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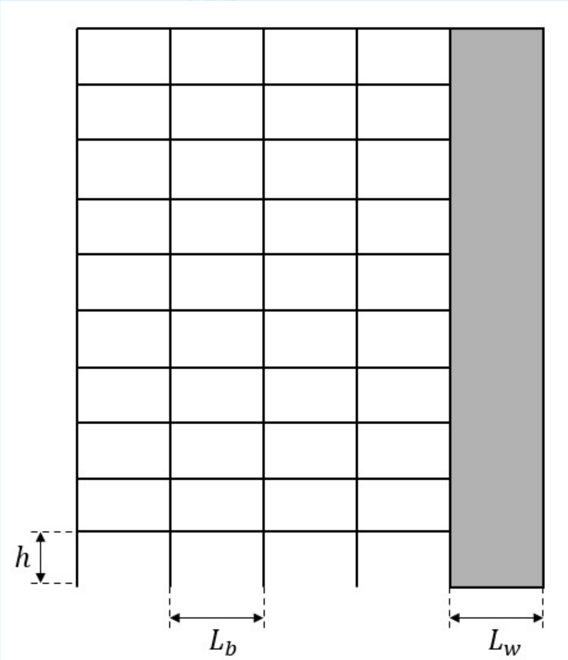
The following MathCAD file provides a worked example of the procedure outlined in the paper to estimate the dynamic properties of wall-frame structures.

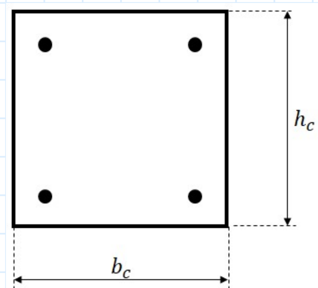
The building parameters are defined within the first two pages, including the number of storeys, beam, column and wall dimensions etc...

This example is for a simple 10 storey wall-frame structure with regular dimensions up the structures' height. Beam, column, and wall sections are rectangular. This file can easily be adapted for steel sections, or I and C shaped walls.

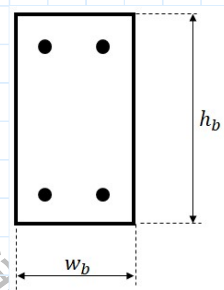
Wall-frame elevation sketch



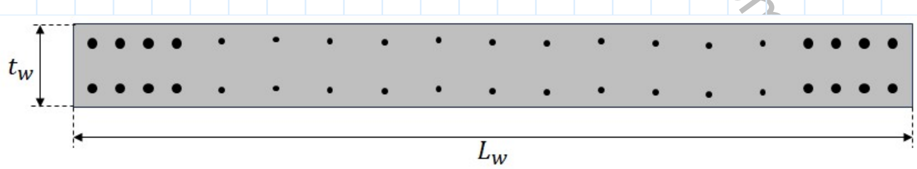
Column cross-section



Beam cross-section



Rectangular wall cross-section



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Building parameters:

$n := 10$ No. of storeys

$i := 1, 2 .. n$

$E_c := 25\ GPa$ Concrete elastic modulus

$f'_c := 30\ MPa$ Concrete strength

$h := 3.6\ m$ Typical storey height

$n_c := 4$ No. of columns

$n_b := n_c$ No. of beams

$A := (6\ m)^2 \cdot (n_c + 1)$ Floor area of a 2D frame

$mass := 8\ kPa \cdot A \cdot \frac{1}{g} = 147\ tonne$ Mass of a single floor

$W := mass \cdot n \cdot g = 14400\ kN$ Building weight

Setting floor heights:

$h_i := 3.6\ m$ Setting floor height array

$h_1 := 4.0\ m$ First floor height

$H := \sum_{i=1}^{n} h_i = 36.4\ m$ Total building height

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Column properties: (Assumed square for this example)

$h_c := 600\ mm$ Column cross-section depth

$b_c := h_c$ Column cross-section width

$I_c := \frac{b_c \cdot h_c^{\ 3}}{12}$ Second moment of area

Beam properties:

$L := 6000\ mm$ Beam length

$h_b := 400\ mm$ Beam depth

$w_b := 300\ mm$ Beam width

$I_b := \frac{w_b \cdot h_b^{\ 3}}{12}$ Second moment of area

Wall properties

$L_w := 6.0\ m$ Wall length

$t_w := 250\ mm$ Wall thickness

$I_w := \frac{L_w^{\ 3} \cdot t_w}{12}$ Second moment of area

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Calculating the storey stiffness of a frame structure. (Schultz, 1992)

$k_{c1} := \frac{I_c}{h_1}$ $\quad k_{c2} := \frac{I_c}{h_2}$ Relative column stiffness

$k_b := \frac{I_b}{L}$ Relative beam stiffness

$k_0 := 24 \cdot \frac{E_c}{h_1^{\ 2}} \cdot \frac{1}{\frac{2}{n_c \cdot k_{c1}} + \frac{1}{n_b \cdot k_b} + 0} = 33.4 \ \frac{kN}{mm}$ First floor storey stiffness

$k := 24 \cdot \frac{E_c}{h_2^{\ 2}} \cdot \frac{1}{\frac{2}{n_c \cdot k_{c2}} + \frac{1}{n_b \cdot k_b} + \frac{1}{n_b \cdot k_b}} = 23 \ \frac{kN}{mm}$ All other floors

$k_{f_i} := k$ $\quad k_{f_1} := k_0$

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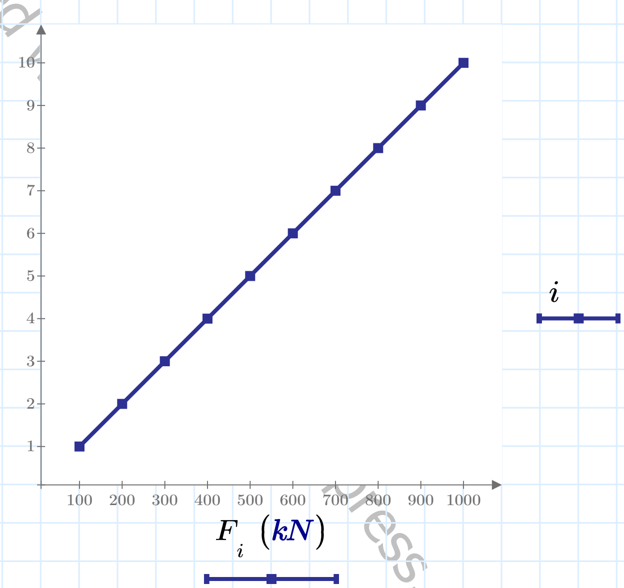
Obtaining storey stiffness of the wall

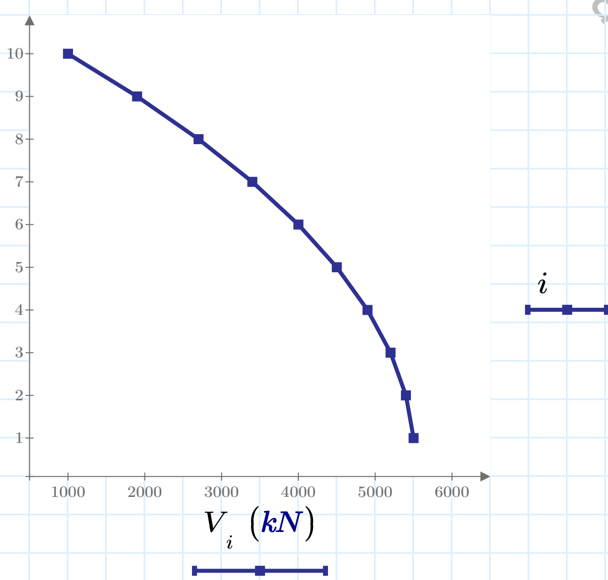
$F_i := i \cdot 100\ kN$

Applying linear force distribution up height



$V_i := \sum_{j=i}^{n} F_j$

Calculating relative shear forces at each floor



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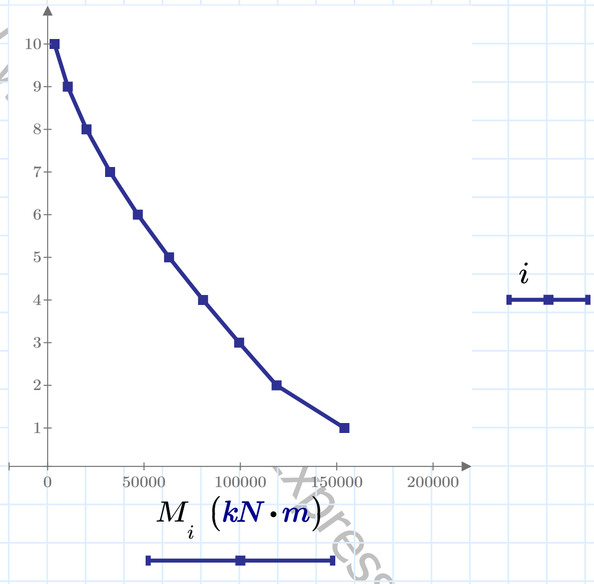
$$M_i := \sum_{j=i}^{n} V_j \cdot h_i$$

Moment demands at each floor



$$\phi_i := \frac{M_i}{E_c \cdot I_w}$$

Curvature at each floor level

$$\Delta\theta_i := \phi_i \cdot h_i$$

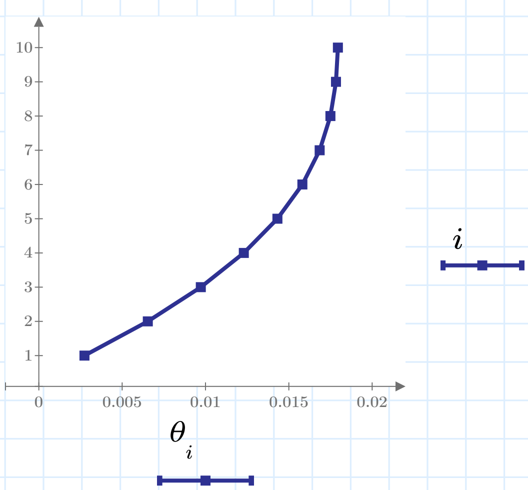
Change in slope (rotations) at each floor

$$\Delta\theta_1 := \phi_1 \cdot \frac{h_1}{2}$$

Change in slope at the base

$$\theta_i := \sum_{j=1}^{i} \Delta\theta_j$$

Storey slope (drift)



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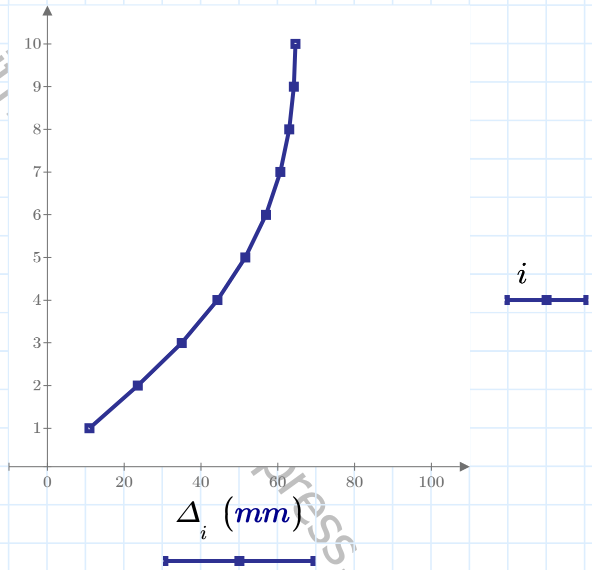
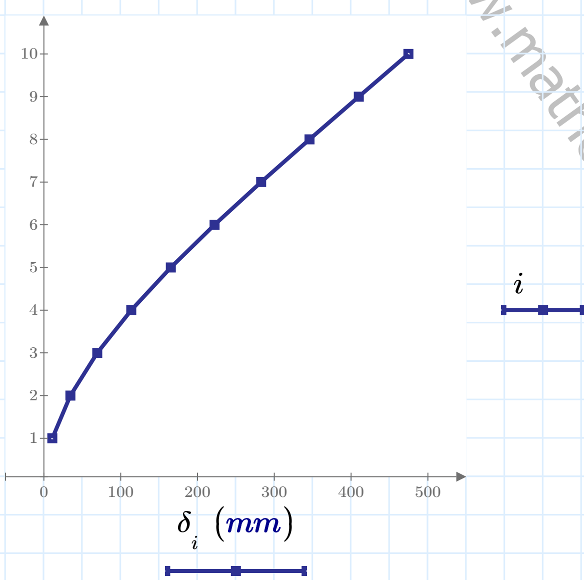
$\Delta_i := \theta_i \cdot h_i$ Inter-storey displacement



$\delta_i := \sum_{j=1}^{i} \Delta_j$ Deflected shape of cantilevered wall



$K_{w_i} := \frac{V_i}{\Delta_i}$ Wall storey stiffness

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The storey stiffness of the wall and frame at each floor is then combined and the period of the structure is estimated using Rayleigh's principle by applying an arbitrary force distribution.

$K_i := K_{w_i} + k_{f_i}$ Sum of wall and frame stiffness'

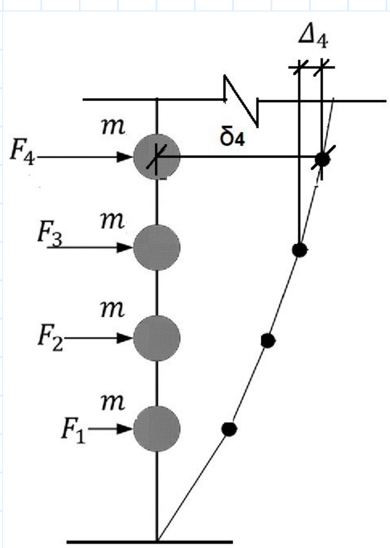
$F_i := i \cdot 100\ kN$ Arbitrary linear force distribution

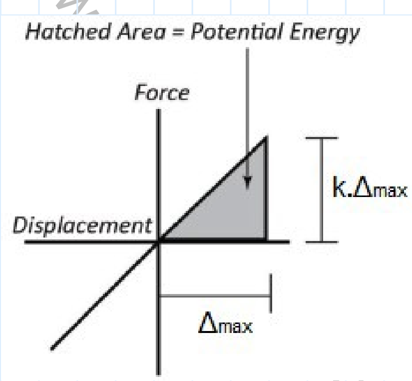
$V_i := \sum_{j=i}^{n} F_j$ Shear demand at each floor

$\Delta_i := \frac{V_i}{K_i}$ Storey deflection

$\delta_i := \sum_{j=1}^{i} \Delta_j$ Deflection at each floor up the height

Check the period of the structure using Rayleigh's method.
Max Potential Energy = PE = Area under the force displacement graph
Since we have MDOF system, potential energy at each floor need to be combined.





Potential energy equation

$$PE = \sum \frac{1}{2} \cdot k \cdot \Delta^2 = \sum \frac{1}{2} \cdot F \cdot \delta^2$$

Kinetic energy is equal to half the mass times velocity squared, where velocity, v, is equal to displacement, δ, multiplied by the angular frequency, ω

$$KE = \sum \frac{1}{2} \cdot mass \cdot v^2 \qquad v = \omega \cdot \delta$$

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Based on conservation of energy, potential energy is equal to the kinetic energy of the system

$PE = KE$ $$\sum \frac{1}{2} \cdot k \cdot \Delta^2 = \sum \frac{1}{2} \cdot mass \cdot (\omega \cdot \delta)^2$$

Angular frequency is then computed by rearranging the equation for conservation of energy

$$PE := \sum_{i=1}^{n} \left(\frac{1}{2} \cdot F_i \cdot \delta_i \right) = \left(6.1 \cdot 10^5\right) J$$ Potential energy of MDOF system

$$KE_r := \sum_{i=1}^{n} \left(\frac{1}{2} \cdot mass \cdot {\delta_i}^2 \right) = \left(2.87 \cdot 10^4\right) J \cdot s^2$$ Relative kinetic energy

$$\omega := \sqrt{\frac{PE}{KE_r}} = 5 \ \frac{1}{s}$$ Circular frequency

$$T_1 := \frac{2 \cdot \pi}{\omega} = 1.36 \ s$$ 1st mode elastic period

$$\Phi_i := \frac{\delta_i}{\delta_n} = \begin{bmatrix} 0.03 \\ 0.1 \\ 0.19 \\ 0.3 \\ 0.42 \\ 0.55 \\ 0.69 \\ 0.81 \\ 0.92 \\ 1 \end{bmatrix}$$ Mode-shape of the wall-frame structure