Trinidad & Tobago Mathematics Olympiad

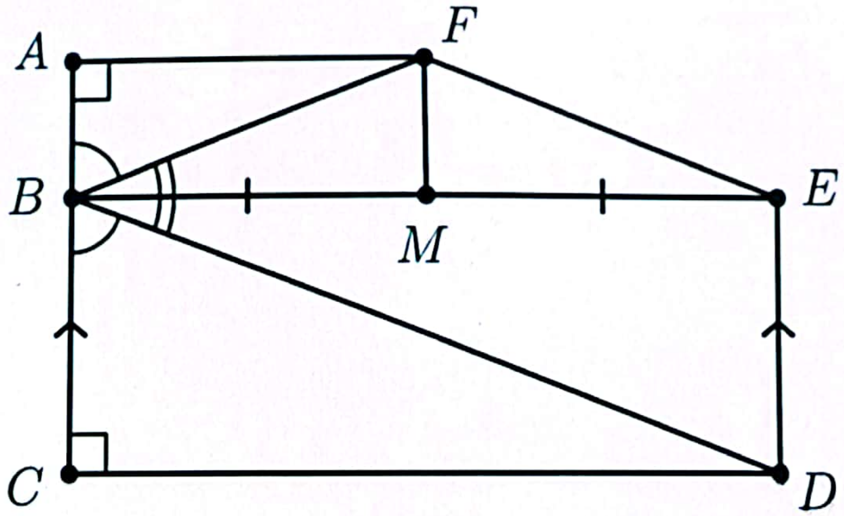
THE THIRTY-SEVENTH TRINIDAD AND TOBAGO MATHEMATICS OLYMPIAD

LEVEL 2

DATE: March 1st, 2024
TIME: 9:00 am - 12:00 noon

Write carefully and legibly, showing all your working. Marks will be awarded for correct deductions and inferences. Marks are indicated next to each question. Begin each answer on a new page. Calculators are NOT allowed.

1. In the following diagram (not drawn to scale), ABC is a straight line with $AB = \frac{1}{2}BC$. D, E and F are points such that $\angle BAF = \angle BCD = 90^\circ$, $\angle ABF = \angle CBD$, $\angle FBE = \angle DBE$ and $DE \parallel CA$. M is the midpoint of BE.



Show that:

(a) $\angle ABE = 90^\circ$

(b) $AF = \frac{1}{2}CD$

(c) $AF = BM$

(d) $\angle BMF = 90^\circ$

(e) Triangle BEF is isosceles. **[25]**

2. Find the smallest integer n, for which there exist integers a, b and c, where b and c are consecutive positive integers, such that

$$n = a \times b \times c,$$

$$n + 1 = (b + c)^2$$

and $n - 1000$ is a power of a. [25]

3. Alex constructed a system of five equations, which has a unique solution (x, y, z). However, Jason changed the constant on the right hand side of **one** of the equations, resulting in the following system, which has no solution:

$$\begin{aligned} 4x - 4y + 3z &= 2 \\ x - y + z &= 3 \\ 2x - y + z &= 4 \\ 4x - 4y + 5z &= 6 \\ 2x + y - z &= 8 \end{aligned}$$

(a) Determine the equation that Jason changed.

(b) Determine the value of the constant on the right hand side of this equation in Alex's system.

(c) Give the solution of Alex's system. [25]

4. Eleven girls stand in a circle. They hold several rounds of voting, on whether to go to the beach, or to the cinema.

If in a given round of voting, a girl's vote is the same as that of **at least one** of the two girls next to her, then her vote is unchanged in the following round. Otherwise, her vote changes.

Show that regardless of how the girls voted in the first round, there is a round after which nobody's vote changes. [25]

END OF PAPER