

**CISC/CMPE452/COGS 400**  
**Mathematics and Calculus**

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# Differential Calculus and Partial Derivative

- Differential Calculus: a) If a linear function  $y = mx + b$ , where  $x$  is the independent variable,  $y$  is the dependent variable,  $b$  is the  $y$ -intercept, and slope  $m = dy/dx$  indicates change in  $x$  is equal to change in  $y$  with respect to  $x$   
b) If  $f$  is a function that takes a time as input and gives the position of a ball at that time as output, then the derivative of  $f$  is how the position is changing in time, that is, it is the velocity of the ball.
- Partial derivative – A partial derivative of a function of several variables is its derivative with respect to one of those variables, with the others held constant (as opposed to the total derivative, in which all variables are allowed to vary). If  $f(x,y,...) = x^2 + xy + y^2$ , assuming  $y$  is constant  $\partial f / \partial x = 2x + y$

# Chain Rule

- The chain rule may be written, in Leibniz's notation, in the following way. We consider  $z$  to be a function of the variable  $y$ , which is itself a function of  $x$  ( $y$  and  $z$  are therefore dependent variables), and so,  $z$  becomes a function of  $x$  as well:

$$\partial z / \partial x = \partial z / \partial y \cdot \partial y / \partial x$$

# Matrix Calculation

- Matrix  $w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$  Transpose  $w^T = \begin{bmatrix} w_1 & w_2 & w_3 \end{bmatrix}$
- Matrix multiplication  $w^T x = \begin{bmatrix} w_1 & w_2 & w_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = w_1 x_1 + w_2 x_2 + w_3 x_3$
- In linear algebra, an  $n$ -by- $n$  square matrix  $\mathbf{A}$  is called **invertible** (also **nonsingular** or **nondegenerate**) if there exists an  $n$ -by- $n$  square matrix  $\mathbf{B}$  such that  $\mathbf{AB} = \mathbf{BA} = \mathbf{I}_n$  where  $\mathbf{I}_n$  denotes the  $n$ -by- $n$  **identity** matrix and the multiplication used is ordinary matrix multiplication. If this is the case, then the matrix  $\mathbf{B}$  is uniquely determined by  $\mathbf{A}$  and is called the **inverse** of  $\mathbf{A}$ , denoted by  $\mathbf{A}^{-1}$ .
- A square matrix that is not invertible is called **singular** or degenerate. A square matrix is **singular** if and only if its **determinant** is 0.

# Matrix

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- Covariance Matrix
- Eigenvalue and Eigenvector

# Distance

- Euclidean distance =  $\sqrt{|(a-x)|^2 + |(b-x)|^2 + |(c-x)|^2}$
- Manhattan distance =  $|a-x| + |b-x| + |c-x|$
- Hamming distance = Gives the number of elements whose values disagree between  $u$  and  $v$ . If  $u = (1 \ 0 \ 1)$  and  $v = (0 \ 0 \ 1)$  then the hamming distance between them is 2.

<https://reference.wolfram.com/language/guide/DistanceAndSimilarityMeasures.html>