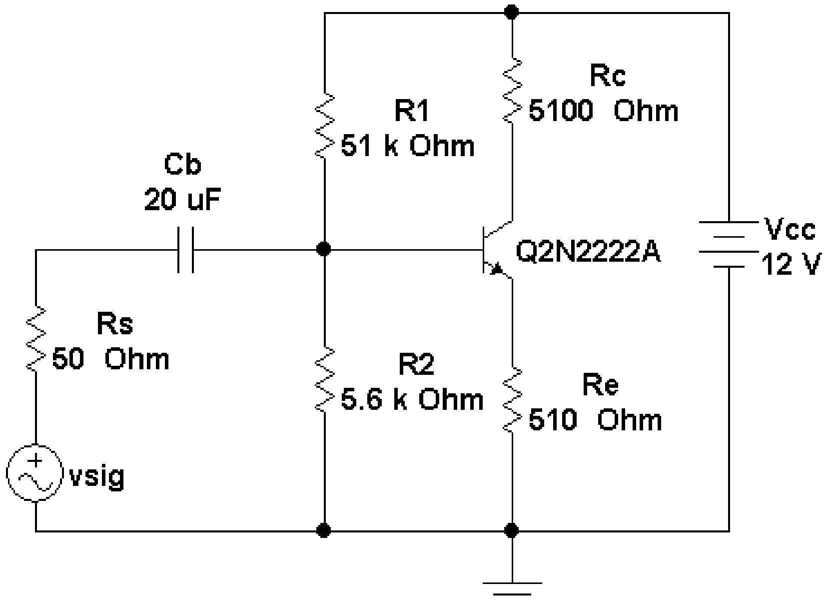
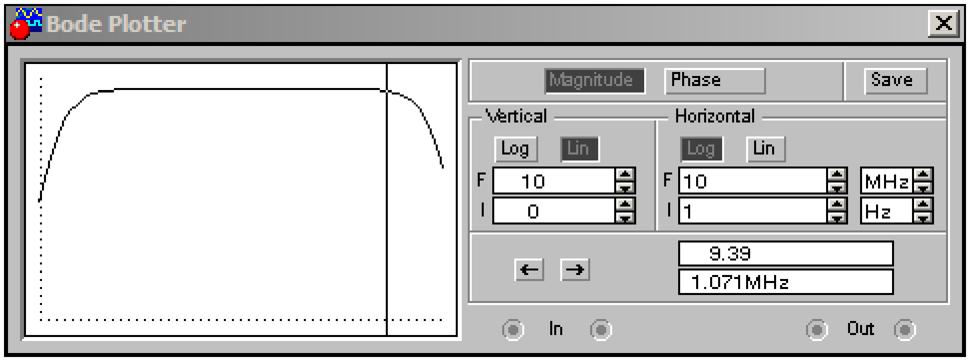
High Frequency BJT Model

Gain of 10 Amplifier – Non-ideal Transistor

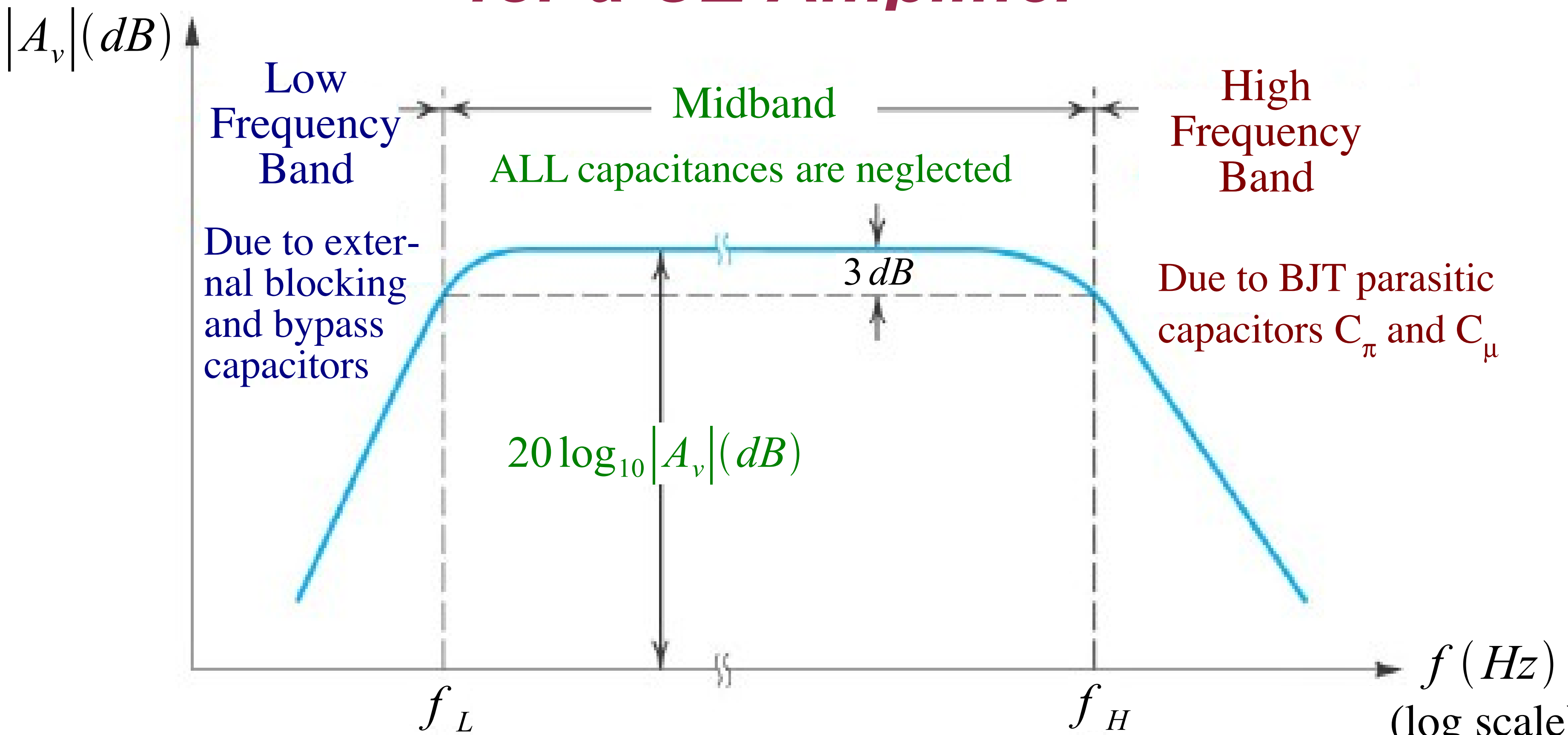




Gain starts dropping at about 1MHz.

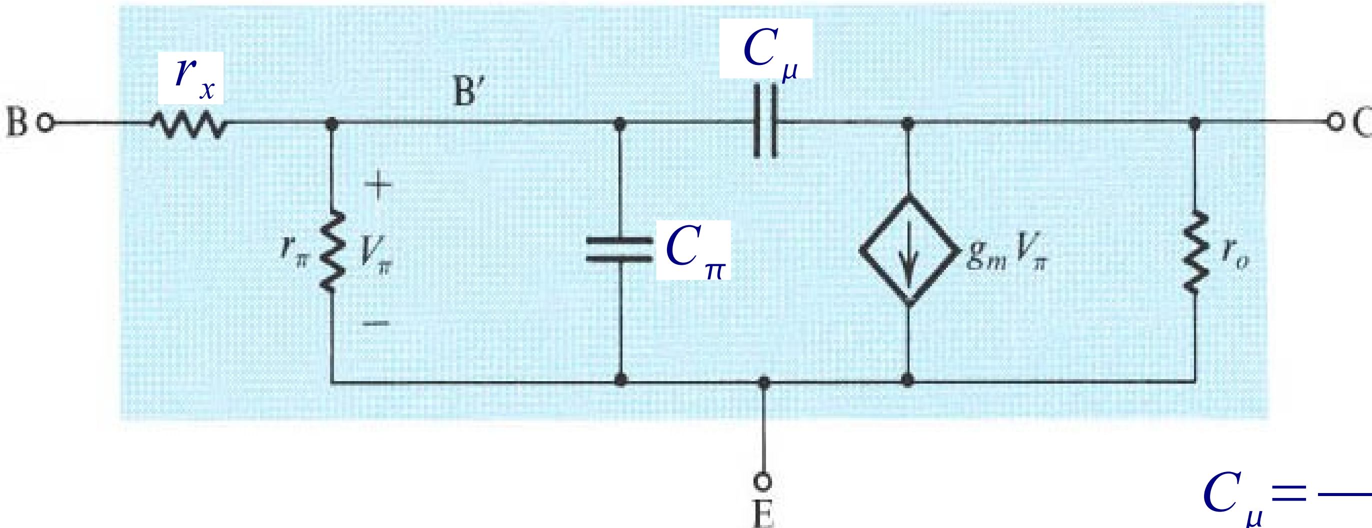
Why!
Because of internal transistor capacitances that we have ignored in our models.

Sketch of Typical Voltage Gain Response for a CE Amplifier



$BW = f_H - f_L \approx f_H$ $\qquad$ $GBP = |A_v|BW$

High Frequency Small-signal Model



SPICE
CJC = $C_{\mu 0}$
CJE = C_{je0}
TF = τ_F
RB = r_x

$$C_\mu = \frac{C_{\mu 0}}{\left(1+\frac{V_{CB}}{V_{0c}}\right)^m}$$

Two capacitors and a resistor added.

A base to emitter capacitor, C_π

A base to collector capacitor, C_μ

A resistor, r_x, representing the base terminal resistance ($r_x << r_\pi$)

$$C_\pi = C_{de} \frac{+C_{je0}}{\left(1-\frac{V_{BE}}{V_{0e}}\right)^m} \approx C_{de} + 2C_{je0}$$

$$C_{de} = \tau_F g_m$$

τ_F = forward-base transit time

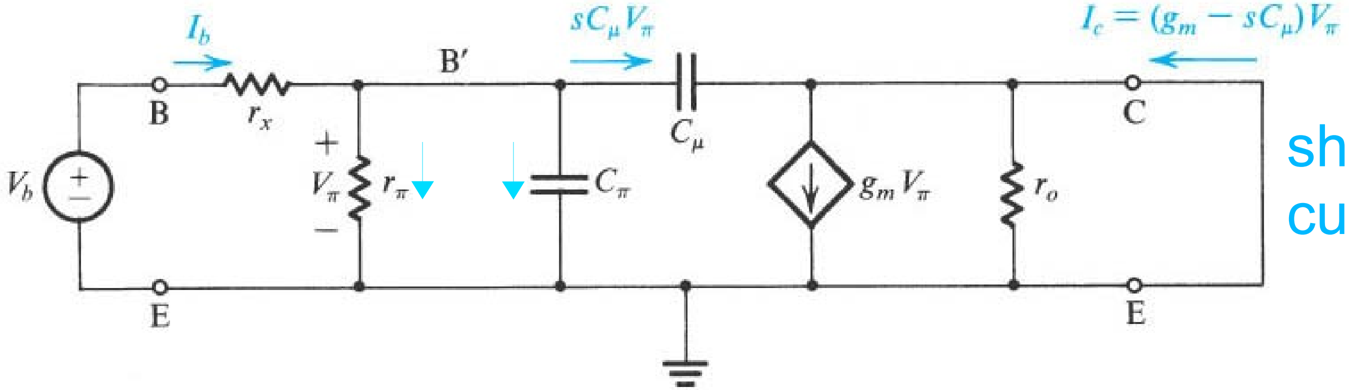
High Frequency Small-signal Model

The internal capacitors on the transistor have a strong effect on circuit high frequency performance! They attenuate base signals, decreasing v_{be} since their <u>reactance approaches zero</u> (short circuit) as frequency increases.

As we will see later C_μ is the principal cause of this gain loss at high frequencies. At the base C_μ looks like a capacitor of value $k\,C_\mu$ connected between base and emitter, where $k > 1$ and may be $>> 1$.

This phenomenon is called the *Miller Effect*.

Frequency-dependent "beta" h_{fe}



short-circuit current

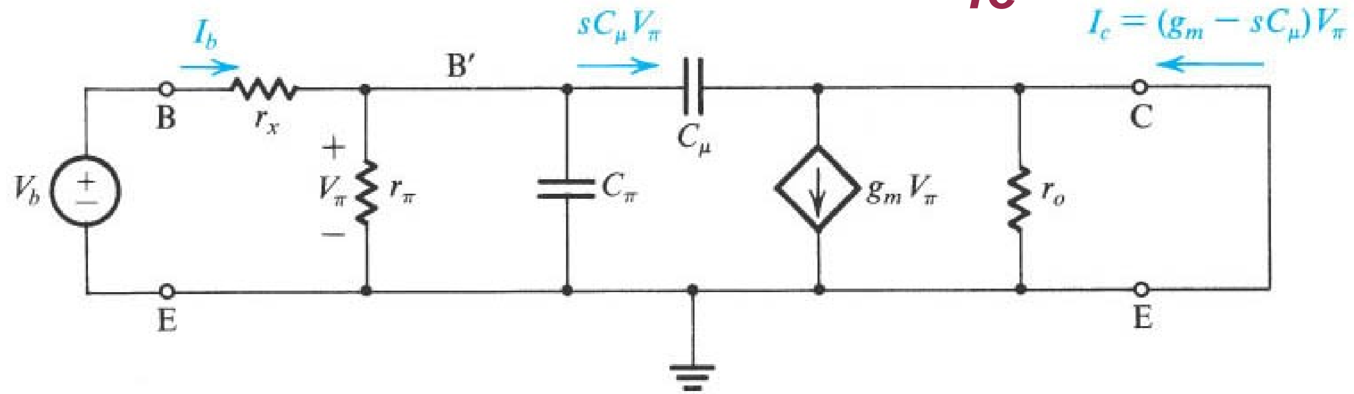
The relationship $i_c = \beta i_b$ does not apply at high frequencies $f > f_H$!

Using the relationship – $i_c = f(V_\pi)$ – find the new relationship between i_b and i_c. For i_b (using *phasor notation* (I_x & V_x) *for frequency domain analysis*):

@ node B': $I_b = \left(\dfrac{1}{r_\pi} + sC_\pi + sC_\mu\right) V_\pi$ where $r_x \approx 0$ (ignore r_x)

NOTE: $s = \sigma + j\omega$, in sinusoidal steady-state $s = j\omega$.

Frequency-dependent h_{fe} or "beta"



$$I_b = \left(\frac{1}{r_\pi} + sC_\pi + sC_\mu\right)V_\pi$$ @ node C: $I_c = (g_m - sC_\mu)V_\pi$ (ignore r_o)

Leads to a new relationship between the I_b and I_c:

$$\boxed{h_{fe} = \frac{I_c}{I_b} = \frac{g_m - sC_\mu}{\frac{1}{r_\pi} + sC_\pi + sC_\mu}}$$

Frequency Response of h_{fe}

$$h_{fe}=\frac{g_m-sC_\mu}{\frac{1}{r_\pi}+sC_\pi+sC_\mu}$$

multiply N&D by r_π and set $s=j\omega$

$$h_{fe}=\frac{(g_m-j\omega C_\mu)r_\pi}{1+j\omega(C_\pi+C_\mu)r_\pi}$$

factor N to isolate g_m

$$h_{fe}=\frac{(1-j\omega\frac{C_\mu}{g_m})\overbrace{g_m r_\pi}^{\beta}}{1+j\omega(C_\pi+C_\mu)r_\pi}$$

$$\boxed{g_m=\frac{I_C}{V_T} \qquad r_\pi=\beta\frac{V_T}{I_C}}$$

For small $\omega=\omega_{low}$: $\omega_{low}\frac{C_\mu}{g_m}\ll 1<\frac{1}{10}$

and: $\omega_{low}(C_\pi+C_\mu)r_\pi\ll 1<\frac{1}{10}$

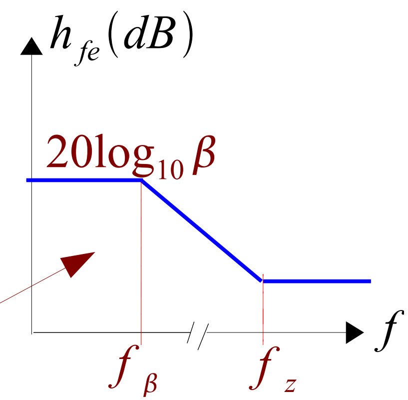
Note: $\omega_{low}(C_\pi+C_\mu)r_\pi=\omega_{low}(C_\pi+C_\mu)\frac{\beta}{g_m}\gg\omega_{low}\frac{C_\mu}{g_m}$

We have: $\boxed{h_{fe}=g_m r_\pi=\beta}$

Frequency Response of h_{fe} cont.

$$h_{fe}=\frac{(1-j\omega\frac{C_{\mu}}{g_m})g_m r_{\pi}}{1+j\omega(C_{\pi}+C_{\mu})r_{\pi}}=\frac{\left(1-j\frac{\omega}{\omega_z}\right)}{\left(1+j\frac{\omega}{\omega_{\beta}}\right)}g_m r_{\pi}=\frac{\left(1-j\frac{f}{f_z}\right)}{\left(1+j\frac{f}{f_{\beta}}\right)}\beta$$

ω_{β}



$$(C_{\pi}+C_{\mu})r_{\pi}=(C_{\pi}+C_{\mu})\frac{\beta}{g_m}\gg\frac{C_{\mu}}{g_m} \text{ => } f_z\gg f_{\beta}$$

Hence, the lower break frequency or $-3dB$ frequency is f_{β}

$$f_{\beta}=\frac{1}{2\pi(C_{\pi}+C_{\mu})r_{\pi}}=\frac{g_m}{2\pi(C_{\pi}+C_{\mu})\beta}$$ the upper: $$f_z=\frac{1}{2\pi C_{\mu}/g_m}=\frac{g_m}{2\pi C_{\mu}}$$

where $f_z>10 f_{\beta}$

Frequency Response of h_{fe} cont.

Using Bode plot concepts, for the range where: $f < f_{\beta}$

$$h_{fe} = g_m r_{\pi} = \beta$$

For the range where: $f_{\beta} < f < f_z$ s.t. $|1 - j f / f_z| \approx 1$

We consider the frequency-dependent numerator term to be 1 and focus on the response of the denominator:

$$h_{fe} = \frac{g_m r_{\pi}}{\left(1 + j\dfrac{f}{f_{\beta}}\right)} = \frac{\beta}{\left(1 + j\dfrac{f}{f_{\beta}}\right)}$$

Frequency Response of h_{fe} cont.

Neglecting numerator term: $$h_{fe}=\frac{g_m r_\pi}{\left(1+j\frac{f}{f_\beta}\right)}=\frac{\beta}{\left(1+j\frac{f}{f_\beta}\right)}$$

And for $f/f_\beta >> 1$ (but $< f/f_z$): $$|h_{fe}|\approx\frac{\beta}{\left(\frac{f}{f_\beta}\right)}=\beta\frac{f_\beta}{f}$$

Unity gain bandwidth: $$|h_{fe}|=1\Rightarrow\beta\frac{f_\beta}{f}\bigg|_{f=f_T}=1\Rightarrow f_T=\beta f_\beta$$

$$\boxed{f_T=\frac{\omega_T}{2\pi}=\beta f_\beta}$$

BJT unity-gain frequency or GBP

Frequency Response of h_{fe} cont.

$$\beta = 100 \quad r_\pi = 2500\,\Omega \quad C_\pi = 12\,pF \quad C_\mu = 2\,pF \quad g_m = 40 \cdot 10^{-3}\,S$$

$$\omega_\beta = \frac{1}{\left(C_\pi + C_\mu\right) r_\pi} = \frac{10^{12} \cdot 10^{-3}}{(12+2) \cdot 2.5} = 28.57 \cdot 10^6\,rps$$

$$f_\beta = \frac{\omega_\beta}{2\pi} = \frac{28.57}{6.28} 10^6\,Hz = 4.55\,MHz \qquad f_T = \beta f_\beta = 455\,MHz$$

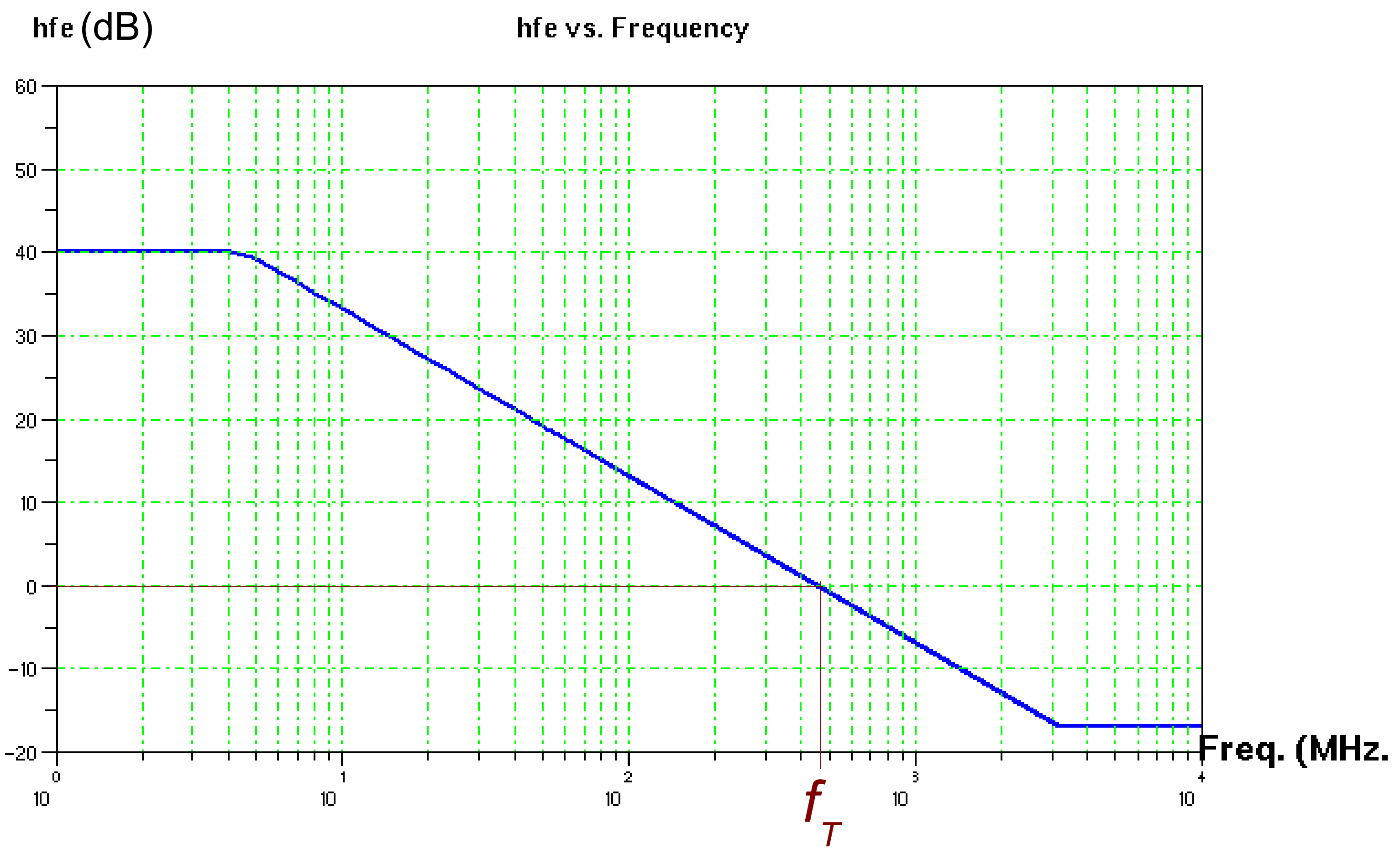
$$\omega_z = \frac{g_m}{C_\mu} = \frac{40 \cdot 10^{-3} \cdot 10^{12}}{2}\,Hz = 20 \cdot 10^9\,rps$$

$$f_z = \frac{\omega_z}{2\pi} = 3.18 \cdot 10^9\,Hz = 3180\,MHz$$

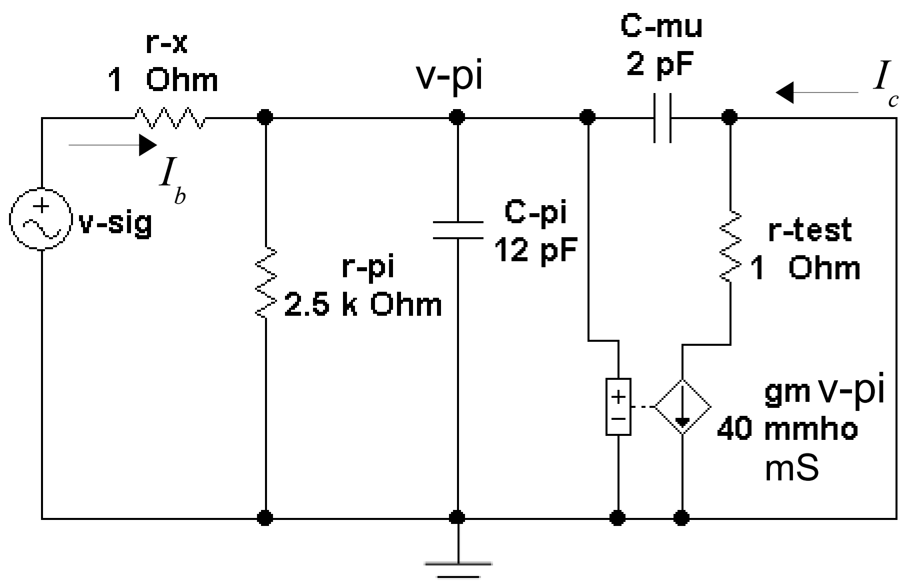
Scilab f_T Plot

```
//fT Bode Plot
Beta=100;
KdB= 20*log10(Beta);
fz=3180;
fp=4.55;
f= 1:1:10000;
term1=KdB*sign(f); //Constant array of len(f)
term2=max(0,20*log10(f/fz));   //Zero for f < fz;
term3=min(0,-20*log10(f/fp));  //Zero for f < fp;
BodePlot=term1+term2+term3;
plot(f,BodePlot);
```

h_{fe} Bode Plot



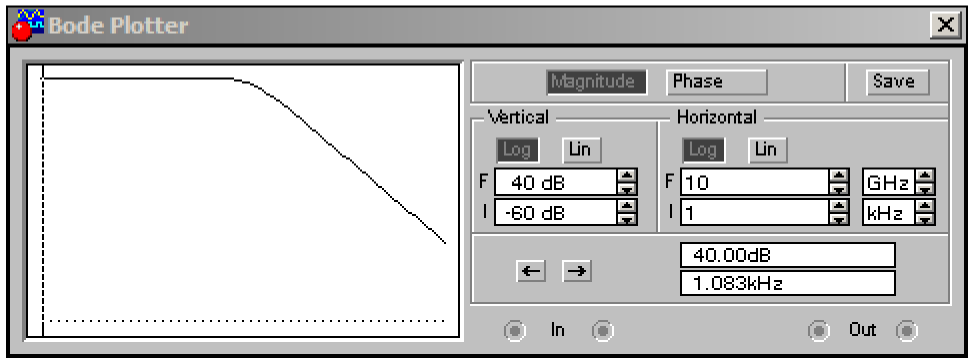
Multisim Simulation



Insert 1 ohm resistors – we want to measure a current ratio.

$$h_{fe}=\frac{I_c}{I_b}=\frac{g_m-s\,C_\mu}{\frac{1}{r_\pi}+s\,(C_\pi+C_\mu)}$$

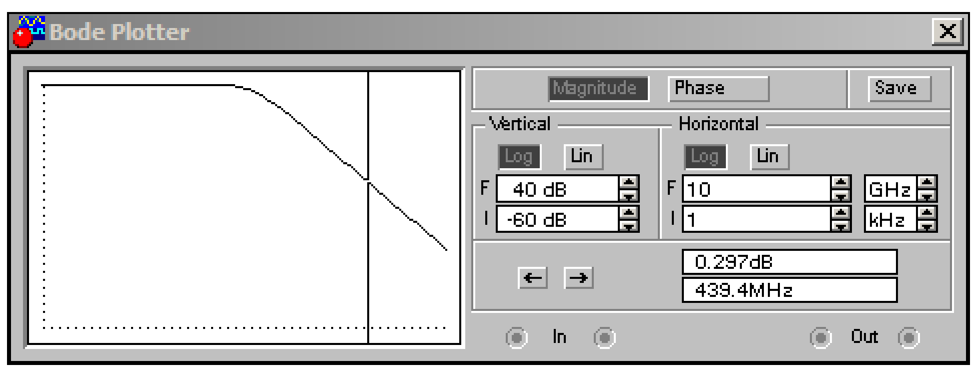
Simulation Results



Low frequency $|h_{fe}|$

Theory:

$$f_T = \beta f_\beta = 455\ MHz$$



Unity Gain frequency about 440 MHz