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Pricing less liquid FX options via the intrinsic currency framework

1 Introduction

The intrinsic currency valuation framework has been developed by RBS over the last 5 years. The major developments have been published in Doust [2007], Chen & Doust [2008] and Doust & Chen [2010]. Under this framework the value of a currency is modelled in its own right, rather than relative to another currency in a currency pair. This captures people's intuition when they talk about individual currencies strengthening or weakening, and also provides a new way to model and represent FX rates. In this framework, the observable FX rates are simply the ratios of the latent intrinsic currency variables.

The original work of Doust [2007] modelled the intrinsic currency variables using simple lognormal stochastic processes with constant or deterministic volatility. The more recent work by Doust & Chen [2010] extends this by using a lognormal process for the volatility, which results in a SABR-like stochastic volatility model, along the lines of

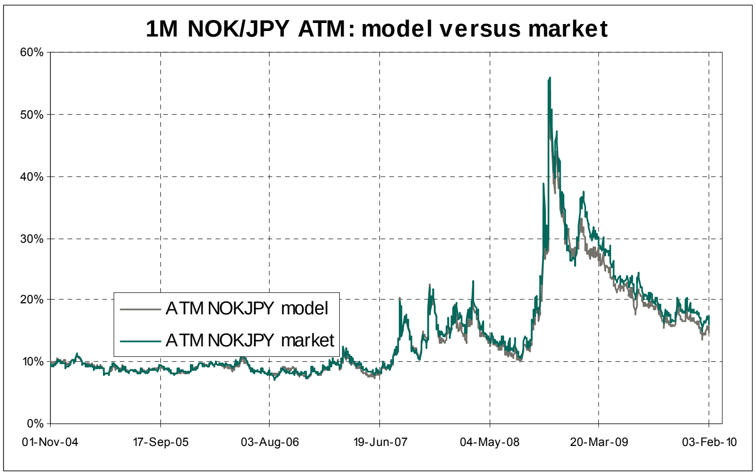

Figure 1: The 1M at-the-money volatilities of NOKJPY from the market, compared with values calculated using the SticVol model. The SticVol model was calibrated to EURNOK and USDNOK, plus all 28 currency pairs between USD, EUR, JPY, GBP, CHF, CAD, AUD and NZD. Model and market are in good agreement.

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Hagan *et al.* [July 2002]. This recent work on the intrinsic currency framework is known as the Stochastic Intrinsic Currency Volatility model, and is commonly referred to by the acronym "SticVol". Making the intrinsic currency volatilities stochastic is an important development, because it allows the framework to be calibrated to FX option smiles and skews. Although the original SABR model can be used to model individual FX volatility curves, the SticVol model has several advantages over SABR. In particular, it preserves the natural symmetries of the FX market relating to changes of measure associated with different numéraire currencies. Another good feature is that the model can simultaneously be calibrated to the volatility curves from many different currency pairs, whereas SABR is only able to model a single currency pair.

One application of the SticVol model is to calculate the volatility curves of less liquid currency pairs such as NOKJPY, SEKBRL, and so on. The purpose of this article is to show how to do that, given the limited availability of data relating to such currency pairs. Volatility curves predicted by the SticVol model will be compared to the historical curves held in the RBS trading systems to illustrate that where market data is available, the SticVol model is a good match to the market. An example of this, for the historical at-the-money NOKJPY volatility, is shown in figure 1.

This article is arranged as follows. Section 2 is a recap of the SticVol model. Section 3 shows how to calibrate the SticVol model to include data on less liquid currencies. Section 4 presents some illustrative results. Section 5 is the conclusion. Some technical details are contained in the appendices starting on page 11.

2 SticVol model summary

Suppose that there are N currencies, so there are N intrinsic currency values X_i and N intrinsic currency volatilities σ_i where $1 \leqslant i \leqslant N$. In this framework, FX rates are given by the ratios of the latent intrinsic currency variables, i.e.

$$X_{ij} = \frac{X_i}{X_j} \tag{1}$$

is the FX rate corresponding to the number of units of currency j which has the same value as 1 unit of currency i. To model the value of any financial contract, a currency must be chosen for measuring value, so without loss of generality choose currency k where $1 \leqslant k \leqslant N$. With this choice of valuation currency (also known as numéraire) and its associated risk-neutral measure, it was shown in Doust & Chen [2010] that the stochastic processes

$$\frac{dX_i}{X_i} = \left(\tilde{\lambda} - r_i + \rho_{ik}\sigma_i\sigma_k\right) dt + \sigma_i dW_i \tag{2}$$

and

$$\frac{d\sigma_i}{\sigma_i} = \tilde{\rho}_{ik}\upsilon_i\sigma_k dt + \upsilon_i dZ_i \tag{3}$$

produce the usual risk-neutral processes for all the FX rates X_{ij}, as well as having the right symmetries in terms of change of numéraire currency. Consequently, these processes for X_i are consistent with all the natural symmetries relating to the inverse and product operations that are possible with FX rates X_{ij}. In (2) and (3), $\tilde{\lambda}$ is a variable which is the same for all the X_i, r_i is the risk-free interest rate in currency i, υ_i is the volatility of σ_i, and dW_i and dZ_i are Wiener processes. Define the column vectors $\mathbf{dW}$ and $\mathbf{dZ}$ whose elements are dW_i and dZ_i. Then write the correlation matrix between the stochastic processes as

$$\boldsymbol{\varrho} dt = E\left[\begin{pmatrix} \mathbf{dW} \\ \mathbf{dZ} \end{pmatrix} \begin{pmatrix} \mathbf{dW}' & \mathbf{dZ}' \end{pmatrix}\right] = \begin{pmatrix} \boldsymbol{\rho} & \tilde{\boldsymbol{\rho}}' \\ \tilde{\boldsymbol{\rho}} & \mathbf{r} \end{pmatrix} dt \quad , \tag{4}$$

where $'$ denotes matrix and vector transpose.

Looking at (2)-(4), the stochastic intrinsic currency volatility framework has the following variables:

- N intrinsic currency volatilities σ_i. These variables were present in the original work Doust [2007], however now they are stochastic quantities.
- An $N \times N$ symmetric matrix $\boldsymbol{\rho}$ of correlations between the N intrinsic currency values X_i. Again, these correlations were present in the original work.

- N volatility of volatility variables υ_i. It turns out that there is a significant tenor dependency to the υ_i in the FX option market, so that the 1 month υ_i variables are typically around 200%-230%, with the 1 year variables around 65%-85%.
- An $N \times N$ symmetric matrix of correlations $\mathbf{r}$ between the N intrinsic currency volatilities σ_i. These are typically all positive, because when there is increased or decreased uncertainty in the market in connection with one currency, that means that there is likely to be increased or decreased uncertainties in the market in connection with other currencies too.
- An $N \times N$ matrix of $\tilde{\rho}$ between all the intrinsic currency values X_i and all the intrinsic currency volatilities σ_i. These variables are closely connected with the risk reversals.

Given the above model, Doust & Chen [2010] developed an approximation formula for the implied option volatility curve, which is given here in formula (8) in the appendix on page 11. For calculating the volatility curves of less liquid currency pairs, the idea is to use the approximation formula to calibrate the model to the volatility curves of currency pairs where data is available, and then use the approximation formula to calculate the volatility curves of all other currency pairs.

3 Calibrating intrinsic currency processes of less liquid currencies

The calibration procedure for FX options on major currencies was described in detail in Doust & Chen [2010], where the assumption was that the at-the-money volatility, the risk reversal and market strangle were available for all currency pairs. For example, for $N = 8$ major currencies the assumption was that data is available for all 28 currency pairs. However, quotes for less liquid currencies are often only reliable when against USD and EUR. For this reason, a two-step approach has been developed to get reliable calibrations of the intrinsic currency parameters when less liquid currencies are included in the calibration. In the first step, the model is calibrated to the 8 major currencies USD, EUR, JPY, GBP, CAD, CHF, AUD and NZD. This was described in Doust & Chen [2010] so no further explanation is required. In the second step, the parameters relating to the major currencies are fixed and the parameters relating to the less liquid currencies are calibrated using all available data for those currencies. As will be shown below, this methodology gives stable calibrations.

3.1 Parmeterising the correlation matrix ϱ

To make the calibration easier, the correlation matrix as specified in (4) is reordered into a matrix representing the correlations of the vector $(dW_1, dZ_1, dW_2, dZ_2, ..., dW_N, dZ_N)'$. This allows the correlations of major currencies that have already been determined to be plugged into the top-left corner of the full correlation matrix. As in Chen & Doust [2009] and Doust & Chen [2010], the maximum information entropy concept is used to help calibrate the unknown correlation parameters.

To describe in more detail how this all works, suppose that the correlation parameters between M major currencies have been fixed. Then as suggested above, suppose that for each of the remaining $N - M$ currencies only the volatility curves for the less liquid currencies against USD and EUR are available. Adopt the convention that the first currency in the vector $(dW_1, dZ_1, dW_2, dZ_2, ..., dW_N, dZ_N)'$ is USD and the second one is EUR, and decompose the full correlation matrix ϱ from (4) as

$$\varrho = \mathbf{C}\mathbf{C}' \tag{5}$$

where $\mathbf{C}$ is the Cholesky decomposition of ϱ. Then the first $2M$ rows of the $\mathbf{C}$ has been determined by the major currency correlation matrix, so only the last $2\,(N - M)$ rows of $\mathbf{C}$ need to be calibrated. However the market data relating to the less liquid currencies only have traction on the first 4 entries of each of the last $2\,(N - M)$ rows in $\mathbf{C}$, because those are the entries that relate to the correlations with USD and EUR. The other entries in those rows only affect the correlations in ϱ that relate to the currency pairs where there is no market data.

Then appendix B proves that maximum information entropy means that the remaining parameters in the bottom $2\,(N-M)$ rows of $\mathbf{C}$ must be zero, so that $\mathbf{C}$ must have the form

$$\mathbf{C}=\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ C_{21} & C_{22} & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ C_{31} & C_{32} & C_{33} & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ C_{41} & C_{42} & C_{43} & C_{44} & 0 & \cdots & 0 & 0 & \cdots & 0 \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ C_{(2M)1} & C_{(2M)2} & C_{(2M)3} & C_{(2M)4} & C_{(2M)5} & \cdots & C_{(2M)(2M)} & 0 & \cdots & 0 \\ C_{(2M+1)1} & C_{(2M+1)2} & C_{(2M+1)3} & C_{(2M+1)4} & 0 & \cdots & 0 & C_{(2M+1)(2M+1)} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ C_{(2N)1} & C_{(2N)2} & C_{(2N)3} & C_{(2N)4} & 0 & \cdots & 0 & 0 & \cdots & C_{(2N)(2N)} \end{pmatrix} . \tag{6}$$

Additionally, since all entries on the leading diagonal ϱ must be 1, the sums of squares of the non-zero C_{ij} across rows must equal 1. One way of arranging this is to parameterise each row using what Francesco Rapisarda [2007] call the Triangular Angle Parameterisation of Rebonato & Jäckel [1999]. These parameterisation techniques significantly reduce the number of correlation parameters that need to be determined and hence improve the stability of the calibration.

3.2 Data model

The data model for the 8 major currencies in Doust & Chen [2010] assumed that:

- The intrinsic volatility σ_i^T was both currency and tenor dependent;
- The volatility of volatility was tenor dependent but currency independent, so that given the tenor all currencies have the same υ^T; and
- The correlation matrix was tenor independent.

Since there was data at 5 tenors for 8 currencies, this meant that there were $5\times 8=40$ σ_i^T parameters, 5 υ^T parameters, and $16\times 15/2=120$ correlation parameters to calibrate.

For the less liquid currencies too, it will be assumed that σ_i^T is both currency and tenor dependent. However since the less-liquid currencies each have different characteristics, here it will be assumed that each tenor for each currency has it's own volatility of volatility υ_i^T, i.e. the currency independence assumption is being dropped for the less liquid currencies. However it will still be assumed that the correlation matrix is tenor independent.

3.3 Calibration procedure

As in Doust & Chen [2010], the idea is to minimise the difference between the market data and the model values, where the model values are calculated using formula (8) in the appendix. This is achieved by minimising the target function Z where Z is defined as the weighted sum of 5 terms, i.e.

$$Z=\omega_{ATM}\chi^2_{ATM}+\omega_{RR}\chi^2_{RR}+\omega_{MS}\chi^2_{MS}+\omega_H H+\omega_{r>0}Z_{r>0} \quad . \tag{7}$$

This is a simplified version of the target function Z which was used in Doust & Chen [2010]. The first three terms in (7) are the χ^2 statistics associated with the difference between market and model values of the at-the-money volatility, the risk reversal and the Market Strangle quotes. The fourth term is the information entropy of the correlation matrix, which helps stabilise the correlation matrix, and the fifth term is to make sure the correlations between the volatilities of volatilities are positive. All these terms were used and justified in Doust & Chen [2010]. The terms in (7) are formally defined by

$$\chi^2_{ATM}=\sum_{ij,T}\left(\frac{[\varepsilon_\sigma]^T_{ij}}{[\Delta_\sigma]^T_{ij}}\right)^2 \quad , \text{ where } T \text{ is tenor, } [\varepsilon_\sigma]^T_{ij}=\left[\sigma^{ATM}_{\text{Market}}\right]^T_{ij}-\left[\sigma^{ATM}_{B}\right]^T_{ij} \quad ,$$

$$\chi^2_{RR}=\sum_{ij,T}\left(\frac{[\varepsilon_{RR}]^T_{ij}}{[\Delta_{RR}]^T_{ij}}\right)^2 \quad , \text{ where } T \text{ is tenor, } [\varepsilon_{RR}]^T_{ij}=[RR_{\text{Market}}]^T_{ij}-[RR_{\text{Model}}]^T_{ij} \quad ,$$

$$\chi^2_{MS} = \sum_{ij,T} \left(\frac{[\varepsilon_{MS}]^T_{ij}}{[\Delta_{MS}]^T_{ij}} \right)^2 \quad , \quad \text{where } T \text{ is tenor, } [\varepsilon_{MS}]^T_{ij} = [MS_{\text{Market}}]^T_{ij} - [MS_{\text{Model}}]^T_{ij} \quad ,$$

$$H = -\ln \left(\det \begin{pmatrix} \boldsymbol{\rho} & \tilde{\boldsymbol{\rho}}' \\ \tilde{\boldsymbol{\rho}} & \mathbf{r} \end{pmatrix} \right) \quad ,$$

$$Z_{r>0} = \sum_{ij} \left(\psi_{\min}\left(r_{ij}, 0; 0.004 \right) \right)^2 \quad ,$$

$$\text{where } \psi_{\min}(x, y; \sigma) = xN\left(\frac{y-x}{\sigma}\right) + yN\left(\frac{x-y}{\sigma}\right) - \frac{\sigma}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-y}{\sigma}\right)^2 \right) \quad ,$$

and where Δ_σ, Δ_{RR} and Δ_{MS} are the error-bars associated with the error terms $[\varepsilon_\sigma]^T_{ij}$, $[\varepsilon_{RR}]^T_{ij}$ and $[\varepsilon_{MS}]^T_{ij}$, and are determined via the bid-offer spreads in the market.

4 Results

To illustrate the methodology, 8 major currencies were used namely USD, EUR, JPY, GBP, CHF, AUD, CAD and NZD, and the fixed values for all the corresponding intrinsic currency parameters were the ones calculated in Doust & Chen [2010]. To these 8 currencies, the 3 less liquid currencies SEK, NOK and BRL were added. On each day the data for the less liquid currencies consisted of quotes for the at-the-money volatility, the 25 delta risk reversal and 25 delta market strangle, for 5 tenors (1 month, 2 month, 3 month, 6 month and 1 year), and on 6 currency pairs, namely USDSEK, USDNOK, EURSEK, EURNOK, USDBRL and EURBRL. This means that there are $3 \times 5 \times 6 = 90$ market data points on each day, which were sourced from the RBS trading systems. The data covers the period from 1st November 2004 to 3rd February 2010. Given the data model described in section 3.2, and given the fact that some of the elements of matrix C in (6) are zero, this means that on each day $5 \times 3 + 5 \times 3 + 4 \times 6 = 54$ parameters need to be calibrated. The weights used in equation (7) were adjusted according to the magnitude of the corresponding terms. The values used here were $\omega_{ATM} = 10$, $\omega_{RR} = 30$, $\omega_{MS} = 2000$, $\omega_H = 500$, $\omega_{r>0} = 10000$, which were found to give stable results.

Figures 2 – 4 show how some of the parameters varied over time. Figure 2 shows the six ρ_{ij} between SEK, NOK and BRL versus USD and EUR, figure 3 shows υ^T_{BRL}, and figure 4 shows σ^T_{BRL}.

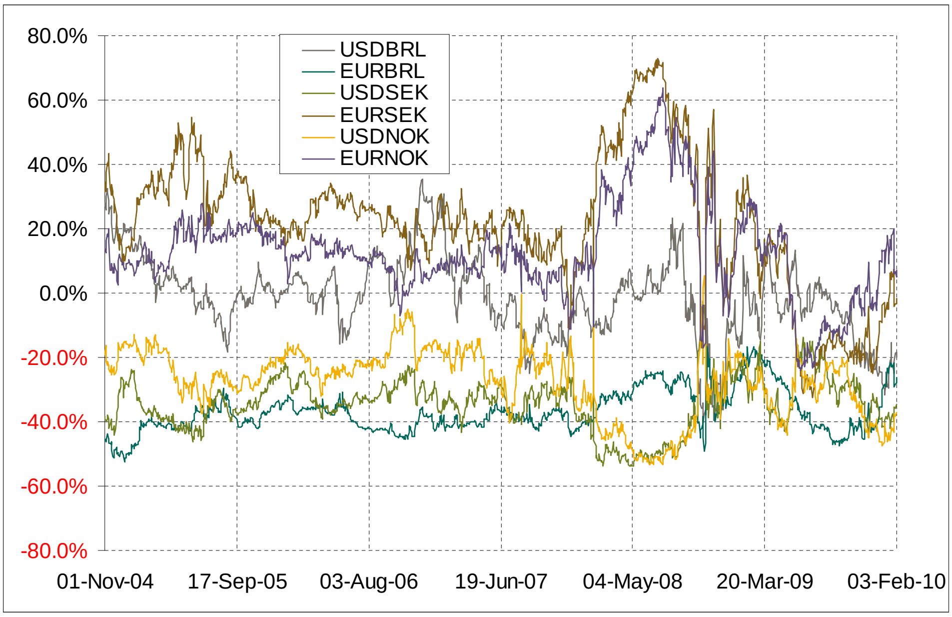

Figure 2: This graph shows the values of ρ_{ij} from (4) between USD, EUR and SEK, NOK, BRL.

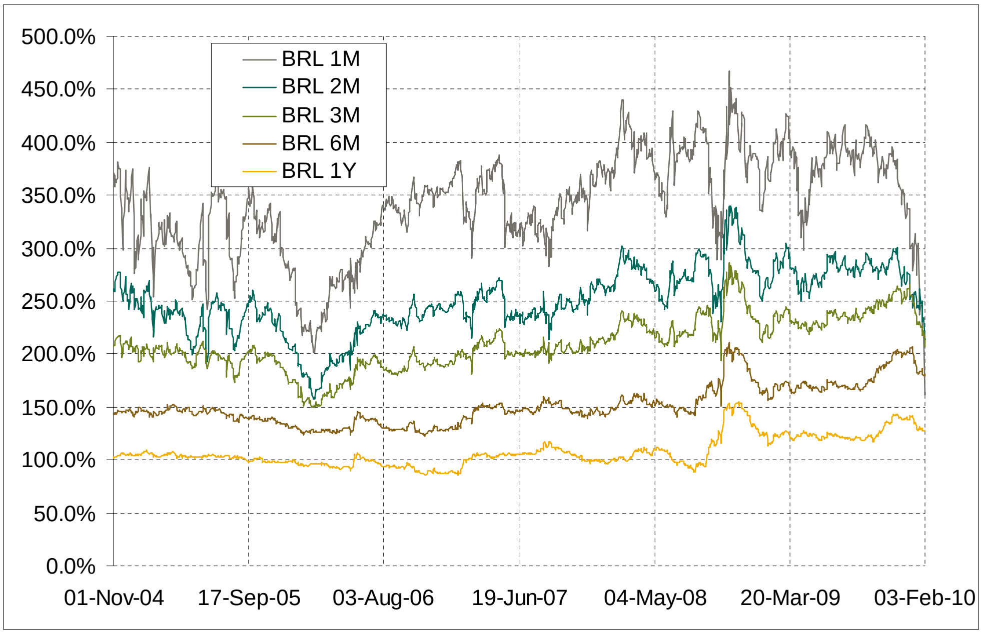

Figure 3: This graph shows υ^T_{BRL} for the different tenors. Because BRL is a volatile emerging market currency, it has very high volatility of volatility parameters compared with the values found for the eight major market currencies.

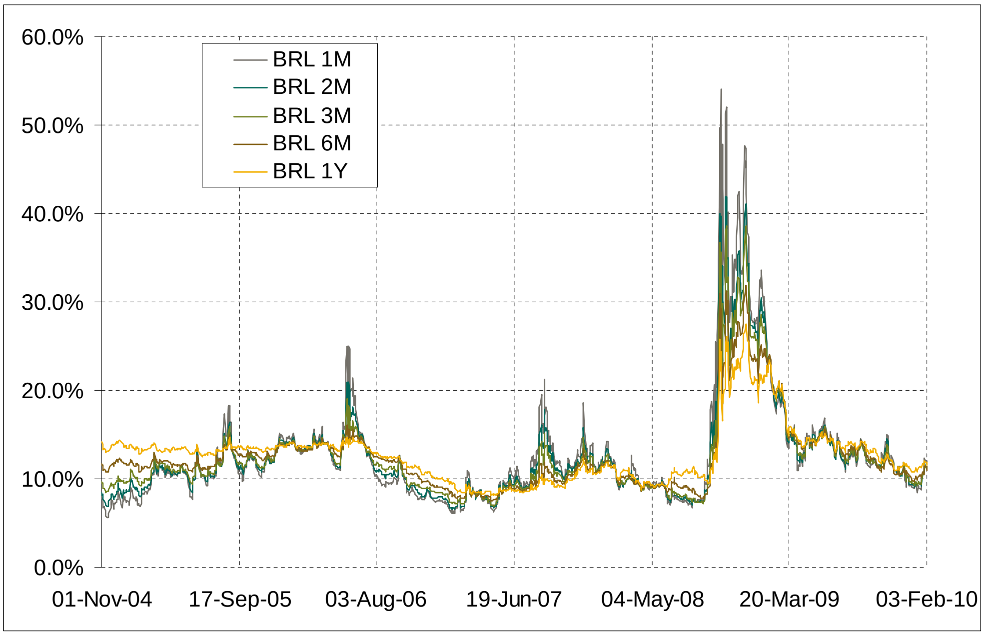

Figure 4: This graph shows the σ^T_{BRL}, the intrinsic volatilities of BRL for different tenors. Because BRL is a volatile emerging market currency, its volatilities have relatively high levels.

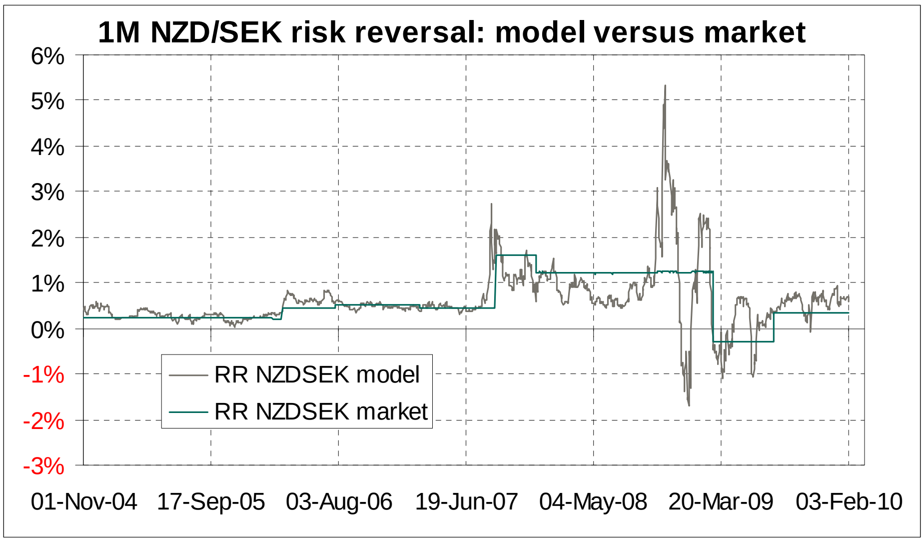

Figure 5: The model and market quotes of 1M risk reversal of NZDSEK. It seems implausible that the NZD/SEK was unchanged during the turmoil in late 2008, so in this example the model is probably doing a better job than the market data.

Once the calibration is done, the SticVol model can be used to imply the option volatilities of less liquid currency pairs such as SEKBRL. Figures 5 -12 show some illustrative comparisons between the model calculated values and market quotes. In all cases model and market lines match each other very well. However it is important to realise that the data for the market quotes may not be completely accurate since, by definition, reliable data for less liquid currency pairs is hard to find.

As an illustration of this, figure 5 shows the NZD/SEK risk reversal. The market data is clearly only updated sporadically, but nonetheless the model value usually reflects the general level of the market data. However there are clear differences during the market turmoil in late 2008, and it seems implausible that the NZD/SEK was unchanged during this period, so in this example the model is probably doing a better job than the data.

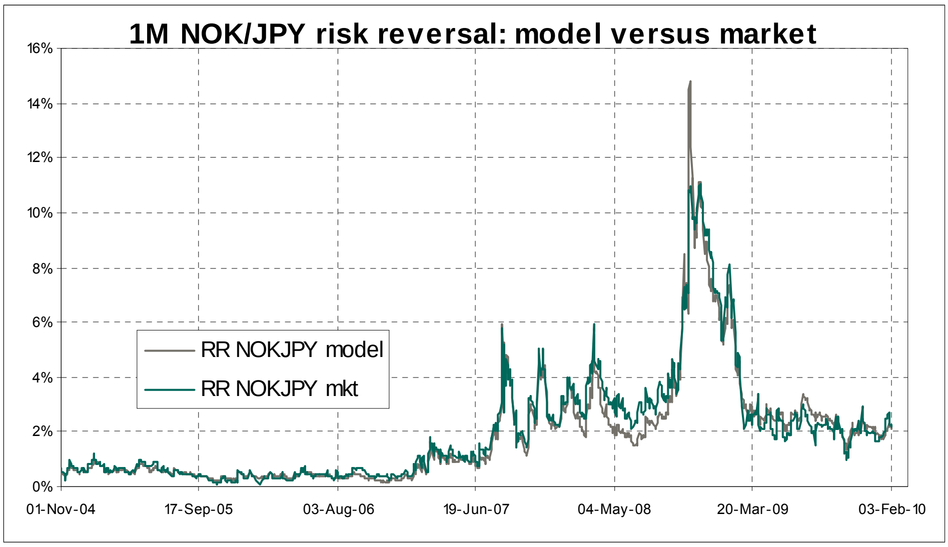

Figure 6: The model and market quotes of 1M risk reversal of NOKJPY.

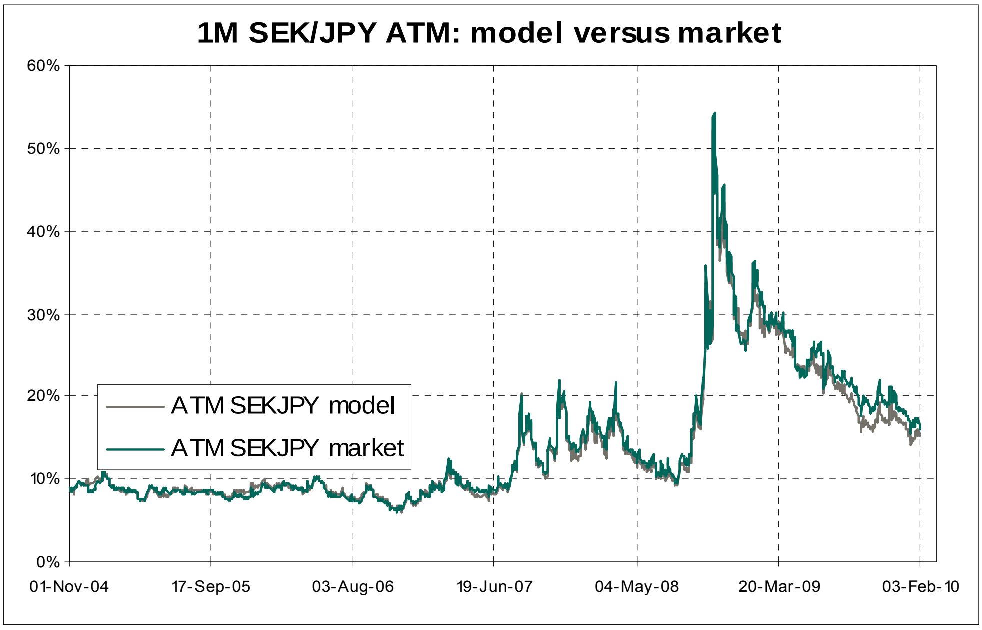

Figure 7: The model and market 1M at-the-money volatilities of SEKJPY.

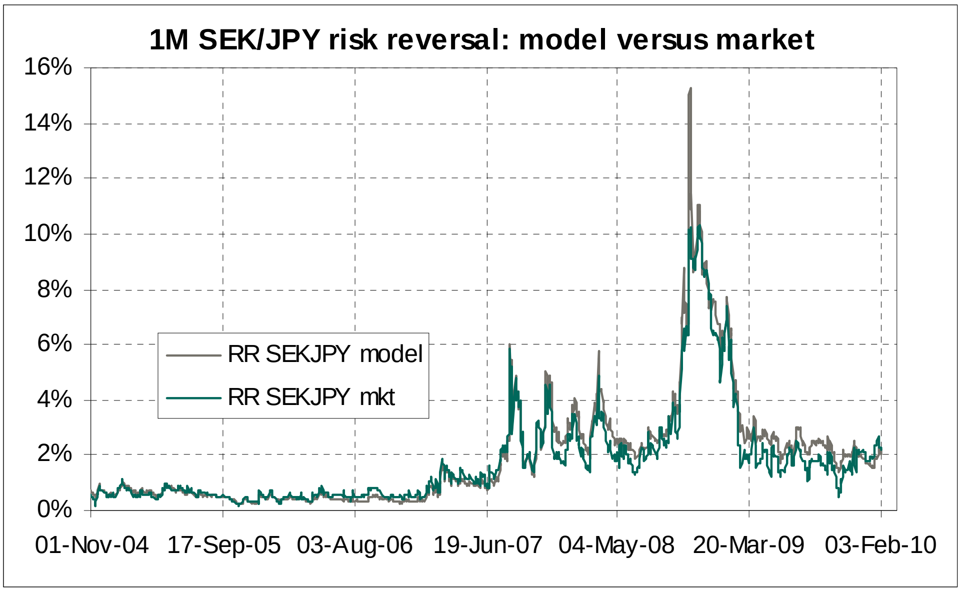

Figure 8: The model and market quotes of 1M risk reversal of SEKJPY.

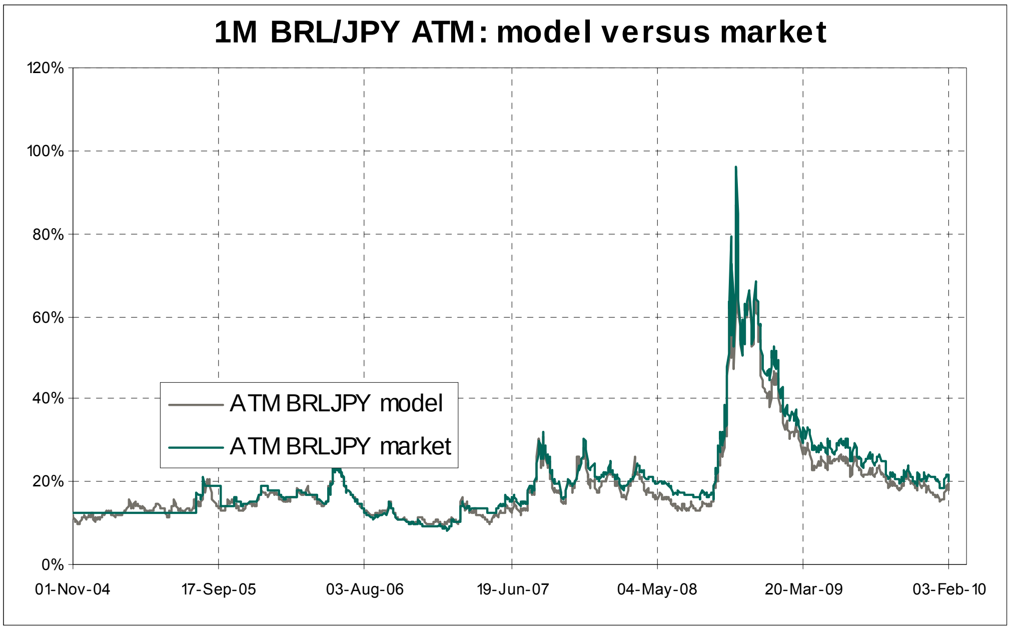

Figure 9: The model and market 1M at-the-money volatilities of BRLJPY.

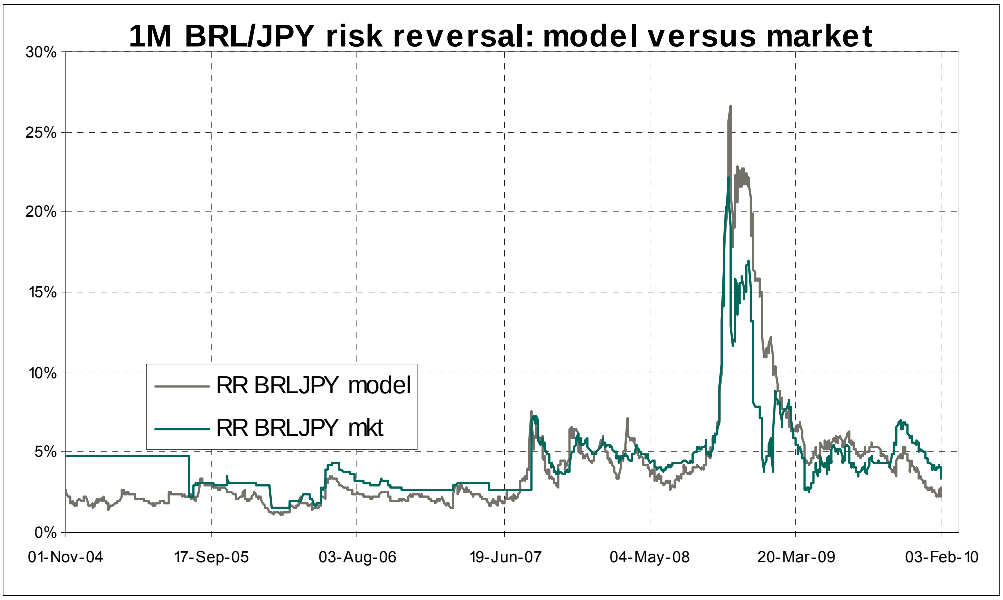

Figure 10: The model and market quotes of 1M risk reversal of BRLJPY.

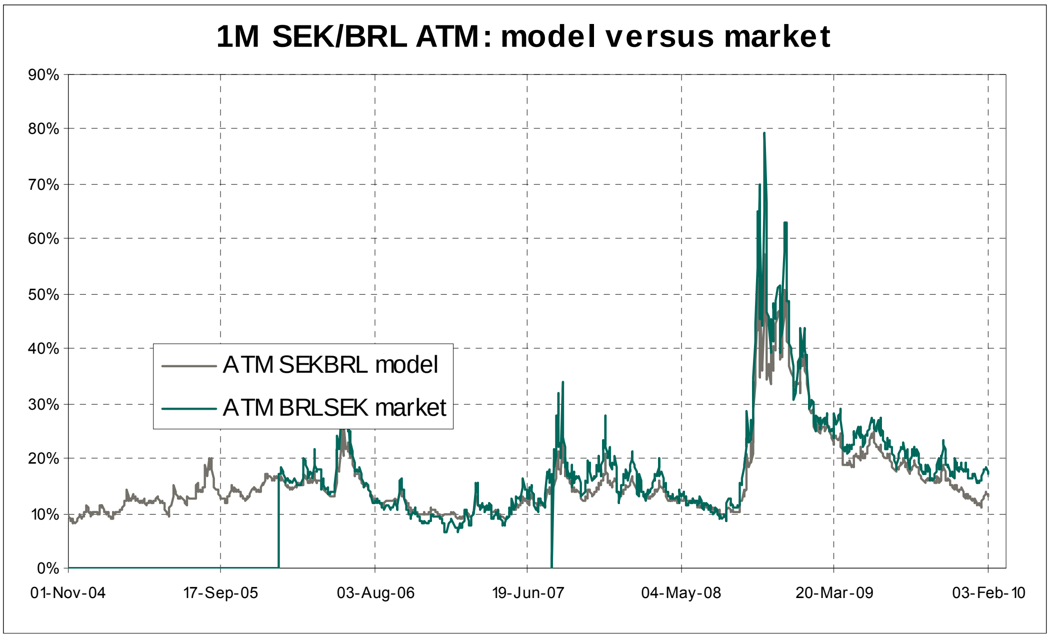

Figure 11: The model and market 1M at-the-money volatilities of SEKBRL.

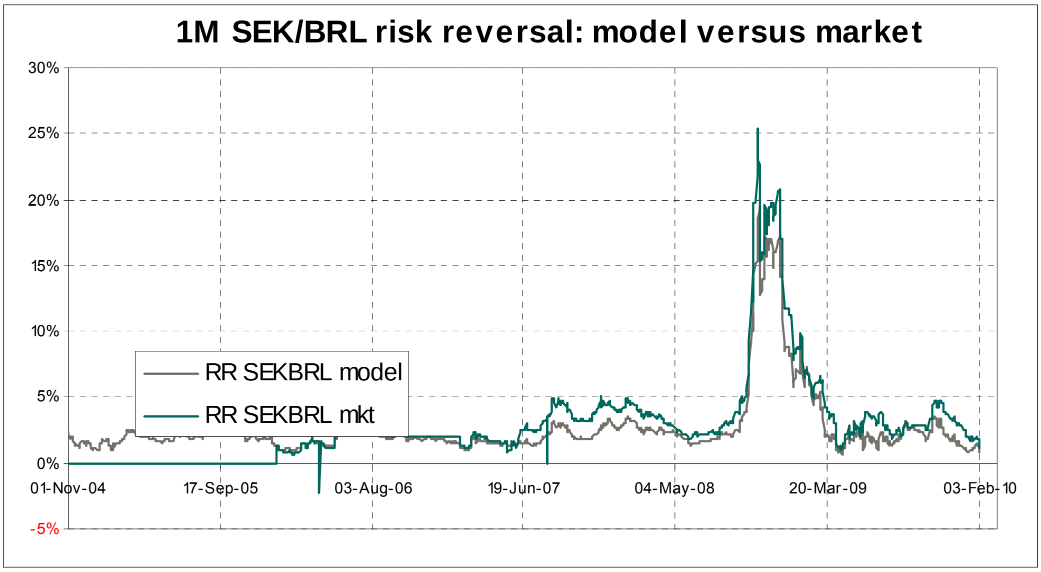

Figure 12: The model and market quotes of 1M risk reversal of SEKBRL.

5 Conclusions

This article shows how to use the SticVol model to price FX options involving less liquid currencies. The pricing of less liquid FX options is a difficult task because market quotes can be scarce. The intrinsic currency framework provides a unique solution to this problem, because it's able to put the data from all currency pairs into a single unified system, which utilises the market information more efficiently. The maximum information entropy principle, which ensures minimally prejudiced probability distributions, is then able to fill in the missing data. The last section shows that a stable calibration of intrinsic currency processes for less liquid currencies can be achieved by just providing quotes for the currencies against USD and EUR. The values that the model produced were good matches to the corresponding market data.

Appendix

A : The approximation formula for implied FX volatilities in the SticVol model

One of the main results in Doust & Chen [2010] is an approximation formula for the vanilla FX option volatility σ_B in terms of the parameters of the stochastic intrinsic currency volatility framework. To write down this formula, let X_{ij}^F be the forward rate of the spot FX rate X_{ij}, and use K to denote the strike of a FX option, where K has the same units as X_{ij}^F. Then the implied volatility approximation formula for the stochastic intrinsic currency volatility model defined by (2)-(4) is given by

$$\sigma_B = \sigma_{ij} \frac{z}{x(z)} \frac{1}{\sqrt{1 - \frac{1}{12}\left(8\kappa_1 + \sigma_{ij}^2 \left(\frac{z}{x(z)}\right)^2\right)\tau_{ex}}} \quad , \tag{8}$$

where

$$z = \frac{1}{\sigma_{ij}} \ln\left(\frac{X_{ij}^F}{K}\right) \tag{9}$$

$$\sigma_{ij} = \sigma(\sigma_i, \sigma_j) = \sqrt{\sigma_i^2 - 2\rho_{ij}\sigma_i\sigma_j + \sigma_j^2} \tag{10}$$

$$\begin{aligned} x(z, \sigma_i, \sigma_j) &= \int_0^z \frac{d\zeta}{J(\xi, \sigma_i, \sigma_j)} \\ &= \frac{1}{\sqrt{a_2 + a_5}} \ln\left(\frac{\sqrt{1 - 2a_1 z + (a_2 + a_5) z^2} - \frac{a_1}{\sqrt{a_2+a_5}} + \sqrt{a_2 + a_5} z}{1 - \frac{a_1}{\sqrt{a_2+a_5}}}\right) \quad , \end{aligned} \tag{11}$$

$$J(z, \sigma_i, \sigma_j) = \sqrt{1 - 2a_1 z + (a_2 + a_5) z^2} \quad , \tag{12}$$

$$\kappa_1 = \frac{1}{4}a_2 - \frac{3}{8}a_1^2 - \frac{1}{8}\sigma_{ij}^2 + \frac{3}{4}(a_1\sigma_{ij} + 2a_3 + 2a_4) - \frac{1}{2}a_5 \quad , \tag{13}$$

and where the functions a_1, a_2, a_3, a_4 and a_5 are given by

$$a_1 = a_1(\sigma_i, \sigma_j) = \frac{1}{\sigma_{ij}}\left(\sigma_i\zeta_i\frac{\partial\sigma_{ij}}{\partial\sigma_i} - \sigma_j\zeta_j\frac{\partial\sigma_{ij}}{\partial\sigma_j}\right) \quad , \tag{14}$$

$$a_2 = a_2(\sigma_i, \sigma_j) = \frac{\upsilon_i^2\sigma_i^2}{\sigma_{ij}^2}\left(\frac{\partial\sigma_{ij}}{\partial\sigma_i}\right)^2 + \frac{2r_{ij}\upsilon_i\upsilon_j\sigma_i\sigma_j}{\sigma_{ij}^2}\frac{\partial\sigma_{ij}}{\partial\sigma_i}\frac{\partial\sigma_{ij}}{\partial\sigma_j} + \frac{\upsilon_j^2\sigma_j^2}{\sigma_{ij}^2}\left(\frac{\partial\sigma_{ij}}{\partial\sigma_j}\right)^2 \quad , \tag{15}$$

$$a_3 = a_3(\sigma_i, \sigma_j) = \tilde{\rho}_{ij}\frac{\upsilon_i\sigma_i\sigma_j}{\sigma_{ij}}\frac{\partial\sigma_{ij}}{\partial\sigma_i} + \tilde{\rho}_{jj}\frac{\upsilon_j\sigma_j^2}{\sigma_{ij}}\frac{\partial\sigma_{ij}}{\partial\sigma_j} \quad , \tag{16}$$

$$a_4 = a_4(\sigma_i, \sigma_j) = \frac{\sigma_i^2\sigma_j^2}{\sigma_{ij}^4}\left(1-\rho_{ij}^2\right)\left(\frac{1}{2}\upsilon_i^2 - r_{ij}\upsilon_i\upsilon_j + \frac{1}{2}\upsilon_j^2\right) \tag{17}$$

$$\begin{aligned} a_5 = a_5(\sigma_i, \sigma_j) = {} & \frac{2\sigma_i^2}{\sigma_{ij}^2}\zeta_i^2 - \frac{\rho_{ij}\sigma_i\sigma_j}{\sigma_{ij}^2}(\zeta_i - \zeta_j)^2 + \frac{2\sigma_j^2}{\sigma_{ij}^2}\zeta_j^2 - 3a_1^2 \\ & + \frac{\sigma_i}{\sigma_{ij}^2}\left(\sigma_i\upsilon_i\tilde{\rho}_{ii}\zeta_i + \sigma_j\upsilon_i\tilde{\rho}_{ij}\zeta_j\right)\frac{\partial\sigma_{ij}}{\partial\sigma_i} + \frac{\sigma_j}{\sigma_{ij}^2}\left(\sigma_j\upsilon_j\tilde{\rho}_{jj}\zeta_j + \sigma_i\upsilon_j\tilde{\rho}_{ji}\zeta_i\right)\frac{\partial\sigma_{ij}}{\partial\sigma_j} \quad . \end{aligned} \tag{18}$$

where

$$\zeta_i = \frac{\upsilon_i\left(\tilde{\rho}_{ii}\sigma_i - \tilde{\rho}_{ij}\sigma_j\right)}{\sigma_{ij}} \quad , \quad \zeta_j = \frac{\upsilon_j\left(\tilde{\rho}_{jj}\sigma_j - \tilde{\rho}_{ji}\sigma_i\right)}{\sigma_{ij}} \quad . \tag{19}$$

B : The maximum entropy parameterisation of the Cholesky decomposition of the correlation matrix

In this appendix we prove that the maximum entropy parameterisation in equation (6) is correct.

Since the Wiener processes generate a multivariate normal distribution, the information entropy of the correlation matrix is given by

$$\begin{aligned} H &= \log\left(\sqrt{(2\pi e)^d\,|\varrho|}\right) \\ &= \frac{1}{2}\log|\varrho| + \text{const} \quad , \end{aligned}$$

where const is a constant, and where $|\varrho|$ is the determinant of ϱ which is the correlation matrix of $[dW_1, dZ_1, ..., dW_N, dZ_N]'$. Therefore maximising the entropy H is equivalent to maximising $|\varrho|$, which can be written

$$|\varrho| = \left|\mathbf{C}\mathbf{C}'\right| = |\mathbf{C}|^2 = \prod_{i=1}^{2N} C_{ii}^2 \quad ,$$

where $\mathbf{C}$ is the Cholesky decomposition of ϱ introduced in (5). The top left corner of $\mathbf{C}$ of size $2M \times 2M$ corresponds to the correlations related to the major currencies, and which are fixed. Hence focussing on what can be changed during the calibration of the less liquid currency parameters, $|\varrho|$ can be written

$$|\varrho| = \text{const} \times \prod_{i=2M+1}^{2N} C_{ii}^2 \quad .$$

However, since all entries on the leading diagonal ϱ must be 1, the sums of squares of the C_{ij} across rows must equal 1, so $|\varrho|$ can be written

$$|\varrho| = \text{const} \times \prod_{i=2M+1}^{2N}\left(1 - \sum_{j=1}^{i-1} C_{ij}^2\right) \quad .$$

This means that to maximise $|\varrho|$, all the C_{ij} where $4 < j < i$ where there is no market data must be zero, because any non-zero value would reduce $|\varrho|$ and hence reduce the entropy H. Hence (6) is correct.

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