. Available at Defaultrisk.com, SSRN and arXiv

# Commodities: Futures, Forwards and Swaps

- **Forward**: OTC contract to buy a commodity to be delivered at a maturity date T at a price specified today. The cash/commodity exchange happens at time T.
- **Future**: Listed Contract to buy a commodity to be delivered at a maturity date T. Each day between today and T margins are called and there are payments to adjust the position.
- **Commodity Swap: Oil Example**:



# Commodities: Modeling Approach

## Schwartz-Smith Model

$$ln(S_t) = x_t + l_t + \varphi(t)$$
$$dx_t = -kx_t dt + \sigma_x dW_x$$
$$dl_t = \mu dt + \sigma_l dW_l$$
$$dW_x \, dW_l = \rho_{x,l} dt$$

## Variables

$S_t$: Spot oil price;
$x_t, l_t$: short and long term components of $S_t$;
This can be re-cast in a classic convenience yield model

## Correlation with credit

$$dW_x \, dW_y = \rho_{x,y} dt,$$
$$dW_l \, dW_y = \rho_{l,y} dt$$

## Calibration

$\varphi$: defined to exactly fit the oil forward curve.
Dynamic parameters $k, \mu, \sigma, \rho$ are calibrated to At the money implied volatilities on Futures options.

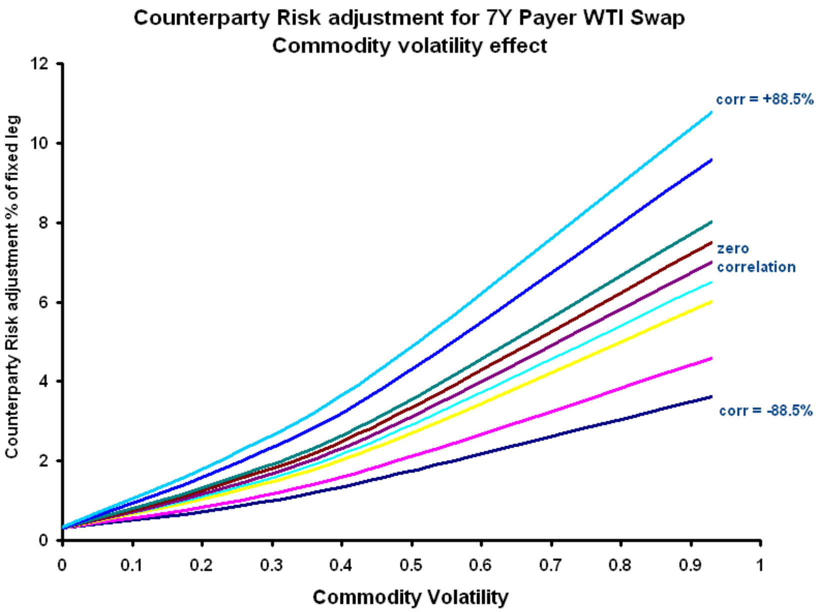
# Commodities

**Total correlation Commodities - Counterparty default**

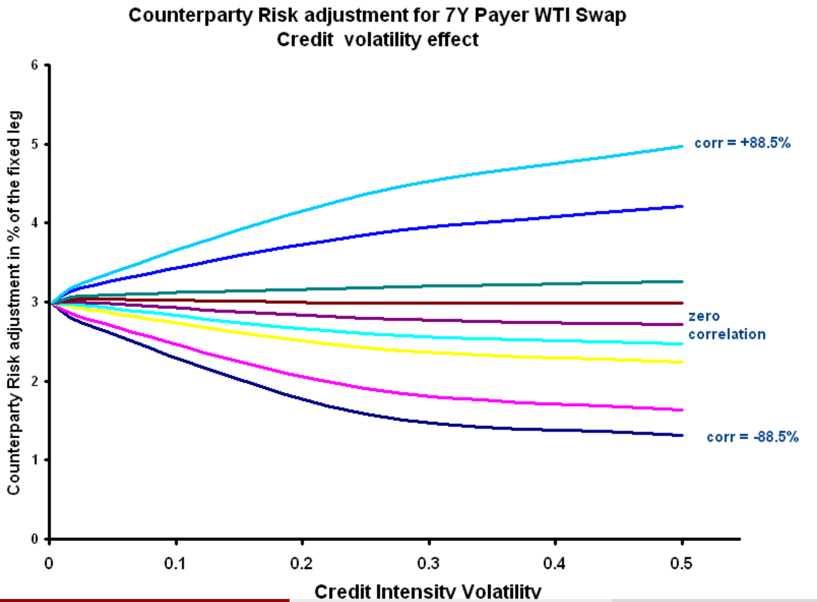
$$\bar{\rho} = \text{corr}(d\lambda_t,\ dS_t) = \frac{\sigma_x \rho_{x,y} + \sigma_L \rho_{L,y}}{\sqrt{\sigma_x^2 + \sigma_L^2 + 2\rho_{x,L}\sigma_x \sigma_L}}$$

We assumed no jumps in the intensity

# Commodities: Commodity Volatility Effect



# Commodities: Credit Volatility Effect



# Commodities[1] : Credit volatility effect

| $\bar{\rho}$ | intensity volatility $\nu_R$ | 0.025 | 0.25 | 0.50 |
|---|---|---|---|---|
| -88.5 | Payer adj | 2.742 | 1.584 | 1.307 |
| | Receiver adj | 1.878 | 2.546 | 3.066 |
| -63.2 | Payer adj | 2.813 | 1.902 | 1.63 |
| | Receiver adj | 1.858 | 2.282 | 2.632 |
| -25.3 | Payer adj | 2.92 | 2.419 | 2.238 |
| | Receiver adj | 1.813 | 1.911 | 2.0242 |
| -12.6 | Payer adj | 2.96 | 2.602 | 2.471 |
| | Receiver adj | 1.802 | 1.792 | 1.863 |
| 0 | Payer adj | 2.999 | 2.79 | 2.719 |
| | Receiver adj | 1.79 | 1.676 | 1.691 |
| +12.6 | Payer adj | 3.036 | 2.985 | 2.981 |
| | Receiver adj | 1.775 | 1.562 | 1.527 |
| +25.3 | Payer adj | 3.071 | 3.184 | 3.258 |
| | Receiver adj | 1.758 | 1.45 | 1.371 |
| +63.2 | Payer adj | 3.184 | 3.852 | 4.205 |
| | Receiver adj | 1.717 | 1.154 | 0.977 |
| +88.5 | Payer adj | 3.229 | 4.368 | 4.973 |
| | Receiver adj | 1.664 | 0.988 | 0.798 |

Fixed Leg Price maturity 7Y: 7345.39 USD for a notional of 1 Barrel per Month

[1] adjusment expressed as % of the fixed leg price

## Commodities[2] : Commodity volatility effect

| $\bar{\rho}$ | Commodity spot volatility $\sigma_S$ | 0.0005 | 0.232 | 0.46 | 0.93 |
|---|---|---|---|---|---|
| -88.5 | Payer adj | 0.322 | 0.795 | 1.584 | 3.607 |
| | Receiver adj | 0 | 1.268 | 2.546 | 4.495 |
| -63.2 | Payer adj | 0.322 | 0.94 | 1.902 | 4.577 |
| | Receiver adj | 0 | 1.165 | 2.282 | 4.137 |
| -25.3 | Payer adj | 0.323 | 1.164 | 2.419 | 6.015 |
| | Receiver adj | 0 | 0.977 | 1.911 | 3.527 |
| -12.6 | Payer adj | 0.323 | 1.246 | 2.602 | 6.508 |
| | Receiver adj | 0 | 0.917 | 1.792 | 3.325 |
| 0 | Payer adj | 0.324 | 1.332 | 2.79 | 6.999 |
| | Receiver adj | 0 | 0.857 | 1.676 | 3.115 |
| +12.6 | Payer adj | 0.324 | 1.422 | 2.985 | 7.501 |
| | Receiver adj | 0 | 0.799 | 1.562 | 2.907 |
| +25.3 | Payer adj | 0.324 | 1.516 | 3.184 | 8.011 |
| | Receiver adj | 0 | 0.742 | 1.45 | 2.702 |
| +63.2 | Payer adj | 0.325 | 1.818 | 3.8525 | 9.581 |
| | Receiver adj | 0 | 0.573 | 1.154 | 2.107 |
| +88.5 | Payer adj | 0.326 | 2.05 | 4.368 | 10.771 |
| | Receiver adj | 0 | 0.457 | 0.988 | 1.715 |

Fixed Leg Price maturity 7Y: 7345.39 USD for a notional of 1 Barrel per Month

[2] adjusment expressed as % of the fixed leg price

# Wrong Way Risk?

Basel 2, under the "Internal Model Method", models wrong way risk by means of a 1.4 multiplying factor to be applied to the zero correlation case, even if banks have the option to compute their own estimate of the multiplier, which can never go below 1.2 anyway.
What did we get in our cases? Two examples:

$$(4.973 - 2.719)/2.719 = 82\% >> 40\%$$

$$(1.878 - 1.79)/1.79 \approx 5\% << 20\%$$

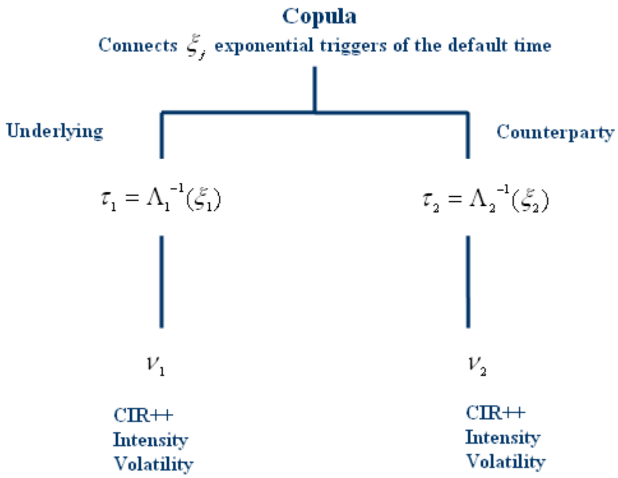
# Credit (CDS)

- Model equations: ("1" = CDS underlying, "2" = counterparty )

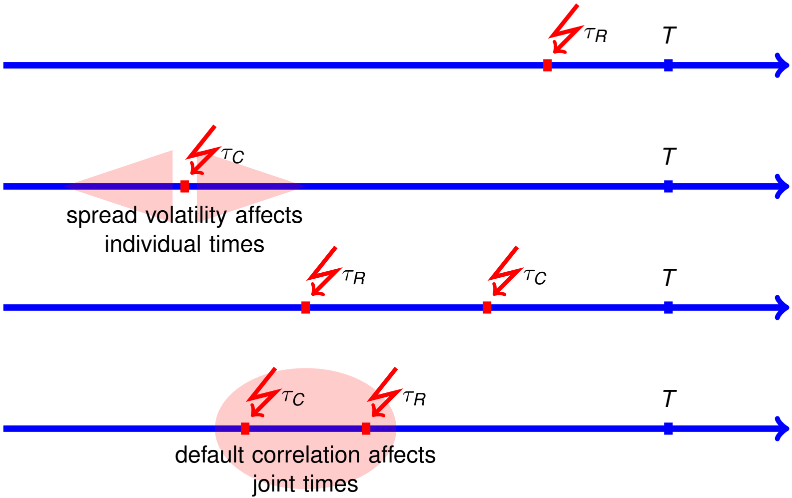
$$d\lambda_j(t) = k_j(\mu_j - \lambda_j(t))dt + \nu_j \sqrt{\lambda_j(t)}dZ_j(t), \quad j = 1, 2$$

- Cumulative intensities are defined as : $\Lambda(t) = \int_0^t \lambda(s)ds$.
- Default times are $\tau_j = \Lambda_j^{-1}(\xi_j)$. Exponential triggers $\xi_1$ and $\xi_2$ are connected through a gaussian copula with correlation parameter $\rho$.
- In our approach, we take into account default correlation between default times $\tau_1$ and $\tau_2$ **and** credit spreads volatility $\nu_j, j = 1, 2$.
- Important: volatility can amplify default time uncertainty, while high correlation reduces conditional default time uncertainty.
  Taking into account $\rho$ and $\nu \Longrightarrow$ better representation of market information and behavior, especially for wrong way risk.

# Credit (CDS) : Overview



# Credit (CDS) Correlation and Volatility Effects

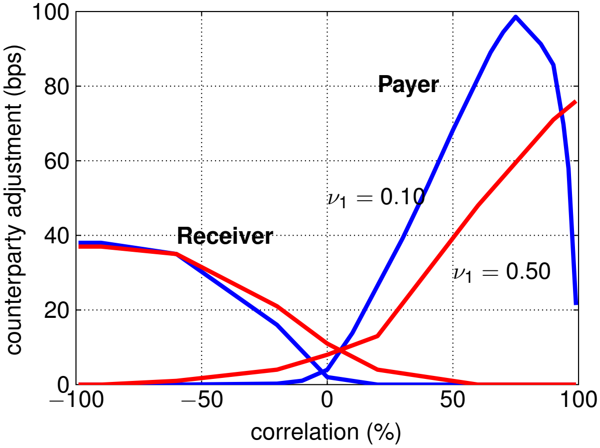


# Moderate counterparty spread $\nu_2 = 0.10$

| $\rho$ | Vol parameter $\nu_1$ | 0.01 | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 |
|---|---|---|---|---|---|---|---|
| | CDS Implied vol | 1.5% | 15% | 28% | 37% | 42% | 42% |
| -99 | Payer adj | 0(0) | 0(0) | 0(0) | 0(0) | 0(0) | 0(0) |
| | Receiver adj | 40(2) | 38(2) | 39(2) | 38(2) | 36(1) | 37(1) |
| -90 | Payer adj | 0(0) | 0(0) | 0(0) | 0(0) | 0(0) | 0(0) |
| | Receiver adj | 39(2) | 38(2) | 38(2) | 38(2) | 35(1) | 37(2) |
| -60 | Payer adj | 0(0) | 0(0) | 0(0) | 0(0) | 0(0) | 1(0) |
| | Receiver adj | 36(1) | 35(1) | 36(1) | 36(1) | 32(1) | 35(1) |
| -20 | Payer adj | 0(0) | 0(0) | 1(0) | 2(0) | 3(0) | 4(1) |
| | Receiver adj | 16(1) | 16(1) | 17(1) | 19(1) | 18(1) | 21(1) |
| 0 | Payer adj | 3(0) | 4(0) | 5(0) | 7(1) | 7(1) | 8(1) |
| | Receiver adj | 0(0) | 2(0) | 5(0) | 8(0) | 10(0) | 11(1) |
| +20 | Payer adj | 27(1) | 25(1) | 23(1) | 20(1) | 16(2) | 13(1) |
| | Receiver adj | 0(0) | 0(0) | 1(0) | 2(0) | 2(0) | 4(0) |
| +60 | Payer adj | 80(4) | 82(4) | 67(4) | 64(4) | 55(3) | 48(3) |
| | Receiver adj | 0(0) | 0(0) | 0(0) | 0(0) | 0(0) | 0(0) |
| +90 | Payer adj | 87(6) | 86(6) | 88(6) | 78(5) | 80(5) | 71(4) |
| | Receiver adj | 0(0) | 0(0) | 0(0) | 0(0) | 0(0) | 0(0) |
| +99 | Payer adj | 10(2) | 21(3) | 52(5) | 68(5) | 73(5) | 76(5) |
| | Receiver adj | 0(0) | 0(0) | 0(0) | 0(0) | 0(0) | 0(0) |

# Large counterparty spread $\nu_2 = 0.20$

| $\rho$ | Vol parameter $\nu_1$ | 0.01 | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 |
|---|---|---|---|---|---|---|---|
| | CDS Implied vol | 1.5% | 15% | 28% | 37% | 42% | 42% |
| -99 | Payer adj | 0(0) | 0(0) | 0(0) | 0(0) | 0(0) | 0(0) |
| | Receiver adj | 41(2) | 40(2) | 39(2) | 40(2) | 40(2) | 40(2) |
| -90 | Payer adj | 0(0) | 0(0) | 0(0) | 0(0) | 0(0) | 0(0) |
| | Receiver adj | 41(2) | 39(2) | 39(2) | 41(2) | 40(2) | 40(2) |
| -60 | Payer adj | 0(0) | 0(0) | 0(0) | 0(0) | 1(0) | 1(0) |
| | Receiver adj | 39(1) | 37(1) | 37(1) | 37(1) | 36(1) | 35(1) |
| -20 | Payer adj | 0(0) | 0(0) | 2(0) | 3(0) | 3(0) | 4(1) |
| | Receiver adj | 17(1) | 17(1) | 17(1) | 19(1) | 21(1) | 20(1) |
| 0 | Payer adj | 3(0) | 5(0) | 6(0) | 7(1) | 6(1) | 6(1) |
| | Receiver adj | 0(0) | 2(0) | 4(0) | 7(0) | 10(0) | 12(1) |
| +20 | Payer adj | 25(1) | 24(1) | 23(1) | 20(1) | 17(1) | 15(1) |
| | Receiver adj | 0(0) | 0(0) | 1(0) | 2(0) | 2(0) | 4(0) |
| +60 | Payer adj | 74(4) | 74(4) | 69(4) | 59(3) | 54(3) | 52(3) |
| | Receiver adj | 0(0) | 0(0) | 0(0) | 0(0) | 0(0) | 1(0) |
| +90 | Payer adj | 91(6) | 90(6) | 88(5) | 80(5) | 81(5) | 81(5) |
| | Receiver adj | 0(0) | 0(0) | 0(0) | 0(0) | 0(0) | 0(0) |
| +99 | Payer adj | 43(4) | 56(5) | 57(5) | 72(5) | 74(5) | 78(5) |
| | Receiver adj | 0(0) | 0(0) | 0(0) | 0(0) | 0(0) | 0(0) |


100
80
60
40
20
0
counterparty adjustment (bps)
−100
−50
0
50
100
correlation (%)
Payer
Receiver
$\nu_1 = 0.10$
$\nu_1 = 0.50$

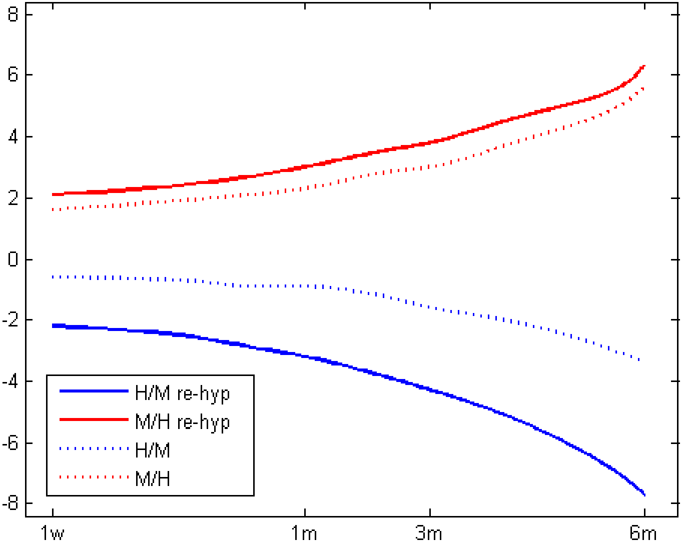
# Collateral Management I

- Risk-neutral evaluation of counterparty risk in presence of collateral management can be a difficult task, due to the complexity of clauses.
- Only few papers in the literature deal with it. Among them we cite Cherubini (2005), Alavian *et al.* (2008), Yi (2009), Assefa *et al.* (2009), Brigo et al (2011) and citations therein.

# Collateral Management II

- Collateralized bilateral CVA for a netted portfolio of IRS with ten year maturity and one year coupon tenor for different default-time correlations with (and without) collateral re-hypothecation. See Brigo, Capponi, Pallavicini and Papatheodorou (2010), forthcoming, from which this presentation is taken.

# Collateral Management III



# Figure explanation

## Bilateral valuation adjustment, margining and rehypotecation

The above figure shows the Bilateral valuation adjustment (DVA-CVA) for a ten-year IRS under collateralization through margining as a function of the update frequency $\delta$ with zero correlation between rates and counterparty spread, zero correlation between rates and investor spread, and zero correlation between the counterparty and the investor defaults. The model allows for nonzero correlations as well. **Continuous lines** represent the re-hypothecation case, while **dotted lines** represent the opposite case. The *red line* represents an investor riskier than the counterparty, while the *blue line* represents an investor less risky than the counterparty. All values are in basis points.

See the full paper by Brigo, Capponi, Pallavicini and Papatheodorou ‘Collateral Margining in Arbitrage-Free Counterparty Valuation Adjustment including Re-Hypotecation and Netting” available at `http://arxiv.org/abs/1101.3926`

# Figure explanation

Again from the figure, we see that the case of an investor riskier than the counterparty (M/H) leads to positive value for Bilateral adjustment DVA-CVA, while the case of an investor less risky than the counterparty has the opposite behaviour. If we inspect the DVA and CVA terms as in the paper we see that when the investor is riskier the DVA part of the correction dominates, while when the investor is less risky the counterparty has the opposite behaviour.
The effect of re-hypothecation is to enhance the absolute size of the correction, a reasonable behaviour, since, in such case, each party has a greater risk because of being unsecured on the collateral amount posted to the other party in case of default.

# Monitoring Counterparty Credit Risk

- When we monitor a (symmetric) risk in a bilateral agreement, we should introduce a “metric” which is shared by both parties.
  - $\rightarrow$ The ISDA Master Agreement defines the term *exposure* to be the netted mid-market mark-to-market value of the transaction.
- We name the exposure priced at time $t$, either by the investor or by the counterparty, with $\varepsilon_t$.
- Notice that the ISDA Master Agreement allows the calculation agent to be a third party.
- Since counterparty risk can be sized in term of exposure, we can operate to mitigate the risk by reducing such quantity.

# Mitigating Counterparty Credit Risk – I

- The ISDA Master Agreement lists two different tools to reduce exposure:
  - → close-out netting rules, which state that if a default occurs, multiple obligations between two parties are consolidated into a single net obligation; and
  - → collateralization, namely the right of recourse to some asset of value that can be sold or the value of which can be applied in the event of default on the transaction.
- We consider that assets used as collaterals are posted on a Collateral Account held by a Collateral Taker, and we name its value at time $t$ with $C_t$.
- Notice that if at time $t$ the investor posts some collateral we consider that $dC_t < 0$, the other way round if the counterparty is posting.

# Mitigating Counterparty Credit Risk – II

- In the following we assume that close-out netting rules are always active, so that we consider the transaction $\Pi(t, T)$ and the collateral account $C_t$ together when calculating the CVA.
- Thus, under close-out netting rules we get

$$\mathrm{C}_{\mathrm{VA}}(t, T; C) := \mathbb{E}_t\left[\, \bar{\Pi}(t, T; C) - \Pi(t, T) - C_T D(t, T) \,\right]$$

  where the expectation is taken under risk-neutral measure, and $\bar{\Pi}(t, T; C)$ will be analyzed in the following slides.
- Furthermore, we assume that mid-market exposure $\varepsilon_t$ can be calculated from the risk-free $\Pi(t, T)$ as

$$\varepsilon_t \doteq \mathbb{E}_t[\, \Pi(t, T) \,]$$

# Re-hypothecation Liquidity Risk – I

- At transaction maturity or after applying close-out netting, the originating party expects to get back the remaining collateral.
- Yet, prevailing legislations may give to the Collateral Taker some rights on the collateral itself.
- For instance, on an early termination date a counterparty to an English CSA will find itself as an unsecured creditor, thus entitled to only a fraction of the value of the collateral it transferred.
- With a New York CSA transferred cash collateral or re-hypothecated collateral are both likely to leave the collateral provider in the same position as an unsecured creditor, but, in this case, the parties may agree on amending the provisions of the CSA which make re-hypothecation possible.

# Re-hypothecation Liquidity Risk – II

- In case of collateral re-hypothecation the surviving party must consider the possibility to recover only a fraction of his collateral.
  - $\rightarrow$ We name such recovery rate $\mathrm{Rec}'_I$, if the investor is the Collateral Taker, or $\mathrm{Rec}'_C$ in the other case (we often use $\mathrm{Lgd}'_I := 1 - \mathrm{Rec}'_I$ and $\mathrm{Lgd}'_C := 1 - \mathrm{Rec}'_C$).
- In the worst case the surviving party has no precedence on other creditors to get back his collateral. In such case the recovery rate of collateral is the one of the transaction. Thus, we get

$$\mathrm{Rec}_I \leq \mathrm{Rec}'_I \leq 1 \ , \quad \mathrm{Rec}_C \leq \mathrm{Rec}'_C \leq 1$$

- If the Collateral Taker is a risk-free third-party we can assume that $\mathrm{Rec}'_I = \mathrm{Rec}'_C = 1$.

# Collateral Choice

*Ideally, firms would like an asset of stable and predictable value, an asset that is not linked to the value of the transaction in any way and an asset that can be sold quickly and easily if the need arises.* [ISDA, Coll. Review, 1.1]

- Thus, in order to achieve an effective collateralization of the transaction, we require that
  - → collaterals hedge investor's exposure on counterparty's default event,
  - → they are liquid assets,
  - → they are not related to the deal's underlying assets or to the counterparty.
- In practice, when collaterals do not match such requirements, their value is reduced by means of corrective factors named haircuts.

# Margining Practice – I

- In general, margining practice consists in a pre-fixed set of dates during the life of a deal when both parties post or withdraw collaterals, according to their current exposure, to or from an account held by the Collateral Taker.
- The Collateral Taker may be a third party or the party of the transaction who is not posting collateral.
- Notice that in legal documents where a pledge or a security interest is in act the Collateral Taker is named the Secured Party, while the other party is the Pledgor.
- We do not consider legal issues which may change collateral arrangement (pledge vs. title transfer) but for re-hypothecation issues.

# Margining Practice – II

- The Collateral Taker remunerates the account (usually) at over-night rate.
  - $\rightarrow$ In the following we consider that the collaterals are risk-free and their account is a cash account accruing at risk-free rate.
- At deal termination date the parties are not forced to close the collateral account, but they may agree to use it for a new deal.
  - $\rightarrow$ We consider that the collateral account is opened anew for each new deal and it is closed upon a default event occurs or maturity is reached.
- If the account is closed any collateral held by the Collateral Taker would be required to be returned to the originating party.
  - $\rightarrow$ We have $C_u = 0$ for all $u \leq t$ or $u \geq T$.
- We do not consider haircuts in the following.

# Margining Practice – III

- A realistic margining practice should allow for collateral posting only on a fixed time-grid ($t_0 = t, \ldots, t_N = T$), and for the presence of independent amounts ($A$), minimum transfer amounts ($M$), and thresholds ($H$), with $H \geq M$.
- Independent amounts represent a further insurance on the transaction and they are often posted as an upfront protection, but they may be updated according to exposure changes. We do not consider them in the following.
- Thresholds represent the amount of permitted unsecured risk, so that they may depend on the credit quality of the counterparties.
- Moving thresholds depending on a deterioration of the credit quality of the counterparties (downgrade triggers) have been a source of liquidity strain during the market crisis.

# Margining Practice – IV

- At each collateral posting date $t_i$, the collateral account is updated according to changes in exposure, otherwise producing an unsecured risk.

1. First, we consider how much collateral the investor should post to or withdraw from the collateral account:

$$1_{\{|(\varepsilon_{t_i}+H_I)^- - C^-_{t_i^-}|>M\}}((\varepsilon_{t_i}+H_I)^- - C^-_{t_i^-})$$

2. Then, we consider how much collateral the counterparty should post to or withdraw from the collateral account:

$$1_{\{|(\varepsilon_{t_i}-H_C)^+ - C^+_{t_i^-}|>M\}}((\varepsilon_{t_i}-H_C)^+ - C^+_{t_i^-})$$

# Margining Practice – V

- By adding the two terms we get how the collateral account is updated during the life of the transaction

$$C_{t_0} := 0\ , \quad C_{t_N^+} := 0\ , \quad C_{u^-} := \frac{C_{\beta(u)^+}}{D(\beta(u),u)}$$

$$\begin{aligned} C_{t_i^+} := C_{t_i^-} & \\ & + 1_{\{|(\varepsilon_{t_i}+H_I)^- - C^-_{t_i^-}|>M\}}((\varepsilon_{t_i}+H_I)^- - C^-_{t_i^-}) \\ & + 1_{\{|(\varepsilon_{t_i}-H_C)^+ - C^+_{t_i^-}|>M\}}((\varepsilon_{t_i}-H_C)^+ - C^+_{t_i^-}) \end{aligned}$$

where $\beta(u)$ is the last update time before $u$, and $t_0 < u \leq t_N$.

- In case of no thresholds ($H_I = H_C = 0$) and no minimum transfer amount ($M = 0$), we obtain a simpler rule

$$C_{t_0} = C_{t_N^+} = 0\ , \quad C_{t^-} = \frac{\varepsilon_{\beta(u)}}{D(\beta(u),u)}\ , \quad C_{t_i^+} = \varepsilon_{t_i}$$

# Close-Out Netting Rules – I

*The effect of close-out netting is to provide for a single net payment requirement in respect of all the transactions that are being terminated, rather than multiple payments between the parties. Under the applicable accounting rules and capital requirements of many jurisdictions, the availability of close-out netting allows parties to an ISDA Master Agreement to account for transactions thereunder on a net basis.* [ISDA, Coll. Review, 2.1.1]

- The occurrence of an event of default gives the parties the right to terminate all transactions that are concluded under the relevant ISDA Master Agreement.
- The ISDA Master Agreement provides for the mechanism of close-out netting to be enforced.

# Close-Out Netting Rules – II

*The Secured Party will transfer to the Pledgor any proceeds and posted credit support remaining after liquidation and/or set-off after satisfaction in full of all amounts payable by the Pledgor with respect to any obligations; the Pledgor in all events will remain liable for any amounts remaining unpaid after any liquidation and/or set-off.* [ISDA, CSA Annex, 8]

- In case of default of one party, the surviving party should evaluate the transactions just terminated, due to the default event occurrence, to claim for a reimbursement after the application of netting rules to consolidate the transactions, inclusive of collateral accounts.
  - → The ISDA Master Agreement defines the term *close-out amount* to be the amount of the losses or costs of the surviving party would incur in replacing or in providing for an economic equivalent.

# Close-Out Netting Rules – III

- Notice that the close-out amount is not a symmetric quantity w.r.t. the exchange of the role of two parties, since it is valued by one party after the default of the other one.
- Instead of the close-out amount we introduce the ”on-default exposure”, namely the price of the replacing transaction or of its economic equivalent.
- We name the on-default exposure priced at time $t$ by the investor on counterparty's default with $\varepsilon_{I,t}$ (and $\varepsilon_{C,t}$ in the other case, namely when the investor is defaulting). Notice that we always consider all prices from the point of view of the investor. Thus,
  - $\rightarrow$ a positive value for $\varepsilon_{I,t}$ means the investor is a creditor of the counterparty, while
  - $\rightarrow$ a negative value for $\varepsilon_{C,t}$ means the counterparty is a creditor of the investor.

# Cash Flows on Counterparty Default Event – I

- We start by listing all the situations may arise on counterparty default event. The case of the investor's default event will be derived accordingly.
- Our goal is to calculate the present value of all cash flows involved by the contract by taking into account:
  - $\rightarrow$ collateral margining operations, and
  - $\rightarrow$ close-out netting rules in case of default.
- Notice that we can safely aggregate the cash flows of the contract with the ones of the collateral account, since on contract termination all the posted collateral are returned to the originating party.
- We introduce the (first) default time $\tau := \min\{\tau_C, \tau_I\}$.

# Cash Flows on Counterparty Default Event – II

1. The investor measures a positive (on-default) exposure on counterparty default ($\varepsilon_{I,\tau_C} > 0$), and some collateral posted by the counterparty is available ($C_{\tau_C} > 0$).
   - $\rightarrow$ Then, the exposure is reduced by netting, and the remaining collateral (if any) is returned to the counterparty. If the collateral is not enough, the investor suffers a loss for the remaining exposure.

$$1_{\{\tau=\tau_C<T\}} 1_{\{\varepsilon_{I,\tau}>0\}} 1_{\{C_\tau>0\}} (\mathrm{Rec}_C(\varepsilon_{I,\tau} - C_\tau)^+ + (\varepsilon_{I,\tau} - C_\tau)^-)$$

# Cash Flows on Counterparty Default Event – III

2. The investor measures a positive (on-default) exposure on counterparty default ($\varepsilon_{I,\tau_C} > 0$), and some collateral posted by the investor is available ($C_{\tau_C} < 0$).
   - $\rightarrow$ Then, the investor suffers a loss for the whole exposure. All the collateral (if any) is returned to the investor if it is not re-hypothecated, otherwise an unsecured claim is needed.

$$1_{\{\tau=\tau_C<T\}} 1_{\{\varepsilon_{I,\tau}>0\}} 1_{\{C_\tau<0\}} (\mathrm{Rec}_C \varepsilon_{I,\tau} - \mathrm{Rec}'_C C_\tau)$$

# Cash Flows on Counterparty Default Event – IV

3. The investor measures a negative (on-default) exposure on counterparty default ($\varepsilon_{I,\tau_C} < 0$), and some collateral posted by the counterparty is available ($C_{\tau_C} > 0$).
   - → Then, the exposure is paid to the counterparty, and the counterparty gets back its collateral in full.

$$1_{\{\tau=\tau_C<T\}} 1_{\{\varepsilon_{I,\tau}<0\}} 1_{\{C_\tau>0\}} (\varepsilon_{I,\tau} - C_\tau)$$

# Cash Flows on Counterparty Default Event – V

4. The investor measures a negative (on-default) exposure on counterparty default ($\varepsilon_{I,\tau_C} < 0$), and some collateral posted by the investor is available ($C_{\tau_C} < 0$).
   - $\rightarrow$ Then, the exposure is reduced by netting and paid to the counterparty. The investor gets back its remaining collateral (if any) in full if it is not re-hypothecated, otherwise an unsecured claim is needed for the part of collateral exceeding the exposure.

$$1_{\{\tau=\tau_C<T\}} 1_{\{\varepsilon_{I,\tau}<0\}} 1_{\{C_\tau<0\}} ((\varepsilon_{I,\tau} - C_\tau)^- + \mathrm{Rec}'_C (\varepsilon_{I,\tau} - C_\tau)^+)$$

# Aggregating Cash Flows – I

- Now, we can aggregate all these cash flows, along with cash flows coming from the default of the investor and the ones due in case of non-default, inclusive of the cash-flows of the collateral account.
- We obtain the cash flows coming from the default of the investor simply by reformulating the previous line of reasoning from the point of view of the counterparty.
- In the following equations we use the risk-free discount factor $D(t, T)$, which is implicitly used also in the definitions of the risk-free discounted payoff $\Pi(t, T)$, and in the accumulation curve used for the collateral account $C_t$.

# Aggregating Cash Flows – II

- We obtain by summing all the contributions

$$
\begin{aligned}
\bar{\Pi}(t, T; C) = \\
& 1_{\{\tau > T\}} \Pi(t, T) \\
& + 1_{\{\tau < T\}} (\Pi(t, \tau) + D(t, \tau) C_\tau) \\
& + 1_{\{\tau = \tau_C < T\}} D(t, \tau) 1_{\{\varepsilon_{I,\tau} < 0\}} 1_{\{C_\tau > 0\}} (\varepsilon_{I,\tau} - C_\tau) \\
& + 1_{\{\tau = \tau_C < T\}} D(t, \tau) 1_{\{\varepsilon_{I,\tau} < 0\}} 1_{\{C_\tau < 0\}} ((\varepsilon_{I,\tau} - C_\tau)^- + \mathrm{Rec}'_C (\varepsilon_{I,\tau} - C_\tau)^+) \\
& + 1_{\{\tau = \tau_C < T\}} D(t, \tau) 1_{\{\varepsilon_{I,\tau} > 0\}} 1_{\{C_\tau > 0\}} ((\varepsilon_{I,\tau} - C_\tau)^- + \mathrm{Rec}_C (\varepsilon_{I,\tau} - C_\tau)^+) \\
& + 1_{\{\tau = \tau_C < T\}} D(t, \tau) 1_{\{\varepsilon_{I,\tau} > 0\}} 1_{\{C_\tau < 0\}} (\mathrm{Rec}_C \varepsilon_{I,\tau} - \mathrm{Rec}'_C C_\tau) \\
& + 1_{\{\tau = \tau_I < T\}} D(t, \tau) 1_{\{\varepsilon_{C,\tau} > 0\}} 1_{\{C_\tau < 0\}} (\varepsilon_{C,\tau} - C_\tau) \\
& + 1_{\{\tau = \tau_I < T\}} D(t, \tau) 1_{\{\varepsilon_{C,\tau} > 0\}} 1_{\{C_\tau > 0\}} ((\varepsilon_{C,\tau} - C_\tau)^+ + \mathrm{Rec}'_I (\varepsilon_{C,\tau} - C_\tau)^-) \\
& + 1_{\{\tau = \tau_I < T\}} D(t, \tau) 1_{\{\varepsilon_{C,\tau} < 0\}} 1_{\{C_\tau < 0\}} ((\varepsilon_{C,\tau} - C_\tau)^+ + \mathrm{Rec}_I (\varepsilon_{C,\tau} - C_\tau)^-) \\
& + 1_{\{\tau = \tau_I < T\}} D(t, \tau) 1_{\{\varepsilon_{C,\tau} < 0\}} 1_{\{C_\tau > 0\}} (\mathrm{Rec}_I \varepsilon_{C,\tau} - \mathrm{Rec}'_I C_\tau)
\end{aligned}
$$

# Aggregating Cash Flows – III

- Hence, by a straightforward calculation we get

$$
\begin{aligned}
\bar{\Pi}(t, T; C) = \Pi(t, T) \\
& - 1_{\{\tau < T\}} D(t, \tau) \left( \Pi(\tau, T) - 1_{\{\tau = \tau_C\}} \varepsilon_{I,\tau} - 1_{\{\tau = \tau_I\}} \varepsilon_{C,\tau} \right) \\
& - 1_{\{\tau = \tau_C < T\}} D(t, \tau)(1 - \mathrm{REC}_C)(\varepsilon^+_{I,\tau} - C^+_\tau)^+ \\
& - 1_{\{\tau = \tau_C < T\}} D(t, \tau)(1 - \mathrm{REC}'_C)(\varepsilon^-_{I,\tau} - C^-_\tau)^+ \\
& - 1_{\{\tau = \tau_I < T\}} D(t, \tau)(1 - \mathrm{REC}_I)(\varepsilon^-_{C,\tau} - C^-_\tau)^- \\
& - 1_{\{\tau = \tau_I < T\}} D(t, \tau)(1 - \mathrm{REC}'_I)(\varepsilon^+_{C,\tau} - C^+_\tau)^-
\end{aligned}
$$

- Notice that the collateral account enters only as a term reducing the exposure of each party upon default of the other one, keeping into account which is the party who posted the collateral.

# Collateralized Bilateral CVA

- Now, by taking risk-neutral expectation of both sides of the above equation, and by plugging in the definition of mid-market exposure, we obtain the general expression for collateralized bilateral CVA.

$$
\begin{aligned}
\mathrm{CVA}(t, T; C) = & -\mathbb{E}_t\Big[ 1_{\{\tau<T\}} D(t,\tau) \big(\varepsilon_\tau - 1_{\{\tau=\tau_C\}}\varepsilon_{I,\tau} - 1_{\{\tau=\tau_I\}}\varepsilon_{C,\tau}\big) \Big] \\
& - \mathbb{E}_t\Big[ 1_{\{\tau=\tau_C<T\}} D(t,\tau) \mathrm{LGD}_C (\varepsilon^+_{I,\tau} - C^+_\tau)^+ \Big] \\
& - \mathbb{E}_t\Big[ 1_{\{\tau=\tau_C<T\}} D(t,\tau) \mathrm{LGD}'_C (\varepsilon^-_{I,\tau} - C^-_\tau)^+ \Big] \\
& - \mathbb{E}_t\Big[ 1_{\{\tau=\tau_I<T\}} D(t,\tau) \mathrm{LGD}_I (\varepsilon^-_{C,\tau} - C^-_\tau)^- \Big] \\
& - \mathbb{E}_t\Big[ 1_{\{\tau=\tau_I<T\}} D(t,\tau) \mathrm{LGD}'_I (\varepsilon^+_{C,\tau} - C^+_\tau)^- \Big]
\end{aligned}
$$

- Now, we need a recipe to calculate on-default exposures $\varepsilon_{I,\tau_C}$ and $\varepsilon_{C,\tau_I}$, that, in the practice, are approximated from today exposure corrected for haircuts or add-ons.

# Formulae for Collateralized Bilateral CVA – I

- We consider all the exposures being evaluated at mid-market, namely we consider:

$$\varepsilon_{I,t} \doteq \varepsilon_{C,t} \doteq \varepsilon_t$$

- Thus, in such case we obtain for collateralized bilateral CVA

$$\begin{aligned}
\mathrm{CVA}(t,T;C) = &-\mathbb{E}_t\big[\, 1_{\{\tau=\tau_C<T\}} D(t,\tau)\mathrm{LGD}_C(\varepsilon_\tau^+ - C_\tau^+)^+ \,\big] \\
&- \mathbb{E}_t\big[\, 1_{\{\tau=\tau_C<T\}} D(t,\tau)\mathrm{LGD}'_C(\varepsilon_\tau^- - C_\tau^-)^+ \,\big] \\
&- \mathbb{E}_t\big[\, 1_{\{\tau=\tau_I<T\}} D(t,\tau)\mathrm{LGD}_I(\varepsilon_\tau^- - C_\tau^-)^- \,\big] \\
&- \mathbb{E}_t\big[\, 1_{\{\tau=\tau_I<T\}} D(t,\tau)\mathrm{LGD}'_I(\varepsilon_\tau^+ - C_\tau^+)^- \,\big]
\end{aligned}$$

- After this section we show a possible way to relax such approximation.

# Formulae for Collateralized Bilateral CVA – II

- If collateral re-hypothecation is not allowed ($\mathrm{LGD}'_C \doteq \mathrm{LGD}'_I \doteq 0$) the above formula simplifies to

$$\begin{aligned}\mathrm{CVA}(t,T;C) = &-\mathbb{E}_t\left[\, 1_{\{\tau=\tau_C<T\}} D(t,\tau)\mathrm{LGD}_C(\varepsilon_\tau^+ - C_\tau^+)^+ \,\right] \\ &-\mathbb{E}_t\left[\, 1_{\{\tau=\tau_I<T\}} D(t,\tau)\mathrm{LGD}_I(\varepsilon_\tau^- - C_\tau^-)^- \,\right]\end{aligned} \tag{1}$$

- On the other hand, if re-hypothecation is allowed and the surviving party always faces the worst case ($\mathrm{LGD}'_C \doteq \mathrm{LGD}_C$ and $\mathrm{LGD}'_I \doteq \mathrm{LGD}_I$), we get

$$\begin{aligned}\mathrm{CVA}(t,T;C) = &-\mathbb{E}_t\left[\, 1_{\{\tau=\tau_C<T\}} D(t,\tau)\mathrm{LGD}_C(\varepsilon_\tau - C_\tau)^+ \,\right] \\ &-\mathbb{E}_t\left[\, 1_{\{\tau=\tau_I<T\}} D(t,\tau)\mathrm{LGD}_I(\varepsilon_\tau - C_\tau)^- \,\right]\end{aligned} \tag{2}$$

# Formulae for Collateralized Bilateral CVA – III

- If we remove collateralization ($C_t = 0$), we recover the result of Brigo and Capponi (2008), and used in Brigo, Pallavicini and Papatheodorou (2009).

$$\begin{aligned}\mathrm{CVA}^{\mathrm{BC}}(t,T) = &-\mathbb{E}_t\left[\,1_{\{\tau=\tau_C<T\}} D(t,\tau)\mathrm{LGD}_C \varepsilon_\tau^+\,\right] \\ &-\mathbb{E}_t\left[\,1_{\{\tau=\tau_I<T\}} D(t,\tau)\mathrm{LGD}_I \varepsilon_\tau^-\,\right]\end{aligned} \tag{3}$$

- If we remove collateralization ($C_t = 0$) and we consider a risk-free investor ($\tau_I \to \infty$), we recover the result of Brigo and Pallavicini (2007), but see also Canabarro and Duffie (2004).

$$\mathrm{CVA}^{\mathrm{BP}}(t,T) = -\mathbb{E}_t\left[\,1_{\{\tau_C<T\}} D(t,\tau_C)\mathrm{LGD}_C \varepsilon_{\tau_C}^+\,\right] \tag{4}$$

# An Example: Perfect Collateralization

- We consider, for this example, updating the collateral account continuously. We obtain the following (perfect) collateralization rule.

$$C_t^{\text{perfect}} := \varepsilon_t$$

- Thus, if we plug it into the collateralized bilateral CVA equation (with all exposure at mid-market), we get that all terms drop, as expected, leading to

$$\mathrm{C}_{\mathrm{VA}}(t, T; C^{\text{perfect}}) = 0$$

$$\mathbb{E}_t\left[\,\bar{\Pi}(t, T; C)\,\right] = \mathbb{E}_t[\,\Pi(t, T)\,] = \varepsilon_t = C_t^{\text{perfect}}$$

- Thus, the proper discount curve for pricing the deal is the collateral accrual curve (see also Fujii et al. (2010) or Piterbarg (2010)).

# Conclusions

- Counterparty Risk adds one level of optionality.
- Analysis including underlying asset/ counterparty default correlation requires a credit model.
- Highly specialized hybrid modeling framework.
- Accurate scenarios for wrong way risk.
- Outputs vary and can be very different from Basel II multipliers
- Bilateral CVA brings in symmetry but also paradoxical statements
- Inclusion of Collateral and netting rules is possible

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