1.4 - SMALL SIGNAL MODEL OF THE BJT

INTRODUCTION

Objective

The objective of this presentation is:

1.) Concept of the small signal model

2.) The small signal model for the BJT

Outline

- Transconductance small signal model
- Input resistance, output resistance of the common emitter model
- Extensions of the small signal BJT model
- BJT frequency response

TRANSCONDUCTANCE SMALL SIGNAL MODEL

Categorization of Electrical Models

<table>
<tr><td colspan="2" rowspan="2"></td><td colspan="2">Time Dependence</td></tr>
<tr><td>Time Independent</td><td>Time Dependent</td></tr>
<tr><td rowspan="2">Linearity</td><td>Linear</td><td>Small-signal, midband
R_{in}, A_v, R_{out}
(.TF)</td><td>Small-signal frequency response - poles and zeros
(.AC)</td></tr>
<tr><td>Nonlinear</td><td>DC operating point
$i_D = f(v_D,v_G,v_S,v_B)$
(.OP)</td><td>Large-signal transient response - Slew rate
(.TRAN)</td></tr>
</table>

Based on the simulation capabilities of SPICE.

What is a Small Signal Model?

• A small signal model is a linear model which is independent of amplitude. It may or may not have time dependence (i.e. capacitors).

• The small signal model for a nonlinear component such as a BJT is a linear model about some nominal operating point. The deviations from the operating point are small enough that it approximates the nonlinear component over a limited range of amplitudes.

Illustration of the *pn* diode:

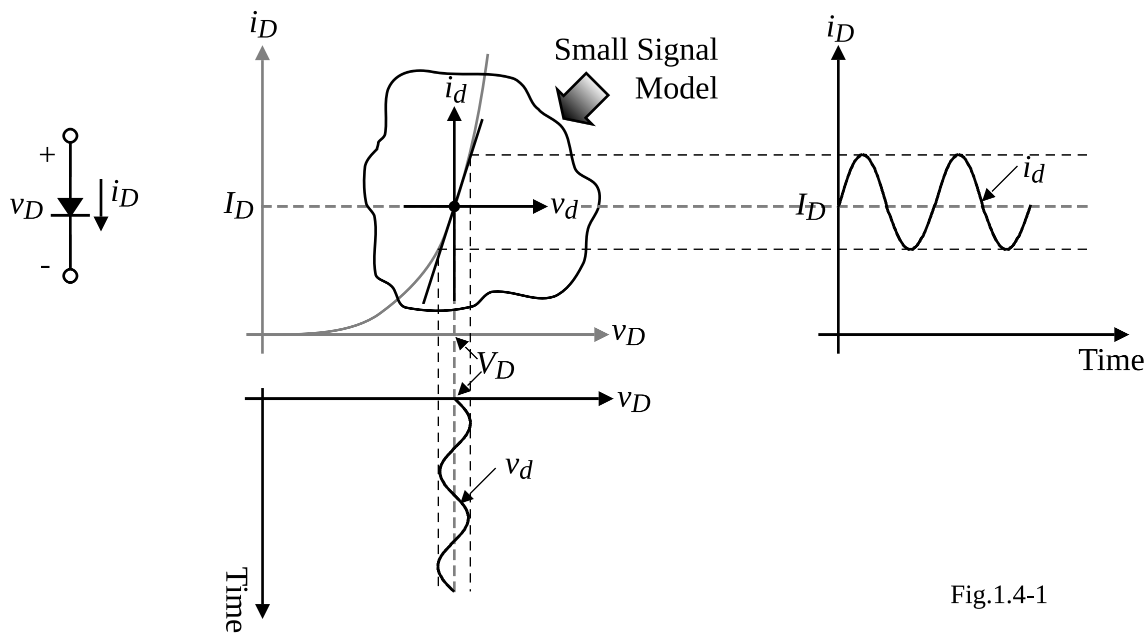
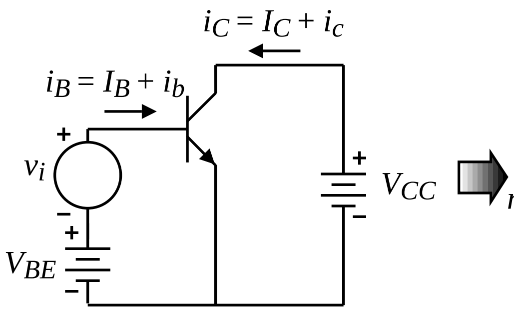


Fig.1.4-1

BJT, Common-Emitter, Forward-Active Region

Effect of a small-signal input voltage applied to a BJT.



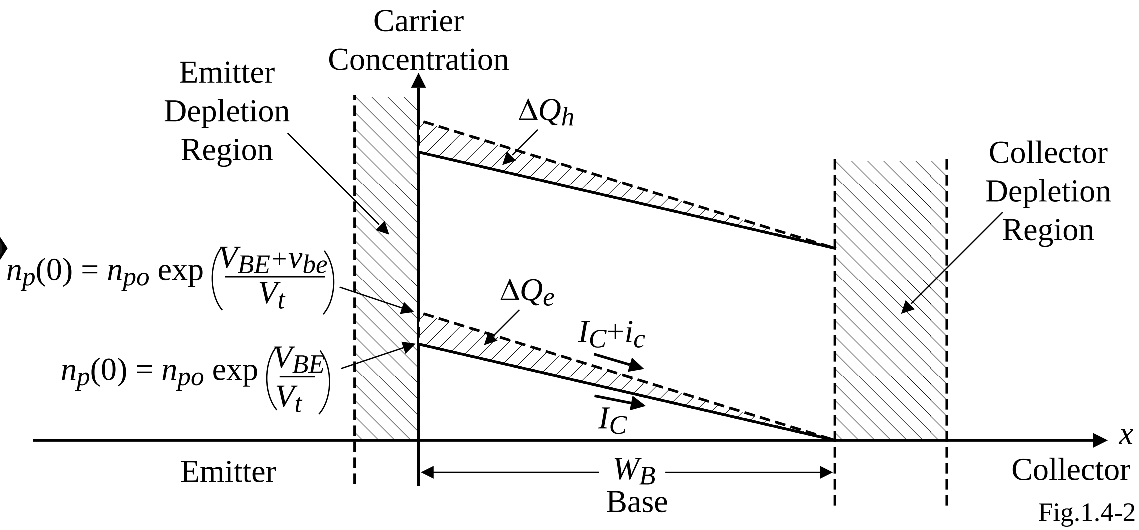


Fig.1.4-2

$v_i \Rightarrow i_b \Rightarrow i_c$

Transconductance of the Small Signal BJT Model

The small signal transconductance is defined as

$$g_m \equiv \frac{di_C}{dv_{BE}}\bigg|_Q = \frac{\Delta i_C}{\Delta v_{BE}} = \frac{i_c}{v_{be}} = \frac{i_c}{v_i} \qquad \Rightarrow \qquad i_c = g_m v_i$$

The large signal model for i_C is

$$i_C = I_S \exp\frac{v_{BE}}{V_t} \qquad \Rightarrow \qquad g_m = \left(\frac{d}{dv_{BE}} I_S \exp\frac{v_{BE}}{V_t}\right)\bigg|_Q = \frac{I_S}{V_t}\exp\frac{V_{BE}}{V_t} = \frac{I_C}{V_t}$$

$$\therefore \qquad \boxed{g_m = \frac{I_C}{V_t}}$$

Another way to develop the small signal transconductance

$$i_C = I_S \exp\left(\frac{V_{BE}+v_i}{V_t}\right) = I_S \exp\left(\frac{V_{BE}}{V_t}\right)\exp\left(\frac{v_i}{V_t}\right) = I_C \exp\left(\frac{v_i}{V_t}\right) \approx I_C\left[1 + \frac{v_i}{V_t} + \frac{1}{2}\left(\frac{v_i}{V_t}\right)^2 + \frac{1}{6}\left(\frac{v_i}{V_t}\right)^3 + \cdots\right]$$

But

$$i_C = I_C + i_c$$

$$\therefore \qquad i_c \approx I_C \frac{v_i}{V_t} + \frac{I_C}{2}\left(\frac{v_i}{V_t}\right)^2 + \frac{I_C}{6}\left(\frac{v_i}{V_t}\right)^3 + \cdots \approx \frac{I_C}{V_t} v_i = g_m v_i$$

INPUT AND OUTPUT RESISTANCE SMALL SIGNAL MODEL

Input Resistance of the Small Signal BJT Model

In the forward-active region, we can write that

$$i_B = \frac{i_C}{\beta_F}$$

Small changes in i_B and i_C can be related as

$$\Delta i_B = \frac{d}{di_C}\left(\frac{i_C}{\beta_F}\right)\Delta i_C$$

The small signal current gain, β_o, can be written as

$$\beta_o = \frac{\Delta i_C}{\Delta i_B} = \frac{1}{\frac{d}{di_C}\left(\frac{i_C}{\beta_F}\right)} = \frac{i_c}{i_b}$$

Therefore, we define the small signal input resistance as

$$r_\pi \equiv \frac{v_i}{i_b} = \frac{\beta_o v_i}{i_c} = \frac{\beta_o}{g_m}$$

$$\boxed{r_\pi = \frac{\beta_o}{g_m}}$$

Output Resistance of the Small Signal BJT Model

In the forward-active region, we can write that the small signal output conductance, g_o ($r_o = 1/g_o$) is

$$g_o \equiv \frac{di_C}{dv_{CE}}\Big|_Q = \frac{\Delta i_C}{\Delta v_{CE}} = \frac{i_c}{v_{ce}} \qquad \Rightarrow \qquad i_c = g_o v_{ce}$$

The large signal model for i_C , including the influence of v_{CE}, is

$$i_C = I_S\left(1 + \frac{v_{CE}}{V_A}\right)\exp\frac{v_{BE}}{V_t}$$

$$g_o \equiv \frac{di_C}{dv_{CE}}\Big|_Q = I_S\left(\frac{1}{V_A}\right)\exp\frac{V_{BE}}{V_t} \approx \frac{I_C}{V_A}$$

$$\therefore \qquad \boxed{r_o = \frac{V_A}{I_C}}$$

Simple Small Signal BJT Model

Implementing the above relationships, $i_c = g_m v_i$, $i_c = g_o v_{ce}$, and $v_i = r_\pi i_b$, into a schematic model gives,

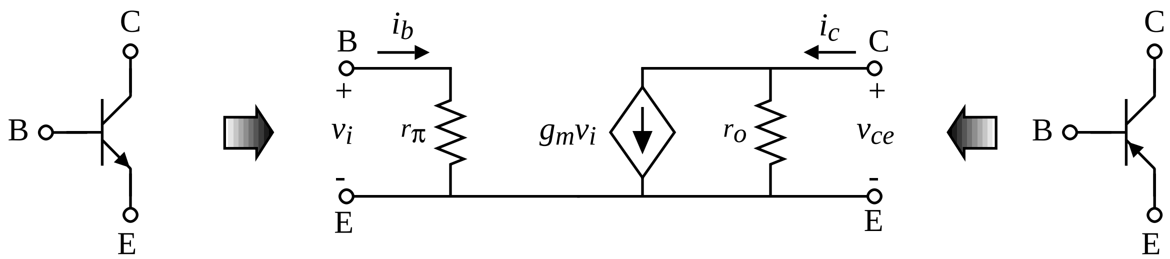


Fig. 1.4-3

Note that the small signal model is the same for either a *npn* or a *pnp* BJT.

Example:

Find the small signal input resistance, R_{in}, the output resistance, R_{out}, and the voltage gain of the common emitter BJT if the BJT is unloaded ($R_L = \infty$), v_{out}/v_{in}, the dc collector current is 1mA, the Early voltage is 100V, and β_o at room temperature.

$$g_m = \frac{I_C}{V_t} = \frac{1\text{mA}}{26\text{mV}} = \frac{1}{26} \text{ mhos or Siemans}$$

$$R_{in} = r_\pi = \frac{\beta_o}{g_m} = 100\cdot 26 = 2.6\text{k}\Omega$$

$$R_{out} = r_o = \frac{V_A}{I_C} = \frac{100\text{V}}{1\text{mA}} = 100\text{k}\Omega$$

$$\frac{v_{out}}{v_{in}} = -g_m\, r_o = -\,26\text{mS}\cdot 100\text{k}\Omega = -2600\text{V/V}$$

EXTENSIONS OF THE SMALL SIGNAL BJT MODEL

Collector-Base Resistance of the Small Signal BJT Model

Recall the influence of V on the base width:

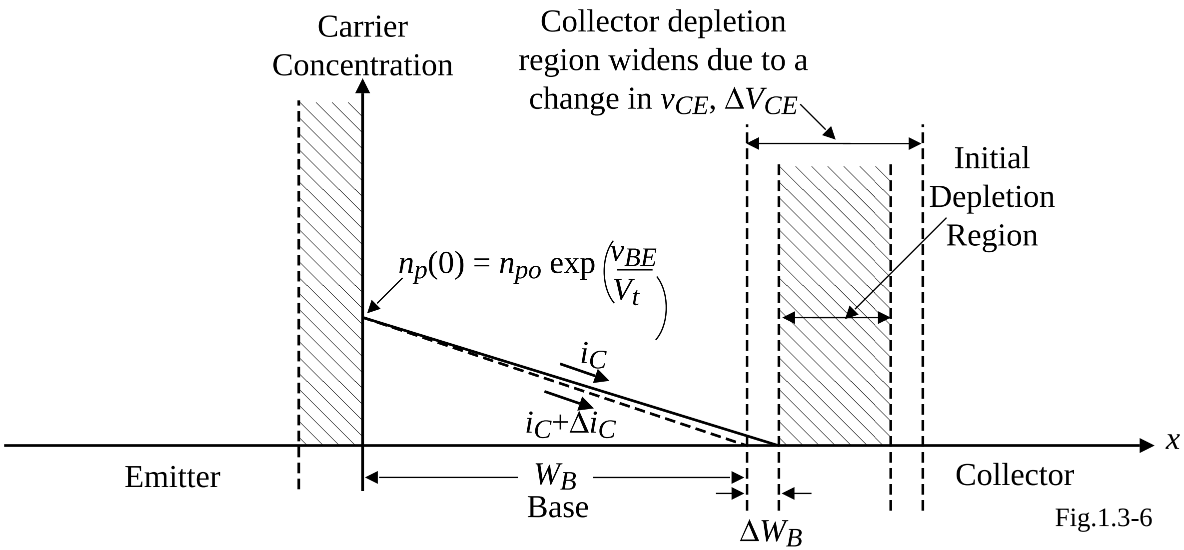


Fig.1.3-6

We noted that an increase in v_{CE} causes and increase in the depletion width and a decrease in the total minority-carrier charge stored in the base and therefore a decrease in the base recombination current, i_{B1}.

This influence is modeled by a collector-base resistor, r_μ, defined as

$$r_\mu = \frac{\Delta v_{CE}}{\Delta i_{B1}} = \frac{\Delta v_{CE}}{\Delta i_C}\frac{\Delta i_C}{\Delta i_{B1}} = r_o \frac{\Delta i_C}{\Delta i_{B1}} \approx \beta_o r_o \quad \text{(lower limit if base current is all recombination current)}$$

In general, $r_\mu \geq 10\ \beta_o r_o$ for the *npn* BJT and about 2-5 $\beta_o r_o$ for the lateral *pnp* BJT.

Base-Charging Capacitance of the Small Signal BJT Model

Consider changes in base-carrier concentrations once again.

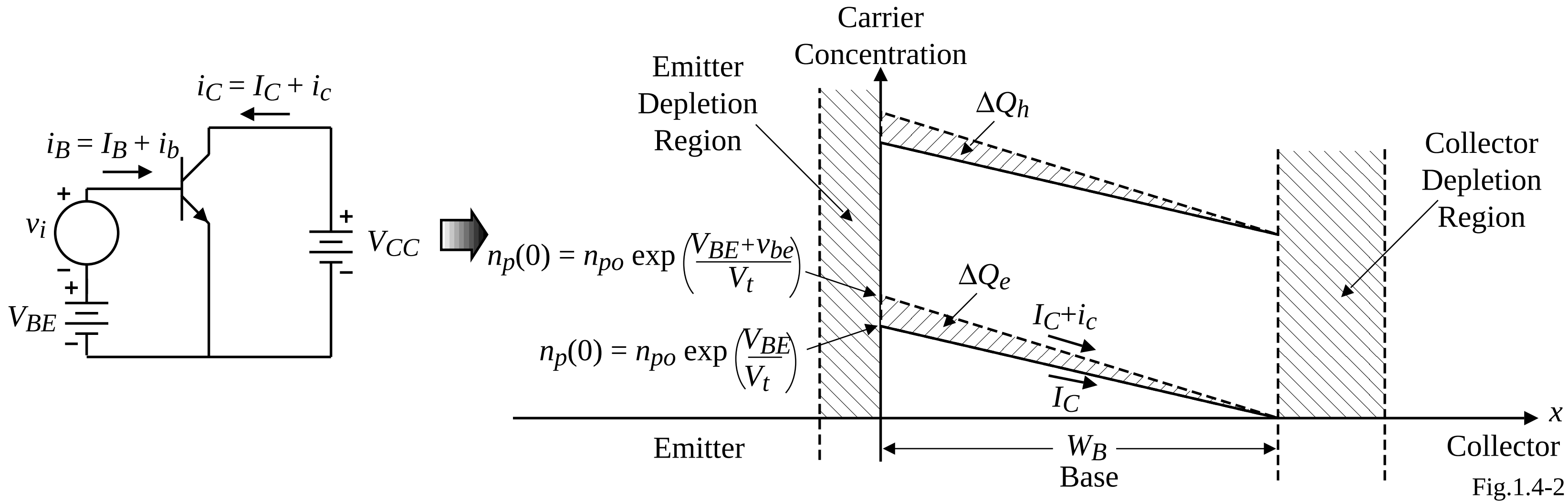


Fig.1.4-2

The Δv_{BE} change causes a change in the minority carriers, $\Delta Q_e = q_e$, which must be equal to the change in majority carriers, $\Delta Q_h = q_h$. This charge can be related to the voltage across the base, v_i, as

$$q_h = C_b v_i$$

where C_b is the base-charging capacitor and is given as

$$C_b = \frac{q_h}{v_i} = \frac{\tau_F\, i_c}{v_i} = \tau_F\, g_m = \tau_F \frac{I_C}{V_t}$$

The base transit time τ_F is defined as $\frac{W_B{}^2}{2D_n}$

Parasitic Elements of the BJT Small Signal Model

Typical cross-section of the *npn* BJT:

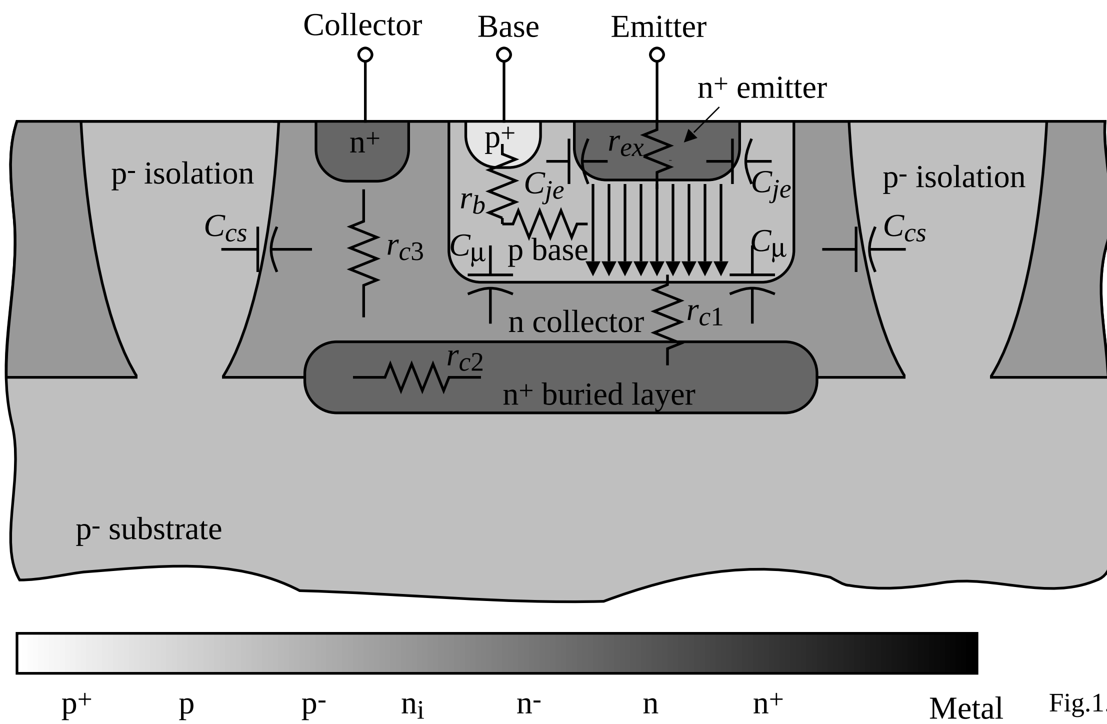


Fig.1.4-4

C_{je} = base-emitter depletion capacitance (forward biased)

$$C_\mu = \frac{C_{\mu 0}}{\left(1 - \frac{v_{CB}}{\psi_0}\right)^m} = \text{collector-base depletion capacitance (reverse biased)}$$

Resistances are all bulk ohmic resistances. Of importance are r_b, r_c, and r_{ex}. Also, r_b is a function of I_C.

Complete Small Signal BJT Model

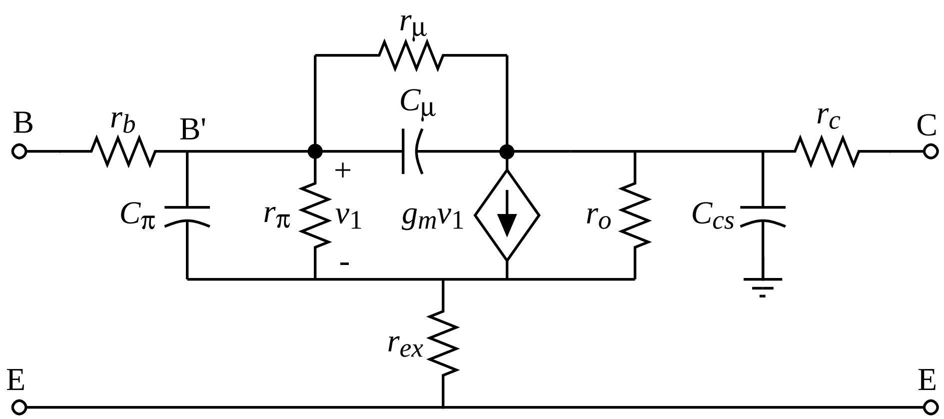


Fig. 1.4-5

The capacitance, C_π, consists of the sum of C_{je} and C_b.

$$C_\pi = C_{je} + C_b$$

Example

Derive the complete small signal equivalent circuit for a BJT at I_C = 1mA, V_{CB} = 3V, and V_{CS} = 5V. The device parameters are $Cje0$ = 10fF, n_e = 0.5, ψ_{0e} = 0.9V, $C_{\mu0}$ = 10fF, n_c = 0.3, ψ_{0c} = 0.5V, C_{cs0} = 20fF, n_s = 0.3, ψ_{0s} = 0.65V, β_o = 100, τ_F = 10ps, V_A = 20V, r_b = 300Ω, r_c = 50Ω, r_{ex} = 5Ω, and r_μ = $10\beta_o r_o$.

Solution

Because C_{je} is difficult to determine and usually an insignificant part of C_π, let us approximate it as $2C_{je0}$.

$\therefore\ C_{je} = 20\text{fF}$

$$C_\mu = \frac{C_{\mu0}}{\left(1+\frac{V_{CB}}{\psi_{0c}}\right)^{n_e}} = \frac{10\text{fF}}{\left(1+\frac{3}{0.5}\right)^{0.3}} = 5.6\text{fF} \quad \text{and} \quad C_{cs} = \frac{C_{cs0}}{\left(1+\frac{V_{CS}}{\psi_{0s}}\right)^{n_s}} = \frac{20\text{F}}{\left(1+\frac{5}{0.65}\right)^{0.3}} = 10.5\text{fF}$$

$$g_m = \frac{I_C}{V_t} = \frac{1\text{mA}}{26\text{mV}} = 38\text{mA/V} \qquad C_b = \tau_F\, g_m = (10\text{ps})(38\text{mA/V}) = 0.38\text{pF}$$

$\therefore\ C_\pi = C_b + C_{je} = 0.38\text{pF}+0.02\text{pF} = 0.4\text{pF}$

$$r_\pi = \frac{\beta_o}{g_m} = 100\cdot 26\Omega = 2.6\text{k}\Omega \qquad r_o = \frac{V_A}{I_C} = \frac{20\text{V}}{1\text{mA}} = 20\text{k}\Omega \quad \text{and} \quad r_\mu = 10\beta_o r_o = 10\cdot 100\cdot 20\text{k}\Omega = 20\text{M}\Omega$$

FREQUENCY RESPONSE OF THE BJT

Transition Frequency, f_T

f_T is the frequency where the magnitude of the short-circuit, common-emitter current equal unity.

Circuit and model:

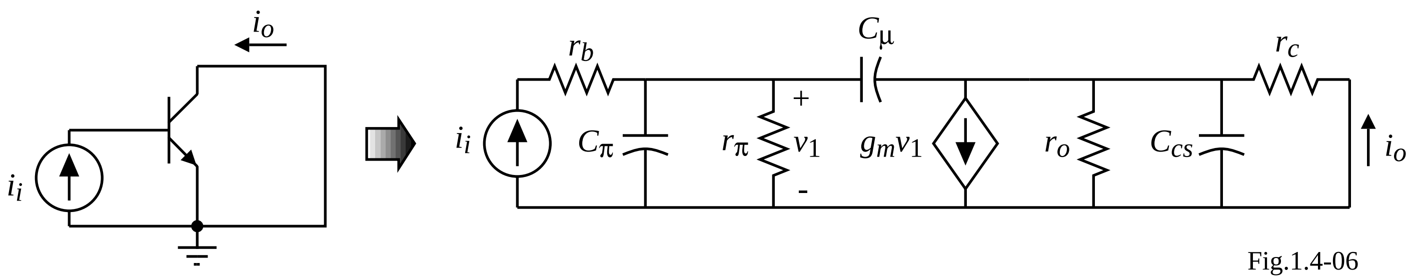


Fig.1.4-06

Assume that $r_c \approx 0$. As a result, r_o and C_{cs} have no effect.

$$\therefore \quad V_1 \approx \frac{r_\pi}{1+ r_\pi(C_\pi+C_b)s} I_i \quad \text{and} \quad I_o \approx g_m V_1 \quad \Rightarrow \quad \frac{I_o(j\omega)}{I_i(j\omega)} = \frac{g_m r_\pi}{1+ g_m r_\pi \frac{(C_\pi+C_b)s}{g_m}} = \frac{\beta_o}{1+ \beta_o \frac{(C_\pi+C_b)s}{g_m}}$$

Now,

$$\beta(j\omega) = \frac{I_o(j\omega)}{I_i(j\omega)} = \frac{\beta_o}{1+ \beta_o \frac{(C_\pi+C_b)j\omega}{g_m}}$$

At high frequencies,

$$\beta(j\omega) \approx \frac{g_m}{j\omega\,(C_\pi+C_b)} \quad \Rightarrow \quad \text{When } |\beta(j\omega)| = 1 \text{ then } \omega_T = \frac{g_m}{C_\pi+C_b} \quad \text{or} \quad f_T = \frac{1}{2\pi}\,\frac{g_m}{C_\pi+C_b}$$

Illustration of the BJT Transition Frequency

β as a function of frequency:

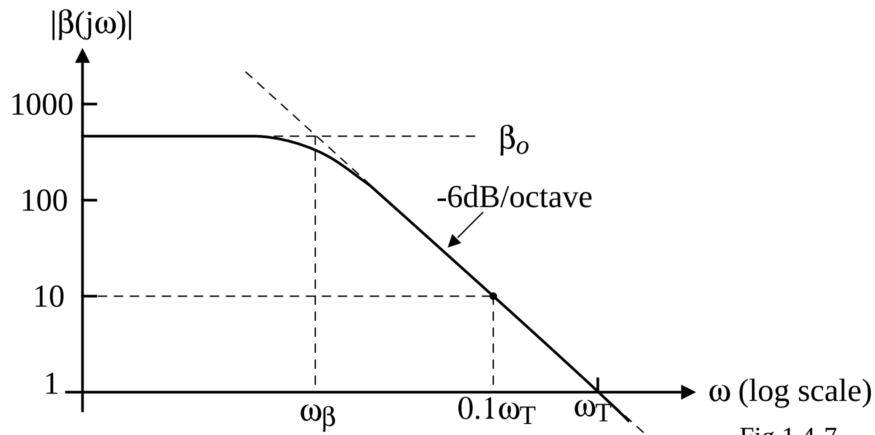


Fig.1.4-7

Note that the product of the magnitude and frequency at any point on the -6dB/octave curve is equal to ω_T.

For example,

$$0.1\ \omega_T \text{ x}10 = \omega_T$$

In measuring ω_T, the value of $|\beta(j\omega)|$ is measured at some frequency less than ω_T (say ω_x) and ω_T is calculated by taking the product of $|\beta(j\omega_x)|$ and ω_x to get ω_T.

Current Dependence of f_T

Note that $\tau_T = \dfrac{1}{\omega_T} = \dfrac{C_\pi}{g_m} + \dfrac{C_\mu}{g_m} = \dfrac{C_b}{g_m} + \dfrac{C_{je}}{g_m} + \dfrac{C_\mu}{g_m} = \tau_F + \dfrac{C_{je}}{g_m} + \dfrac{C_\mu}{g_m}$

At low currents, the C_{je} and C_μ terms dominate causing τ_T to rise and ω_T to fall.

At high currents, τ_T approaches τ_F which is the maximum value of ω_T.

For further increases in collector current, ω_T decreases because of high-level injection effects and the Kirk effect.

Typical frequency dependence of f_T:

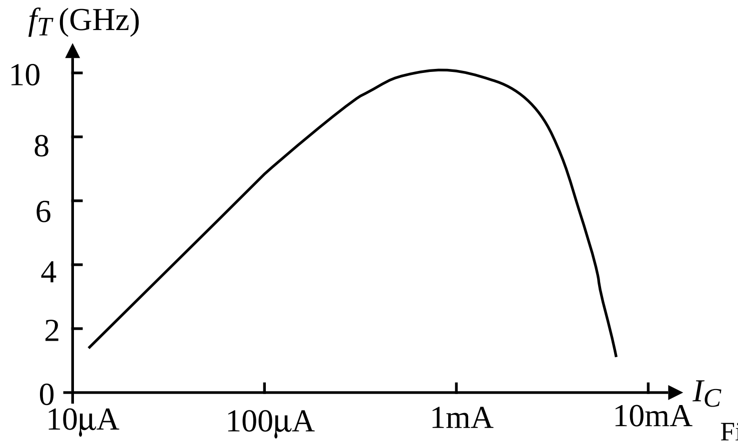


Fig.1.4-8