

# Lecture 8

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## ANNOUNCEMENTS

- A summary of frequently misunderstood/missed concepts is now posted on the class website, and will be updated regularly.
- Graded HW assignments can be picked up in lab (353 Cory).  
→ **Please indicate your lab section on your HW assignments!**

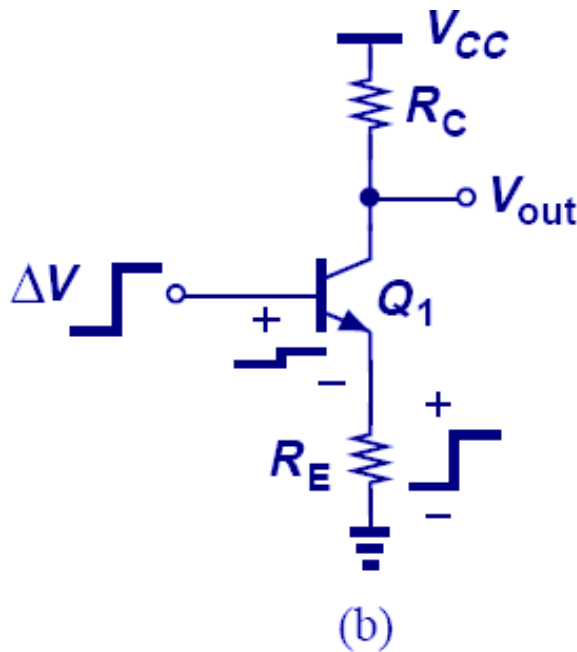
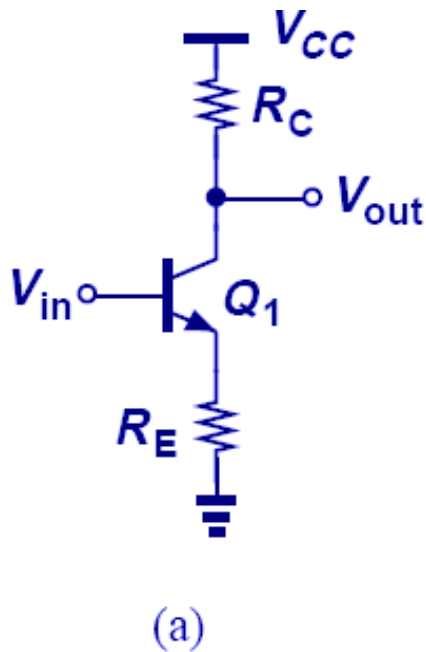
## OUTLINE

- BJT Amplifiers (cont'd)
  - Common-emitter topology
    - CE stage with emitter degeneration
    - Impact of Early effect ( $r_o$ )

Reading: Finish Chapter 5.3.1

# Emitter Degeneration

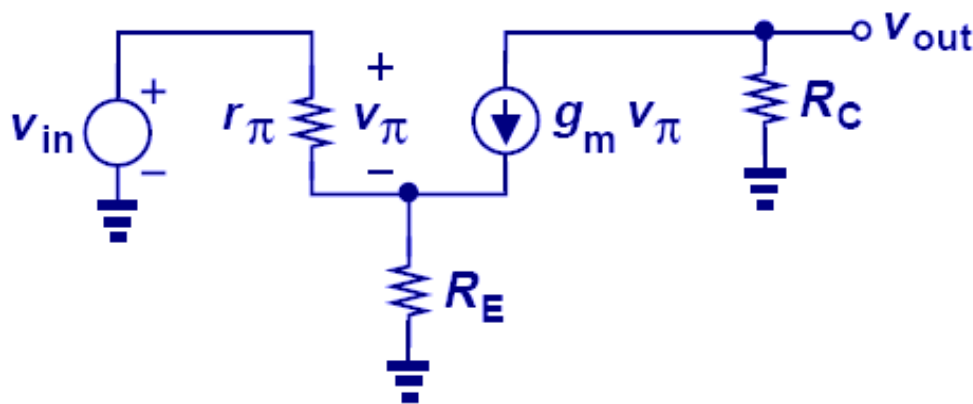
- By inserting a resistor in series with the emitter, we “degenerate” the CE stage.
- This topology will decrease the gain of the amplifier but improve other aspects, such as linearity, and input impedance.



# Small-Signal Analysis

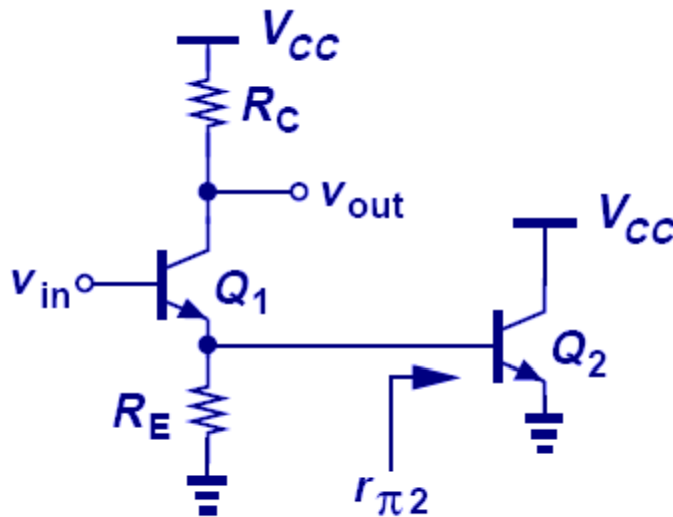
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- The gain of a degenerated CE stage = the total load resistance seen at the collector divided by  $1/g_m$  plus the total resistance placed in series with the emitter.

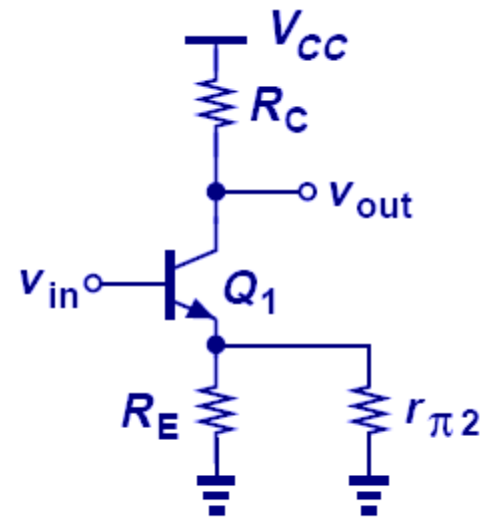


$$A_v = \frac{-g_m R_C}{1 + g_m R_E} = \frac{-R_C}{\frac{1}{g_m} + R_E}$$

# Emitter Degeneration Example 1



(a)

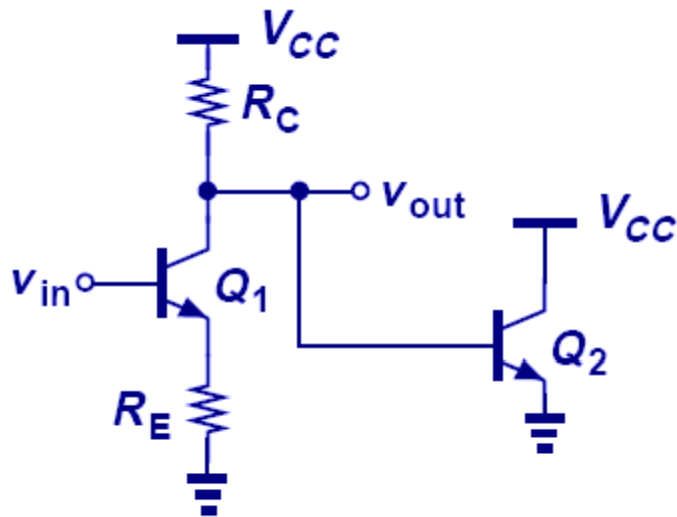


(b)

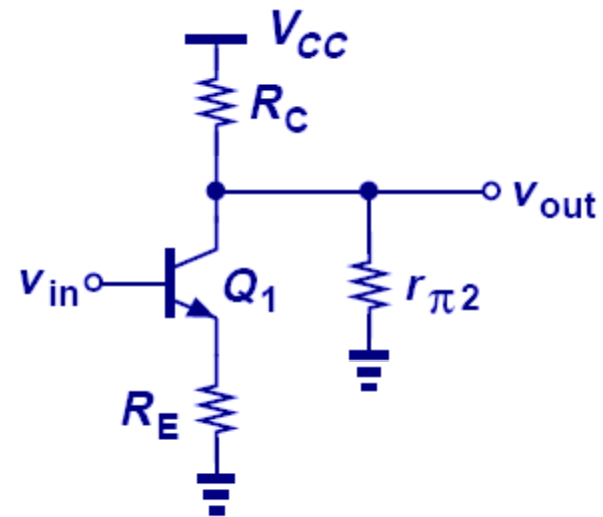
Note that the input impedance of  $Q_2$  is in parallel with  $R_E$ .

$$A_v = -\frac{R_C}{\frac{1}{g_{m1}} + R_E \parallel r_{\pi 2}}$$

# Emitter Degeneration Example 2



(a)



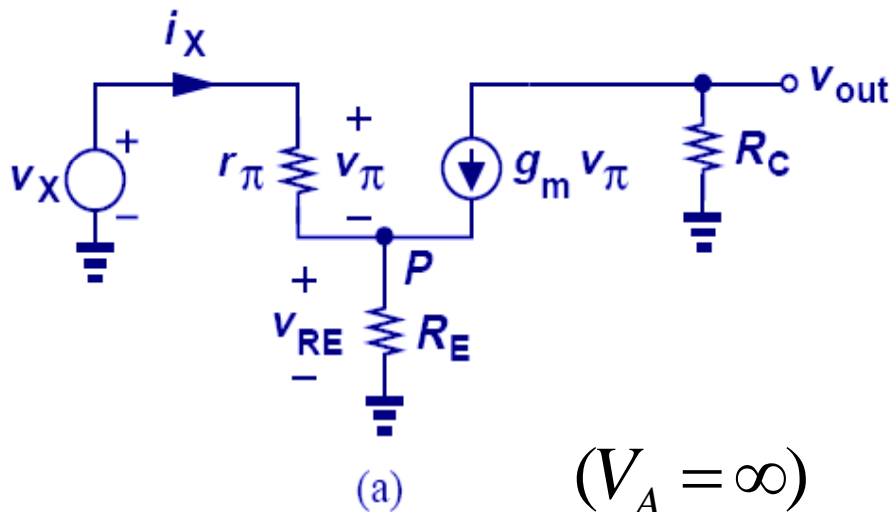
(b)

Note that the input impedance of  $Q_2$  is in parallel with  $R_C$ .

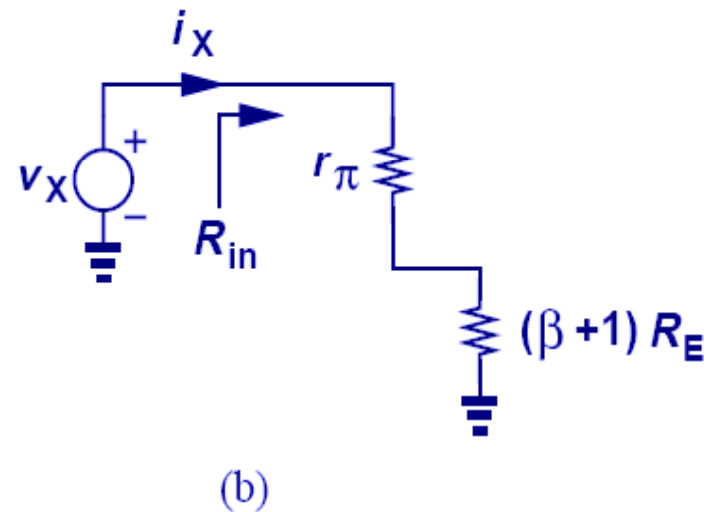
$$A_v = - \frac{R_C \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + R_E}$$

# Input Impedance of Degenerated CE Stage

- With emitter degeneration, the input impedance is increased from  $r_\pi$  to  $r_\pi + (\beta+1)R_E$  — a desirable effect.



$$(V_A = \infty)$$

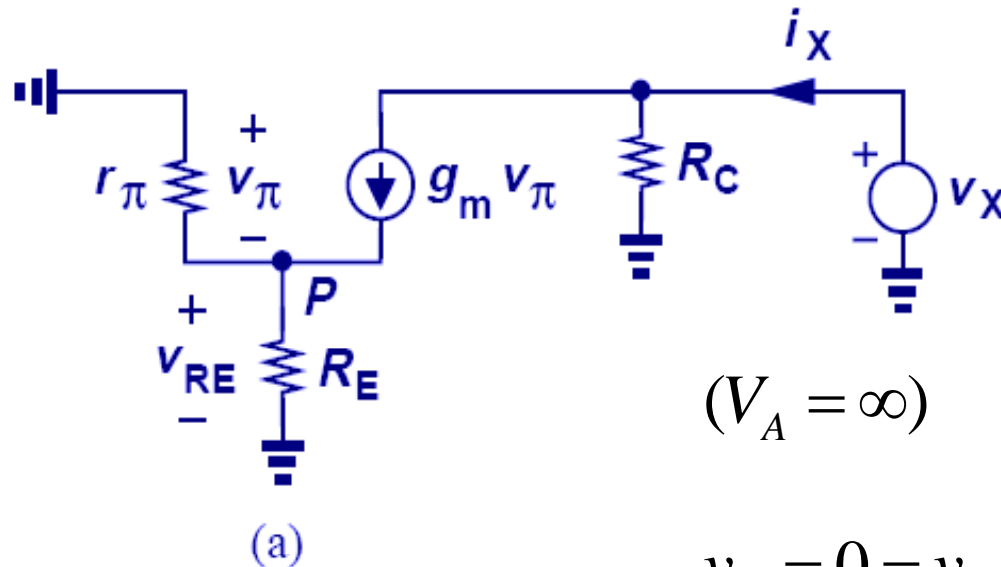


$$v_x = r_\pi i_x + R_E (1 + \beta) i_x$$

$$R_{in} \equiv \frac{v_x}{i_x} = r_\pi + (\beta + 1) R_E$$

# Output Impedance of Degenerated CE Stage

- Emitter degeneration does not alter the output impedance, if the Early effect is negligible.

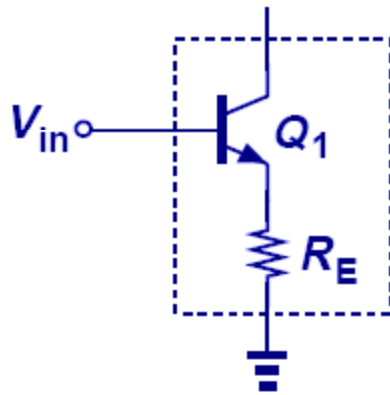


$$(V_A = \infty)$$

$$v_{in} = 0 = v_\pi + \left( \frac{v_\pi}{r_\pi} + g_m v_\pi \right) R_E \Rightarrow v_\pi = 0$$

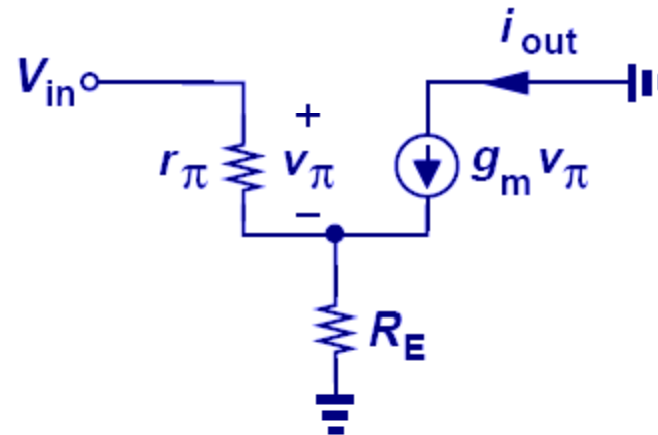
$$R_{out} \equiv \frac{v_x}{i_x} = R_C$$

# Degenerated CE Stage as a “Black Box”



$$(V_A = \infty)$$

(a)



(b)

$$i_{out} = g_m \frac{v_{in}}{1 + (r_\pi^{-1} + g_m)R_E}$$

$$G_m \equiv \frac{i_{out}}{v_{in}} \approx \frac{g_m}{1 + g_m R_E}$$

- If  $g_m R_E \gg 1$ ,  $G_m$  is more linear.



# Degenerated CE Stage with Base Resistance

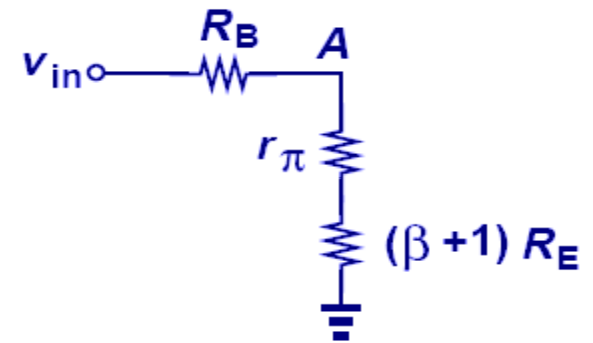
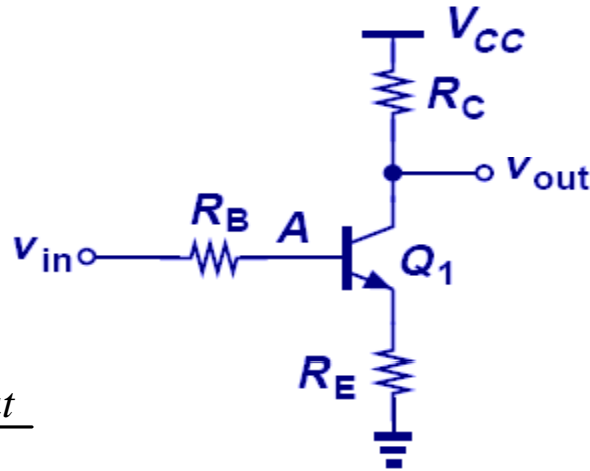
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$$(V_A = \infty)$$

$$\frac{v_{out}}{v_{in}} = \frac{v_A}{v_{in}} \cdot \frac{v_{out}}{v_A}$$

$$\frac{v_{out}}{v_{in}} = \frac{-\beta R_C}{r_\pi + (\beta + 1)R_E + R_B}$$

$$A_v \approx \frac{-R_C}{\frac{1}{g_m} + R_E + \frac{R_B}{\beta + 1}}$$

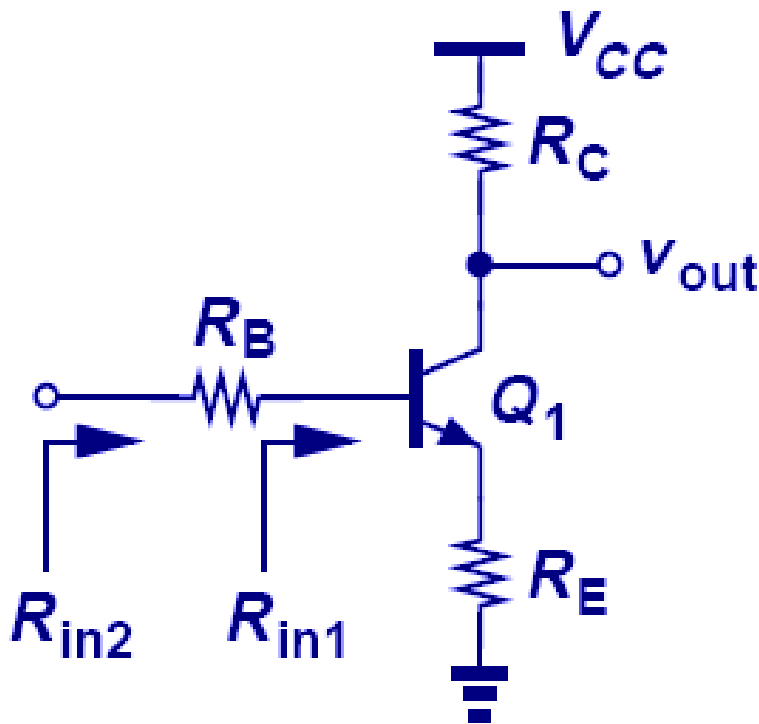


# Degenerated CE Stage:

## Input/Output Impedances

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- $R_{in1}$  is more important in practice, because  $R_B$  is often the output impedance of the previous stage.



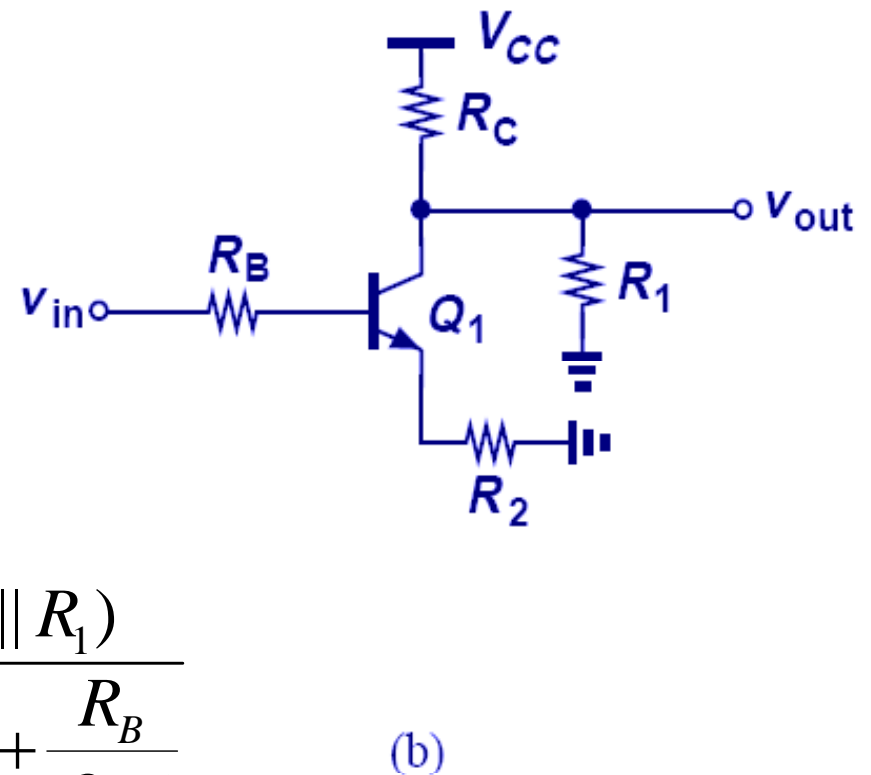
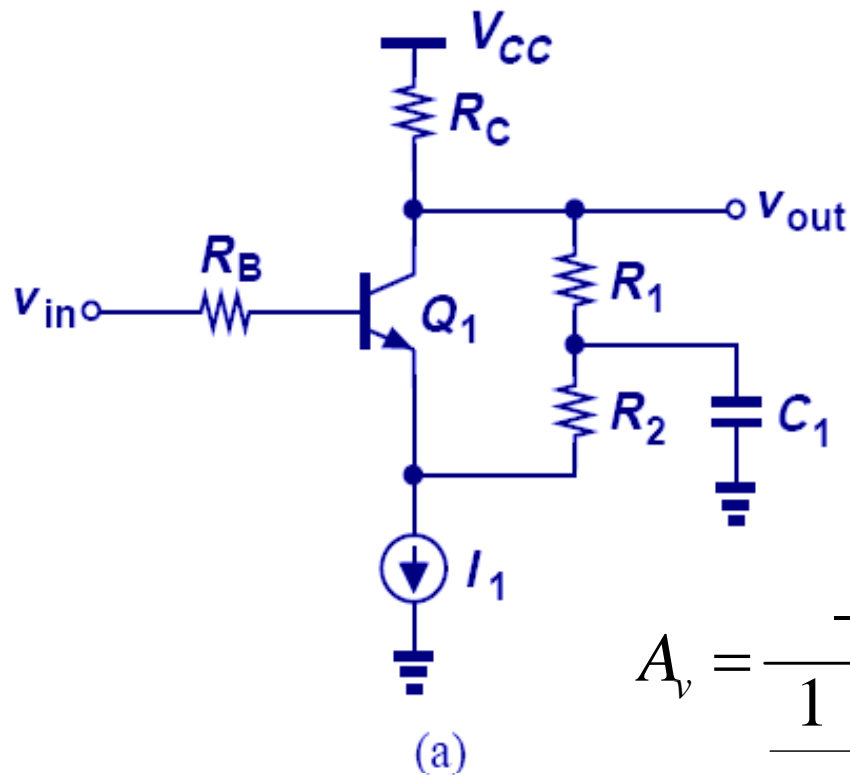
$$(V_A = \infty)$$

$$R_{in1} = r_{\pi} + (\beta + 1)R_E$$

$$R_{in2} = R_B + r_{\pi} + (\beta + 1)R_E$$

$$R_{out} = R_C$$

# Emitter Degeneration Example 3



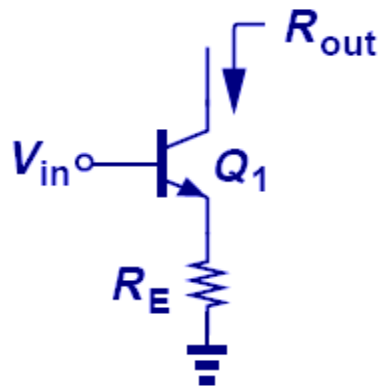
$$A_v = \frac{-(R_C \parallel R_1)}{\frac{1}{g_m} + R_2 + \frac{R_B}{\beta + 1}}$$

$$R_{in} = r_{\pi} + (\beta + 1)R_2 + R_B$$

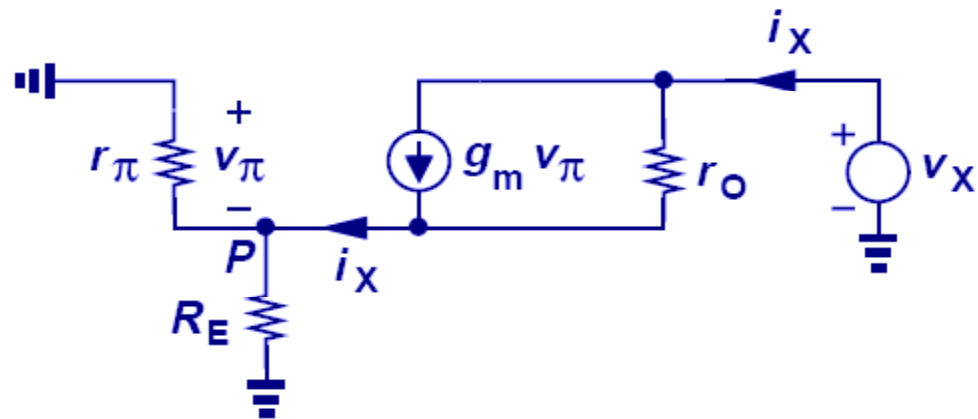
$$R_{out} = R_C \parallel R_1$$

# Output Impedance of Degenerated CE Stage with $V_A < \infty$

- Emitter degeneration boosts the output impedance.
  - This improves the gain of the amplifier and makes the circuit a better current source.



(a)



(b)

$$R_{out} = [1 + g_m (R_E \parallel r_\pi)] r_O + R_E \parallel r_\pi$$

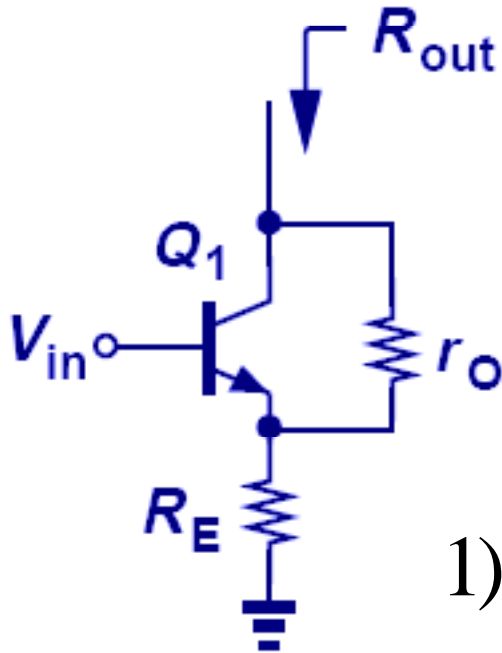
$$R_{out} = r_O + (g_m r_O + 1)(R_E \parallel r_\pi)$$

$$R_{out} \approx r_O [1 + g_m (R_E \parallel r_\pi)]$$

# Two Special Cases

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Stage with explicit depiction of  $r_o$ :

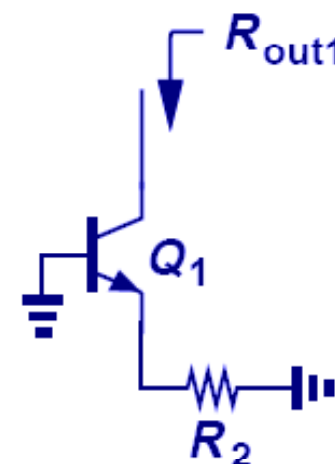
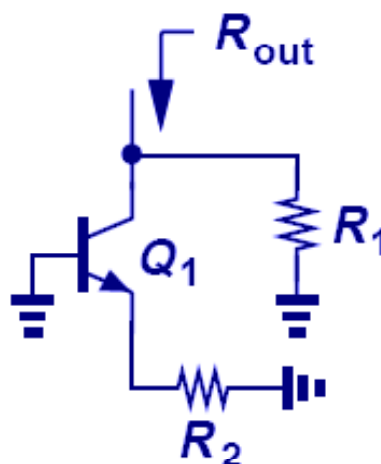
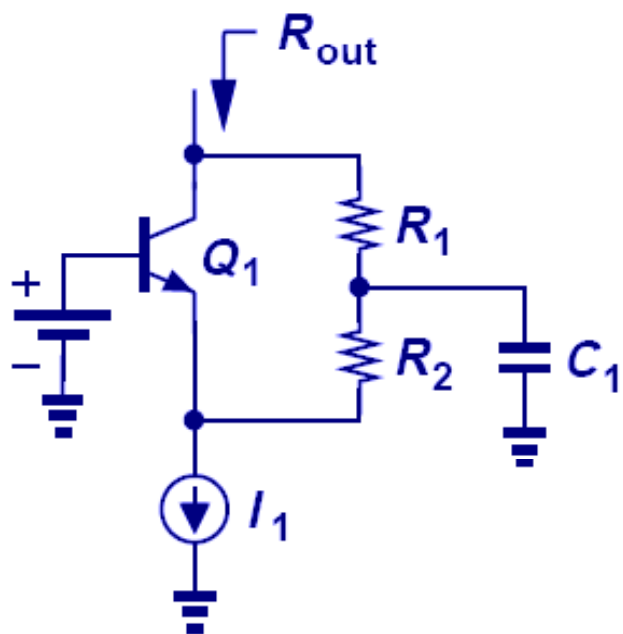


$$1) R_E \gg r_\pi : R_{out} \approx r_o (1 + g_m r_\pi) \approx \beta r_o$$

$$2) R_E \ll r_\pi : R_{out} \approx (1 + g_m R_E) r_o$$

# Analysis by Inspection

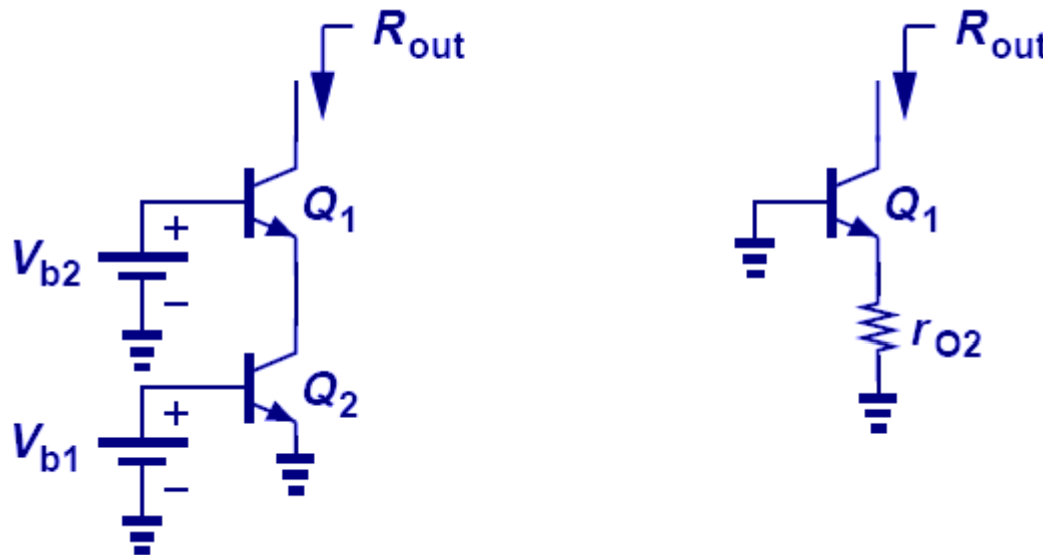
- This seemingly complicated circuit can be greatly simplified by first recognizing that the capacitor creates an AC short to ground, and gradually transforming the circuit to a known topology.



$$R_{out} = R_1 \parallel R_{out1} \Rightarrow R_{out1} = [1 + g_m (R_2 \parallel r_\pi)] r_o \Rightarrow R_{out} = [1 + g_m (R_2 \parallel r_\pi)] r_o \parallel R_1$$

# Example: Degeneration by Another BJT

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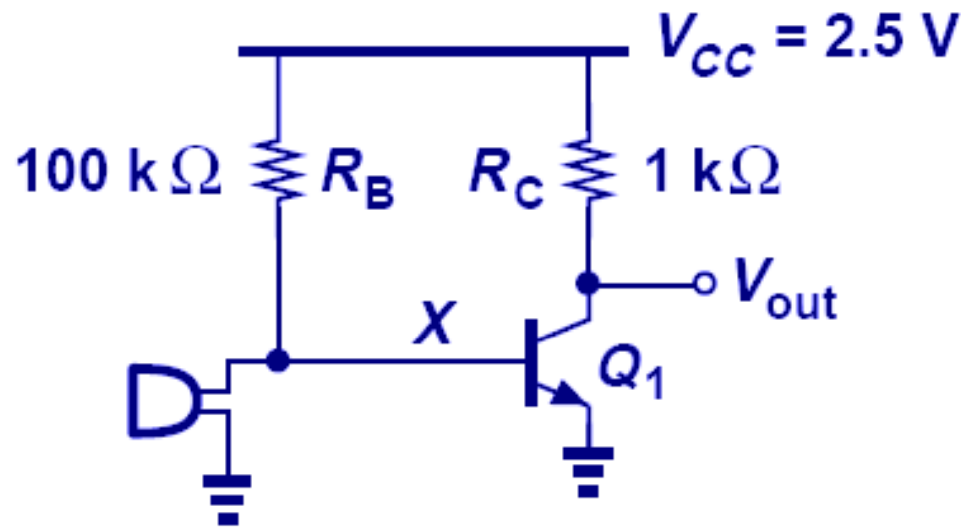


$$R_{out} = [1 + g_{m1}(r_{O2} \parallel r_{\pi1})]r_{O1}$$

- Called a “cascode”, this circuit offers many advantages that we will study later...

# Bad Input Connection

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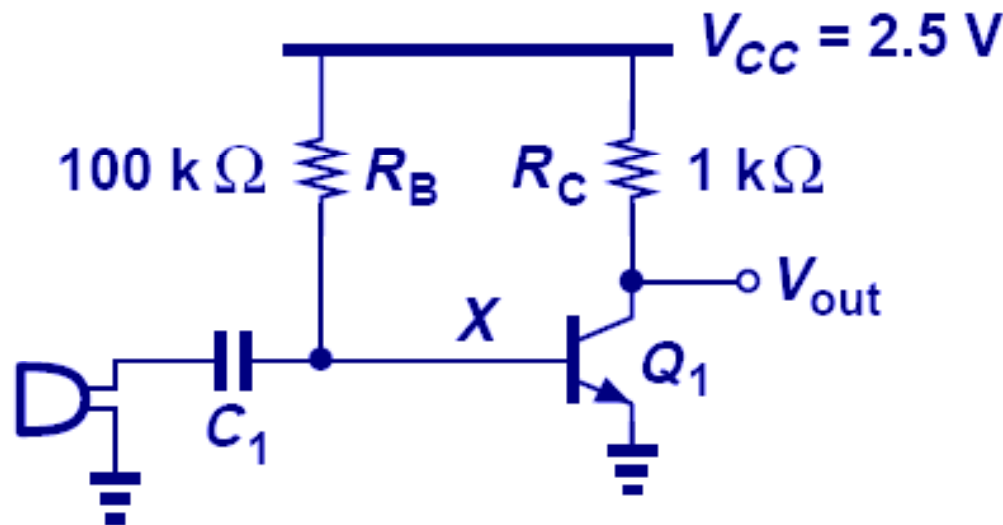
- Since the microphone has a very low resistance (connecting the base of  $Q_1$  to ground), it attenuates the base voltage and renders  $Q_1$  with a very small bias current.



# Use of Coupling Capacitor

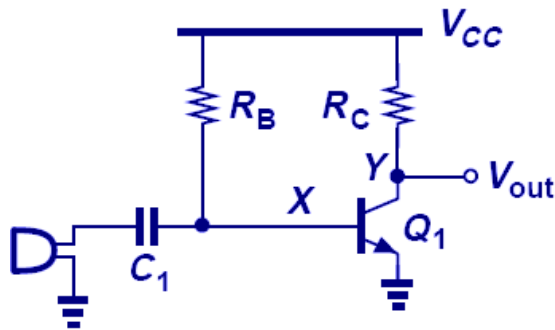
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- A capacitor is used to isolate the DC bias network from the microphone, and to short (or “couple”) the microphone to the amplifier at higher frequencies.

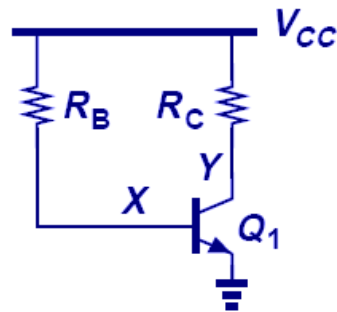


# DC and AC Analysis

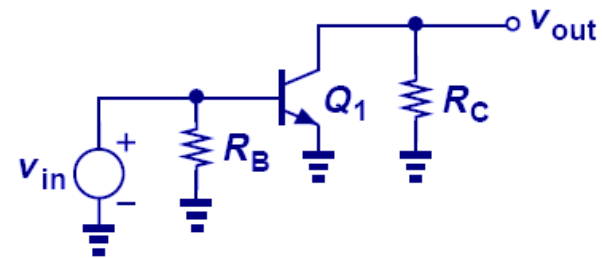
- The coupling capacitor is replaced with an open circuit for DC analysis, and then replaced with a short circuit for AC analysis.



(a)

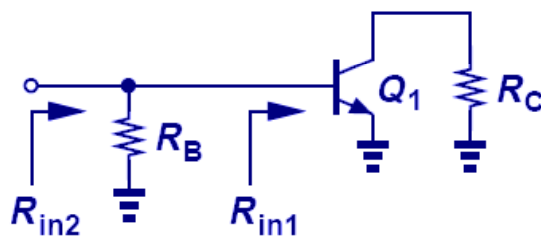


(b)

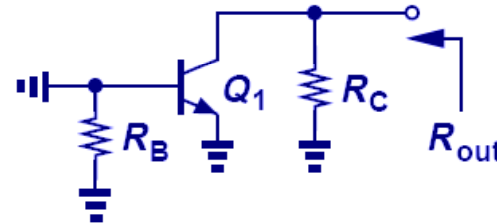


(c)

$$A_v = -g_m (R_C \parallel r_o)$$



(d)



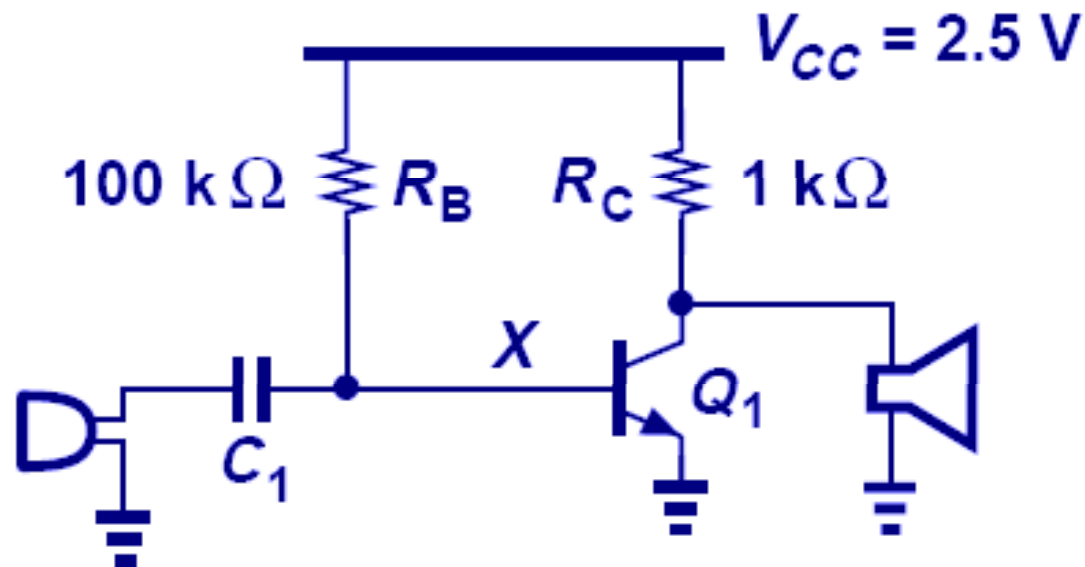
(e)

$$R_{in} = r_\pi \parallel R_B$$

$$R_{out} = R_C \parallel r_o$$

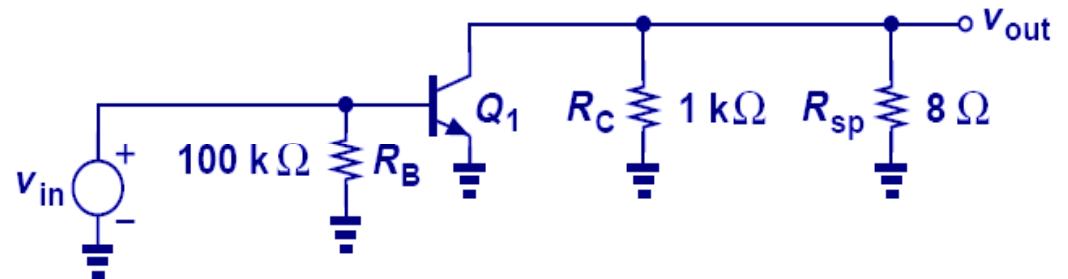
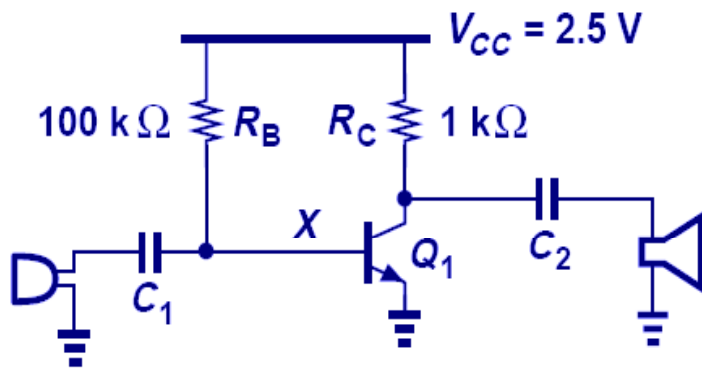
# Bad Output Connection

- Since the speaker has an inductor with very low DC resistance, connecting it directly to the amplifier would ~short the collector to ground, causing the BJT to go into deep saturation mode.



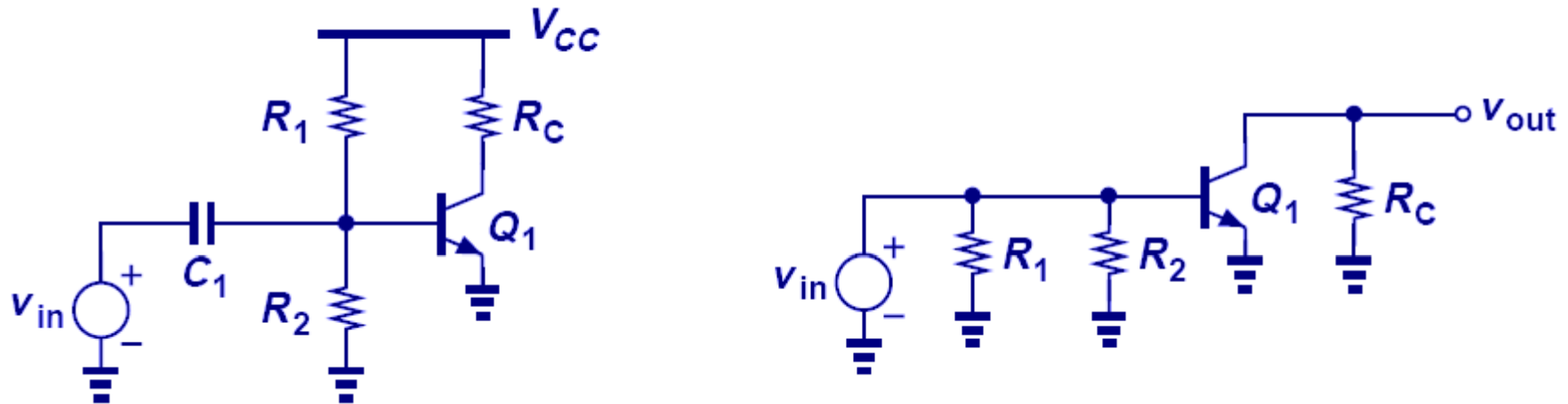
# Use of Coupling Capacitor at Output

- The AC coupling indeed allows for correct biasing. However, due to the speaker's small input impedance, the overall gain drops considerably.



# CE Stage with Voltage-Divider Biasing

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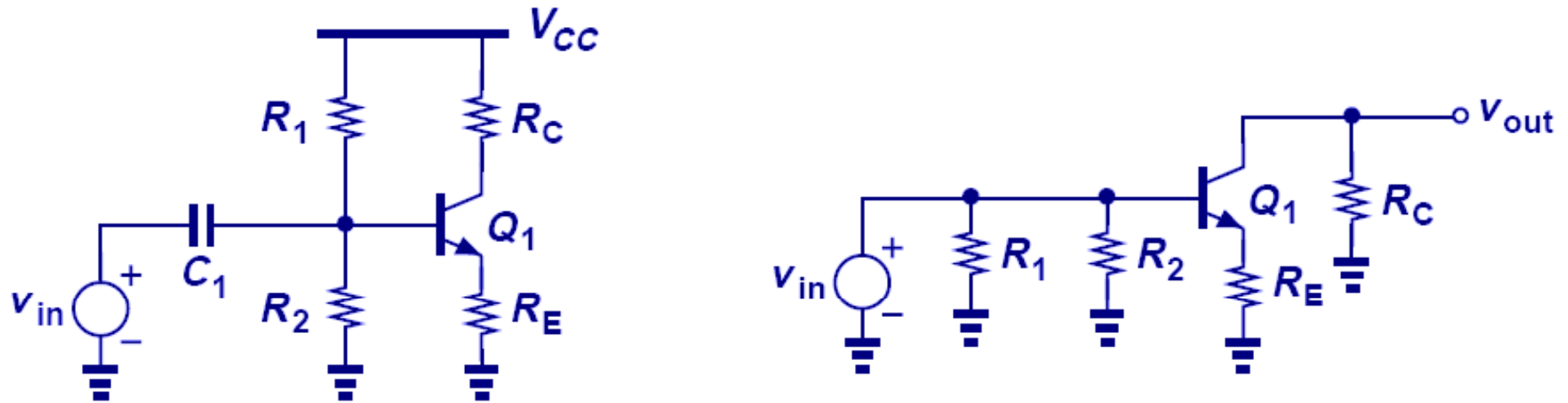


$$A_v = -g_m(R_C \parallel r_o)$$

$$R_{in} = r_\pi \parallel R_1 \parallel R_2$$

$$R_{out} = R_C \parallel r_o$$

# CE Stage with Robust Biasing



$$(V_A = \infty)$$

$$A_v = \frac{-R_C}{\frac{1}{g_m} + R_E}$$

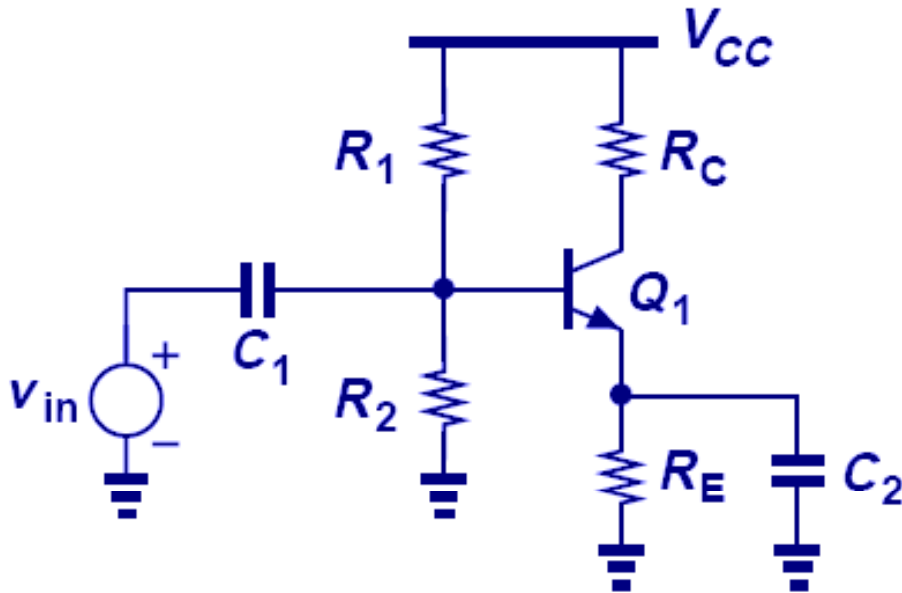
$$R_{in} = [r_\pi + (\beta + 1)R_E] \parallel R_1 \parallel R_2$$

$$R_{out} = R_C$$

# Elimination of Emitter Degeneration for AC Signals

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- The capacitor  $C_2$  shorts out  $R_E$  at higher frequencies to eliminate the emitter degeneration.



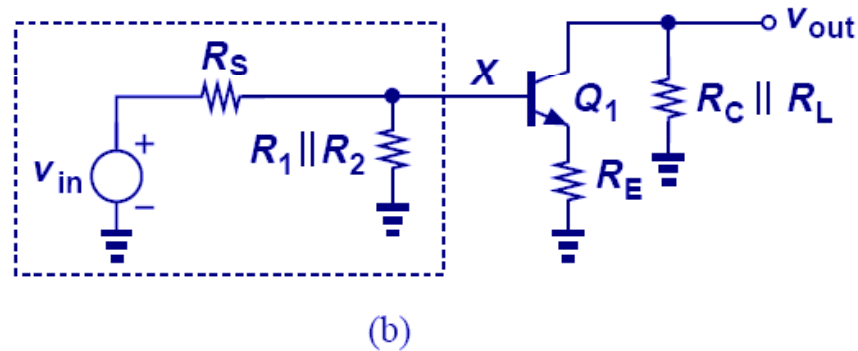
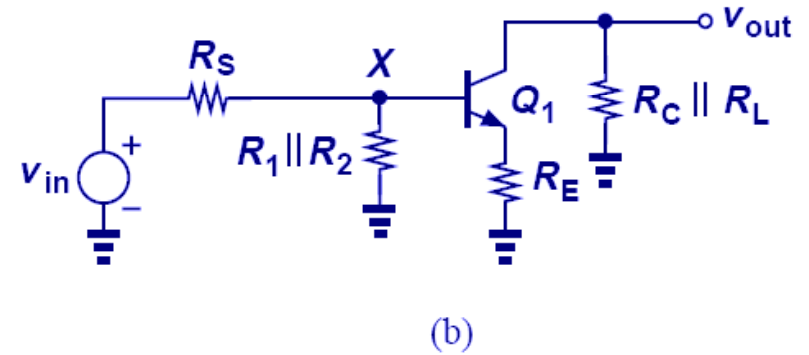
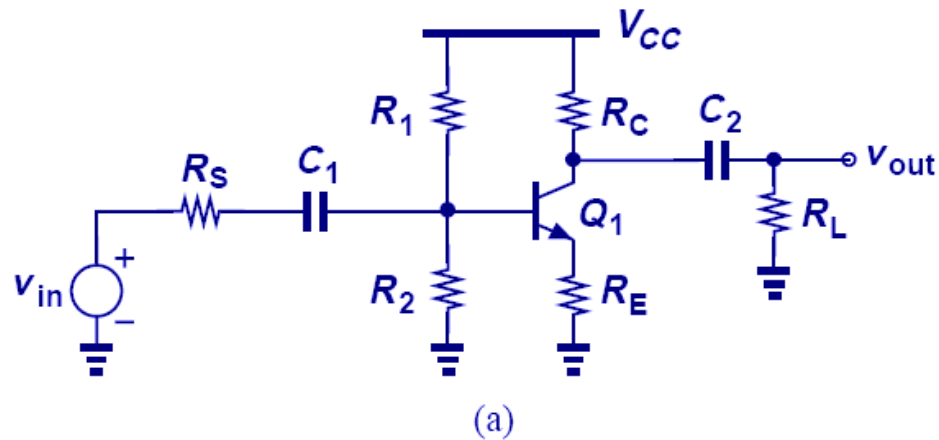
$$(V_A = \infty)$$

$$A_v = -g_m R_C$$

$$R_{in} = r_\pi \parallel R_1 \parallel R_2$$

$$R_{out} = R_C$$

# Complete CE Stage



$$A_v = \frac{-R_C \parallel R_L}{\frac{1}{g_m} + R_E + \frac{R_s \parallel R_1 \parallel R_2}{\beta + 1}} \cdot \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_s}$$



# Summary of CE Concepts

