

$$5.1.1 - f'(x) = \frac{f(x+h) - f(x)}{h} \quad \text{or} \quad \frac{h}{2} f''(c)$$

$$b. f'(1) = \frac{f(1.1) - f(1)}{.1} = \frac{\ln 1.1 - \ln 1}{.1} = .99503$$

$$e(1) = \frac{h^2}{2} f''(c) = \frac{.01}{2} \cdot f''(c) = .005 f''(c)$$

$$a. f'(1) = \frac{\ln 1.01 - \ln 1}{.01} = .99510$$

$$e(1) = \frac{1}{2} f''(c) = .05 \cdot f''(c)$$

$$c. f'(1) = \frac{\ln 1.001 - \ln 1}{.001} = .99950$$

$$e(1) = \frac{.001}{2} f''(c) = .0005 \cdot f''(c)$$

$$5.1.7 - f(x+h) = f(x) + h f'(x) + \frac{h^2}{2} f''(c)$$

$$h f'(x) = f(x) - f(x-h) + \frac{h^2}{2} f''(c)$$

$$f'(x) = \frac{f(x) - f(x-h)}{h} + \frac{h}{2} f''(c)$$

$$f'(x) \approx \frac{f(x) - f(x-h)}{h}$$

$$e(x) = \frac{h}{2} f''(c)$$

$$5.2.1. a. \int_0^1 x^2 dx = \frac{h}{2} (y_0 + y_n + 2 \sum_{i=1}^{m-1} y_i) \quad h = (b-a)/m = (1-0)/1 = 1$$

$$m=2 \quad h = (1-0)/2 = \frac{1}{2} \quad \int_0^1 x^2 dx = \frac{1}{4} (0^2 + 1^2 + 2 \cdot \sum_{i=1}^1 (\frac{1}{2})^2) = \frac{1}{4} (1 + 2 \cdot \frac{1}{4}) = \frac{1}{4} \cdot \frac{3}{2} = \frac{3}{8}$$

$$m=4 \quad h = (1-0)/4 = \frac{1}{4} \quad \int_0^1 x^2 dx = \frac{1}{8} (0^2 + 1^2 + 2 \cdot (\frac{1}{4})^2 + 2 \cdot (\frac{3}{4})^2) = \frac{1}{8} (1 + \frac{1}{8} + \frac{9}{8} + \frac{9}{8}) = \frac{1}{8} (\frac{22}{8}) = \frac{22}{64} = \frac{11}{32}$$

$$\text{actual } \int_0^1 x^2 dx = \frac{1}{3} x^3 \Big|_0^1 = \frac{1}{3} \cdot 1 - \frac{1}{3} \cdot 0 = \frac{1}{3}$$

$$\text{error } (m=4) = \left| \frac{1}{3} - \frac{11}{32} \right| = \frac{1}{96} = .0104$$

$$\text{error } (m=2) = \left| \frac{1}{3} - \frac{3}{8} \right| = .0417$$

$$\text{error } (m=1) = \left| \frac{1}{3} - \frac{1}{2} \right| = .1667$$

$$b. m=1 \quad h = (\frac{\pi}{2} - 0)/1 = \frac{\pi}{2} \quad \int_0^{\pi/2} \cos x dx = \frac{\pi}{4} (\cos 0 + \cos \frac{\pi}{2} + 0) = \frac{\pi}{4} = .7854$$

$$m=2 \quad h = (\frac{\pi}{2} - 0)/2 = \frac{\pi}{4} \quad \int_0^{\pi/2} \cos x dx = \frac{\pi}{8} (\cos 0 + \cos \frac{\pi}{2} + 2 \cdot \cos \frac{\pi}{4}) = .9481$$

$$m=4 \quad h = (\frac{\pi}{2} - 0)/4 = \frac{\pi}{8} \quad \int_0^{\pi/2} \cos x dx = \frac{\pi}{16} (\cos 0 + \cos \frac{\pi}{2} + 2 \cdot \cos \frac{\pi}{8} + 2 \cdot \cos \frac{3\pi}{8}) = .9871$$

$$\text{actual } \int_0^{\pi/2} \cos x dx = \sin x \Big|_0^{\pi/2} = \sin \frac{\pi}{2} - \sin 0 = 1$$

$$\text{error } (m=4) = |1 - .9871| = .0129$$

$$\text{error } (m=2) = |1 - .9481| = .0519$$

$$\text{error } (m=1) = |1 - .7854| = .2146$$