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SAILBOAT

(1) EGIM02 3/10/19

Advanced Computational Methods for engineers

Intro

Prof Michael G edwards
engineering central C131

Prerequisites

- Undergrad Maths
- Knowledge of MATLAB assumed.

Part 1

- Newton's method
- Numerical Integration
- Discretization of Ordinary Differential Equations (ODEs)

Part 2

- Discretation of Partial Differential Equations (All types Elliptic, Hyperbolic and Parabolic)
- Finite difference and Finite Volume methods
- Consistency, stability and convergence.
- An introduction to the Solutions of Linear systems
- Gaussian elimination
- Relaxation methods.

Assessment

- Weekly formative assignments.

If you do all the formatives, then exam should be fine.

Course text books

Check slides
↳ The key site one.   sp?

Exam
↳ 70% | 2 assignments 15% each.

Assignment 1 week 5/6 due week 8/9
Assignment 2 week 8/9 due week 11/12

collaboration is considered plagerism

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Notes 1

Newton's Method - A root finding method.

Objective: find the root of a non-linear eqn

eg. $f(x) = x^3 - 6x^2 + 11x - 6$.

The method begins with an initial guess
↳ Relies on the concept of iteration to calculate the root.

example

Projectile motion

$$\frac{d^2x}{dt^2} = 0$$

$$\frac{d^2y}{dt^2} = -g$$

equations of motion without wind resistance.

Initial data

$x(0)$, $\frac{dx}{dt} = V\cos\alpha$

$y(0) = 0$, $\frac{dy}{dt}(0) = V\sin\alpha$

~~Exact solution~~

Projectile motion with wind resistance

$$\frac{d^2x}{dt^2} = -k\frac{dx}{dt}$$

$$\frac{d^2y}{dt^2} = -g - k\frac{dy}{dt}$$

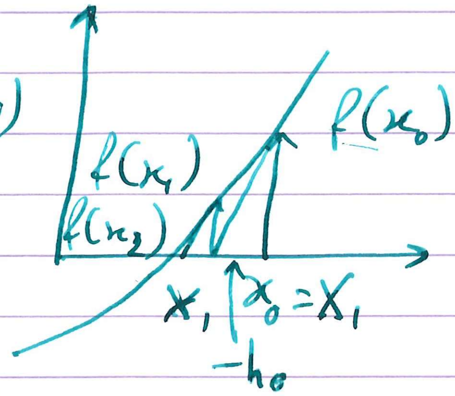
Exact solution.

$$x = \frac{V\cos\alpha}{k}\left(1 - e^{-kt}\right)$$

$$y = \frac{(Vk\sin\alpha + g)}{k^2}\left(1 - e^{(-kt)}\right) - \frac{g}{k}t$$

§. Rest of Slide, pg 5 Notes 1

Newton method: geometric construction



$f(x)$

$$f'(x_0) = \frac{f(x_0) - 0}{-h_0}$$

h_0

★ Get very familiar with Taylor Series

$$f(x+h) = f(x) + h f'(x) + \frac{h^2}{2} f''(\theta) \qquad x < \theta < x+h$$

$$f(x+h) \simeq f(x) + hf'(x) \qquad f'(x) = \frac{df(x)}{dx}$$

$$f(x) + hf'(x) = 0 \qquad f''(x) = \frac{d^2f(x)}{dx^2}$$

$$f(x_r + h_r) \simeq 0 \qquad f(x_r) + h_r f'(x_r) = 0$$

$\boxed{\text{Suffix } r \text{ notation}}$

Algorithm

Rest on Slide 6, as above. complete

Initial data $x_0 = X_1$

Choose convergence tolerance ε

loop

$$h_r = \frac{-f(x_r)}{f'(x_r)}$$

$$x_{r+1} = x_r + h_r$$

loop until $|f(x_r + h_r)| \leq \varepsilon$

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Other root finding methods.

~~Secant~~ - Secant Method. ← wont be used.

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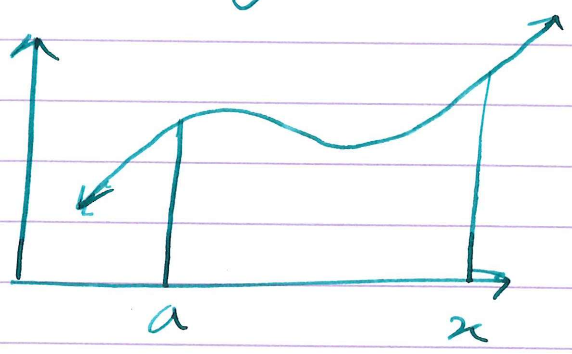
Part 2

Notes 2:

Integration.

Interpolation · Trapezoidal rule · Interval Transform ·
· Method of undetermined coefficients:
Interpolation and Numerical Integration Error
· examples
ASIDE: Error analysis.

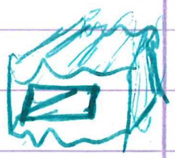
Why do we need numerical integration?
Integrate functions that cannot be integrated analytically
E.g. Error function



We can approximate $f(x)$ by an n^{th} order polynomial

Numerical integration involves the use of interpolation.
(Think trapezium rule.

☆ A lot of derivations on slides.



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Finite element Method. 1/10/19.
EGIM09.

Dr. Will Harrison.

Intro
10 credits
Lecture
Tuesday 2-4pm.] in GH
example class Thursday 1-2pm]
Optional PC lab sessions
Weeks 5-10 Monday 4-6 pm EC C109
Office hour
Monday 10-11 AM.
engineering EN 307
North

Module Objectives.
- Understand FEM in 1D and 2D context.
- and apply to simple engineering problems
All scenarios will be steady state.

Assessment
2 written assignments 10% each.
1D problems
due 4pm friday November ~~8th~~. 10th check.
2D problems
due 4pm friday Nov 29th

Exam in Jan → 80%.

full course notes on blackboard.
Chp 1 - 5 currently available.
6 - 7 to come soon.

☆ Q&A board on Blackboard ~~for~~ to get any questions answered, as apposed to emailing him.

1 EGM09 Finite element Method. 1/10/19

2 Module proper

FEM is the method.
FEA is the application of FEM

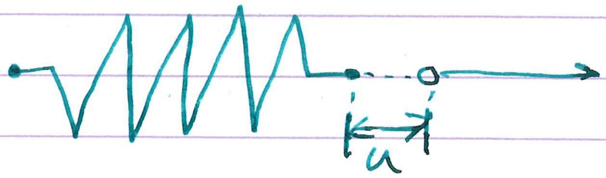
☆ code_aster
An open source FEM package.
→ all docs are in french however.

Local DoF = DoF per node.

Global DoF = DoF per node × number of nodes

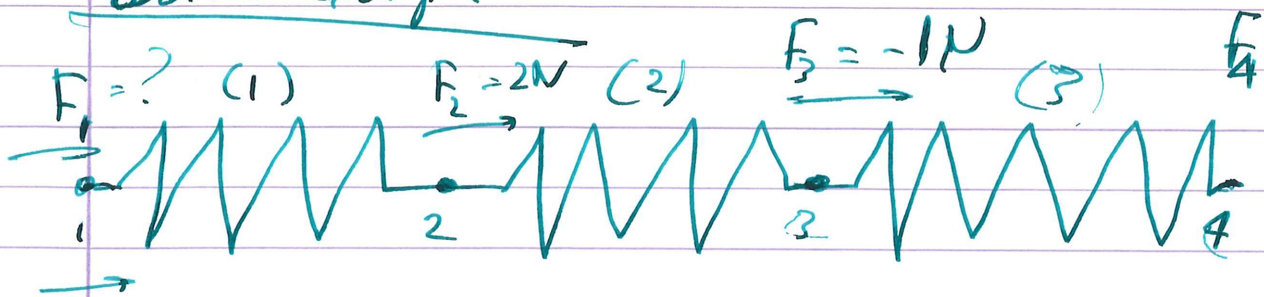
Dirichlet Boundary condition
"Fixed boundary, such as applied displacement or temperature.

Stiffness Matrix.
Spring element.



Hookes Law
$F = ku$.

Worked example.



Fixed at both ends.

$u_1 = 0$ $\quad$ $u_2 = ?$ $\quad$ $u_3 = ?$ $\quad$ $u_4 = 0$

Spring array example.

$$[K]\{u\} = \{F\}$$

Element stiffness

$$K^{(1)} = \begin{bmatrix} 20 & -20 \\ -20 & 20 \end{bmatrix}$$

$$K^{(2)} = \begin{bmatrix} 15 & -15 \\ -15 & 15 \end{bmatrix}$$

$$K^{(3)} = \begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix}$$

Assemble problem

3 FEM 1/10/19.

Assemble problem.

$$\begin{bmatrix} k^{(1)} & -k^{(1)} & 0 & 0 \\ -k^{(1)} & k^{(1)}+k^{(3)} & -k^{(3)} & 0 \\ 0 & -k^{(2)} & k^{(2)}+k^{(3)} & -k^{(3)} \\ 0 & 0 & -k^{3} & k^{3} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix}$$

$$\begin{bmatrix} 20 & -20 & 0 & 0 \\ -20 & 35 & -15 & 0 \\ 0 & -15 & 25 & -10 \\ 0 & 0 & -10 & 10 \end{bmatrix} \begin{Bmatrix} 0 \\ u_2 \\ u_3 \\ 0 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ 2 \\ -1 \\ F_4 \end{Bmatrix}$$

Ignore F_1 and F_4 rows for now.
Since ignore rows, also columns.
So ignore col{1, 4}

This produces a condensed matrix.

simplified subset.
$$\begin{bmatrix} 35 & -15 \\ -15 & 25 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 2 \\ -1 \end{Bmatrix}$$

Gaussian elimination..

row 2 $-$ $\left(\frac{-15}{35}\right)$ Row 1

$$\begin{bmatrix} 35 & -15 \\ 0 & 18.57 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 2 \\ -0.143 \end{Bmatrix}$$

Back substitute.

$$u_3 = \frac{-0.143}{18.57} = -0.00769 \text{ mm.}$$

$$35u_2 - 15u_3 = 2$$

$$u_2 = 0.0538 \text{ mm.}$$

Force balance

$$F_1 = 20u_1 = 20u_2$$
$$= -1.077\ N$$

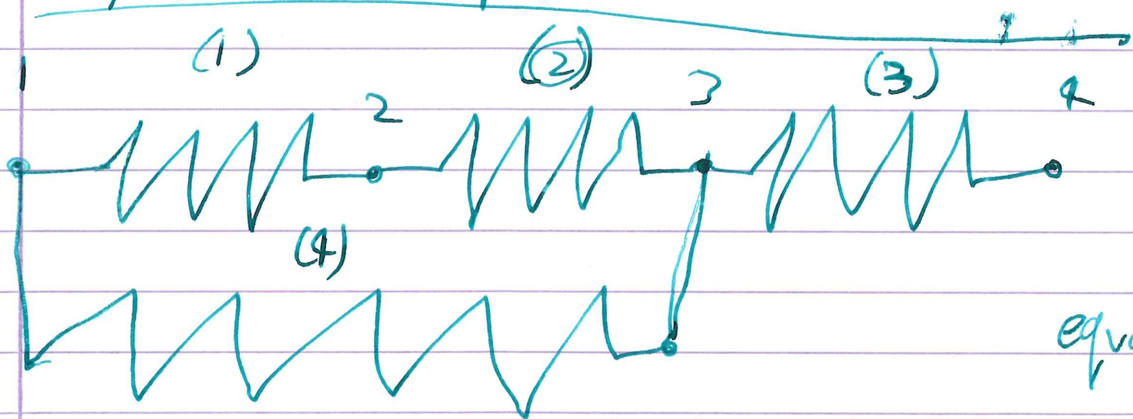
$$F_4 = -10u_3 + 10u_4$$
$$= 0.0769\ N.$$

☆ <u>for equilibrium.</u>

$$F_1 + F_2 + F_3 + F_4 = 0.$$
$$-1.077 + 2 + (-1) + 0.0769 = 0.$$

approximation.

example sheets are on Blackboard.
transient problems are not on exam,
ignore last chapter of notes.



equals

$$[K] = \begin{bmatrix} k^{(1)} + k^{(4)} & -k^{(1)} & -k^{(4)} & 0 \\ -k^{(1)} & k^{(1)} + k^{(2)} & -k^{(2)} & 0 \\ -k^{(4)} & -k^{(2)} & k^{(2)} + k^{(3)} + k^{(4)} & -k^{(3)} \\ 0 & 0 & -k^{(3)} & k^{(3)} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix}$$

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