## Homework 3

#### MSBA 6450

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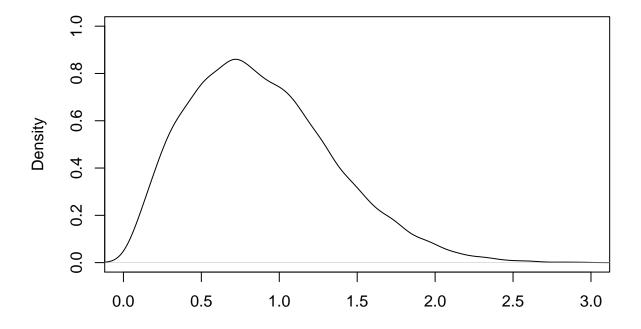
#### Problem 1

Simulate 10,000 random samples from the distribution with the following CDF starting with numbers generated from a uniform distribution  $U[0\ ,1]$ 

```
set.seed(124345)
NSim=10^4
U=runif(NSim)
Z=(-log(1-U))^(0.5) # The Inverse transformation of the F(x) function

#Validation
set.seed(124345)
Y=rweibull(2000,shape=2,scale=1)
plot(density(Z), xlim =c(0,3), ylim = c(0,1), main = "From Uniform", xlab=NA)
```

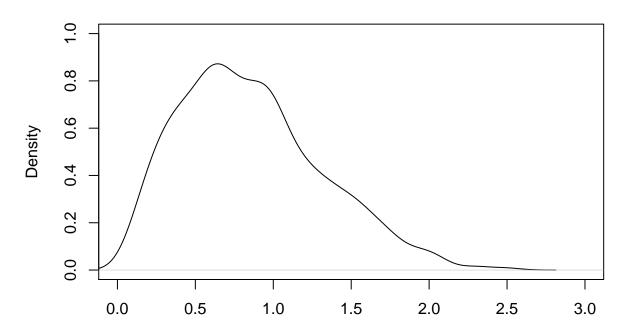
#### From Uniform

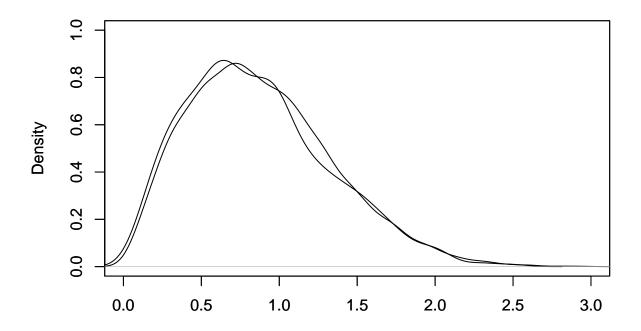


```
par(mfrow=c(1,1))

plot(density(Y), xlim = c(0,3), ylim = c(0,1), main = "From Weibull", xlab=NA)
```

### **From Weibull**



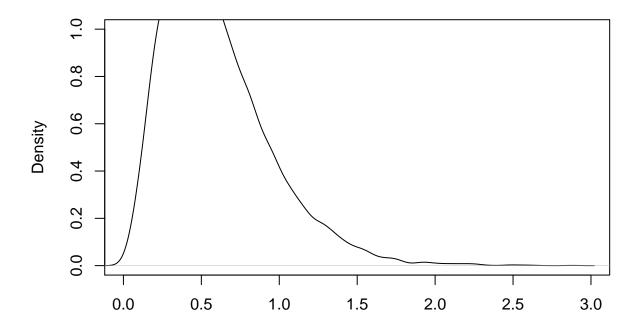


#### Problem 2: General Transformation

```
set.seed(123456)
NSim=3*10^4
U=runif(NSim)
U=matrix(data=U, nrow=3)
X=-log(U) #uniform to exponential
X=1/5*apply(X,2,sum) #sum up to get chisquares

#Validation
set.seed(123456)
Y=rgamma(10000, 3, 5)
plot(density(X), xlim =c(0,3), ylim = c(0,1), main = "From Uniform", xlab=NA)
```

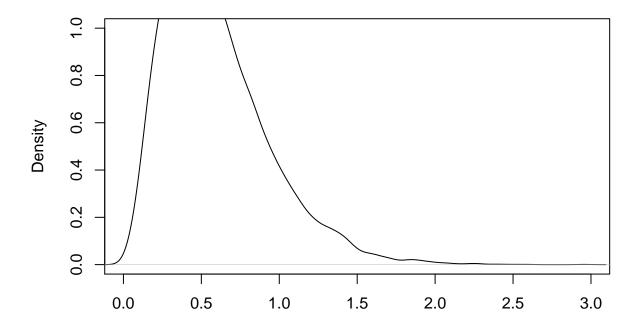
### **From Uniform**



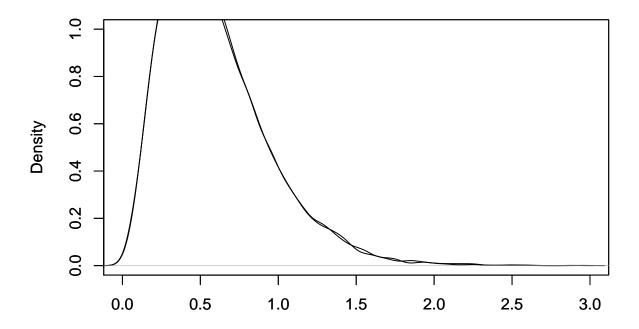
```
par(mfrow=c(1,1))

plot(density(Y), xlim = c(0,3), ylim = c(0,1), main = "From Gamma", xlab=NA)
```

### From Gamma



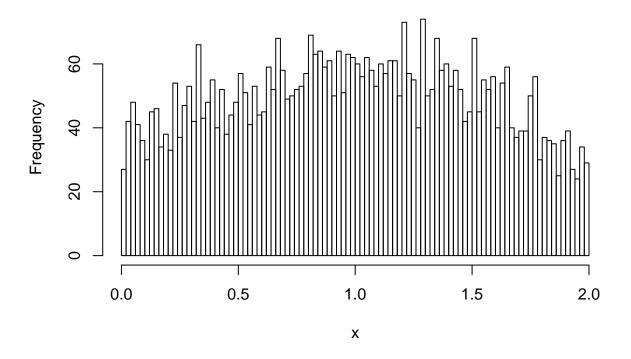
```
#Alternate View
                            =c(0,3),
                                        ylim = c(0,1),
                                                                         "",
plot(density(X),
                     xlim
                                                             main
xlab=NA)
par(new=TRUE)
plot(density(Y),
                            =c(0,3),
                                                                         Шη,
                                         ylim = c(0,1),
                     xlim
                                                             main
xlab=NA)
```



## Problem 3: Accept Reject Method

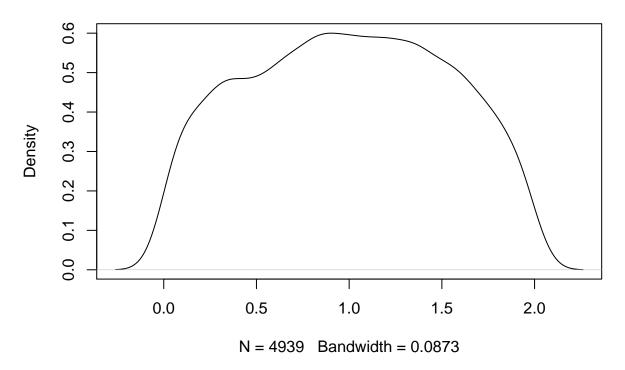
```
set.seed(123456)
u<-runif(10000, 0, 2)
y<-runif(10000, 0, 1)
x<-u[y<(3/10)*(-u^2+2*u+1)]
hist(x, breaks = 100)</pre>
```

# Histogram of x



plot(density(x))

### density.default(x = x)

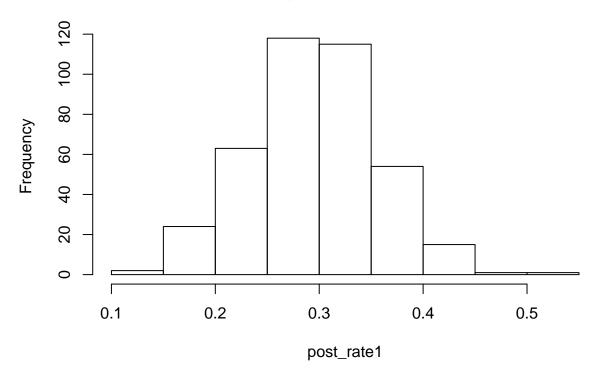


### Problem 4: Bayesian Data Analysis

```
# Data = 15 out of 50, beta prior
# Data = 25 our of 75, beta prior
set.seed(123456)
#Priors
n_draw <- 10000
prior_rate1 <- rbeta(n_draw,2,5)

gen_model1 <- function(rate) {
    subscribers <- rbinom(1, size = 50, prob = rate)
    subscribers
}
subscribers1 <- rep(NA, n_draw)
for(i in 1:n_draw) {
    subscribers1[i] <- gen_model1(prior_rate1[i])
}
post_rate1 <- prior_rate1[subscribers1 == 15]
hist(post_rate1)</pre>
```

## Histogram of post\_rate1



```
#Chaining Updates in Beliefs. Yesterday's post is today's prior

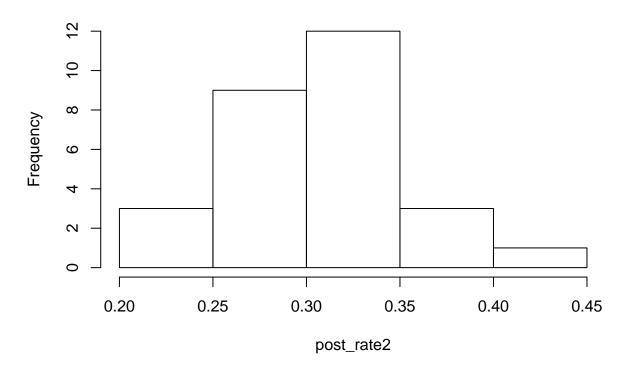
prior_rate2 <- post_rate1

gen_model2 <- function(rate) {
    subscribers <- rbinom(1, size = 75, prob = rate)
    subscribers
}
subscribers2 <- rep(NA, length(prior_rate2))
for(i in 1:length(prior_rate2)) {
    subscribers2[i] <- gen_model2(prior_rate2[i])
}

post_rate2 <- prior_rate2[subscribers2 == 25]</pre>

hist(post_rate2)
```

## Histogram of post\_rate2



```
quantile(post_rate2, c(0.025, 0.975))

## 2.5% 97.5%
## 0.2367445 0.4062711

quantile(post_rate1, c(0.025, 0.975))

## 2.5% 97.5%
## 0.1766791 0.4071628
```