Due: 17th of October 2016 at 11:59pm

COMP 2007 – Assignment 4

All submitted work must be done individually without consulting someone elses solutions in accordance with the Universitys Academic Dishonesty and Plagiarism policies.

Assignment should be submitted via Blackboard as pdf (no handwriting!).

Questions

1. **[60 points]** Together with some friends you organise a fruit and vegetable buyers group to get good quality produce from the markets at a reasonable price by bulk buying every week. The idea is that every weekend one person in the group buys the fruit and vegetables for everyone in the group that wants to be part of the shop that week. As a member of the group you do not have to be part of the shop every week, only the weeks you sign up for.

   Of course, the buying requires quite a lot of time and driving, so you want to make sure that the buying arrangement is *fair* and does not overload any individual. Note that since not everyone participates every week a round-robin approach is out of the question.

   Here is one way to define what is fair. Let the n people in the group be labelled $S = \{p_1, \ldots, p_n\}$. For each week i, $1 \leq i \leq d$, let S_i be the set of people in the group that signed up for the shop. If person p_j signed up at week i then the expected probability that p_j will be doing the buying that week is $1/|S_i|$. So the total *buying obligation* Δ_j for p_j, is the sum of the expected probability over all the weeks that p_j signed up to be part of the shop.

   More concretely, suppose the group plans for exactly d weeks, and on the ith week a subset of $S_i \subseteq S$ of the people want to take part. Then the above definition of the total buying obligation for person p_j can be rewritten as

$$\Delta_j = \sum_{i | p_j \in S_i} \frac{1}{|S_i|}.$$

   Ideally, we require that p_j does the buying at most $\lceil \Delta_j \rceil$ times over all the d weeks.

   A *working schedule* is a choice of a buyer for each week – that is, a sequence $p_{i_1}, p_{i_2}, \ldots, p_{i_d}$ with $p_{i_t} \in S_t$ – and that a *fair buying schedule* is one where each person p_j is chosen to do the buying at most $\lceil \Delta_j \rceil$ times.

   (a) Give an algorithm to compute a fair buying schedule with running time polynomial in n and d.

   (b) Prove that your algorithm correctly computes a fair buying schedule for any sequence of sets $S_1, \ldots, S_d$.

   (c) Prove the complexity of your algorithm

2. **[40 points]** We define an *essential* edge of a flow network (V, E, c) to be an edge whose deletion causes the largest decrease in the maximum $s-t$-flow value. Let f be an arbitrary maximum $s-t$-flow. Either prove the following claims or show through counterexamples that they are false:

   (a) An essential edge is always an edge e with a highest capacity $c(e)$ in the network.

(b) An essential edge is always an edge e with a maximum flow $f(e)$ in the network.

(c) An essential edge is always an edge e with a maximum value of $f(e)$ among the edges belonging to some minimum cut.

(d) An edge that does not belong to some minimum cut cannot be an essential edge.

(e) A network might contain several essential edges.

(f) An essential edge e must have $f(e) = c(e)$.

(g) In the special case when all the edges in the network has unit capacity, that is $\forall e \in E : c(e) = 1$, every edge in any min cut is an essential edge.