
Table of Contents

Problem 1	1
Problem 2	2
Problem 3	2
Problem 4	4
Problem 5	5
Problem 6	9

Problem 1

```
clear; close all; clc;
syms x1 x h del1 del2 del3 del4 del5 del6 phi;
xmatrix = [ 1, x1, x1^2, x1^3, x1^4, x1^5;
            0, 1, 2*x1, 3*x1^2, 4*x1^3, 5*x1^4;
            0, 0, 2, 6*x1, 12*x1^2, 20*x1^3;
            1, x1+h, (x1+h)^2, (x1+h)^3, (x1+h)^4, (x1+h)^5;
            0, 1, 2*(x1+h), 3*(x1+h)^2, 4*(x1+h)^3, 5*(x1+h)^4;
            0, 0, 2, 6*(x1+h), 12*(x1+h)^2, 20*(x1+h)^3];
delvec = [del1;del2;del3;del4;del5;del6];
cvec = inv(xmatrix)*delvec;
wcluttered = cvec(1) + cvec(2)*x + cvec(3)*x^2 + cvec(4)*x^3 +
    cvec(5)*x^4 + cvec(6)*x^5;
w = collect(wcluttered, [del1, del2, del3, del4, del5, del6]);
phi1 = simplify(solve( del1*phi == subs(w,[del2, del3, del4, del5,
    del6, x1], [0 0 0 0 0 0]),phi));
phi2 = simplify(solve( del2*phi == subs(w,[del1, del3, del4, del5,
    del6, x1], [0 0 0 0 0 0]),phi));
phi3 = simplify(solve( del3*phi == subs(w,[del1, del2, del4, del5,
    del6, x1], [0 0 0 0 0 0]),phi));
phi4 = simplify(solve( del4*phi == subs(w,[del1, del2, del3, del5,
    del6, x1], [0 0 0 0 0 0]),phi));
phi5 = simplify(solve( del5*phi == subs(w,[del1, del2, del3, del4,
    del6, x1], [0 0 0 0 0 0]),phi));
phi6 = simplify(solve( del6*phi == subs(w,[del1, del2, del3, del4,
    del5, x1], [0 0 0 0 0 0]),phi));
fprintf("Problem 1 \n")
fprintf("phi1 = %s \n",phi1)
fprintf("phi2 = %s \n",phi2)
fprintf("phi3 = %s \n",phi3)
fprintf("phi4 = %s \n",phi4)
fprintf("phi5 = %s \n",phi5)
fprintf("phi6 = %s \n",phi6)
fprintf("phi1, phi2, phi3 are for node 1\n")

Problem 1
phi1 = ((h - x)^3*(3*h*x + h^2 + 6*x^2))/h^5
phi2 = (x*(h - x)^3*(h + 3*x))/h^4
phi3 = (x^2*(h - x)^3)/(2*h^3)
phi4 = (x^3*(10*h^2 - 15*h*x + 6*x^2))/h^5
phi5 = -(x^3*(4*h^2 - 7*h*x + 3*x^2))/h^4
phi6 = (x^3*(h - x)^2)/(2*h^3)
```

phi1, phi2, phi3 are for node 1

Problem 2

```
clear;
syms x1 x h del1 del2 del3 del4 del5 del6 phi;
xmatrix = [ 1, x1, x1^2, x1^3, x1^4, x1^5;
            0, -1, -2*x1, -3*x1^2, -4*x1^3, -5*x1^4;
            1, (x1+h/2), (x1+h/2)^2, (x1+h/2)^3, (x1+h/2)^4,
            (x1+h/2)^5;
            0, -1, -2*(x1+h/2), -3*(x1+h/2)^2, -4*(x1+h/2)^3,
            -5*(x1+h/2)^4;
            1, (x1+h), (x1+h)^2, (x1+h)^3, (x1+h)^4, (x1+h)^5;
            0, -1, -2*(x1+h), -3*(x1+h)^2, -4*(x1+h)^3, -5*(x1+h)^4];
delvec = [del1;del2;del3;del4;del5;del6];
cvec = inv(xmatrix)*delvec;
wcluttered = cvec(1) + cvec(2)*x + cvec(3)*x^2 + cvec(4)*x^3 +
            cvec(5)*x^4 + cvec(6)*x^5;
w = collect(wcluttered, [del1, del2, del3, del4, del5, del6]);
phi1 = simplify(solve( del1*phi == subs(w,[del2, del3, del4, del5,
            del6, x1], [0 0 0 0 0 0]),phi));
phi2 = simplify(solve( del2*phi == subs(w,[del1, del3, del4, del5,
            del6, x1], [0 0 0 0 0 0]),phi));
phi3 = simplify(solve( del3*phi == subs(w,[del1, del2, del4, del5,
            del6, x1], [0 0 0 0 0 0]),phi));
phi4 = simplify(solve( del4*phi == subs(w,[del1, del2, del3, del5,
            del6, x1], [0 0 0 0 0 0]),phi));
phi5 = simplify(solve( del5*phi == subs(w,[del1, del2, del3, del4,
            del6, x1], [0 0 0 0 0 0]),phi));
phi6 = simplify(solve( del6*phi == subs(w,[del1, del2, del3, del4,
            del5, x1], [0 0 0 0 0 0]),phi));
fprintf("\nProblem 2 \n")
fprintf("phi1 = %s \n",phi1)
fprintf("phi2 = %s \n",phi2)
fprintf("phi3 = %s \n",phi3)
fprintf("phi4 = %s \n",phi4)
fprintf("phi5 = %s \n",phi5)
fprintf("phi6 = %s \n",phi6)
fprintf("phi1, phi2 are for node 1\n")
```

Problem 2

```
phi1 = ((h + 6*x)*(h^2 - 3*h*x + 2*x^2)^2)/h^5
phi2 = -(x*(h^2 - 3*h*x + 2*x^2)^2)/h^4
phi3 = (16*x^2*(h - x)^2)/h^4
phi4 = (8*x^2*(h - x)^2*(h - 2*x))/h^4
phi5 = (x^2*(h - 2*x)^2*(7*h - 6*x))/h^5
phi6 = (x^2*(h - x)*(h - 2*x)^2)/h^4
phi1, phi2 are for node 1
```

Problem 3

```
clear;
```

```

syms x1 x h del1 del2 del3 del4 del5 del6 phi q1 q2;
xmatrix = [ 1, x1, x1^2, x1^3, x1^4, x1^5;
            0, 1, 2*x1, 3*x1^2, 4*x1^3, 5*x1^4;
            0, 0, 2, 6*x1, 12*x1^2, 20*x1^3;
            1, x1+h, (x1+h)^2, (x1+h)^3, (x1+h)^4, (x1+h)^5;
            0, 1, 2*(x1+h), 3*(x1+h)^2, 4*(x1+h)^3, 5*(x1+h)^4;
            0, 0, 2, 6*(x1+h), 12*(x1+h)^2, 20*(x1+h)^3];
delvec = [del1;del2;del3;del4;del5;del6];
cvec = inv(xmatrix)*delvec;
wcluttered = cvec(1) + cvec(2)*x + cvec(3)*x^2 + cvec(4)*x^3 +
            cvec(5)*x^4 + cvec(6)*x^5;
w = collect(wcluttered, [del1, del2, del3, del4, del5, del6]);
phi1 = simplify(solve( del1*phi == subs(w,[del2, del3, del4, del5,
            del6, x1], [0 0 0 0 0 0]),phi));
phi2 = simplify(solve( del2*phi == subs(w,[del1, del3, del4, del5,
            del6, x1], [0 0 0 0 0 0]),phi));
phi3 = simplify(solve( del3*phi == subs(w,[del1, del2, del4, del5,
            del6, x1], [0 0 0 0 0 0]),phi));
phi4 = simplify(solve( del4*phi == subs(w,[del1, del2, del3, del5,
            del6, x1], [0 0 0 0 0 0]),phi));
phi5 = simplify(solve( del5*phi == subs(w,[del1, del2, del3, del4,
            del6, x1], [0 0 0 0 0 0]),phi));
phi6 = simplify(solve( del6*phi == subs(w,[del1, del2, del3, del4,
            del5, x1], [0 0 0 0 0 0]),phi));
fprintf("\nProblem 3 \n")
fprintf("phi1 = %s \n",phi1)
fprintf("phi2 = %s \n",phi2)
fprintf("phi3 = %s \n",phi3)
fprintf("phi4 = %s \n",phi4)
fprintf("phi5 = %s \n",phi5)
fprintf("phi6 = %s \n",phi6)
q = [ int(((q1+x*(q2-q1)/h)*phi1),x,0,h);
      int(((q1+x*(q2-q1)/h)*phi2),x,0,h);
      int(((q1+x*(q2-q1)/h)*phi3),x,0,h);
      int(((q1+x*(q2-q1)/h)*phi4),x,0,h);
      int(((q1+x*(q2-q1)/h)*phi5),x,0,h);
      int(((q1+x*(q2-q1)/h)*phi6),x,0,h)];
for ii = 1:6
    fprintf("q_%i = %s \n",ii, q(ii))
end
fprintf("q_1, q_2, q_3 are the components of the source vector for
        node 1\n")

```

Problem 3

```

phi1 = ((h - x)^3*(3*h*x + h^2 + 6*x^2))/h^5
phi2 = (x*(h - x)^3*(h + 3*x))/h^4
phi3 = (x^2*(h - x)^3)/(2*h^3)
phi4 = (x^3*(10*h^2 - 15*h*x + 6*x^2))/h^5
phi5 = -(x^3*(4*h^2 - 7*h*x + 3*x^2))/h^4
phi6 = (x^3*(h - x)^2)/(2*h^3)
q_1 = (h*(5*q1 + 2*q2))/14
q_2 = (h^2*(13*q1 + 8*q2))/210
q_3 = (h^3*(4*q1 + 3*q2))/840

```

```

q_4 = (h*(2*q1 + 5*q2))/14
q_5 = -(h^2*(8*q1 + 13*q2))/210
q_6 = (h^3*(3*q1 + 4*q2))/840
q_1, q_2, q_3 are the components of the source vector for node 1

```

Problem 4

```

clear;
syms x1 x h del1 del2 del3 del4 del5 del6 phi q1 q2;
xmatrix = [ 1, x1, x1^2, x1^3, x1^4, x1^5;
            0, -1, -2*x1, -3*x1^2, -4*x1^3, -5*x1^4;
            1, (x1+h/2), (x1+h/2)^2, (x1+h/2)^3, (x1+h/2)^4,
            (x1+h/2)^5;
            0, -1, -2*(x1+h/2), -3*(x1+h/2)^2, -4*(x1+h/2)^3,
            -5*(x1+h/2)^4;
            1, (x1+h), (x1+h)^2, (x1+h)^3, (x1+h)^4, (x1+h)^5;
            0, -1, -2*(x1+h), -3*(x1+h)^2, -4*(x1+h)^3, -5*(x1+h)^4];
delvec = [del1;del2;del3;del4;del5;del6];
cvec = inv(xmatrix)*delvec;
wcluttered = cvec(1) + cvec(2)*x + cvec(3)*x^2 + cvec(4)*x^3 +
            cvec(5)*x^4 + cvec(6)*x^5;
w = collect(wcluttered, [del1, del2, del3, del4, del5, del6]);
phi1 = simplify(solve( del1*phi == subs(w,[del2, del3, del4, del5,
            del6, x1], [0 0 0 0 0 0]),phi));
phi2 = simplify(solve( del2*phi == subs(w,[del1, del3, del4, del5,
            del6, x1], [0 0 0 0 0 0]),phi));
phi3 = simplify(solve( del3*phi == subs(w,[del1, del2, del4, del5,
            del6, x1], [0 0 0 0 0 0]),phi));
phi4 = simplify(solve( del4*phi == subs(w,[del1, del2, del3, del5,
            del6, x1], [0 0 0 0 0 0]),phi));
phi5 = simplify(solve( del5*phi == subs(w,[del1, del2, del3, del4,
            del6, x1], [0 0 0 0 0 0]),phi));
phi6 = simplify(solve( del6*phi == subs(w,[del1, del2, del3, del4,
            del5, x1], [0 0 0 0 0 0]),phi));
fprintf("\nProblem 4 \n")
fprintf("phi1 = %s \n",phi1)
fprintf("phi2 = %s \n",phi2)
fprintf("phi3 = %s \n",phi3)
fprintf("phi4 = %s \n",phi4)
fprintf("phi5 = %s \n",phi5)
fprintf("phi6 = %s \n",phi6)
q = [ int(((q1+x*(q2-q1)/h)*phi1),x,0,h);
      int(((q1+x*(q2-q1)/h)*phi2),x,0,h);
      int(((q1+x*(q2-q1)/h)*phi3),x,0,h);
      int(((q1+x*(q2-q1)/h)*phi4),x,0,h);
      int(((q1+x*(q2-q1)/h)*phi5),x,0,h);
      int(((q1+x*(q2-q1)/h)*phi6),x,0,h)];
for ii = 1:6
    fprintf("q_%i = %s \n",ii, q(ii))
end
fprintf("The source vector components for node 1 are q_1 and q_2\n")
fprintf("The source vector components for node 2 are q_3 and q_4\n")
fprintf("The source vector components for node 1 are q_5 and q_6\n")

```

Problem 4

$\phi_1 = ((h + 6x)(h^2 - 3hx + 2x^2)^2)/h^5$

$\phi_2 = -(x(h^2 - 3hx + 2x^2)^2)/h^4$

$\phi_3 = (16x^2(h - x)^2)/h^4$

$\phi_4 = (8x^2(h - x)^2(h - 2x))/h^4$

$\phi_5 = (x^2(h - 2x)^2(7h - 6x))/h^5$

$\phi_6 = (x^2(h - x)(h - 2x)^2)/h^4$

$q_1 = (h(79q_1 + 19q_2))/420$

$q_2 = -(h^2(5q_1 + 2q_2))/420$

$q_3 = (4h(q_1 + q_2))/15$

$q_4 = (2h^2(q_1 - q_2))/105$

$q_5 = (h(19q_1 + 79q_2))/420$

$q_6 = (h^2(2q_1 + 5q_2))/420$

The source vector components for node 1 are q_1 and q_2

The source vector components for node 2 are q_3 and q_4

The source vector components for node 1 are q_5 and q_6

Problem 5

```
clear;
syms xbar x
L = 144;
h = L/2;
E = 30e6;
b = 3;
c = 2;
I = b*(2*c)^3/12;
EI = E*I;
q0 = 50;

phi(1) = 1 - 3.*(xbar/h).^2 + 2.*(xbar/h).^2;
phi(2) = -xbar.*(1-xbar/h).^2;
phi(3) = 3.*(xbar/h).^2 - 2.*(xbar/h).^3;
phi(4) = -xbar.*((xbar/h).^2 - (xbar/h));

q1 = zeros(4,1);
q2 = zeros(4,1);
for ii = 1:4
    q1(ii) = int(q0*phi(ii),xbar,0,h);
    q2(ii) = int(q0*phi(ii),xbar,0,h);
end
q = [q1(1); q1(2); q1(3) + q2(1); q1(4) + q2(2); q2(3); q2(4)];

k = @(ii,jj) int(EI*diff(diff(phi(ii)))*diff(diff(phi(jj))),xbar,0,h);
K1 = zeros(4);
K2 = zeros(4);
for ii = 1:4
    for jj = 1:4
        K1(ii,jj) = k(ii,jj);
        K2(ii,jj) = k(ii,jj);
    end
end
end
```

```

Kg = zeros(6);
for ii = 1:4
    for jj = 1:4
        Kg(ii,jj) = K1(ii,jj);
    end
end

for ii = 1:4
    for jj = 1:4
        Kg(ii+2,jj+2) = Kg(ii+2,jj+2) + K2(ii,jj);
    end
end

Qtop = Kg(1:3,1:3)*[0;0;0]-q(1:3);
Q = [Qtop; 0; 0; 0];
DELbot = inv(Kg(4:6,4:6))*(q(1:3)+Q(4:6));
DEL = [0;0;0;DELbot];

w1 = 0;
w2 = 0;
for ii = 1:4
    w1 = w1 + DEL(ii).*phi(ii);
    w2 = w2 + DEL(ii+2).*phi(ii);
end
w1 = simplify(w1);
w2 = simplify(w2);
M1 = EI*diff(diff(w1));
M2 = EI*diff(diff(w2));
sigma1 = M1*c/I;
sigma2 = M2*c/I;
fprintf("\nProblem5\n")
fprintf("Q_equil force at left: %f lb\n", Q(1))
fprintf("Q_equil moment at left: %f in*lb\n", Q(2))
fprintf("Q_equil force at right: %f lb\n", Q(5))
fprintf("Q_equil moment at right: %f in*lb\n", Q(6))
Qdef(1) = subs(diff(M1),xbar,0);
Qdef(2) = subs(M1,xbar,0);
Qdef(3) = -subs(diff(M2),xbar,L/2);
Qdef(4) = -subs(M2,xbar,L/2);
fprintf("Q_def force at left: %f lb\n", Qdef(1))
fprintf("Q_def moment at left: %f in*lb\n", Qdef(2))
fprintf("Q_def force at right: %f lb\n", Qdef(3))
fprintf("Q_def moment at right: %f in*lb\n", Qdef(4))

xvec1 = linspace(0,L/2);
xvec2 = linspace(L/2,L);
figure
hold on
title("Problem 5 w(x)")
plot(xvec1, subs(w1, xbar, xvec1))
plot(xvec2, subs(w2, xbar, xvec2-L/2))
xlabel("x (in)")
ylabel("Vertical displacement (in)")
hold off

```

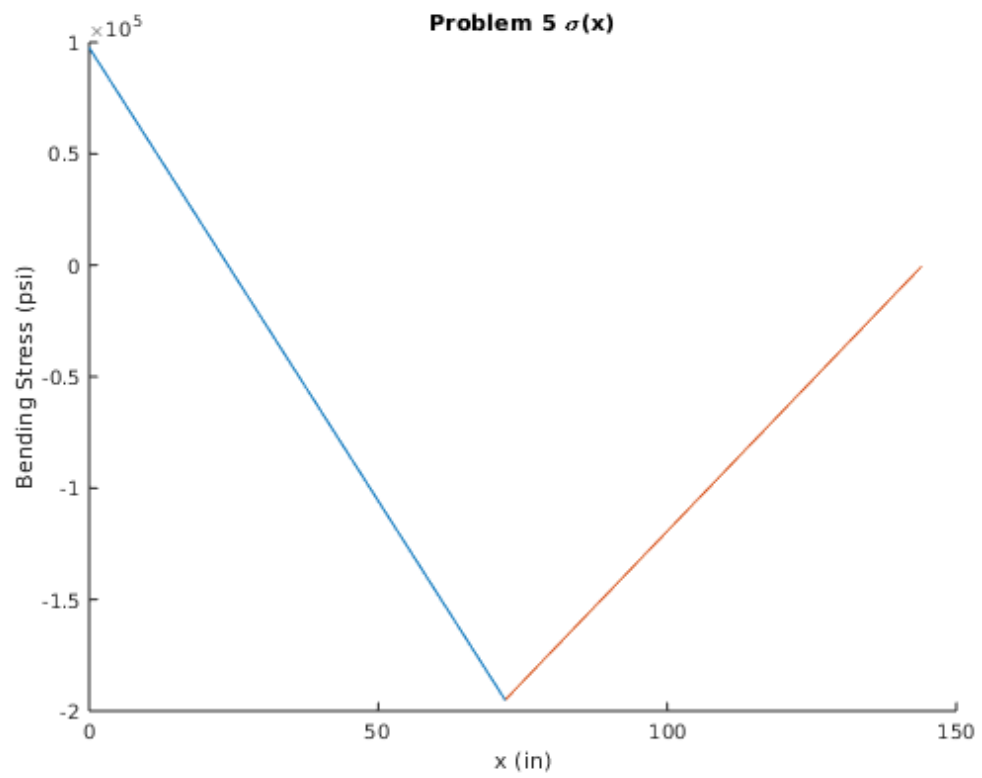
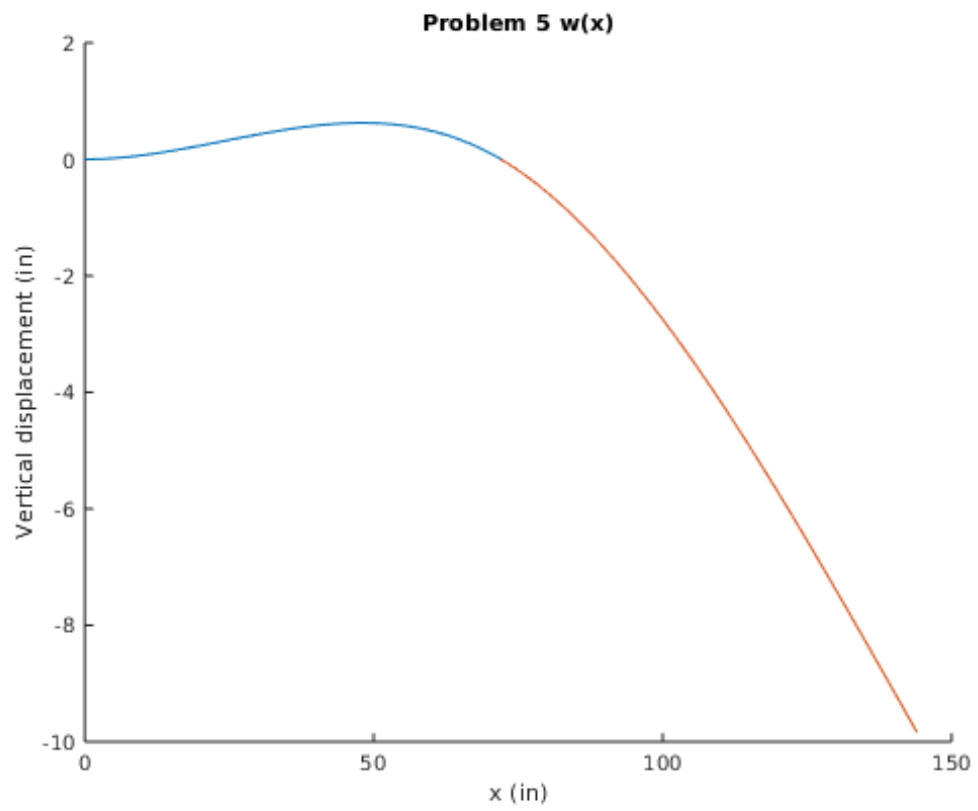
```

figure
hold on
title("Problem 5 \sigma(x)")
plot(xvec1, subs(sigma1, xbar, xvec1))
plot(xvec2, subs(sigma2, xbar, xvec2-L/2))
xlabel("x (in)")
ylabel("Bending Stress (psi)")
hold off

fprintf("vertical displacement for 0<=x<=L/2 is w(x)=%s\n",
    subs(w1,xbar,x))
fprintf("vertical displacement for L/2<=x<=L is w(x)=%s\n",
    simplify(subs(w2,xbar,x-L/2)))
fprintf("bending stress for 0<=x<=L/2 is sigma(x)=%s\n",
    subs(sigma1,xbar,x))
fprintf("bending stress for L/2<=x<=L is sigma(x)=%s\n",
    simplify(subs(sigma2,xbar,x-L/2)))

Problem5
Q_equil force at left: -2400.000000 lb
Q_equil moment at left: 21600.000000 in*lb
Q_equil force at right: 0.000000 lb
Q_equil moment at right: 0.000000 in*lb
Q_def force at left: -32537.500000 lb
Q_def moment at left: 780900.000000 in*lb
Q_def force at right: -21600.000000 lb
Q_def moment at right: 4200.000000 in*lb
vertical displacement for 0<=x<=L/2 is w(x)=-((1406744379605447*x^2*(x
- 72))/124515522497539473408
vertical displacement for L/2<=x<=L is w(x)=((x -
72)*(15758995816094758739*x^2 - 5682431241353511222624*x +
204378416738933075165952))/2101199442145978613760000
bending stress for 0<=x<=L/2 is
sigma(x)=109901904656675546875/1125899906842624 -
(109901904656675546875*x)/27021597764222976
bending stress for L/2<=x<=L is
sigma(x)=(1969874477011844842375*x)/729583139634020352 -
1315022943694508844875/3377699720527872

```



Problem 6

```
clear;
syms xbar x
L = 144;
h = L/2;
E = 30e6;
b = 3;
c = 2;
I = b*(2*c)^3/12;
EI = E*I;

q0 = 50;
Ks = 2000;
M0 = 10000;
F0 = 2000;

phi(1) = 1 - 3.*(xbar/h).^2 + 2.*(xbar/h).^2;
phi(2) = -xbar.*(1-xbar/h).^2;
phi(3) = 3.*(xbar/h).^2 - 2.*(xbar/h).^3;
phi(4) = -xbar.*((xbar/h).^2 - (xbar/h));

q1 = zeros(4,1);
q2 = zeros(4,1);
for ii = 1:4
    q1(ii) = int(q0*phi(ii),xbar,0,h);
    q2(ii) = 0;
end
q = [q1(1); q1(2); q1(3) + q2(1); q1(4) + q2(2); q2(3); q2(4)];

k = @(ii,jj) int(EI*diff(diff(phi(ii)))*diff(diff(phi(jj))),xbar,0,h);
K1 = zeros(4);
K2 = zeros(4);
for ii = 1:4
    for jj = 1:4
        K1(ii,jj) = k(ii,jj);
        K2(ii,jj) = k(ii,jj);
    end
end
K2(3,3) = K2(3,3) + Ks;
Kg = zeros(6);
for ii = 1:4
    for jj = 1:4
        Kg(ii,jj) = K1(ii,jj);
    end
end

for ii = 1:4
    for jj = 1:4
        Kg(ii+2,jj+2) = Kg(ii+2,jj+2) + K2(ii,jj);
    end
end
```

```

Qtop = Kg(1:3,1:3)*[0;0;0]-q(1:3);
Q = [Qtop; -M0; F0; 0];
DELbot = inv(Kg(4:6,4:6))*(q(1:3)+Q(4:6));
DEL = [0;0;0;DELbot];
Q(5) = Q(5) - Ks*DEL(5);

w1 = 0;
w2 = 0;
for ii = 1:4
    w1 = w1 + DEL(ii).*phi(ii);
    w2 = w2 + DEL(ii+2).*phi(ii);
end
w1 = simplify(w1);
w2 = simplify(w2);
M1 = EI*diff(diff(w1));
M2 = EI*diff(diff(w2));
sigma1 = M1*c/I;
sigma2 = M2*c/I;
fprintf("\nProblem 6\n")
fprintf("Q_equil force at left: %f lb\n", Q(1))
fprintf("Q_equil moment at left: %f in*lb\n", Q(2))
fprintf("Q_equil force at right: %f lb\n", Q(5))
fprintf("Q_equil moment at right: %f in*lb\n", Q(6))
Qdef(1) = subs(diff(M1),xbar,0);
Qdef(2) = subs(M1,xbar,0);
Qdef(3) = -subs(diff(M2),xbar,L/2);
Qdef(4) = -subs(M2,xbar,L/2);
fprintf("Q_def force at left: %f lb\n", Qdef(1))
fprintf("Q_def moment at left: %f in*lb\n", Qdef(2))
fprintf("Q_def force at right: %f lb\n", Qdef(3))
fprintf("Q_def moment at right: %f in*lb\n", Qdef(4))

xvec1 = linspace(0,L/2);
xvec2 = linspace(L/2,L);
figure
hold on
plot(xvec1, subs(w1, xbar, xvec1))
plot(xvec2, subs(w2, xbar, xvec2-L/2))
title("Problem 6 w(x)")
xlabel("x (in)")
ylabel("Vertical displacement (in)")
hold off
figure
hold on
plot(xvec1, subs(sigma1, xbar, xvec1))
plot(xvec2, subs(sigma2, xbar, xvec2-L/2))
title("Problem 6 \sigma(x)")
xlabel("x (in)")
ylabel("Bending Stress (psi)")
hold off

fprintf("vertical displacement for 0<=x<=L/2 is w(x)=%s\n",
    subs(w1,xbar,x))

```

```

fprintf("vertical displacement for L/2<=x<=L is w(x)=%s\n",
    simplify(subs(w2,xbar,x-L/2)))
fprintf("bending stress for 0<=x<=L/2 is sigma(x)=%s\n",
    subs(sigma1,xbar,x))
fprintf("bending stress for L/2<=x<=L is sigma(x)=%s\n",
    simplify(subs(sigma2,xbar,x-L/2)))

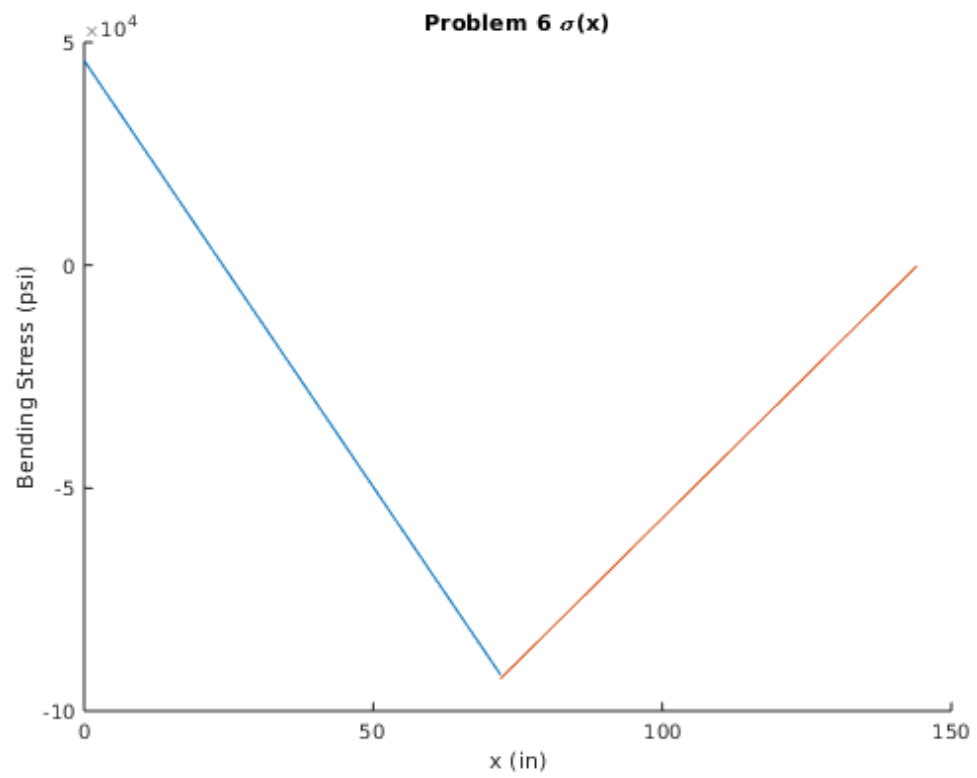
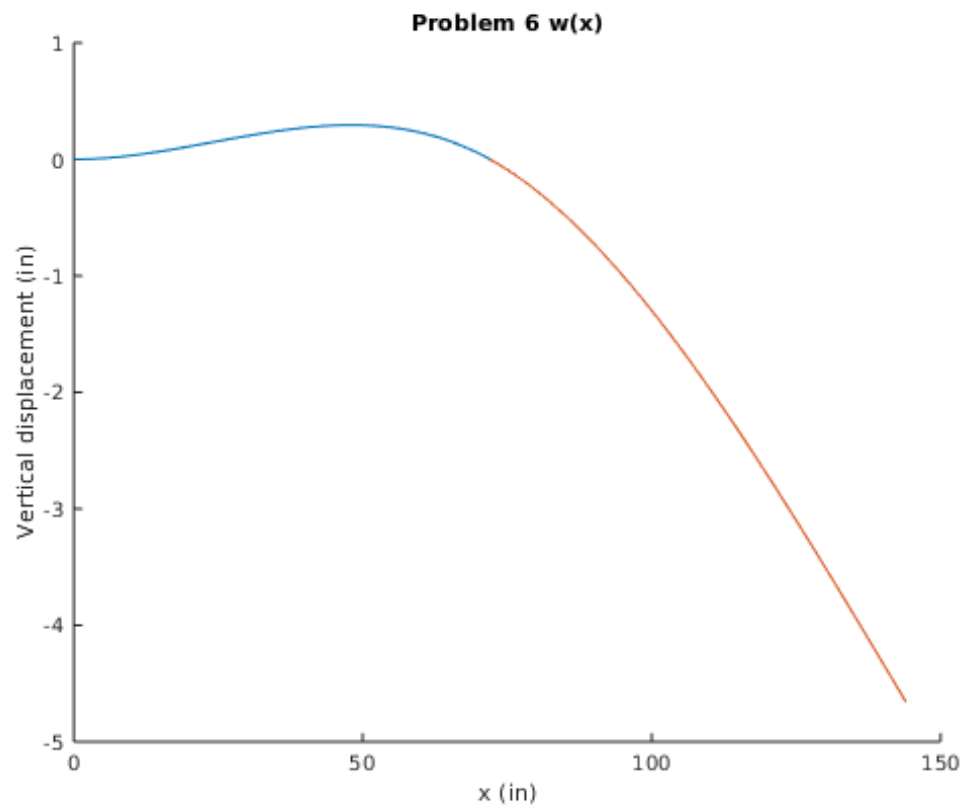
```

Problem 6

```

Q_equil force at left: -2400.000000 lb
Q_equil moment at left: 21600.000000 in*lb
Q_equil force at right: 11316.925336 lb
Q_equil moment at right: 0.000000 in*lb
Q_def force at left: -15303.778663 lb
Q_def moment at left: 367290.687919 in*lb
Q_def force at right: -10283.074664 lb
Q_def moment at right: 1800.000000 in*lb
vertical displacement for 0<=x<=L/2 is w(x)=-((294067592044577*x^2*(x
- 72))/55340232221128654848
vertical displacement for L/2<=x<=L is w(x)=((x -
72)*(48015090552099481*x^2 - 17310646992061559232*x +
627015879496690373376))/13447676429734263128064
bending stress for 0<=x<=L/2 is
sigma(x)=206766275656343203125/4503599627370496 -
(68922091885447734375*x)/36028797018963968
bending stress for L/2<=x<=L is
sigma(x)=(3751178949382771953125*x)/2918332558536081408 -
2503825896003655703125/13510798882111488

```



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