Laboratory Assignment #3:
Inviscid Flow over Airfoils and Wings

Part 1 Assigned Date: Friday, October 31, Week 11
Part 1 Due Date: Thursday, November 13, Week 13

Part 2 Assigned Date: Friday, November 7, Week 12
Part 2 Due Date: Thursday, November 20, Week 14

Part 3 Assigned Date: Friday, November 14, Week 13
Part 3 Due Date: Thursday, December 4, Week 16

Learning Outcomes and Policies

Policies

The following policies are included at the end of this document. You must understand and comply with all of these policies. If you have any questions on these policies, you must ask the teaching team before you turn in the submissions associated with this assignment.

1. Group Collaboration Policy
2. MATLAB Code Policy
3. Plagiarism Notes
4. Individual Contribution Evaluation
5. Grading Rubrics

Learning Outcomes:

1. Understand the difference between the application and results of thin airfoil theory and the vortex panel method.
2. Understand how changes in wing section camber and thickness alter the sectional lift slope and zero-lift angle of attack.
3. Compare theoretical predictions for lift and drag for airfoils with experimentally measured results.
4. Practice using Prandtl Lifting Line Theory to calculate lift and drag on a finite wing.
5. Show how the number of terms in Prandtl Lifting Line Theory affects the resulting error in the solution.
6. Analyze a finite wing planform for an aircraft and demonstrate how the wing performance can be estimated using the Vortex Panel Method and Prandtl Lifting Line Theory.

Assignment Overview:

Project Description

This lab encourages you to put the theories that you have learned in your aerodynamics class (ASEN 3711) into practice by using simple MATLAB codes to analyze both 2D airfoils and 3D finite wings. You will use both provided computer codes and your own codes to do this analysis. The lab is divided into three parts, each with their own submission to keep you on track during the semester.

Submission Requirements

There is no formal lab report required for this lab. Instead, the lab is broken up into three parts that will be graded separately. For Parts 1 and 3, you will generate and plot aerodynamic data and answer questions based on these results. For Part 2, you will write your own finite-wing code (which you will then also use for Part 3).

For each of Parts 1, 2, and 3, there are several Tasks, and for each Task, there is one or more deliverables. A deliverable is marked as PLOT if it requires you turn in one or more plots, TABLE if it requires you to turn in one or more tables, DISCUSSION if it requires you turn in a (one or two paragraph) reflection of attained results in which you answer a series of related questions, and/or CODE if it requires that you turn in a documented MATLAB code. The marking rubrics for the PLOT and CODE deliverables are available at the end of this document.

There is no specific format required for the submissions. Simply submit your **Plots, Tables, Codes, and Discussion Paragraphs** as described in each part. Note that while there is no format required, you will be graded on the construction of your plots as well as the clarity of your discussion.

Grading Breakdown

Part	**Due**	**Description**	**Turnin**	**Weight**
Part 1	Thursday, Week 13	Task 1	4-Digit Airfoil Generator	5%
		Task 2	Effect of Airfoil Thickness on Lift	15%
		Task 3	Effect of Airfoil Camber on Lift	10%
Part 2	Thursday, Week 14	Task 1	Prandtl Lifting Line Code	20%
		Task 2	Effect of Taper on Induced Drag	10%
Part 3	Thursday, Week 16	Task 1	Effect of Angle of Attack on Coefficient of Lift	5%
		Task 2	Estimation of Profile Drag Coefficient	15%
		Task 3	Effect of Angle of Attack on Coefficient of Drag	10%
		Task 4	Effect of Airspeed on Steady, Level Flight Thrust	10%
Total				100%

Part 1: Analysis of 2D Airfoils

Overview

As the thickness of an airfoil is increased, it is no longer accurate to approximate the flow around it using thin airfoil theory, with a continuous vortex sheet located on the chord line. Instead, a vortex sheet can be "wrapped" around the airfoil surface to better account for the thickness of the body and to directly enforce the boundary conditions on the surface, including no penetration of the surface streamline.

This process can be accomplished with a classical "Vortex Panel Method" as outlined by Anderson in Section 4.10 of the book "Fundamentals of Aerodynamics". The approach discussed by Anderson represents a first-order method which approximates the vortex sheet strength as constant along each panel. It can be error prone and unstable. A better, and more complicated, second-order method is to allow the vortex sheet strength to linearly vary across each panel. This is described in Section 5.10 of the textbook by Kuethe and Chow. A pdf copy of this reading is provided on the course website along with a MATLAB function `Vortex_Panel.m` which implements the FORTRAN code outlined in Kuethe and Chow, along with a few minor modifications. Note that this code computes the flow for any arbitrary two-dimensional body defined by a set of (x, y) coordinates that define its surface.

Part 1 Task 1: NACA 4-Digit Airfoil Generator

You will need to build 4-digit NACA airfoils repeatedly throughout this lab. As such, it is advantageous to build a MATLAB function to construct panels for a given NACA airfoil. The formula for the shape of a NACA 4-digit series airfoil with camber is a bit involved. The first ingredient is the thickness distribution of the airfoil normal to the mean camber line, which is given by:

$$y_t = \frac{t}{0.2}c\left[0.2969\sqrt{\frac{x}{c}} - 0.1260\left(\frac{x}{c}\right) - 0.3516\left(\frac{x}{c}\right)^2 + 0.2843\left(\frac{x}{c}\right)^3 - 0.1036\left(\frac{x}{c}\right)^4\right]$$

where c is the chord length, x is the position along the chord from 0 to c, y_t is the half thickness at a given value of x (mean camber line to surface), and t is the maximum thickness as a fraction of the chord. As with the case of a symmetric NACA airfoil, the last two digits in the NACA XXXX description gives $100t$. The second ingredient is the formula for the mean camber line, which is:

$$y_c = \begin{cases} m\frac{x}{p^2}\left(2p - \frac{x}{c}\right), & 0 \le x < pc \\ m\frac{c-x}{(1-p)^2}\left(1 + \frac{x}{c} - 2p\right), & pc \le x \le c \end{cases}$$

where m is the maximum camber and p is the location of maximum camber. The first digit in the NACA XXXX description gives $100m$ while the second digit gives $10p$. Then, the coordinates (x_U, y_U) and (x_L, y_L) of the upper and lower airfoil surface, respectively, become:

$$x_U = x - y_t \sin\xi \qquad y_U = y_c + y_t \cos\xi$$
$$x_L = x + y_t \sin\xi \qquad y_L = y_c - y_t \cos\xi$$

where the thickness is applied perpendicular to the camberline, so the local angle is computed via:

$$\xi = \arctan\left(\frac{dy_c}{dx}\right).$$

As an example, $m = 4/100$, $p = 4/10$, and $t = 15/100$ for the NACA 4415 airfoil.

Your function should output a vector containing the x-locations of the boundary points and a vector containing the y-locations of the boundary points. The x- and y-locations should be ordered such that they move around the surface in a clockwise direction, starting from and ending at the trailing edge (as required by the vortex panel code developed from Kuethe and Chow (see Figure 5.23)).

Remark: *We are not requiring you to turn in your code for this portion of the lab. However, it is strongly recommended that you confirm with the teaching team in lab that your code is working correctly before proceeding on to the next tasks.*

Part 1 Task 1 Deliverable: Airfoil Coordinates (PLOT)
Grade Percentage: 5% of Overall Grade

With the aid of your NACA 4-digit airfoil generator, generate a plot showing the shape of two NACA airfoils, a NACA 0018 and a NACA 2418. For the cambered airfoil, also plot the camberline.

Part 1 Task 2: Effect of Airfoil Thickness on Lift

The provided MATLAB vortex panel code estimates the sectional coefficient of lift using the second-order vortex panel method. The provided code takes the form of a MATLAB function:

```
function c_l = Vortex_Panel(x_b,y_b,alpha)
```

where `c_l` is the sectional coefficient of lift (to be computed and returned), `x_b` and `y_b` are the vectors containing the x- and y-locations of the panel boundary points (provided by your NACA 4-digit function, ordered around the surface in a clockwise direction starting from and ending at the trailing edge to be consistent with the code from Kuethe and Chow), and `alpha` is the angle of attack (in degrees). Since the MATLAB vortex panel code can be called for several different airfoils types, it can easily be used to study the effect of airfoil thickness on lift.

Part 1 Task 2 Deliverable 1: Convergence Study (PLOT and DISCUSSION)
Grade Percentage: 5% of Overall Grade

Using the provided MATLAB vortex panel code, calculate the sectional coefficient of lift, c_l, experienced by a NACA 0012 airfoil at an angle of attack of $\alpha = 5°$ for a variety of values for the number of total panels (i.e., the number of panels for both the upper and lower surfaces), generate a plot of the predicted c_l versus the number of total panels, and determine the number of total panels required to achieve a predicted c_l within 1 percent relative error to an "exact" solution attained using a large number of panels.

Does the "exact" sectional lift coefficient make sense based on your understanding of airfoil theory? Is the number of panels you needed to achieve 1% relative error more or less than you expected? Do you believe it is worth using even more panels to further reduce the error? Why or why not?

Part 1 Task 2 Deliverable 2: Effect of Thickness (PLOT, TABLE, and DISCUSSION) Grade Percentage: 10% of Overall Grade

Using the provided MATLAB vortex panel code, calculate and plot the sectional coefficient of lift, c_l, versus angle of attack, α, for the following three airfoils:

- NACA 0006 (Relatively Thin Airfoil)
- NACA 0012 (Moderate Thickness Airfoil)
- NACA 0018 (Relatively Thick Airfoil)

Include all of these plots in the same figure to provide a clearer comparison. Estimate the zero lift angle of attack $\alpha_{L=0}$ (in degrees) and the lift slope a_0 (in per degree) for all three airfoils using the plotted data. In addition, estimate $\alpha_{L=0}$ (in degrees) and a_0 (in per degree) using thin airfoil theory. Create a table comparing the values of $\alpha_{L=0}$ and a_0 attained via the vortex panel method and thin airfoil theory for all three airfoils, and for the NACA 0006 and NACA 0012 airfoils, also compare these values with available experimental data in the table (electronic access to *Theory of Wing Sections* from Abbott and von Doenhoff (1949) is available through the University Libraries to aid you in this endeavor).

How do changes in the wing section thickness alter the sectional lift slope? How accurate is the assumption of thin airfoil theory for each wing section? How well do the two different theoretical methods (thin airfoil theory and vortex panel method) compare with the experimental data, and where do you think any disagreements might come from? The experimental data was collected at precise Reynolds numbers - what are the Reynolds numbers for the two theoretical methods?

Remark: *The equation for $\alpha_{L=0}$ using thin airfoil theory is given by Equation 4.61 in Anderson:*

$$\alpha_{L=0} = -\frac{1}{\pi}\int_0^{\pi} \frac{dz}{dx}\left(\cos\theta_0 - 1\right) d\theta_0$$

where $\frac{dz}{dx}$ is the slope of the camberline (note that the NACA equations for the camberline above use y instead of z). θ_0 may be computed from the x-coordinate along the chord line and chord length c using the following formula:

$$x = \frac{c}{2}\left(1 - \cos\theta_0\right)$$

All you need to compute this is to numerically integrate the equation, which is only a function of the slope of the camberline and the x-coordinate.

Remark: *The experimental results appearing in* Theory of Wing Sections *from Abbott and von Doenhoff (1949) was the result of years of wind tunnel testing carried out by researchers at the National Advisory Committee for Aeronautics (NACA), which was the predecessor to NASA. You will want to pull out the representative experimental results for the NACA 0006 and 0012 airfoils from this text. An easy way to do this is to use a digitizer - see MATLAB Digitizer for an example. Note there is no available experimental data for the NACA 0018 airfoil in this text, so we are not asking you to compare with experimental data for this airfoil.*

Part 1 Task 3: Effect of Airfoil Camber on Lift

The provided MATLAB vortex panel code can also be used to study the effect of airfoil camber on lift.

Part 1 Task 3 Deliverable: Effect of Camber (PLOT, TABLE, and DISCUSSION)
Grade Percentage: 10% of Overall Grade

Using the provided MATLAB vortex panel code, calculate and plot the sectional coefficient of lift, c_l, versus angle of attack, α, for the following three airfoils:

- NACA 0012 (Symmetric Airfoil)
- NACA 2412 (Moderately Cambered Airfoil)
- NACA 4412 (Significantly Cambered Airfoil)

Include all of these plots in the same figure to provide a clearer comparison. Estimate the zero lift angle of attack $\alpha_{L=0}$ (in degrees) and the lift slope a_0 (in per degree) for all three airfoils using the plotted data. In addition, estimate $\alpha_{L=0}$ (in degrees) and a_0 (in per degree) using thin airfoil theory. Create a table comparing the values of $\alpha_{L=0}$ and a_0 attained via the vortex panel method and thin airfoil theory for all three airfoils, and also compare these values with available experimental data in the table (again, leverage *Theory of Wing Sections* for this purpose).

How do changes in camber alter the sectional lift slope and the zero-lift angle of attack? How accurate is the assumption of thin airfoil theory for each wing section? How well do the two different theoretical methods (thin airfoil theory and vortex panel method) compare with the experimental data, and where do you think any disagreements might come from?

1 Grading Rubrics:

PLOT Grading Rubric

Item	Grading Criteria
Plot Data - 60%	
Plot Data	Is all expected data included? Is the data that is included correct? Is there any data included that is not relevant?
Plot Format - 40%	
Axes	Are the axes properly labeled (with units)? Are the axes properly scaled and divided?
Linetypes and Markers	Do the color choices of the lines add/detract from the clarity of the plot? Are the markers appropriate for the data plotted? Do the line weights and format add/detract from the clarity of the plot?
Caption	Does the caption just repeat the axis information? Does the caption give all necessary information for the plot to stand alone? Is the caption well-formatted and concise?
Annotations and Legend	Is the legend properly formatted and limits set? Do annotations (if present) improve/detract from the plot? Is the plot over/under annotated?
Overall Format	Does the font size of the plot match the rest of the paper? Is the plot legible and clear with no obscuration of data? Is the plot suitable for publication in a professional setting?
Total	**100%**

CODE Grading Rubric

Item	Weight	Grading Criteria
Code Execution	75%	Does the code run successfully out-of-the box? Does the code produce correct results?
Documentation	25%	Is the code thoroughly documented so it can be understood by others? Is the documentation concise and clear?
Total	**100%**	