Wichita State University

Flight Control System: Module 1, Lecture 2: Aircraft Dynamic Model

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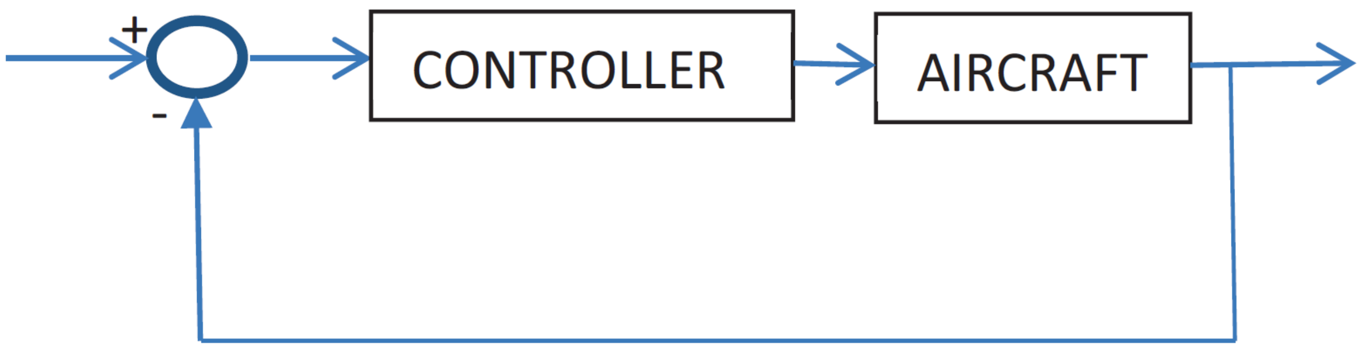
Sec: 1.4, 1.5, 2.3

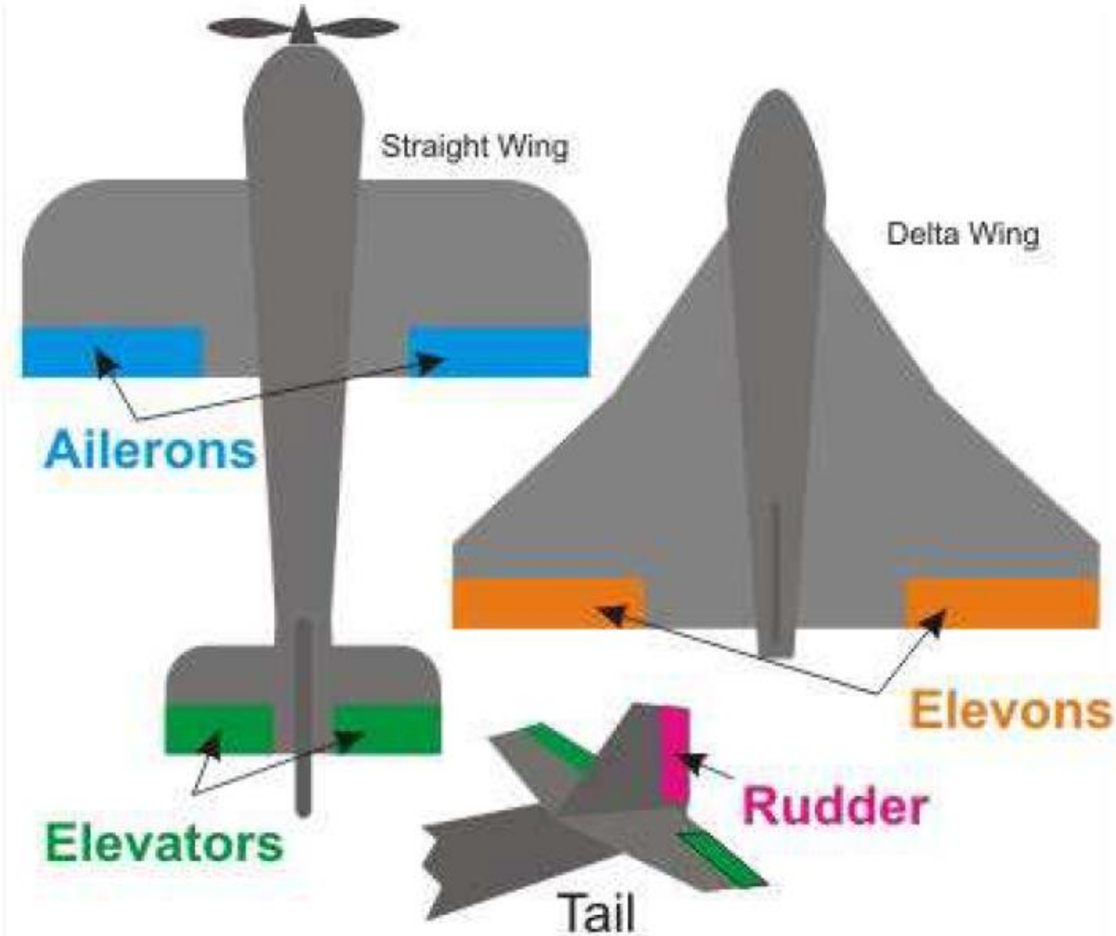
Aircraft as a Dynamic System

Control System Block Diagram

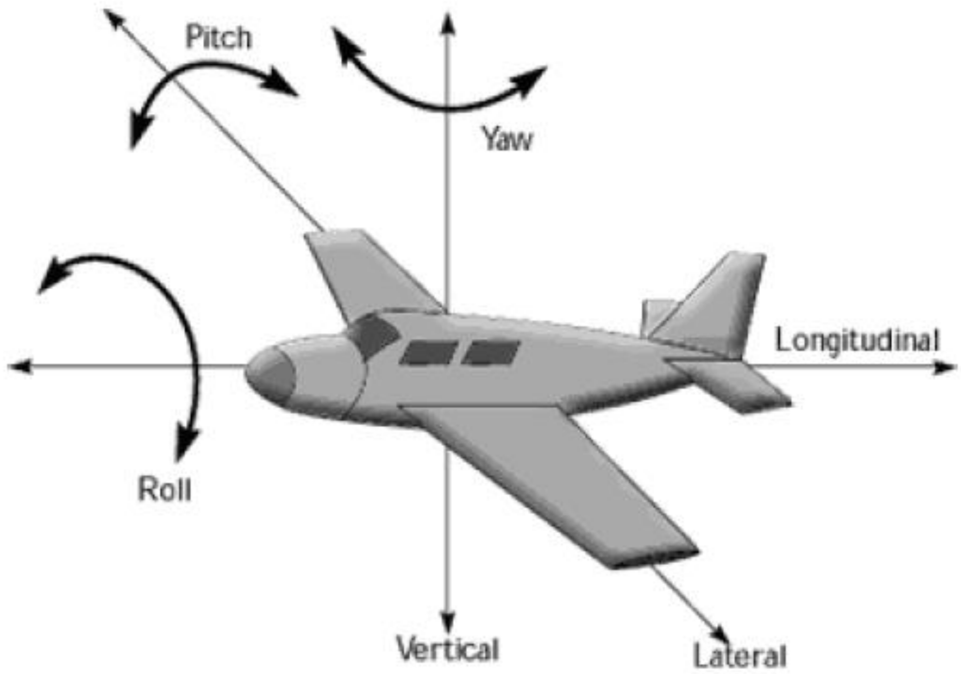


- Dynamic system that can be modified
- Desired behavior
- Actual behavior
- Corrections to the behavior of the dynamic system

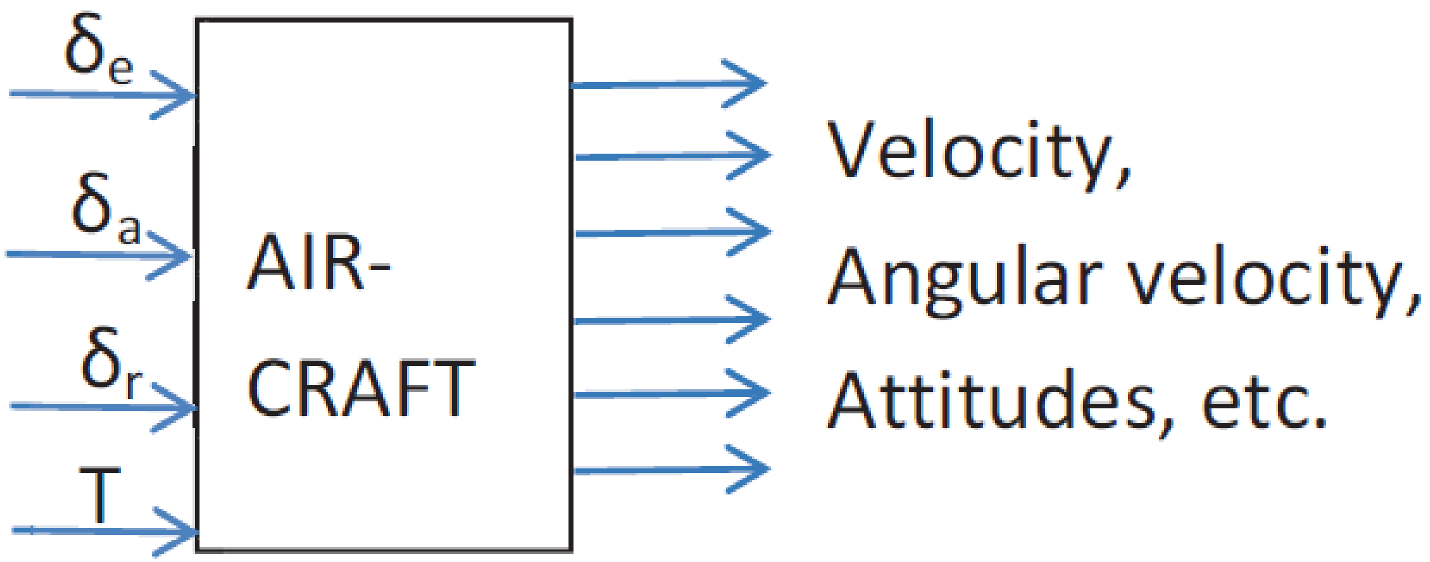
AIRCRAFT CONTROL SURFACES



Aircraft Attitude Angles



Aircraft as a System



Aircraft Center of Mass Equations of Motion

BODY-FIXED REFERENCE FRAME

- Origin, center of mass
- X-axis, Fuselage reference line (FRL)
- Z-axis, plane of symmetry

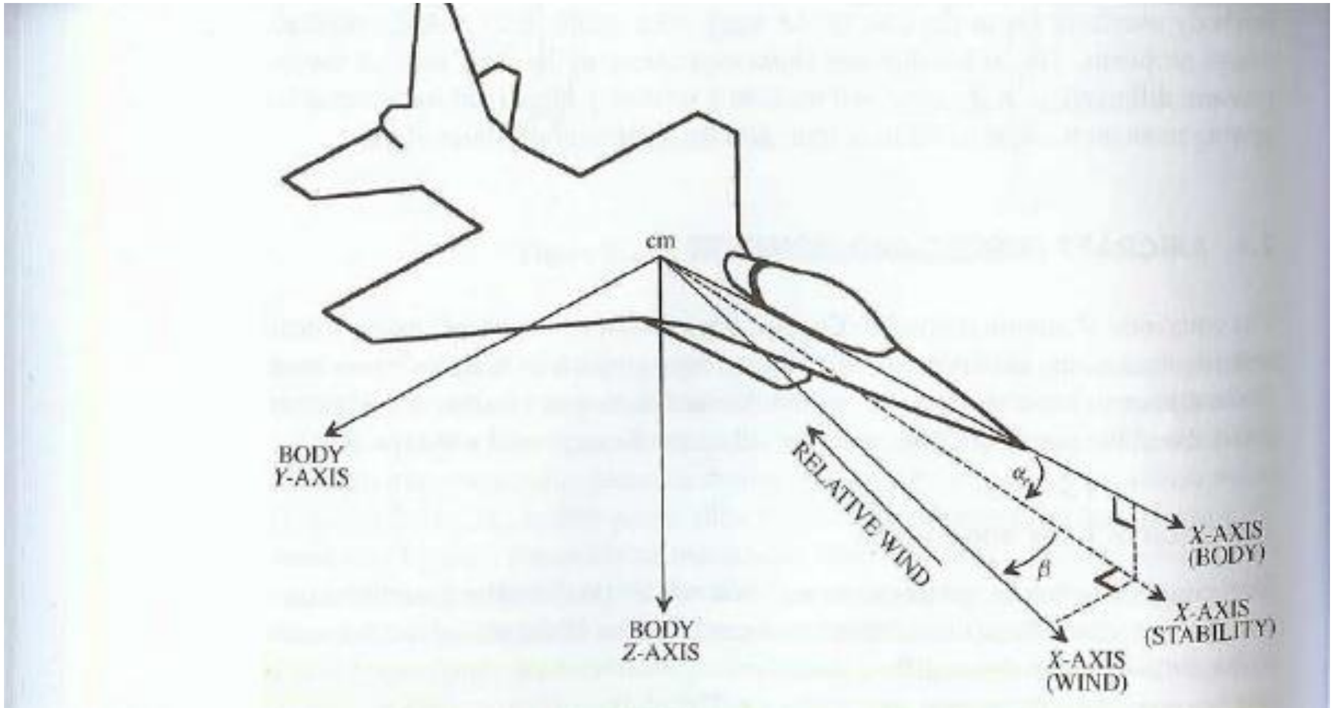


Figure 2.3-1 Definitions of axes and aerodynamic angles.

Translational Dynamics

- Application of Newtonian principles

$$\underline{\boldsymbol{F}} = m\underline{\boldsymbol{a}}$$

- Non-inertial reference frame

$$\underline{\boldsymbol{F}} = m\frac{{}^{\mathcal{I}}d}{dt}\underline{\boldsymbol{v}}$$

$$= m\left(\frac{{}^{\mathcal{B}}d}{dt}\underline{\boldsymbol{v}} + {}^{\mathcal{I}}\underline{\boldsymbol{\omega}}^{\mathcal{B}} \times \underline{\boldsymbol{v}}\right)$$

TRANSLATIONAL DYNAMICS

- Scalar equations of motion

$$\begin{bmatrix} X_A + X_T + X_g \\ Y_A + Y_T + Y_g \\ Z_A + Z_T + Z_g \end{bmatrix} = m\left(\begin{bmatrix} \dot{U} \\ \dot{V} \\ \dot{W} \end{bmatrix} + \begin{vmatrix} \hat{\boldsymbol{i}} & \hat{\boldsymbol{j}} & \hat{\boldsymbol{k}} \\ P & Q & R \\ U & V & W \end{vmatrix}\right)$$

$$\boxed{\begin{aligned} X_A + X_T + X_g &= m\left(\dot{U} + QW - VR\right) \\ Y_A + Y_T + Y_g &= m\left(\dot{V} + UR - PW\right) \\ Z_A + Z_T + Z_g &= m\left(\dot{W} + PV - QU\right) \end{aligned}}$$

Translational Dynamics

- Scalar equations of motion

$$\boxed{\dot{U} = \frac{X_A + X_T + X_g}{m} + VR - QW}$$

$$\boxed{\dot{V} = \frac{Y_A + Y_T + Y_g}{m} + PW - UR}$$

$$\boxed{\dot{W} = \frac{Z_A + Z_T + Z_g}{m} + QU - PV}$$

Aircraft Rotational Dynamics

Moment Equation

- Center of mass is the point of interest

$$\underline{\boldsymbol{M}} = \frac{{}^{\mathcal{I}}d}{dt}\underline{\boldsymbol{H}} = \left(\frac{{}^{\mathcal{B}}d}{dt}\underline{\boldsymbol{H}} + {}^{\mathcal{I}}\underline{\boldsymbol{\omega}}^{\mathcal{B}} \times \underline{\boldsymbol{H}}\right)$$

- Angular momentum captures the rotational kinematics of the aircraft

$$\underline{\boldsymbol{H}} = [\mathbb{I}]\ {}^{\mathcal{I}}\underline{\boldsymbol{\omega}}^{\mathcal{B}}$$

$$\begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix} \begin{bmatrix} P \\ Q \\ R \end{bmatrix}$$

Angular Momentum

- Typically, we have $I_{xy} = I_{yz} = 0$
- We therefore have

$$\begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = \begin{bmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & 0 \\ -I_{xz} & 0 & I_{zz} \end{bmatrix} \begin{bmatrix} P \\ Q \\ R \end{bmatrix}$$

$$H_x = I_{xx}P - I_{xz}R$$
$$H_y = I_{yy}Q$$
$$H_z = -I_{xz}P + I_{zz}R$$

Scalar Equations

- Moments about the center of mass

$$L_A + L_T = \ell, \quad M_A + M_T = m, \quad N_A + N_T = n$$

- We therefore have

$$\begin{bmatrix} \ell \\ m \\ n \end{bmatrix} = \begin{bmatrix} I_{xx}\dot{P} - I_{xz}\dot{R} \\ I_{yy}\dot{Q} \\ -I_{xz}\dot{P} + I_{zz}\dot{R} \end{bmatrix}$$

$$+ \begin{bmatrix} \hat{\boldsymbol{i}} & \hat{\boldsymbol{j}} & \hat{\boldsymbol{k}} \\ P & Q & R \\ I_{xz}P - I_{xz}R & I_{yy}Q & -I_{xz}P + I_{zz}R \end{bmatrix}$$

Moment Equations

- We define $$\Gamma = I_{xx}I_{zz} - I_{xz}^2$$
- Scalar equations

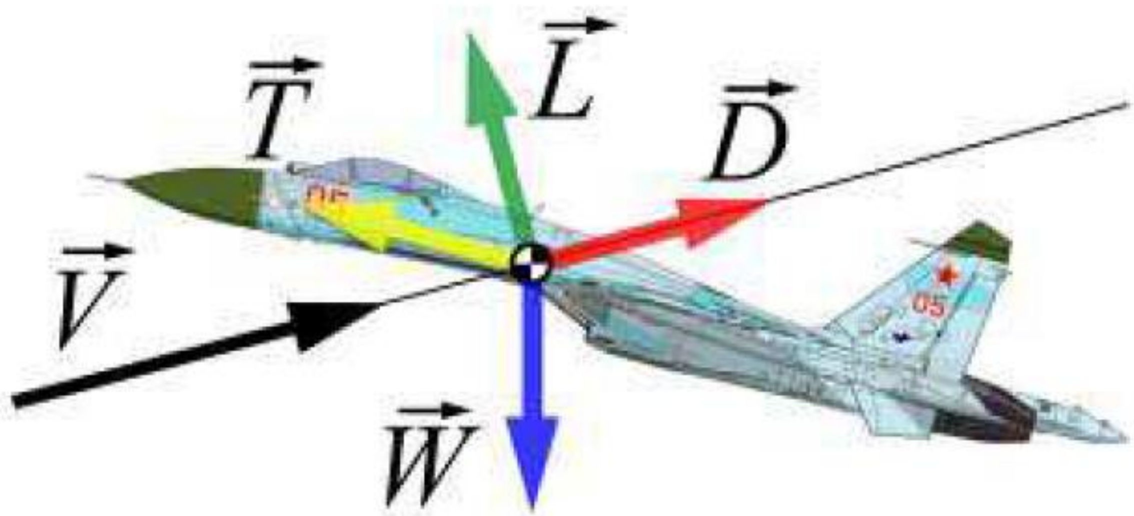
$$\boxed{\begin{aligned}\Gamma\dot{P} = {} & I_{xz}\left(I_{xx} - I_{yy} + I_{zz}\right)PQ \\ & - \left[I_{zz}\left(I_{zz} - I_{yy}\right) + I_{xz}^2\right]QR + I_{zz}\ell + I_{xz}n\end{aligned}}$$

$$\boxed{I_{yy}\dot{Q} = \left(I_{zz} - I_{xx}\right)PR - I_{xz}\left(P^2 - R^2\right) + m}$$

$$\boxed{\begin{aligned}\Gamma\dot{R} = {} & \left[\left(I_{xx} - I_{yy}\right)I_{xx} + I_{xz}^2\right]PQ \\ & - I_{xz}\left(I_{xx} - I_{yy} + I_{zz}\right)QR + I_{xz}\ell + I_{xx}n\end{aligned}}$$

Aerodynamic Forces and Moments

Aerodynamic Forces



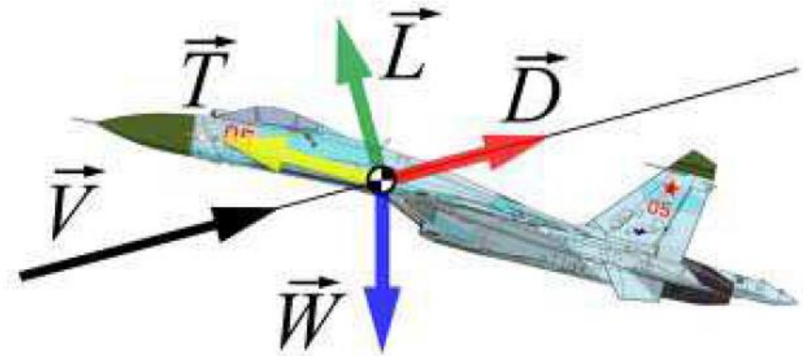
$$\underline{\boldsymbol{V}} \equiv \underline{\boldsymbol{v}}_{\mathrm{rel}} = \begin{bmatrix} U' \\ V' \\ W' \end{bmatrix} \qquad V_t = \sqrt{U'^2 + V'^2 + W'^2}$$

ANGLE OF ATTACK

$$X = -D\cos\alpha + L\sin\alpha$$
$$Z = -D\sin\alpha - L\cos\alpha$$

- Angle of attack

$$\tan\alpha = \frac{W'}{U'}$$



- Important parameter that influences the forces and moments on an aircraft

Aerodynamic Forces and Moments

$$X_A = \frac{1}{2}\rho V_t^2 S C_x,\ Y_A = \frac{1}{2}\rho V_t^2 S C_y,\ Z_A = \frac{1}{2}\rho V_t^2 S C_z$$

$$L_A = \frac{1}{2}\rho V_t^2 S \bar{c} C_\ell,\ M_A = \frac{1}{2}\rho V_t^2 S \bar{c} C_m,\ N_A = \frac{1}{2}\rho V_t^2 S \bar{c} C_n$$

- Relative velocity of aircraft
- Wing area
- Density of air varies with altitude
- Wing chord length

Sideslip Angle

- Angle of sideslip

$$\sin \beta = \frac{V'}{V_t}$$

- Mach number

$$M = \frac{V_t}{a}$$

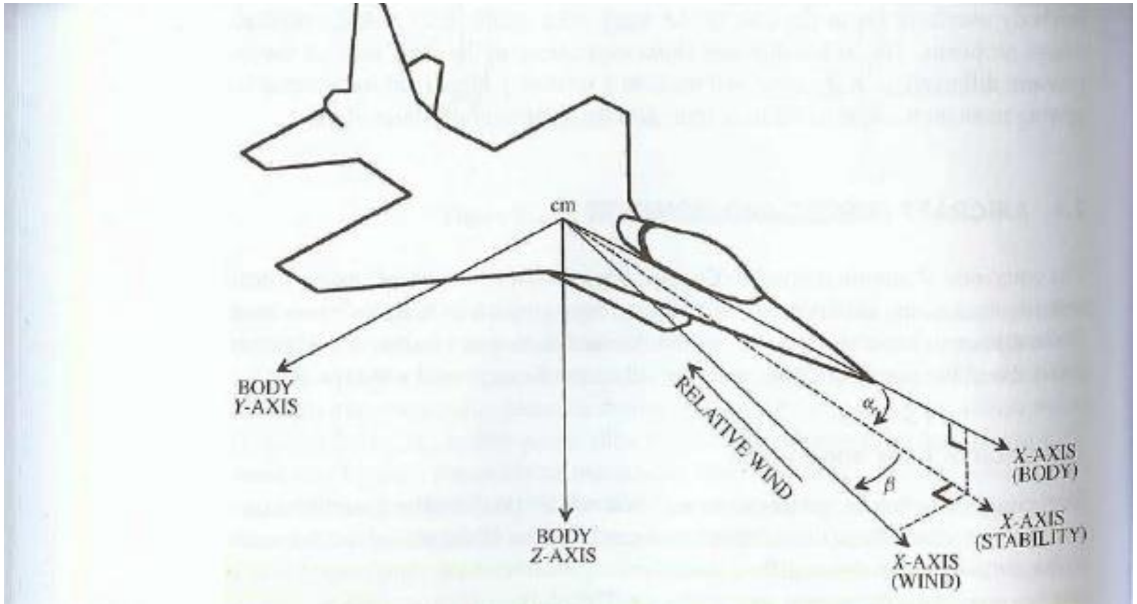


Figure 2.3-1 Definitions of axes and aerodynamic angles.

- Wind reference frame
  – 2-3 Euler rotation sequence

State-Space Equations

State Space Equations (1 of 2)

- An ordinary differential equation can be reduced to the state-space model
- State vector

$$\underline{\boldsymbol{x}} = \begin{bmatrix} U \\ V \\ W \\ P \\ Q \\ R \end{bmatrix}$$

- The force and moment equations are nonlinear and have the general structure:

$$\dot{\underline{\boldsymbol{x}}} = \underline{\boldsymbol{f}}\left(\underline{\boldsymbol{x}}, \underline{\boldsymbol{u}}\right)$$

State Space Equations (2 of 2)

- Control vector

$$\underline{\boldsymbol{u}} = \begin{bmatrix} \delta_e \\ \delta_a \\ \delta_r \end{bmatrix}$$

- Depending on the type of aircraft, there may be additional control inputs available

Linearization of State Equations

- Determine equilibrium points of the aircraft

$$\underline{\mathbf{0}} = \underline{\boldsymbol{f}}\left(\underline{\boldsymbol{x}}, \underline{\boldsymbol{u}}\right) \qquad \Longrightarrow \; \underline{\boldsymbol{x}}_e, \underline{\boldsymbol{u}}_e$$

- Determine a linear model about each equilibrium point

$$\delta\dot{\underline{\boldsymbol{x}}} = \left[\frac{\partial \underline{\boldsymbol{f}}}{\partial \underline{\boldsymbol{x}}}\right]_{(\underline{\boldsymbol{x}}_e, \underline{\boldsymbol{u}}_e)} \delta\underline{\boldsymbol{x}} + \left[\frac{\partial \underline{\boldsymbol{f}}}{\partial \underline{\boldsymbol{u}}}\right]_{(\underline{\boldsymbol{x}}_e, \underline{\boldsymbol{u}}_e)} \delta\underline{\boldsymbol{u}}$$

CAVEAT

- Each linear model is valid only about the particular equilibrium point about which it was constructed

- Each linear model is valid only for small perturbations in the states and the controls, about its equilibrium point