

Trinidad & Tobago Mathematics Olympiad

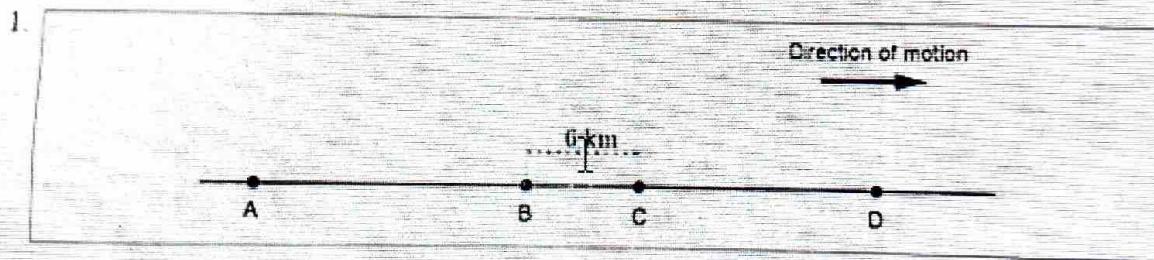
THE THIRTY-FIFTH TRINIDAD AND TOBAGO MATHEMATICS OLYMPIAD

LEVEL 2

DATE: February 7th, 2020

TIME: 9:00 am - 12:00 noon

Write carefully and legibly, showing all your working. Marks will be awarded for correct deductions and inferences. Marks are indicated next to each question. Begin each answer on a new page. Calculators are NOT allowed.



Consider the above diagram (not drawn to scale), which shows the relative positions of points A , B , C and D along a straight road. The distance between B and C is 6 km.

Three drivers - Sun, Moon and Comet all driving in the direction shown.

At 8:00 pm, Comet was at A .

At 8:10 pm, Comet overtook Sun at B .

At 8:14 pm, Comet overtook Moon at C .

At 8:29 pm, Sun overtook Moon at D .

Assume that Sun's speed, Moon's speed and Comet's speed were all constant.

(a) Show that the distance between A and B is 15 km. [6]

(b) Given that Sun's speed was 8 km/hour faster than Moon's speed, show that the distance between C and D is 13 km. [9]

(c) At what time, before Comet arrived at D , was Sun's distance from C the same as Comet's distance from D ? Justify your answer. [10]

2. Let ABC be an equilateral triangle. Let X be a point on the side BC , between B and C . Extend the line AX to a point P on the circumscribed circle of triangle ABC .

Note: The circumscribed circle of a triangle is the circle passing through its three vertices.

(a) Prove:

- (i) Triangles PAB and PCX are similar.
- (ii) Triangles PCA and PXB are similar.

[12]

(b) Using the results from Part (a), show that

$$PA = PB + PC.$$

[8]

(c) Is it possible to choose the point X , so that the line CP is tangent to the circumscribed circle of triangle PBX ? Justify your answer.

[5]

3. Let n belong to the set of positive integers.

(a) Show that $3^n + 169$ is divisible by 5 if and only if n is a multiple of 4.

[7]

(b) Using the result from Part (a), show that there is no value of n such that $3^n + 169$ is divisible by 20.

[9]

(c) Find all values of n such that $3^n + 169$ is a perfect square.

[9]

4. There are 20 spies in Port of Spain. Each spy is tracking at least one other spy, but no two spies are tracking each other (that is, if a spy A is tracking a spy B , then B is not tracking A).

A set of spies is called a *Spy Ring* if the spies can be arranged around a circle in some order so that the first is tracking the second, the second is tracking the third, and so on, and the last is tracking the first.

Given that every set of 12 spies (chosen from among the 20) is a *Spy Ring*,

(a) Show that:

- (i) Each spy is tracking at most 10 other spies.
- (ii) Each spy is tracking at least 9 other spies.

[3]

[4]

(b) Using the results from Part (a), show that every set of 13 spies is also a *Spy Ring*.

[18]

END OF PAPER