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Faculty of Engineering, Built Environment and Information Technology

ENE310

Filter Design

Practical 2

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1 Introduction

Filter design is an important process for deigning unique high quality filters. The designer is able to create and optimize the filter based on the topology and hyper parameters it is made up of. The different design parameters require optimisation and fine tuning to maximise the design goals and to compensate for physical deficiencies which may exist. The design process and parameters are investigated.

2 Design objectives

Design two eighth order filters. One where a second order filter is copy and pasted (CP) four times to make an eighth order system. The second is designed using cascaded design with frequency and quality factor scaling. The filter is required to be a Bessel filter with a cutoff (design) frequency of 56.8KHz. The topology of the circuit must be researched and chosen (KRC, Multifeedback). The quality factor and frequency may then be observed from the simulations. The component tolerances must be investigated with different resistor and capacitor standards. The hyper parameters such as the quality may be affected by the component tolerance. Monte Carlo may also be used to investigate the tolerances. Physical op-amps are also investigated to simulate affects the likes of voltage offset and gain bandwidth product on the filter and it's response.

3 Detail Design

3.1 Design Choice

A multifeedback low pass filter was chosen for design and synthesis. The multifeedback topology allows potentially easier manipulation of the quality factor (Q) and gain at the expense of more components. Multifeedback circuit's also do not lose accuracy as fast as sallen-key circuits do at higher gains. A gain of ten is chosen for the design. Multifeedback circuits are also less sensitive to component changes (variance) which is beneficial when the tolerance of the component is unpredictable. At higher Q values the sallen-key may ripple more than the multifeedback around the resonance frequency. Albeit a Bessel filter is being designed for thus an extremely high Q will be required for any ripple to exist.

Sallen-keys however do possess more accuracy around the cut-off frequency. They will begin to attenuate closer to the value than a multifeedback would. However Bessel as it is normalized to be cascaded it loses a bit of accuracy around the cut off frequency as well. An alternate design with KRC will be included to observe the accuracy difference around the cut-off. The KRC also possess a lower noise gain than the multifeedback at lower frequencies.

The trade-off of a slightly less accurate low pass with multifeedback around the design frequency but better scaling with Q, and the gain due to more customization with components deems multifeedback as the choice for the design. The noise gain may be reduced into the circuit through physical measures such as buffers, shorter wires and the ordering of the cascade discussed further in the report.

3.2 Transfer function

The low pass transfer function of a second order circuit is given[1]:

$$H(jf) = \frac{H_0}{1 - (\frac{f}{f_c})^2 + (\frac{jf}{f_c})/Q} \tag{1}$$

The function is realized within python, code is attached in appendix B.

3.3 Second order Copy paste

From the cascade design table for a second order Bessel filter cascaded through copy and pasting (CP) to an eigth order:
Q = 0.577
f_{scale} = 1.274

$$H_{CP}(jf) = \left(\frac{H_0}{1 - (\frac{f}{f_{scale}*f_c})^2 + (\frac{jf}{f_{scale}*f_c})/Q}\right)^4 \tag{2}$$

The transfer function is multiplied by itself four times in identical cascaded design.

3.4 Eight order from cascaded design

Table of the different scaling frequencies and quality factors for each stage of eighth order Bessel:

Stage (n)	$f_{scale-n}$	Q_n
1	1.784	0.506
2	1.838	0.560
3	1.958	0.711
4	2.196	1.226

The transfer function follows the form of a second order above with each function being a stage with different Q and scaling frequencies:

$$\begin{aligned} H(jf) = &\left(\frac{\sqrt[4]{H_0}}{1 - (\frac{f}{f_{scale-1}*f_c})^2 + (\frac{jf}{f_{scale-1}*f_c})/Q_1}\right) * \left(\frac{\sqrt[4]{H_0}}{1 - (\frac{f}{f_{scale-2}*f_c})^2 + (\frac{jf}{f_{scale-2}*f_c})/Q_2}\right) * \\ &\left(\frac{\sqrt[4]{H_0}}{1 - (\frac{f}{f_{scale-3}*f_c})^2 + (\frac{jf}{f_{scale-3}*f_c})/Q_3}\right) * \left(\frac{\sqrt[4]{H_0}}{1 - (\frac{f}{f_{scale-4}*f_c})^2 + (\frac{jf}{f_{scale-4}*f_c})/Q_4}\right) \end{aligned} \tag{3}$$

3.5 Multifeedback implementation

3.5.1 Design equations

The equations are applied in the following order to derive components values:
Determining the capacitance spread value n:

$$n \geq 4 * Q^2(1 + H_0) \tag{4}$$

Choosing capacitor values such that:

$$n = \frac{C_1}{C_2} \tag{5}$$

R_3 may then be found:

$$R_3 = \frac{1 + \sqrt{1 - 4 * Q^2 * (1 + H_0)/n}}{2 * \omega_0 * Q * C_2} \tag{6}$$

and subsequently R_1 and R_2:

$$R_1 = \frac{R_3}{H_0} \tag{7}$$

$$R_2 = \frac{1}{\omega_0^2 * R_3 * C_1 * C_2} \tag{8}$$

Smaller values for C_1 and C_2 allow for larger values for the resistors which prevents excessive power draw from small resistances and large currents.

3.6 Ideal component design

The formulae above used in conjunction with the design frequency, frequency scaling factors, and Q are used to design an ideal filter realization. The ideal filter realizations do not use any set component standard. Raw numerical values are used for each component where possible.
The design frequency is 56.8KHz and a value of 100 pico-farads for capacitor C_2 is chosen.

3.6.1 Copy and paste second order filter

The gain is spread evenly among each stage as each stage is a duplicate.

Table showing ideal values of copy and paste filter:

H0	Fn	Q	n_chosen	C1	C2	R1	R2	R3
1.778	1.274	0.577	3.699	3.699E-10	1.0E-10	1.07E+04	6.86E+03	1.91E+04
1.778	1.274	0.577	3.699	3.699E-10	1.0E-10	1.07E+04	6.86E+03	1.91E+04
1.778	1.274	0.577	3.699	3.699E-10	1.0E-10	1.07E+04	6.86E+03	1.91E+04
1.778	1.274	0.577	3.699	3.699E-10	1.0E-10	1.07E+04	6.86E+03	1.91E+04

3.6.2 Cascaded second order filter

Gain is also spread between each stage evenly.

Table showing ideal values of cascaded filter:

H0	Fn	Q	n_chosen	C1	C2	R1	R2	R3
1.778	1.784	0.506	2.845	2.845E-10	1.000E-10	8.73E+03	5.59E+03	1.55E+04
1.778	1.838	0.56	3.485	3.485E-10	1.000E-10	7.65E+03	4.90E+03	1.36E+04
1.778	1.958	0.711	5.617	5.618E-10	1.000E-10	5.66E+03	3.62E+03	1.01E+04
1.778	2.196	1.226	16.703	1.670E-09	1.000E-10	2.93E+03	1.87E+03	5.20E+03

3.6.3 LTspice ideal copy-paste filter diagram

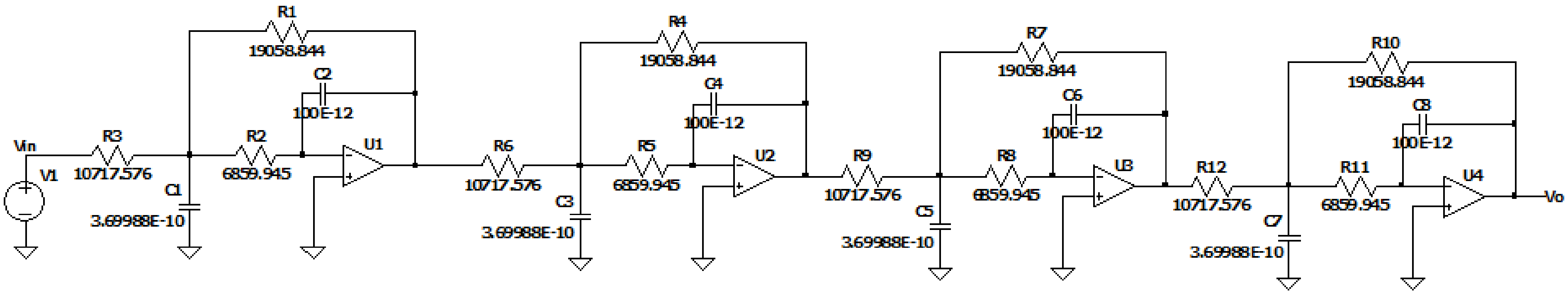

Figure 1: Ideal copy-paste filter diagram

3.6.4 LTspice ideal cascade filter diagram

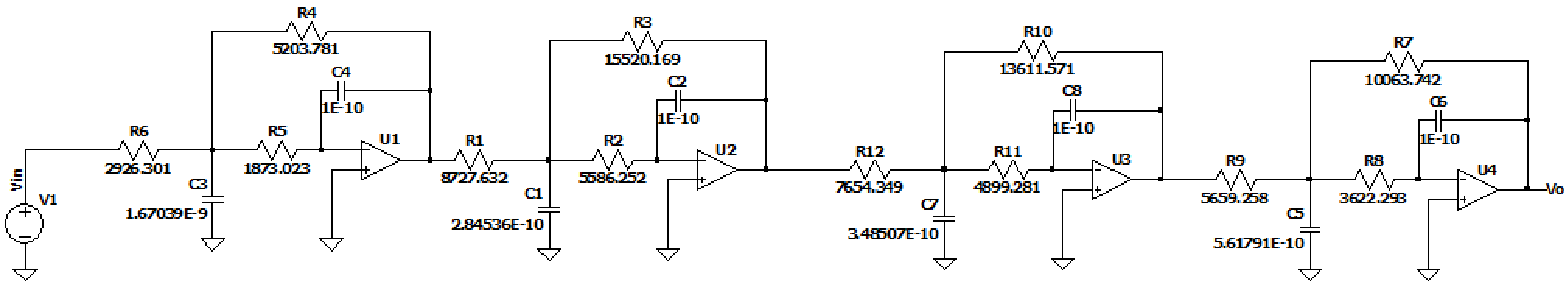

Figure 2: Ideal cascade filter diagram

3.7 E6/E12 Resistor and Capacitor component design

The values from the ideal component design tables are approximated with real world components using the E6 resistor and E12 Capacitor standards.

3.7.1 Copy and paste second order filter

Table showing E6/E12 values of copy and paste filter:

C1	C2	R1	R2	R3
3.60E-10	1E-10	1.00E+04	6.81E+03	1.91E+04
3.60E-10	1E-10	1.00E+04	6.81E+03	1.91E+04
3.60E-10	1E-10	1.00E+04	6.81E+03	1.91E+04
3.60E-10	1E-10	1.00E+04	6.81E+03	1.91E+04

3.7.2 Cascaded filter

Table showing E6/E12 values of cascaded filter:

C1	C2	R1	R2	R3
2.70E-10	1E-10	1.00E+04	4.70E+03	1.50E+04
3.30E-10	1E-10	6.80E+03	4.70E+03	1.50E+04
5.60E-10	1E-10	4.70E+03	3.30E+03	1.00E+04
1.80E-09	1E-10	3.30E+03	2.20E+03	4.70E+03

3.7.3 LTspice E6/E12 copy-paste filter diagram

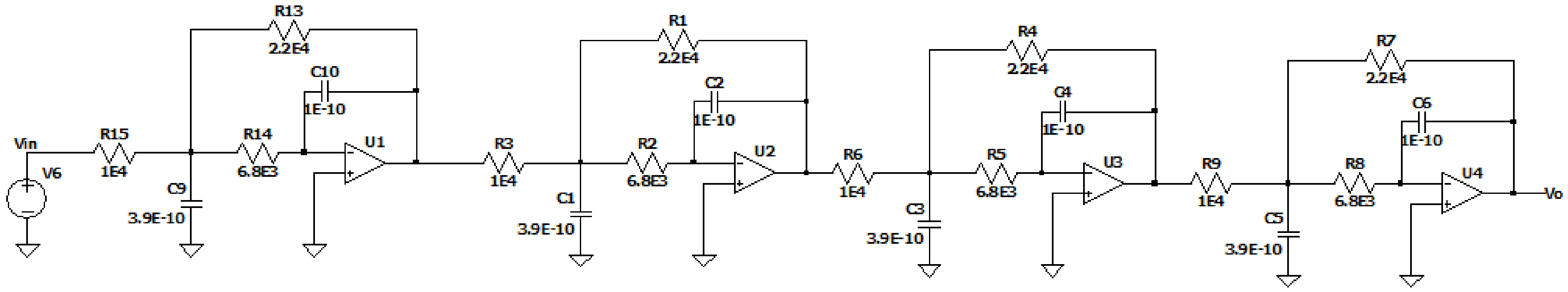

Figure 3: Ideal copy-paste filter diagram

3.7.4 LTspice E6/E12 cascade filter diagram

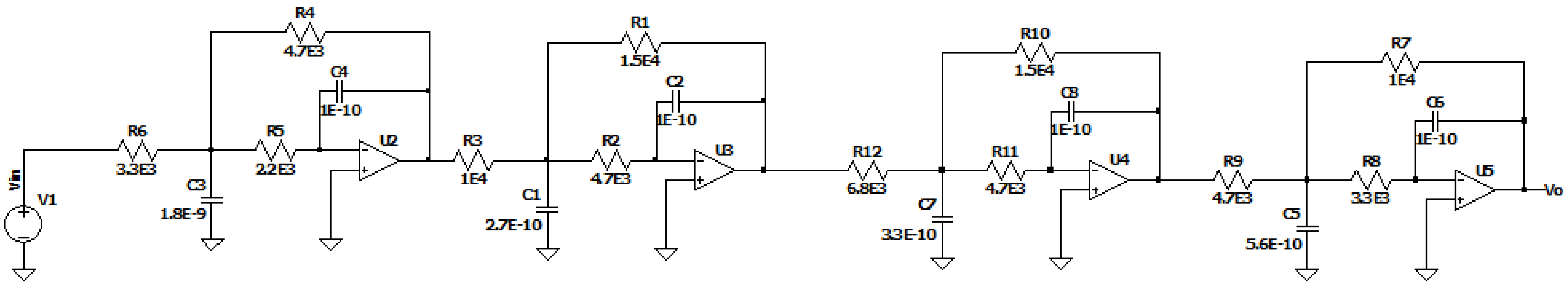

Figure 4: Ideal cascade filter diagram

3.8 Alternate tolerance: E96/E24 Resistor and Capacitor component design

In addition to the E6/E12 standard, the values from the ideal component design tables are approximated with real world components using the E96 resistor and E24 Capacitor standards. These values are used in conjunction with non ideal op-amps for testing.

3.8.1 Copy and paste second order filter

Table showing E6/E12 values of copy and paste filter:

C1	C2	R1	R2	R3
3.90E-10	1E-10	1.00E+04	6.80E+03	2.20E+04
3.90E-10	1E-10	1.00E+04	6.80E+03	2.20E+04
3.90E-10	1E-10	1.00E+04	6.80E+03	2.20E+04
3.90E-10	1E-10	1.00E+04	6.80E+03	2.20E+04

3.8.2 Cascaded filter

Table showing E6/E12 values of cascaded filter:

C1	C2	R1	R2	R3
2.70E-10	1E-10	1.00E+04	4.70E+03	1.50E+04
3.30E-10	1E-10	6.80E+03	4.70E+03	1.50E+04
5.60E-10	1E-10	4.70E+03	3.30E+03	1.00E+04
1.80E-09	1E-10	3.30E+03	2.20E+03	4.70E+03

3.8.3 LTspice E96/E24 copy-paste filter diagram

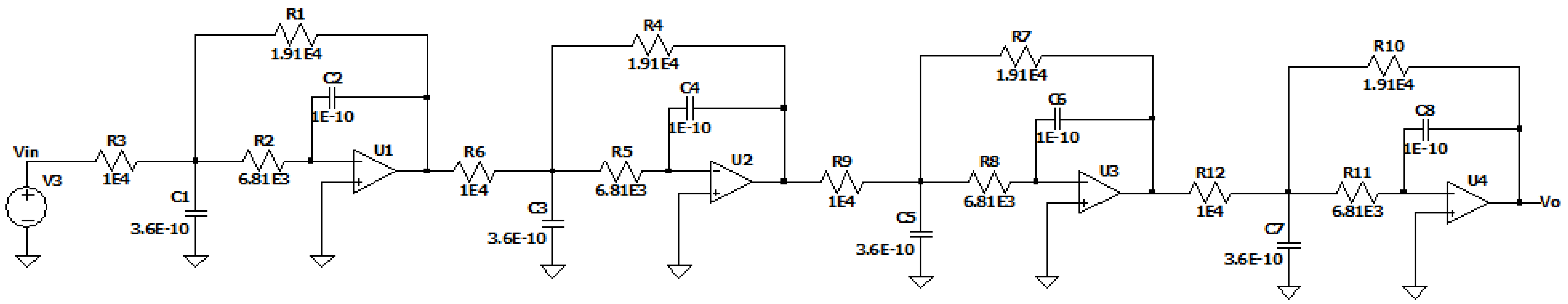

Figure 5: Ideal copy-paste filter diagram

3.8.4 LTspice E96/E24 cascade filter diagram

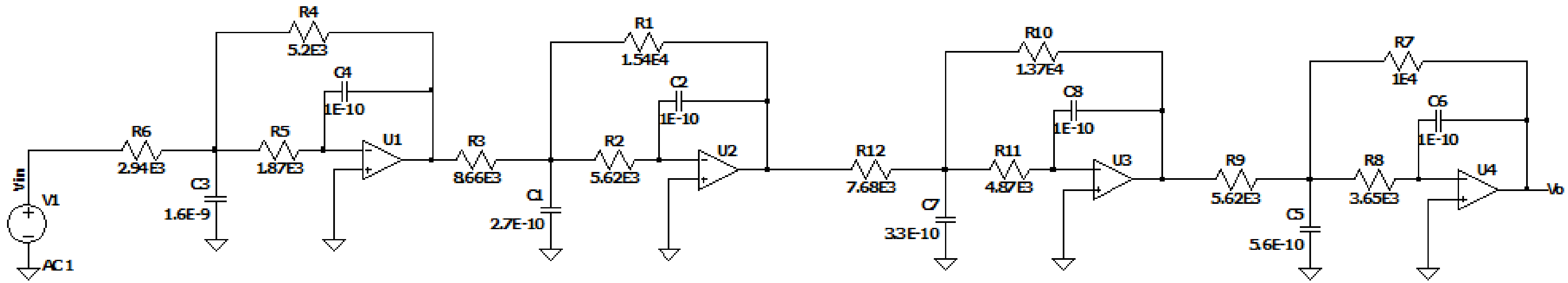

Figure 6: Ideal cascade filter diagram

3.9 Op-amp considerations

Two operational amplifier designs are considered within the investigation for component variations:

1. Ideal universal op-amp for ideal and single parameter testing considerations:
   - Gain bandwidth product is fixed at 10MHz
   - Voltage off-set is set to zero
2. LF347 op-amp for practical-real-life considerations
   - Gain bandwidth product is 4MHz
   - Voltage off-set is given as 5mV typically
3. Additionally the MCP604's frequency response is observed
   - Gain bandwidth product is 10MHz
   - Voltage off-set is given as 5μV typically

3.10 Non-ideal circuits

The E24/E96 resistor and capacitor standard circuit used in conjunction with the MCP604 and LF347 op-amps for non-ideal simulations.

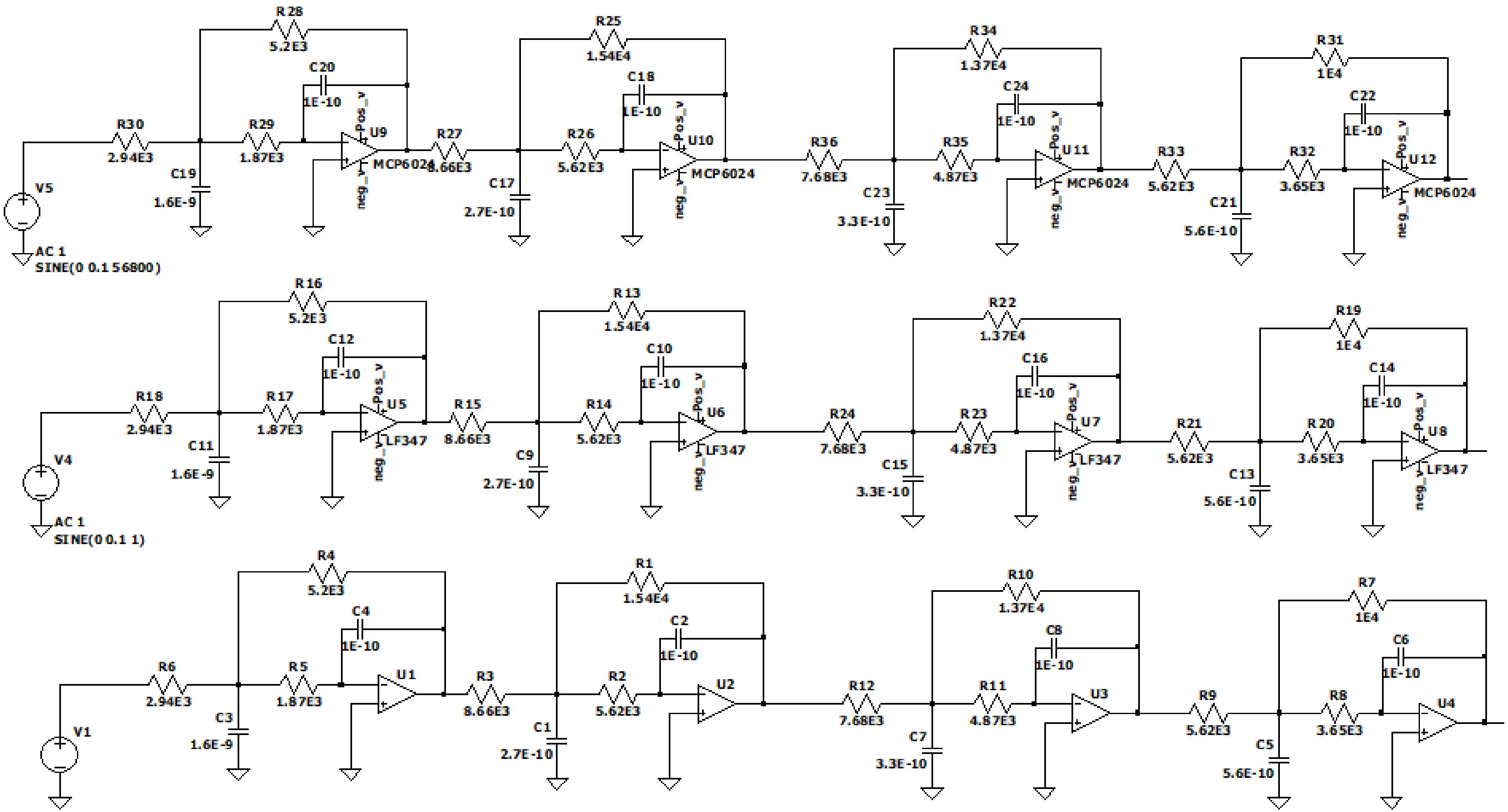

Figure 7: MCP604 (top), LF347 (middle), Ideal (bottom) filter circuit diagrams

4 Results and simulations

4.1 Copy-paste transfer function

4.1.1 Transfer function amplitude response of copy-paste filter

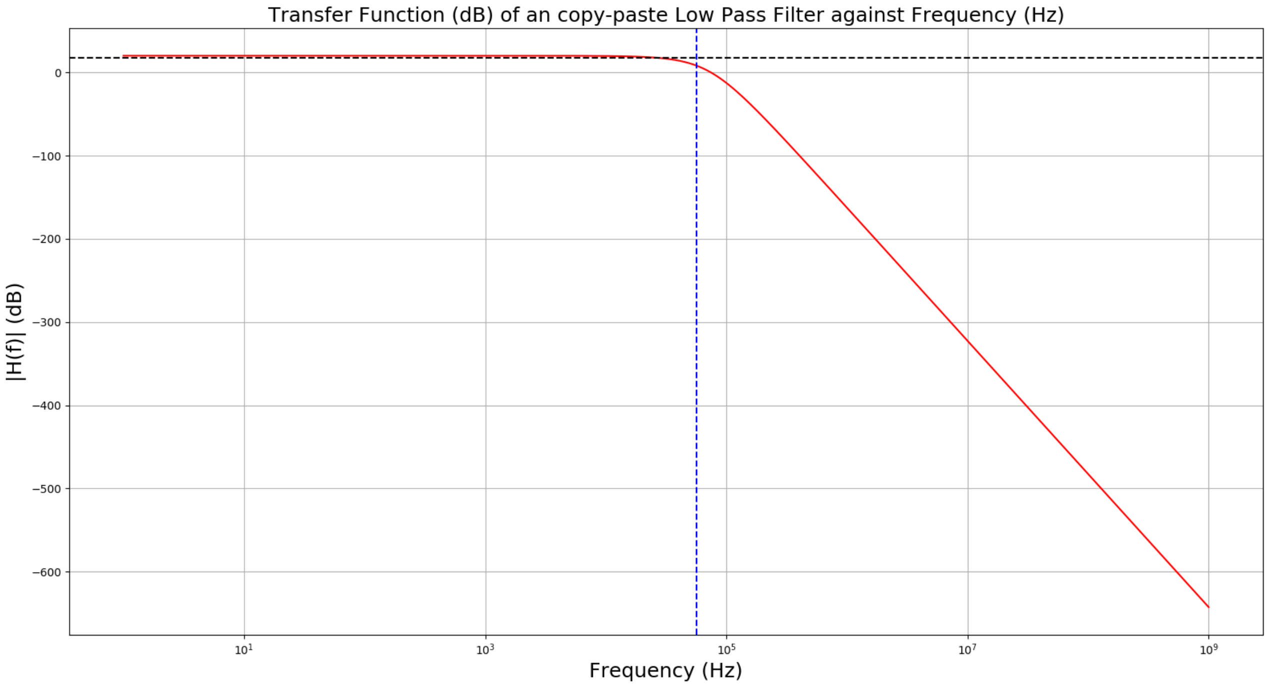

Figure 8: Copy-paste python simulation

4.1.2 Copy-paste cut off and design frequency

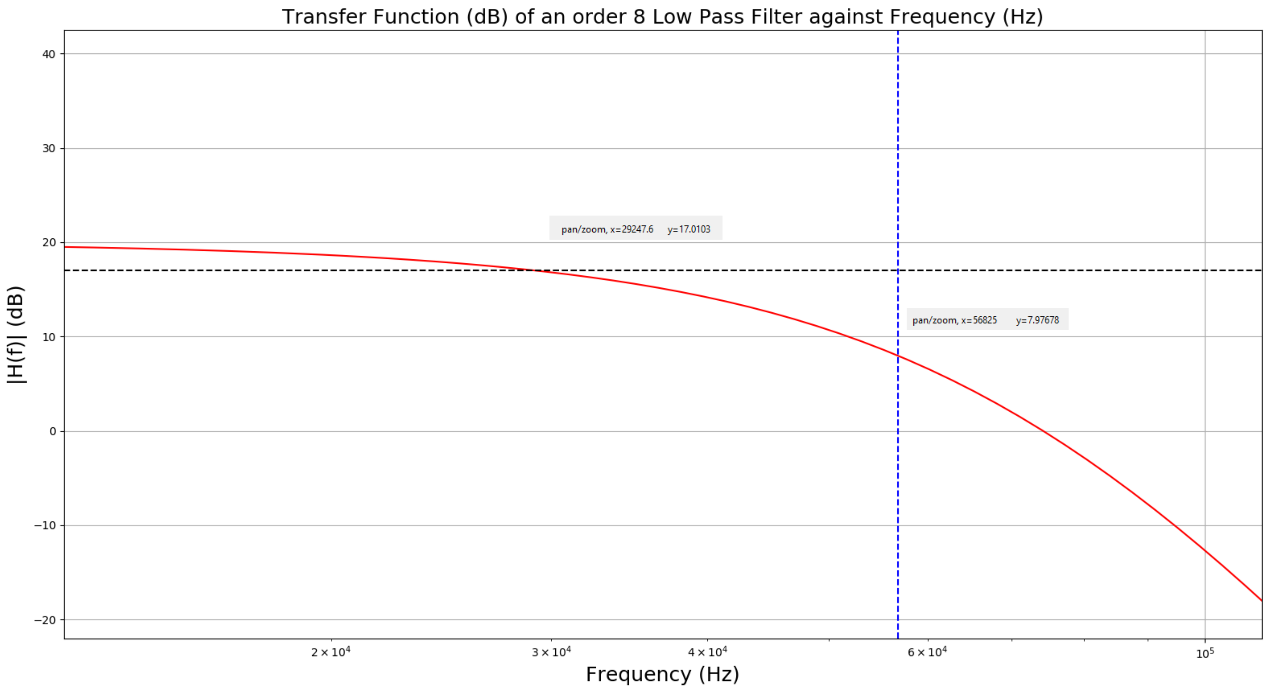

Figure 9: Copy-paste cut-off and design frequency

4.2 Cascaded transfer function

4.2.1 Transfer function amplitude response of cascaded filter

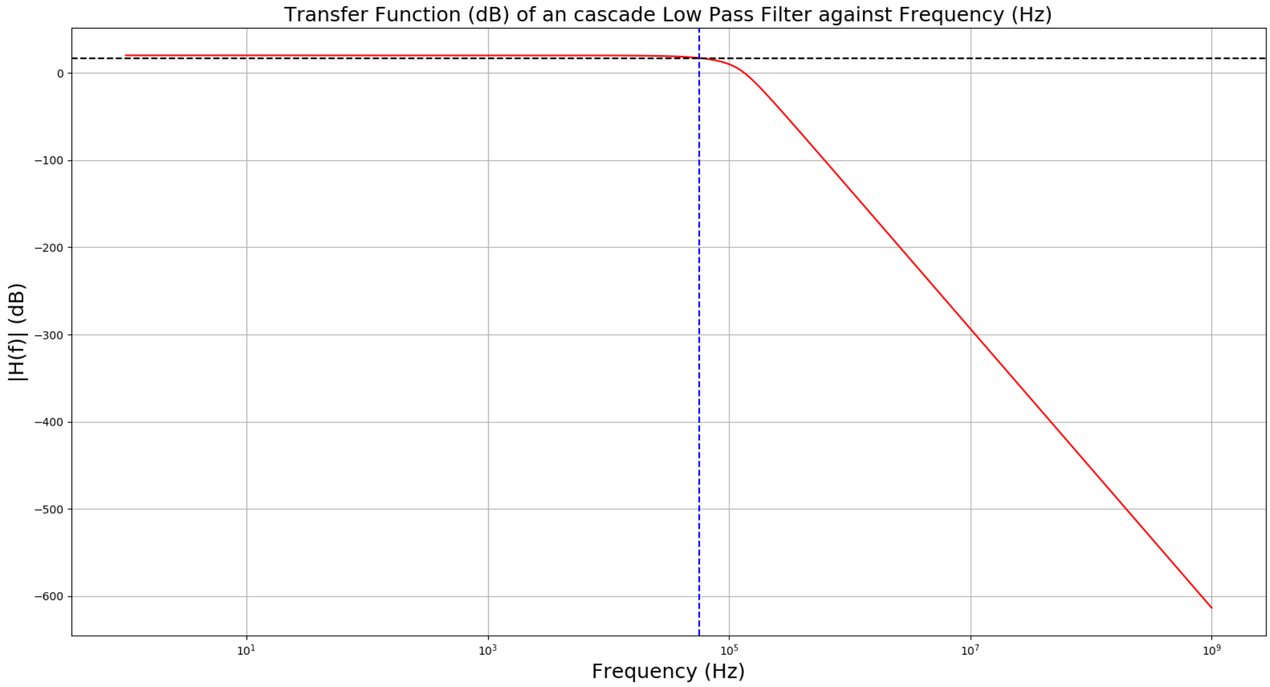

Figure 10: Cascade python simulation

4.2.2 Copy-paste cut off and design frequency

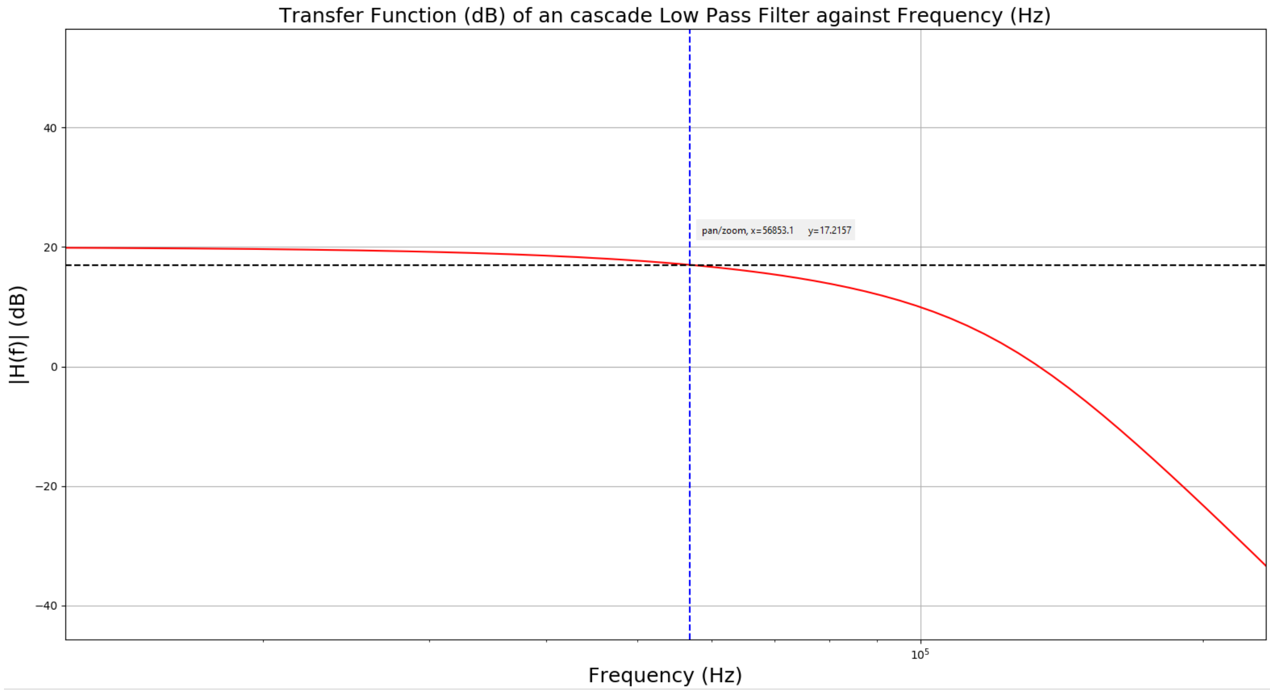

Figure 11: Cascade cut-off and design frequency

4.3 Comparison of transfer functions

4.3.1 Amplitude response

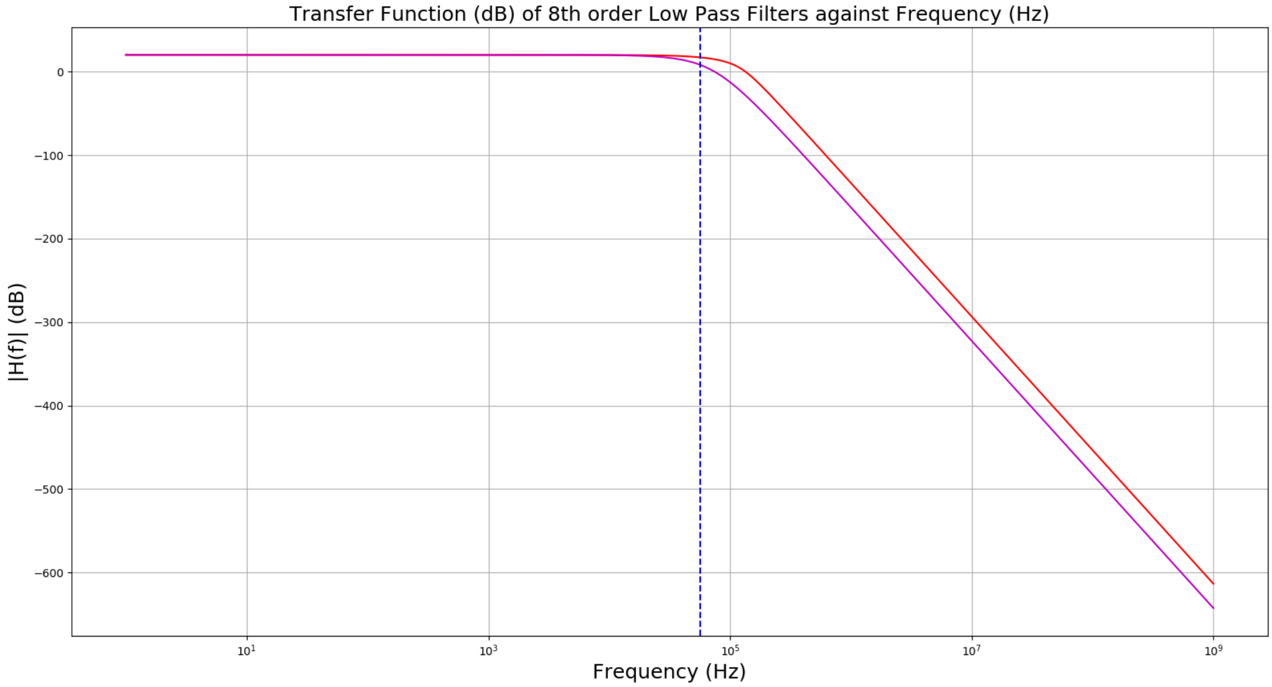

Figure 12: Comparison of amplitude responses, purple is CP, red is Cas

4.3.2 Unity frequency

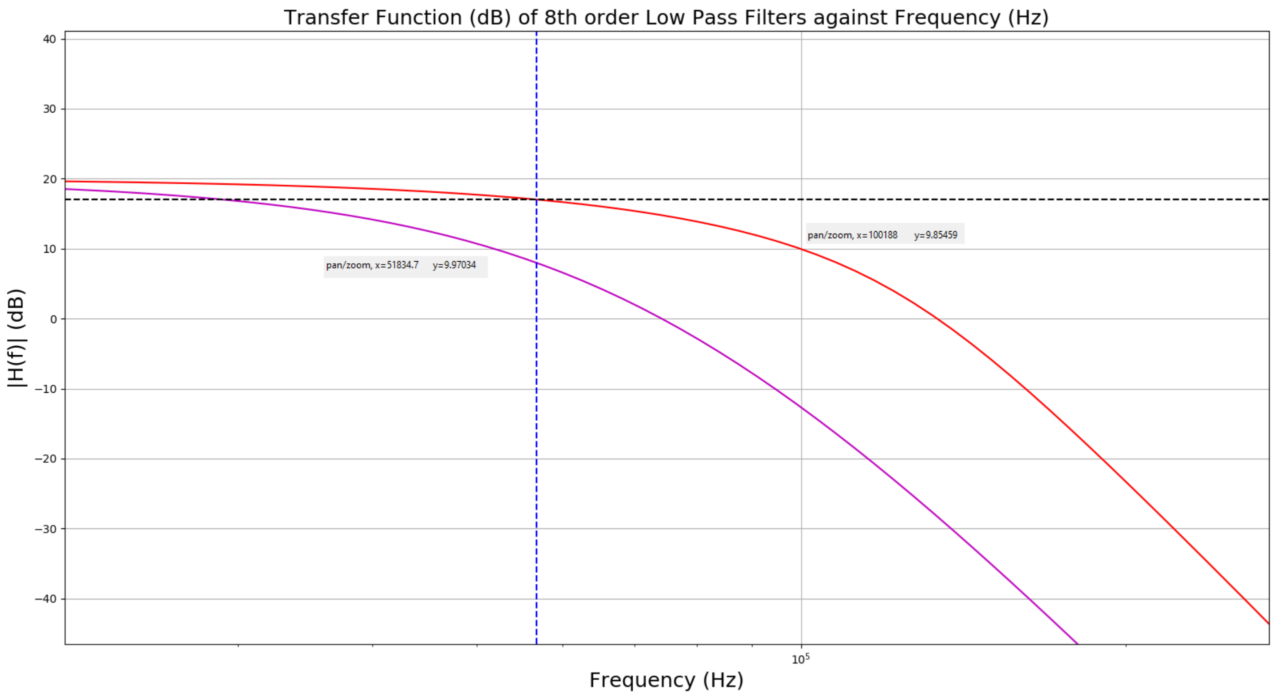

Figure 13: Comparison of unity frequencies, purple is CP, red is Cas

4.3.3 Phase comparison

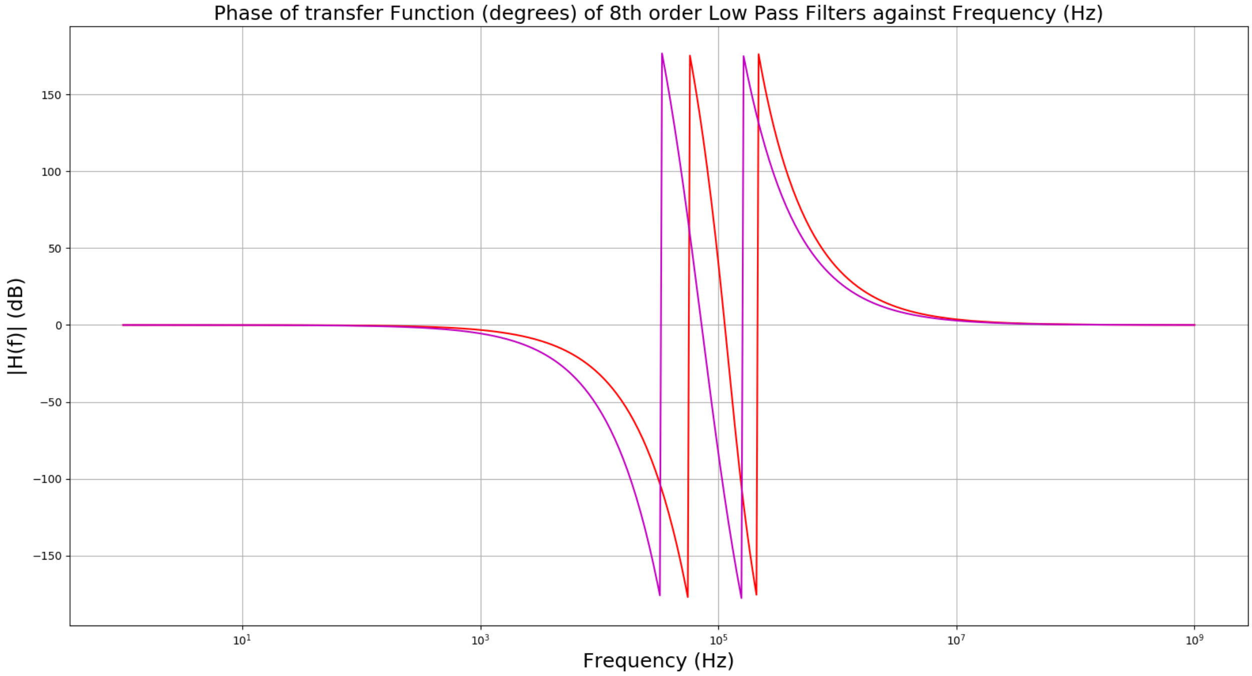

Figure 14: Comparison of phase, purple is CP, red is Cas

4.4 Ideal circuit

4.5 Copy and paste Bessel multi-feedback low-pass filter

4.5.1 Frequency response

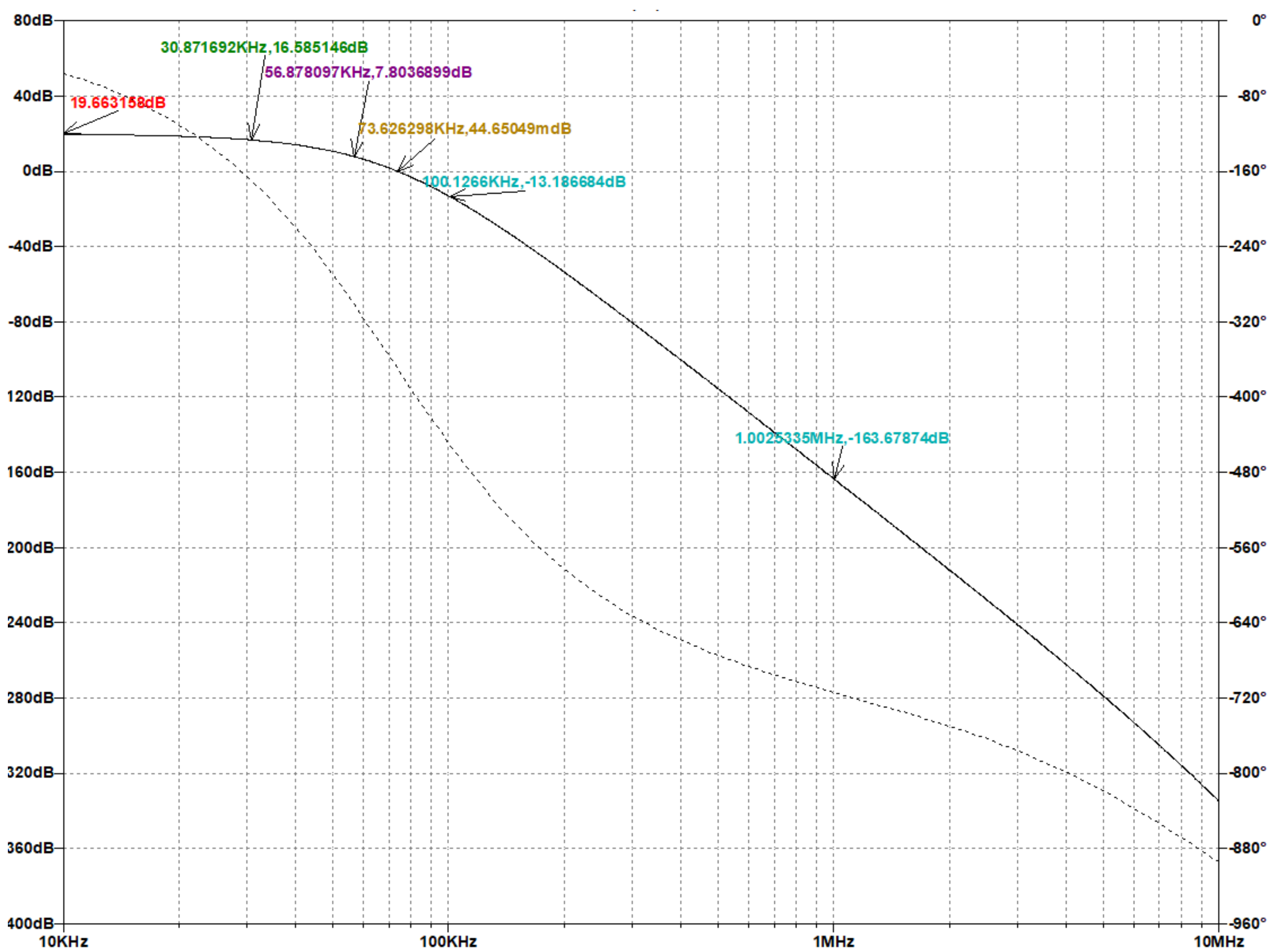

Figure 15: Frequency response

4.5.2 Transient response

Gain response

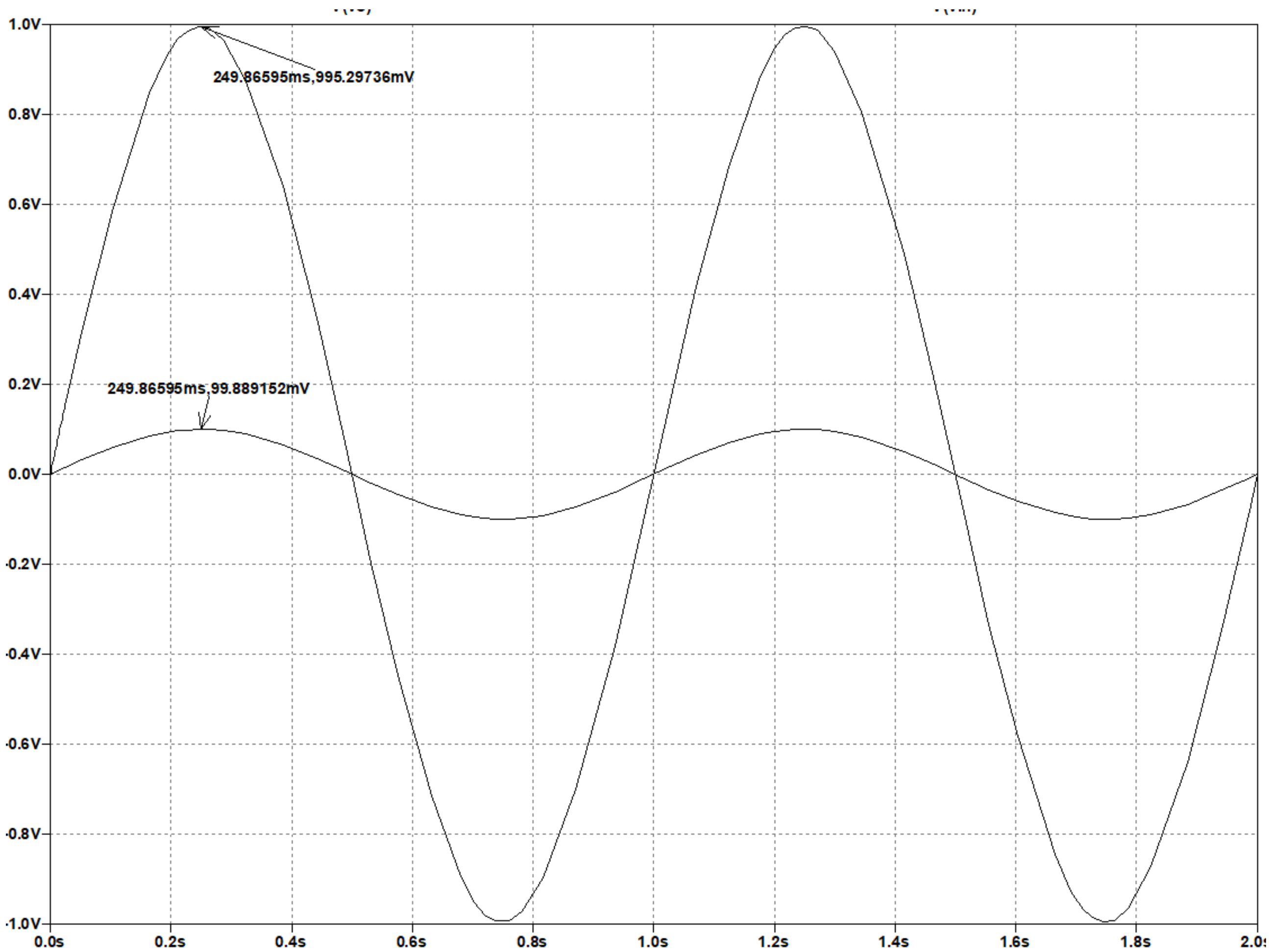

Figure 16: Transient response of voltage gain

Cut-off frequency transient response

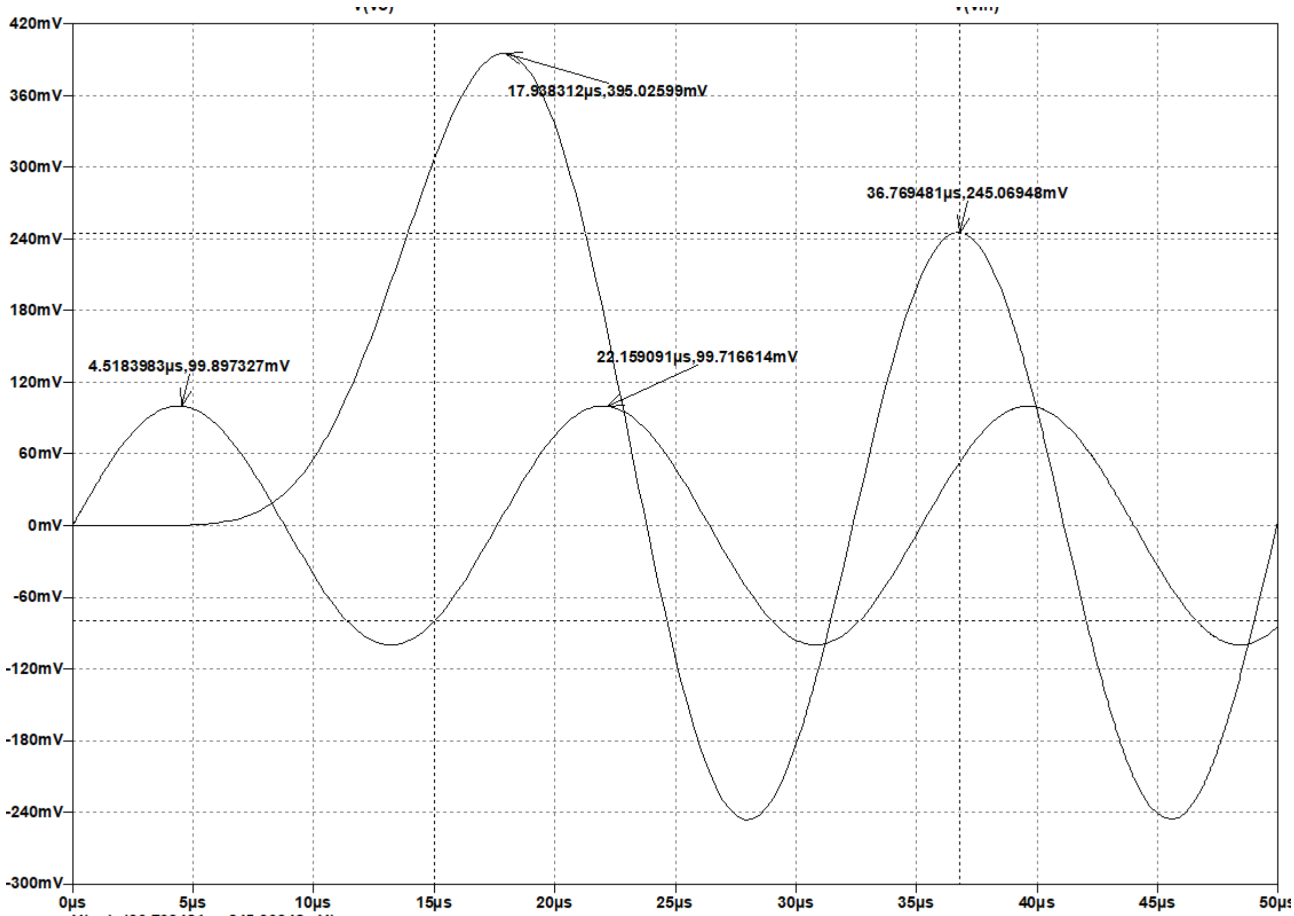

Figure 17: Transient response of phase and gain at cut-off frequency

4.5.3 Interpretation of results for ideal copy-paste filter

Gain

Roll-off

Cut-off

Design frequency and natural frequency

Final Q factor

Unity frequency

Phase shift

4.6 Cascaded Bessel multi-feedback low-pass filter

4.6.1 Frequency response

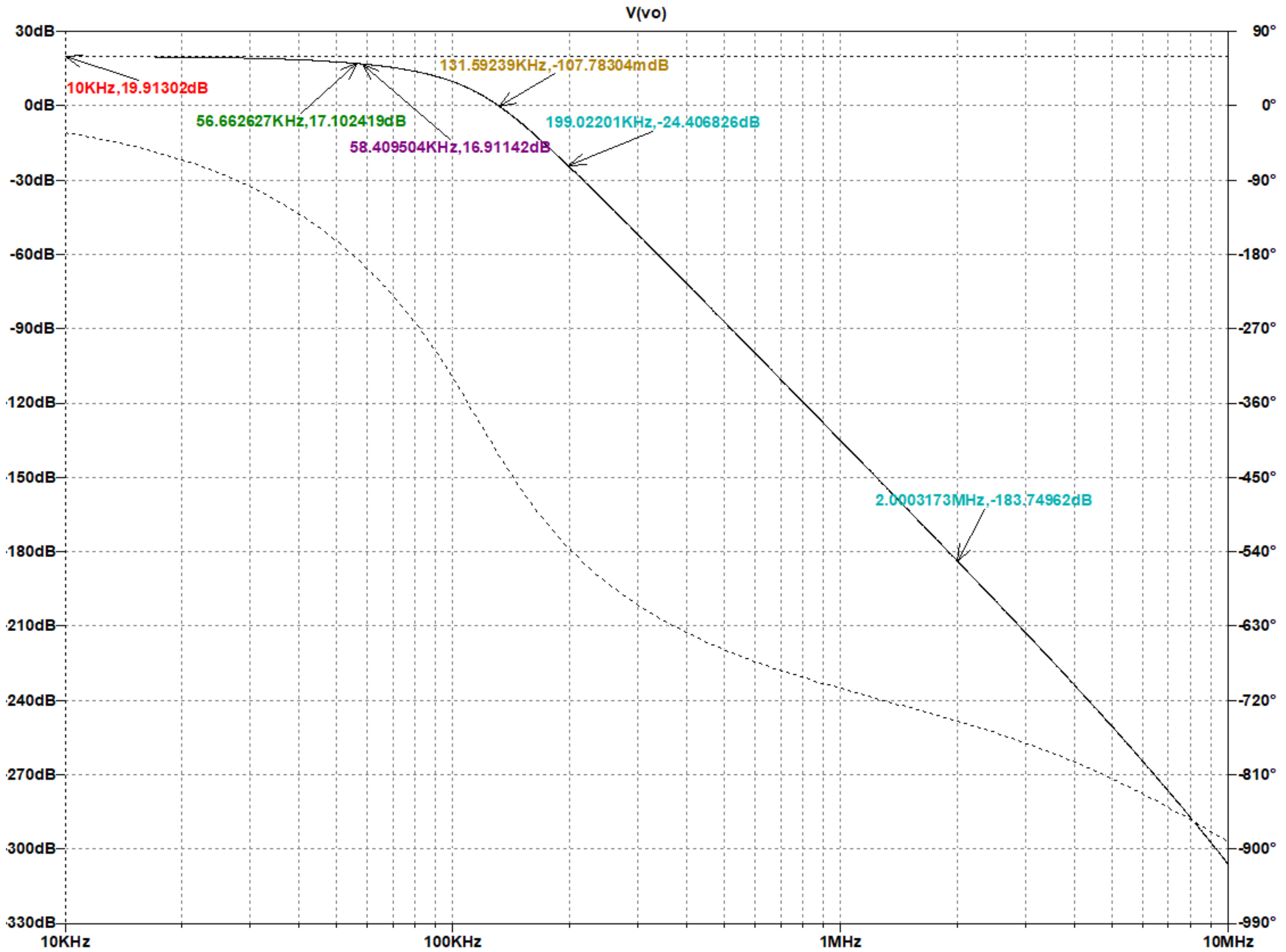

Figure 18: Frequency response

4.6.2 Transient response

Gain response

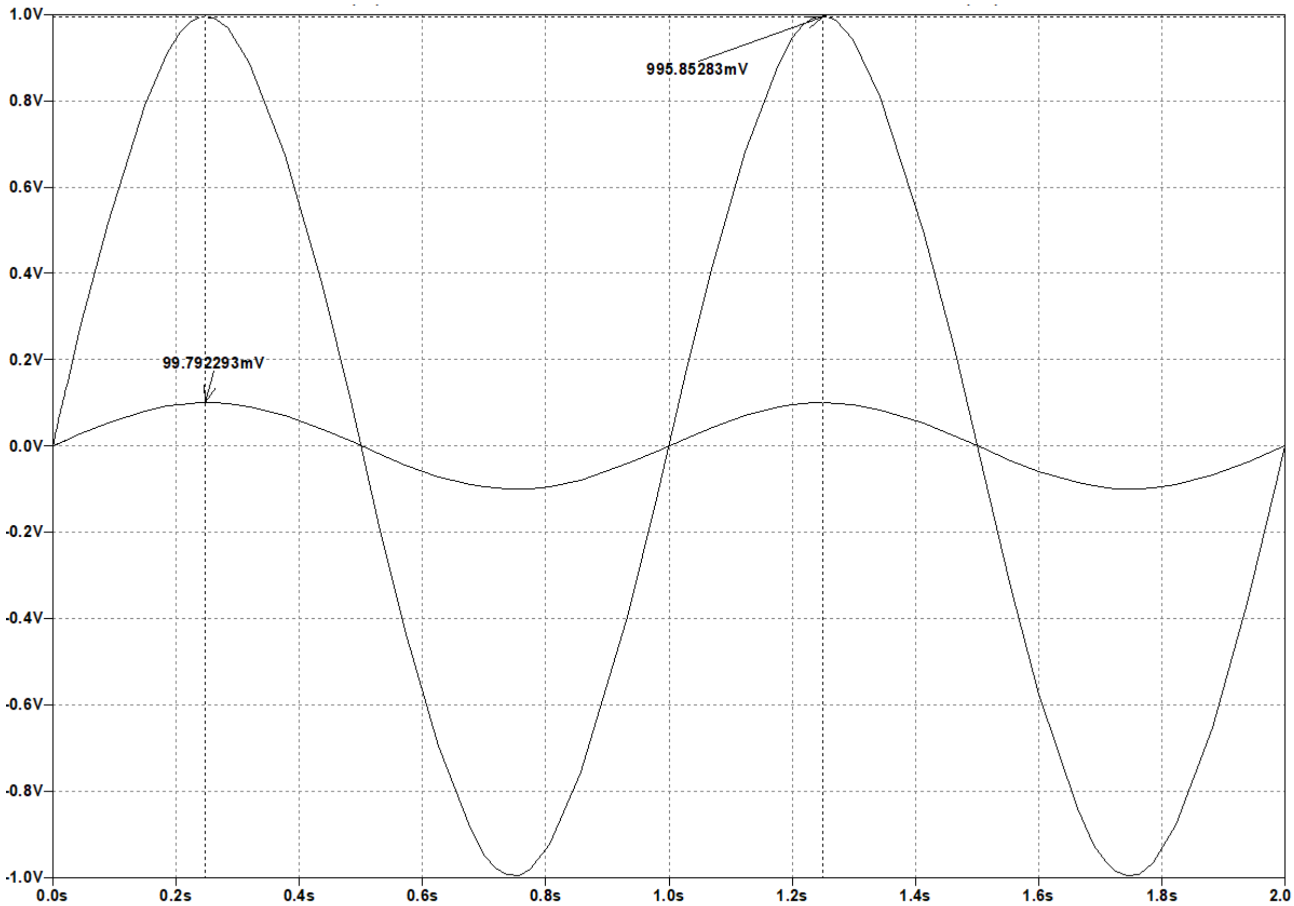

Figure 19: Transient response of voltage gain

Cut-off frequency transient response

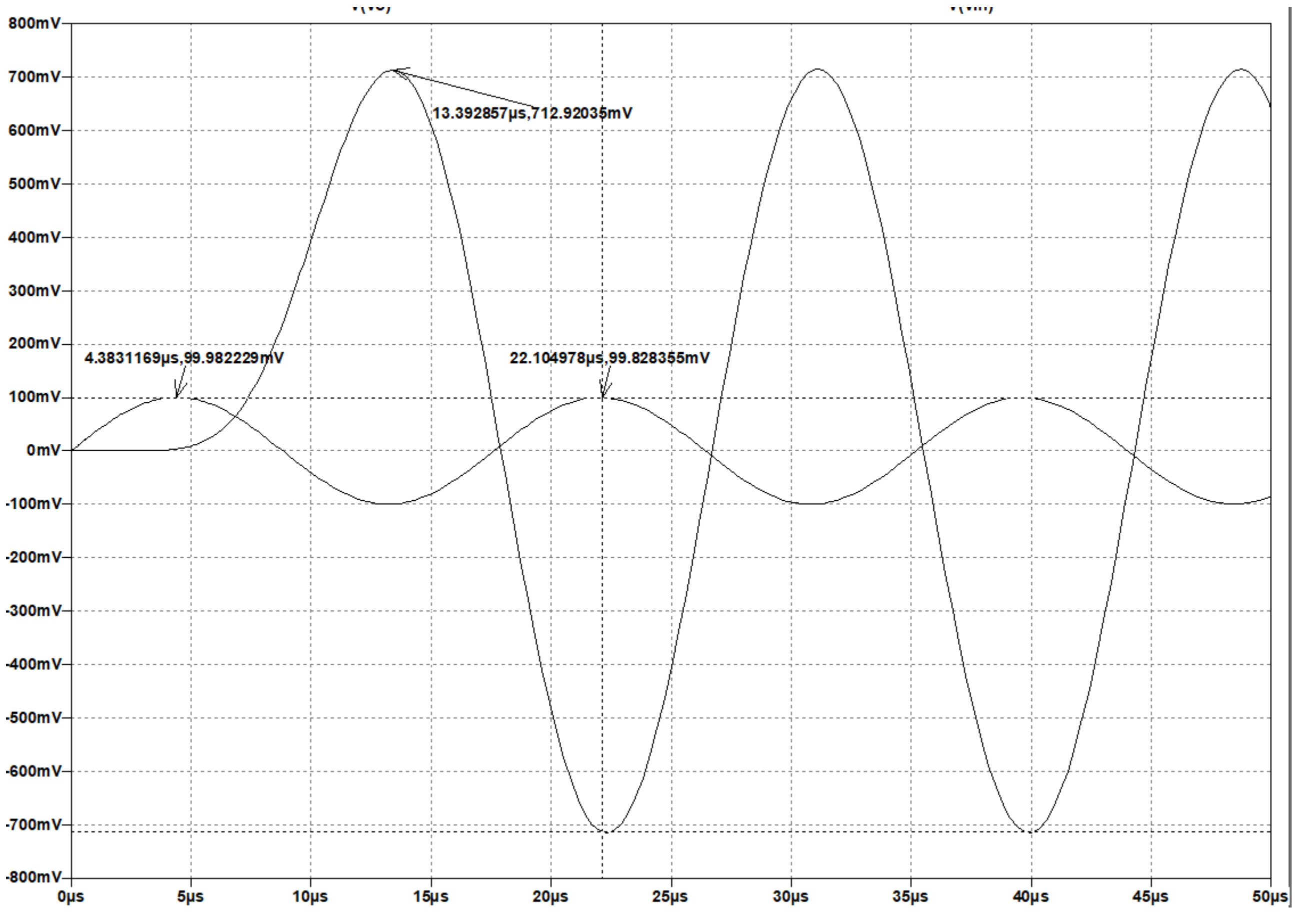

Figure 20: Transient response of phase and gain at cut-off frequency

4.6.3 Interpretation of results for ideal cascaded filter

Gain

Roll-off

Cut-off

Design frequency and natural frequency

Final Q factor

Unity frequency

Phase shift

4.7 E6/E12 circuit

4.8 Copy and paste Bessel multi-feedback low-pass filter

4.8.1 Frequency response

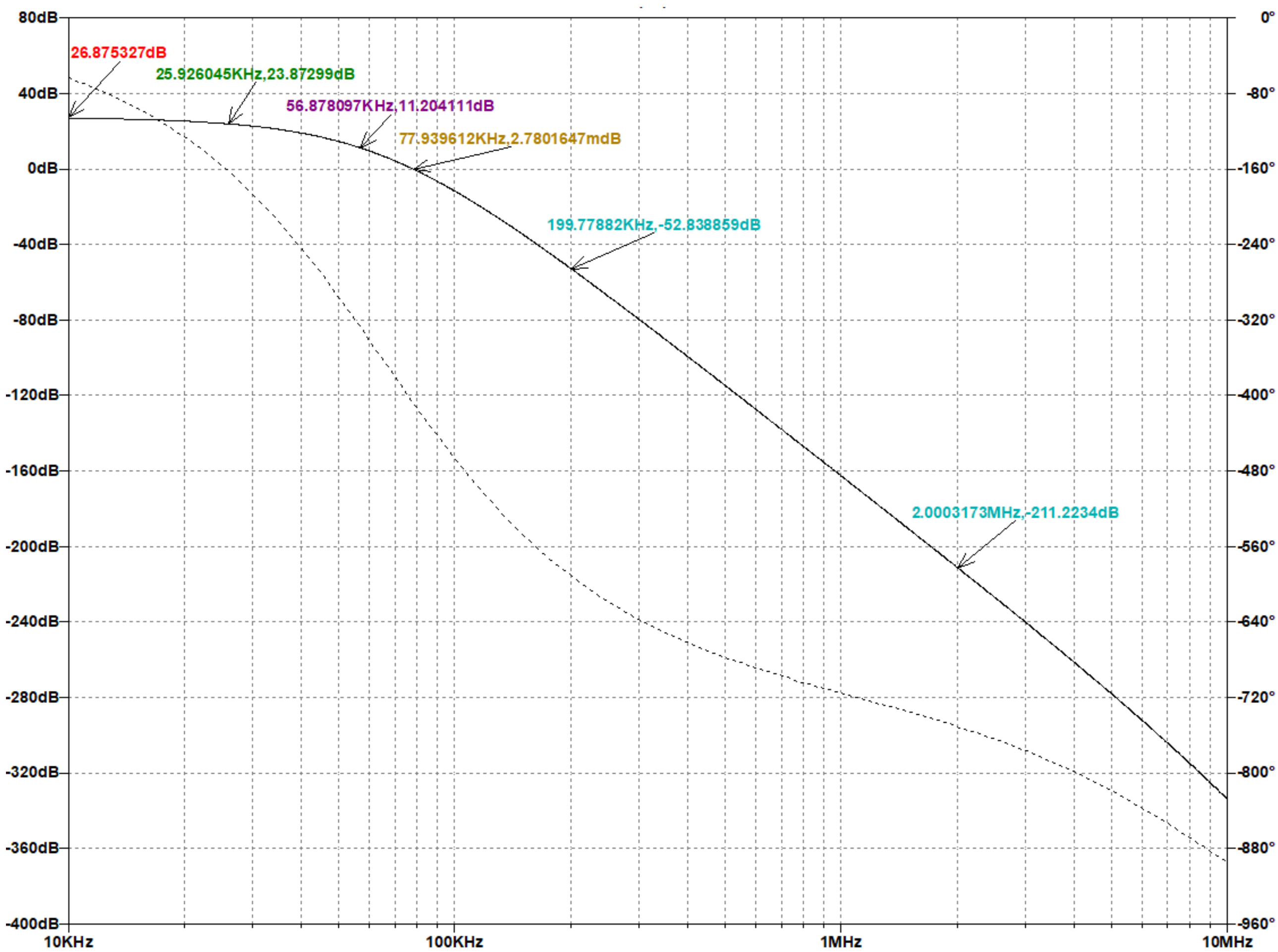

Figure 21: Frequency response

4.8.2 Transient response

Gain response

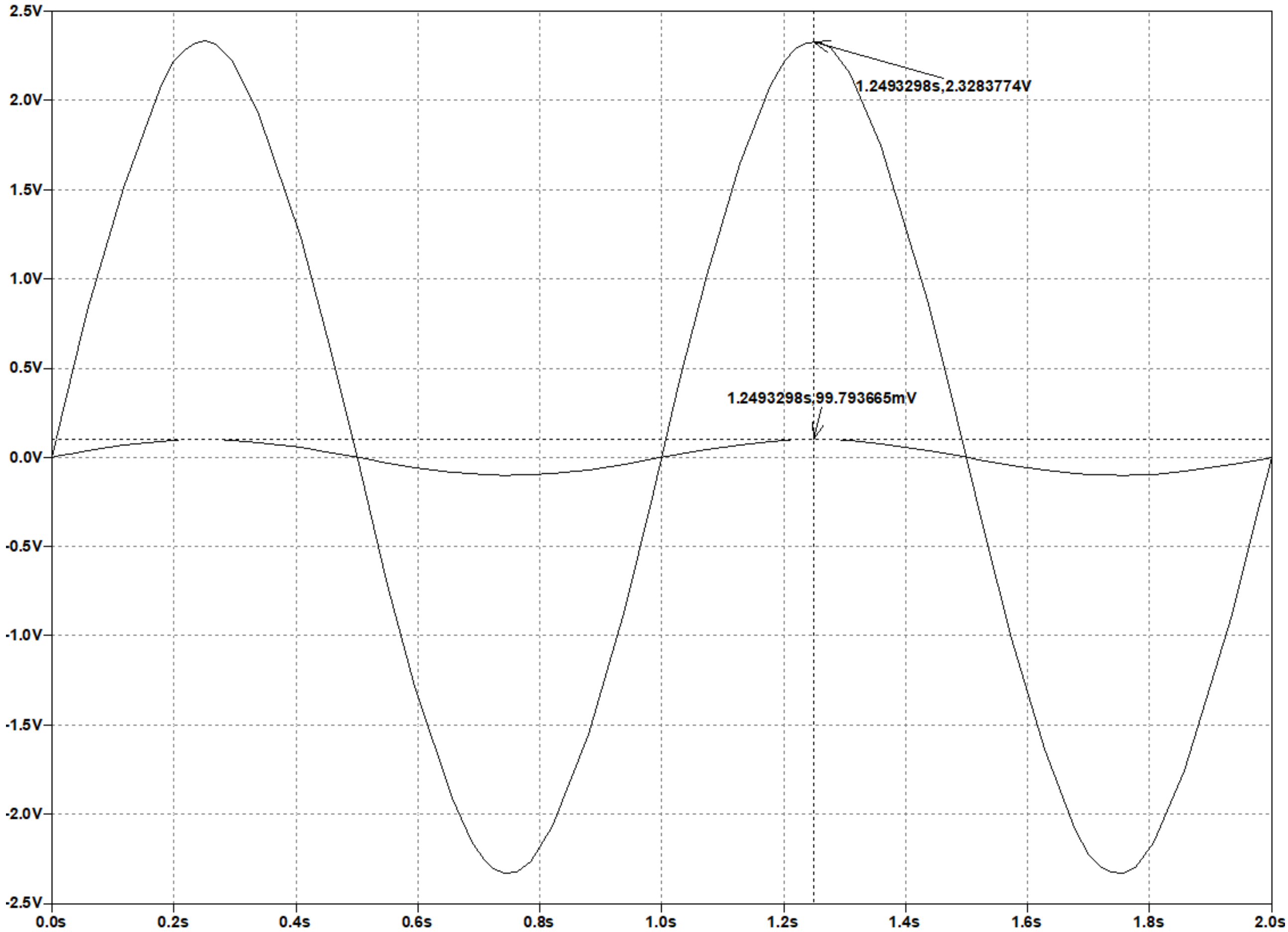

Figure 22: Transient response of voltage gain

Cut-off frequency transient response

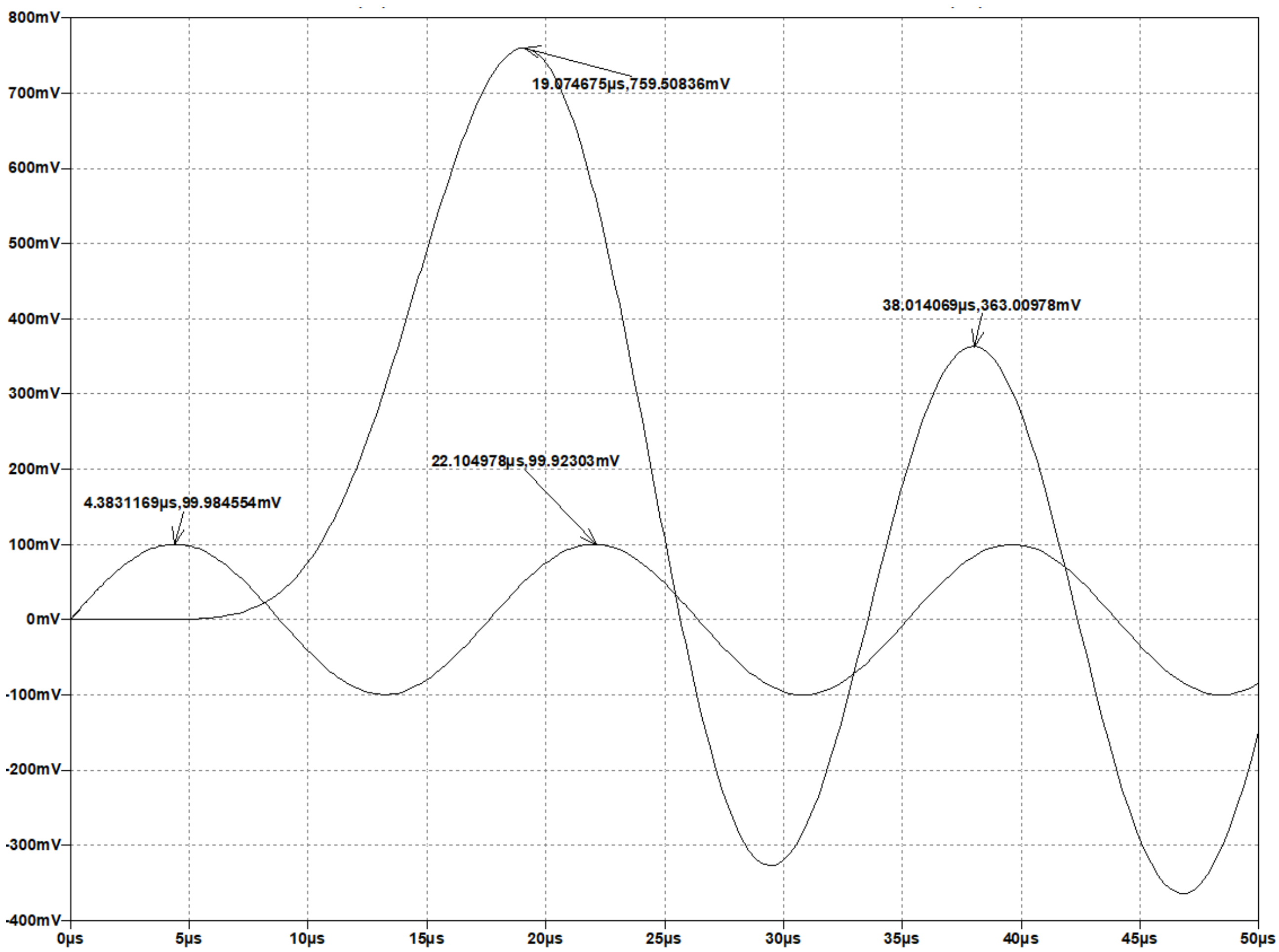

Figure 23: Transient response of phase and gain at cut-off frequency

4.9 Cascaded Bessel multi-feedback low-pass filter

4.9.1 Frequency response

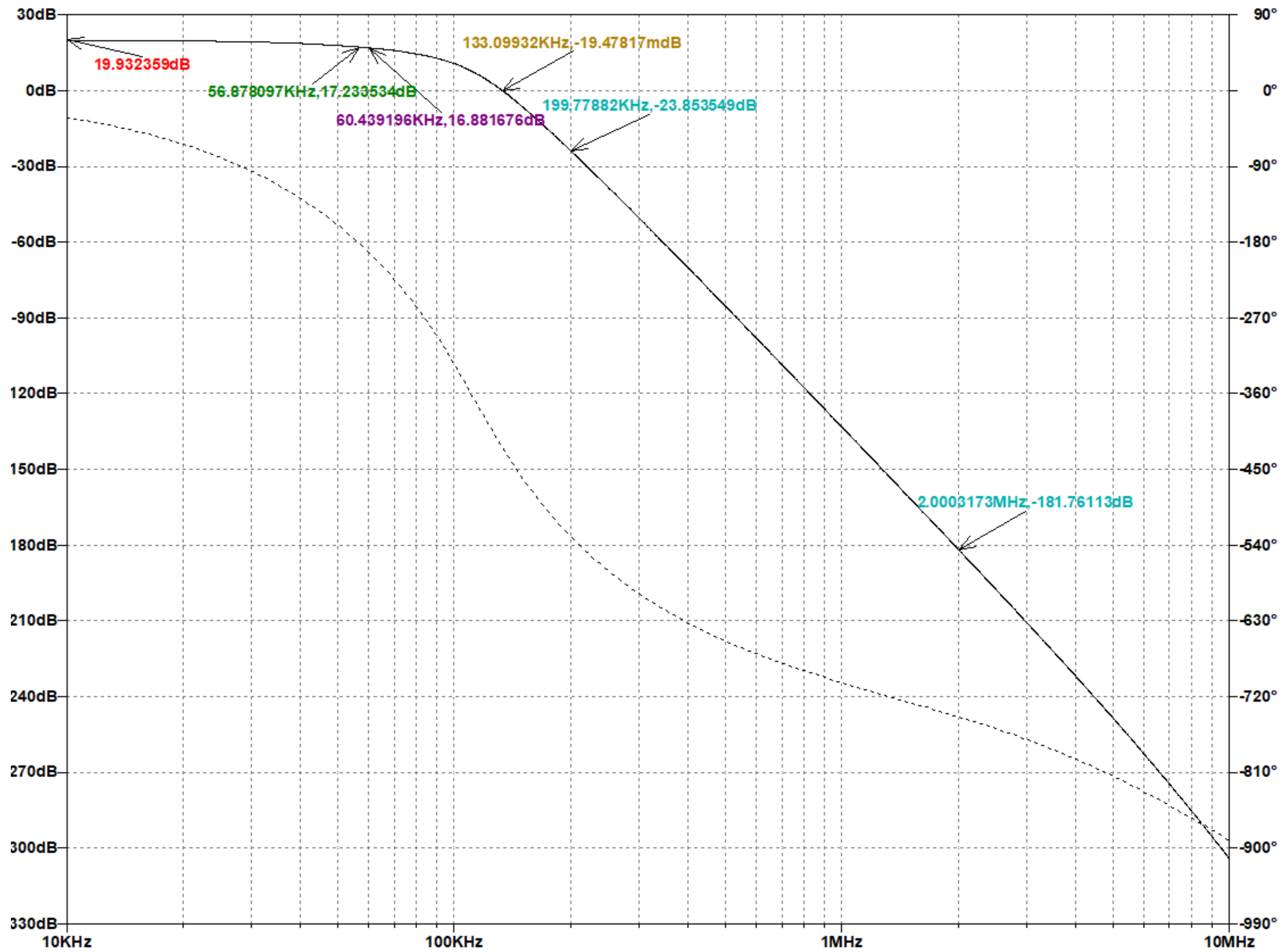

Figure 24: Frequency response

4.9.2 Transient response

Gain response

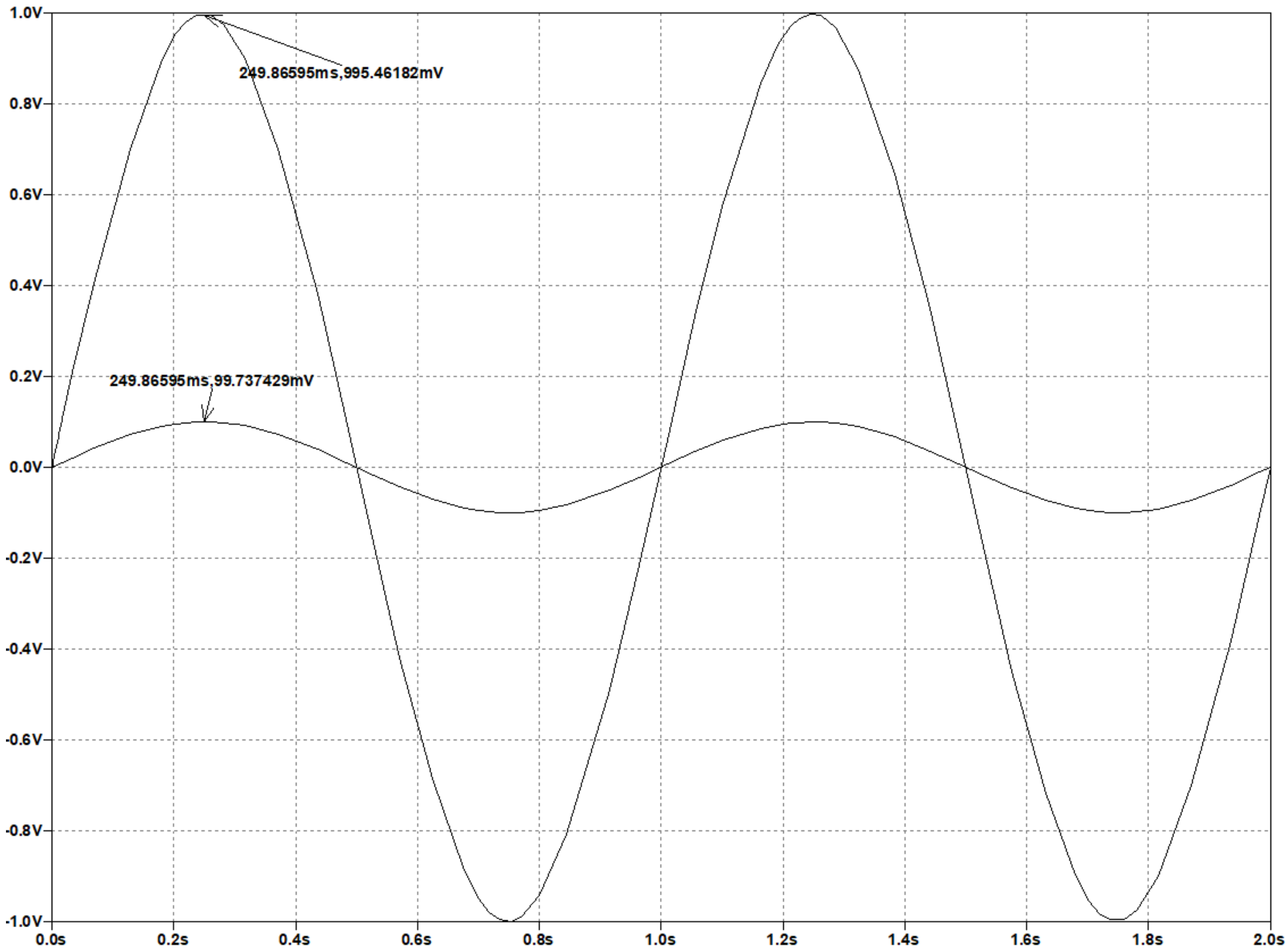

Figure 25: Transient response of voltage gain

Cut-off frequency transient response

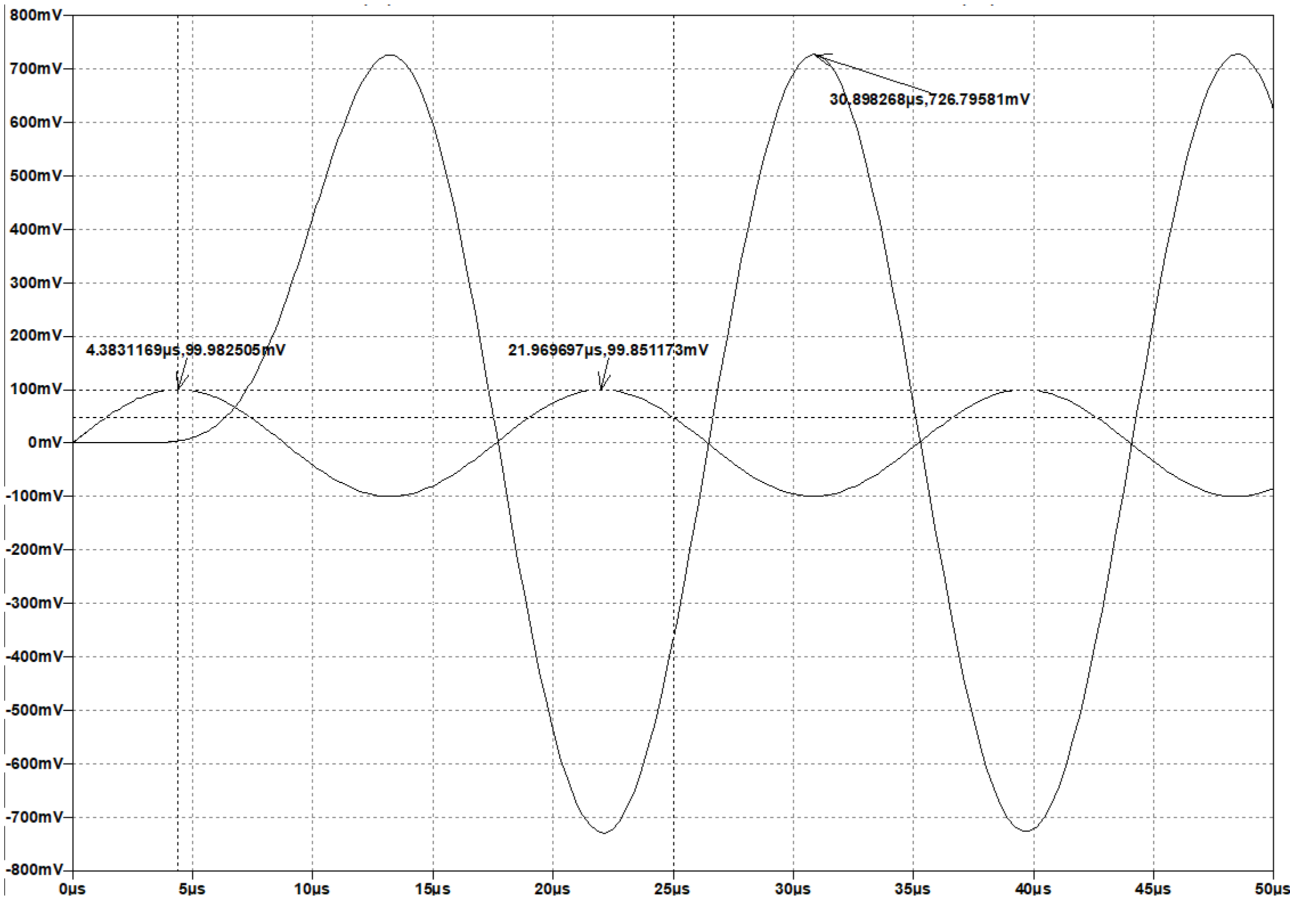

Figure 26: Transient response of phase and gain at cut-off frequency

4.10 E96/E24 circuit

4.11 Copy and paste Bessel multi-feedback low-pass filter

4.11.1 Frequency response

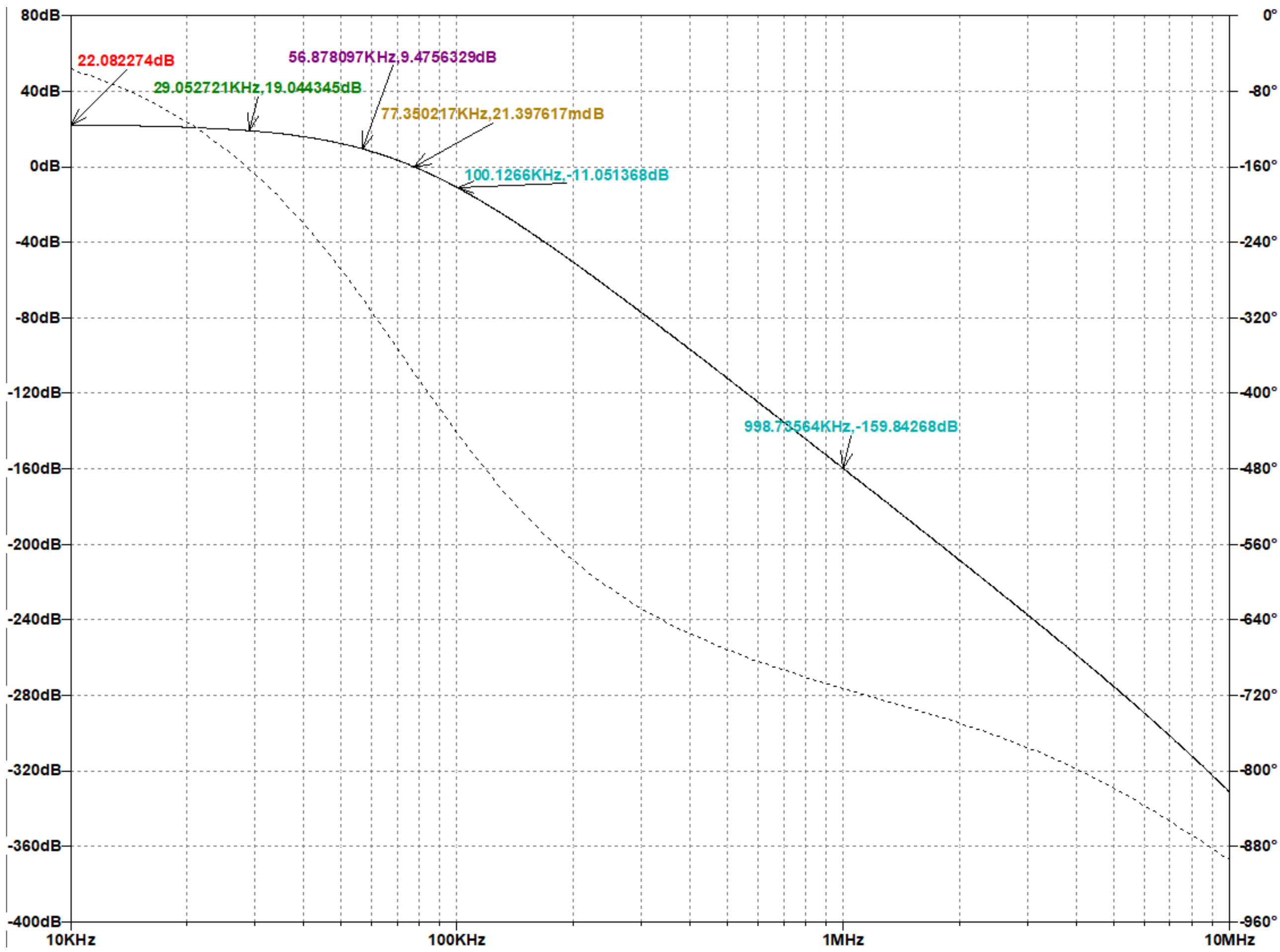

Figure 27: Frequency response

4.11.2 Transient response

Gain response

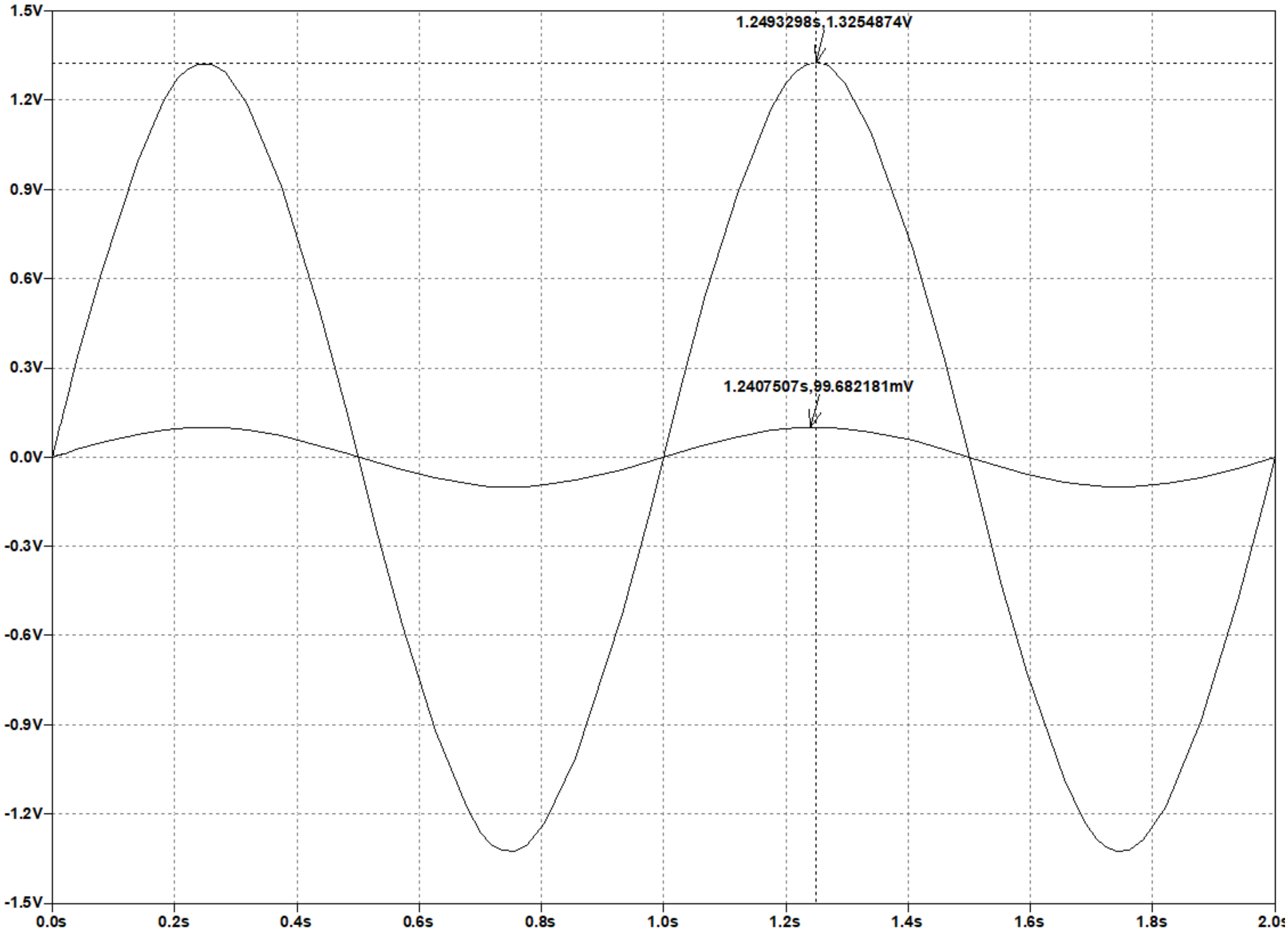

Figure 28: Transient response of voltage gain

Cut-off frequency transient response

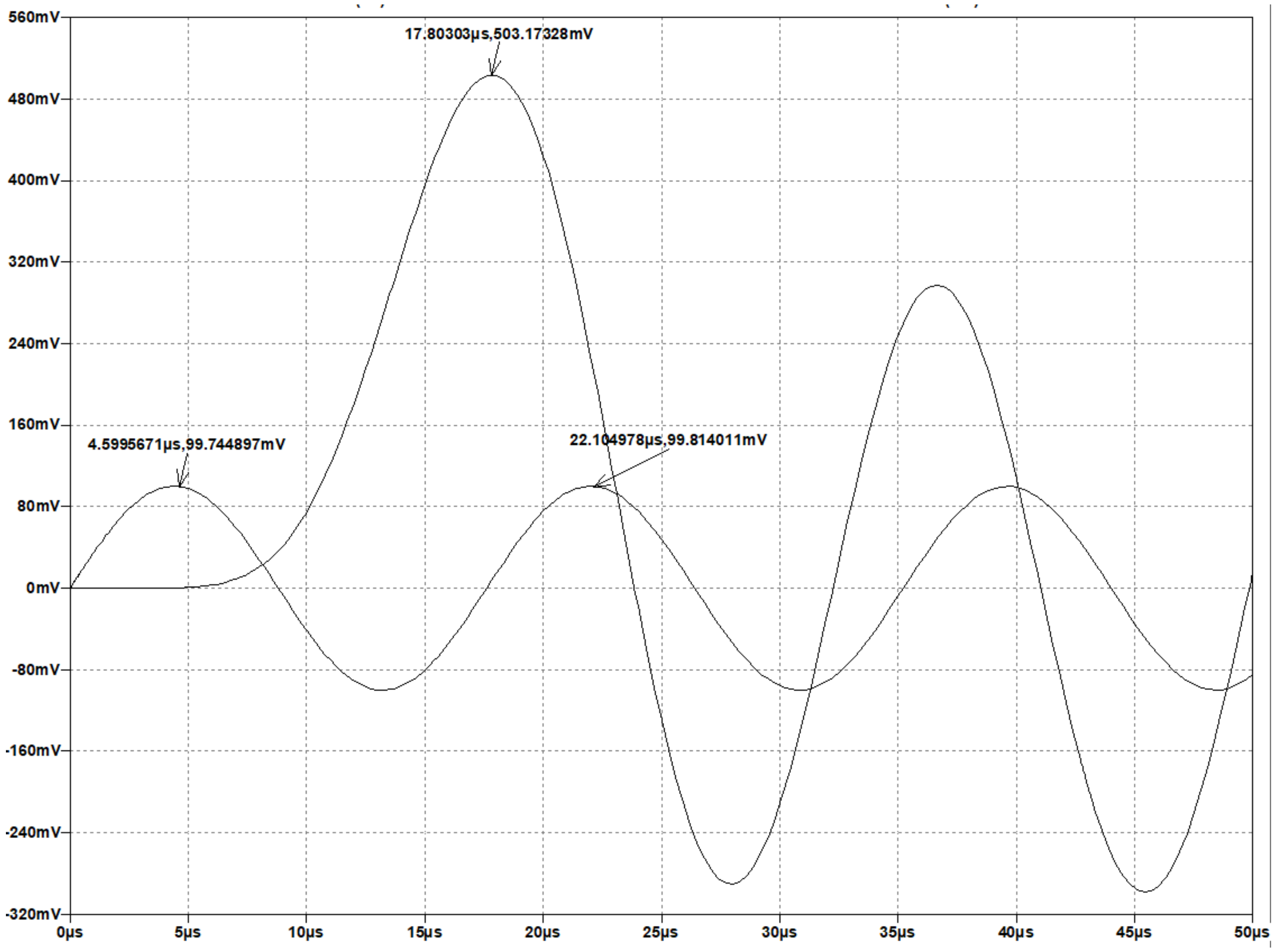

Figure 29: Transient response of phase and gain at cut-off frequency

4.12 Cascaded Bessel multi-feedback low-pass filter

4.12.1 Frequency response

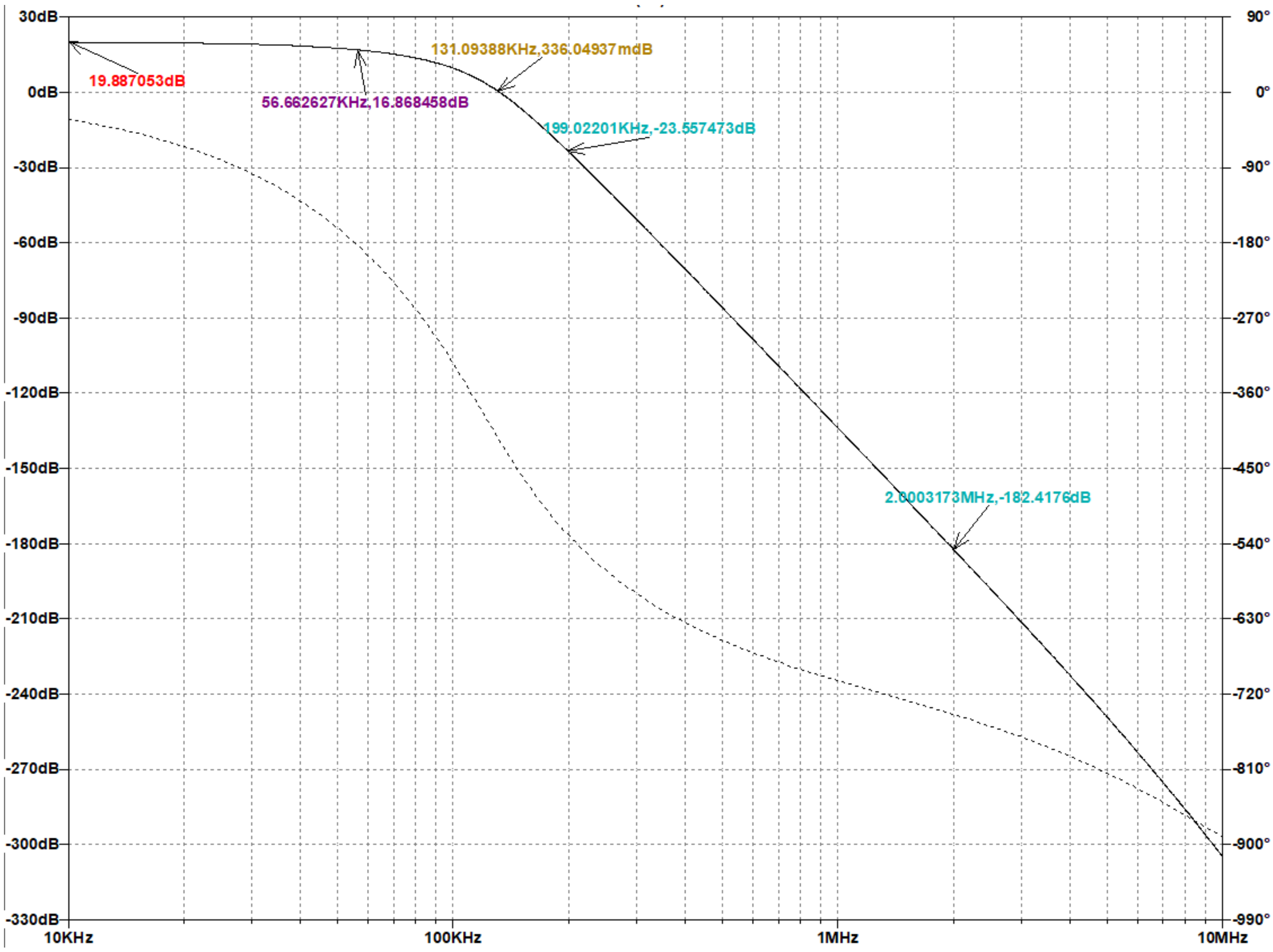

Figure 30: Frequency response

4.12.2 Transient response

Gain response

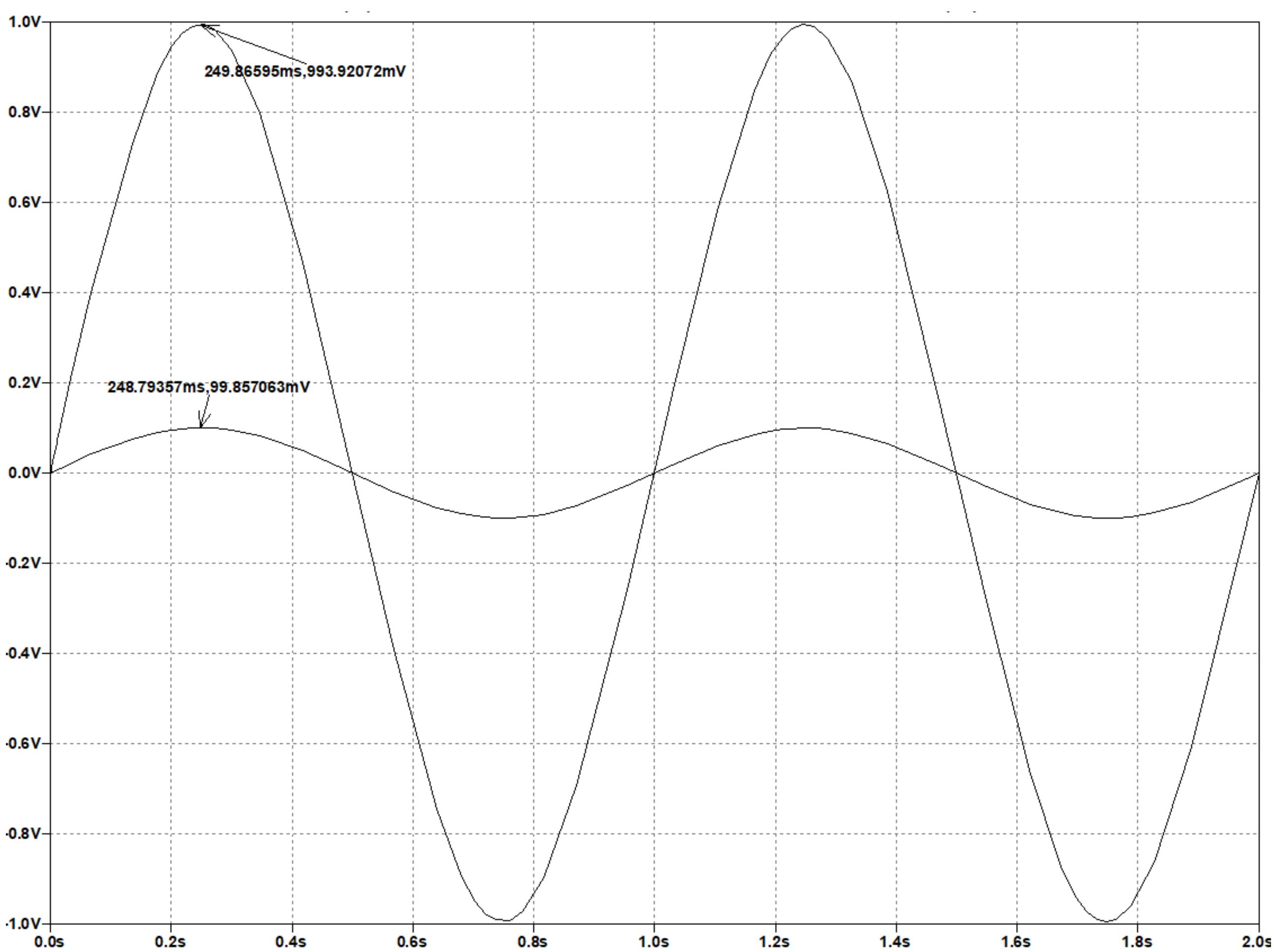

Figure 31: Transient response of voltage gain

Cut-off frequency transient response

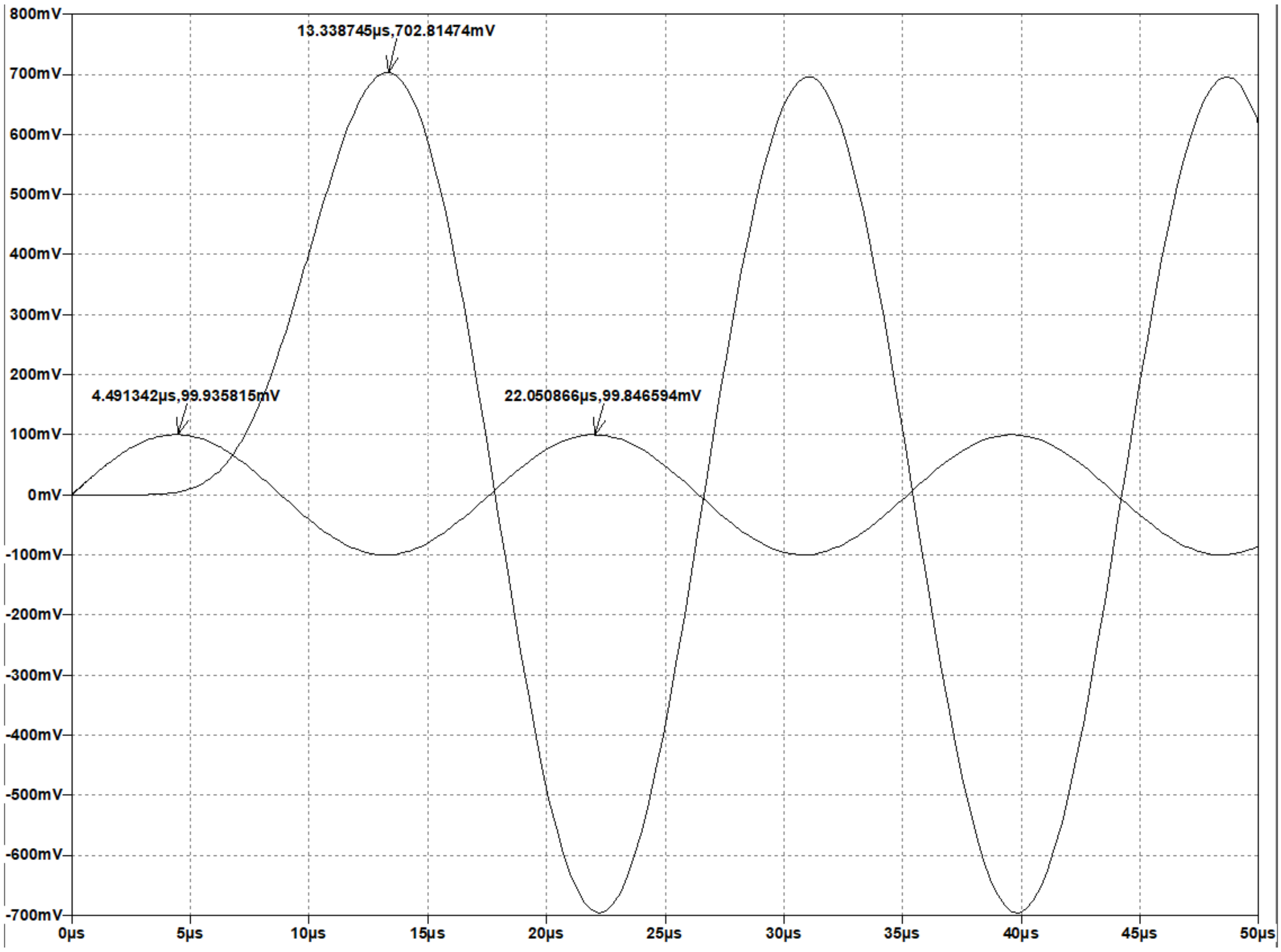

Figure 32: Transient response of phase and gain at cut-off frequency

4.13 Non-ideal components circuit

The circuits are comprised of E96/E24 resistors and capacitors with the op-amp being LF347.

4.14 Copy and paste Bessel multi-feedback low-pass filter

4.14.1 Frequency response of LM347

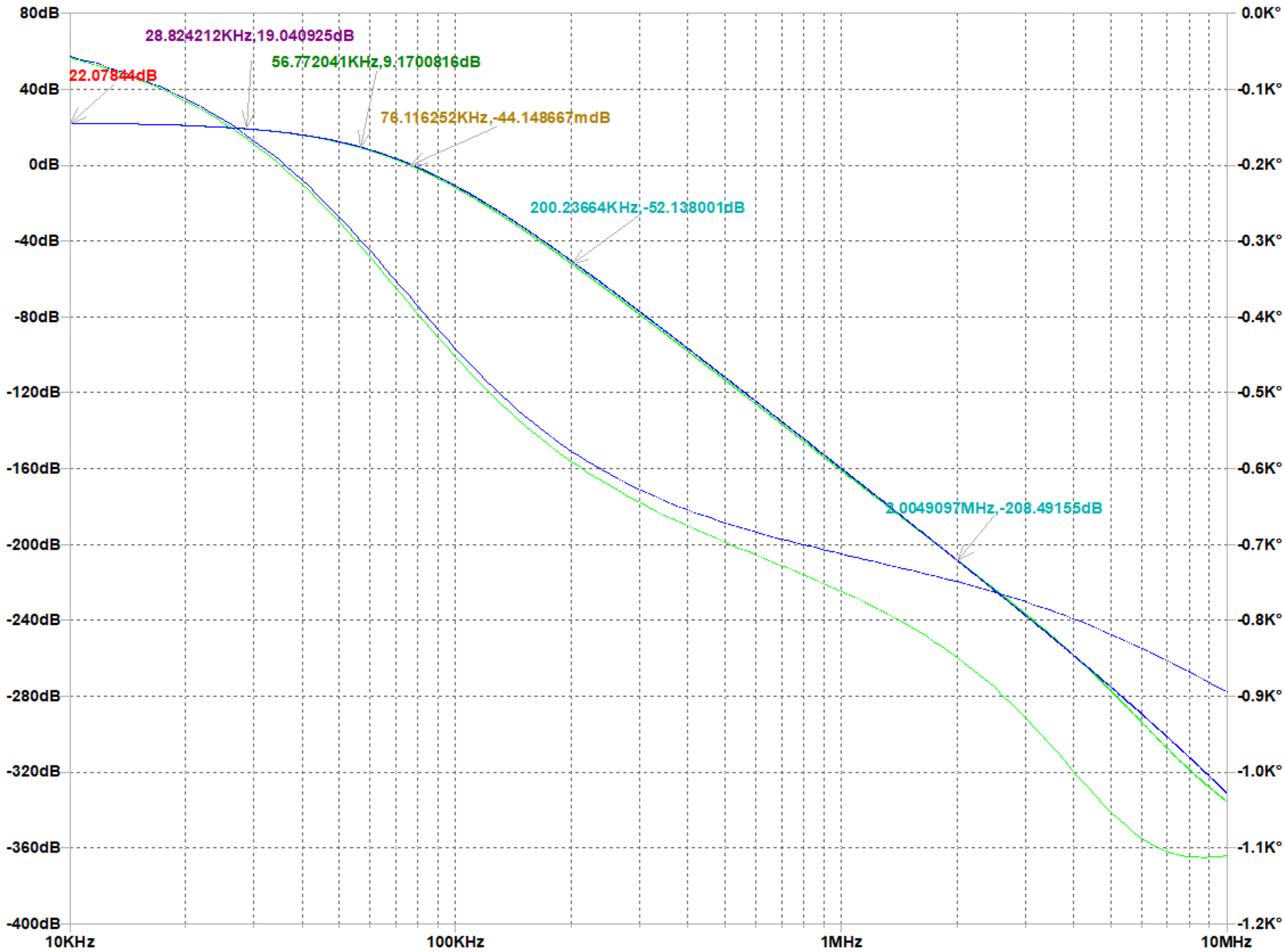

Figure 33: Frequency response

4.14.2 Frequency response of MCP604

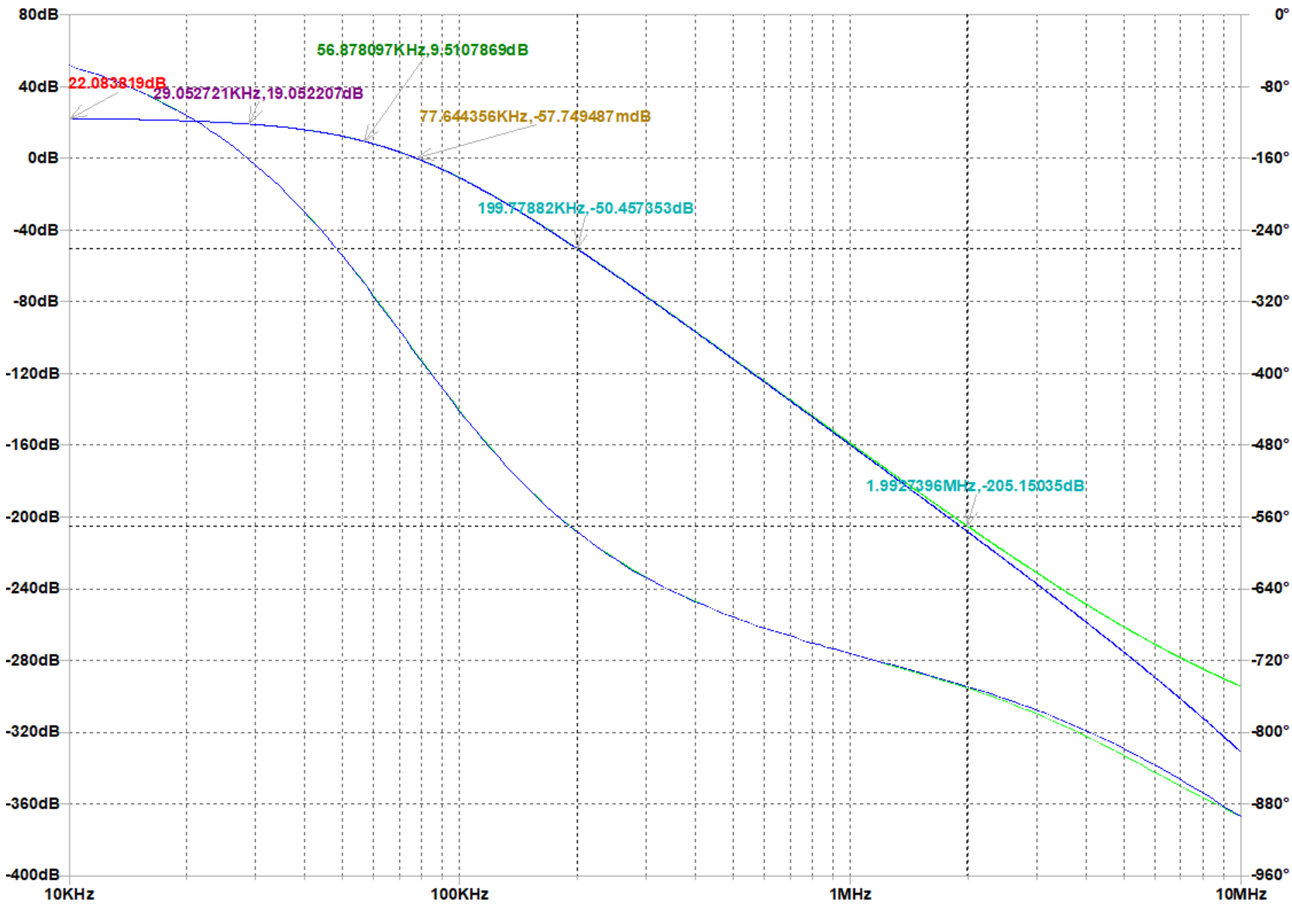

Figure 34: Frequency response

4.14.3 Transient response

Gain response

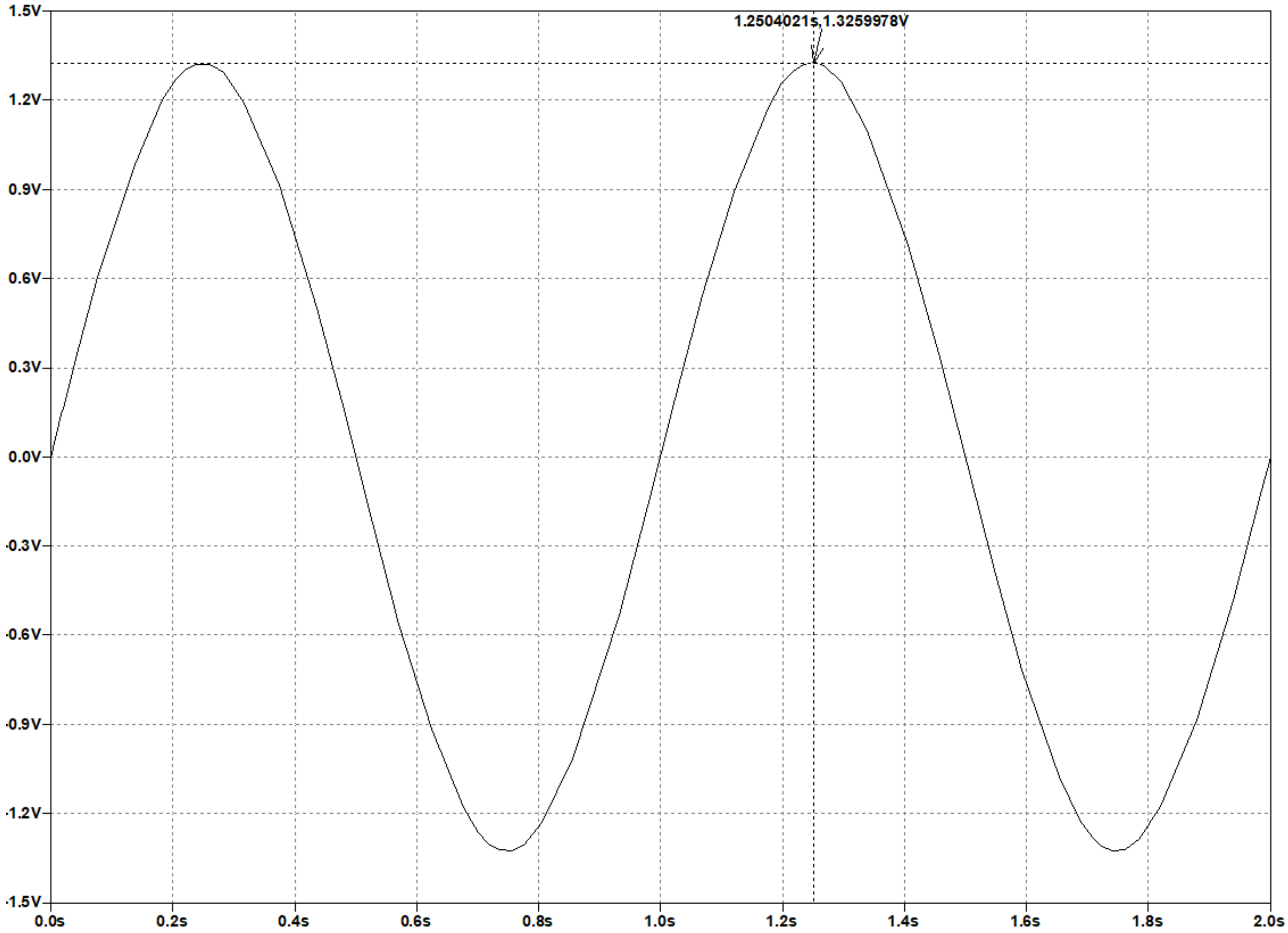

Figure 35: Transient response of voltage gain

Cut-off frequency transient response

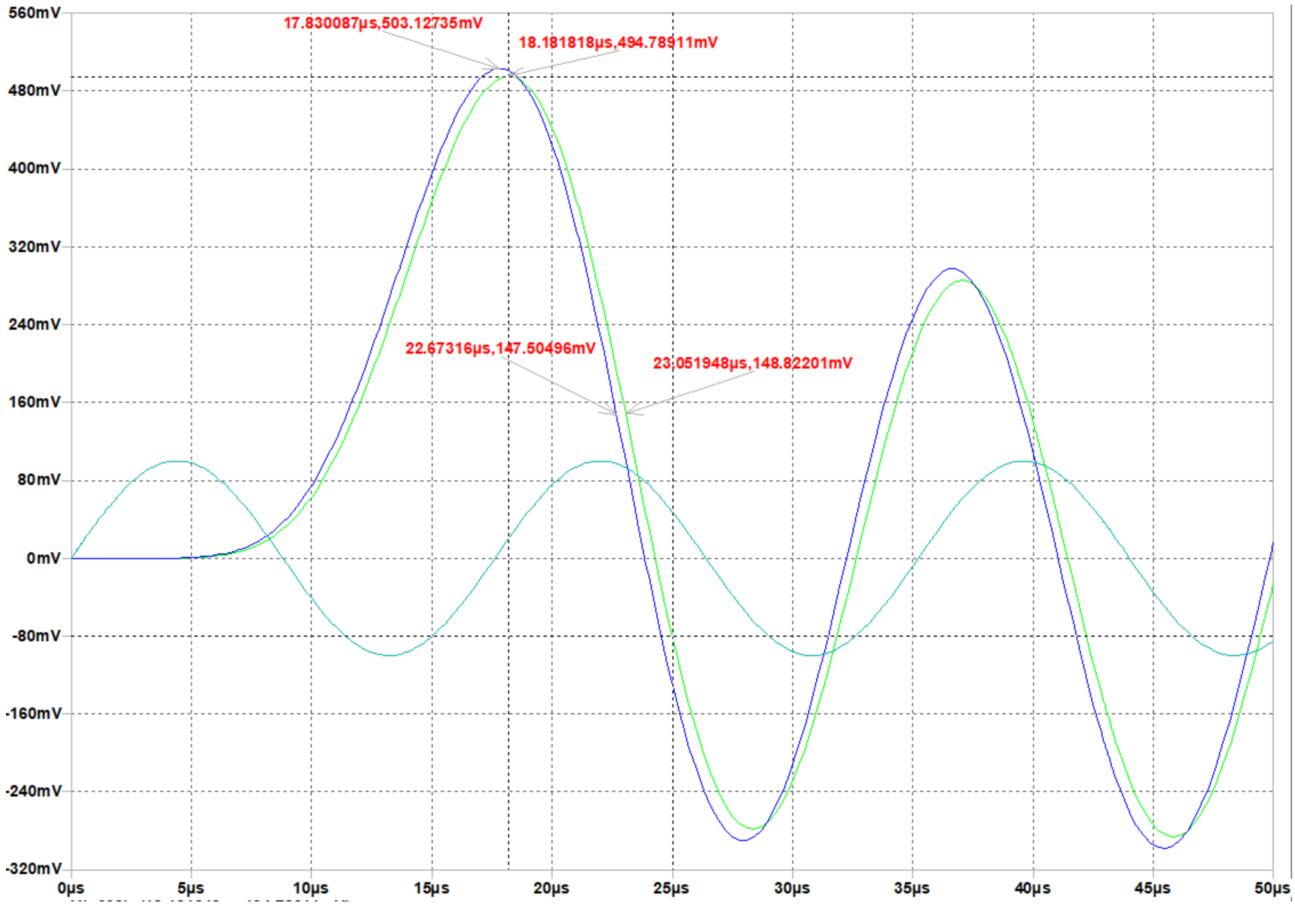

Figure 36: Transient response of phase and gain at cut-off frequency

4.15 Cascaded Bessel multi-feedback low-pass filter

4.15.1 Frequency response vs LF347

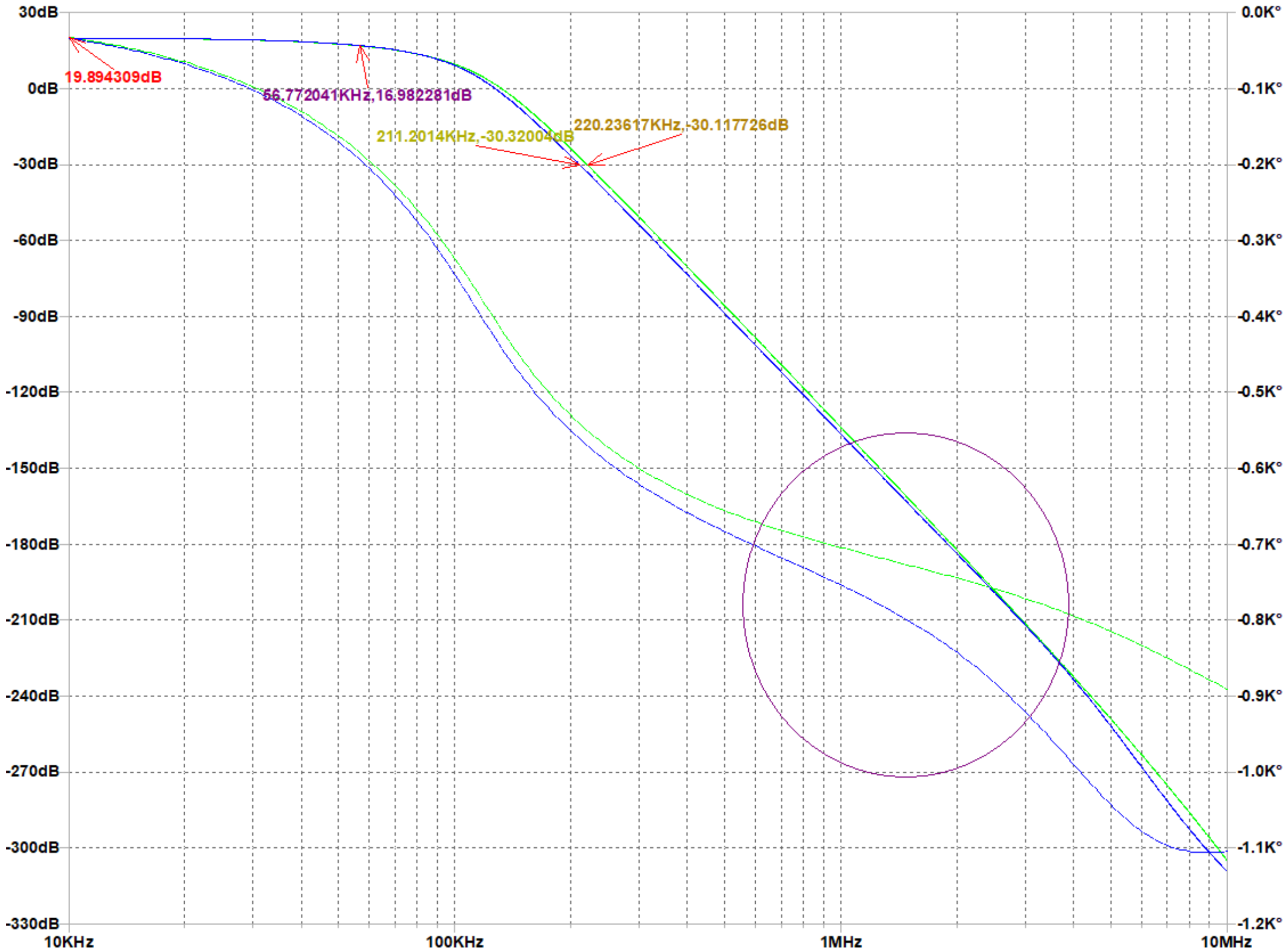

Figure 37: Frequency response

4.15.2 Frequency response vs MCP604

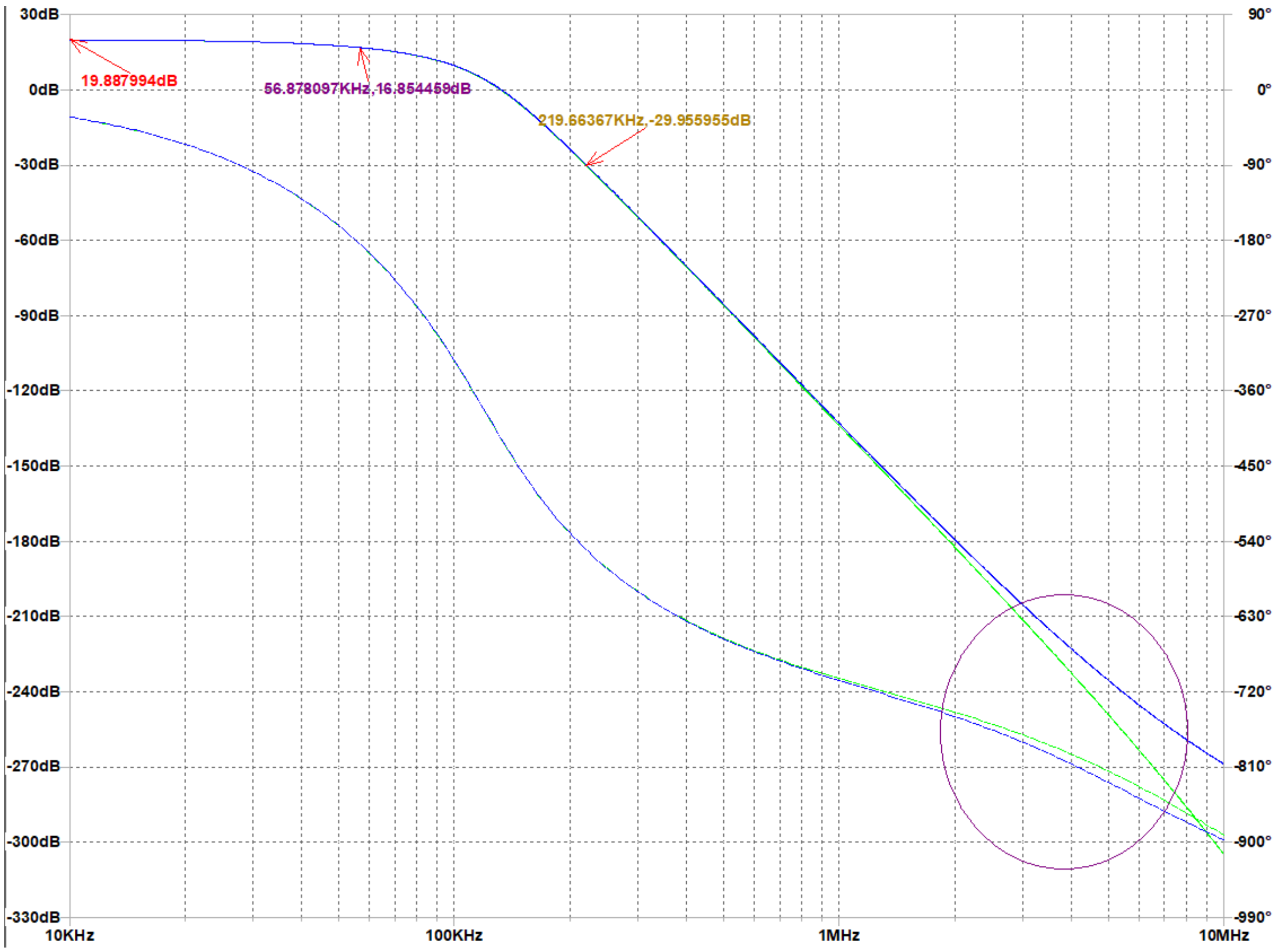

Figure 38: Frequency response

4.15.3 Transient response

Cut-off frequency transient response

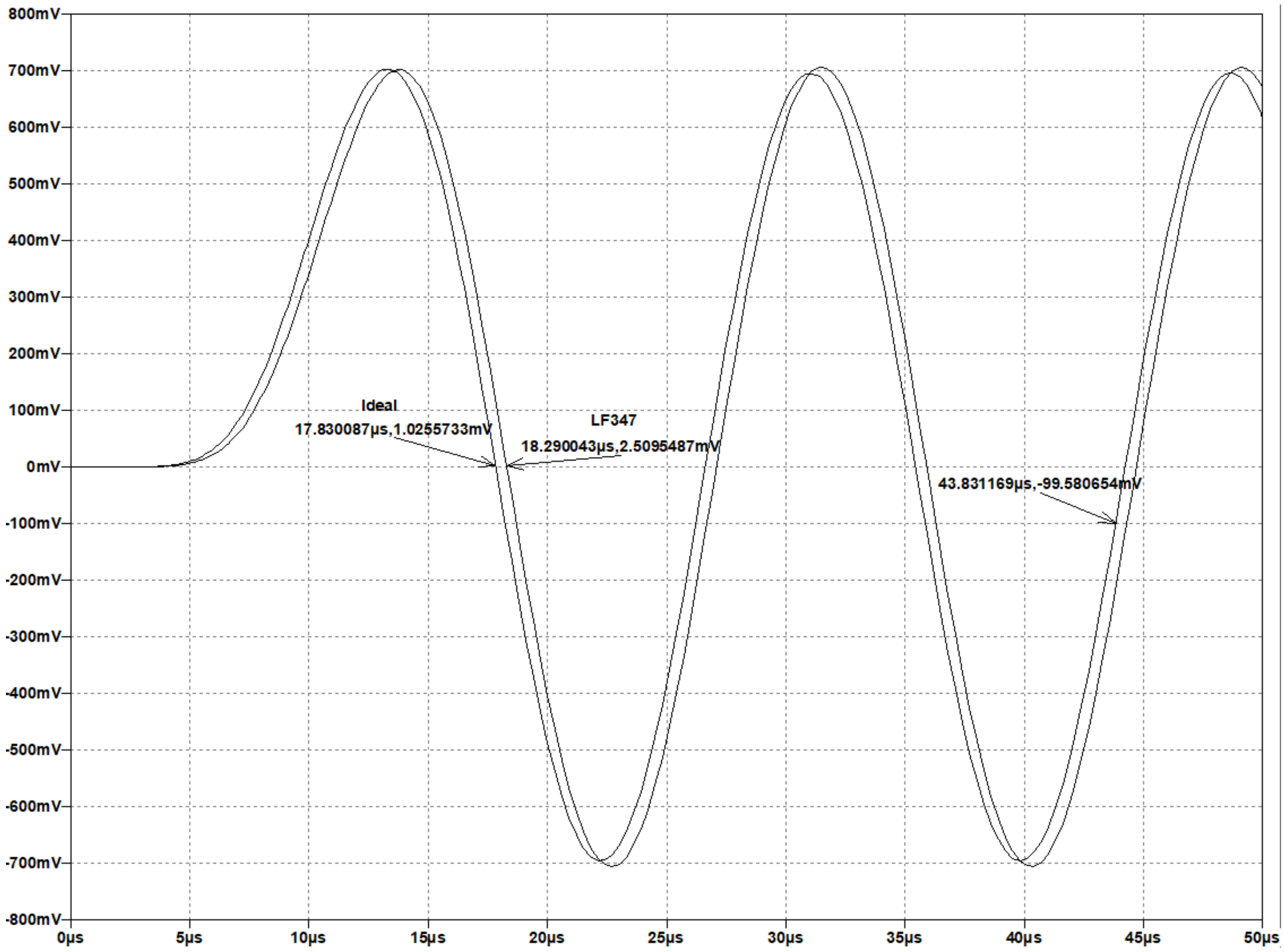

Figure 39: Transient response of phase and gain at cut-off frequency

4.16 Monte Carlo

Monte carlo analysis of the cascaded circuit for further observations

4.16.1 Frequency response

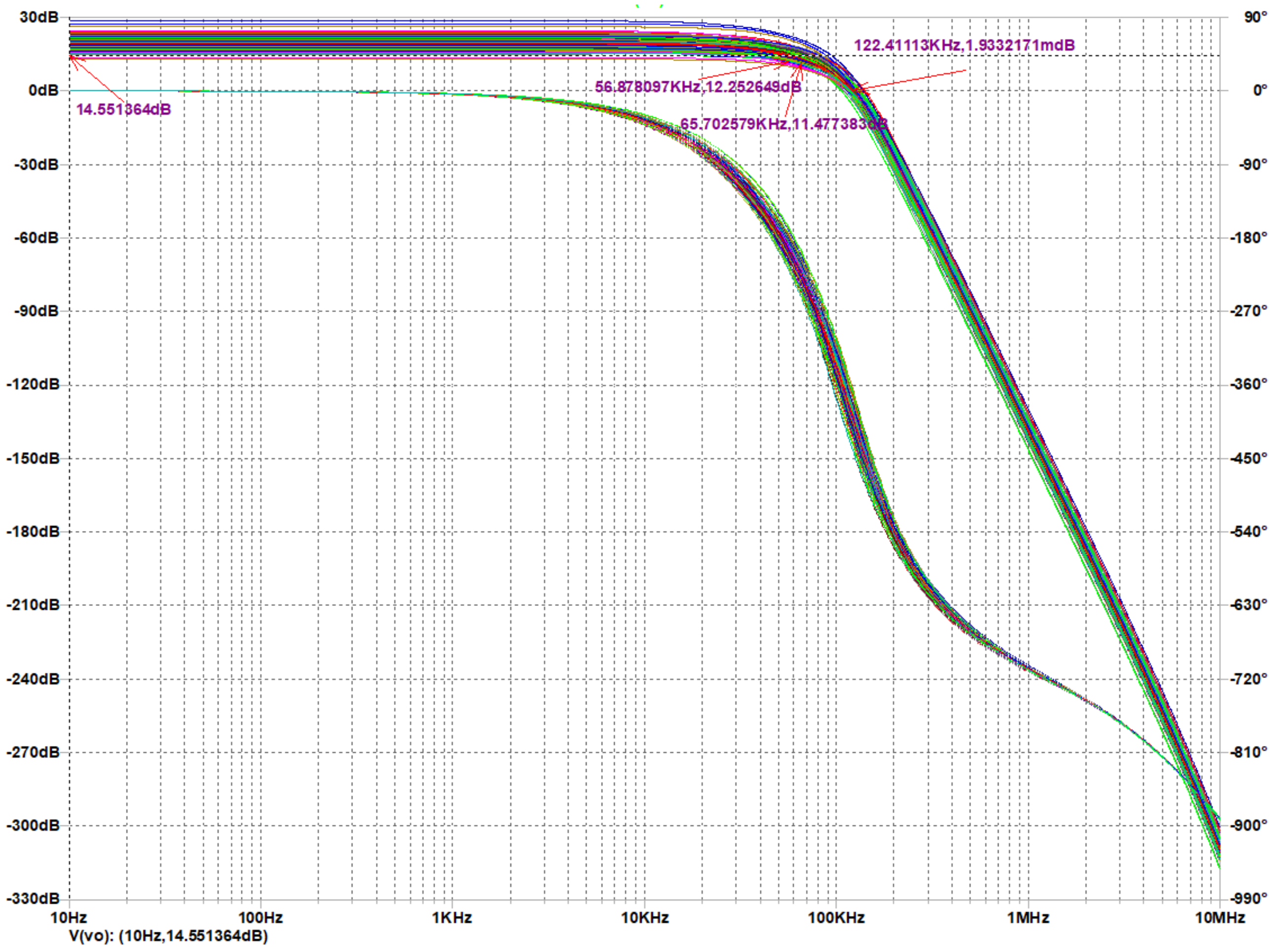

Figure 40: Monte Carlo analysis of Frequency response

4.16.2 Amplitude response

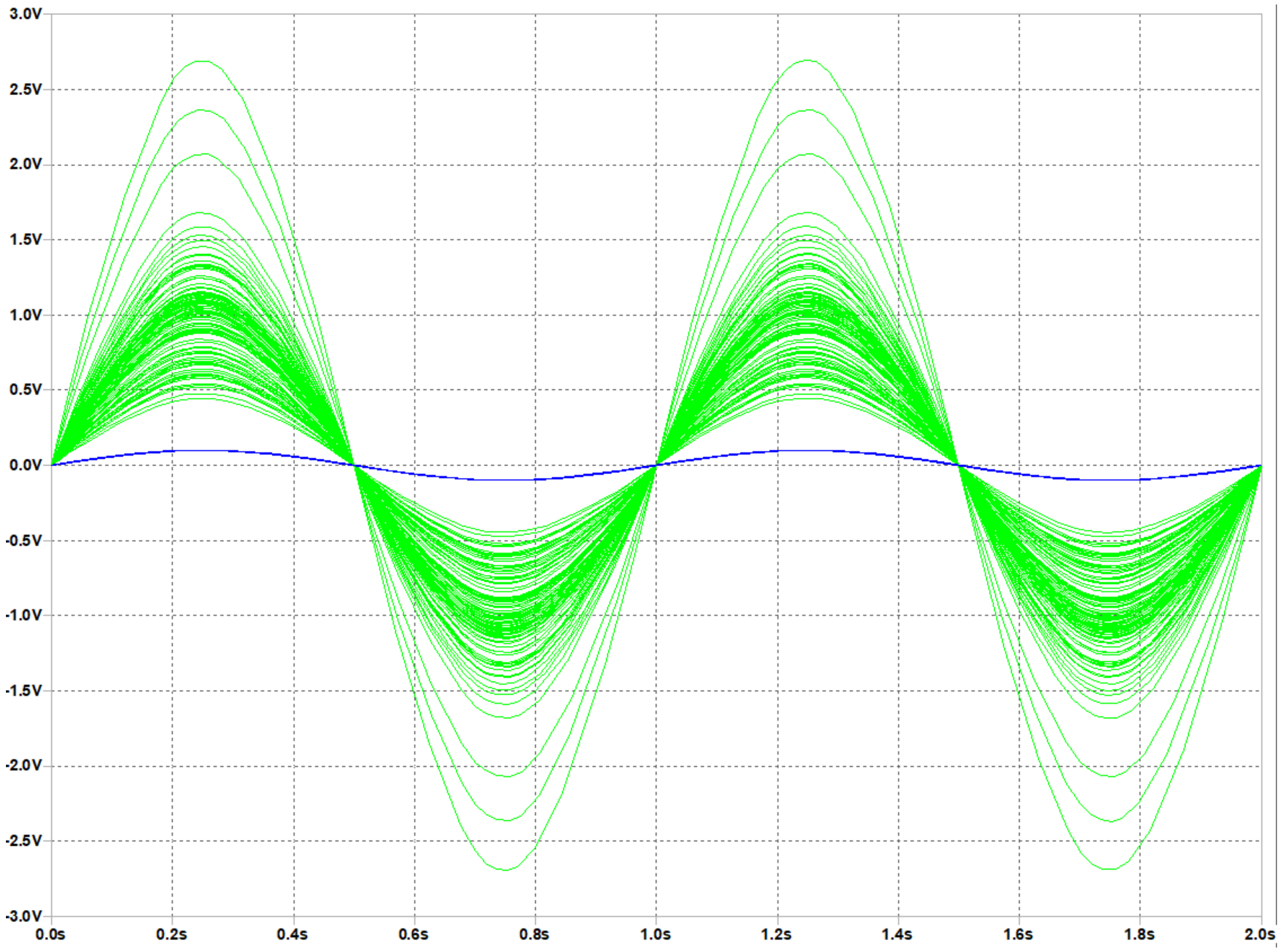

Figure 41: Monte Carlo analysis of Amplitude response

4.16.3 Cut off amplitude response

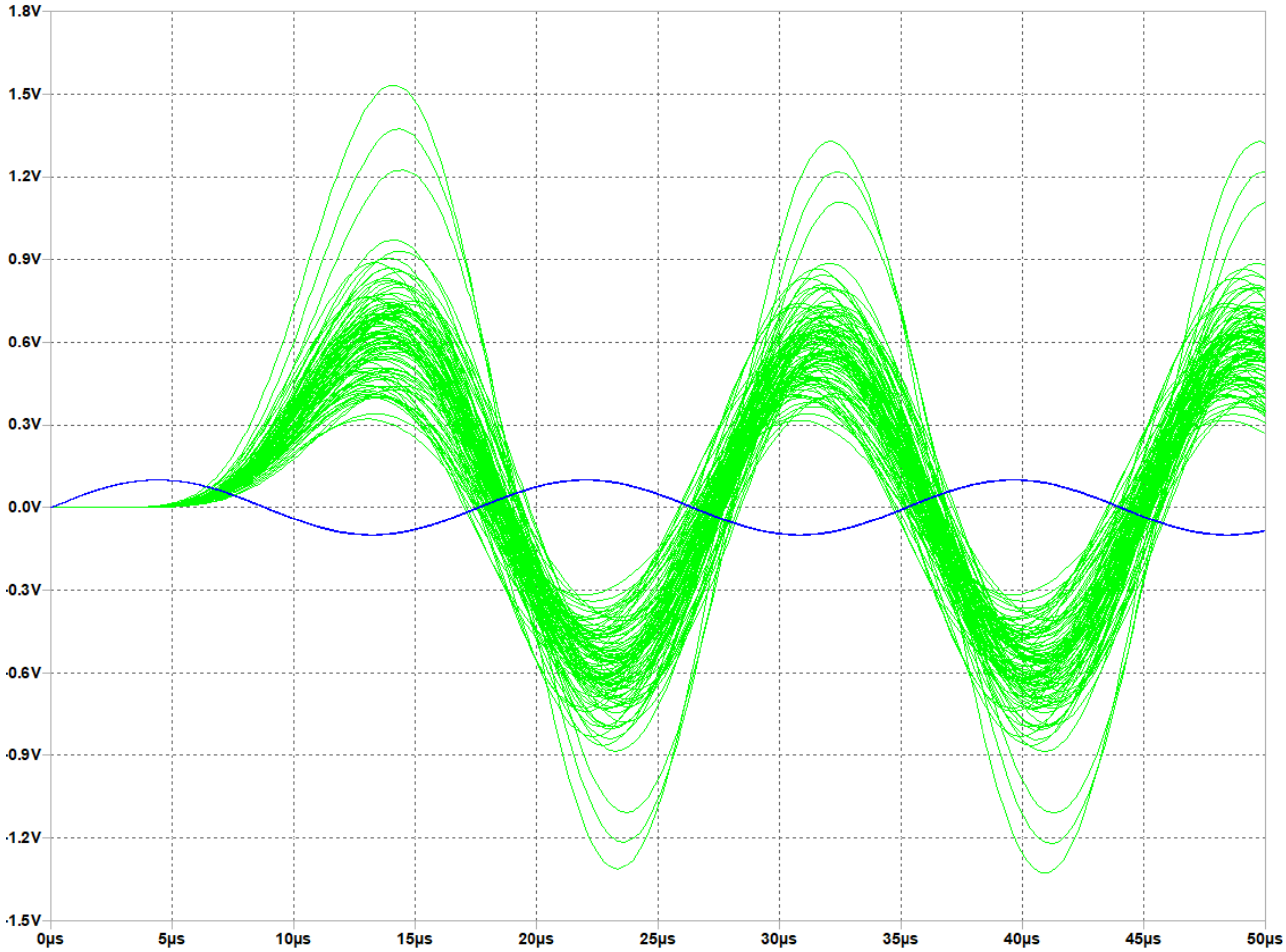

Figure 42: Monte Carlo analysis of Cut off amplitude response

4.16.4 Interpretation of results for approximated and non-ideal filters

Gain

Roll-off

Cut-off

Design frequency and natural frequency

Final Q factor

Unity frequency

Phase shift

5 Discussion of results

6 Conclusion

Two eighth order filters were successfully implemented. A second order Bessel multifeedback filter was copy and pasted four times to make an eighth order filter to a certain degree of success. A high order cascaded filter was successfully designed and simulated. A multifeedback circuit was successfully designed and implemented for both filters. Both circuits were investigated using varying physical components and different resistor-capacitor standards. The standards being the E96/E24 and E6/E12 standards and the MCP604 and LM347 op-amps investigated. Monte Carlo analysis was also successfully used for further observation of the cascaded circuit for varying component tolerances. The frequency error was observed for the Bessel filter of the cascaded filter. The cascaded filter was found to have a a closer resemblance to the ideal transfer function response than the copy-paste filter which performed well as a filter but not at the required design requirements.

A Theory questions

A.1 Explain the difference between the natural frequency and the cut-off frequency. How does the relationship between the two vary for Chebyshev and Butterworth responses?

The natural frequency of a filter is the frequency at which the the signal leaves the passband. This is the case for the Chebyshev filter which ripples before leaving the passband past the cut off frequency. The natural frequency may be the same as the cut off frequency. Which is the case for the Butterworth filter. The cut off frequency is the frequency at which the signal lowers to 3dB less than the maximum gain (in dB). This is where the gain is $1/\sqrt{2}$ the maximum gain. No distortion may be picked up by a human ear before this point as the gain reduces.

A.2 Designers need to choose between topologies when implementing their filters. What are some reasons that a designer may choose a KRC filter over a Multiple Feedback filter?

The KRC topology is less sensitive to noise for low freqeuncy operations with low Q values. The sallen key does not invert its input at each stage. Gain accuracy is higher especially when used as a unity gain filter. The amount of components may also be potentially less in this topology.

A.3 Designers need to choose between topologies when implementing their filters. What are some reasons that a designer may choose a Multiple Feedback filter over a KRC filter?

The mutlifeedback is less sensitive to component variations and high Q values being used in it's design. The filter topology is less sensitive to noise as frequency increases. The low pass multifeedback circuit at higher frequencies remains linear in attenuation at higher frequencies while the KRC loses attenuation linearity (and thus accuracy) from the topology of the shunt capacitor. The gain is easier to scale against Q.

A.4 In cascade design, how would one order the different sections?

The recommended order in the prescribed textbook [1]: the sections are ordered in ascending Q values for each section to avoid signal clipping and reduced dynamic range. Additional recommendations were found to be to place the highest Q stage first as to reduce the amount of internal noise which will be affected by the largest Q. Placing first allows the noise not be present or significantly reduced at the final stages output.

B Python code

```
import numpy as np
import matplotlib.pyplot as plt
import scipy as sp
from cmath import phase

def results(freq, mag, phase, title, xlab, ylab):
    plt.title(title, size=18)
    plt.xlabel(xlab, size=18)
    plt.ylabel(ylab, size=18)
    plt.grid(1)
    plt.xscale('log')
    plt.plot(freq, mag, 'r')
    plt.show()
    return

freq = np.logspace(0, 9, 500)
w_freq = freq * 2 * np.pi


fc = 56.8 * 10**3.0
wc = fc * 2 * np.pi
Gain = float(1.77827941)

def LP(Gain, Q, fn):
    wnc = float(fn * wc)
    mag = Gain / (1.0 - (w_freq / wnc) ** 2.0 + (1j * w_freq / wnc) / Q)
    return mag


mag_8_cp = LP(Gain, 0.577, 1.274)*LP(Gain, 0.577, 1.274)*LP(Gain, 0.577,
   red↪ 1.274)*LP(Gain, 0.577, 1.274)
mag_8_ca = LP(Gain, 0.506, 1.784)*LP(Gain, 0.560, 1.838)*LP(Gain, 0.711,
   red↪ 1.958)*LP(Gain, 1.226, 2.196)
#ph_1 = sp.angle(mag_8)
mag_8_ca = 20 * np.log10(np.abs(mag_8_ca))
mag_8_cp = 20 * np.log10(np.abs(mag_8_cp))

#mag_2 = LP(2, Q)
#mag_2 = 20 * np.log10(np.abs(mag_2))

#results(freq, mag, phase, "Test", "Hz", "dB")

plt.title("Transfer␣Function␣(dB)␣of␣8th␣order␣Low␣Pass␣Filters␣against␣
   red↪ Frequency␣(Hz)", size=18)
plt.xlabel("Frequency␣(Hz)", size=18)
plt.ylabel("|H(f)|␣(dB)", size=18)
plt.grid(1)
plt.xscale('log')
```

```
plt.plot(freq, mag_8_ca, 'r')
plt.plot(freq, mag_8_cp, 'm')
#plt.axvline(x = 10**5, color = 'k', linestyle = '--')
#plt.axvline(x = 10**6, color = 'k', linestyle = '--')
plt.axvline(x = fc, color = 'b', linestyle = '--')
#plt.axhline(y = -3, color = 'k', linestyle = '--')
plt.axhline(y = mag_8_ca[0] - 3, color = 'k', linestyle = '--')
plt.show()

#mag_8 = LP(2.0, 0.506)*LP(1.0, 0.56)*LP(1.0, 0.711)*LP(1.0, 1.226)
#ph_2 = sp.angle(mag_8)
#mag_8 = 20 * np.log10(np.abs(mag_8))

#mag_2 = LP(1, 0.506)
#mag_2 = 20 * np.log10(np.abs(mag_2))

#plt.plot(freq, mag_8, 'g')
#plt.plot(freq, mag_2, 'm')
#plt.show()

#plt.title("Phase of Transfer Function (degrees) of an order 8 Low Pass
   red↪ Filter against Frequency (Hz)", size=18)
#plt.xlabel("Frequency (Hz)", size=18)
#plt.ylabel("|H(f)| (dB)", size=18)
#plt.grid(1)
#plt.xscale('log')
#plt.plot(freq, ph_1, 'r')
#plt.plot(freq, ph_2, 'm')
#plt.show()
```