March 18, 2011

# The stochastic intrinsic currency volatility framework

## FX basket option pricing

The Stochastic intrinsic currency volatility framework, commonly known as "SticVol", is an FX derivative pricing model that has been developed at RBS. One advantage of the SticVol model is that for derivatives involving several different currencies, the model can be simultaneously calibrated to the implied volatility curves of all currency pairs under consideration. This short article looks at the pricing differences between the SticVol model, and two Gaussian copula models for basket options involving eight currencies.

# 1 Introduction

In 2006 RBS introduced the concept of intrinsic currency values[1, 2]. The basic concept is to model spot FX rates $X_{ij}$ between currency $i$ and currency $j$ as ratios of the intrinsic currency values $X_i$ and $X_j$, so that

$$X_{ij} = \frac{X_i}{X_j} \quad . \tag{1}$$

The idea behind $(1)$ is that $X_i$ and $X_j$ represent the intrinsic value of the individual currencies $i$ and $j$. It is possible to give an interpretation to the intrinsic value of a currency (see e.g. [2]), and such an

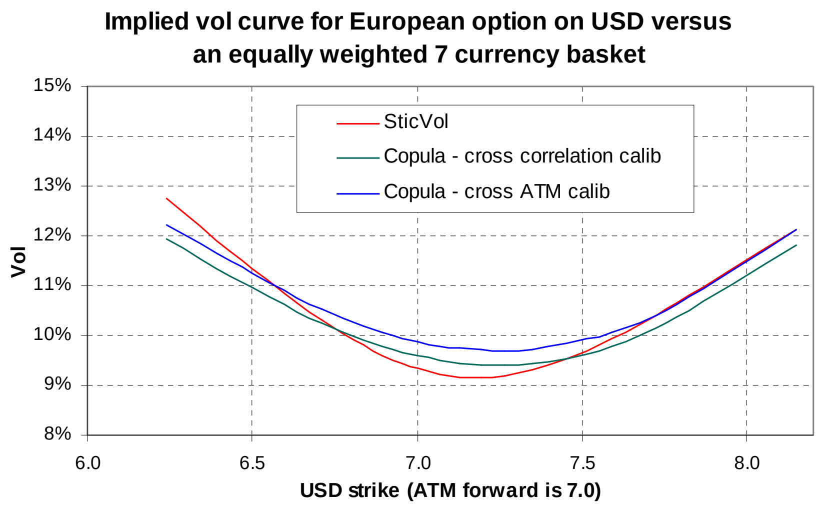


Figure 1: For a 3 month FX option to exchange USD versus an equally weighted basket of EUR, JPY, GBP, CHF, CAD, AUD and NZD, the SticVol model produces lower values for at-the-money options, and slightly higer values for deep out-of-the-money options.

**Paul Doust**
Head of Client
Quantitative Analysis
+44-20-7085-6015
paul.doust@rbs.com

**Quantitative Solutions**
+44-20-7085-9300
GBMQuantSolutions@rbs.com

**Links**
www.rbsm.com

interpretation can be useful to aid intuition. However, the work presented here does not hinge on the correctness of any particular interpretation. Note that it is not necessary to specify the units of the $X_i$ because they cancel out in the ratio[1]. Although the $X_i$ and their volatilities $\sigma_i$ are not directly observable, various methods can be used to distill information about them from the quantities that are observable in the FX market (see e.g. [3]). The convention where each $X_i$ increases when the currency it represents becomes more valuable is adopted. This implies that $X_{ij}$ is the number of units of currency $j$ corresponding to one unit of currency $i$. The traditional approach to modelling the FX market is to focus on the spot FX rates $X_{ij}$. However, the intrinsic currency approach has various advantages, because it can be used to create a more fundamental and symmetric description of the FX markets.

The SticVol model[4] extended the intrinsic currency framework into the derivative pricing arena. The idea is that each $X_i$ follows a log-normal stochastic process with volatility $\sigma_i$, where the intrinsic currency volatilities $\sigma_i$ are themselves stochastic variables. The model is similar to the SABR model [5] introduced by Hagan et al in the interest rate option markets. However, for products which involve more than two currencies it is not possible to apply SABR to all the spot FX rates $X_{ij}$ in a fully consistent way. In a situation where both $X_{ij}$ and $X_{jk}$ follow SABR stochastic processes, the inconsistency is that the process for $X_{ik}$ cannot also be of the SABR form. SticVol solves that problem by modelling the intrinsic currency variables $X_i$ instead.

One use for the SticVol model is in calculating the implied volatility curves for less liquid vanilla FX options. The idea here is to calibrate the model to the implied volatility curves which are available, which allows the intrinsic volatilities of each currency to be determined. The maximum information entropy concept can then be used to help determine the full correlation matrix between all the intrinsic currency values and their volatilities, from which the implied volatility curves for all currency pairs can be calculated. The results shown in [6] confirm that this methodology works well.

The use for the SticVol model that is investigated in this article is the pricing of derivative products that involve many currencies. A good example of this is a basket option, which gives the right to exchange an amount of one currency for a basket of other currencies on the option expiry date. Such products can be useful for companies that deal with a lot of currencies which are different from their main reporting currency.

The calibration methodology described in [4] enforces a positive definite covariance matrix between all the $X_i$ and the $\sigma_i$, so once the calibration is complete, it is straightforward to calculate the value of any product involving the chosen currencies using Monte-Carlo simulations. Because the calibration is to the implied volatility curves of all currency pairs, an entire FX option portfolio could be valued consistently from a single calibration to the SticVol model across all currencies in the portfolio. Other methodologies for producing valuations that are consistent with all the implied volatility curves for a large set of currencies frequently have problems relating to maintaining a positive definite covariance matrix in certain situations. For example, with a local volatility model the covariance matrix depends on the spot rates, and extreme spot rate values can end up breaking the positive definiteness. The standard SABR formula for implied volatilities, which can produce probability distributions which go negative for low strikes, shows how easy it is to create models of a single quantity which produce inconsistent results. These kind of problems are even worse when there are with multiple implied volatility curves for different currency pairs, which are related in subtle ways, so a consistent framework such as SticVol is valuable for building reliable models.

The layout of this short article is as follows. Section 2 describes the details of the FX basket option product that will be analysed in this article. Section 3 describes the two alternative models that will be used to value the product, in addition to the SticVol model, and section 4 compares the pricing results from the models. Section 5 is the conclusion. The appendix starting on page 7 is a brief summary of the SticVol model.

[1] However, using the intepretation of intrinsic currency values from [2], it is possible to understand the $X_i$ as measuring currency values in 'goods'. In other words, one can imagine there exists a universal basket of goods defined implicitly by the FX market, which can be used to measure the value of currencies all over the world.

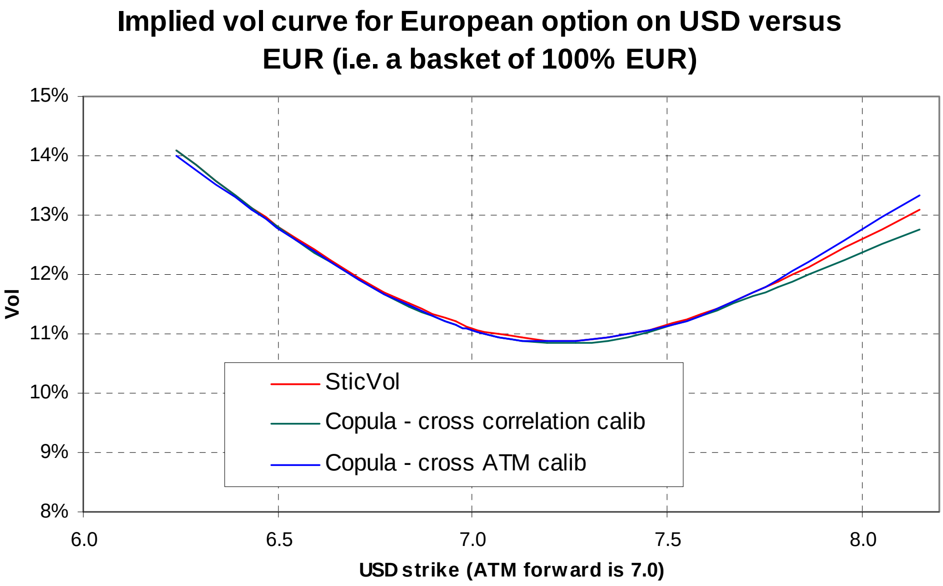


Figure 2: The EUR/USD implied volatility curves for all three models. This shows reasonably good agreement between all the models, although with some divergence for deep out-of-the-money EUR calls.

# 2 FX basket options

Given a basket containing fixed amounts $A_€$, $A_{JPY}$, $A_£$, $A_{CHF}$, $A_{CAD}$, $A_{AUD}$ , $A_{NZD}$ of EUR, JPY, GBP, CHF, CAD, AUD and NZD respectively, the value $B_\$$ of that basket in USD is given by

$$B_\$ = A_€ X_{€,\$} + \frac{A_{JPY}}{X_{\$,JPY}} + A_£ X_{£,\$} + \frac{A_{CHF}}{X_{\$,CHF}} + \frac{A_{CAD}}{X_{\$,CAD}} + A_{AUD} X_{AUD,\$} + A_{NZD} X_{NZD,\$} \tag{2}$$

where all the conventional market FX rates have been used to calculate $B_\$$. The reason that some of the exchange rates divide the amount, and others multiply the amounts, is that for some currencies (JPY, CHF, CAD) the convention is to quote the FX rate as the number of currency units per USD and for other currencies (EUR, GBP, AUD, NZD) the convention is to quote the FX rate as the number of USD for one non-USD currency unit. Given all the amounts $A_i$ which define the basket $B_\$$, the option under consideration is the right to exchange that basket for a fixed amount of USD, $K_\$$, i.e.

$$\text{USD value of call option on basket at expiry} = \max\left(B_\$ - K_\$, 0\right) \quad , \tag{3}$$

$$\text{USD value of put option on basket at expiry} = \max\left(K_\$ - B_\$, 0\right) \quad . \tag{4}$$

$K_\$$ can be thought of as the strike price of the option. To value this product, one needs a model which can handle all eight currencies simultaneously.

To illustrate pricing, the amounts $A_i$ were calculated so that at option expiry the forward value in USD of all $A_i$ were equal to the same amount of USD, which will be denoted by $A_\$$. Since there are 7 currencies, that means that the at-the-money strike $K_\$^{ATM}$ is $7A_\$$. The x-axis for all figures in this article is the USD strike $K_\$$, measured in units $A_\$$, so that the at-the-money point on the figures is $7.0$. Then regarding the basket $B_\$$ as a new currency, a convenient way to present the results is in terms of the implied volatility input required in the Black-Scholes vanilla FX option model to produce the correct option price.

# 3 Basket option models

The ideal models to price currency derivatives that involve a lot of currencies should be able to be calibrated to the implied volatility curves of all possible currency pairs involved. The basket option described in the previous

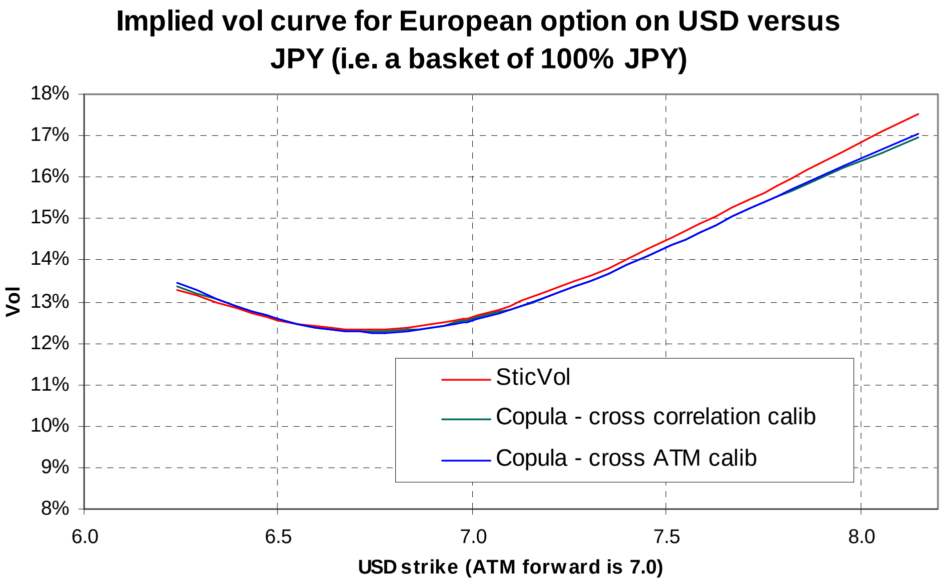


Figure 3: The JPY/USD implied volatility curves for all three models. This shows reasonably good agreement between all the models, with the SticVol model having slightly higher volatilities for JPY calls. Even though the SticVol implied volatiltiy curve is slightly higher here, in figures 1 and 4 the SticVol curves are generally lower than the implied volatility curves of the Gaussian copula models.

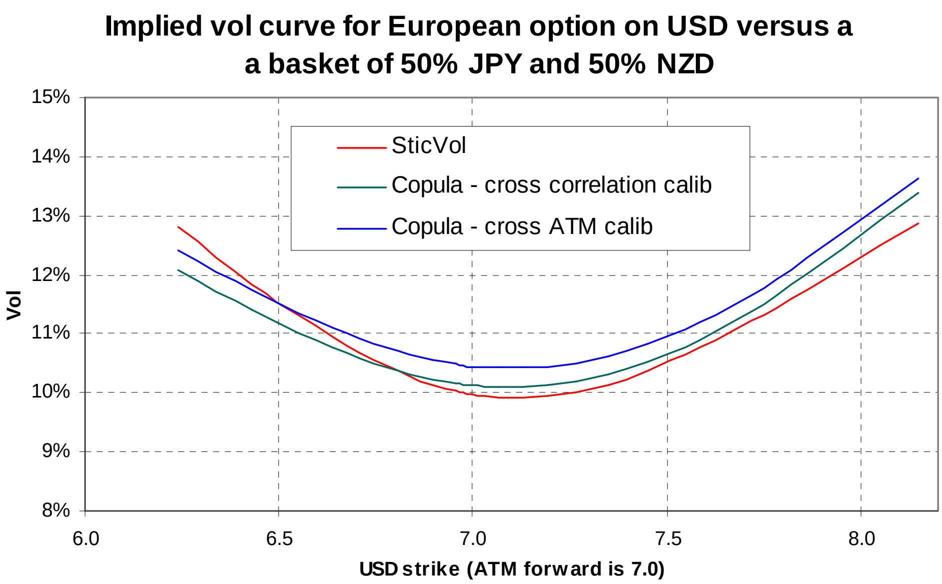


Figure 4: For a 3 month FX option to exchange USD versus an equally weighted basket of JPY and NZD, the SticVol model generally produces lower values than the two Gaussian copula models, except for deep out-of-the-money put options on the basket where the implied volatility is slightly higher with the SticVol model.

section involves 8 currencies, and for 8 currencies there are 8×7/2=28 possible pairs. Apart from SticVol, it is difficult to find models that can handle that many currency pairs at the same time. For example, when trying to calibrate simultaneously to as many as 28 currency pairs, some models have problems ensuring that the covariance matrix between all the FX rates remains positive definite in the tails of the probability distributions.

One model that can be used to price such products is a Gaussian copula model. A copula is a joint multivariate distribution where all the marginal distributions are uniform. It is a standard technique for stitching together univariate distributions into a multivariarte distribution where the joint behaviour is specified by the copula. In this case, the variables underlying the copula would be the 7 FX rates of the 7 non-USD currencies against the USD. The univariate distribution of each of those 7 FX rates is modelled exactly, which means that the corresponding implied volatility curves are fitted exactly. The copula parameters, which in this case are the 21 free parameters of a 7×7 correlation matrix, can then be determined by reference to each of the other 21 currency pairs. The two variations of this model will be used are as follows:

- Cross correlation calibration: here, each correlation $\rho_{XY}$ in the 7×7 correlation matrix is set using the currency triangle formula
$$\rho_{XY} = \frac{\sigma_{X\$}^2 + \sigma_{Y\$}^2 - \sigma_{XY}^2}{2\sigma_{X\$}\sigma_{Y\$}} \quad , \tag{5}$$
where $\sigma_{X\$}$, $\sigma_{Y\$}$ and $\sigma_{XY}$ are the at-the-money volatilities of the currency pairs $X$-\$, $Y$-\$ and $X$-$Y$ respectively.
- Cross ATM calibration: here, the correlation $\rho_{XY}$ is set so that the model implies the correct at-the-money volatility for each currency pair $X$-$Y$.

Although the Gaussian copula model can't calibrate to the full implied volatility curves between all 28 currency pairs, by calibrating to the full implied volatility curves for the 7 non-USD currencies against the USD it is calibrating to the most important currency pairs involved in the product.

# 4 Basket option model comparison

Market data from 2-Feb-2010 was used to compare the SticVol model with the two variations of the Gaussian copula model that were discussed in the previous section. However, the market data used here was slightly different from the market data for 2-Feb-2010 which was shown in detail in table 1 in the SticVol article [4]. The procedure adopted here was to take the SticVol parameters ($\sigma_i$, $\upsilon_i$, $\boldsymbol{\rho}$, $\tilde{\boldsymbol{\rho}}$, $\mathbf{r}$) which were output from the calibration on 2-Feb-2010 (shown in tables 3 and 4 in [4]), and then use SticVol Monte Carlo simulations to recalculate the at-the-money volatilities, risk-reversals and smile strangles. The results of doing this are shown in table 1. The data in table 1 was then used to calibrate the Gaussian copula models. This procedure eliminated calibration discrepancies between the SticVol and Gaussian copula models, as illustrated in figures 2 and 3 which show that the volatility curves for EUR/USD and JPY/USD and in reasonably good agreement between all the models.

Two basket options were considered, namely an equally forward weighted basket between all 7 non-USD currencies, and a smaller basket consisting of equally forward weighted amounts of just JPY and NZD. The reason for choosing JPY and NZD is because the NZD/JPY currency pair has one of the biggest implied volatility skews in the market. Figure 1 on the front page shows the different implied volatility curves for the equally weighted basket, and figure 4 shows the curves for the JPY and NZD basket. All SticVol prices were calculated using 1,000,000 Monte-Carlo paths, which took around 20 seconds to run on a workstation capable of running 12 threads simultaneously.

The general conclusion is that the SticVol model produces slightly lower at-the-money basket option prices, even though it's calibrated to the same data as the Gaussian copula models. Furthermore, the deep out-of-the-money option prices given by SticVol can be higher, especially for puts on the basket. Because the SticVol model is calibrated to the implied volatility curves for all 28 currency pairs, the SticVol price is likely to be more reliable.

# 5 Conclusion

For pricing FX option products which involve more than two currencies, it is important to have a model which calibrates to the implied volatility curves of all possible currency pairs between the currencies in a consistent way. The SticVol model is capable of doing that, however as the number of currencies rises, it becomes increasingly hard to find other models which can do that reliably. One problem that models can have is that with many currency pairs the tails of the distributions of different currency pairs, as specified by smile models like SABR, can be inconsistent with each other. If that happens, then those models are liable to fail.

To illustrate using SticVol for derivative pricing, a basket option involving 8 currencies was considered. The SticVol model produced lower values for at-the-money options when compared to Gaussian copula models. However, because the SticVol model was calibrated to all currency pairs, the valuation should be more reliable than those kinds of models which only calibrate to a subset of the market data.

| | Spot | Forward | Domestic rate | Foreign rate | ATM vol | 15$\delta$ Put Strike | 15$\delta$ Put vol | 15$\delta$ Call Strike | 15$\delta$ Call vol |
|---|---|---|---|---|---|---|---|---|---|
| EURUSD | 1.3948 | 1.3945 | 0.34% | 0.25% | 11.08% | 1.3074 | 12.52% | 1.4772 | 10.99% |
| GBPUSD | 1.5941 | 1.5930 | 0.52% | 0.25% | 11.44% | 1.4899 | 13.01% | 1.6895 | 11.22% |
| AUDUSD | 0.8824 | 0.8741 | 4.01% | 0.26% | 14.66% | 0.8008 | 17.18% | 0.9392 | 13.74% |
| NZDUSD | 0.7081 | 0.7036 | 2.80% | 0.26% | 15.31% | 0.6422 | 17.87% | 0.7587 | 14.40% |
| USDJPY | 90.6200 | 90.5871 | 0.25% | 0.10% | 12.62% | 84.0264 | 14.63% | 96.7091 | 12.45% |
| EURJPY | 126.3968 | 126.3196 | 0.34% | 0.10% | 13.99% | 115.8512 | 16.86% | 135.2154 | 12.95% |
| GBPJPY | 144.4535 | 144.3044 | 0.52% | 0.11% | 15.73% | 131.2499 | 18.52% | 155.8343 | 14.61% |
| CHFJPY | 85.8207 | 85.8351 | 0.04% | 0.11% | 13.48% | 78.9600 | 16.26% | 91.7100 | 12.60% |
| AUDJPY | 79.9631 | 79.1830 | 4.01% | 0.12% | 19.57% | 70.3180 | 23.41% | 86.8526 | 17.63% |
| CADJPY | 85.7251 | 85.6888 | 0.27% | 0.10% | 16.64% | 77.6092 | 19.34% | 93.1247 | 15.80% |
| NZDJPY | 64.1680 | 63.7343 | 2.80% | 0.11% | 19.94% | 56.4610 | 23.85% | 70.0542 | 17.99% |
| EURGBP | 0.8750 | 0.8754 | 0.34% | 0.51% | 10.02% | 0.8304 | 10.21% | 0.9281 | 11.15% |
| USDCHF | 1.0559 | 1.0554 | 0.25% | 0.04% | 10.64% | 0.9978 | 10.88% | 1.1227 | 11.78% |
| EURCHF | 1.4728 | 1.4717 | 0.34% | 0.04% | 4.03% | 1.4374 | 4.55% | 1.5027 | 4.00% |
| GBPCHF | 1.6832 | 1.6812 | 0.52% | 0.04% | 10.22% | 1.5850 | 11.44% | 1.7740 | 10.26% |
| AUDCHF | 0.9317 | 0.9225 | 4.01% | 0.05% | 11.47% | 0.8620 | 13.27% | 0.9778 | 11.16% |
| CADCHF | 0.9989 | 0.9983 | 0.27% | 0.04% | 11.14% | 0.9391 | 11.87% | 1.0625 | 11.87% |
| NZDCHF | 0.7477 | 0.7425 | 2.80% | 0.05% | 11.70% | 0.6923 | 13.66% | 0.7876 | 11.27% |
| USDCAD | 1.0571 | 1.0572 | 0.25% | 0.27% | 12.15% | 0.9925 | 12.25% | 1.1353 | 13.56% |
| EURCAD | 1.4744 | 1.4742 | 0.34% | 0.27% | 11.00% | 1.3866 | 11.89% | 1.5667 | 11.59% |
| GBPCAD | 1.6851 | 1.6841 | 0.52% | 0.27% | 12.26% | 1.5728 | 13.29% | 1.8006 | 12.74% |
| AUDCAD | 0.9328 | 0.9241 | 4.01% | 0.28% | 11.16% | 0.8651 | 12.89% | 0.9782 | 10.90% |
| NZDCAD | 0.7485 | 0.7438 | 2.80% | 0.28% | 11.55% | 0.6947 | 13.31% | 0.7891 | 11.31% |
| EURAUD | 1.5807 | 1.5953 | 0.34% | 3.99% | 11.04% | 1.5083 | 10.88% | 1.7063 | 12.80% |
| GBPAUD | 1.8065 | 1.8224 | 0.52% | 4.00% | 13.05% | 1.7022 | 13.27% | 1.9669 | 14.50% |
| EURNZD | 1.9698 | 1.9820 | 0.34% | 2.79% | 11.25% | 1.8731 | 10.96% | 2.1244 | 13.21% |
| GBPNZD | 2.2512 | 2.2642 | 0.52% | 2.80% | 13.48% | 2.1105 | 13.67% | 2.4503 | 15.01% |
| AUDNZD | 1.2462 | 1.2424 | 4.01% | 2.81% | 7.54% | 1.1917 | 8.11% | 1.2968 | 8.23% |

Table 1: Market data on 2-Feb-2010 for 3-month options on the 28 currency pairs that can be constructed from the 8 currencies USD, EUR, JPY, GBP, CHF, AUD, CAD, NZD. The interest rates are continuously compounding rates with day count 92/365.

# Appendix

## A : Summary of the SticVol model

Suppose that there are $N$ currencies, so there are $N$ intrinsic currency values $X_i$ and $N$ intrinsic currency volatilities $\sigma_i$ where $1 \leqslant i \leqslant N$. To model the value of any financial contract, a currency must be chosen for measuring value, so without loss of generality choose currency $k$ where $1 \leqslant k \leqslant N$. With this choice of valuation currency (also known as numéraire) and its associated risk-neutral measure, the SticVol stochastic processes are given by

$$\frac{dX_i}{X_i} = \left(\tilde{\lambda} - r_i + \rho_{ik}\sigma_i\sigma_k\right) dt + \sigma_i dW_i \tag{6}$$

and
$$\frac{d\sigma_i}{\sigma_i} = \tilde{\rho}_{ik}\upsilon_i\sigma_k dt + \upsilon_i dZ_i \quad . \tag{7}$$

Here, $\tilde{\lambda}$ is a variable which is the same for all the $X_i$, $r_i$ is the risk-free interest rate in currency $i$, $\upsilon_i$ is the volatility of $\sigma_i$, and $dW_i$ and $dZ_i$ are Weiner processes. Regarding $\rho_{ik}$ and $\tilde{\rho}_{ik}$, define the column vectors $\mathbf{dW}$ and $\mathbf{dZ}$ whose the elements are $dW_i$ and $dZ_i$. Then write the correlation matrix between the stochastic processes as

$$\begin{pmatrix} \mathbf{dW} \\ \mathbf{dZ} \end{pmatrix} \begin{pmatrix} \mathbf{dW}' & \mathbf{dZ}' \end{pmatrix} = \begin{pmatrix} \boldsymbol{\rho} & \tilde{\boldsymbol{\rho}}' \\ \tilde{\boldsymbol{\rho}} & \mathbf{r} \end{pmatrix} dt \quad , \tag{8}$$

where $'$ denotes matrix and vector transpose. This defines $\rho_{ik}$ and $\tilde{\rho}_{ik}$. This framework produces the usual risk-neutral processes for all the FX rates $X_{ij} = X_i/X_j$, as well as having the right symmetries in terms of change of numéraire currency.

Looking at $(6)$-$(8)$, the stochastic intrinsic currency volatility framework has the following variables:

- $N$ intrinsic currency volatilities $\sigma_i$. These variables were present in the original work[1], however now they are stochastic quantities.
- An $N \times N$ symmetric matrix $\boldsymbol{\rho}$ of correlations between the $N$ intrinsic currency values $X_i$. Again, these correlations were present in the original work.
- $N$ volatility of volatility variables $\upsilon_i$. It turns out that there is a significant tenor dependency to the $\upsilon_i$ in the FX option market, so that the 1 month $\upsilon_i$ variables are typically around 200%-230%, with the 1 year variables around 65%-85%.
- An $N \times N$ symmetric matrix of correlations $\mathbf{r}$ between the $N$ intrinsic currency volatilities $\sigma_i$. These are typically all positive, because when there is increased or decreased uncertainty in the market in connection with one currency, that means that there is likely to be increased or decreased uncertainties in the market in connection with other currencies too.
- An $N \times N$ matrix of $\tilde{\boldsymbol{\rho}}$ between all the intrinsic currency values $X_i$ and all the intrinsic currency volatilities $\sigma_i$. These variables are closely connected with the risk reversals.

Given $(6)$-$(8)$, a SABR style formula for the implied volatility of an FX option can be derived. Details of the SABR style formula, together with a full discussion of the SticVol model can be found in [4].

# References

[1] Paul Doust, "The intrinsic currency valuation framework", *RISK magazine*, March 2007.

[2] Jian Chen and Paul Doust, "Estimating Intrinsic Currency Values", *RISK magazine*, July 2008.

[3] Jian Chen and Paul Doust, "Improving intrinsic currency analysis: using information entropy and beyond", *RBS research article*, 12th November 2009.

[4] Paul Doust and Jian Chen, "The stochastic intrinsic currency volatility framework: a consistent model of multiple forex rates and their volatilities", *RBS research article*, 22nd April 2010.

[5] P. S. Hagan, D. Kumar, A. S. Lesniewski, and D. E. Woodward, "Managing smile risk", *Wilmott Magazine*, pages 84–108, July 2002.

[6] Paul Doust and Jian Chen, "Pricing less liquid FX options via the intrinsic currency framework", *RBS research article*, 12th January 2011.