Solutions

Group: __________ __________ __________ Name: __________

Math 231 A. Worksheet 3.

The goal is to learn how to evaluate the trig integrals of the form $\int \sin^n x \cos^m x \, dx$

1. Use substitution to evaluate $\int \sin^2 x \cos x \, dx$.

$u = \sin x \quad du = \cos x \, dx$

$$\int \sin^2 x \cos x \, dx = \int u^2 \, du = \frac{1}{3}u^3 + C = \frac{1}{3}\sin^3 x + C$$

2. The goal is to evaluate $\int \sin^2 x \cos^3 x \, dx$.

Technique: Rewrite as $\int \sin^2 x \cos^3 x \, dx = \int \sin^2 x \cos^2 x (\cos x \, dx)$. Use the identity $\sin^2 x + \cos^2 x = 1$ to write $\cos^2 x$ in terms of $\sin x$. Then make the substitution $u = \sin x$.

$$\int \sin^2 x \cos^3 x \, dx = \int \sin^2 x (1 - \sin^2 x) \cos x \, dx \qquad u = \sin x \quad du = \cos x \, dx$$

$$= \int u^2 (1 - u^2) \, du$$

$$= \int (u^2 - u^4) \, du$$

$$= \frac{1}{3}u^3 - \frac{1}{5}u^5 + C = \frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x \, dx + C$$

3. Use the idea in problem 2 to evaluate $\int \sin^3 x \cos^2 x \, dx$.

$$\int \sin^3 x \cos^2 x \, dx = \int (1 - \cos^2 x)\cos^2 x \sin x \, dx \qquad u = \cos x \quad du = -\sin x \, dx$$

$$= \int (u^4 - u^2) \, du$$

$$= \frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C$$

These ideas work on any integral $\int \sin^n x \cos^m x \, dx$ **where one of** n **or** m **is odd.**

4. The goal is to evaluate $\int \cos^2 x\,dx$.

Technique: Use the identity $\cos^2 x = \frac{1}{2}(1+\cos 2x)$ to rewrite the integral as the sum of two simpler ones.

$$\int \cos^2 x\,dx = \int \frac{1}{2}(1+\cos 2x)\,dx$$

$u = 2x \quad du = 2\,dx$

$$= \int \frac{1}{2}\,dx + \int \frac{1}{2}\cos 2x\,dx$$

$$= \frac{x}{2} + \frac{1}{4}\sin 2x + C$$

The identity $\sin^2 x = \dfrac{1}{2}(1-\cos 2x)$ **can be used to evaluate integrals like** $\int \sin^2 x\,dx$ **.**

5. Evaluate $\int \cos^4 x\,dx$.

$$\int \cos^4 x\,dx = \int \frac{1}{4}(1+\cos 2x)^2\,dx$$

$$= \frac{1}{4}\int \left(1 + 2\cos 2x + \cos^2 2x\right) dx$$

$$= \frac{x}{4} + \frac{1}{4}\sin 2x + \frac{1}{4}\int \cos^2 2x\,dx$$

$$= \frac{x}{4} + \frac{1}{4}\sin 2x + \frac{1}{8}\int (1+\cos 4x)\,dx$$

$$= \frac{3x}{8} + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$$

6. For which values of A are the integrals $\int_0^A \cos^2 x\, dx$ and $\int_0^A \sin^2 x\, dx$ equal?

$$\int_0^A \cos^2 x\, dx = \int_0^A \sin^2 x\, dx$$

$$\Leftrightarrow \int_0^A \cos^2 x\, dx = \int_0^A (1-\cos^2 x)\, dx$$

$$\Leftrightarrow 2\int_0^A \cos^2 x\, dx = \int_0^A dx$$

$$\Leftrightarrow \int_0^A \cos^2 x\, dx = \frac{A}{2}$$

$$\Leftrightarrow \left[\frac{x}{2} + \frac{1}{4}\sin 2x\right]_0^A = \frac{A}{2}$$

$$\Leftrightarrow \sin 2A = 0$$

$$\Leftrightarrow A = k\frac{\pi}{2} \text{ for } k \in \mathbb{Z}.$$

7. How might you solve $\int \sec^2 x \tan x\, dx$? How about $\int \sec^3 x \tan x$?

- Recall $\frac{d}{dx}(\tan x) = \sec^2 x$

$\int \sec^2 x \tan x\, dx$, $u = \tan x$ $du = \sec^2 x\, dx$

$= \int u\, du$

$= \frac{1}{2}u^2 + C$

$= \frac{1}{2}\tan^2 x + C$

- Recall $\frac{d}{dx}(\sec x) = \sec x \tan x$

$\int \sec^3 x \tan x\, dx$, $u = \sec x$ $du = \sec x \tan x\, dx$

$= \int u^2\, du$

$= \frac{1}{3}u^3 + C = \frac{1}{3}\sec^3 x + C.$