Table of Contents

Assignment 1

Aero 707 Liam Hood

```
clear; close all; clc
global r2d
r2d = 180/pi;
```

Problem 2

```
problem_2()
```

Problem 3

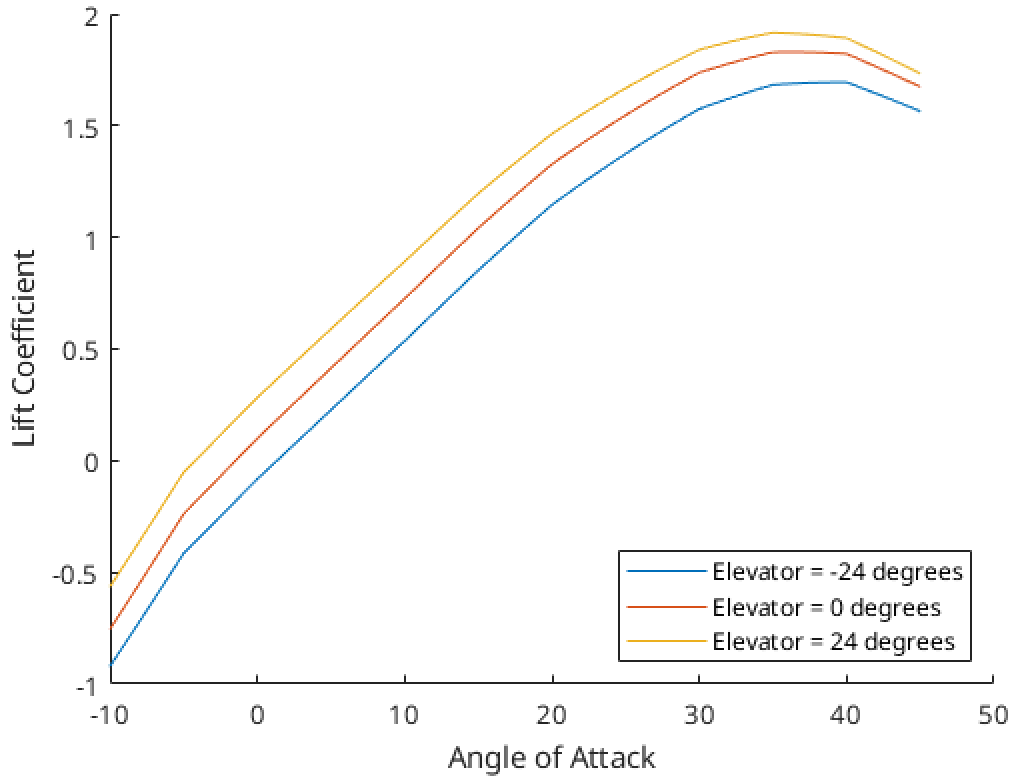
```
problem_3
```

```
******Problem 3******
i. Gust of 20 ft/s left to right (+velocity in y body fixed)
Don't know why this needs to be here
```

Problem 4

```
problem_4
```

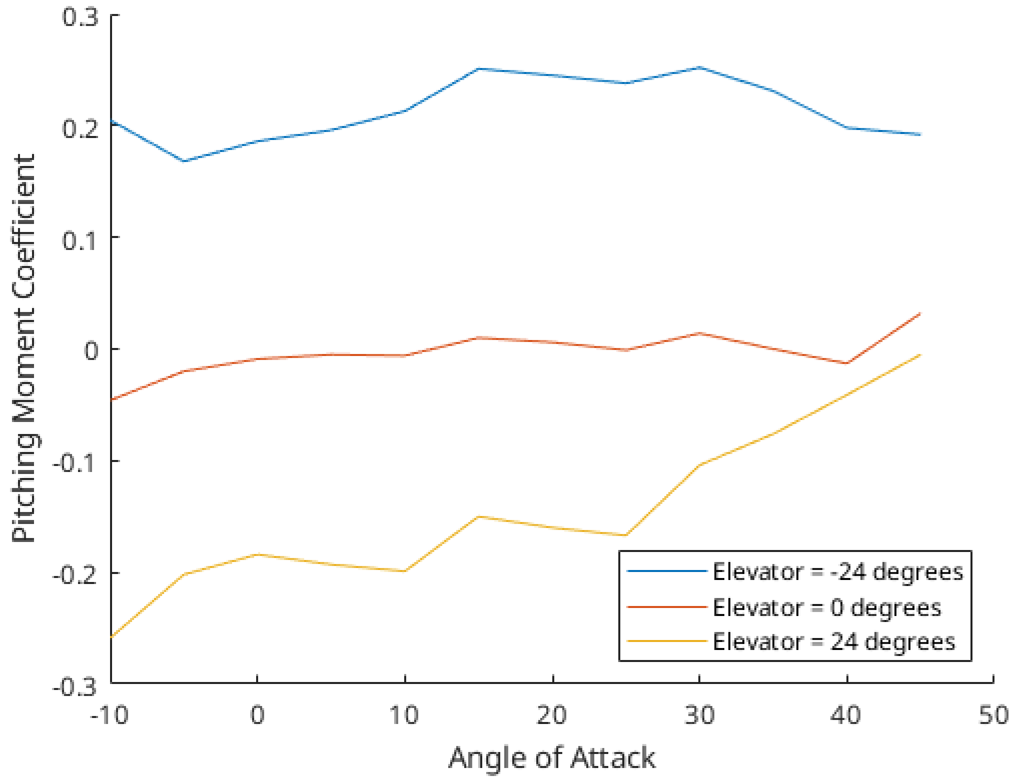
```
******Problem 4******
Max lift occurs at an angle of attack of 35.000000 degrees
```



Problem 5

```
problem_5
```

*******Problem 5*******
We can see that the F16 lacks pitch stiffness as the curves all vary from flat to a slight slope up. The elevator does significantly shift the whole curve showing it has significant control power



Problem 6

```
problem_6
```

```
******Problem 6******
I did not see a what pitch or flight path was used for Table 3.6-1
so I adjusted pitch to match initial cost in the table
TAS: 170.000000 ft/s  AOA: 0.385718 radians  Pitch: 0.217817 radians
Pitch Rate: 0.000000 radians/s  Altitude: 0.000000 ft  Distance 0.000000 ft
 Sum of Weighted Squares: 28.937859
TAS: 500.000000 ft/s  AOA: 0.010123 radians  Pitch: 0.068749 radians
Pitch Rate: 0.000000 radians/s  Altitude: 0.000000 ft  Distance 0.000000 ft
 Sum of Weighted Squares: 3.542947
TAS: 500.000000 ft/s  AOA: 0.094771 radians  Pitch: 0.105418 radians
Pitch Rate: 0.000000 radians/s  Altitude: 0.000000 ft  Distance 0.000000 ft
 Sum of Weighted Squares: 10.812971
****Find derivations for state equations*****
```

Problem 7

```
problem_7
```

```
******Problem 7******
Equilibrium Points:
```

```
 x1=-2    x2=0
 x1=1 - 3^(1/2)*1i    x2=0
 x1=3^(1/2)*1i + 1    x2=0
 x1=0    x2=-2*2^(1/2)
 x1=0    x2=2*2^(1/2)
Linearize around x1=-2, x2=0 using first order Taylor series
The A matrix is:
[12,  0]
[ 0, -2]

 The eigenvalues of this A matrix are lambda= -2.000000 and 12.000000

Linearize around x1=1-3^(1/2)*i, x2=0 using first order Taylor series
The A matrix is:
[3*(- 1 + 3^(1/2)*1i)^2,              0]
[                     0, 1 - 3^(1/2)*1i]

 The eigenvalues of this A matrix are lambda= 1.000000+-1.732051i and
 -6.000000+-10.392305i

Linearize around x1=1+3^(1/2)*i, x2=0 using first order Taylor series
The A matrix is:
[3*(1 + 3^(1/2)*1i)^2,              0]
[                   0, 1 + 3^(1/2)*1i]

 The eigenvalues of this A matrix are lambda= 1.000000+1.732051i and
 -6.000000+10.392305i

Linearize around x1=0, x2=2*2^(1/2) using first order Taylor series
The A matrix is:
[        0, -4*2^(1/2)]
[2*2^(1/2),          0]

 The eigenvalues of this A matrix are lambda= 0.000000+-4.000000i and
 0.000000+4.000000i

Linearize around x1=0, x2=-2*2^(1/2) using first order Taylor series
The A matrix is:
[         0, 4*2^(1/2)]
[-2*2^(1/2),         0]

 The eigenvalues of this A matrix are lambda= 0.000000+-4.000000i and
 0.000000+4.000000i
```

Problems

```
function problem_2()
    fprintf("******Problem 2******\n")
%     % xdot = A*x + B*u
%     A = [8, -2, 3;
%          0, 1, 4;
%          7, 9, 10];
```

```
%     B = [2; 1; 5];
%     % y = C*x+D*u
%     C = [2, 1, 3];
%     D = 0;
%
%     % Transfer function G(s)
    fprintf("Transfer Function\n")
%     syms s
%     G = collect(C*inv((s*eye(3)-A))*B);
%     fprintf("From formula\n")
%     fprintf("G(s) = %s\n", G)
%     fprintf("Built in Function\n")
%     [b,a] = ss2tf(A,B,C,D);
%     G_builtin = collect((b(1)*s^3 + b(2)*s^2 + b(3)*s + b(4))/...
%         (a(1)*s^3 + a(2)*s^2 + a(3)*s + a(4)));
%     fprintf("G(s) = %s\n", G_builtin)
%     % Poles
%     fprintf("\nPoles\n")
%     poles_builtin = eig(A);
%     poles = vpasolve(det(s*eye(3)-A)==0,s);
%     fprintf("From formula\n")
%     fprintf("Poles %f, %f, %f \n", poles)
%     fprintf("Built in Function\n")
%     fprintf("Poles %f, %f, %f \n", poles_builtin)
end

function problem_3()
    global r2d
    fprintf("\n******Problem 3******\n")
    % 2.3-1 Find instantaneous angle of attack and angle of side slip
    % in the gusts

    % Given starting conditions
    v = 500;
    alpha = 8/r2d;
    beta = -5/r2d;
    % Rotation matrices
    % stability from body fixed
    C_s_bf = [cos(alpha), 0, sin(alpha);
                0, 1, 0;
                -sin(alpha), 0, cos(alpha)];
    % wind from stability
    C_w_s = [cos(beta), sin(beta), 0;
            -sin(beta), cos(beta), 0;
            0, 0, 1];
    % wind from body fixed
    C_w_bf = C_w_s*C_s_bf;
    % starting V
    vw = [v;0;0];
    vbf = C_w_bf'*vw;

    % i
    fprintf("i. Gust of 20 ft/s left to right (+velocity in y body fixed)\n")
    vbf1 = vbf + [0;20;0];
```

```
    disp("Don't know why this needs to be here")
    [alpha1, beta1] = find_alpha_beta(vbf1);
    disp("but it won't publish without these strings")
    fprintf("\tAngle of attack is now %f degrees\n", alpha1)
    fprintf("\tAngle of sideslip is now %f degrees\n", beta1)
    % ii
    fprintf("ii. Gust of 50 ft/s from dead astern(+velocity in x body
 fixed)\n")
    vbf2 = vbf + [50;0;0];
    [alpha2, beta2] = find_alpha_beta(vbf2);
    fprintf("\tAngle of attack is now %f degrees\n", alpha2)
    fprintf("\tAngle of sideslip is now %f degrees\n", beta2)

    % iii
    fprintf("iii. Gust of 30 ft/s from right and below (-velocity in y, -
velocity in z\n")
    vbf3 = vbf + [0;-30*cos(70/r2d);-30*sin(70/r2d)];
    [alpha3, beta3] = find_alpha_beta(vbf3);
    fprintf("\tAngle of attack is now %f degrees\n", alpha3)
    fprintf("\tAngle of sideslip is now %f degrees\n", beta3)

end

function problem_4()
    fprintf("\n******Problem 4******\n")
    % 2.3-4
    n = 1e2;
    alpha = linspace(-10,45,n);
    elev = [-24,0,24];
    for ii = 1:3
        cl(ii,:) = zeros(1,n);
        for jj = 1:n
            cl(ii,jj) = CX(alpha(jj),elev(ii))*sind(alpha(jj)) - ...
                CZ(alpha(jj),0,elev(ii))*cosd(alpha(jj));
        end
    end
    figure
    hold on
    plot(alpha, cl(1,:))
    plot(alpha, cl(2,:))
    plot(alpha, cl(3,:))
    legend("Elevator = -24 degrees", "Elevator = 0 degrees", ...
        "Elevator = 24 degrees", "Location","southeast")
    xlabel("Angle of Attack")
    ylabel("Lift Coefficient")
    hold off

    [~, I] = max(cl(3,:));
    fprintf("Max lift occurs at an angle of attack of %f degrees\n", alpha(I))
end

function problem_5()
    fprintf("\n******Problem 5******\n")
    % 2.3-5
```

```
    n = 1e2;
    alpha = linspace(-10,45,n);
    elev = [-24,0,24];
    for ii = 1:3
        cm(ii,:) = zeros(1,n);
        for jj = 1:n
            cm(ii,jj) = CM(alpha(jj),elev(ii));
        end
    end
    figure
    hold on
    plot(alpha, cm(1,:))
    plot(alpha, cm(2,:))
    plot(alpha, cm(3,:))
    legend("Elevator = -24 degrees", "Elevator = 0 degrees", ...
        "Elevator = 24 degrees", "Location","southeast")
    xlabel("Angle of Attack")
    ylabel("Pitching Moment Coefficient")
    hold off
    fprintf("We can see that the F16 lacks pitch stiffness as the curves all
\n")
    fprintf("vary from flat to a slight slope up. The elevator does
 significantly \n")
    fprintf("shift the whole curve showing it has significant control power
\n")
end

function problem_6()
    fprintf("\n******Problem 6******\n")
    global r2d
    % 3.5-1
    fprintf("I did not see a what pitch or flight path was used for Table
 3.6-1\n")
    fprintf("so I adjusted pitch to match initial cost in the table\n")
    x1 = [170; 22.1/r2d;  12.48/r2d; 0; 0; 0];
    u1 = [.297, -25.7, .25, 0];
    [xd1] = transp(x1,u1);
    sos1 = xd1(1)^2 + 100*xd1(2)^2 + 10*xd1(4)^4;
    fprintf("TAS: %f ft/s  AOA: %f radians  Pitch: %f radians  \n", x1(1:3))
    fprintf("Pitch Rate: %f radians/s  Altitude: %f ft  Distance %f ft\n",
 x1(4:6))
    fprintf("\tSum of Weighted Squares: %f\n", sos1)

    x2 = [500; .58/r2d; 3.939/r2d; 0; 0; 0];
    u2 = [.293, 2.46, .25, 0];
    [xd2] = transp(x2,u2);
    sos2 = xd2(1)^2 + 100*xd2(2)^2 + 10*xd2(4)^4;
    fprintf("TAS: %f ft/s  AOA: %f radians  Pitch: %f radians  \n", x2(1:3))
    fprintf("Pitch Rate: %f radians/s  Altitude: %f ft  Distance %f ft\n",
 x2(4:6))
    fprintf("\tSum of Weighted Squares: %f\n", sos2)

    x3 = [500; 5.43/r2d; 6.04/r2d; 0; 0; 0];
    u3 = [.204, -4.10, .25, 0];
```

```
    [xd3] = transp(x3,u3);
    sos3 = xd3(1)^2 + 100*xd3(2)^2 + 10*xd3(4)^4;
    fprintf("TAS: %f ft/s  AOA: %f radians  Pitch: %f radians  \n", x3(1:3))
    fprintf("Pitch Rate: %f radians/s  Altitude: %f ft  Distance %f ft\n",
 x3(4:6))
    fprintf("\tSum of Weighted Squares: %f\n", sos3)

    fprintf("****Find derivations for state equations*****\n")

end

function problem_7()
    fprintf("\n******Problem 7******\n")
    global r2d
    % 3.7-1
    syms x1 x2
    x1dot = x1^3-x2^2+8;
    x2dot = x1*x2;
    sol = solve(x1dot==0,x2dot==0,x1,x2);
    fprintf("Equilibrium Points:\n")
    for ii = 1:5
        fprintf("\tx1=%s    x2=%s\n",[sol.x1(ii),sol.x2(ii)])
    end
    fprintf("Linearize around x1=-2, x2=0 using first order Taylor series\n")
    A = subs([diff(x1dot,x1), diff(x1dot,x2); diff(x2dot,x1), diff(x2dot,x2)],
[x1,x2],[-2,0]);
    fprintf("The A matrix is:\n")
    disp(A)
    [~,D] = eig(A);
    fprintf("\tThe eigenvalues of this A matrix are lambda= %f and %f\n\n",
[D(1,1),D(2,2)])

    fprintf("Linearize around x1=1-3^(1/2)*i, x2=0 using first order Taylor
 series\n")
    A = subs([diff(x1dot,x1), diff(x1dot,x2); diff(x2dot,x1), diff(x2dot,x2)],
[x1,x2],[1-3^(1/2)*i,0]);
    fprintf("The A matrix is:\n")
    disp(A)
    [~,D] = eig(A);
    fprintf("\tThe eigenvalues of this A matrix are lambda= %f+%fi and %f+%fi
\n\n",[real(D(1,1)),imag(D(1,1)),real(D(2,2)),imag(D(2,2))])

    fprintf("Linearize around x1=1+3^(1/2)*i, x2=0 using first order Taylor
 series\n")
    A = subs([diff(x1dot,x1), diff(x1dot,x2); diff(x2dot,x1), diff(x2dot,x2)],
[x1,x2],[1+3^(1/2)*i,0]);
    fprintf("The A matrix is:\n")
    disp(A)
    [~,D] = eig(A);
    fprintf("\tThe eigenvalues of this A matrix are lambda= %f+%fi and %f+%fi
\n\n",[real(D(1,1)),imag(D(1,1)),real(D(2,2)),imag(D(2,2))])

    fprintf("Linearize around x1=0, x2=2*2^(1/2) using first order Taylor
 series\n")
```

```
    A = subs([diff(x1dot,x1), diff(x1dot,x2); diff(x2dot,x1), diff(x2dot,x2)],
[x1,x2],[0,2*2^(1/2)]);
    fprintf("The A matrix is:\n")
    disp(A)
    [~,D] = eig(A);
    fprintf("\tThe eigenvalues of this A matrix are lambda= %f+%fi and %f+%fi
\n\n",[real(D(1,1)),imag(D(1,1)),real(D(2,2)),imag(D(2,2))])

    fprintf("Linearize around x1=0, x2=-2*2^(1/2) using first order Taylor
 series\n")
    A = subs([diff(x1dot,x1), diff(x1dot,x2); diff(x2dot,x1), diff(x2dot,x2)],
[x1,x2],[0,-2*2^(1/2)]);
    fprintf("The A matrix is:\n")
    disp(A)
    [~,D] = eig(A);
    fprintf("\tThe eigenvalues of this A matrix are lambda= %f+%fi and %f+%fi
\n\n",[real(D(1,1)),imag(D(1,1)),real(D(2,2)),imag(D(2,2))])

end
```

*******Problem 2*******
Transfer Function

Functions

```
function [alpha, beta] = find_alpha_beta(vbf)
    global r2d
    syms alphas betas V
    C_s_bf = [cos(alphas), 0, sin(alphas);
                0, 1, 0;
                -sin(alphas), 0, cos(alphas)];
    % wind from stability
    C_w_s = [cos(betas), sin(betas), 0;
            -sin(betas), cos(betas), 0;
            0, 0, 1];
    % wind from body fixed
    C_w_bf = C_w_s*C_s_bf;
    sol = vpasolve([V;0;0]==C_w_bf*vbf,[alphas,betas,V]);
    alpha = sol.alphas*r2d;
    beta = sol.betas*r2d;
end

function [CXI]=CX(alpha,elev)
    % Data and interpolation for F-16 Axial force coefficient.
    alphtabl=[-10,-5,0,5,10,15,20,25,30,35,40,45];
    elevtabl=[-24,-12,0,12,24];
    cxtabl=[-.099, -.081, -.081, -.063, -.025, .044, .097...
    .113, .145, .167, .174, .166;
    -.048, -.038, -.040, -.021, .016, .083, .127...
    .137, .162, .177, .179, .167;
    -.022, -.020, -.021, -.004, .032, .094, .128...
    .130, .154, .161, .155, .138;
    -.040, -.038, -.039, -.025, .006, .062, .087...
```

```
    .085, .100, .110, .104, .091;
    -.083, -.073, -.076, -.072, -.046, .012, .024...
    .025, .043, .053, .047, .040];
    CXI= interp2(alphtabl,elevtabl,cxtabl,alpha,elev);
end

function [CZI]=CZ(alpha,beta,elev)
    % Data and interpolation for F-16 z-force coefficient
    alphtabl=[-10,-5,0,5,10,15,20,25,30,35,40,45];
    cztabl=[.770, .241, -.100, -.416, -.731, -1.053, -1.366...
    -1.646, -1.917, -2.120, -2.248, -2.229];
    C1= interp1(alphtabl,cztabl,alpha);
    CZI= C1*(1-(beta/57.3)^2) - .19*(elev/25.0);
end

function [CMI]=CM(alpha,elev)
    % F-16 model pitching-moment data and interpolation
    alphtabl=[-10,-5,0,5,10,15,20,25,30,35,40,45];
    elevtabl=[-24,-12,0,12,24];
    cmtabl=[.205, .168, .186, .196, .213, .251, .245...
    .238, .252, .231, .198, .192;
    .081, .077, .107, .110, .110, .141, .127...
    .119, .133, .108, .081, .093;
    -.046, -.020, -.009, -.005, -.006, .010, .006...
    -.001, .014, .000, -.013, .032;
    -.174, -.145, -.121, -.127, -.129, -.102, -.097...
    -.113, -.087, -.084, -.069, -.006;
    -.259, -.202, -.184, -.193, -.199, -.150, -.160...
    -.167, -.104, -.076, -.041, -.005];
    CMI = interp2(alphtabl,elevtabl,cmtabl,alpha,elev);
end

function [xd] = transp(x,u)
    % Medium-sized transport aircraft, longitudinal dynamics
    S = 2170.0; CBAR = 17.5; MASS = 5e3; IYY = 4.1e6;
    TSTAT = 6e4; DTDV = -38; ZE = 2; CDCLS = .042;
    CLA = .085; CMA = -.022; CMDE = -.016; % per degree
    CMQ = -16; CMADOT = -6; CLADOT = 0; % per radian
    RTOD = 57.29578; GD = 32.17;
    THTL =  u(1);
    ELEV =  u(2);
    XCG =   u(3);
    LAND =  u(4);
    VT =    x(1);        % TAS in fps
    ALPHA = RTOD*x(2);   % A. O. A.
    THETA = x(3);        % PITCH ATTITUDE
    Q =     x(4);        % PITCH RATE
    H =     x(5);        % ALTITUDE
    [~,QBAR] = ADC(VT,H);
    QS = QBAR*S;
    SALP = sin(x(2)); CALP = cos(x(2));
    GAM = THETA - x(2); SGAM = sin(GAM); CGAM = cos(GAM);
    if LAND == 0         % clean
        CLO = .2; CDO = .016;
```

```
        CMO = .05; DCDG = 0; DCMG = 0;
    elseif LAND == 1    % landing flap and gear
        CLO = 1; CDO = .08;
        CMO = -.2; DCDG = .02; DCMG = -.05;
    else
        disp("Landing Gear & Flaps")
    end
    THR = (TSTAT+VT*DTDV) * max(THTL,0);            % THR
    CL = CLO+CLA*ALPHA;                             % NONDIM LIFT
    CM = DCMG+CMO+CMA*ALPHA+CMDE*ELEV+CL*(XCG-.25); % MOMENT
    CD = DCDG+CDO+CDCLS*CL*CL;                      % DRAG POLAR

    % State Equations
    xd(1) = (THR*CALP-QS*CD)/MASS - GD*SGAM;
    xd(2) = (-THR*SALP-QS*CL+MASS*(VT*Q+GD*CGAM))/(MASS*VT+QS*CLADOT);
    xd(3) = Q;
    D = .5*CBAR*(CMQ*Q+CMADOT*xd(2))/VT;
    xd(4) = (QS*CBAR*(CM+D)+THR*ZE)/IYY;
    xd(5) = VT*SGAM;
    xd(6) = VT*CGAM;
end

function [MACH,QBAR] = ADC(VT,H)
    R0 = 2.377e-3;
    TFAC = 1-.703e-5*H;
    T = 519*TFAC;
    if H >= 35000
        T = 390;
    end
    RHO = R0*(TFAC^4.14);
    MACH = VT/sqrt(1.4*1716.3*T);
    QBAR = .5*RHO*VT*VT;
end
```

```
but it won't publish without these strings
 Angle of attack is now 8.000000 degrees
 Angle of sideslip is now -2.710123 degrees
ii. Gust of 50 ft/s from dead astern(+velocity in x body fixed)
 Angle of attack is now 7.271964 degrees
 Angle of sideslip is now -4.549540 degrees
iii. Gust of 30 ft/s from right and below (-velocity in y, -velocity in z
 Angle of attack is now 4.766735 degrees
 Angle of sideslip is now -6.207827 degrees
```

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