Chaos in Seemingly Simple Optical Systems

Liam Packer

May 3, 2022

Chaos in Seemingly Simple Optical Systems

Liam Packer

A Brief Intro to Nonlinear Optic

Nonlinear Polarizatio ntensity-Dependent Refractive Index A Simple System

Analysis

The Driving Equations

Non-Chaotic Regime

The Road to Chaos

Graphical Orbit Analysis

Back to the Complex Ma

Table of contents

A Brief Intro to Nonlinear Optics

Nonlinear Polarization Intensity-Dependent Refractive Index A Simple System

Analysis

The Driving Equations
Non-Chaotic Regime
The Road to Chaos
Chaos
Graphical Orbit Analysis
Back to the Complex Map

References

Chaos in Seemingly Simple Optical Systems

Liam Packer

A Brief Intro to Nonlinear Optics Nonlinear Polarization Intensity-Dependent Refractive Index

nalysis

Non-Chaotic Regime The Road to Chaos Chaos

raphical Orbit Analy lack to the Complex



A Brief Intro to Nonlinear Optics

Chaos in Seemingly Simple Optical Systems

Liam Packer

A Brief Intro to Nonlinear Optics

Intensity-Depender

Analysis

The Driving Equations

Non-Chaotic Regime

The Road to Chaos

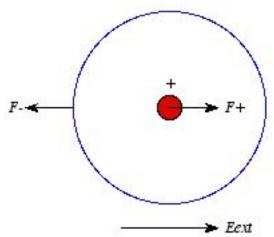
Chaos Graphical Orbit Analy

ack to the comp

deferences

The Linear Polarization

- ▶ In linear optics: $P \propto E$
- ► Refractive index *n* a constant of a material for a given wavelength
- ▶ Most general, $\mathbf{P} = \epsilon_0 \chi^{(1)}(\mathbf{E})$, or $P^i = \epsilon_0 \chi_{ij} E^j$



Chaos in Seemingly Simple Optical Systems

Liam Packer

Brief Intro to

Nonlinear Polarization

Intensity-Dependent Refractive Index A Simple System

nalvsis

Non-Chaotic Regime
The Road to Chaos
Chaos
Graphical Orbit Analysis

Back to the Complex Ma



Nonlinear Polarization

ntensity-Dependent Refractive Index A Simple System

Analysis

The Road to Chaos
Chaos
Graphical Orbit Analysis

Back to the Complex M

References

Nonlinear optics idea: Expand P in iterated products of E

$$\begin{aligned} \mathbf{P} &= \epsilon_0 [\chi^{(1)}(\mathbf{E}) + \chi^{(2)}(\mathbf{E}, \mathbf{E}) + \chi^{(3)}(\mathbf{E}, \mathbf{E}, \mathbf{E}) + \cdots] \\ &= \mathbf{P}^{(1)} + \mathbf{P}^{(2)} + \mathbf{P}^{(3)} + \cdots \\ \text{or } P^i &= \epsilon_0 [\chi^{(1)}_{ii} E^j + \chi^{(2)}_{iik} E^j E^k + \chi^{(3)}_{iikl} E^j E^k E^l + \cdots] \end{aligned}$$

- where $P^{(n)} \equiv \epsilon_0 \chi^{(n)}(\mathbf{E}, \mathbf{E}, \cdots)$
- (assumes instantaneous response of medium to electric field)

Graphical Orbit Analysis Back to the Complex Ma

References

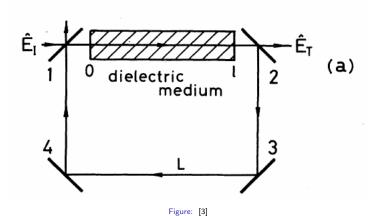
Lets consider some third-order effects. For simplicity, we'll only work with magnitudes and say that $P^{(3)}(t) = \epsilon_0 \chi^{(3)} E^3(t) \propto E^3(t)$.

- Picking a monochromatic wave, $E(t) = \mathcal{E} \cos \omega t$
- Then, using the identity $\cos^3 \omega t = \frac{1}{4} \cos(3\omega t) + \frac{3}{4} \cos \omega t$:

$$P^{(3)}(t) = \frac{1}{4} \epsilon_0 \chi^{(3)} \mathcal{E}^3 \cos 3\omega t + \frac{3}{4} \epsilon_0 \chi^{(3)} \mathcal{E}^3 \cos \omega t$$

Since $n(\omega) = \sqrt{1 + \chi_{eff}(\omega)}$ and $I(\omega) \propto |E(\omega)|^2$, this effects the index of refraction: $n(\omega) = n_0 + n_2 I(\omega)$ (See Boyd §6.2, 2008).

A Simple System



Chaos in Seemingly Simple Optical Systems

Liam Packer

Brief Intro to Ionlinear Optics Nonlinear Polarization Intensity-Dependent Refractive Index

A Simple System

The Driving Equations
Non-Chaotic Regime
The Road to Chaos
Chaos
Graphical Orbit Analysis

) - f - u - u - u - u

A Simple System

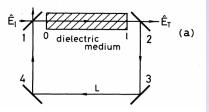
Analysis
The Driving

Non-Chaotic Regime
The Road to Chaos
Chaos
Graphical Orbit Analysis

Graphical Orbit Ana

- Linear case: constant phase shift $\phi_0 = \frac{2\pi L}{\lambda} n_0$
- Nonlinear case: variable phase shift $\phi(\omega) = \frac{2\pi L}{\lambda} n(\omega)$
- Third-order effects: $n(\omega) = n_0 + n_2 I(\omega)$
- Equation of motion guess:

$$\dot{E}(t) \propto E(t) \exp(i\phi)$$



Analysis

Chaos in Seemingly Simple Optical Systems

Liam Packer

A Brief Intro to Nonlinear Option

Intensity-Dependent

A Simple System

Analysis

Non-Chaotic Regime
The Road to Chaos

Chaos

Graphical Orbit Analysis

Back to the Complex Ma

Analysis The Driving Equations

Non-Chaotic Regime
The Road to Chaos
Chaos
Graphical Orbit Apalesis

Graphical Orbit Analysis Back to the Complex Map

References

 $E(t) = A + BE(t - t_R) \exp\{i[\phi(t) - \phi_0]\},$ (1)

 $\gamma^{-1}\dot{\phi}(t) = -\phi(t) + \operatorname{sgn}(n_2)|E(t - t_R)|^2.$ (2)

[3]

- $ightharpoonup E(t) \equiv$ dimensionless (complex) field amplitude at top-left corner of ring
- $lack \phi(t) \equiv$ time-dependent phase shift of the electric field
- ► $A \propto \sqrt{(2\pi/\lambda)n_2}E_{in}$, $B \propto \sqrt{(2\pi/\lambda)n_2}\hat{E}(t,0)$ a dissipation parameter
- $t_R \equiv$ time delay due to propagation of light through cavity

nalvsis

The Driving Equations Non-Chaotic Regime

Chaos

Craphical Orbit Analysis

Graphical Orbit Analysis

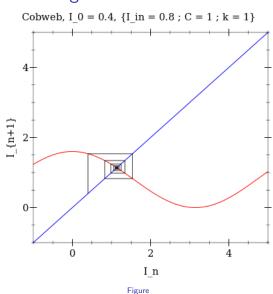
Back to the Complex Map

References

 $I_{n+1} = I_{in} \left(1 + C \cos \left(\frac{2\pi L}{\lambda} (n_0 + n_2 I_n) \right) \right)$ (3)

[2]

- $ightharpoonup I_n \equiv$ Intensity of electric field at start of n^{th} round trip
- $ightharpoonup I_{in} \equiv$ Intensity of input electric field
- ▶ $n_0 \equiv$ linear index of refraction, $n_2 \equiv$ second-order index of refraction
- ▶ $L \equiv$ length of cavity, $\lambda \equiv$ wavelength of monochromatic input light



Chaos in Seemingly Simple Optical Systems

Liam Packer

A Brief Intro t

Intensity-Dependent Refractive Index A Simple System

Analysis

The Driving Equation

Non-Chaotic Regime

Chaos Graphical Orbit Analysis

The Road to Chaos

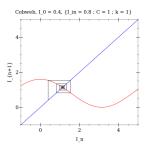


Figure: $\lambda \approx -0.3205$, $f'(x_i) \approx 0.72575$ $\lambda \approx -0.10966$, $f'(x_i) \approx 0.64491$

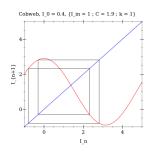
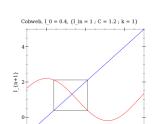
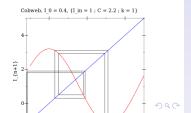


Figure: $\lambda \approx -0.39309$, $f'(x_i) \approx 0.45558$ $\lambda \approx -0.08662$, $f'(x_i) \approx 0.50001$





Chaos in Seemingly Simple Optical Systems

Liam Packer

A Brief Intro to Nonlinear Optics

Intensity-Dependent Refractive Index A Simple System

nalysis

The Driving Equations
Non-Chaotic Regime

The Road to Chaos Chaos

Graphical Orbit Analysis
Back to the Complex Map

Figure: $\lambda \approx 0.3937816$, $f'(x_i) \approx$ really big.. (10^8)

Chaos in Seemingly Simple Optical Systems

Liam Packer

A Brief Intro to Nonlinear Option Nonlinear Polarization Intensity-Dependent Refractive Index

\nalveie

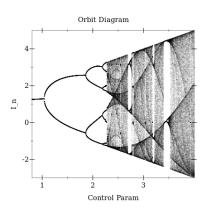
The Driving Equations
Non-Chaotic Regime
The Road to Chaos

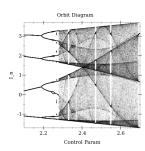
Chaos

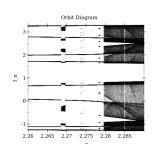
Graphical Orbit Analysis

Back to the Complex Map

Orbit Diagram







Chaos in Seemingly Simple Optical Systems

Liam Packer

A Brief Intro to Nonlinear Option

Intensity-Dependent Refractive Index A Simple System

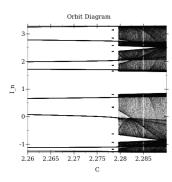
Δnalveie

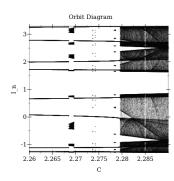
The Driving Equations Non-Chaotic Regime The Road to Chaos

Graphical Orbit Analysis

. .

Two Different Orbit Diagrams





Chaos in Seemingly Simple Optical Systems

Liam Packer

A Brief Intro to Nonlinear Optic

Intensity-Dependent Refractive Index A Simple System

Δnalveie

The Driving Equations
Non-Chaotic Regime
The Road to Chaos
Chaos

Graphical Orbit Analysis

Poforoncos

Liam Packer

A Brief Intro to Nonlinear Opti

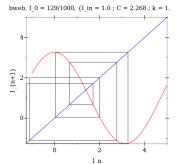
Intensity-Dependent Refractive Index A Simple System

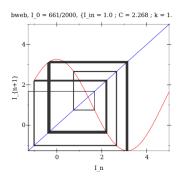
Analysis

Non-Chaotic Regime The Road to Chaos Chaos

Graphical Orbit Analysis

_f___





\nalveie

The Driving Equations

Non-Chaotic Regime

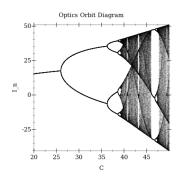
The Road to Chaos

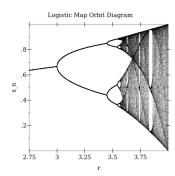
Graphical Orbit Analysis

Back to the Complex Map

Reference

When n_2 is small, the above map reduces to $I_{n+1} \approx I_{in}(1 + C(1 - \frac{1}{2}x^2))$, a quadratic form reminiscent of the logistic map $x_{n+1} = rx_n(1 - x_n)$:





Periodic Harmony in the Chaos

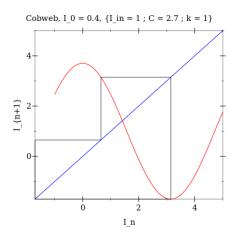


Figure: $\lambda \approx -0.9956$, $f'(x_i) \approx 0.05$

Chaos in Seemingly Simple Optical Systems

Liam Packer

A Brief Intro to Nonlinear Option

Intensity-Dependent Refractive Index A Simple System

nalysis

The Driving Equations
Non-Chaotic Regime
The Road to Chaos

Graphical Orbit Analysis

Liam Packer

A Brief Intro t Nonlinear Opt

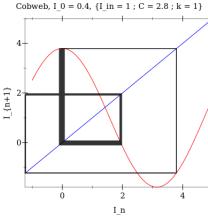
Intensity-Dependent Refractive Index A Simple System

nalvsis

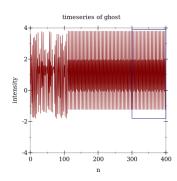
Non-Chaotic Regime
The Road to Chaos
Chaos

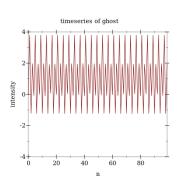
Graphical Orbit Analysis





Spooky Ghosts





Chaos in Seemingly Simple Optical Systems

Liam Packer

A Brief Intro to Nonlinear Opti

Intensity-Dependent Refractive Index A Simple System

\nalveie

The Driving Equations Non-Chaotic Regime The Road to Chaos

Graphical Orbit Analysis

Back to the Complex N

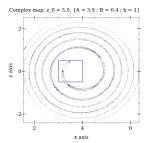
Nonlinear Polarization Intensity-Dependent Refractive Index A Simple System

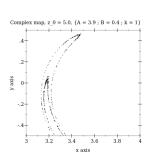
nalvsis

The Driving Equations Non-Chaotic Regime The Road to Chaos

Chaos

Back to the Complex Map





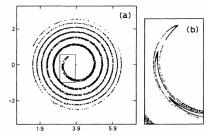
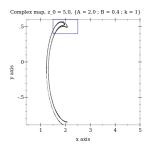
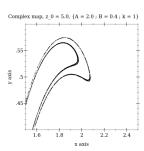
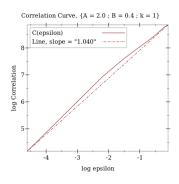


FIG. 2. (a) Plot of 5000 successive points of a series E_n on the complex E plane. The parameter values chosen are B=0.4, A=3.9, and $\varphi_0=0$. (b) Enlargement of the rectangular region of (a).

Fractal Dimension







Chaos in Seemingly Simple Optical Systems

Liam Packer

A Brief Intro to Nonlinear Optic

Intensity-Dependent Refractive Index A Simple System

Analysis

The Driving Equations

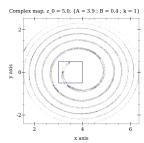
Non-Chaotic Regime

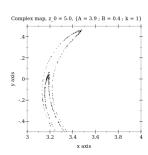
The Road to Chaos

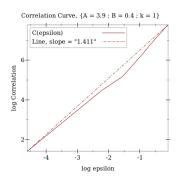
Chaos

Back to the Complex Map

Fractal Dimension







Chaos in Seemingly Simple Optical Systems

Liam Packer

A Brief Intro to

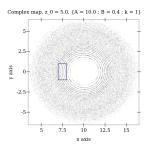
ntensity-Dependent Refractive Index A Simple System

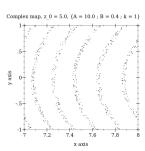
Analysis

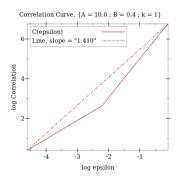
Non-Chaotic Regime
The Road to Chaos
Chaos

Back to the Complex Map

Fractal Dimension







Chaos in Seemingly Simple Optical Systems

Liam Packer

A Brief Intro to

Intensity-Dependent Refractive Index

Analysis

The Driving Equations
Non-Chaotic Regime
The Road to Chaos
Chaos

Back to the Complex Map



References

Chaos in Seemingly Simple Optical Systems

Liam Packer

Nonlinear Option

Intensity-Dependent

Analyci

The Driving Equations
Non-Chaotic Regime
The Road to Chaos

Chaos Graphical Orbit An

Back to the Complex M

Liam Packer

References

Polarization image.

http://teacher.pas.rochester.edu/PHY217/ LectureNotes/Chapter4/LectureNotesChapter4. html.

Accessed: 2022-05-02.

Robert G. Harrison and Dhruba J. Biswas. Chaos in light.

Nature, 321(6068):394-401, May 1986.

Number: 6068 Publisher: Nature Publishing Group.

K. Ikeda, H. Daido, and O. Akimoto.

Optical Turbulence: Chaotic Behavior of Transmitted Light from a Ring Cavity.

Physical Review Letters, 45(9):709-712, September 1980.

Publisher: American Physical Society.