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一個高效率且可泛用於多種隨機過程之歐式多項式選擇權定價模型

An Efficient and General Framework for Pricing European-Style Polynomial Options under Various Stochastic Process

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致謝

時光荏苒，碩一不斷的探索金融的研究領域，一直到完成本論文，回頭看我已經走了一大段路了，還記得一開始想要作傅立葉定價相關的內容，到王之彥老師給我一個可以實現傅立葉定價理論的研究方向，讓我能持續的專研自己有興趣的學問。這段過程中我探索大量的數學理論，也在與老師討論中改進模型的應用性，這樣不斷的增加需求以及滿足論文完整的呈現，我的程式能力在過程中潛移默化地進步，從一開始只為了實現一個特定的功能，到能滿足不同需求而不用大量改動程式，我自然而然地思考降低程式間的耦合度，這段時間的訓練真的很值得，與老師的討論過程中也領悟到做研究追根究底的精神，和數據的呈現與解釋方式，都是難能可貴的技能。對於老師這段時間的指點，我的感謝感謝無以言表，同樣地非常感謝這段時間幫助過我的家人與同學，謝謝你們陪伴我度過這段刺激的碩論旅程，期許自己能記得這段時間的體悟，在未來不斷進步。

摘要

本研究引入了一種創新的方法，用於定價多項式選擇權，並提供了更強的靈活性，以適應各種隨機過程和報酬函數。本研究基於傅立葉餘弦展開方法 (Fang and Oosterlee, 2008)，這是一種利用密度函數的傅里葉餘弦展開中的定價模型。我們通過擴展傅立葉餘弦展開方法，使該方法可以為多項式選擇權進行定價，並且同樣可以為買權與賣權進行定價，達到使用同一種模型就能定價多種選擇權。本文進行了大量的蒙地卡羅模擬，闡述定價模型的準確性，並提出證據證明此模型指數誤差收斂的性質。總體而言，本研究提出了一種高效且穩健的定價方法，尤其適用於定價多項式選擇權。

關鍵詞：多項式選擇權、傅立葉餘弦展開、選擇權定價、隨機過程

Abstract

This research employs an innovative approach for pricing polynomial options, offering enhanced flexibility to adapt to various stochastic processes and payoff functions. The study is based on the Fourier cosine expansion (COS) method, which is a pricing model utilizing the Fourier cosine expansion of density functions. By extending the COS method (Fang and Oosterlee, 2008), I can price polynomial options, encompassing both call and put options, under different stochastic models. Extensive Monte Carlo simulations are conducted to attest to the accuracy of the pricing model and provide evidence of its exponential convergence property. Overall, this research presents an efficient and robust pricing method, particularly suitable for pricing polynomial options.

Keywords: polynomial options, Fourier cosine expansion, options pricing model, stochastic process

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