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Assignment 1

Aero 707 Liam Hood

```
clear; close all; clc
global r2d
r2d = 180/pi;
```

Problem 2

```
problem_2()
```

Problem 3

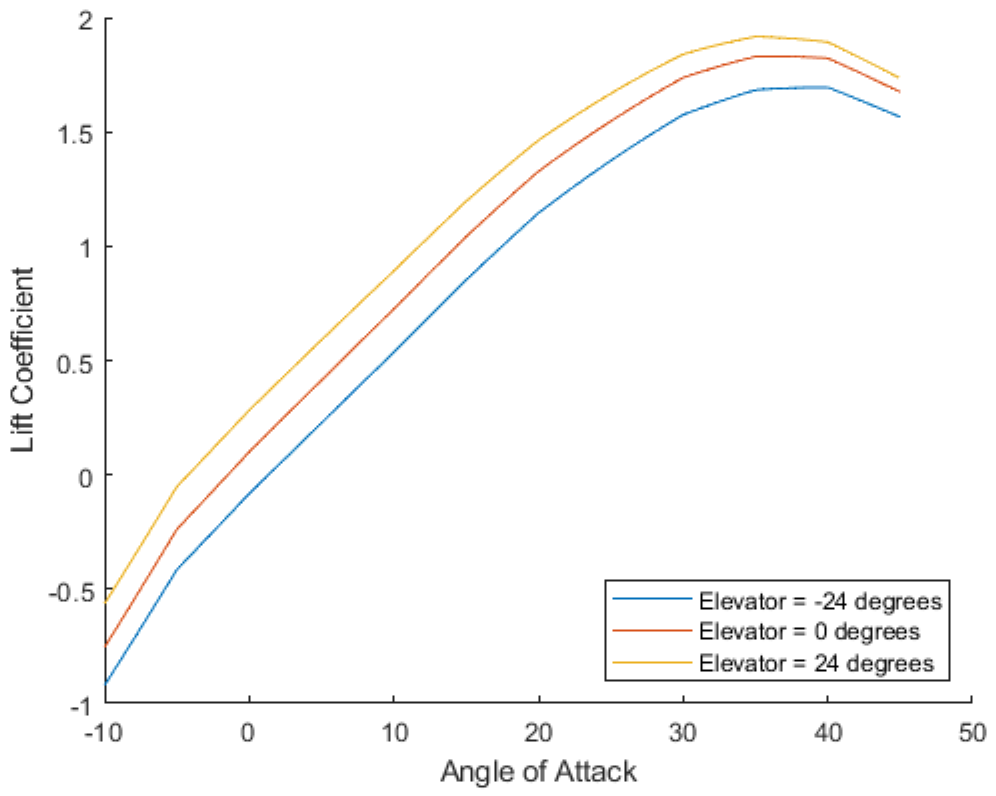
```
problem_3
```

```
******Problem 3******
No idea why this output doesn't publish but it runs
Needs this line to publish and it needs to be this long
What is this text doing??
i. Gust of 20 ft/s left to right (+velocity in y body fixed)
Don't know why this needs to be here
but it won't publish without these strings
 Angle of attack is now 8.000000 degrees
 Angle of sideslip is now -2.710123 degrees
ii. Gust of 50 ft/s from dead astern(+velocity in x body fixed)
 Angle of attack is now 7.271964 degrees
 Angle of sideslip is now -4.549540 degrees
iii. Gust of 30 ft/s from right and below (-velocity in y, -velocity in z
 Angle of attack is now 4.766735 degrees
 Angle of sideslip is now -6.207827 degrees
```

Problem 4

```
problem_4
```

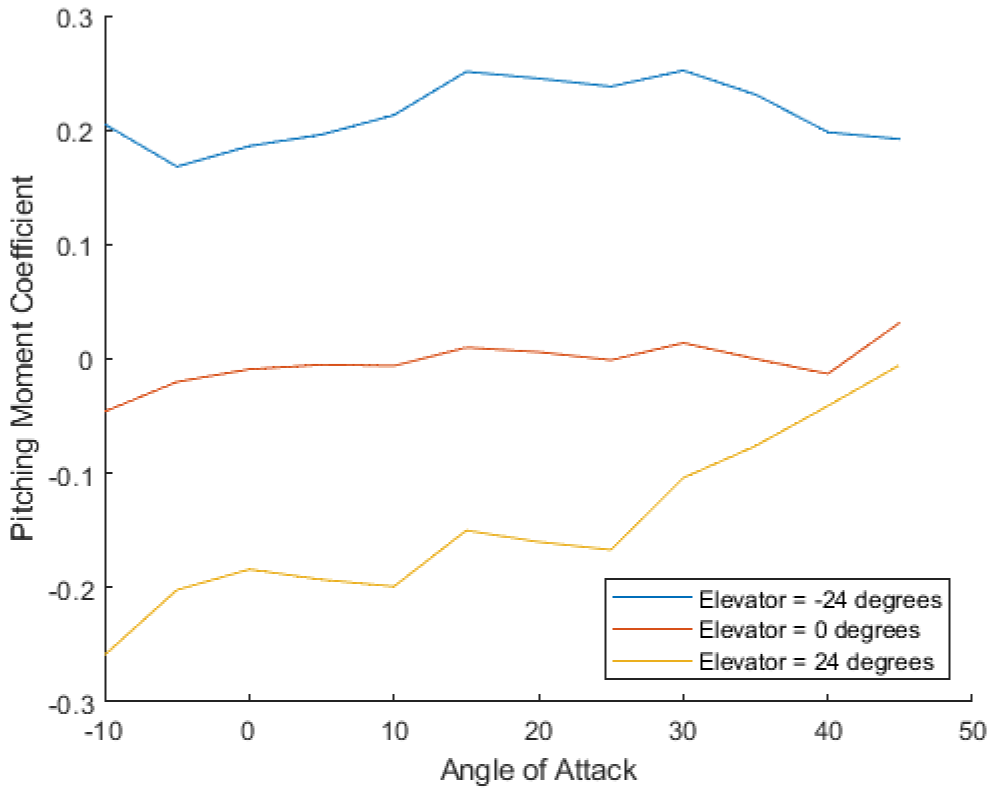
```
******Problem 4******
Max lift occurs at an angle of attack of 35.000000 degrees
```



Problem 5

```
problem_5
```

```
******Problem 5******
We can see that the F16 lacks pitch stiffness as the curves all
vary from flat to a slight slope up. The elevator does significantly
shift the whole curve showing it has significant control power
```



Problem 6

```
problem_6
```

```
******Problem 6******
I did not see a what pitch or flight path was used for Table 3.6-1
so I adjusted pitch to match initial cost in the table
TAS: 170.000000 ft/s  AOA: 0.385718 radians  Pitch: 0.217817 radians
Pitch Rate: 0.000000 radians/s  Altitude: 0.000000 ft  Distance 0.000000 ft
 Sum of Weighted Squares: 28.937859
TAS: 500.000000 ft/s  AOA: 0.010123 radians  Pitch: 0.068749 radians
Pitch Rate: 0.000000 radians/s  Altitude: 0.000000 ft  Distance 0.000000 ft
 Sum of Weighted Squares: 3.542947
TAS: 500.000000 ft/s  AOA: 0.094771 radians  Pitch: 0.105418 radians
Pitch Rate: 0.000000 radians/s  Altitude: 0.000000 ft  Distance 0.000000 ft
 Sum of Weighted Squares: 10.812971
****Find derivations for state equations*****
```

Problem 7

```
problem_7
```

```
******Problem 7******
Equilibrium Points:
 x1=-2    x2=0
 x1=1 - 3^(1/2)*1i    x2=0
 x1=3^(1/2)*1i + 1    x2=0
 x1=0    x2=-2*2^(1/2)
 x1=0    x2=2*2^(1/2)
Linearize around x1=-2, x2=0 using first order Taylor series
The A matrix is:
[12,  0]
[ 0, -2]

 The eigenvalues of this A matrix are lambda= -2.000000 and 12.000000

Linearize around x1=1-3^(1/2)*i, x2=0 using first order Taylor series
The A matrix is:
[3*(- 1 + 3^(1/2)*1i)^2,              0]
[                     0, 1 - 3^(1/2)*1i]

 The eigenvalues of this A matrix are lambda= 1.000000+-1.732051i and
 -6.000000+-10.392305i

Linearize around x1=1+3^(1/2)*i, x2=0 using first order Taylor series
The A matrix is:
[3*(1 + 3^(1/2)*1i)^2,              0]
[                   0, 1 + 3^(1/2)*1i]

 The eigenvalues of this A matrix are lambda= 1.000000+1.732051i and
 -6.000000+10.392305i

Linearize around x1=0, x2=2*2^(1/2) using first order Taylor series
The A matrix is:
[        0, -4*2^(1/2)]
[2*2^(1/2),          0]

 The eigenvalues of this A matrix are lambda= 0.000000+-4.000000i and
 0.000000+4.000000i

Linearize around x1=0, x2=-2*2^(1/2) using first order Taylor series
The A matrix is:
[         0, 4*2^(1/2)]
[-2*2^(1/2),         0]

 The eigenvalues of this A matrix are lambda= 0.000000+-4.000000i and
 0.000000+4.000000i
```

Problems

```
function problem_2()
    fprintf("******Problem 2******\n")
    disp("No idea why this output doesn't publish but it runs")
    % xdot = A*x + B*u
    A = [8, -2, 3;
         0, 1, 4;
         7, 9, 10];
    B = [2; 1; 5];
    % y = C*x+D*u
    C = [2, 1, 3];
    D = 0;

    % Transfer function G(s)
    fprintf("Transfer Function\n")
    syms s
    G = collect(C*inv((s*eye(3)-A))*B);
    fprintf("From formula\n")
    fprintf("G(s) = %s\n", G)
    fprintf("Built in Function\n")
    [b,a] = ss2tf(A,B,C,D);
    G_builtin = collect((b(1)*s^3 + b(2)*s^2 + b(3)*s + b(4))/...
        (a(1)*s^3 + a(2)*s^2 + a(3)*s + a(4)));
    fprintf("G(s) = %s\n", G_builtin)
    % Poles
    fprintf("\nPoles\n")
    poles_builtin = eig(A);
    poles = vpasolve(det(s*eye(3)-A)==0,s);
    fprintf("From formula\n")
    fprintf("Poles %f, %f, %f \n", poles)
    fprintf("Built in Function\n")
    fprintf("Poles %f, %f, %f \n", poles_builtin)
end

function problem_3()
    global r2d
    fprintf("\n******Problem 3******\n")
    disp("No idea why this output doesn't publish but it runs")
    disp("Needs this line to publish and it needs to be this long")
    % 2.3-1 Find instantaneous angle of attack and angle of side slip
    % in the gusts

    % Given starting conditions
    v = 500;
    alpha = 8/r2d;
    beta = -5/r2d;
    % Rotation matrices
    % stability from body fixed
    C_s_bf = [cos(alpha), 0, sin(alpha);
                0, 1, 0;
                -sin(alpha), 0, cos(alpha)];
    % wind from stability
    C_w_s = [cos(beta), sin(beta), 0;
            -sin(beta), cos(beta), 0;
            0, 0, 1];
    % wind from body fixed
    C_w_bf = C_w_s*C_s_bf;
    % starting V
    vw = [v;0;0];
    vbf = C_w_bf'*vw;
    % i
    fprintf("i. Gust of 20 ft/s left to right (+velocity in y body fixed)\n")
    vbf1 = vbf + [0;20;0];
    disp("Don't know why this needs to be here")
    [alpha1, beta1] = find_alpha_beta(vbf1);
    disp("but it won't publish without these strings")
    fprintf("\tAngle of attack is now %f degrees\n", alpha1)
    fprintf("\tAngle of sideslip is now %f degrees\n", beta1)
    % ii
    fprintf("ii. Gust of 50 ft/s from dead astern(+velocity in x body
 fixed)\n")
    vbf2 = vbf + [50;0;0];
    [alpha2, beta2] = find_alpha_beta(vbf2);
    fprintf("\tAngle of attack is now %f degrees\n", alpha2)
    fprintf("\tAngle of sideslip is now %f degrees\n", beta2)

    % iii
    fprintf("iii. Gust of 30 ft/s from right and below (-velocity in y, -
velocity in z\n")
    vbf3 = vbf + [0;-30*cos(70/r2d);-30*sin(70/r2d)];
    [alpha3, beta3] = find_alpha_beta(vbf3);
    fprintf("\tAngle of attack is now %f degrees\n", alpha3)
    fprintf("\tAngle of sideslip is now %f degrees\n", beta3)

end

function problem_4()
    fprintf("\n******Problem 4******\n")
    % 2.3-4 Find instantaneous angle of attack and angle of side slip
    % in the gusts
    n = 1e2;
    alpha = linspace(-10,45,n);
    elev = [-24,0,24];
    for ii = 1:3
        cl(ii,:) = zeros(1,n);
        for jj = 1:n
            cl(ii,jj) = CX(alpha(jj),elev(ii))*sind(alpha(jj)) - ...
                CZ(alpha(jj),0,elev(ii))*cosd(alpha(jj));
        end
    end
    figure
    hold on
    plot(alpha, cl(1,:))
    plot(alpha, cl(2,:))
    plot(alpha, cl(3,:))
    legend("Elevator = -24 degrees", "Elevator = 0 degrees", ...
        "Elevator = 24 degrees", "Location","southeast")
    xlabel("Angle of Attack")
    ylabel("Lift Coefficient")
    hold off

    [~, I] = max(cl(3,:));
    fprintf("Max lift occurs at an angle of attack of %f degrees\n", alpha(I))
end
```

```
function problem_5()
    fprintf("\n******Problem 5******\n")
    % 2.3-5
    n = 1e2;
    alpha = linspace(-10,45,n);
    elev = [-24,0,24];
    for ii = 1:3
        cm(ii,:) = zeros(1,n);
        for jj = 1:n
            cm(ii,jj) = CM(alpha(jj),elev(ii));
        end
    end
    figure
    hold on
    plot(alpha, cm(1,:))
    plot(alpha, cm(2,:))
    plot(alpha, cm(3,:))
    legend("Elevator = -24 degrees", "Elevator = 0 degrees", ...
        "Elevator = 24 degrees", "Location","southeast")
    xlabel("Angle of Attack")
    ylabel("Pitching Moment Coefficient")
    hold off
    fprintf("We can see that the F16 lacks pitch stiffness as the curves all
\n")
    fprintf("vary from flat to a slight slope up. The elevator does
 significantly \n")
    fprintf("shift the whole curve showing it has significant control power
\n")
end

function problem_6()
    fprintf("\n******Problem 6******\n")
    global r2d
    % 3.5-1
    fprintf("I did not see a what pitch or flight path was used for Table
 3.6-1\n")
    fprintf("so I adjusted pitch to match initial cost in the table\n")
    x1 = [170; 22.1/r2d;  12.48/r2d; 0; 0; 0];
    u1 = [.297, -25.7, .25, 0];
    [xd1] = transp(x1,u1);
    sos1 = xd1(1)^2 + 100*xd1(2)^2 + 10*xd1(4)^4;
    fprintf("TAS: %f ft/s  AOA: %f radians  Pitch: %f radians  \n", x1(1:3))
    fprintf("Pitch Rate: %f radians/s  Altitude: %f ft  Distance %f ft\n",
 x1(4:6))
    fprintf("\tSum of Weighted Squares: %f\n", sos1)

    x2 = [500; .58/r2d; 3.939/r2d; 0; 0; 0];
    u2 = [.293, 2.46, .25, 0];
    [xd2] = transp(x2,u2);
    sos2 = xd2(1)^2 + 100*xd2(2)^2 + 10*xd2(4)^4;
    fprintf("TAS: %f ft/s  AOA: %f radians  Pitch: %f radians  \n", x2(1:3))
    fprintf("Pitch Rate: %f radians/s  Altitude: %f ft  Distance %f ft\n",
 x2(4:6))
```

```
    fprintf("\tSum of Weighted Squares: %f\n", sos2)

    x3 = [500; 5.43/r2d; 6.04/r2d; 0; 0; 0];
    u3 = [.204, -4.10, .25, 0];
    [xd3] = transp(x3,u3);
    sos3 = xd3(1)^2 + 100*xd3(2)^2 + 10*xd3(4)^4;
    fprintf("TAS: %f ft/s  AOA: %f radians  Pitch: %f radians  \n", x3(1:3))
    fprintf("Pitch Rate: %f radians/s  Altitude: %f ft  Distance %f ft\n",
 x3(4:6))
    fprintf("\tSum of Weighted Squares: %f\n", sos3)

    fprintf("****Find derivations for state equations*****\n")

end

function problem_7()
    fprintf("\n******Problem 7******\n")
    global r2d
    % 3.7-1
    syms x1 x2
    x1dot = x1^3-x2^2+8;
    x2dot = x1*x2;
    sol = solve(x1dot==0,x2dot==0,x1,x2);
    fprintf("Equilibrium Points:\n")
    for ii = 1:5
        fprintf("\tx1=%s    x2=%s\n",[sol.x1(ii),sol.x2(ii)])
    end
    fprintf("Linearize around x1=-2, x2=0 using first order Taylor series\n")
    A = subs([diff(x1dot,x1), diff(x1dot,x2); diff(x2dot,x1), diff(x2dot,x2)],
[x1,x2],[-2,0]);
    fprintf("The A matrix is:\n")
    disp(A)
    [~,D] = eig(A);
    fprintf("\tThe eigenvalues of this A matrix are lambda= %f and %f\n\n",
[D(1,1),D(2,2)])

    fprintf("Linearize around x1=1-3^(1/2)*i, x2=0 using first order Taylor
 series\n")
    A = subs([diff(x1dot,x1), diff(x1dot,x2); diff(x2dot,x1), diff(x2dot,x2)],
[x1,x2],[1-3^(1/2)*i,0]);
    fprintf("The A matrix is:\n")
    disp(A)
    [~,D] = eig(A);
    fprintf("\tThe eigenvalues of this A matrix are lambda= %f+%fi and %f+%fi
\n\n",[real(D(1,1)),imag(D(1,1)),real(D(2,2)),imag(D(2,2))])

    fprintf("Linearize around x1=1+3^(1/2)*i, x2=0 using first order Taylor
 series\n")
    A = subs([diff(x1dot,x1), diff(x1dot,x2); diff(x2dot,x1), diff(x2dot,x2)],
[x1,x2],[1+3^(1/2)*i,0]);
    fprintf("The A matrix is:\n")
    disp(A)
    [~,D] = eig(A);
    fprintf("\tThe eigenvalues of this A matrix are lambda= %f+%fi and %f+%fi
\n\n",[real(D(1,1)),imag(D(1,1)),real(D(2,2)),imag(D(2,2))])

    fprintf("Linearize around x1=0, x2=2*2^(1/2) using first order Taylor
 series\n")
    A = subs([diff(x1dot,x1), diff(x1dot,x2); diff(x2dot,x1), diff(x2dot,x2)],
[x1,x2],[0,2*2^(1/2)]);
    fprintf("The A matrix is:\n")
    disp(A)
    [~,D] = eig(A);
    fprintf("\tThe eigenvalues of this A matrix are lambda= %f+%fi and %f+%fi
\n\n",[real(D(1,1)),imag(D(1,1)),real(D(2,2)),imag(D(2,2))])

    fprintf("Linearize around x1=0, x2=-2*2^(1/2) using first order Taylor
 series\n")
    A = subs([diff(x1dot,x1), diff(x1dot,x2); diff(x2dot,x1), diff(x2dot,x2)],
[x1,x2],[0,-2*2^(1/2)]);
    fprintf("The A matrix is:\n")
    disp(A)
    [~,D] = eig(A);
    fprintf("\tThe eigenvalues of this A matrix are lambda= %f+%fi and %f+%fi
\n\n",[real(D(1,1)),imag(D(1,1)),real(D(2,2)),imag(D(2,2))])

end
```

```
******Problem 2******
No idea why this output doesn't publish but it runs
Transfer Function
From formula
G(s) = (20*s^2 - 82*s - 345)/(s^3 - 19*s^2 + 41*s + 285)
Built in Function
G(s) = (20*s^2 - 82*s - 345)/(s^3 - 19*s^2 + 41*s + 285)

Poles
From formula
Poles -2.795832, 6.795832, 15.000000
Built in Function
Poles 15.000000, 6.795832, -2.795832
```

Functions

```
function [alpha, beta] = find_alpha_beta(vbf)
    global r2d
    syms alphas betas V
    C_s_bf = [cos(alphas), 0, sin(alphas);
                0, 1, 0;
                -sin(alphas), 0, cos(alphas)];
    % wind from stability
    C_w_s = [cos(betas), sin(betas), 0;
            -sin(betas), cos(betas), 0;
            0, 0, 1];
    % wind from body fixed
    C_w_bf = C_w_s*C_s_bf;
    sol = vpasolve([V;0;0]==C_w_bf*vbf,[alphas,betas,V]);
    alpha = sol.alphas*r2d;
    beta = sol.betas*r2d;
end

function [CXI]=CX(alpha,elev)
    % Data and interpolation for F-16 Axial force coefficient.
    alphtabl=[-10,-5,0,5,10,15,20,25,30,35,40,45];
    elevtabl=[-24,-12,0,12,24];
    cxtabl=[-.099, -.081, -.081, -.063, -.025, .044, .097...
    .113, .145, .167, .174, .166;
    -.048, -.038, -.040, -.021, .016, .083, .127...
    .137, .162, .177, .179, .167;
    -.022, -.020, -.021, -.004, .032, .094, .128...
    .130, .154, .161, .155, .138;
    -.040, -.038, -.039, -.025, .006, .062, .087...
    .085, .100, .110, .104, .091;
    -.083, -.073, -.076, -.072, -.046, .012, .024...
    .025, .043, .053, .047, .040];
    CXI= interp2(alphtabl,elevtabl,cxtabl,alpha,elev);
end

function [CZI]=CZ(alpha,beta,elev)
    % Data and interpolation for F-16 z-force coefficient
    alphtabl=[-10,-5,0,5,10,15,20,25,30,35,40,45];
    cztabl=[.770, .241, -.100, -.416, -.731, -1.053, -1.366...
    -1.646, -1.917, -2.120, -2.248, -2.229];
    C1= interp1(alphtabl,cztabl,alpha);
    CZI= C1*(1-(beta/57.3)^2) - .19*(elev/25.0);
end

function [CMI]=CM(alpha,elev)
    % F-16 model pitching-moment data and interpolation
    alphtabl=[-10,-5,0,5,10,15,20,25,30,35,40,45];
    elevtabl=[-24,-12,0,12,24];
    cmtabl=[.205, .168, .186, .196, .213, .251, .245...
    .238, .252, .231, .198, .192;
    .081, .077, .107, .110, .110, .141, .127...
    .119, .133, .108, .081, .093;
    -.046, -.020, -.009, -.005, -.006, .010, .006...
    -.001, .014, .000, -.013, .032;
    -.174, -.145, -.121, -.127, -.129, -.102, -.097...
    -.113, -.087, -.084, -.069, -.006;
    -.259, -.202, -.184, -.193, -.199, -.150, -.160...
    -.167, -.104, -.076, -.041, -.005];
    CMI = interp2(alphtabl,elevtabl,cmtabl,alpha,elev);
end

function [xd] = transp(x,u)
    % Medium-sized transport aircraft, longitudinal dynamics
    S = 2170.0; CBAR = 17.5; MASS = 5e3; IYY = 4.1e6;
    TSTAT = 6e4; DTDV = -38; ZE = 2; CDCLS = .042;
    CLA = .085; CMA = -.022; CMDE = -.016; % per degree
    CMQ = -16; CMADOT = -6; CLADOT = 0; % per radian
    RTOD = 57.29578; GD = 32.17;
    THTL =  u(1);
    ELEV =  u(2);
    XCG =   u(3);
    LAND =  u(4);
    VT =    x(1);        % TAS in fps
    ALPHA = RTOD*x(2);  % A. O. A.
    THETA = x(3);        % PITCH ATTITUDE
    Q =     x(4);        % PITCH RATE
    H =     x(5);        % ALTITUDE
    [~,QBAR] = ADC(VT,H);
    QS = QBAR*S;
    SALP = sin(x(2)); CALP = cos(x(2));
    GAM = THETA - x(2); SGAM = sin(GAM); CGAM = cos(GAM);
    if LAND == 0         % clean
        CLO = .2; CDO = .016;
        CMO = .05; DCDG = 0; DCMG = 0;
    elseif LAND == 1    % landing flap and gear
        CLO = 1; CDO = .08;
        CMO = -.2; DCDG = .02; DCMG = -.05;
    else
        disp("Landing Gear & Flaps")
    end
    THR = (TSTAT+VT*DTDV) * max(THTL,0);             % THR
    CL = CLO+CLA*ALPHA;                              % NONDIM LIFT
    CM = DCMG+CMO+CMA*ALPHA+CMDE*ELEV+CL*(XCG-.25); % MOMENT
    CD = DCDG+CDO+CDCLS*CL*CL;                       % DRAG POLAR

    % State Equations
    xd(1) = (THR*CALP-QS*CD)/MASS - GD*SGAM;
    xd(2) = (-THR*SALP-QS*CL+MASS*(VT*Q+GD*CGAM))/(MASS*VT+QS*CLADOT);
    xd(3) = Q;
    D = .5*CBAR*(CMQ*Q+CMADOT*xd(2))/VT;
    xd(4) = (QS*CBAR*(CM+D)+THR*ZE)/IYY;
    xd(5) = VT*SGAM;
    xd(6) = VT*CGAM;
end

function [MACH,QBAR] = ADC(VT,H)
    R0 = 2.377e-3;
    TFAC = 1-.703e-5*H;
    T = 519*TFAC;
    if H >= 35000
        T = 390;
    end
    RHO = R0*(TFAC^4.14);
    MACH = VT/sqrt(1.4*1716.3*T);
    QBAR = .5*RHO*VT*VT;
end
```

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