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RF & MICROWAVE CIRCUITS

CIRCUITS RF & HYPER

2018-2019

AXEL FLAMENT
DAMIEN DUCATTEAU

REMINDER

$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

Reflection coefficient is referenced to 50Ω

If Z → Γ then Z* → Γ*

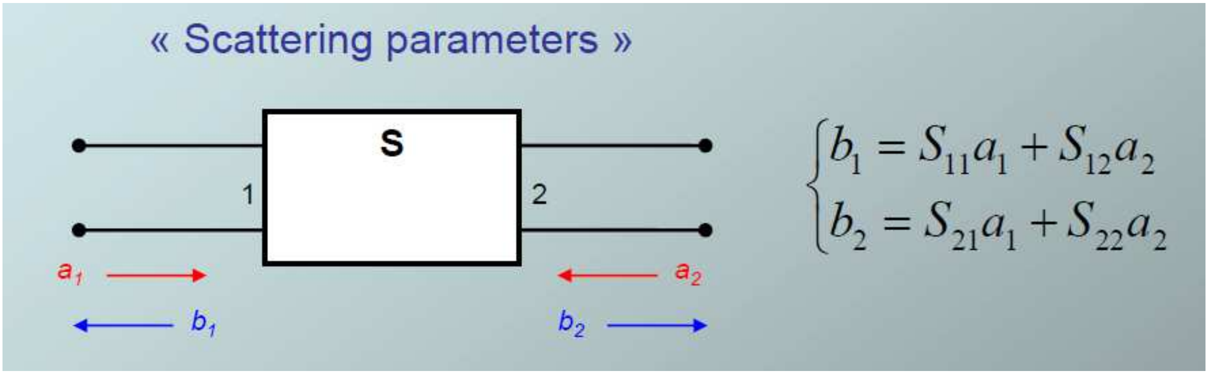
$$Z = Z_0 \frac{1+\Gamma}{1-\Gamma}$$

$a_2 = b_2\Gamma_L$

$a_1 = b_1\Gamma_S$

S-PARAMETERS

REMINDER



- $S_{11} = (b_1/a_1)_{a2=0}$= input reflection coefficient when the output is loaded by Z_0
- $S_{12} = (b_1/a_2)_{a1=0}$= Reverse transmission coefficient when the input is loaded by Z_0
- $S_{21} = (b_2/a_1)_{a2=0}$= Direct transmission coefficient when the output is loaded by Z_0
- $S_{22} = (b_2/a_2)_{a1=0}$= output reflection coefficient when the input is loaded by Z_0

REMINDER-S parameters calculation

- To calculate the S-parameters matrix of a (passive) quadripole, it is more convenient to work with known elements (voltage and currents) rather than with incident and reflected power waves…

- 1^{st} step : write equations with input and output currents and voltages
- 2^{nd} step : Use the relationships : $a_i+b_i = V_i/\sqrt{Z_0}$ and $a_i-b_i = I_i\sqrt{Z_0}$
- 3^{rd} step : Use the definitions of S parameters : $S_{11}= (b_1/a_1)_{a2=0}$ for the 4 coefficients

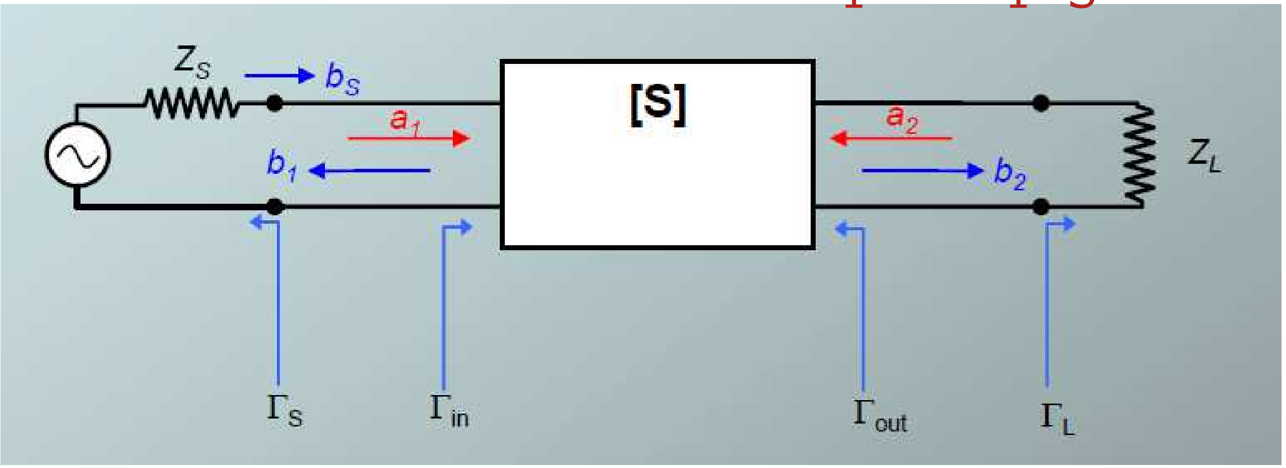
S-PARAMETERS

REMINDER

$$\begin{cases} b_1 = S_{11}a_1 + S_{12}a_2 \\ b_2 = S_{21}a_1 + S_{22}a_2 \end{cases}$$

$a_2 = b_2\Gamma_L$

$a_1 = b_1\Gamma_S$



- $\Gamma_{in} = b_1/a_1 = S_{11} + S_{12}\, a_2/a_1 = S_{11} + S_{12}\, \Gamma_L\, b_2/a_1$
- $\Gamma_{in} = S_{11}+S_{12}S_{21}\, \Gamma_L / (1-S_{22}\, \Gamma_L)$
- $\Gamma_{out} = b_2/a_2 = S_{22} + S_{21}\, a_1/a_2 = S_{22} + S_{21}\, \Gamma_S\, b_1/a_2$
- $\Gamma_{out} = S_{22} + S_{21}\, S_{12}\, \Gamma_S / (1-S_{11}\, \Gamma_S)$

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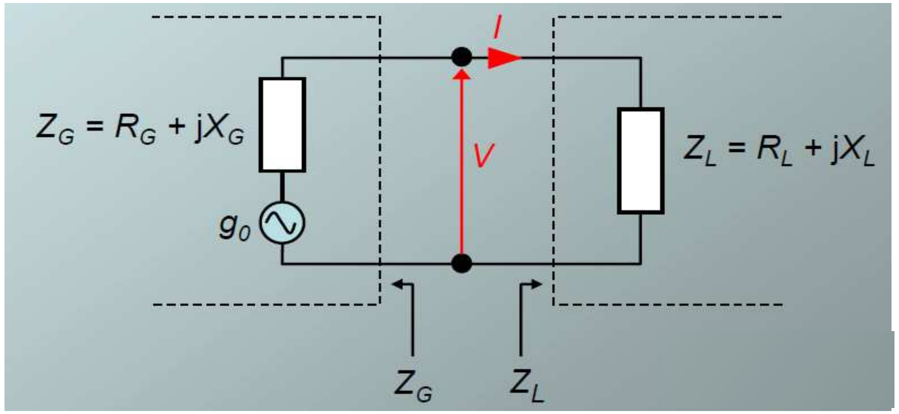
REMINDER – Power in AC circuits



- Instantaneous power = v(t) x i(t) [VA]
- Complex power = V x I* [VA], only for sinusoid signals
- Real or average power = Re (complex power) = $\frac{1}{T}\int_{t0}^{t0+T} u(t)i(t)dt$ [W]
  - dissipated as heat
- Reactive power = Im(complex power) [VAR]
  - No work
- Apparent power = |complex power| [VA]
- Power factor = cosΦ = real power/apparent power (ideally close to 1)

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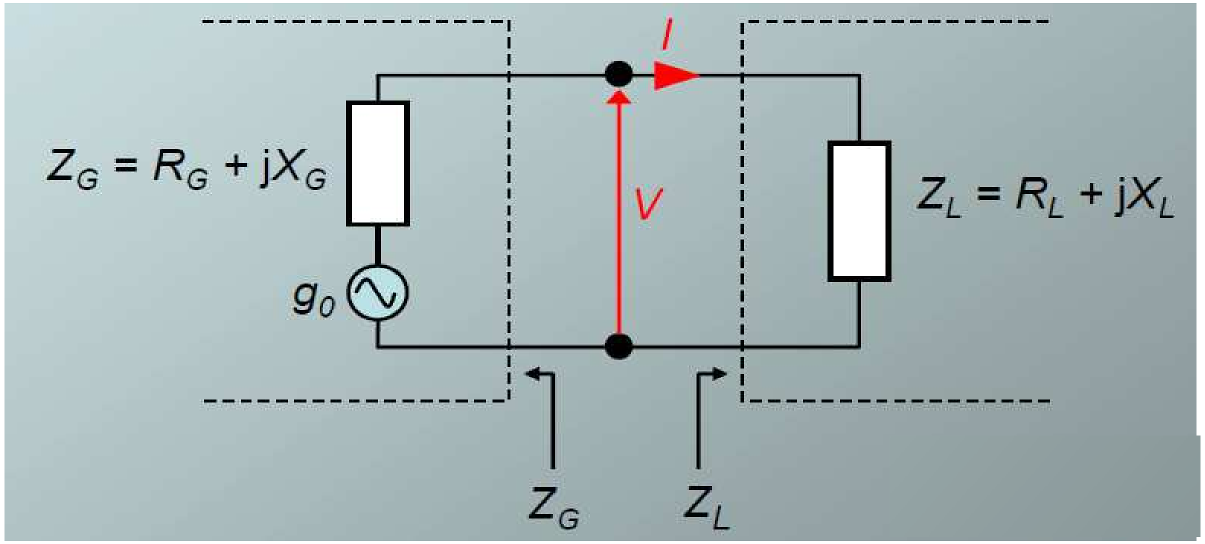
REMINDER

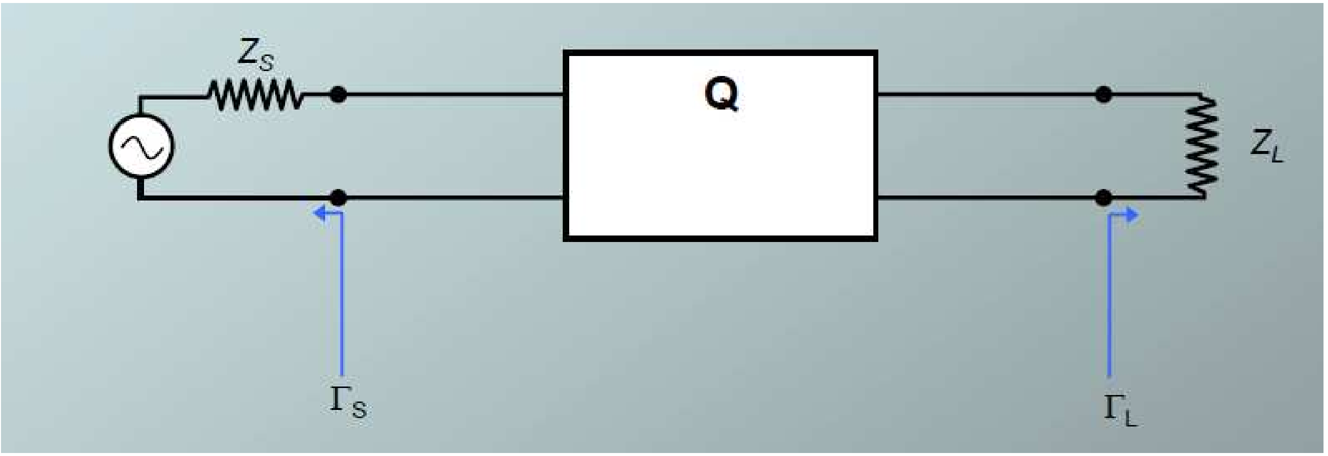


- Maximizing power transfer ?
- P = 0.5 Re (VI*) maximized if $Z_L = Z_G$ *
- In this case, the load receives all the available power from the source and delivered power to the load is maximized

OUTLINE

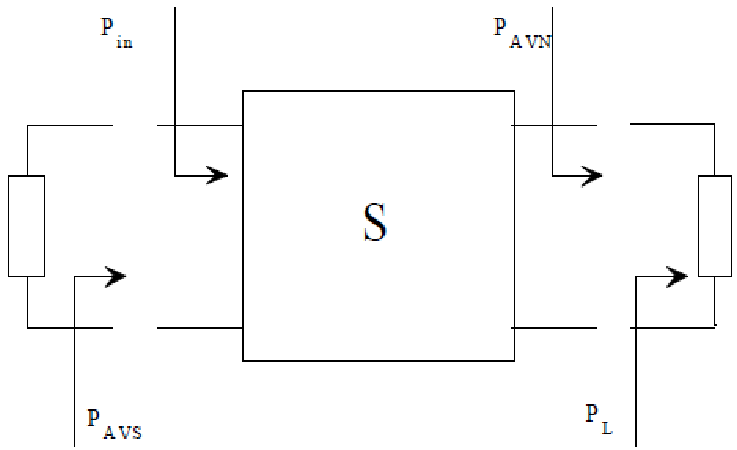
- Transducic power gain and gain definitions
- G_T calculation / Maximizing G_T
- Unilaterality
- Stability
- Noise
- Methodology

Power transducic gain



- Power Transducic Gain = G_T = Power delivered to the load/power available at the source

Other gain definitions



- $G_T = P_L/P_{AVS}$ = transducic gain = power delivered to the load /power available at the source
- $G_P = P_L/P_{in}$ = power gain = power delivered to the load / power at the input of the quadripole
- $G_A = P_{AVN}/P_{AVS}$ = available gain = power available at the output of the quadripole / power available at the source

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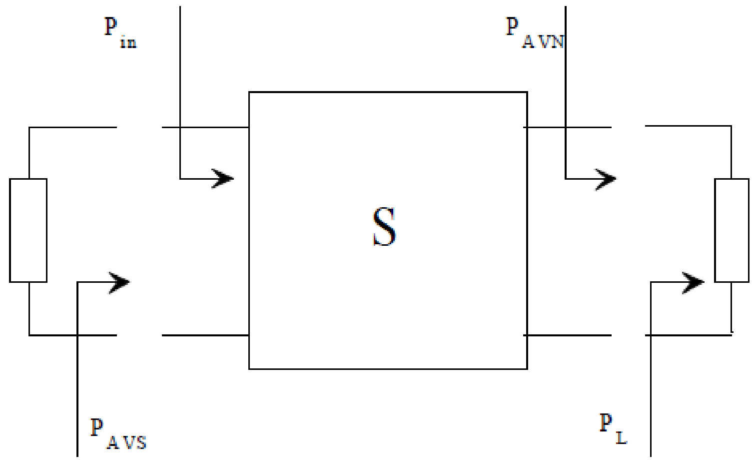
Other gain definitions



- $G_T \leq G_P$
- $G_T = G_P$ when the input is matched

- $G_T \leq G_A$
- $G_T = G_A$ when the output is matched

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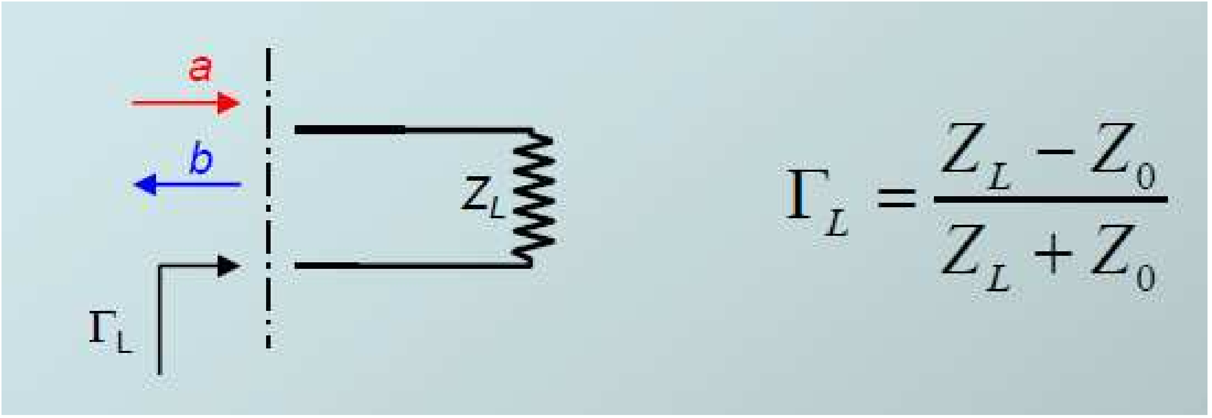
POWER TRANSDUCIC GAIN

Power delivered to the load



- Delivered Power = Incident Power – Reflected Power
- $= |a|^2 - |b|^2 = |a|^2 - |\Gamma_L|^2|a|^2 = |a|^2 (1-|\Gamma_L|^2)$

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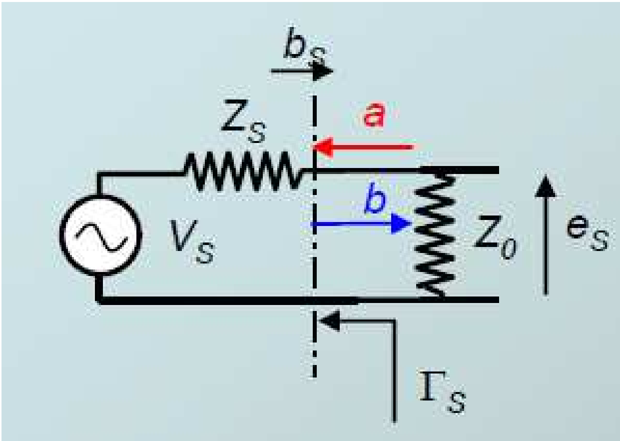
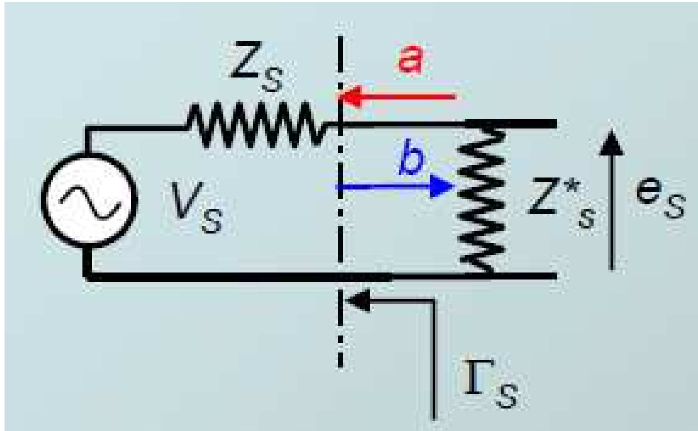
POWER TRANSDUCIC GAIN

Power available at the source

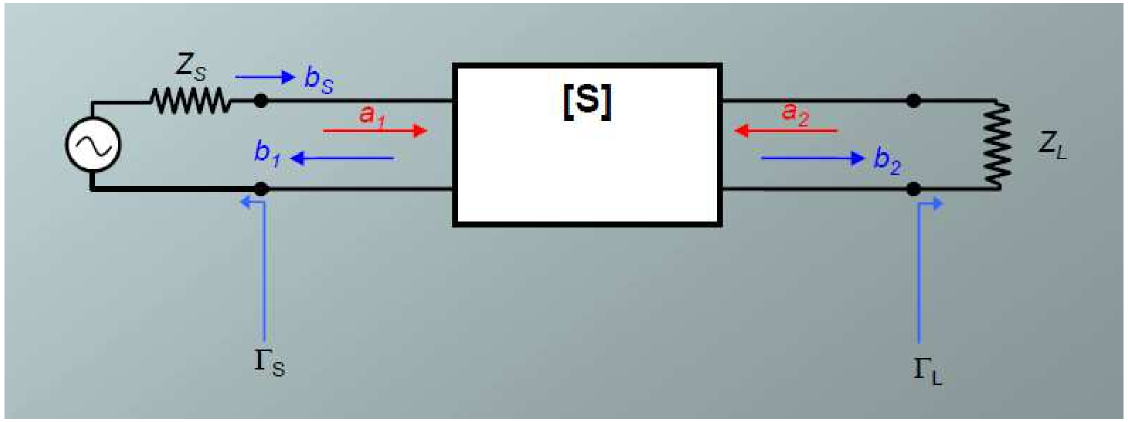




- Power available at the source = power delivered to the conjugate load
- $b=b_s + \Gamma_S\, a$
- $a=b\, \Gamma_S^*$
- So $b=b_s/(1-|\Gamma_S|^2)$
- $P_{avs} = |b|^2 - |a|^2 = |b_S|^2 / (1-|\Gamma_S|^2)$

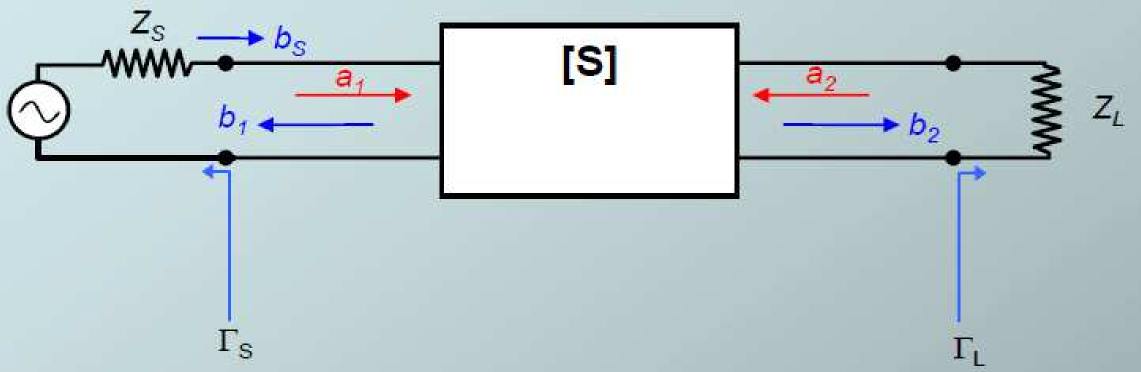
POWER TRANSDUCIC GAIN

G_T calculation



$$G_T = \frac{P_{del}}{P_{avs}} = \frac{|b_2|^2\left(1-|\Gamma_L|^2\right)}{\dfrac{|b_s|^2}{1-|\Gamma_S|^2}} = \frac{|b_2|^2}{|b_S|^2}\left(1-|\Gamma_L|^2\right)\left(1-|\Gamma_S|^2\right)$$

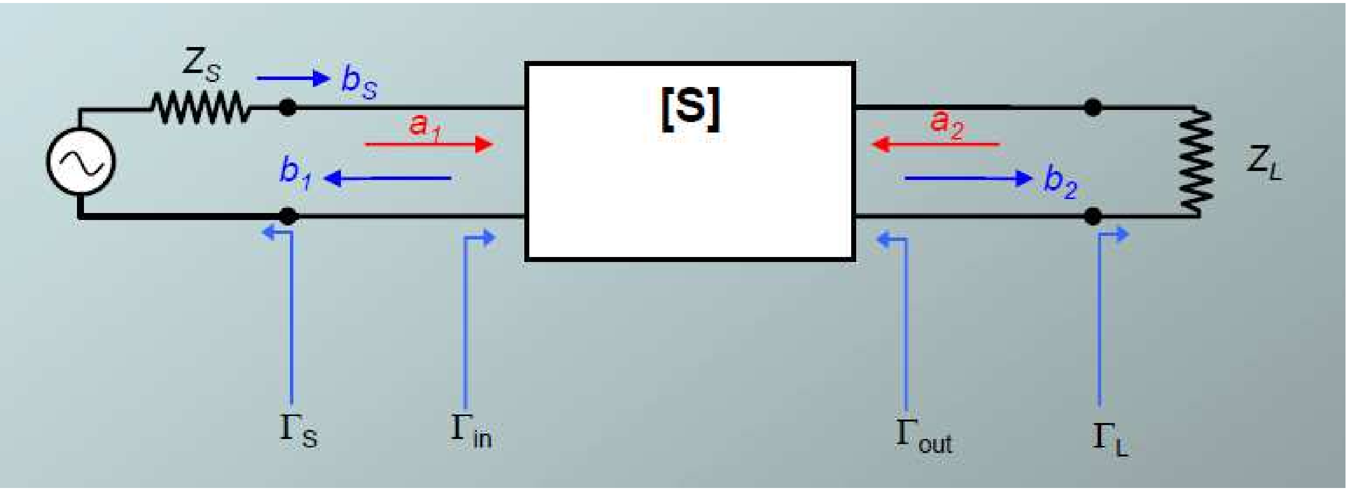
G_T calculation



$$\begin{cases} b_1 = S_{11}a_1 + S_{12}a_2 & 1 \\ b_2 = S_{21}a_1 + S_{22}a_2 & 2 \end{cases} \qquad \begin{aligned} a_2 &= \Gamma_L b_2 \quad 3 \\ a_1 &= b_S + \Gamma_S b_1 \quad 4 \end{aligned}$$

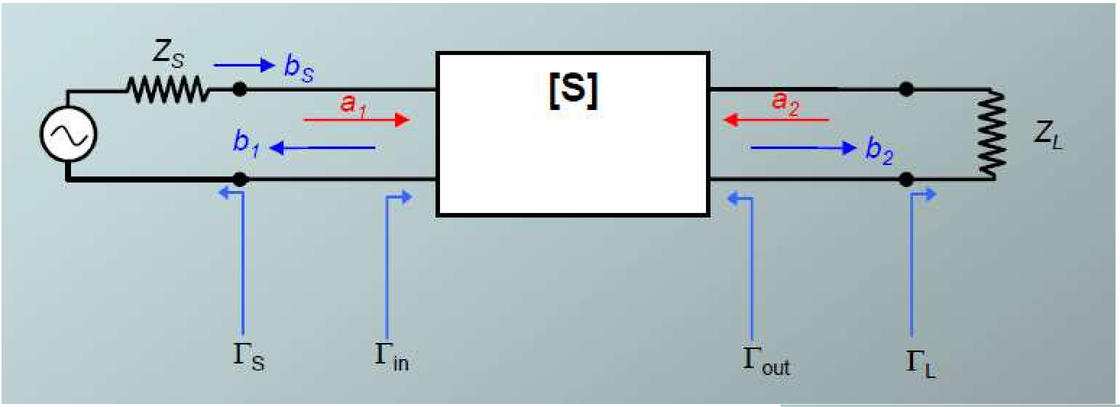
- (3) in (2) : $b_2 = S_{21}\, a_1 / (1 - S_{22}\, \Gamma_L)$ (5)
- (1) in (4) : $a_1 = b_S + \Gamma_S S_{11} a_1 + \Gamma_S S_{12} a_2 = b_S + \Gamma_S S_{11} a_1 + \Gamma_S S_{12} \Gamma_L b_2$
- So $a_1 = (b_S + \Gamma_S S_{12}\, \Gamma_L b_2) / (1 - \Gamma_S S_{11})$
- In (5) …

G_T calculation



$$G_T = \frac{|S_{21}|^2\left(1-|\Gamma_S|^2\right)\left(1-|\Gamma_L|^2\right)}{\left|(1-S_{11}\Gamma_S)(1-S_{22}\Gamma_L)-S_{21}S_{12}\Gamma_S\Gamma_L\right|^2}$$

How to maximize G_T?



$$G_T = \frac{|S_{21}|^2\left(1-|\Gamma_S|^2\right)\left(1-|\Gamma_L|^2\right)}{\left|(1-S_{11}\Gamma_S)(1-S_{22}\Gamma_L)-S_{21}S_{12}\Gamma_S\Gamma_L\right|^2}$$

- In maximizing :
  - Power transfer from the source to the quadripole input
  ($\Gamma_{in} = S_{11}+S_{12}S_{21}\,\Gamma_L / (1-S_{22}\,\Gamma_L) = \Gamma_S{}^*$) « Maximum power gain »
  - Power transfer from the quadripole output to the load
  ($\Gamma_{out} = S_{22}+S_{12}S_{21}\,\Gamma_S / (1-S_{11}\,\Gamma_S) = \Gamma_L{}^*$) « Maximum available gain »

Maximizing G_T (K>1)

- When both input and output are matched (and if it is possible), we have the Maximum Available Gain (MAG)
- $\Gamma_{in} = S_{11}+S_{12}S_{21}\,\Gamma_L / (1-S_{22}\,\Gamma_L) = \Gamma_S{}^*$
- $\Gamma_{out} = S_{22}+S_{12}S_{21}\,\Gamma_S / (1-S_{11}\,\Gamma_S) = \Gamma_L{}^*$

$$G_{MAG} = \left|\frac{S_{21}}{S_{12}}\right| \left|K - \sqrt{K^2 - 1}\right| \quad \text{où}$$

$$K = \frac{1 + |\Delta|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12}S_{21}|} > 1$$

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

- We MUST have K>1 for MAG adaptation !

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Maximizing G_T (K>1)

- $\Gamma_{in} = \Gamma_S^* \Leftrightarrow$

$$\Gamma_S = M^*\left(\frac{B_1 \pm \sqrt{B_1^2 - 4|M|^2}}{2|M|^2}\right)$$

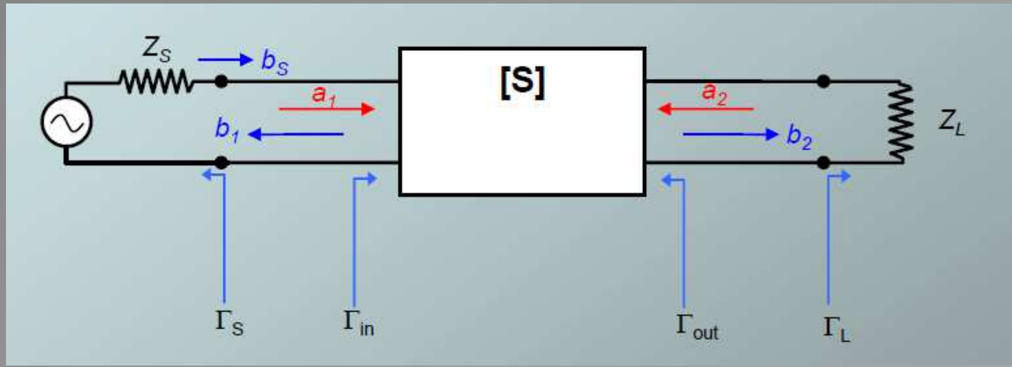
- $\Gamma_{out} = \Gamma_L^* \Leftrightarrow$

$$\Gamma_L = N^*\left(\frac{B_2 \pm \sqrt{B_2^2 - 4|N|^2}}{2|N|^2}\right)$$

Use minus sign when B_1 is positive and plus sign when B_1 is negative

- $B_1 = 1+|S_{11}|^2 - |S_{22}|^2 - |\Delta|^2$
- $B_2 = 1+|S_{22}|^2 - |S_{11}|^2 - |\Delta|^2$
- $M = S_{11} - \Delta S_{22}^*$
- $N = S_{22} - \Delta S_{11}^*$
- $\Delta = S_{11}S_{22} - S_{21}S_{12}$

Examples



- Let's consider a GaAs MESFET transistor whose S-parameters @ 1GHz are the following :
- S_{11} = 0.94 exp –j 45° S_{12} = 0.04 exp –j 64°
- S_{21} = 4.61 exp j 142° S_{22} = 0.52 exp –j 20°
- What is the transducic gain of the transistor when $Z_S = Z_L = 50\Omega$?
- What is the transducic gain of the transistor when $Z_S = 50\Omega$ and $Z_L = 100\Omega$?
- What is the transducic gain of the transistor when $Z_S = 100\Omega$ and $Z_L = 50\Omega$?
- What is the transducic gain of the transistor when $Z_S = Z_L = 100\Omega$?
- What is the maximum transducic gain of the transistor when $Z_S = 50\Omega$? What must be the output impedance in this case?
- What is the maximum transducic gain of the transistor when $Z_L = 50\Omega$? What must be the input impedance in this case?
- What is the maximum transducic gain of the transistor?

S_{11} = 0.94 exp –j 45° =0.665-j0.665 S_{12} = 0.04 exp –j 64° = 0.0175 –j0.036
S_{21} = 4.61 exp j 142° = -3.63 + j2.84 S_{22} = 0.52 exp –j 20° = 0.49 –j0.18

ANSWERS

- $Z_L = Z_S = 50\Omega$ so $\Gamma_L = \Gamma_S = 0$: $G_T = |S_{21}|^2 = 21.2 = 13.3$dB
- $Z_S = 50\Omega$ and $Z_L = 100\Omega$ so $\Gamma_L = 1/3$ and $\Gamma_S = 0$: $G_T = |S_{21}|^2 (1-|\Gamma_L|^2)/|1-S_{22}\Gamma_L|^2 = 21.2 * (8/9) / (0.7036) = 26.8 = 14.3$dB
- $Z_S = 100\Omega$ and $Z_L = 50\Omega$ so $\Gamma_L = 0$ and $\Gamma_S = 1/3$: $G_T = |S_{21}|^2 (1-|\Gamma_S|^2)/|1-S_{11}\Gamma_S|^2 = 21.2 * (8/9) / (0.6568) = 28.7 = 14.6$dB
- $Z_L = Z_S = 100\Omega$ so $\Gamma_L = \Gamma_S = 1/3$:
  - $G_T = |S_{21}|^2 (1-|\Gamma_L|^2)(1-|\Gamma_S|^2)/|(1-S_{22}\Gamma_L)(1-S_{11}\Gamma_S) - S_{21}S_{12}\Gamma_L\Gamma_S|^2$
  - $G_T = 21.2 (8/9)(8/9)/(0.448) = 37.4 = 15.73$dB
- When $Z_S = 50\Omega$, $\Gamma_{out} = S_{22}$. To achieve the highest gain, Γ_L should be set equal to $\Gamma_{out}* = S_{22}* = 0.49 + j0.18$ (ZL = 124+61.5jΩ)
  - $G_T = G_T = |S_{21}|^2 (1-|\Gamma_L|^2)/|1-S_{22}\Gamma_L|^2 = 21.2 * 1.37 = 29.06 = 14.63$dB
- When $Z_L = 50\Omega$, $\Gamma_{in} = S_{11}$. To achieve the highest gain, Γ_S should be set equal to $\Gamma_{in}* = S_{11}* = 0.66 + j0.66$ (Z_S = 11.7+119.7jΩ)
  - $G_T = G_T = |S_{21}|^2 (1-|\Gamma_S|^2)/|1-S_{11}\Gamma_S|^2 = 21.2 * 8.59 = 182.1 = 22.6$dB
- MAG but K<1 so MSG = $|S_{21}/S_{12}| = 115.25 = 20.6$dB

Unilaterality

- One common assumption is to assume that the component is unilateral (i.e. $S_{12} = 0$)
- In this case :

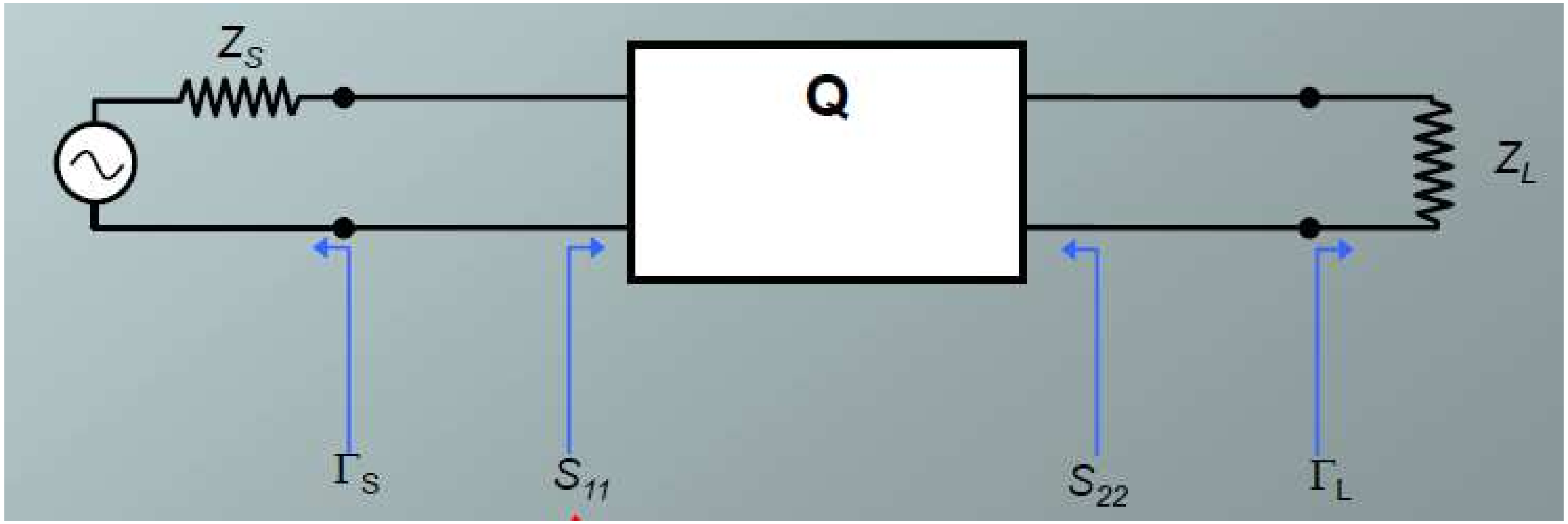
$$\boxed{G_T = \frac{|S_{21}|^2\left(1-|\Gamma_S|^2\right)\left(1-|\Gamma_L|^2\right)}{\left|(1-S_{11}\Gamma_S)(1-S_{22}\Gamma_L)-S_{21}S_{12}\Gamma_S\Gamma_L\right|^2}}$$

$$\boxed{G_{Tu} = |S_{21}|^2 \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}}$$

$$G_{Tu}(\mathrm{dB}) = G_0(\mathrm{dB}) + G_S(\mathrm{dB}) + G_L(\mathrm{dB})$$

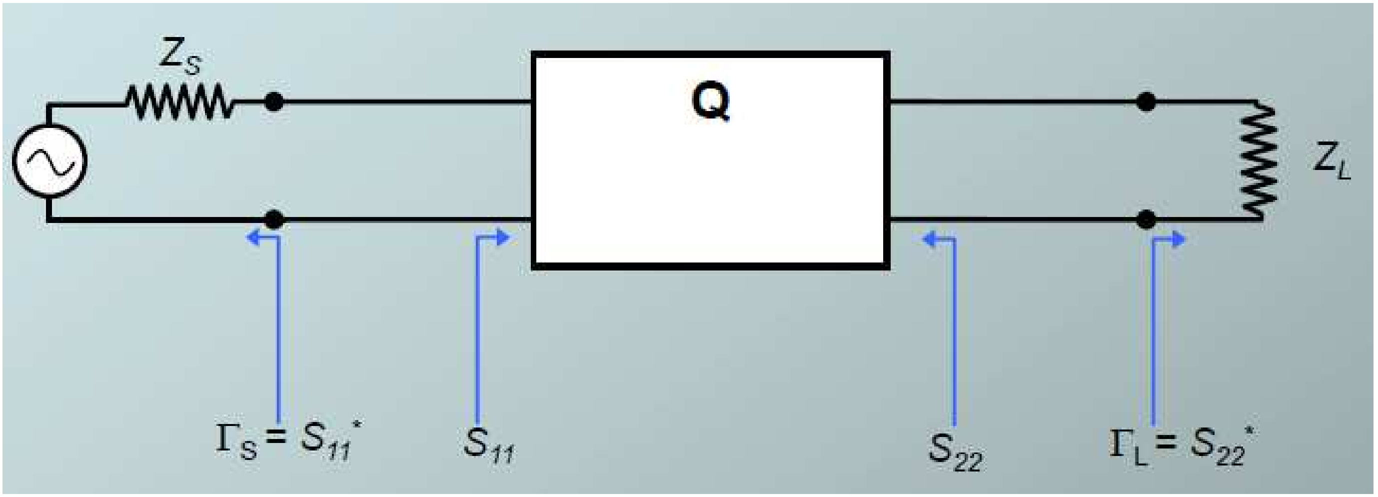
Unilaterality

- When a component is unilateral :
  - Γ_{in} is independent of output load ($\Gamma_{in} = S_{11}$)
  - Γ_{out} is independent of input load ($\Gamma_{out} = S_{22}$)



Maximizing G_{Tu}

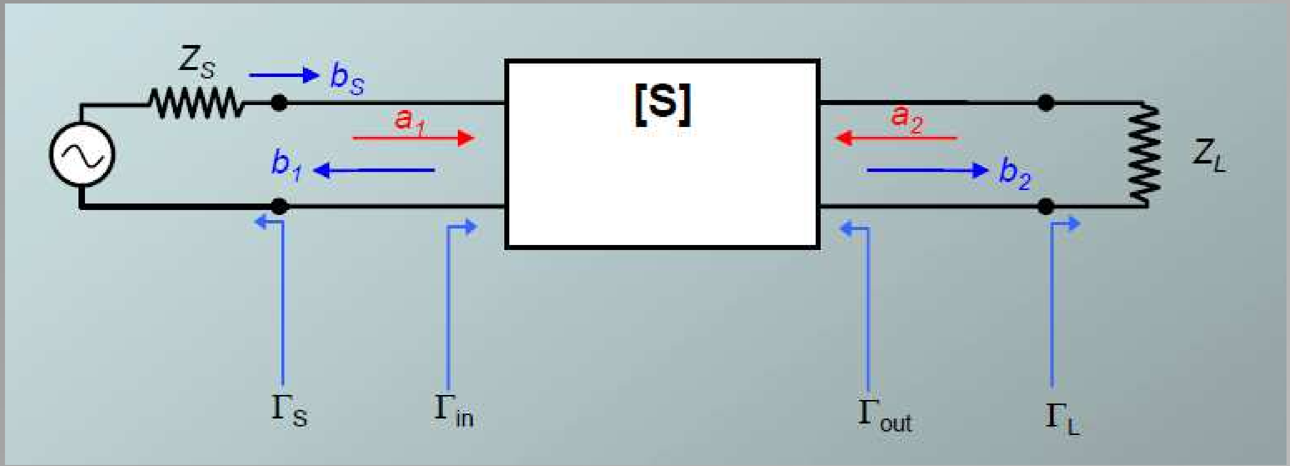
- To maximize G_{Tu}, input and output must be matched



$$G_{Tu\max} = |S_{21}|^2 \frac{1}{\left(1-|S_{11}|^2\right)} \frac{1}{\left(1-|S_{22}|^2\right)}$$

Example

- Let's consider a GaAs MESFET transistor whose S-parameters @ 1GHz are the following :
- S_{11} = 0.94 exp –j 45° S_{12} = 0.04 exp –j 64°
- S_{21} = 4.61 exp j 142° S_{22} = 0.52 exp –j 20°
- What is the maximum unilateral transducic gain?



Answer

$$G_{Tu\max} = |S_{21}|^2 \frac{1}{\left(1-|S_{11}|^2\right)} \frac{1}{\left(1-|S_{22}|^2\right)}$$

- If the component is supposed to be unilateral, $G_{Tu} = |4.61|^2/(1-0.94^2)(1-0.52)^2 = 250.2 = 24dB$

- But we have to check the unilaterality assumption !!!

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Unilaterality criterion

$$G_T = \frac{|S_{21}|^2\left(1-|\Gamma_S|^2\right)\left(1-|\Gamma_L|^2\right)}{\left|(1-S_{11}\Gamma_S)(1-S_{22}\Gamma_L)-S_{21}S_{12}\Gamma_S\Gamma_L\right|^2}$$

$$G_{Tu} = |S_{21}|^2 \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

- When assuming $S_{12} = 0$, the G_T expression turns into the G_{Tu} expression
- By doing so, you introduce an error (because S_{12} is never equals to 0), which is :

$$\frac{1}{(1+u)^2} < \frac{G_T}{G_{Tu}} < \frac{1}{(1-u)^2}$$

- With

$$u = \frac{\left|s_{11}s_{22}s_{12}s_{21}\right|}{\left|(1-|s_{11}|^2)(1-|s_{22}|^2)\right|}$$

- You have to check whether this error is big or not !!
- 10% error ⇔ u<0.05 , 1% error ⇔ u<0.005
- u=0.1 ⇔ 20% error, u = 0.3 ⇔ 100% error !

Example

- Let's consider a GaAs MESFET transistor whose S-parameters @ 1GHz are the following :
- S_{11} = 0.94 exp –j 45° S_{12} = 0.04 exp –j 64°
- S_{21} = 4.61 exp j 142° S_{22} = 0.52 exp –j 20°

- Check the unilaterality of the component

- Answer : u = 0.94*4.61*0.04*0.52 / $(1-0.94^2)(1-0.52^2)$
- u= 1.06
- The device is non-unilateral (we say bilateral), the assumption S_{12} = 0 is wrong !

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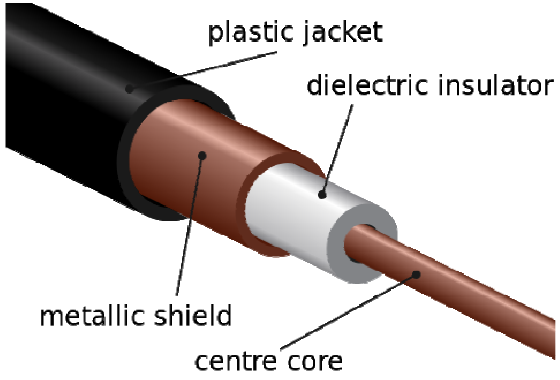
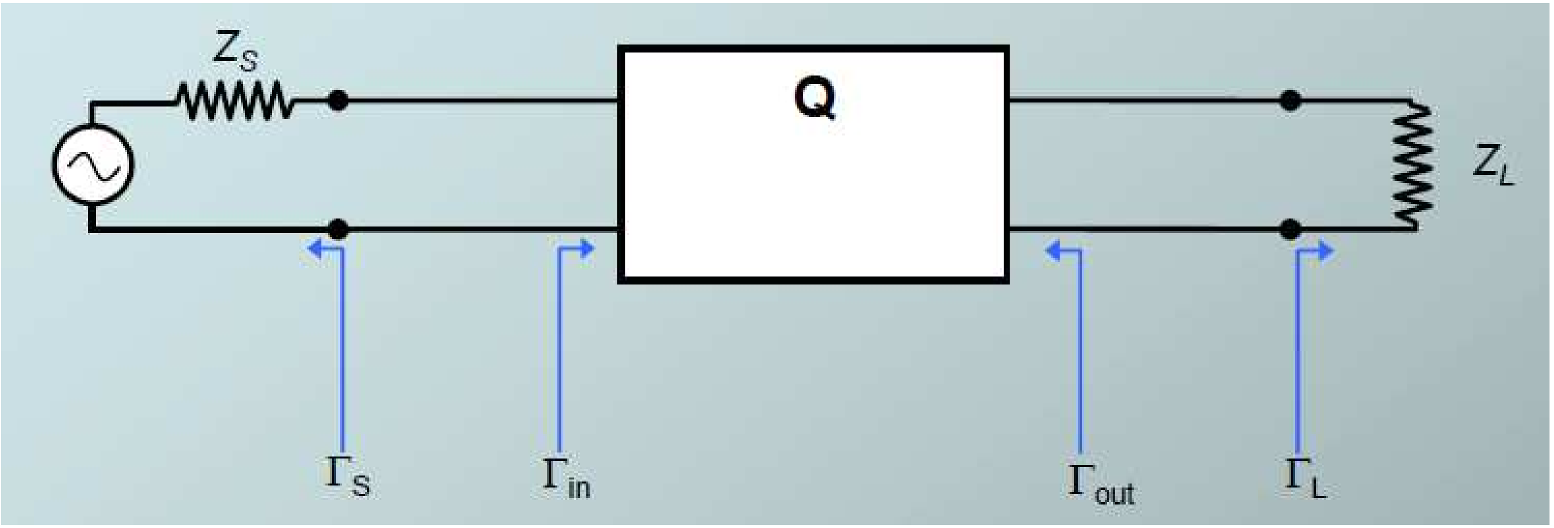


Interlude – why 50Ω ?

- Let's consider an air-dielectric coaxial cable
  - $Z_0 = 60/\sqrt{\varepsilon_r}$ x ln(outer radius/inner radius)
  - Peak electric field = E_{max} = V /((inner radius) x ln(outer radius/inner radius))
- The max power « capability » of the cable is E_{max}^2/Z_0
  - We find a maximum at the condition : outer radius/inner radius = $\sqrt{e}$
  - Setting this into Z_0 expression, we find 30Ω
- The attenuation constant in a transmission line is $R/2Z_0$ where R is the series resistance per unit length
  - R = (outer radius $^{-1}$ + inner radius^{-1})/(2pi*skin depth*metal conductivity)
  - We find a minimum at the condition : outer radius/inner radius = 3.6
  - Setting this into Z_0 expression, we find 77Ω
- An arithmetic or geometric average between these 2 values gives… 50Ω !!

Stability

- When we want to optimize power transfer, there is a risk that the input wave get reflected at the output and come back to the input : oscillation…
- Unless the component is unilateral !
- instability exists because $S_{12} \neq 0$



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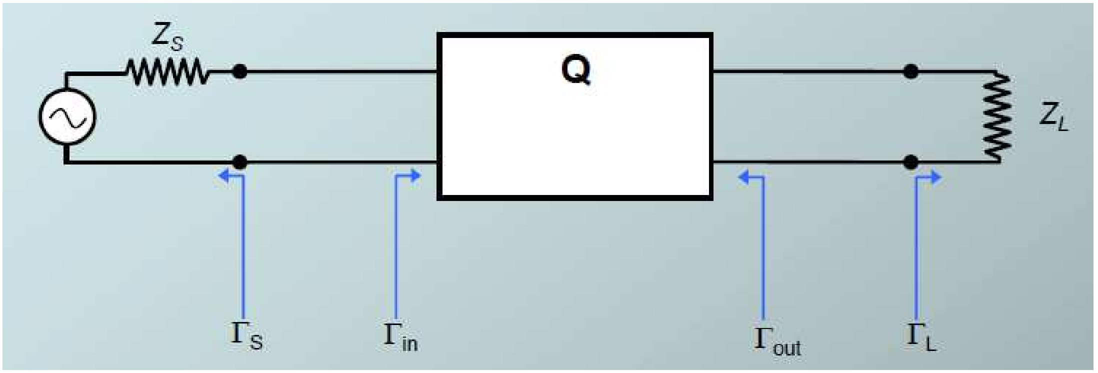
Stability



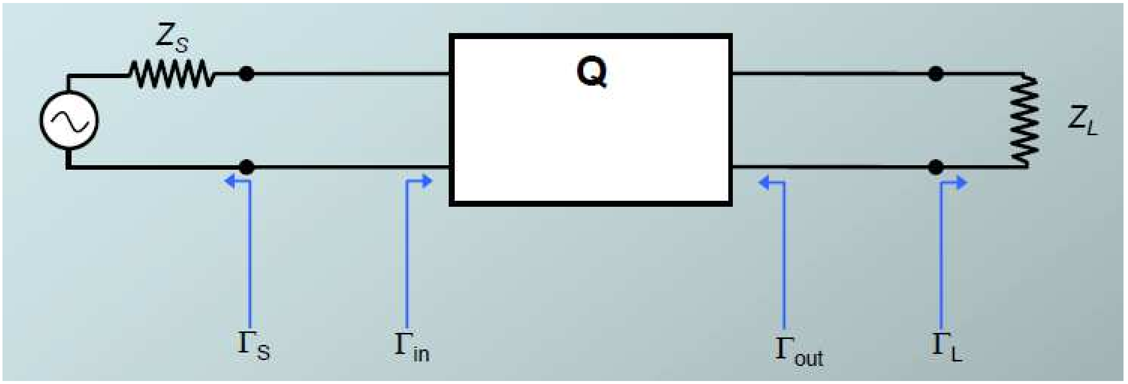
- Instability occurs because energy at one port grows each time a wave is reflected at that port
  - It means that the reflection coefficient has a module higher than 1
- In particular, passive quadripole are always stable

- We have to check the input and output reflection coefficients of the considered component

- $\Gamma_{in} = S_{11}+S_{12}S_{21}\,\Gamma_L\,/\,(1-S_{22}\,\Gamma_L)$
- $\Gamma_{out} = S_{22} + S_{21}\,S_{12}\,\Gamma_S\,/\,(1-S_{11}\,\Gamma_S)$

- →Stability depends on input and output loads

Stability



- Stability ⇔ $|\Gamma_{in}| < 1$ and $|\Gamma_{out}| < 1$
- Method : calculate K (Rollett factor):

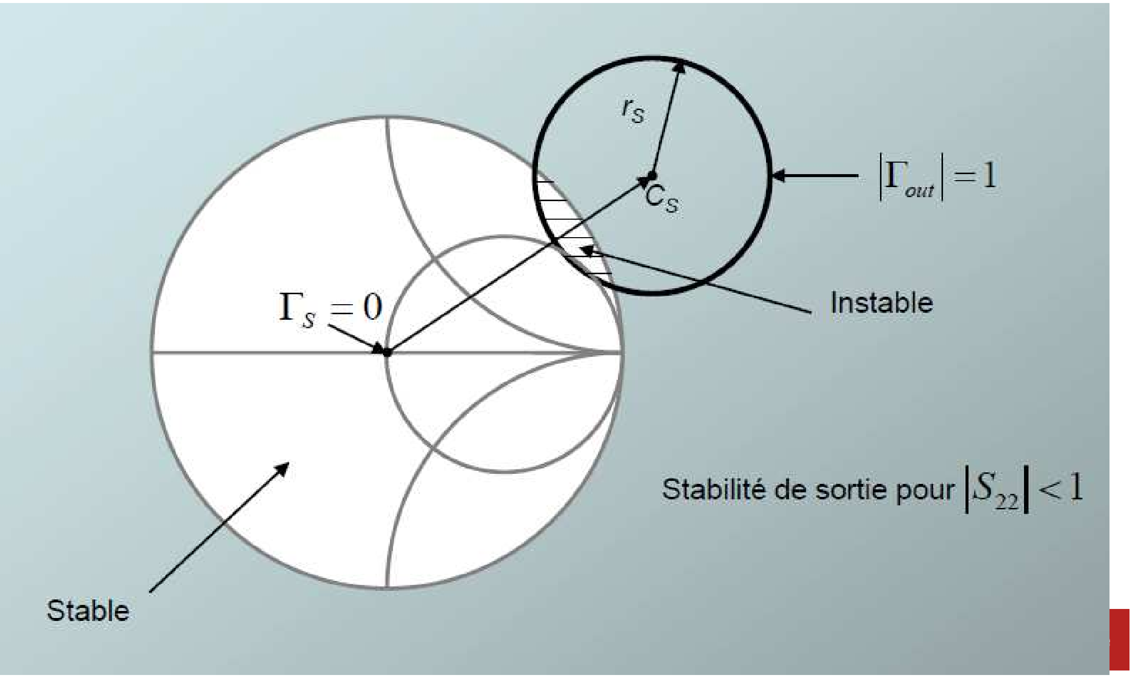
$$K = \frac{1 + |S_{11}S_{22} - S_{12}S_{21}|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12}S_{21}|}$$

- If K > 1 : the component is unconditionally stable ⇔ any loads can be placed both at input and outputs, you can find the MAG
- If K < 1 : the component is potentially unstable ⇔ you can not choose any load/source impedance you want ➔ you have to study stability in deeper details

Stability study

$$\left|\Gamma_{in}\right| = \left|S_{11}^{'}\right| = \left|S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}\right| < 1 \qquad \left|\Gamma_{out}\right| = \left|S_{22}^{'}\right| = \left|S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}\right| < 1$$

- The study of the stability is to find the places of the (im)possible loads in the Smith chart



Input and output stability circles

$$\left|\Gamma_{in}\right| = \left|S'_{11}\right| = \left|S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}\right| < 1$$

$$\left|\Gamma_{out}\right| = \left|S'_{22}\right| = \left|S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}\right| < 1$$

$$r_L = \left|\frac{S_{12}S_{21}}{\left|S_{22}\right|^2 - \left|\Delta\right|^2}\right|$$

$$c_L = \frac{\left(S_{22} - \Delta S_{11}^*\right)^*}{\left|S_{22}\right|^2 - \left|\Delta\right|^2}$$

$$r_S = \left|\frac{S_{12}S_{21}}{\left|S_{11}\right|^2 - \left|\Delta\right|^2}\right|$$

$$c_S = \frac{\left(S_{11} - \Delta S_{22}^*\right)^*}{\left|S_{11}\right|^2 - \left|\Delta\right|^2}$$

$$\Delta = S_{11}S_{22} - S_{21}S_{12}$$

Example

- Let's consider a GaAs MESFET transistor whose S-parameters @ 1GHz are the following :
- S_{11} = 0.94 exp –j 45° S_{12} = 0.04 exp –j 64°
- S_{21} = 4.61 exp j 142° S_{22} = 0.52 exp –j 20°

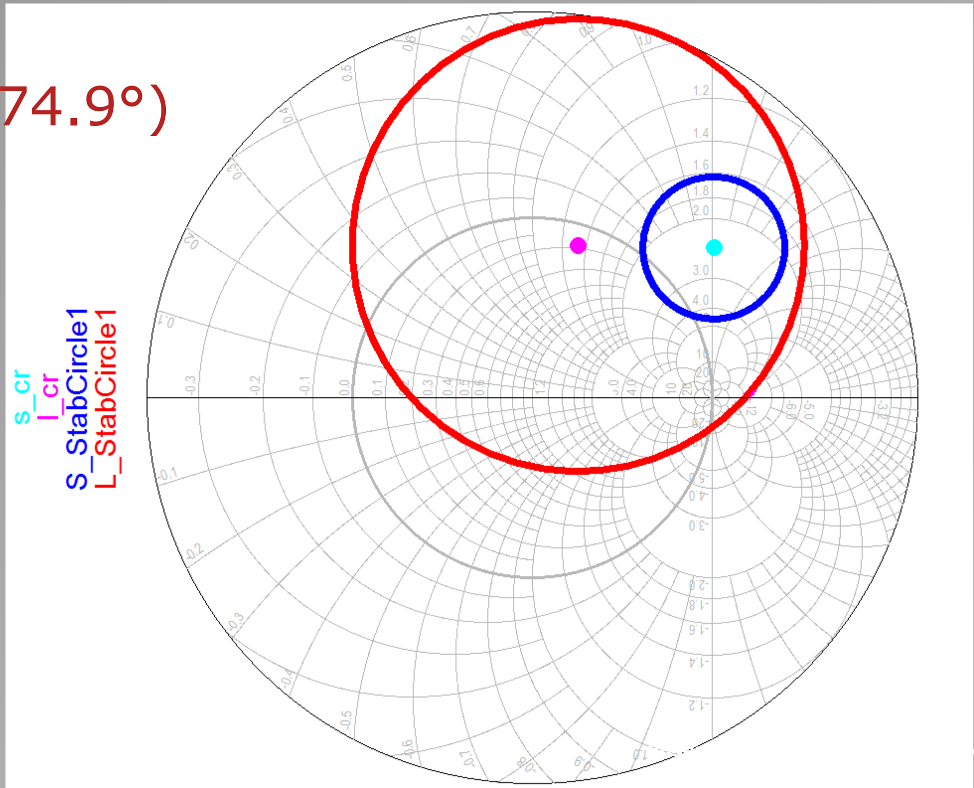
- Is this transistor stable @ 1GHz?
- If unstable, draw the input and output stability circles

Answers

- $K = 0.2629/0.3688 = 0.7129$
- <1 conditionally stable

- $\Delta = 0.17 - j0.62 = 0.65\exp(-j74.9°)$
- $r_L = 1.26$
- $c_L = 0.89 \exp(j73.2°)$

- $r_S = 0.39$
- $c_S = 1.31 \exp(j39.6°)$



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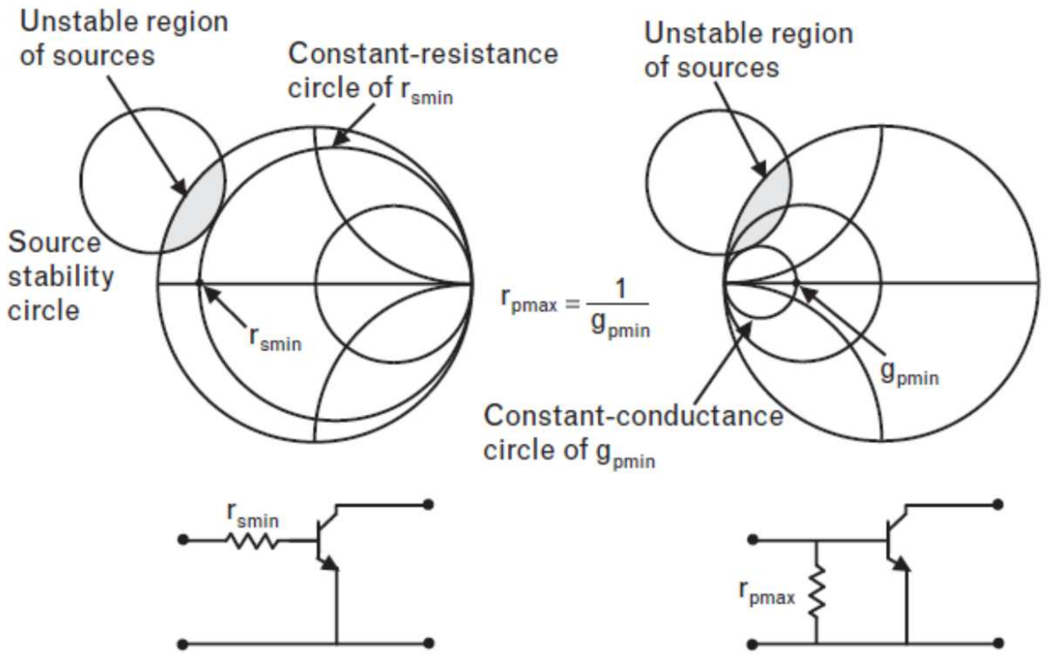
How to stabilize an unstable component

- Many ways to do it…
- The easiest (but not the most performant…) consists in introducing a lossy element (in series or in parallel) at the input and/or the output of the component
- This element will dissipate energy and will stabilize the component
  - Less gain
  - More noise
- To calculate the value :



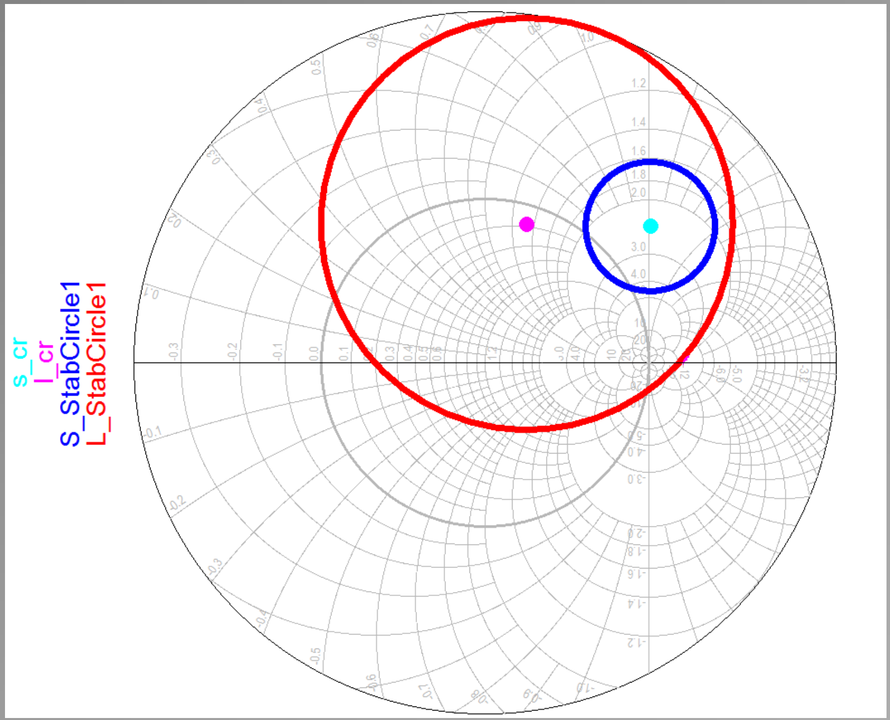
Example

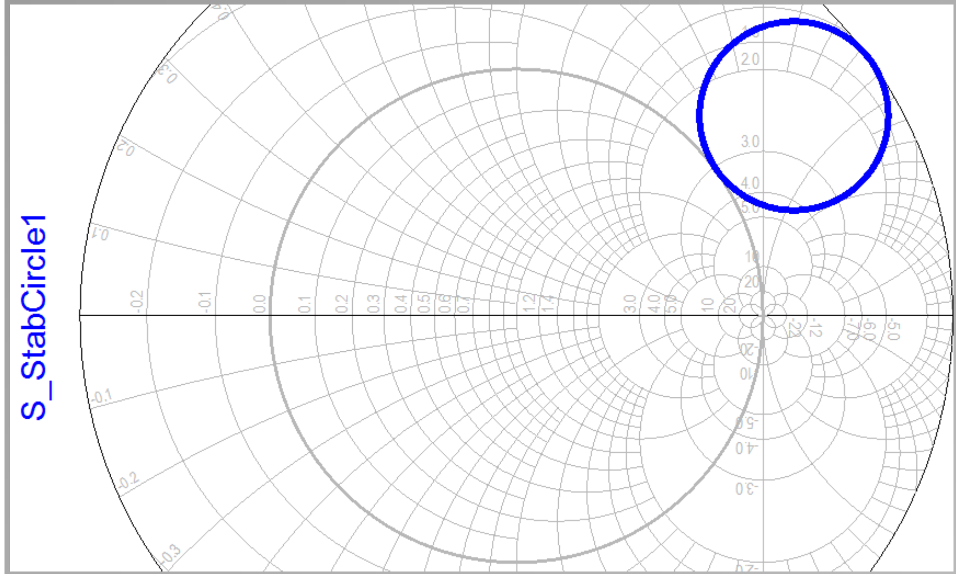
- Let's consider a GaAs MESFET transistor whose S-parameters @ 1GHz are the following :
- $S_{11} = 0.94 \exp -j\, 45°$ $S_{12} = 0.04 \exp -j\, 64°$
- $S_{21} = 4.61 \exp j\, 142°$ $S_{22} = 0.52 \exp -j\, 20°$

- Calculate the series resistor (inserted at the input) used to stabilize the component @ 1GHz

Answers

- r_{smin} = 0,4
- R_{smin} = 0,4*50 = 20 Ohms





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$$\begin{cases} V_1 = AV_2 - BI_2 \\ I_1 = CV_2 - DI_2 \end{cases}$$

BUT…

- Introducing a resistor in series or in parallel modifies the S-parameters of the overall component…
- Gain calculation should be done with this new configuration of parameters
- Unfortunately, S parameters matrices can not be multiplied…
  - Go into ABCD form for the extra resistor and the component
  - Multiply the matrices
  - Go back into S-parameters form… !

$$A = \frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{2S_{21}} \qquad S_{11} = \frac{A+B-C-D}{A+B+C+D}$$

$$B = \frac{(1+S_{11})(1+S_{22})-S_{12}S_{21}}{2S_{21}} \qquad S_{12} = \frac{2\Delta_c}{A+B+C+D}$$

$$C = \frac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{2S_{21}} \qquad S_{21} = \frac{2}{A+B+C+D}$$

$$D = \frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{2S_{21}} \qquad S_{22} = \frac{-A+B-C+D}{A+B+C+D}$$

New S-parameters matrix

- In particular, the new S-parameters matrix is (for a series resistance at the input) :
- New S =

$$\frac{1}{1+\frac{r}{2}(1-S_{11})}\begin{pmatrix} S_{11}+\frac{r}{2}(1-S_{11}) & S_{12} \\ S_{21} & S_{22}+\frac{r}{2}(S_{22}-S_{11}S_{22}+S_{12}S_{21}) \end{pmatrix}$$

EXAMPLE

- Let's consider a GaAs MESFET transistor whose S-parameters @ 1GHz are the following :
- $S_{11} = 0.94 \exp -j\ 45°$ $S_{12} = 0.04 \exp -j\ 64°$
- $S_{21} = 4.61 \exp j\ 142°$ $S_{22} = 0.52 \exp -j\ 20°$

- Find the new S-parameters including the extra resistor of 22 Ohms (+10% safety margin)
- What is the MAG?

Answers

- S_{11} = 0.94 exp –j 45° S_{12} = 0.04 exp –j 64°
- S_{21} = 4.61 exp j 142° S_{22} = 0.52 exp –j 20°

- S_{11} = 0.84 exp –j 43° S_{12} = 0.037 exp –j 71°
- S_{21} = 4.28 exp j 135° S_{22} = 0.52 exp –j 16°

- K = 1,015
- MAG = 4,28/0,04 * (1,015-sqrt(1,015²-1)) = 107 * 0,841 = 90 = 19,6dB

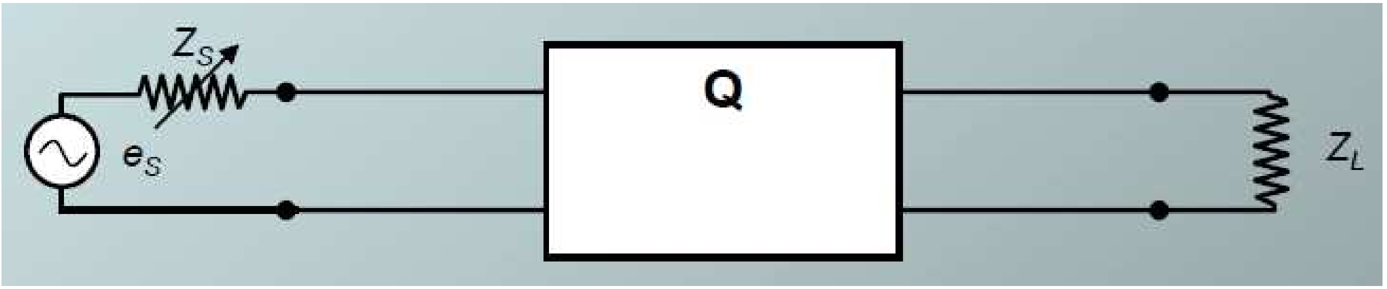
Noise

- Noise Factor : N or F
- Shows the degradation of the SNR between the input and the output of the component

$$F = \frac{(Signal / noise)_{input}}{(Signal / noise)_{output}}$$

- To fully characterize a noisy component : 3 parameters :
  - F_{min} : Minimum Noise factor of the quadripole
  - Γ_{opt} (or Z_{opt}) : the reflection coefficient leading to Fmin
  - R_n : Equivalent noise resistance (shows how fast noise factor increases as we move away from Γ_{opt})

Noise



$$F = F_{\min} + 4R_n \frac{\left|\Gamma_S - \Gamma_{opt}\right|^2}{\left(1-\left|\Gamma_S\right|^2\right)\left|1+\Gamma_{opt}\right|^2}$$

F only depends on Z_S !

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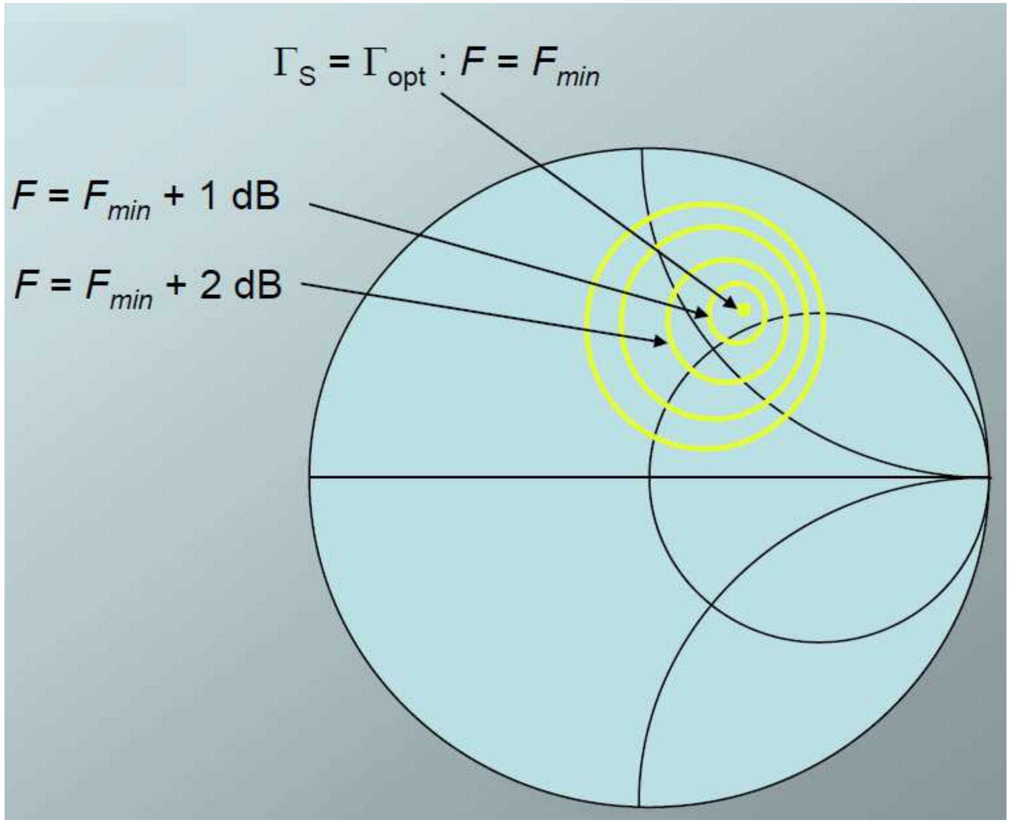
Noise

- F = cst : give circles in Smith chart

$$N_i = \frac{F_i - F_{MIN}}{4r_n}\left|1+\Gamma_{opt}\right|^2 = \frac{\left|\Gamma_S - \Gamma_{opt}\right|^2}{1-\left|\Gamma_S\right|^2}$$

$$r_{Fi} = \frac{1}{1+N_i}\sqrt{N_i^2 + N_i\left(1-\left|\Gamma_{opt}\right|^2\right)}$$

$$c_{Fi} = \frac{\Gamma_{opt}}{1+N_i}$$



Example

- $F_{min} = 0.66$ dB
- $\Gamma_{opt} = 0.47 \exp(j30°)$
- $R_n = 12\Omega$

- Draw the 1dB noise circle

$$N_i = \frac{F_i - F_{MIN}}{4r_n}\left|1+\Gamma_{opt}\right|^2 = \frac{\left|\Gamma_S - \Gamma_{opt}\right|^2}{1-\left|\Gamma_S\right|^2}$$

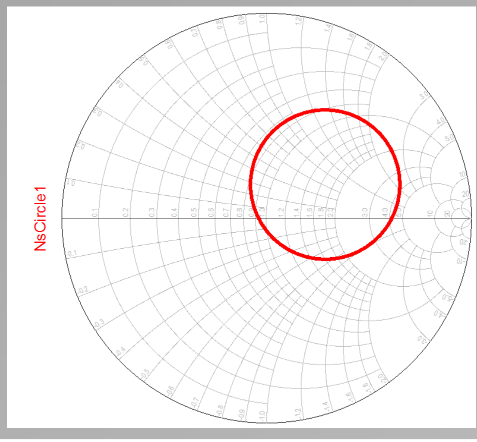
$$r_{Fi} = \frac{1}{1+N_i}\sqrt{N_i^2 + N_i\left(1-\left|\Gamma_{opt}\right|^2\right)}$$

$$c_{Fi} = \frac{\Gamma_{opt}}{1+N_i}$$

Answer

- F_{min} = 0.66dB = 1.164
- F=1dB =1.26
- r_N =R_N/50 =0.24

- N_{1dB} = (1.26-1.164)/(4*0.24) x abs (1+0.407+j0.235)²
  = 0.1 x 2.035 = 0.203
- r_{F1dB} = (1/1.203) x sqrt(0.203²+0.203(1-0.47²))= 0.371
- c_{F1dB} = 0.39 exp(j30°)



Methodology of design

- Stability factor calculation
  - OK → all the impedances with a positive real part are possible
  - Not OK : draw the circles and check the possible source and load impedances
  - Stabilize the component with dissipative components (Resistors)
  - Calculate the new S-parameters
- Unilateral factor calculation
  - OK : unilateral design
  - Not OK : bilateral design
- Choose a desirable NF (if required) and draw the circle

Noise-oriented design :

- Once Γ_S is fixed, calculate $\Gamma_{out} = S_{22} + S_{21}S_{12}\Gamma_S / (1-S_{11}\Gamma_S)$
- Choose $\Gamma_L = \Gamma_{out}$ * for maximum power transfer

- Γ_S and Γ_L are now fixed, let's see how we build them !
- Next lesson : impedance adaptation networks