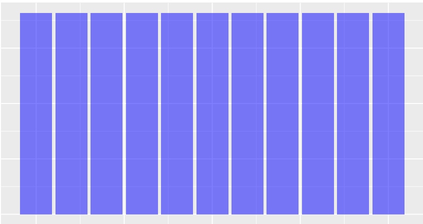
Probability Distributions

(Discrete) Uniform

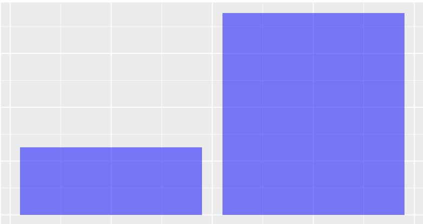
Each integer value in a specified range is assigned the same value.



Parameters	lower bound $a \in \mathbb{Z}$, upper bound $b \in \mathbb{Z}$ s.t. $a < b$
Possible values	$\{a, a+1, \dots, b-1, b\}$
Notation	$U(a,b)$
Probability function	$p(x) = \frac{1}{b-a+1}$
Expectation	$\frac{a+b}{2}$
Variance	$\frac{(b-a)(b-a+2)}{12}$
MGF	$M(t) = \frac{1}{b-a+1} \sum_{k=a}^{b} e^{kt}$
Kernel	1

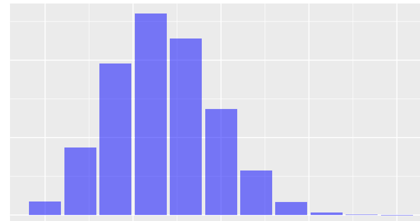
Bernoulli

Represents the success of a single experiment as a binary outcome.



Parameters	probability of success $0 \leq p \leq 1$
Possible values	$\{0, 1\}$
Notation	$\text{Bern}\,(p)$
Probability function	$p(x) = p^x (1-p)^{1-x}$
Expectation	p
Variance	$p(1-p)$
MGF	$M(t) = 1 + p\left(e^t - 1\right)$
Kernel	$p^x (1-p)^{-x}$

Binomial



Represents the number of successes in a fixed number of independent and repeated trials of the same Bernoulli experiment.

Parameters	number of trials $n \in \mathbb{N}$, probability of success on a single trial $0 \leq p \leq 1$
Possible values	$\{0, 1, 2, \dots, n\}$
Notation	$\text{Bin}\,(n, p)$
Probability function	$p(x) = \binom{n}{x} p^x (1-p)^{n-x}$
Expectation	np
Variance	$np(1-p)$
MGF	$M(t) = \left[1 + p\left(e^t - 1\right)\right]^n$
Kernel	$\binom{n}{x} p^x (1-p)^{-x}$

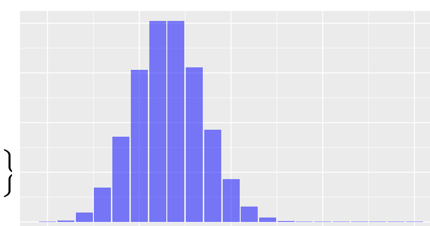
Related Distributions

- If $X \sim \text{Bin}\,(n, p)$, then

$$X = \sum_{i=1}^{n} X_i,$$

  where the $X_i \sim \text{Bern}\,(p)$ independently.

Hypergeometric

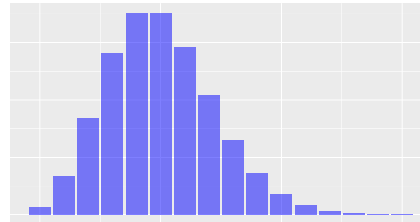


Represents the number of when drawing a fixed number of samples from a population containing a known number of successes.

Possible values	$\left\{\max\left\{0, n - (n - (N - k))\right\}, \dots, \min\left\{n, k\right\}\right\}$
Parameters	size of population $N \in \mathbb{N}$, number of successes in population $k \in \mathbb{N}$, number of samples drawn $n \in \mathbb{N}$
Notation	$\text{Hypergeometric}\,(N, k, n)$
Probability function	$p(x) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$
Expectation	$\frac{nk}{N}$
Variance	$n\left(\frac{k}{N}\right)\left(\frac{N-k}{N}\right)\left(\frac{N-n}{N-1}\right)$
MGF	No useful expression
Kernel	$\binom{k}{x}\binom{N-k}{n-x}$

Poisson

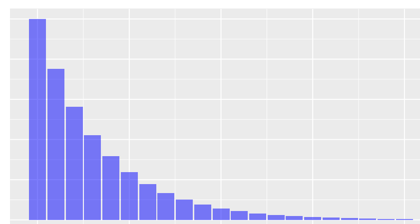
Represents the number of events occuring in a fixed interval.



Possible values	$\{0, 1, 2, \ldots\}$
Parameters	average number of events in interval $\lambda > 0$,
Notation	$\mathrm{Po}\,(\lambda)$
Probability function	$p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$
Expectation	λ
Variance	λ
MGF	$M(t) = e^{\lambda\left(e^t - 1\right)}$
Kernel	$\frac{\lambda^x}{x!}$

Geometric

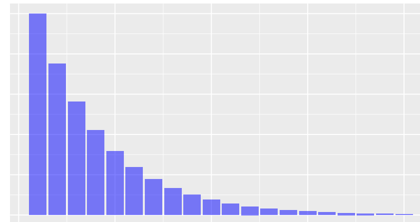
Represents the number of failed Bernoulli trials preceeding the first success.



Parameters	probability of success on a single trial $0 \leq p \leq 1$,
Possible values	$\{0, 1, 2, \ldots\}$
Notation	$\mathrm{Geom}\,(p)$
Probability function	$p(x) = (1-p)^x p$
Expectation	$\frac{1-p}{p}$
Variance	$\frac{1-p}{p^2}$
MGF	$M(t) = \frac{p}{1-(1-p)e^t}$ for $t < -\ln(1-p)$
Kernel	$(1-p)^x$

Geometric (alternative)

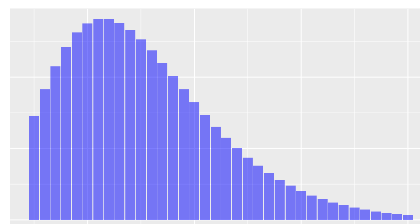
Represents the position of the first success in a sequence of Bernoulli trials.



Parameters	probability of success on a single trial $0 \leq p \leq 1$,
Possible values	$\{1, 2, \ldots\}$
Notation	$\text{Geom}\,(p)$
Probability function	$p(x) = (1-p)^{x-1}p$
Expectation	$\frac{1}{p}$
Variance	$\frac{1-p}{p^2}$
MGF	$M(t) = \frac{pe^t}{1-(1-p)e^t}$ for $t < -\ln(1-p)$
Kernel	$(1-p)^x$

Negative Binomial

Represents the number of Bernoulli trials preceeding the rth success.



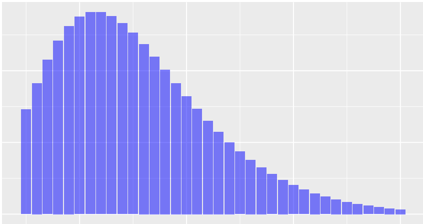
Parameters	number of desired successes $r \in \mathbb{N}$, probability of success on a single trial $0 \leq p \leq 1$,
Possible values	$\{r, r+1, \ldots\}$
Notation	$\text{NB}\,(r, p)$
Probability function	$p(x) = \binom{x-1}{r-1}(1-p)^{x-r}p^r$
Expectation	$\frac{r}{p}$
Variance	$\frac{r(1-p)}{p^2}$
MGF	$M(t) = \left[\frac{pe^t}{1-(1-p)e^t}\right]^r$ for $t < -\ln p$
Kernel	$\binom{x-1}{r-1}(1-p)^x$

Related Distributions

- If $X \sim \text{NB}\,(r, p)$, then
$$X = \sum_{i=1}^{n} X_i,$$
where the $X_i \sim \text{Geom}\,(p)$ independently.

Negatve Binomial (Alternative)

Represents the number of failures preceeding the rth success in a sequence of Bernoulli trials.

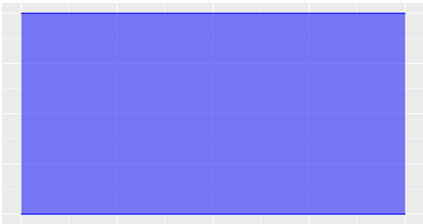


Parameters	number of desired successes $r \in \mathbb{N}$, probability of success on a single trial $0 \leq p \leq 1$,
Possible values	$\{0, 1, 2, \ldots\}$
Notation	$\mathrm{NB}(r, p)$
Probability function	$p(x) = \binom{x+r-1}{x}(1-p)^x p^r$
Expectation	$\frac{r(1-p)}{p}$
Variance	$\frac{r(1-p)}{p^2}$
MGF	$M(t) = \left[\frac{p}{1-(1-p)e^t}\right]^r$ for $t < -\ln p$
Kernel	$\binom{x+r-1}{x}(1-p)^x$

Continuous

(Continuous) Uniform

Each value in a specified interval has the same probability density.



Parameters	lower bound $a \in \mathbb{Z}$, upper bound $b \in \mathbb{Z}$ s.t. $a < b$
Possible values	(a, b)
Notation	$\mathrm{U}(a, b)$
Probability density function	$f(x) = \frac{1}{b-a}$
Cumulative distribution function	$F(x) = \begin{cases} 0, & \text{if } x \leq a, \\ \frac{x-a}{b-a}, & \text{if } a < x < b, \\ 1, & \text{if } b \leq x. \end{cases}$
Expectation	$\frac{a+b}{2}$
Variance	$\frac{(b-a)^2}{12}$
MGF	$M(t) = \begin{cases} 1, & \text{if } t = 0, \\ \frac{e^{tb}-e^{ta}}{t(b-a)}, & \text{if } t \neq 0. \end{cases}$
Kernel	1

Multivariate

Discrete

Continuous