Homework 1

MSBA 6450

William Wu (wuxx1066@umn.edu), Jingjing Olivia Liang (liang625@umn.edu)

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```r
#install.packages("lpSolve")
library(lpSolve)
```

```
## Warning: package 'lpSolve' was built under R version 3.5.3
```

Problem 1

A manufacturing firm in California, Calmetal, produces four different metal products, each of which must be machined, polished, and assembled. The specific time requirements (in hours) for each product are as follows:

The firm has available to it on a weekly basis 480 hours of machine time, 400 hours of polishing time, and 400 hours of assembly time. The unit profits on the products are \$6, \$4, \$6, and \$8, respectively. The firm has a contract with a distributor to provide 50 units of product I and 100 units of any combination of products II and III each week. Through other customers, the firm can sell each week as many units of products I, II, and III as it can produce, but only a maximum of 30 units of product IV. Moreover, local Government regulations mandate that at most 25 units of product IV can be produced weekly. How many units of each product should the firm manufacture each week to meet all contractual and regulatory obligations while maximizing its total profit? Assume that any unfinished pieces can be completed the following week (i.e., integer values of units is not required). What is the maximum profit?

Machining:

3a + 2b + 2c + 4d <= 480

Polishing:

a + b + 2c + 3d <= 400

Assembling:

2a + b + 2c + d <= 400

Where Product I = a, Product II = b, etc.

Additional Constraints:

Distribute 50 units of Product I

a >= 50

100 units of any combo of II and III

b + c >= 100

Government mandates no more than 25 units (overrides the 30 unit constraint)

d <= 25

Objective Function:

6a + 4b + 6c + 8d = Total Profit

```
obj_fun <- c(6,4,6,8)
constr_eq <- matrix(c(3,2,2,4,
                      1,1,2,3,
                      2,1,2,1,
                      1,0,0,0,
                      0,1,1,0,
                      0,0,0,1), ncol = 4, byrow=TRUE)
constr_dir <- c("<=","<=","<=",">=",">=","<=")
constr_rhs <- c(480, 400, 400, 50, 100, 25)
solution <- lp("max", obj_fun, constr_eq, constr_dir, constr_rhs)

solution$objval
```

```
## [1] 1250
```

```
solution$solution
```

```
## [1]  50   0 145  10
```

The firm should create 50 of Product A, 0 of Product B, 145 of Product C, and 10 of Product D The total profit is $1250.

Problem 2

The manager of a supermarket meat department finds she has 200lb of round steak, 800 lb of chuck steak, and 150lb of pork in stock on Saturday morning, which she will use to make hamburger meat, picnic patties, and meat loaf. The demand for each of these items always exceeds the supermarket's supply. Hamburger meat must be at least 20% ground round and 50% ground chuck (by weight); picnic patties must be at least 20% ground pork and 50% ground chuck; and meat loaf must be at least 10% ground round, 30% ground pork, and 40% ground chuck. The remainder of each product is an inexpensive nonmeat filler which the store has in unlimited supply. How many pounds of each product should be made if the manager desires to minimize the amount of meat that must be stored in the supermarket over Sunday? How much meat needs to be stored over Sunday?

Round Steak

$0.2a + 0.1c <= 200$

Chuck Steak

$0.5a + 0.5b + 0.4c <= 800$

Pork

$0.2b + 0.3c <= 150$

Where Hamburger = a, Picnic Patties = b, and Meat Loaf = c

Objective Function

a+b+c= Total Pounds of Meat

Because the demand exceeds the supply, all the meat will be sold.

To minimize the amount of meat stored, we need to maximize the amount of meat products created.

By maximizing the amount of meat products created, we will also maximize profits as a byproduct.

Assuming: You can sell a half pound of meat. No integers are necesarry. I can totally buy half a pound of hamburger from a local butcher.

```r
obj_fun <- c(1,1,1)
constr_eq <- matrix(c(0.2, 0.0, 0.1,
                      0.5, 0.5, 0.4,
                      0.0, 0.2, 0.3), ncol = 3, byrow=TRUE)
constr_dir <- c("<=","<=","<=")
constr_rhs <- c(200, 800, 150)
solution <- lp("max", obj_fun, constr_eq, constr_dir, constr_rhs)

solution$objval
```

```
## [1] 1625
```

```r
solution$solution
```

```
## [1] 937.5 562.5 125.0
```

Based on the solution, we create 1625 total pounds of meat products. 937.5 lbs Hamburger, 562.5 lbs Picnic Patties, and 125 lbs Meat Loaf.

Plugging these values back into the original meat equations gives the amount of each meat left over. The total amount of meat stored over Sunday is as follows.

```r
#Round Steak
#Since a hamburger is 20% Round Steak, and a Meatloaf is 10%, using the following equation calculates t
200 - (927.5*0.2 + 0.1*125)
```

```
## [1] 2
```

```r
#Repeat for the other meats
#Chuck Steak
800 - (927.5*0.5 + 562.5*0.5 + 125*0.4)
```

```
## [1] 5
```

```r
#Pork
150 - (562.5*0.2 + 125*0.3)
```

```
## [1] 0
```

With this distribution, there is a total of 2lbs of Round Steak and 5lbs of Chuck Steak, for a total of 7lbs remaining.

The constraining resource is pork, since there isn't a product that only uses Round and Chuck, likely cannot use any more.

However, since the recipes for each of the meat products calls for "at least" x% of meat, you could theoretically replace some of the filler meat in any of the products with the leftover meats to completely use it all. In this product mix, there is at least 400 lbs of filler.

Problem 3

The Aztec Refining Company of Texas produces two types of unleaded gasoline, regular and premium, which it sells to its chain of service stations around the state for \$12 and \$14 per barrel, respectively. Both types are blended from Aztec's inventory of refined domestic oil and refined foreign oil, and must meet the following specifications:

The company wants to decide what quantities (i.e., blended barrels) of the oils should it blend into the two gasolines in order to maximize weekly profit? What is the maximum weekly profit?

Question Outline

a = Regular

b = Premium

c = Domestic

d = Foreign

Vapor Pressure and Octane Ratings Constraints

23a + 23b - 25c - 15d >= 0

88a + 93b - 87c - 98d >= 0

Capacity Constraints

a >= 50000

b >= 5000

c <= 40000

d <= 60000

a <= 100000

b <= 20000

```
obj_fun <- c(12,14,-8,-15)
constr_eq <- matrix(c(23,23,-25,-15,
                      88,93,-87,-98,
                      1,0,0,0,
                      0,1,0,0,
                      0,0,1,0,
                      0,0,0,1,
                      1,0,0,0,
                      0,1,0,0), ncol = 4, byrow = TRUE)
constr_dir <- c('>=', '<=', '>=','>=', '<=','<=','<=','<=')
constr_rhs <- c(0, 0, 50000,5000, 40000,60000,100000,20000)

solution <- lp('max', obj_fun, constr_eq, constr_dir, constr_rhs)
solution$objval
```

```
## [1] 138010.2
```

```
solution$solution
```

```
## [1] 50000.00  5000.00 40000.00 14132.65
```

Conclusion: The company should blend 40,000 bbl Domestic oil and 14132.65 bbl foreign oil into the two gasolines to maximize weekly profit. The maximum weekly profit is \$138010.20.