

6.3.4.6

$$y'' - 2ty' + 2y = 0 \quad y_1 = y \quad y_2 = y' = y_1' \quad y_1(0) = 1 \quad y_2(0) = 1$$

$$y_2' = 2ty_2 - 2y_1 \quad y_1' = y_2$$

1 step $h = \frac{1}{4}$

$$y_2\left(\frac{1}{4}\right) = 1 + \frac{1}{8} \left((2 \cdot 0 \cdot 1 - 2 \cdot 1) + (2 \cdot \frac{1}{4} \cdot 1 - 2 \cdot (\frac{1}{4}(2 \cdot 0 \cdot 1 - 2 \cdot 1))) \right)$$

$$y_2\left(\frac{1}{4}\right) = 1 + \frac{1}{8} \left((0 - 2) + \left(\frac{1}{2} - 2 \left(\frac{1}{4}(0 - 2) \right) \right) \right)$$

$$y_2\left(\frac{1}{4}\right) = 1 + \frac{1}{8} \left((-2) + \left(\frac{1}{2} - 2 \left(-\frac{1}{2} \right) \right) \right)$$

$$y_2\left(\frac{1}{4}\right) = 1 + \frac{1}{8} \left((-2) + \left(\frac{1}{2} + 1 \right) \right) = 1 + \frac{1}{8} \left(-2 + \frac{3}{2} \right) = 1 + \frac{1}{8} \left(-\frac{1}{2} \right)$$

$$y_2\left(\frac{1}{4}\right) = 1 - \frac{1}{16} = \frac{15}{16} = \boxed{.9375}$$

$$y_1\left(\frac{1}{4}\right) = 1 + \frac{1}{8} (1 + .9375) = \boxed{1.2422}$$

$$y_2\left(\frac{1}{2}\right) = .9375 + \frac{1}{8} \left((2 \cdot \frac{1}{4} \cdot .9375 - 2 \cdot 1.2422) + (2 \cdot \frac{1}{2} \cdot (\dots) - 2 \cdot 1.2422) \right)$$

$$y_2\left(\frac{1}{2}\right) = \boxed{.34350}$$

$$y_1\left(\frac{1}{2}\right) = 1.2422 + \frac{1}{8} (.9375 + .34350)$$

$$y_1\left(\frac{1}{2}\right) = \boxed{1.4023}$$