Gurobi Formulation – Phase-Model

Sets:

- P – phase (INT1_NS, INT1_EW, INT2_NS, INT2_EW, INT3_NS, INT3_EW, INT4_NB, INT4_EB, INT4_WB)
- I – Incompatible phases
- G– Intersection Groups – which phase is associated with which intersection
- C – Congested Phase

Data:

- T – simulation length.
- min_p – minimum time a phase can be on (20 seconds).
- max_p – maximum time a phase can be on (90 seconds).
- $current_{percantage}$ – The current percentage a lane can be "on" for.
- max_{cap} – the maximum a lane percentage can be turned "on" for.
- $current\_score$ – the score the simulation returns based on the light schedule.

Variables:

- $x_{p,t} - 1\ if\ phase\ p \in P\ is\ "on"\ in\ time\ t \in T, 0\ if\ not$
- $\theta -$ score (cars exited)

Objective:

- $maximise: \theta$

Constraints:

- Incompatible phases cannot be on at the same time:

$$x_{p_i,t} + x_{p_j,t} \leq 1 \qquad \forall\ t \in T\ , \forall\ p \in P, \forall\ i,j \in I$$

- One phase at each intersection must always be on – relaxed for final 90 seconds:

$$\sum_{p \in g} x_{p,t} = 1 \quad \forall\ t \in [T - \max_p)\ ,\ \ \forall\ g \in G$$

$$\sum_{p \in g} x_{p,t} \leq 1 \quad \forall\ t \in [T - \max_p, T]\ ,\ \ \forall\ g \in G$$

- A phase cannot be on longer than max_p

$$\sum_{t=s}^{s+max_p} x_{p,t} \leq max_p \qquad \forall\ p \in P, \forall\ s \in [0, T - max_p - 1]$$

- If a light turns on, the phase must be on for at least min_p

$$\sum_{t=s}^{s+min_p} x_{p,t} \geq min_p * (x_{p,s} - x_{p,s-1}) \qquad \forall p \in P, \forall\ s \in [1, T - min_p - 1]$$

- Lights cannot be green at the end of the simulation

$$x_{p,s} - x_{p,s-1} \leq 0 \quad \forall\ p \in P, \forall s \in [T - min_p + 1, T]$$

Lazy Constraints:

- Optimality cut: Bounds the maximum potential score (θ) based on the $current_{score}$ returned by the simulation.

  $$\theta \leq current_{score} + 50 * dist_{expr}$$

  Where $dist_{expr}$ is the Hamming Distance Expression and is the difference between the generated schedules and is represented as:

  $$dist_{expr} = old_{on} + new_{on} - 2 * matches$$

  Where old_{on} is the number of lights that are green in the old schedule, new_{on} is the number of lights that are on in the new schedule and $matches$ is the number of lights that stay the same between the two schedules.

- Feasibility cut: if a lane during the simulation reaches above 75% maximum capacity, add a feasibility cut that forces that lanes "on" time to be increased by at least $new\_min_{percentage}$.

  $$\sum_{t \in T} x_{c,t} \geq new\_min_{percentage} * T$$

  where $new\_min_{percentage}$ = $\min\left(\max\left(current_{percantage} + 0.025, 0.01\right), \max_{cap}\right)$.