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```
clear; close all; clc;
```

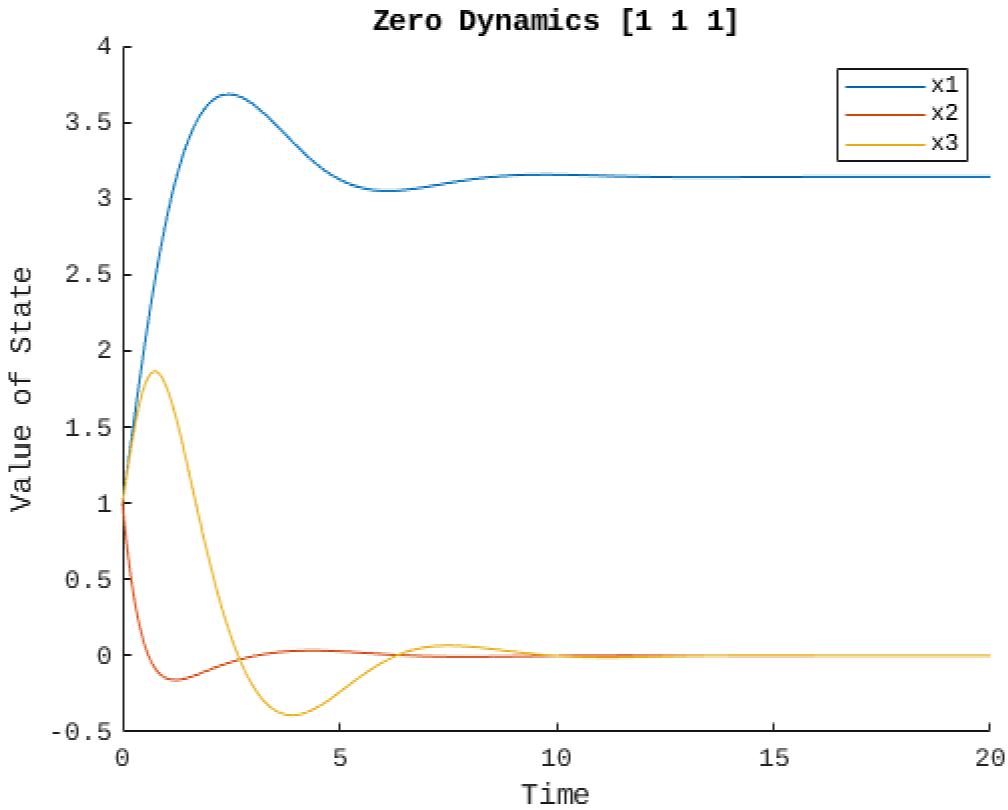
Problem 1

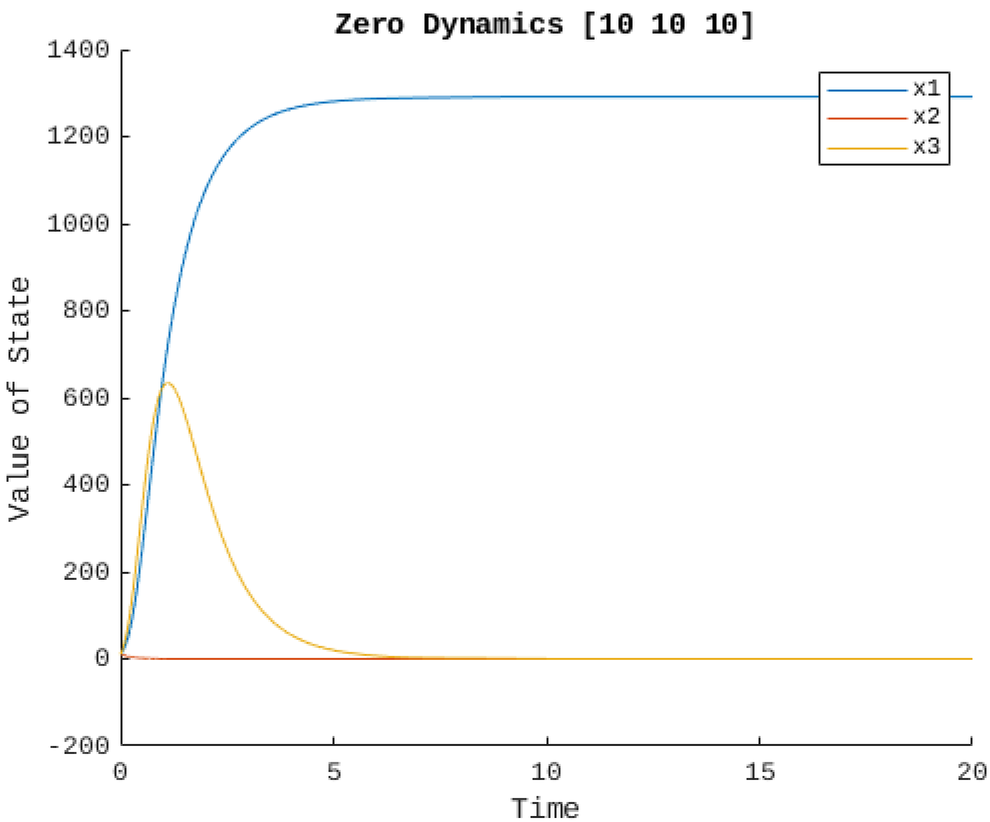
```
fprintf("Problem 1\n")
tol = 1e-8;
opts = odeset( 'RelTol' , tol , 'AbsTol' , tol ) ;
[ t , xz ] = ode45( @zerodynamics , [0, 20] , [1, 1, 1] , opts) ;
figure
hold on
plot(t, xz(:, 1))
plot(t, xz(:, 2))
plot(t, xz(:, 3))
hold off
title("Zero Dynamics [1 1 1]")
xlabel("Time")
ylabel("Value of State")
legend("x1", "x2", "x3")

[ t , xz ] = ode45( @zerodynamics , [0, 20] , [10, 10, 10] , opts) ;
figure
hold on
plot(t, xz(:, 1))
plot(t, xz(:, 2))
plot(t, xz(:, 3))
hold off
title("Zero Dynamics [10 10 10]")
xlabel("Time")
ylabel("Value of State")
legend("x1", "x2", "x3")

[ t , xz ] = ode45( @zerodynamics , [0, 20] , [10, 1, 1] , opts) ;
figure
hold on
plot(t, xz(:, 1))
plot(t, xz(:, 2))
plot(t, xz(:, 3))
hold off
title("Zero Dynamics [10 1 1]")
xlabel("Time")
ylabel("Value of State")
```

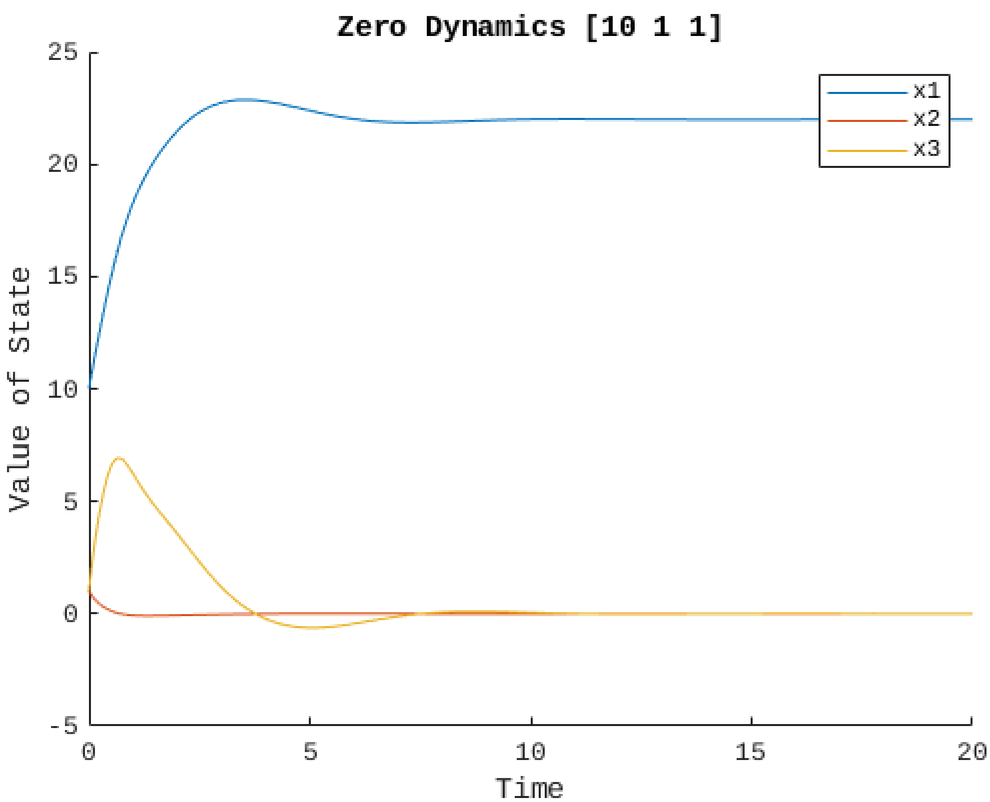
```
legend("x1", "x2", "x3")

[ t , xz ] = ode45( @zerodynamics , [0, 20] , [1, 10, 1] , opts) ;
figure
hold on
plot(t, xz(:, 1))
plot(t, xz(:, 2))
plot(t, xz(:, 3))
hold off
title("Zero Dynamics [1 10 1]")
xlabel("Time")
ylabel("Value of State")
legend("x1", "x2", "x3")

[ t , xz ] = ode45( @zerodynamics , [0, 20] , [1, 1, 10] , opts) ;
figure
hold on
plot(t, xz(:, 1))
plot(t, xz(:, 2))
plot(t, xz(:, 3))
hold off
title("Zero Dynamics [1 1 10]")
xlabel("Time")
ylabel("Value of State")
legend("x1", "x2", "x3")

fprintf("For the zero dynamics, x2 and x3 return to zero but x1 just steadies
  \n")
fprintf("out at whatever value it is at when the x2 and x3 values steady out
\n")
fprintf("But the control is for designed for x1 so with the full closed loop
\n")
fprintf("x2 and x3 go to zero and x1 goes where commanded")

[ t , x ] = ode45( @prop_to_ref , [0, 20] , [10, 10, 10] , opts, 500) ;
figure
hold on
plot(t, x(:, 1))
plot(t, x(:, 2))
plot(t, x(:, 3))
hold off
title("Command x1 to 500 with initial states of [10, 10, 10]")
xlabel("Time")
ylabel("Value of State")
legend("x1", "x2", "x3")

[ t , x ] = ode45( @prop_to_ref , [0, 20] , [10, 10, 10] , opts, 0) ;
figure
hold on
plot(t, x(:, 1))
plot(t, x(:, 2))
plot(t, x(:, 3))
hold off
title("Command x1 to zero with initial states of [10, 10, 10]")
```

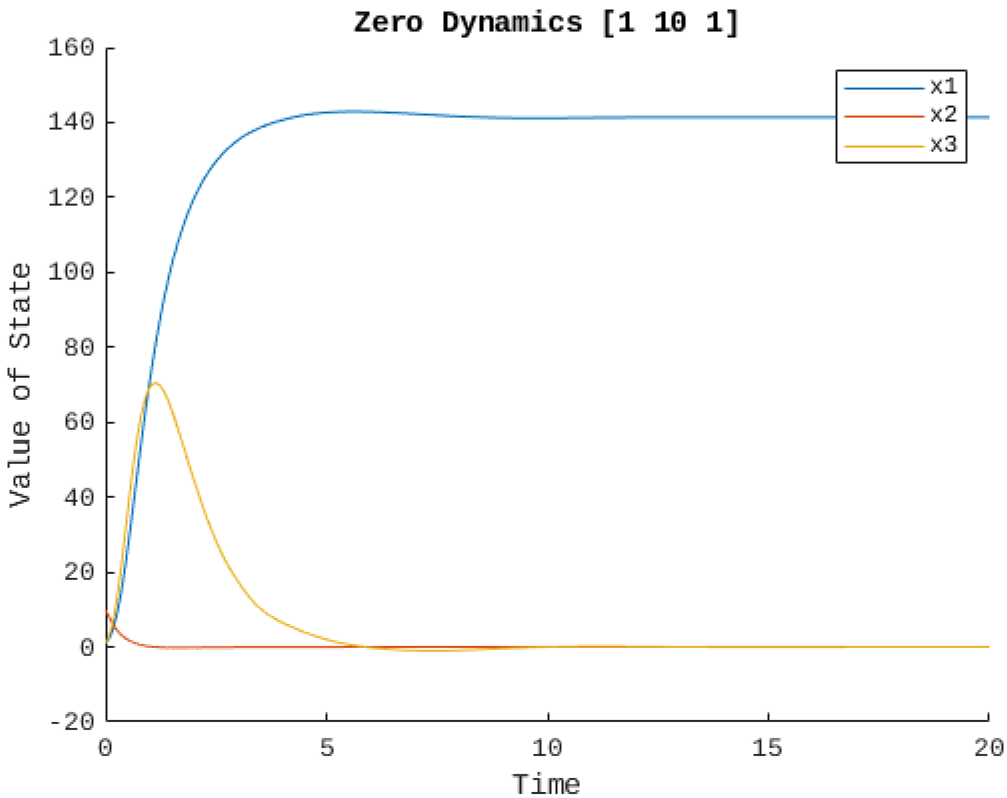
```
xlabel("Time")
ylabel("Value of State")
legend("x1", "x2", "x3")
```

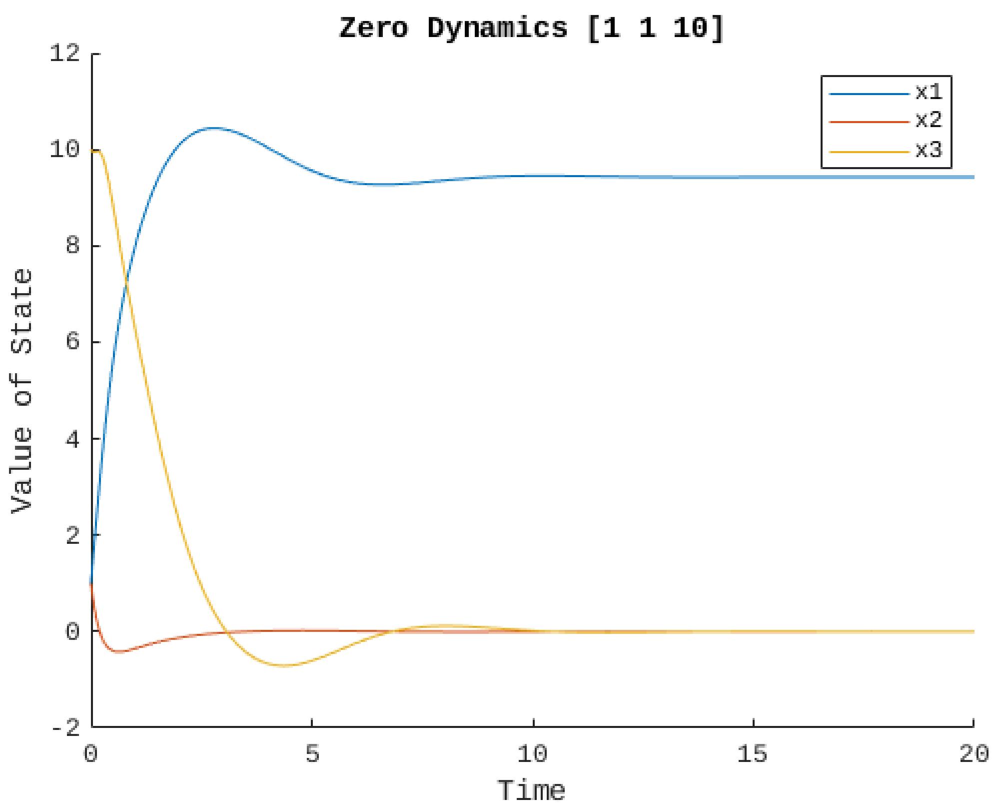
Problem 1
For the zero dynamics, x2 and x3 return to zero but x1 just steadies out at whatever value it is at when the x2 and x3 values steady out
But the control is for designed for x1 so with the full closed loop x2 and x3 go to zero and x1 goes where commanded

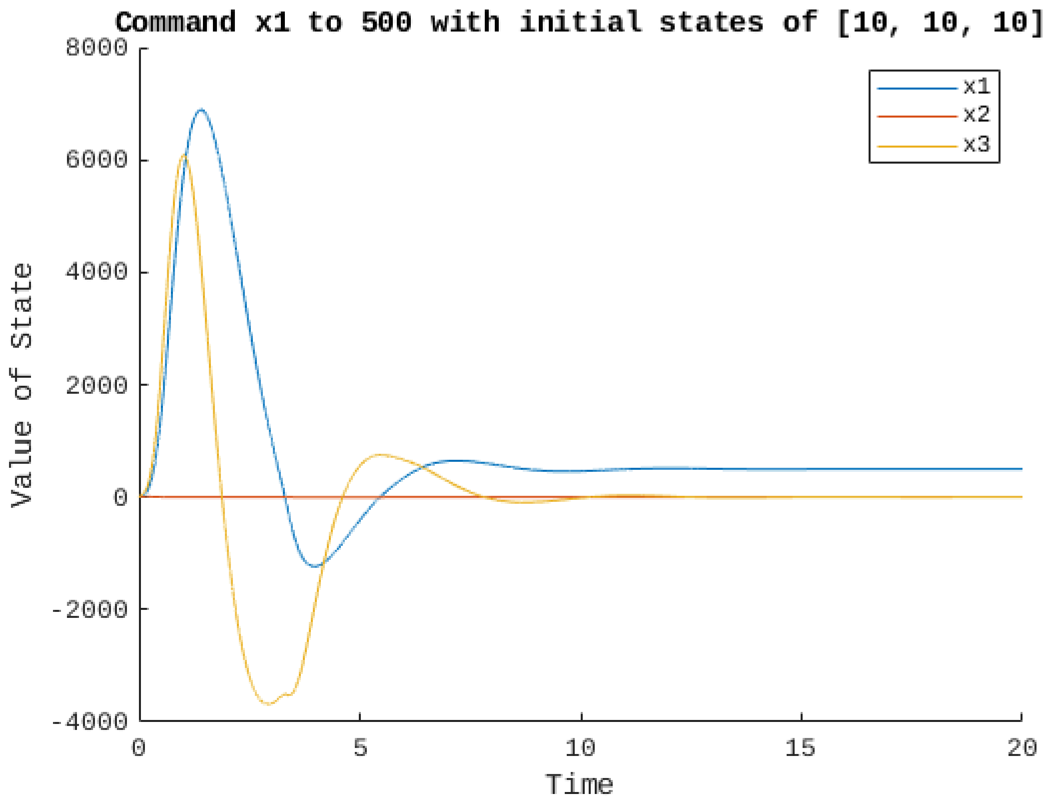


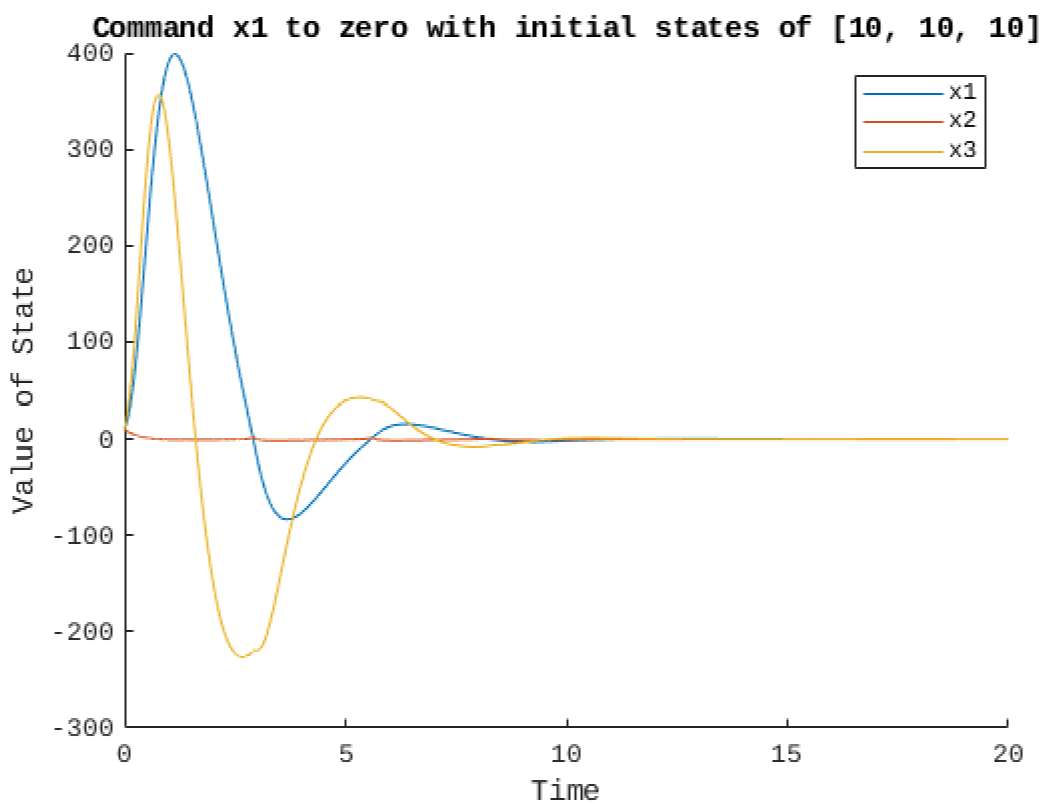

Zero Dynamics [10 10 10]
1400
1200
1000
800
600
400
200
0
-200
Value of State
0
5
10
15
20
Time
x1
x2
x3


Zero Dynamics [10 1 1]
25
20
15
10
5
0
-5
Value of State
0
5
10
15
20
Time
x1
x2
x3


Zero Dynamics [1 10 1]
160
140
120
100
80
60
40
20
0
-20
0
5
10
15
20
Time
Value of State
x1
x2
x3


Zero Dynamics [1 1 10]
12
10
8
6
4
2
0
-2
0
5
10
15
20
Time
Value of State
x1
x2
x3


Command x1 to 500 with initial states of [10, 10, 10]
8000
6000
4000
2000
0
-2000
-4000
Value of State
0
5
10
15
20
Time
x1
x2
x3

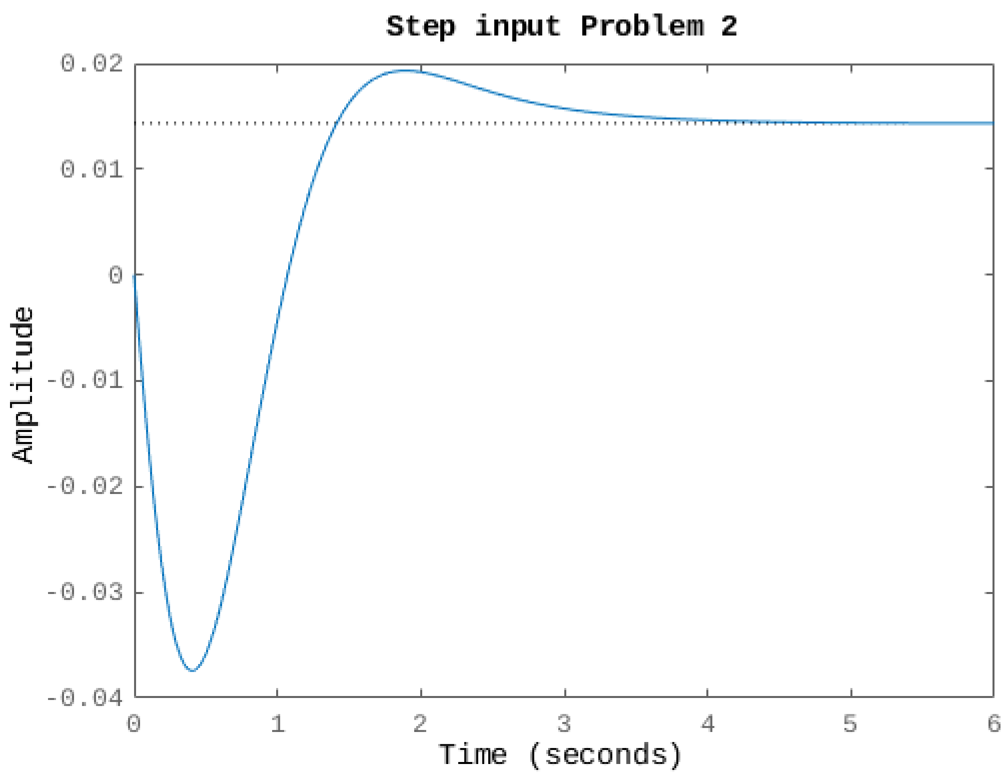

Command x1 to zero with initial states of [10, 10, 10]
400
300
200
100
0
-100
-200
-300
Value of State
0
5
10
15
20
Time
x1
x2
x3

Problem 2

```
A = [-1.01887 0.90506;
     0.82225 -1.07741];
B = [-.00215; -.17555];
C = [0 1];
D = 0;
sys0 = ss(A, B, C, D);
fprintf("Problem 2\n")
fprintf("Open Loop Poles")
eig(sys0)

%a
Qn = [4 2;
      2 1];
Rn = .7;
Umat = [B eye(2)];
sys2 = ss(A, Umat, C, D);
fprintf("a. observer gains\n")
[Kest, L, P] = kalman(sys2, Qn, Rn);
disp(L)

%b
n = -3.5*(1/sqrt(2));
w = 3.5*sqrt(1-(1/sqrt(2))^2);
des_poles = [n+i*w; n-i*w];
Kack = acker(A, B, des_poles);
fprintf("b. controller gains by ackermann's formula \n")
disp(Kack)

%c
reg = lqgreg(Kest,Kack);
closed_loop = feedback(sys0, -reg);
fprintf("c. closed loop poles \n")
disp(eig(closed_loop))
figure()
step(closed_loop)
title("Step input Problem 2")
```

```
Problem 2
Open Loop Poles
ans =

   -0.1850
   -1.9113

a. observer gains
    1.8568
    1.2981

b. controller gains by ackermann's formula
  -56.3591  -15.5642
```

```
c. closed loop poles
  -2.4749 + 2.4749i
  -2.4749 - 2.4749i
  -1.6972 + 0.5679i
  -1.6972 - 0.5679i
```



Problem 3

```
clear;
fprintf("Problem 3\n")
fprintf("Linearization about (0,0) and Jacobian calculated on paper\n")
A = [0 1; -1 0];
B = [0; 1];
G = eye(2);
Q = eye(2);
R = 1;
C = [1 0];
D = 0;
Umat = [B G];
sys = ss(A, Umat, C, D);
[Kest, L, P] = kalman(sys, Q, R);
[K,S,clp] = lqr(A,B,Q,R);
fprintf("Augmented Closed loop system is \n")
fprintf("d/dt[x; x_est] = A_aug*[x; x_est] + B_aug*r + G_aug*w - L_aug*nu\n")
fprintf("y = C_aug*[x; x_est] + nu\n")
```

```
fprintf("where \n")
A_aug = [A-B*K, B*K; zeros(2), A-L*C]
B_aug = [B; 0; 0]
G_aug = [G; G]
L_aug = [0; 0; L]
C_aug = [ C 0 0]
```

```
Problem 3
Linearization about (0,0) and Jacobian calculated on paper
Augmented Closed loop system is
d/dt[x; x_est] = A_aug*[x; x_est] + B_aug*r + G_aug*w - L_aug*nu
y = C_aug*[x; x_est] + nu
where

A_aug =

         0    1.0000         0         0
   -1.4142   -1.3522    0.4142    1.3522
         0         0   -1.3522    1.0000
         0         0   -1.4142         0


B_aug =

     0
     1
     0
     0


G_aug =

     1     0
     0     1
     1     0
     0     1


L_aug =

         0
         0
    1.3522
    0.4142


C_aug =

     1     0     0     0
```

Functions

```
function xdot = zerodynamics(t, x)
    x1 = x(1);
    x2 = x(2);
    x3 = x(3);
    f = [x1*x2+x3; -2*x2; sin(x1)+2*x1*x2];
    g = [0; x1; 1];
    G = x1^2+1;
    dhdx = [1 0 0];
    xdot = (eye(3)-g*(1/G)*dhdx)*f;
end

function xdot = prop_to_ref(t, x, r)
    x1 = x(1);
    x2 = x(2);
    x3 = x(3);
    rdd = 0;
    e = r-x1;
    k = 2;
    f = [x1*x2+x3; -2*x2; sin(x1)+2*x1*x2];
    g = [0; x1; 1];
    G = x1^2+1;
    dhdx = [1 0 0];
    xdot = (eye(3)-g*(1/G)*dhdx)*f+g*(1/G)*(rdd + k*e);
end
```

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