Assessing the Impact of the Comprehensive Risk Measure (CRM) on the Correlation Business under Extensions to Basel II.

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Global Derivatives Trading and Risk Management

ICBI Paris, Tuesday 12th April 2011.

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Acknowledgements

- Thanks to Alex Lipton, Leif Andersen and the Credit Quant Group at BAML.
- Note: the material presented here should not be taken as representative of the views of BAML or BAML's approach to the CRM calculation.

Overview

- Impact of Revisions to Basel II on Correlation Books
- Relating Modelling Assumptions Back to the Empirical Data
- Modelling the Dynamics of the Correlation Skew
- Implementational Challenges
- Implications for Pricing and Risk Management

Impact of Basel II on Correlation Books

- Impact of Revisions to Basel II on Correlation Books:

$$\text{Charge} = \text{VaR} + \text{sVaR} + \text{IRC} + \text{CRM} + \text{Standard Charge}$$

“General Risk” :

- **VaR**: 10 day 99% Value at Risk.
- **sVaR** (stressed VaR): As above, but applying scenarios calibrated to a 1 year period of “significant financial stress” to the current portfolio.

“Specific Risk”: Risk not normally captured by VaR:

- **IRC** (Incremental Risk Charge) for unsecuritised Credit Derivatives books to capture default and rating migration, illiquidity (account for “liquidity horizon” of at least 3 months). 99.9% confidence level at a 1 year time-horizon.
- **CRM** (Comprehensive Risk Measure) for Credit Correlation books. 99.9% confidence level at a 1 year time-horizon.

Impact of Basel II on Correlation Books: CRM

CRM (Comprehensive Risk Measure). (from *Revisions to the Basel II market risk framework*, *July 2009*, *BIS*). Supposed to capture the effect of:

- "Cumulative risk arising from multiple defaults, including … ordering of defaults in tranched products.
- Credit Spread risk including gamma and cross-gamma effects.
- Volatility of Implied Correlations, including the cross effect between spreads and correlations.
- Basis risk … both index and … its constituent single names; and … the basis between the implied correlation of an index and that of bespoke portfolios.
- Recovery rate volatility, as it relates to the propensity for recovery rates to affect tranche prices.
- [if you include] benefits from dynamic hedging… [you also need to include] risk of hedge slippage … potential costs of rebalancing."

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Impact of Basel II on Correlation Books: CRM

- The alternative to successfully implementing CRM is to treat entire book according to standard model (!)
- Standard model charge for securitised products is rating and seniority based and does not allow for any netting unless portfolio, maturity, attachment, series, etc. perfectly match. Not based on economic risk, long and short treated separately.
- Hence the urgent need for greater clarity on the impact of these measures on correlation books and the need for an implementation that will satisfy the regulators.

CRM: Relating Modelling Assumptions Back to the Empirical Data

- Back-testing a one in a thousand year loss is challenging…

- …Hence to a greater or lesser degree testing and justification of the approach will have to be model based.

Modelling the Dynamics of the Correlation Skew

- One of the requirements for CRM is ability to compute an extreme loss percentile incorporating the effect of default correlation.

- Therefore it is necessary to have a methodology for generating base correlation scenarios. There is no universally accepted model for the base correlation skew, and those approaches that exist tend to have unintuitive parameters so it is unclear whether or not they are capable of generating realistic looking time-series. Can we work directly with either prices or correlations?

- First let us sketch out a few of the constraints. Consider an index on which we specify attachment points $\{K_0,\ldots,K_N: K_0=0\%, K_N=100\%\}$. Define the exceedance (loss on a K_i-100% tranche) where L_T is the loss on the portfolio by a given time-horizon:

$$\bar{L}_{K_i}(T) = \mathbb{E}\left[\max(L_T - K_i, 0)\right]$$

Modelling the Dynamics of the Correlation Skew

- In this presentation we will treat discounted and undiscounted loss interchangeably since it will not change our conclusions. Since:

$$\bar{L}_{K_i}(T) - \bar{L}_{K_j}(T) = \int_{K_i}^{K_j} \mathrm{Prob}(L_T > K')dK'$$

- We can show on very general grounds that the Exceedance must be **monotonically decreasing and positively convex** with respect to the K_i.

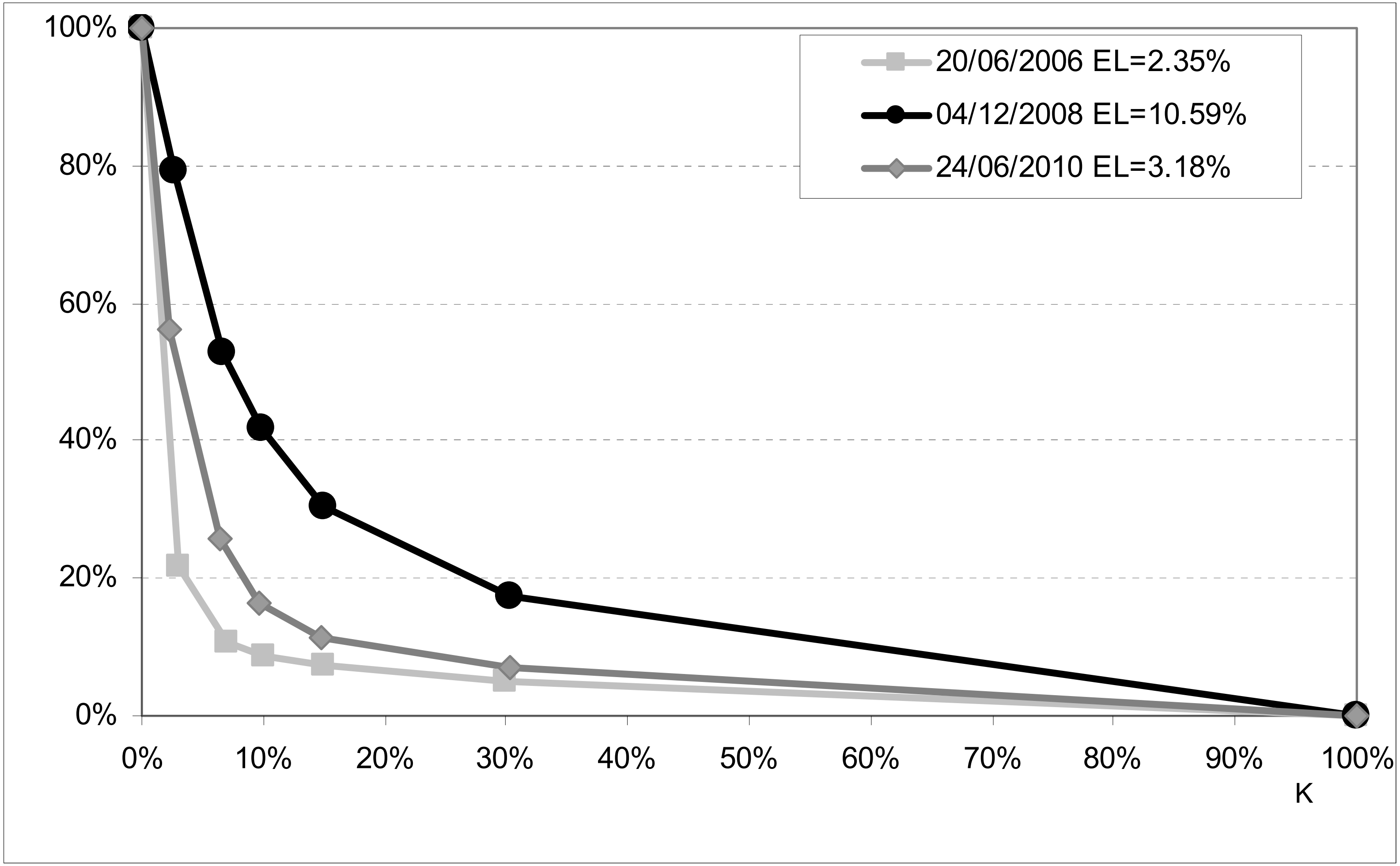
- However, we take it as axiomatic that we are interested in scenarios that can mostly be calibrated by the chosen correlation model (which must have stochastic recovery), we also have a set of constraints in correlation space.

- For example, consider 30-100% tranche CDX.IG9.5Y as of Aug 2010:

Current Implied Spread:	9bp
Model Bounds (Deterministic Recovery):	0bp - 20bp
Model Bounds (Stochastic Recovery):	0bp - 29bp

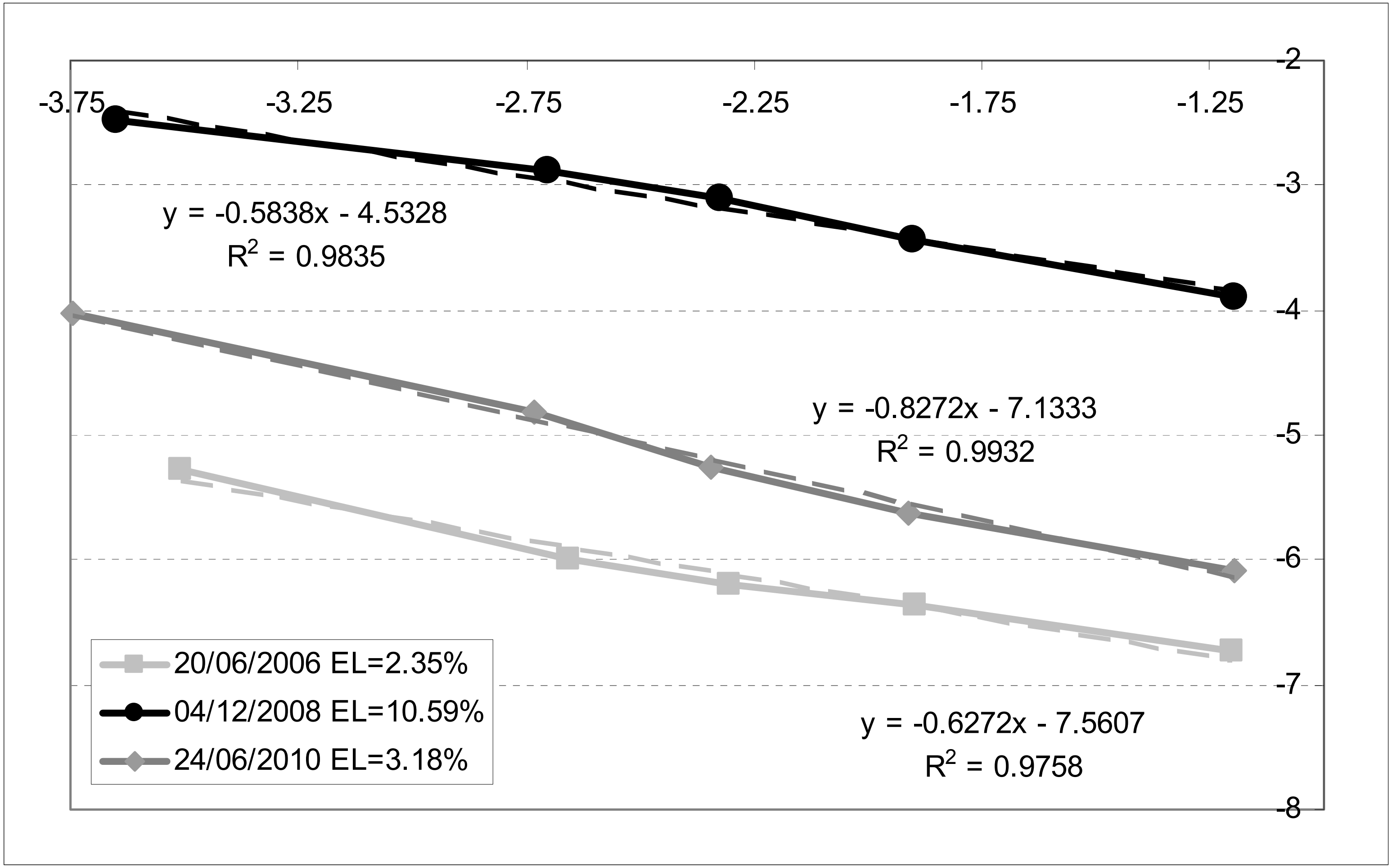
- Need to impose constraints in **2** completely different bases. Whichever basis we choose to work with, may hit constraints in the other one.

Let's Look at the Empirical Data: Price Space



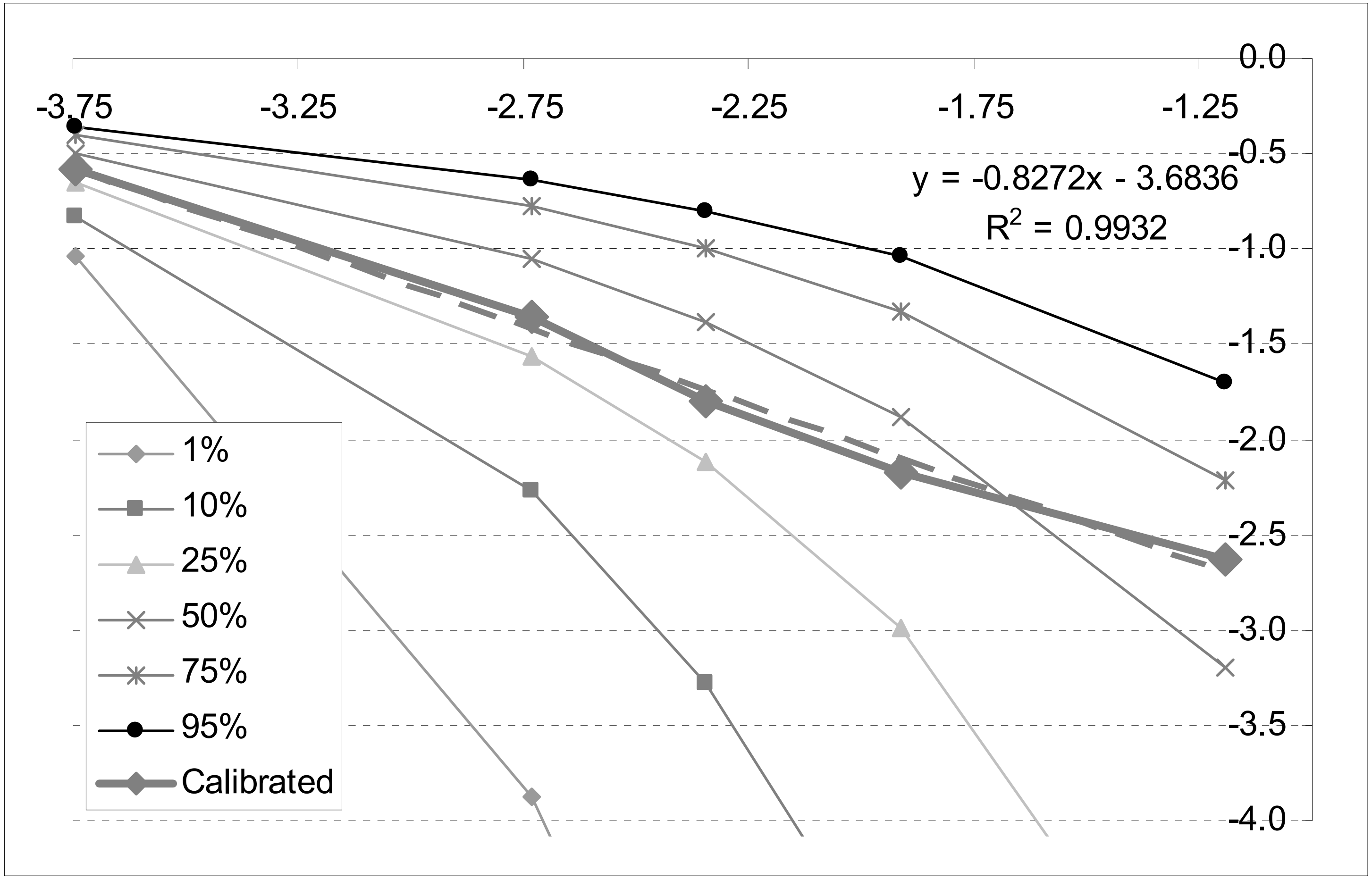
- Exceedances (normalised by portfolio expected loss) calibrated to 5Y CDX IG on the run tranches on a series of dates corresponding to very different market conditions.

Dynamics of the Correlation Skew: Price Space



- Log Exceedances for the same 3 dates (x axis is ln(K))

Dynamics of the Correlation Skew: Price Space



- This behaviour in price space cannot be replicated by Gaussian Copula (+stochastic recovery) here plotted for a range of flat correlations.

Dynamics of the Correlation Skew: Extreme Value Theory

- A Possible Explanation: Extreme Value Theory (EVT) (Chavez-Dumoulin & Embrechts 2011). Analogue of central limit theorem applied to extremal events/tails.

- Excess Distribution function $F_u(X)$ of a sum of iid random variables $X = X_1 + X_2 + \ldots + X_n$ with distribution function $F(x) = \mathrm{Prob}(X_i < x)$ can be approximated above a threshold u via a generalised Pareto distribution (GPD):

$$F_u(x) = P\left(X - u \le x | X > u\right) \to G_\xi\left(\frac{x}{\beta(u)}\right)$$

$$G_\xi(x) = \begin{cases} 1 - (1+\xi x)^{-1/\xi} & \text{if } \xi \neq 0 \\ 1 - \exp(-x) & \text{if } \xi = 0 \end{cases}$$

$\xi > 0 \rightarrow$ Fat Tailed

- Value of ξ depends on "Maximal Domain of Attraction" (MDA) of F. Limit depends strongly on distribution of underlying iid variables F (whereas central limit theorem does not, although variables must have finite variance).

Modelling the Dynamics of the Correlation Skew

- Exceedance (integral of Pareto tail). Define $\bar{F}(X) = 1 - F(X)$. Hypothesise that for sufficiently large $K_i > u$, tail of loss distribution can be approximated by GPD, where $0 \leq \xi < 1$:

$$\ln\left(\bar{L}_{K_i}\right) = \ln\left(\frac{\beta \bar{F}_0(u)}{1-\xi}\right) + \left(1 - \frac{1}{\xi}\right) \ln\left(1 + \frac{\xi}{\beta}(K_i - u)\right) \qquad 0 < \xi < 1$$

$$\ln\left(\bar{L}_{K_i}\right) = \left(\ln\left(\beta \bar{F}_0(u)\right) + \frac{u}{\beta}\right) - \frac{1}{\beta} K_i \qquad \xi = 0$$

- Empirical exceedance data shows evidence of Pareto (fat) tail, for which $\xi > 0$. This is the MDA of a sum of iid variables with fat-tailed distribution *F* such as Pareto, student-t.

- Loss in Gaussian Copula (+stochastic recovery) can be expressed as a sum of conditionally independent loss variables, but their distribution is not fat-tailed, hence MDA for tail distribution corresponds to $\xi = 0$.

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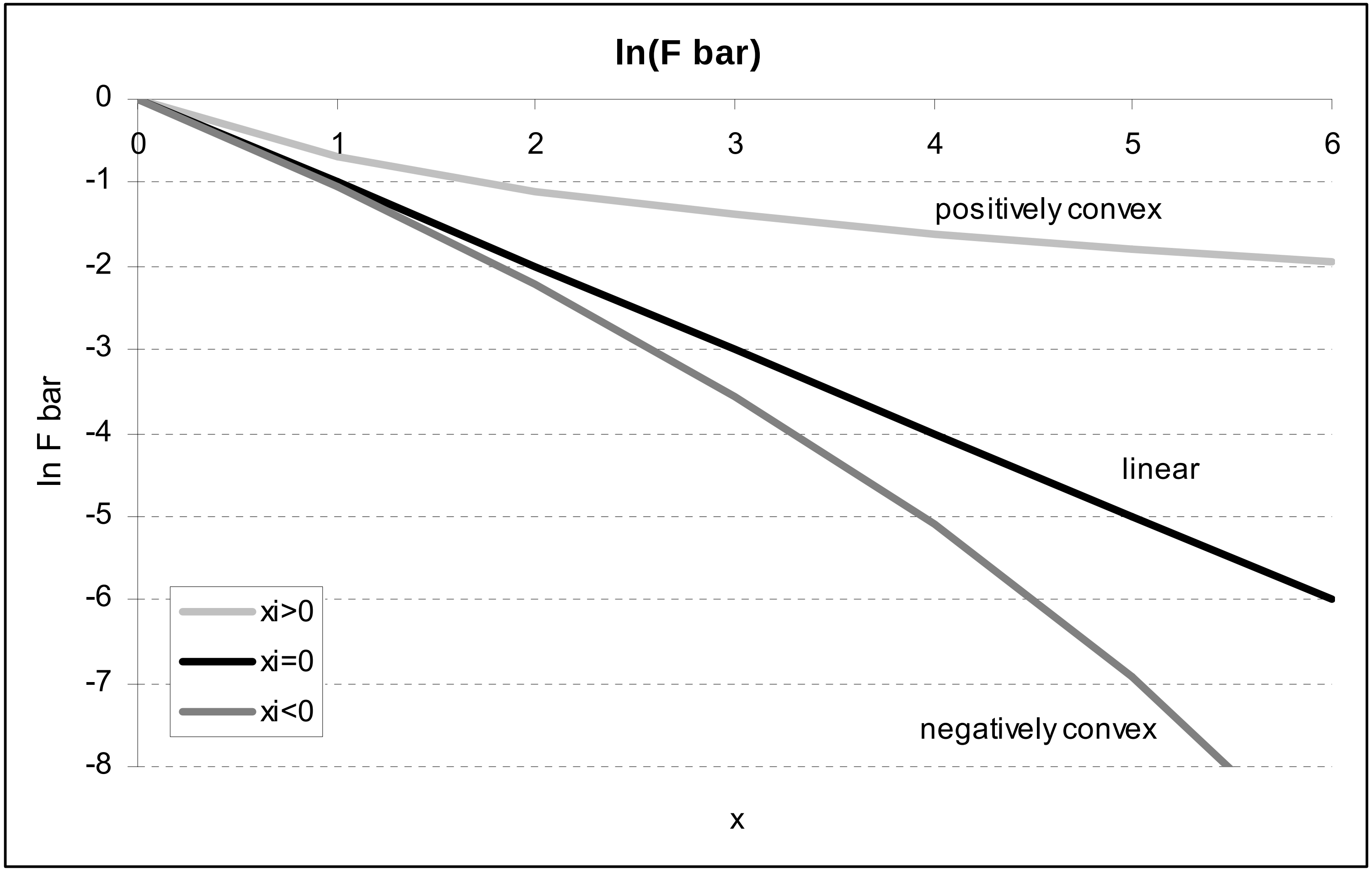
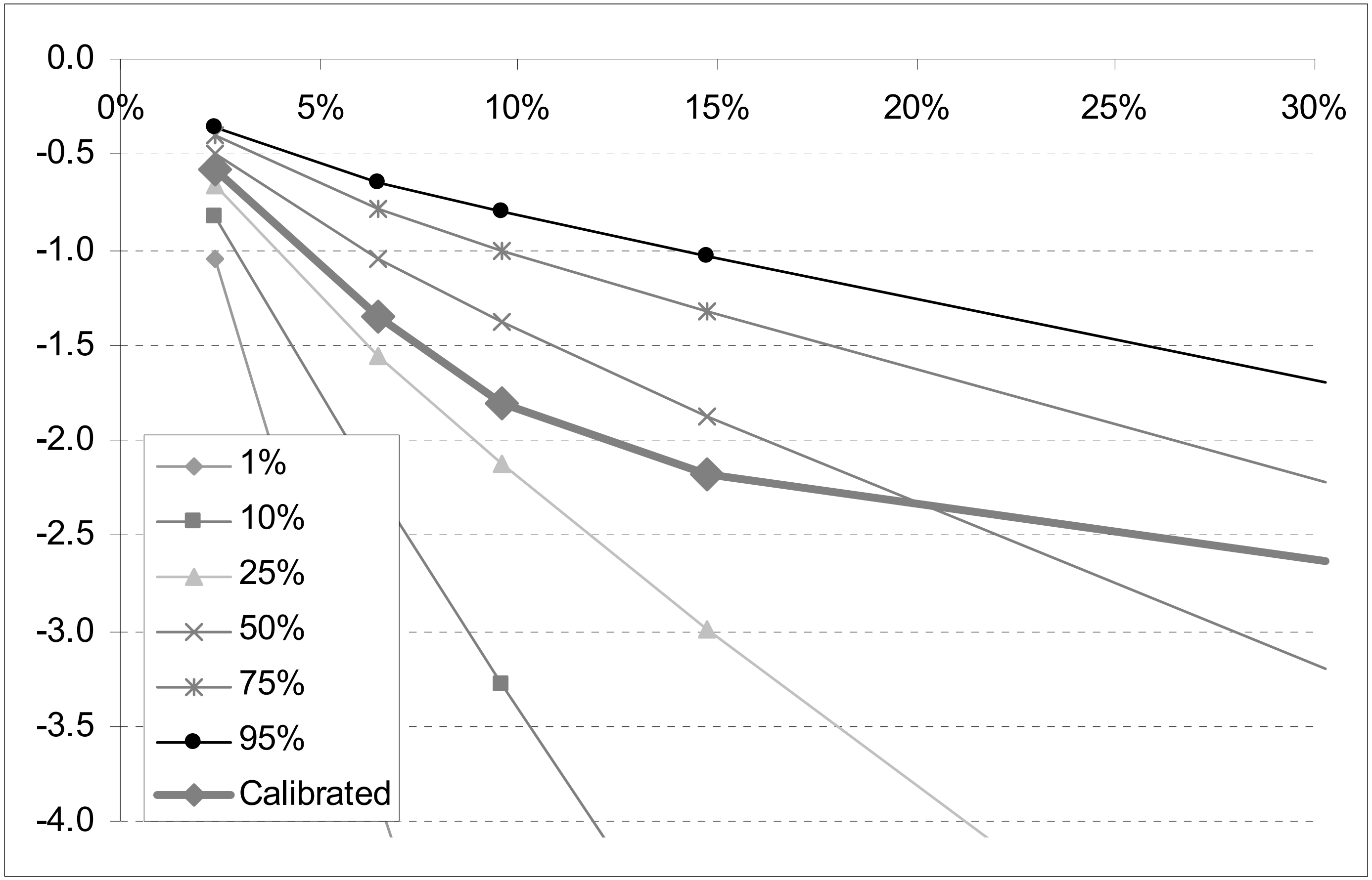
Modelling the Dynamics of the Correlation Skew

- Pareto tails for the 3 cases (x axis is K):



Modelling the Dynamics of the Correlation Skew

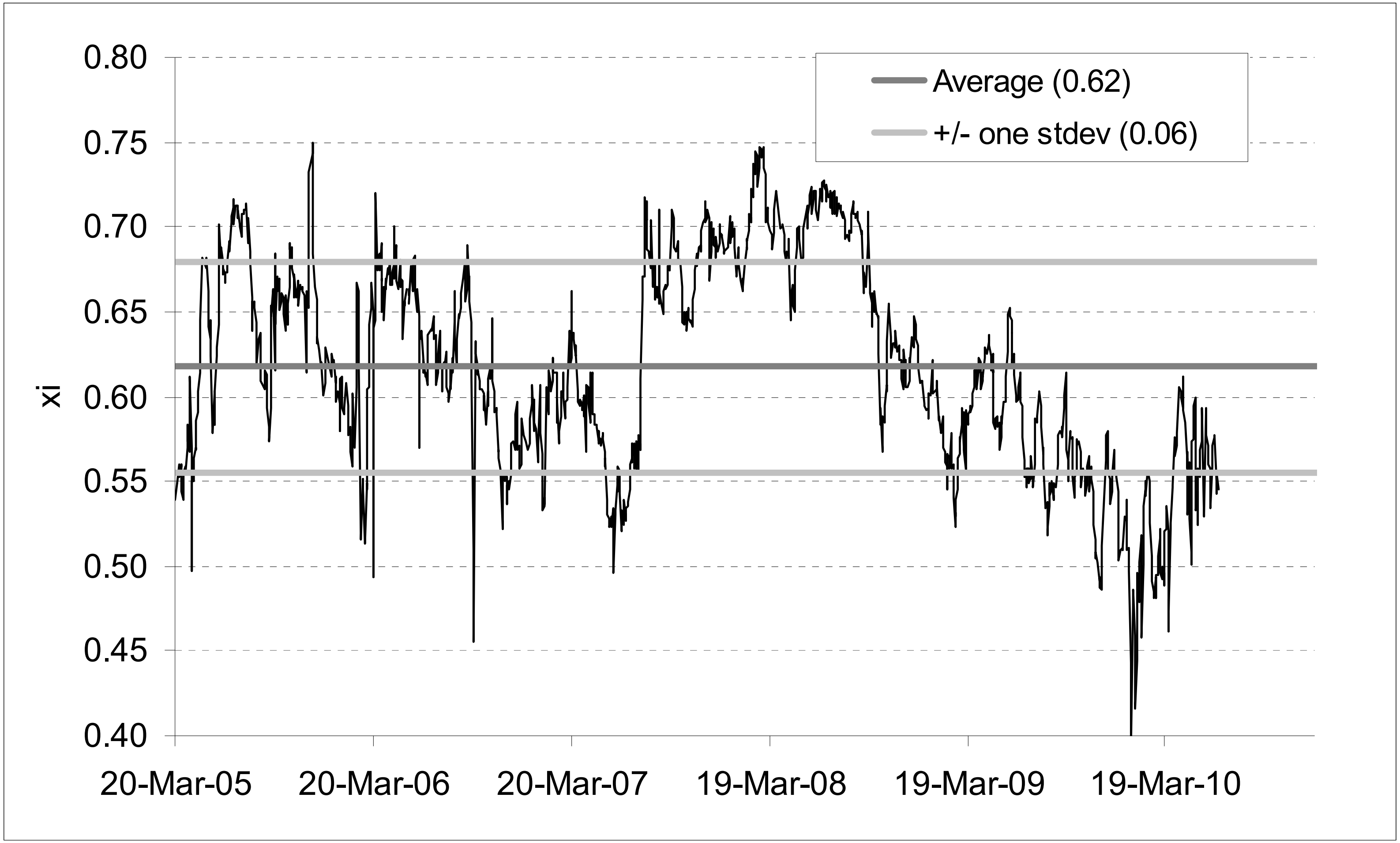
- In Gaussian Copula, ln(Exceedance) very close to linear function of K: ξ=0



- …whereas empirical data clearly is not: positively convex => 0<ξ<1. (same data as a few slides back, plotted against K rather than ln(K)).

Modelling the Dynamics of the Correlation Skew

- Time Series of xi values fitted to CDX.IG.5Y covering over 1200 data points:



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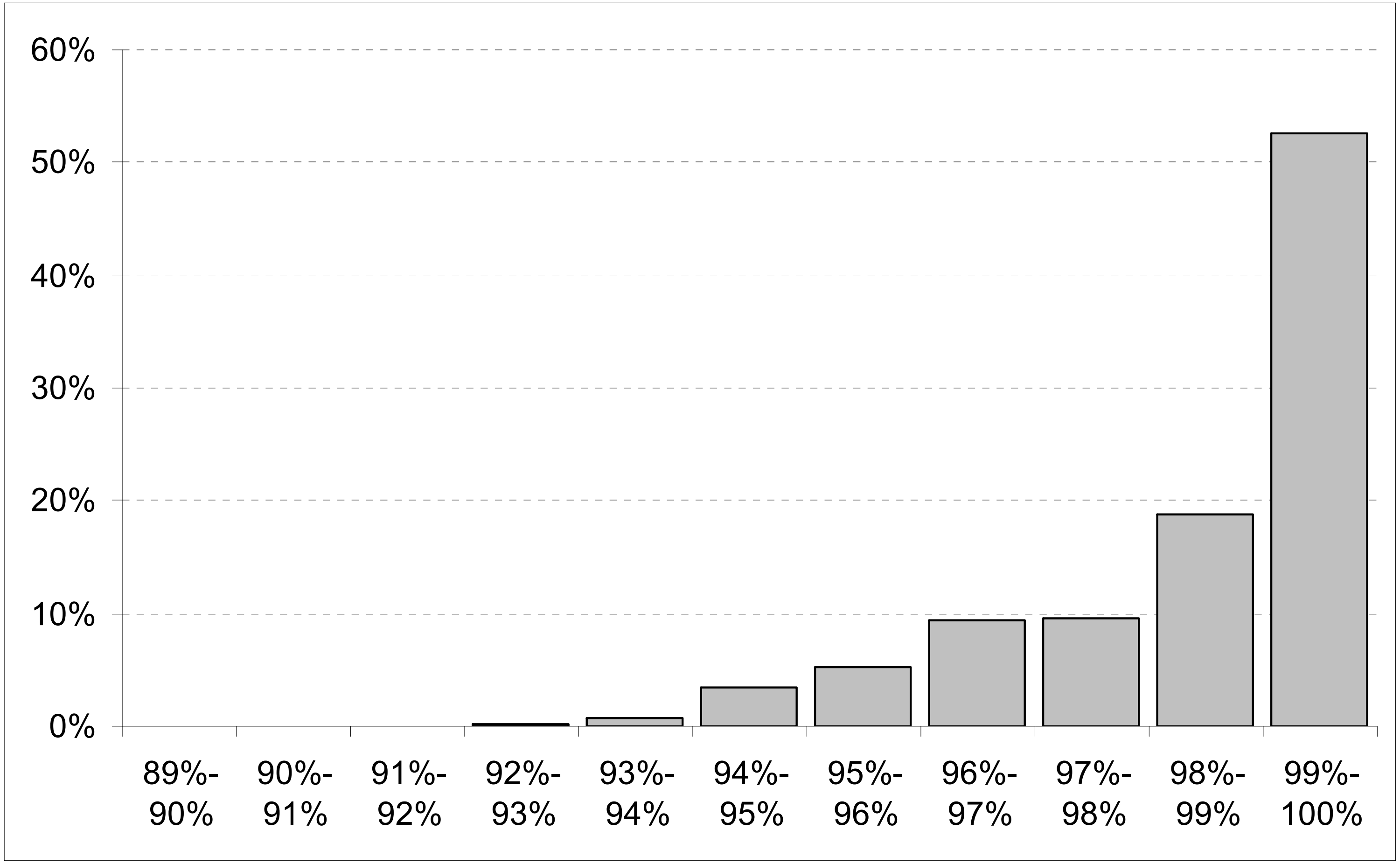
Modelling the Dynamics of the Correlation Skew

- Distribution of R^2 values obtained in the fits plotted on the previous slide.



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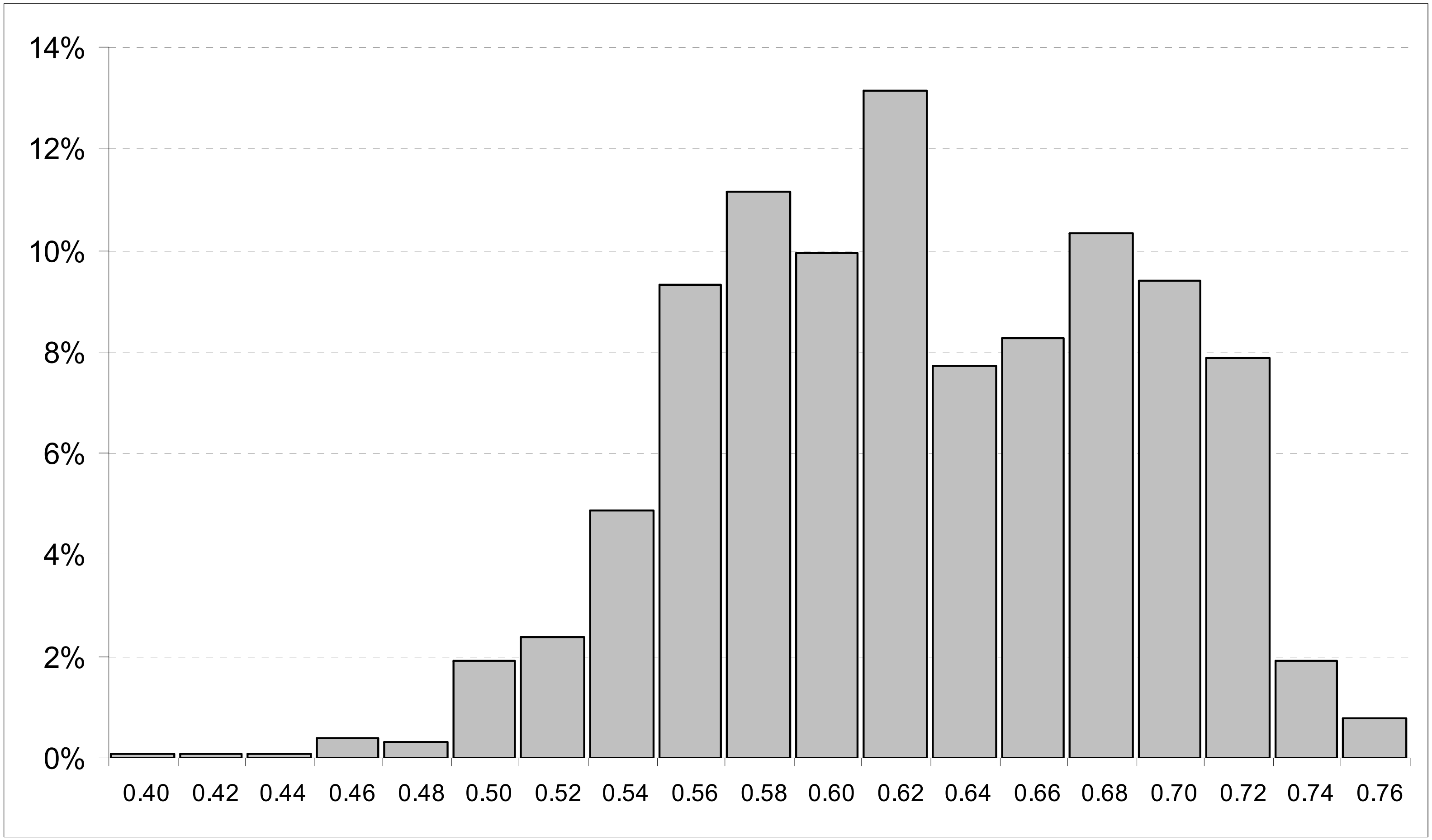
Modelling the Dynamics of the Correlation Skew

- Distribution of xi values obtained in the fits plotted on the previous slide.



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Modelling the Dynamics of the Correlation Skew

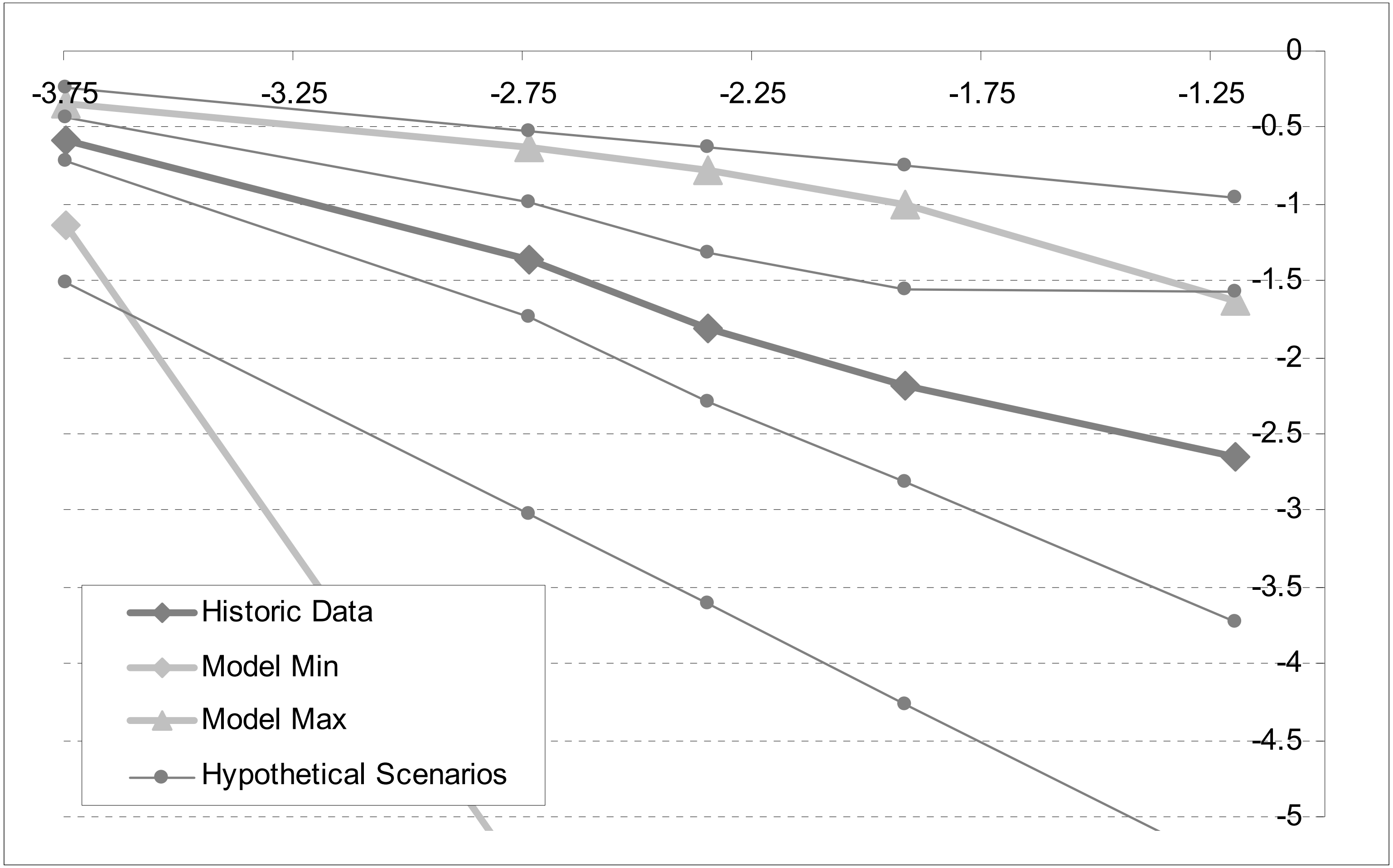
- Qualitative description: Empirically, across a wide range of market conditions, prices imply Pareto (fat) tails in loss dist. with ξ ~ 0.5 - 0.7.

- Pareto functional form with 0< ξ <1 automatically takes care of monotonicity and positive convexity requirements:

$$
\begin{aligned}
\bar{L}_{K_i} &= \frac{\beta \bar{F}_0(u)}{1-\xi}\left(1+\frac{\xi}{\beta}(K_i-u)\right)^{1-\frac{1}{\xi}} \\
\partial_{K_i}\bar{L}_{K_i} &= -\bar{F}_0(u)\left(1+\frac{\xi}{\beta}(K_i-u)\right)^{-\frac{1}{\xi}} && \leq 0 \\
\partial^2_{K_i}\bar{L}_{K_i} &= \frac{\bar{F}_0(u)}{\beta}\left(1+\frac{\xi}{\beta}(K_i-u)\right)^{-1-\frac{1}{\xi}} && \geq 0
\end{aligned}
$$

- Modelling exceedances as a best fit power law is too simplistic: tranche prices are sensitive to small “wiggles” in distribution. However, it does suggest a sensible basis in which to decompose price movements: In(exceedance).

Modelling the Dynamics of the Correlation Skew



- A series of hypothetical expected loss scenarios that correspond to theoretically non-arbitrageable prices but which nonetheless can violate model bounds i.e. correlation cannot be calibrated.

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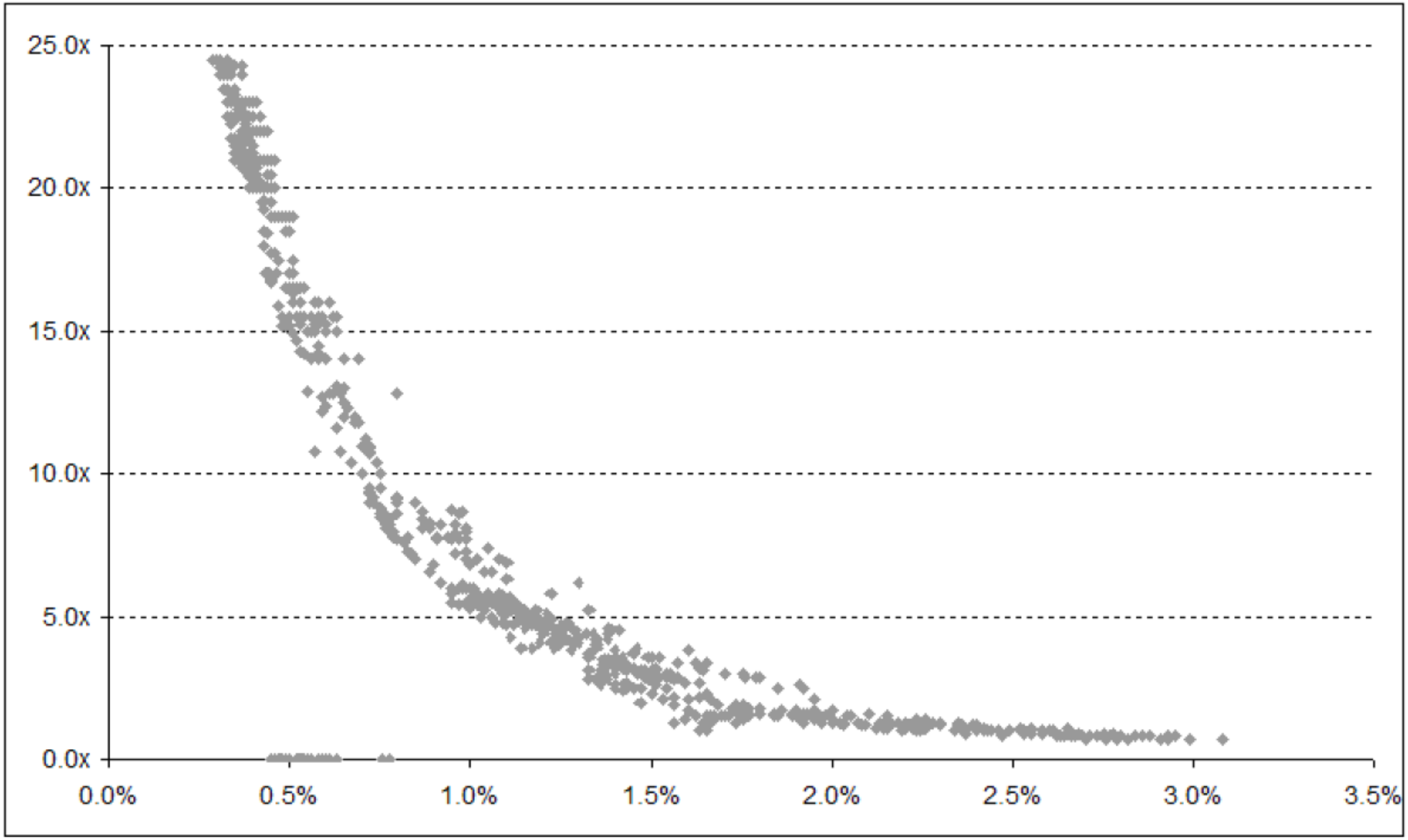
Constraints in Correlation Space

- Sets of prices that do not exhibit theoretical arbitrage can still fail to calibrate.
- Nonetheless, some people have suggested operating purely in price/exceedance space. See for example (Kainth *et al* 2010).
- One problem with this though, if you are going to perform a PCA in loss space (effectively a kind of linearisation), is that the relationship of tranche prices to underlying index is highly convex.

Equity Tranche Leverage: Convexity



- Scatter plot of equity tranche leverage against index reference level for CDX.IG.5Y on the run from Mar-05 to Jun-10.

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Constraints in Correlation Space

- There are also significant correlation convexity effects and cross convexity of spread and correlation.
- The net result is that generating scenarios in loss space based on a PCA, especially if the index level has changed significantly, can be very problematic.
- In this presentation we will therefore consider an alternative approach: performing a PCA in base correlation space.

Correlation: Simplest possible Basis

- Need to capture "*Volatility of Implied Correlations, including the cross effect between spreads and correlations*" hence need to include a variable representing index level. Simple and obvious suggestion:

$$Y_0 = \ln\left(\bar{L}_{K_0}(T)\right)$$
$$Y_i = \rho_i(T) : \quad i = 1, ..., N-1$$

- Where $\{\rho_i(T)\}$ are the base correlations for maturity T.

- Ultimately we want to be able to generate scenarios via principal components Z for which:

$$\rho_i = A_i + \sum_j B_{ij} Z_j$$

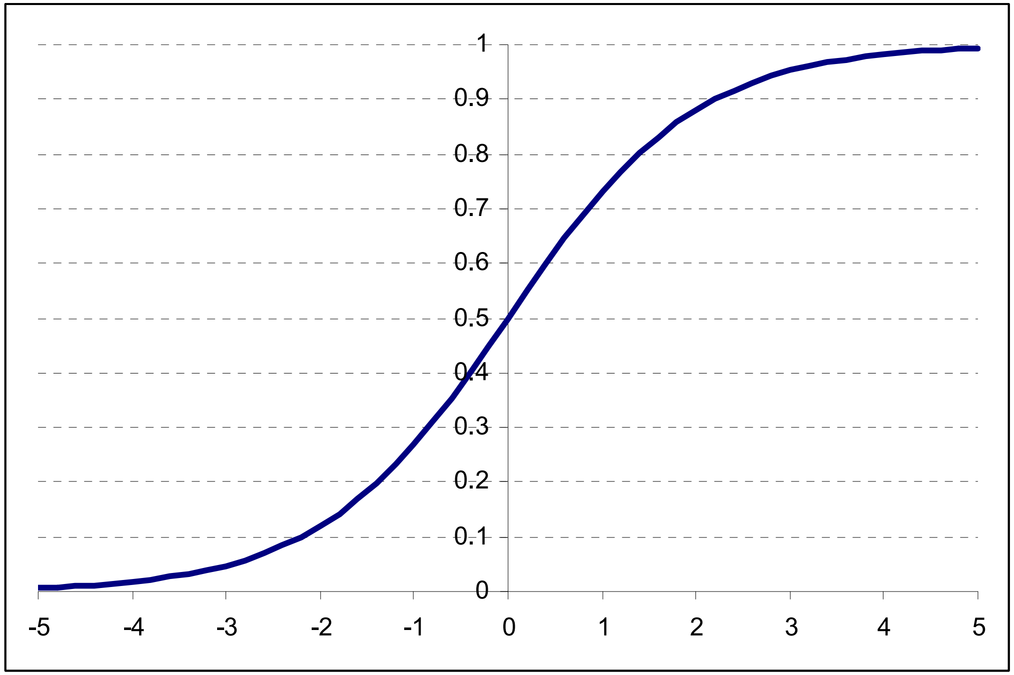
- However, scenarios generated in this basis are not guaranteed to be in [0,1].

Correlation: Logistic Transformation

- One approach: introduce the logistic transformation and write:

$$\rho_i = \Lambda(A_i + \sum_j B_{ij} Z_j) = \frac{1}{1 + \exp\left(-A_i - \sum_j B_{ij} Z_j\right)}$$

- Where the logistic transformation is defined to be:



$$\Lambda(z) = \frac{1}{(1 + \exp(-z))},$$

$$\Lambda^{-1}(y) = \ln\left(\frac{y}{1-y}\right)$$

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Correlation: Logistic Transformation

- Suggests we define the basis:

$$Y_0 = \ln\left(\bar{L}_{K_0}(T)\right)$$
$$Y_i = \Lambda^{-1}(\rho_i) = \ln\left(\frac{\rho_i(T)}{1-\rho_i(T)}\right): \;\; i = 1, ..., N-1$$

- Assuming this Decomposition we can then generate Scenarios according to:

$$\rho_i' = \rho_i + \Delta\rho_i = \frac{\rho_i}{\rho_i + (1-\rho_i)\exp\left(-\sum_j B_{ij}\Delta Z_j\right)}$$

- This guarantees correlations remain in [0,1]. However, it is rather non-linear for very low or very high correlations.

Relation between Correlation Skew and Index Level

- Given a time-series, how should we decompose the PCA so that in scenarios we can incorporate co-movement of index and base correlation skew?

- Suppose we take the change in index level ΔY_0 as given. If we assume the increments are normally distributed we can evaluate the *Schur complement* of the full covariance matrix Σ and determine the conditional distribution of the remaining increments for $i,j=1,\ldots,N-1$:

$$\mathbb{E}\left[\Delta Y_i \middle| \Delta Y_0\right] = \rho_{i0}\frac{\sigma_i}{\sigma_0}\Delta Y_0$$

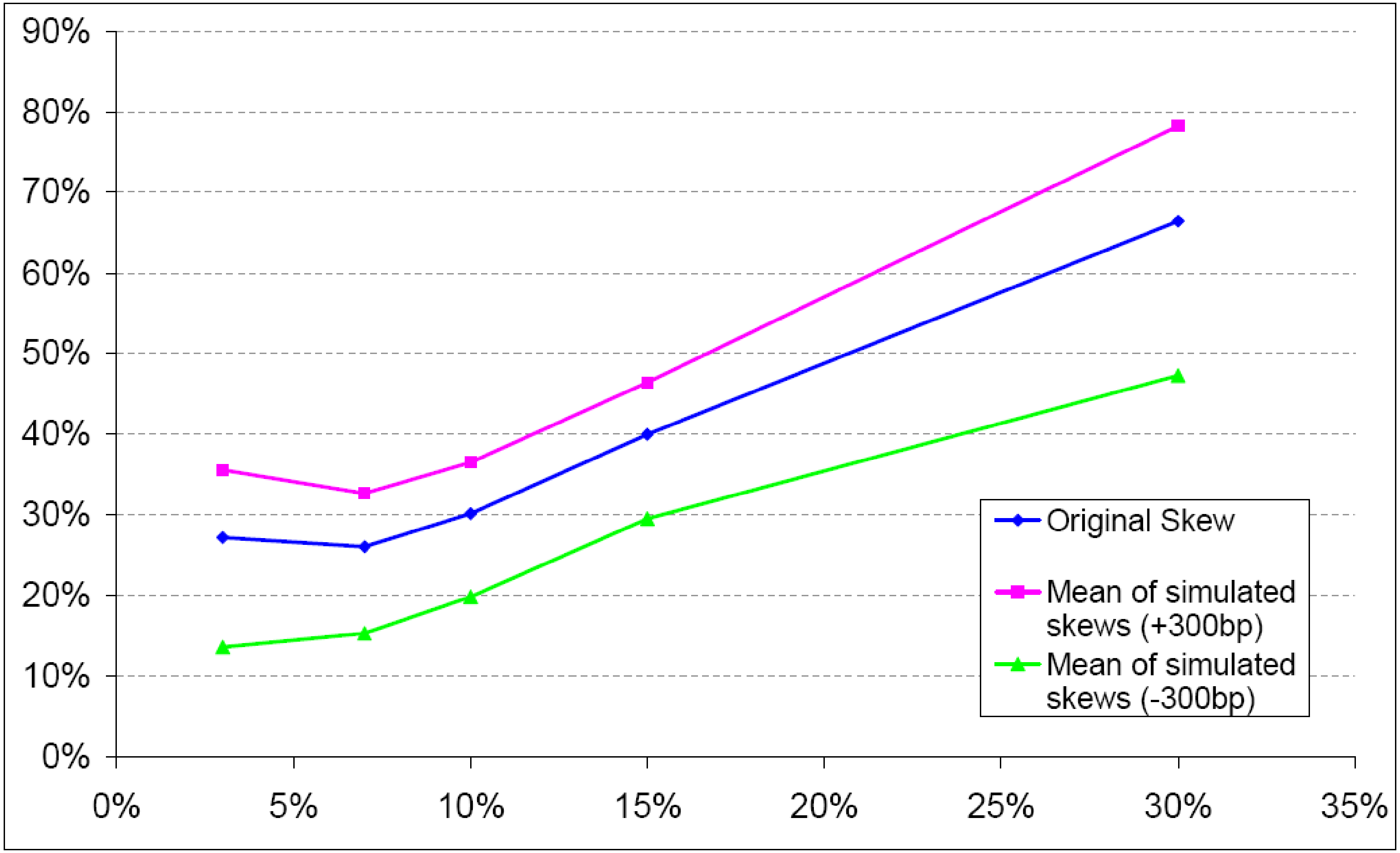
$$\hat{\Sigma}_{ij} = \mathrm{covar}\left(\Delta Y_i, \Delta Y_j \middle| \Delta Y_0\right) = \left(\rho_{ij} - \rho_{i0}\rho_{j0}\right)\sigma_i\sigma_j$$

- Where ρ_{ij} is the full correlation matrix of the increments and σ_i their standard deviation.

- This approach generalises nicely to multiple indices and maturities.

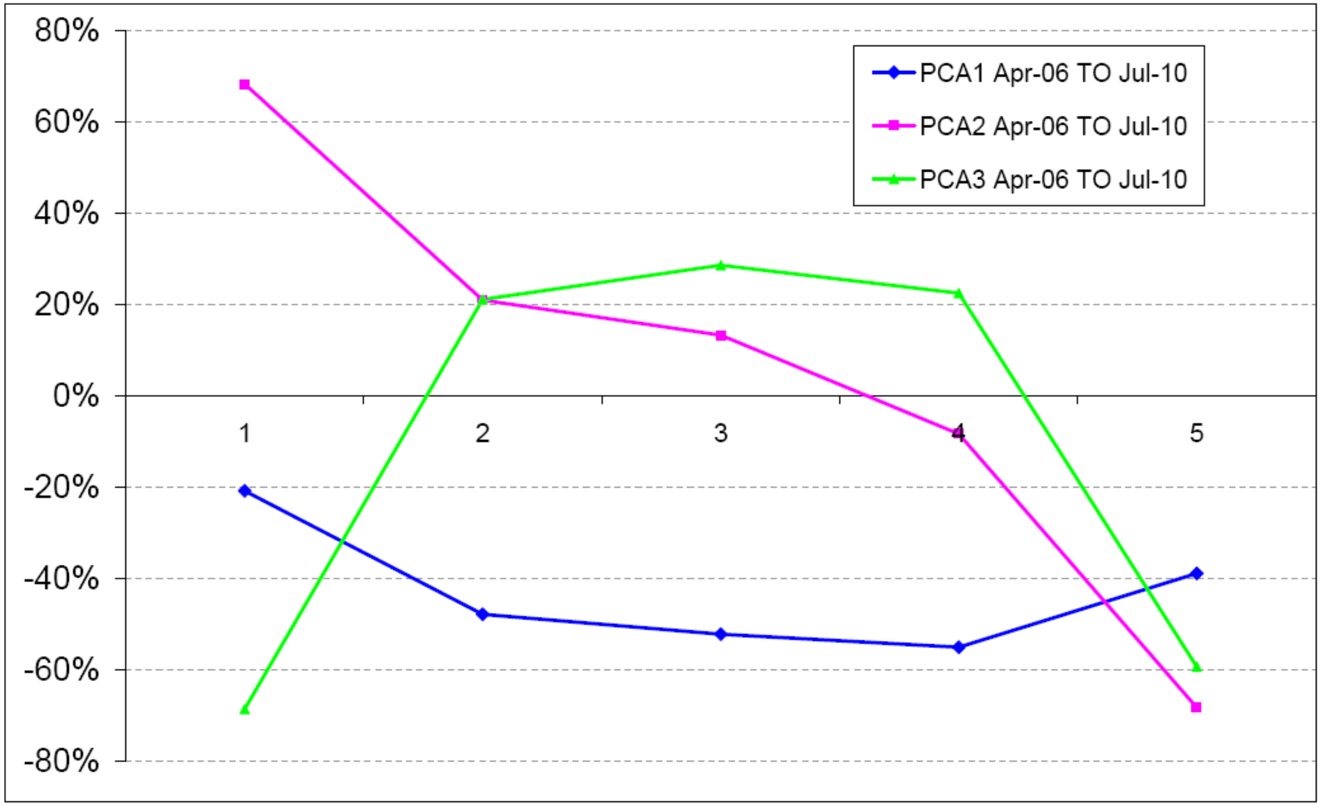
Relation between Correlation Skew and Index Level

- Hence can simulate "mean" skew conditional on a given change in index expected loss (here for CDX.IG.7Y, July 2010)



PCA Approach

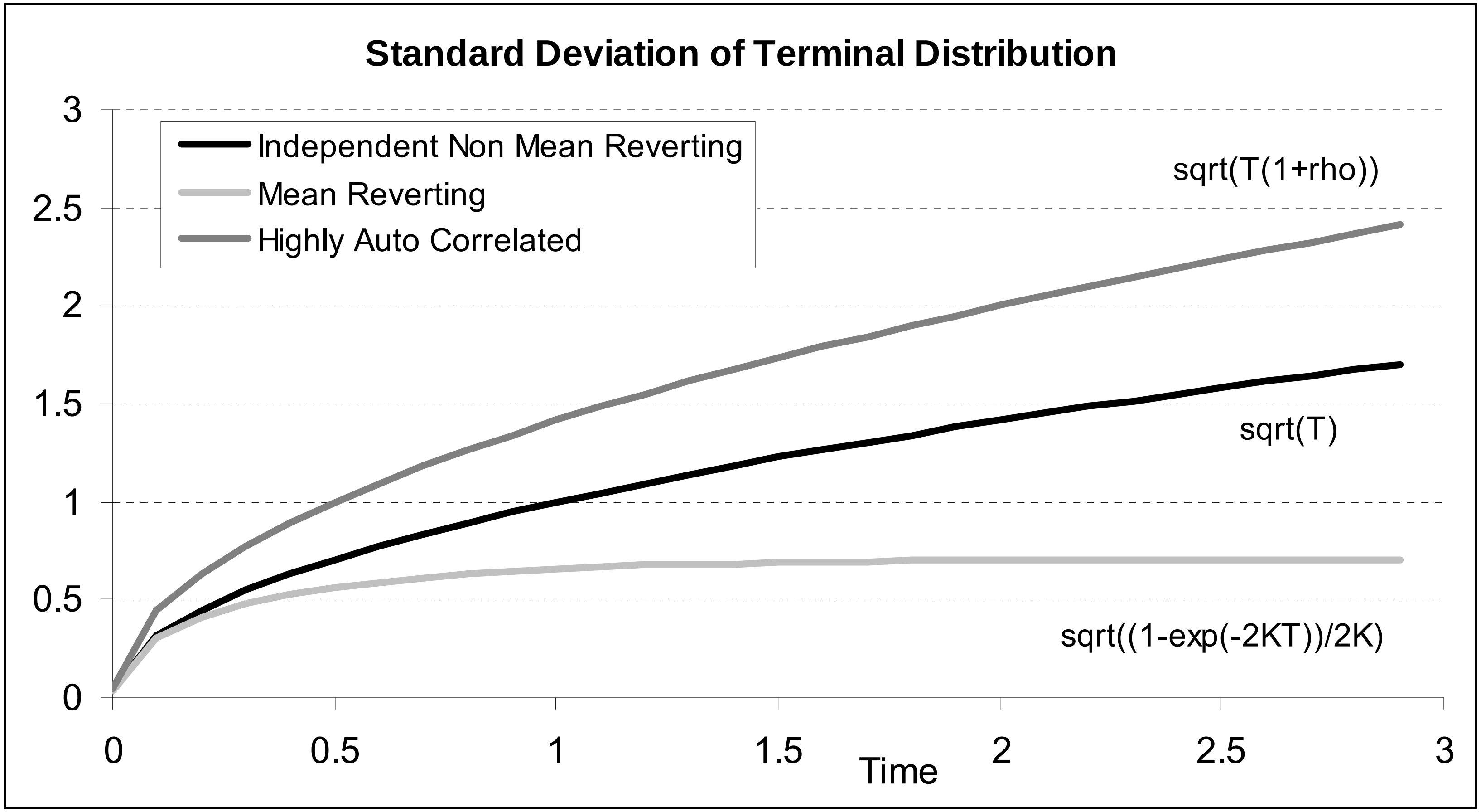
- Since we are interested in the one year time-horizon, use one year overlapping increments. Data is shown for logistic basis only (other basis similar):



component	% variance
1	87.13%
2	7.90%
3	4.57%
4	0.32%
5	0.08%

PCA Approach: One Year Time Horizon

- Offsetting Effects: Auto Correlation and Mean Reversion. Graph shows 3 different Gaussian processes with same local volatility:



- Very difficult to Isolate these offsetting effects. Expect mean reversion of base correlations as their values are bounded in [0,1].

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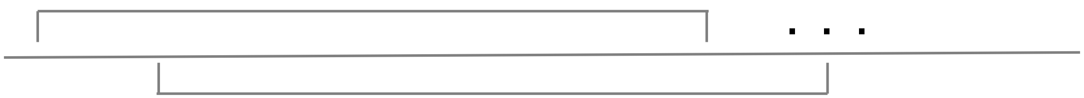
PCA Approach: Overlapping Increments/One Year Time Horizon

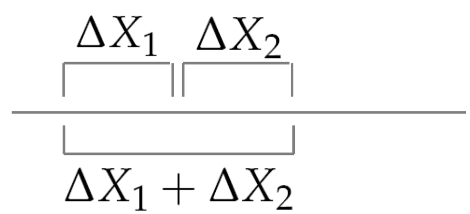
- If we use short increments and extrapolate to one year we tend to overstate volatility since correlations are bounded in [0,1], not endlessly diffusive. Also need to make further model assumptions about auto-corr, mean-rev.

- Alternative choice: overlapping one year increments (Hansen & Hodrick 1980), (Müller 1993)



- Suppose we have two zero mean equal time increments ΔX_1 and ΔX_2 with stdev σ and autocorrelation ρ and we form an estimator for the variance of their sum:

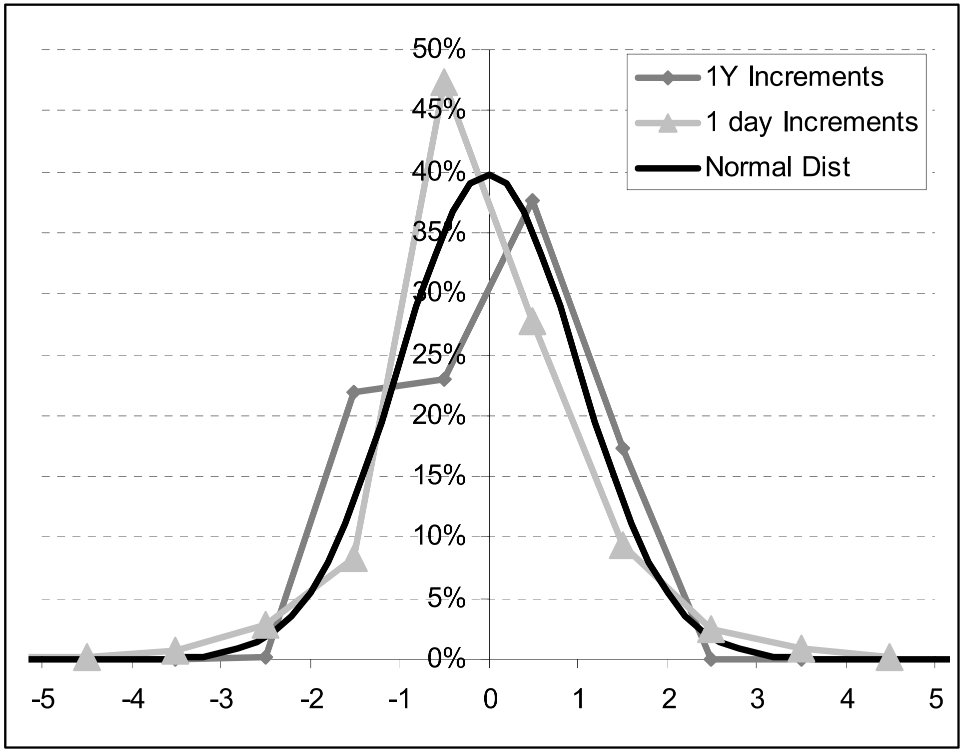
$$\mathbb{E}\left[(\Delta X_1 + \Delta X_2)^2\right] = 2\sigma^2 (1+\rho)$$

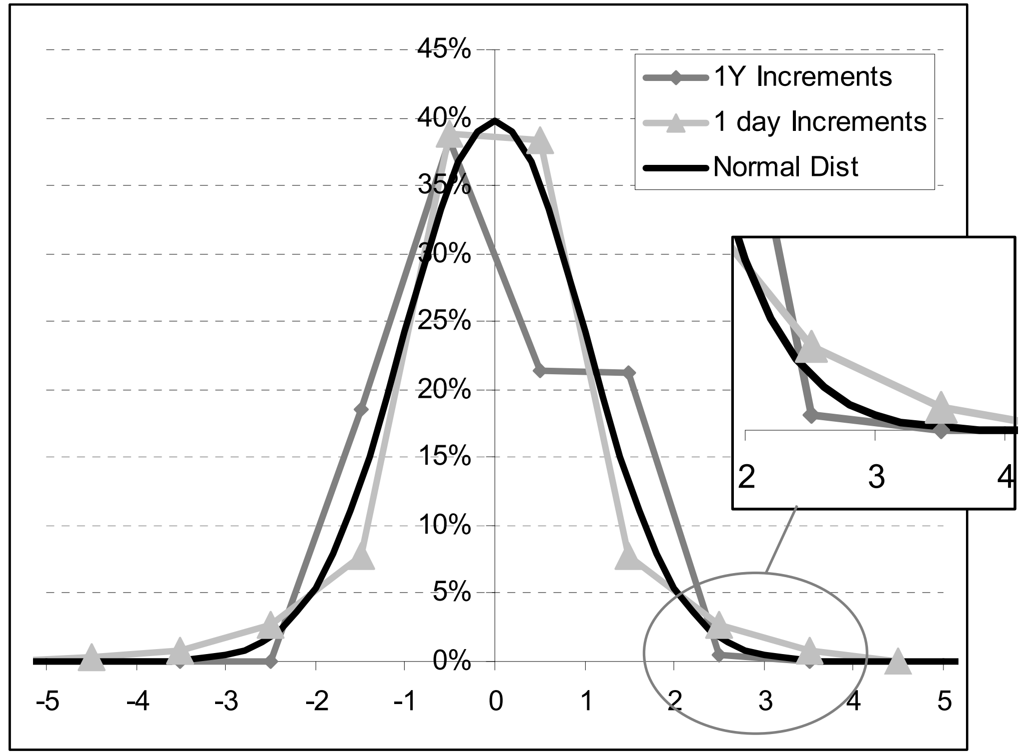


- This is a *biased* estimator of the local volatility σ but an *unbiased* estimator of the term volatility across the entire time increment. In the current context, we need an estimate of term volatility, not local volatility.

PCA Approach: Overlapping Increments/One Year Time Horizon

- Short Increments also exhibit more Leptokurtic behaviour. One year increments are closer to normal.





- Left Graph is log increments of index, Right Graph is increments in 30% base correlation. Data is coarsely bucketed but clearly shows fat tails in short time increments. We prefer normality from modelling point of view.

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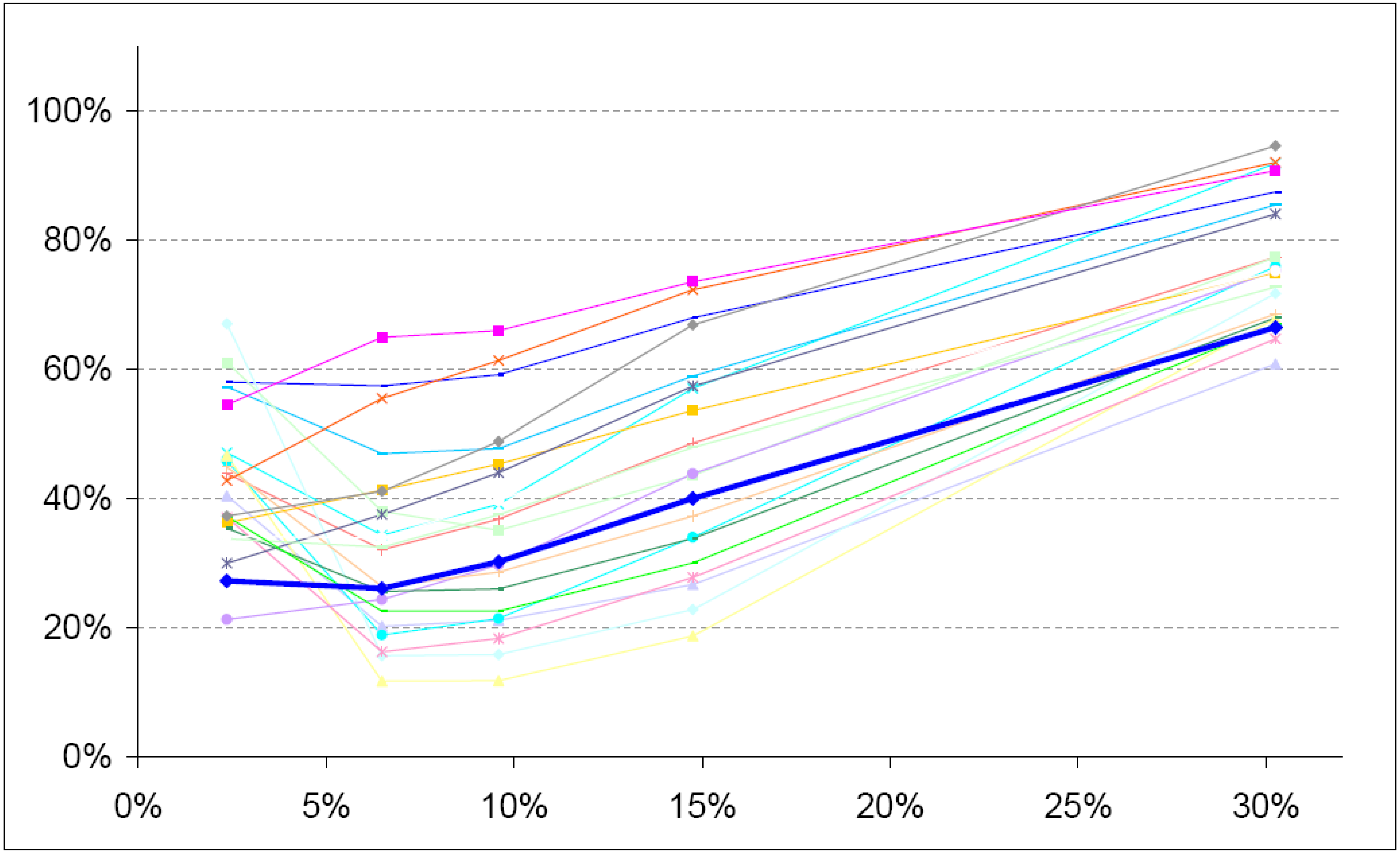
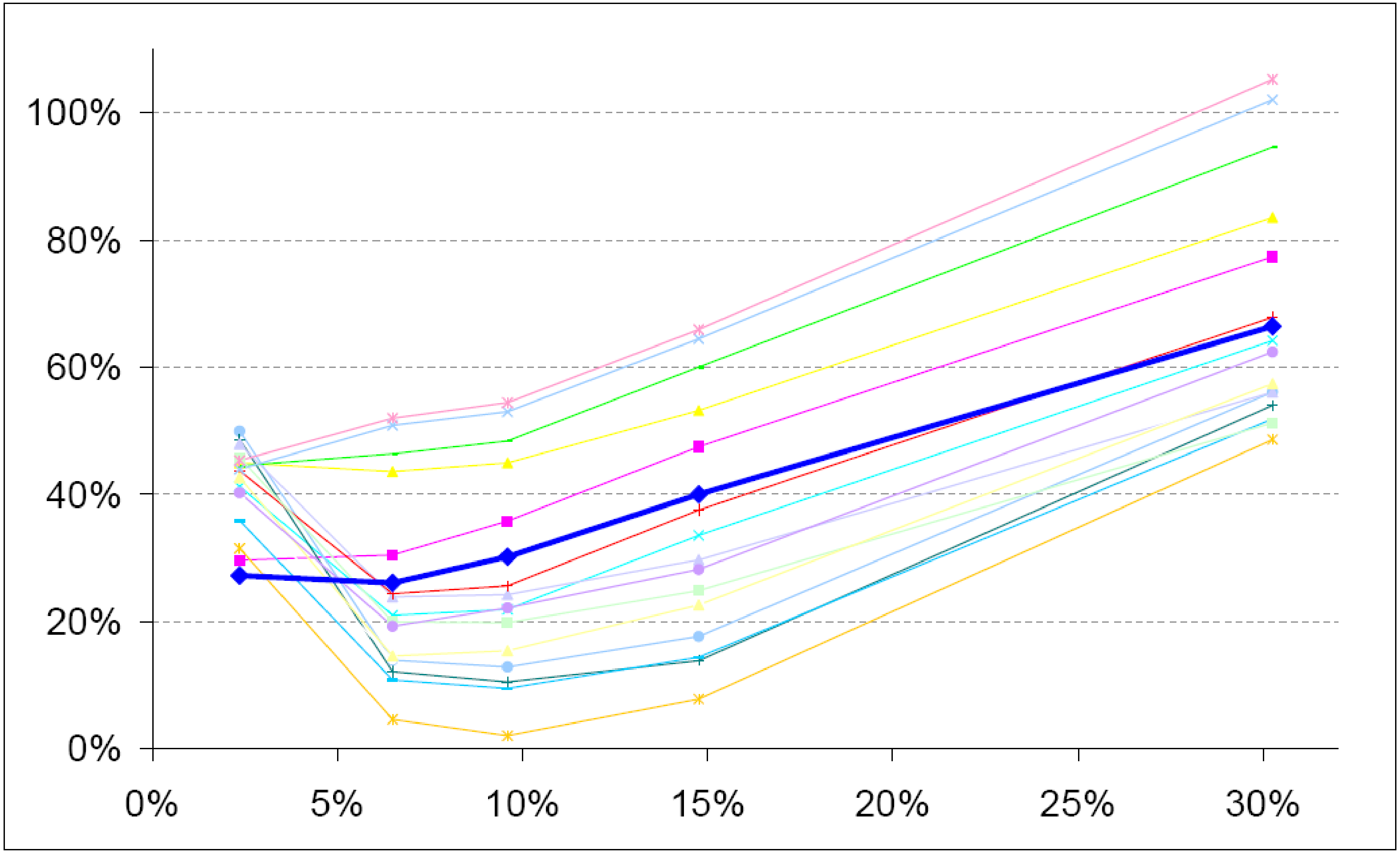
PCA Approach: Example Scenarios

- Example Scenarios for a +200bp move in underlying index:



PCA Approach: Example Scenarios

- Application of Historical shifts to current data (for similar index move):



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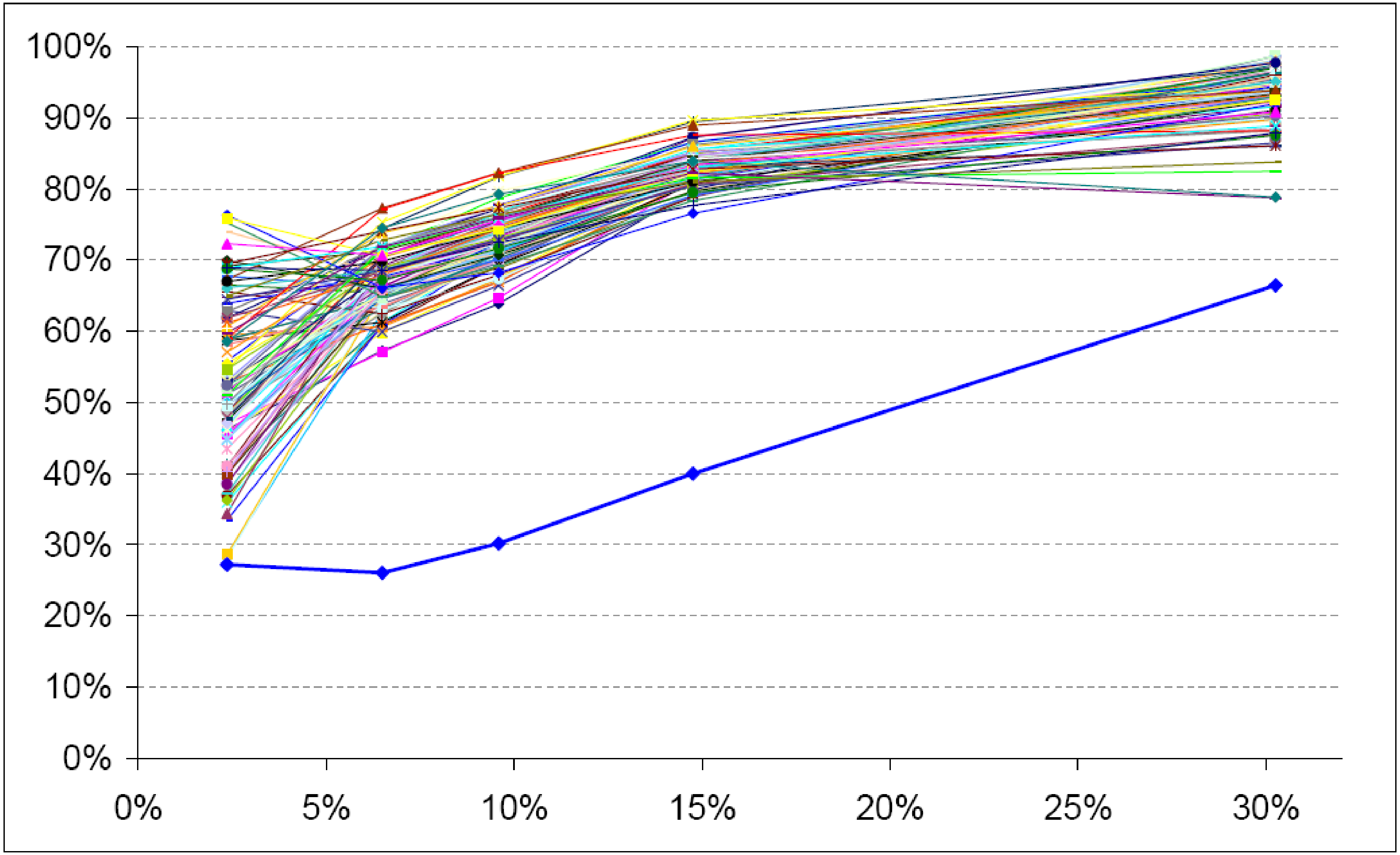
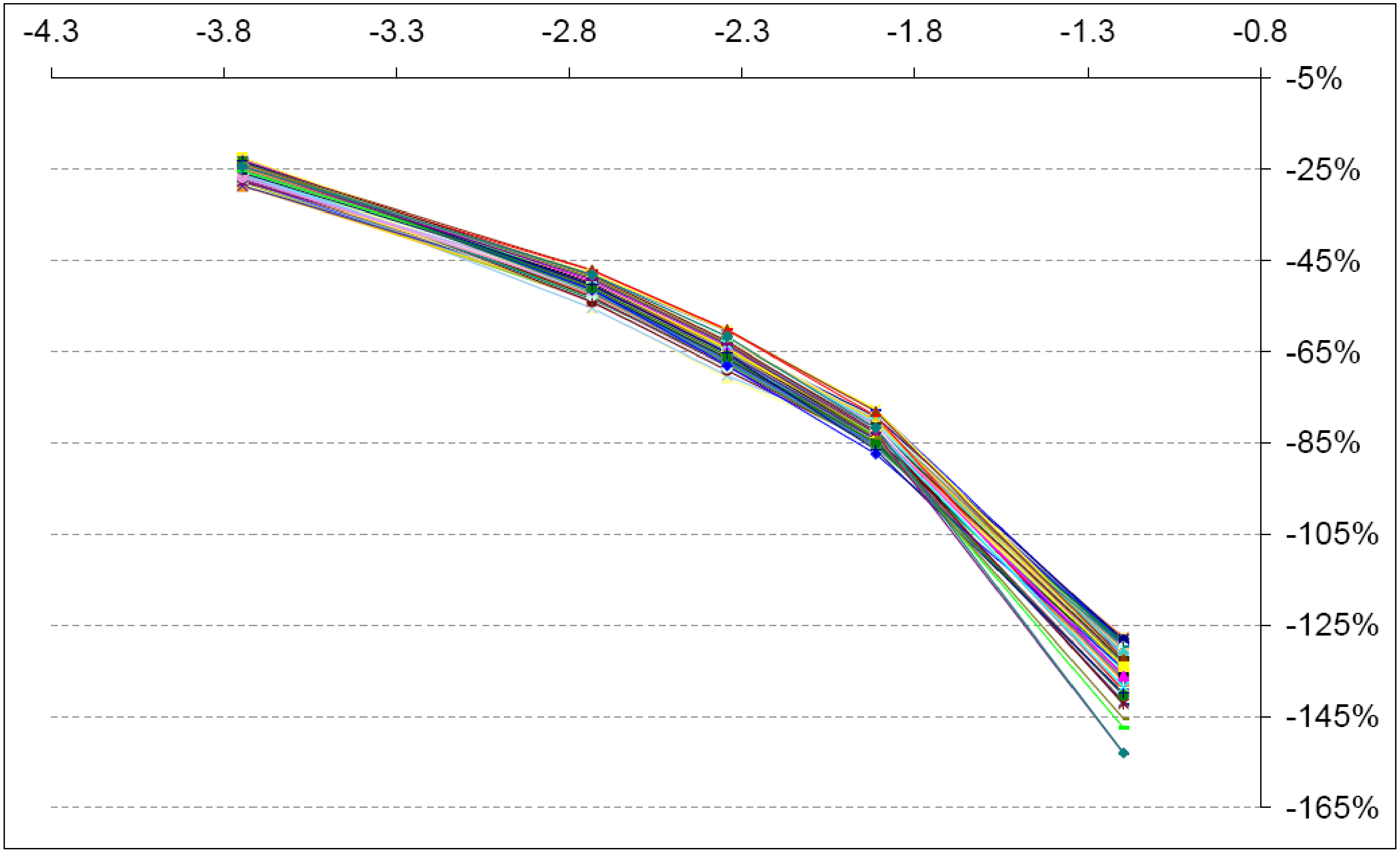
PCA Approach: Example Scenarios

- Skews for +200bp index shift at the >3σ percentile for PCA1



PCA Approach: Example Scenarios

- Corresponding log-log plot of exceedances for +200bp index shift at the >3σ percentile for PCA1



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PCA Approach: Example Scenarios

- “Failures” in Price Space at different Percentiles:

	Correlation failures	Price failures	Probability weighted
$< -3\sigma$	-	18%	0.02%
$-3\sigma \to -2\sigma$	-	15%	0.32%
$-2\sigma \to -1\sigma$	-	11%	1.49%
$-\sigma \to \sigma$	-	0%	0%
$1\sigma \to 2\sigma$	-	0%	0%
$2\sigma \to 3\sigma$	-	0%	0%
$> 3\sigma$	-	0%	0%
Total			1.84%

- Very few failures, and in this case they all occur for large negative moves i.e. low correlation scenarios. Arguably these are of less importance for CRM calculation since we are primarily concerned about massive spread widening and increased correlation, not the reverse.

Implementational Challenges: Monte Carlo, Parallelisation

- Estimating the 0.1% percentile of the distribution in a Monte Carlo simulation requires many paths. Even with 100,000 paths, only 100 scenarios on average would reach this percentile. Obvious avenues to explore are the usual arsenal of variance reduction techniques, although there are several simpler things we can do first:
- Parallelisation is essential in order to value of order 10^5 trades of order 10^5 times.
- Most pseudorandom number generators are serial and incompatible with parallelisation. However (L'Ecuyer *et al* 2001) present an approach that allows generation of multiple streams of random pseudo-random numbers.

Implementational Challenges: Optimising Individual Paths

- Avoid calibration steps: operate directly in hazard rate/correlation space. This is another reason for working in correlation space rather than tranche price space.
- Trade compression/decompression. Compress fungible trades such as index tranches, index default swaps, single name default swaps to drastically reduce trade population. Ideally implement so that it allows for "decompression" too.
- Performance Optimisation of Quant code is essential. Use fast approximations, such as conditional normal for tranche pricing (Shelton 2004), reduced numbers of time steps and integration nodes.

CRM: Implications for Pricing and Risk Management

- Selling tail risk can result in punitive charges.
- Seek cheap ways of buying tail risk. Avoid the 99.9% loss being in the high default, high spread tail. This also reduces model uncertainty- since the tail we are estimating will then no longer be the complete meltdown scenario where model assumptions are most hard to justify.
- Linearisation of risk does not work for CRM. This is indeed one of the motivations for regulators and BIS to introduce this measure in the first place. However, exposures can be understood by decomposing positions into equivalent tranches/capital structure exposures. Hence the calculation is not a complete black box.

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