1. Introduction

When estimating the sample covariance matrix of stock returns as the input to a mean-variance optimization problem and the number of stocks under consideration is large relative to the number of historical return observations, the most extreme values in the matrix have a lot of error.

The mean-variance optimization routines will place large bets on the extreme value which is unreliable because these extreme values are largely error.

The authors demonstrate a method to “pull downward” the values that have large positive error and “pull upward” the values with large negative error.

1. Formal Description of the Problem

Active portfolio management is all in relation to a benchmark index. Being long a stock means that the weight in the portfolio is greater than the weight in the benchmark. Being short a stock means that the weight in the portfolio is less than the weight in the benchmark.

Example

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | ***WB*** | ***x*** | ***WP = WB + x*** | ***y*** | *** = E(x)*** | *** =  - WB*** |
| Stock | Market Cap | Benchmark Wt | Active Wt | Portfolio Wt | Stock Returns | Expected Stock Returns | Expected Stock Excess Returns |
| A | 550 | 22.8% | -2.8% | 20.0% | 2.00% | 1.90% | 1.47% |
| B | 125 | 5.2% | 2.8% | 8.0% | 0.50% | 0.40% | 0.38% |
| C | 325 | 13.5% | -8.5% | 5.0% | -1.60% | -1.70% | -1.47% |
| D | 478 | 19.8% | 5.2% | 25.0% | 0.90% | 0.80% | 0.64% |
| E | 610 | 25.3% | -3.3% | 22.0% | -0.40% | -0.50% | -0.37% |
| F | 322 | 13.4% | -3.4% | 10.0% | 4.00% | 3.90% | 3.38% |

The optimization problem is subject to the following constraints:

|  |  |
| --- | --- |
| Min(*x'x*) | Minimize the tracking error variance |
| *x' >= g* | Meet or exceed manager's target gain |
| *x'***1** = 0 | Porfolio weights add to unity (fully invested); portfolio deviations must add to zero |
| *x* >= -*w*B | Portfolio is long only |
| *x* <= *c***1** - *WB* | Total position in a stock cannot exceed c% of the portfolio |