Trinidad & Tobago Mathematics Olympiad

THE THIRTY-SIXTH TRINIDAD AND TOBAGO
MATHEMATICS OLYMPIAD

LEVEL 2

DATE: April 21st, 2023
TIME: 9:00 am - 12:00 noon

Write carefully and legibly, showing all your working. Marks will be awarded for correct deductions and inferences. Marks are indicated next to each question. Begin each answer on a new page. Calculators are NOT allowed.

1. Reduce the following fraction to its lowest terms:

$$\frac{93366336633663357}{81188118811881180000}$$

[25]

2. Let $ABCD$ be a rectangle and let P be the midpoint of AB. Let Q be the point on PD such that CQ is perpendicular to PD.

   (a) Show that the angles PDA, QCD and PCB are equal.

   (b) Show that there is a circle passing through the points P, B, C and Q.

   (c) Hence, show that the angles PDA and PQB are equal.

   (d) Hence, show that the triangle BCQ is isosceles. [25]

3. Find all ordered triples (x, y, z) of real numbers that satisfy the following system of equations:

$$xy + 1 = y + z$$
$$yz + 1 = x + y$$
$$xz + 1 = x + y + z$$

[25]

4. A large circle is drawn, and 2023 distinct dots are marked inside it. Allen has a marker and Sue has an eraser. They play a game, taking turns alternately, with Allen going first.

   On his turn, Allen counts the number of dots, say n, in the circle, then marks $f(n)$ new dots in the circle.

   On her turn, Sue chooses a diameter of the circle, then erases all the dots in one of the semicircles formed by this diameter, including those on the boundary.

   Can Allen ensure that there is always at least one dot in the circle if

   (a) $f(n) = n$?

   (b) $f(n) = n + 1$?

[25]

END OF PAPER