MATH 231. Midterm 1. September 25, 2017. B

Full Name: Solution key **Section Code (from table below):** | | | |

Net ID: ____________

Also fill and bubble your last name and middle initial and your net id on the scantron. Once the test begins, bubble in the test version on your scantron.

Instructions

- **Do not turn this page until instructed to.**
- You must not communicate with other students during this test.
- No written materials of any kind allowed.
- No phones, calculators, smart-watches or electronic devices of any kind allowed for any reason, including checking the time (you may use a simple wristwatch).
- There are many different versions of this exam.
- **When we end the exam, you must stop writing. Stay in your seat until we collect your exams.**
- Violations of academic integrity (in other words, cheating) will be taken extremely seriously.

Trig identities

$\sin^2 x + \cos^2 x = 1$, $\tan^2 x + 1 = \sec^2 x$, $\sin 2x = 2\sin x \cos x$,
$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$, $\frac{d}{dx}(\arctan x) = \frac{1}{x^2+1}$

TA	Section, Start time, Location	TA	Section, Start time, Location
Ochoa de Alazia, Itziar	ADJ, 10:00, 1 ILLINI HALL ADK, 11:00, 140 HENRY	Huynh, Chi	ADP, 8:00, 345 ALTGELD ADV, 9:00, 1 ILLINI HALL
Kim, Hee Yeon	ADC, 10:00, 147 ALTGELD ADE, 12:00, 147 ALTGELD	Mastroeni, Matthew	ADM, 1:00, 137 HENRY BLD ADR, 12:00, 143 HENRY BLD
Shin, Brian	ADL, 12:00, 136 BURRILL ADF, 1:00, 147 ALTGELD	Shinkle, Emily	ADA, 8:00, 137 HENRY BLD ADB, 9:00, 140 HENRY BLD
Valletta, Justin	ADN, 2:00, 143 HENRY BLD ADU, 3:00, 143 HENRY BLD	Wojtalewicz, Nikolas	ADG, 2:00, 145 ALTGELD ADH, 3:00, 145 ALTGELD
Merriman, Claire	ADI, 9:00, 111 DKH	Obeidin, Malik	AD1, 3:00, 173 ALTGELD
Quan, Hadrian	AD3,1:00, 159 ALTGELD	Rasekh, Nima	AD2, 9:00, 159 ALTGELD
Zhu, Heyi	ADD, 11:00, 143 HENRY BLD ADT, 1:00, 443 ALTGELD		

Question	12	13	14	Total
Score				
Grader				

This is version B. Bubble in test code B now on your scantron (bottom right corner).

For the following six multiple choice questions mark your answers on the Scantron form. Each is worth 6 points.

(1) Evaluate $\int_0^{\pi/4} \tan^5 x \sec^4 x \, dx$.

$u = \tan\theta$

$du = \sec^2\theta \, d\theta$

A. 5/12

B. 7/24

C. 11/24

D. 7/12

E. 1/24

$$= \int_0^1 u^5 (u^2+1) \, du$$

$$= \int_0^1 (u^7 + u^5) \, du$$

$$= \left[\frac{u^8}{8} + \frac{u^6}{6} \right]_0^1$$

$$= \frac{1}{8} + \frac{1}{6} - 0$$

$$= \frac{7}{24}$$

(2) Evaluate $\int_{1/3}^{1} \ln(3x) \, dx$.

By parts : $u = \ln(3x)$ $\quad dv = dx$

$du = \frac{1}{x} dx \quad v = x$

A. $3\ln 3 - 3$

B. $\ln 3$

C. $2/3$

D. 0

E. $\ln 3 - 2/3$

$$\int_{1/3}^{1} \ln(3x) \, dx = \left[x \ln(3x) \right]_{1/3}^{1} - \int_{1/3}^{1} 1 \, dx$$

$$= \ln 3 - \tfrac{1}{3} \ln(1) - 1 + \tfrac{1}{3}$$

$$= \ln 3 - \tfrac{2}{3}$$

(3) Evaluate $\int x\cos^2 x\,dx$.

A. $\frac{x^2}{2} + \frac{x\sin 2x}{2} + \frac{\cos 2x}{4} + C$

B. $\frac{x^2}{4} + \frac{x\sin 2x}{4} + \frac{\cos 2x}{8} + C$

C. $\frac{x\cos^3 x}{3\sin x} + C$

D. $\frac{x^2}{4} - \frac{x\sin 2x}{4} - \frac{\cos 2x}{4} + C$

E. $\frac{x^2}{2}\cos^2 x + C$

By parts: $u = x \qquad dv = \cos^2 x\,dx$

$du = dx \qquad v = \int \cos^2 x\,dx$

$$= \int \tfrac{1}{2}(1+\cos 2x)\,dx$$

$$= \left(\frac{x}{2} + \frac{\sin 2x}{4}\right)$$

$$\int x\cos^2 x\,dx = \frac{x^2}{2} + \frac{x\sin 2x}{4} - \int\left(\frac{x}{2} + \frac{\sin 2x}{4}\right)dx$$

$$= \frac{x^2}{2} + \frac{x\sin 2x}{4} - \frac{x^2}{4} + \frac{\cos 2x}{8} + C$$

$$= \frac{x^2}{4} + \frac{x\sin 2x}{4} + \frac{\cos 2x}{8} + C$$

(4) Which is the correct partial fraction decomposition of $\dfrac{3}{x^2(x^2+1)}$?

A. $\dfrac{A}{x} + \dfrac{Bx+C}{x^2} + \dfrac{Dx+E}{x^2+1}$, with $B = 3, D = 3$

B. $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx+D}{x^2+1}$, with $B = 3, D = -3$

C. $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx+D}{x^2+1}$, with $B = 3, D = -4$

D. $\dfrac{A}{x} + \dfrac{Bx+C}{x^2} + \dfrac{Dx+E}{x^2+1}$, with $B = 4, D = 4$

E. $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx+D}{x^2+1}$, with $B = 3, D = 1$

$$\frac{3}{x^2(x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{(x^2+1)}$$

$$\Rightarrow \quad 3 = Ax(x^2+1) + B(x^2+1) + (Cx+D)x^2$$

$$\Rightarrow \quad 3 = (A+B)x^3 + (B+D)x^2 + Ax + B$$

$\Rightarrow \quad B = 3$ and $D = -B = -3$.

(5) Using a trig substitution, the integral $\int \frac{x^2}{(x^2+4)^{3/2}}\,dx$ can be transformed into one of the integrals below. Select the correct one.

A. $\int \frac{\tan^2\theta}{\sec\theta}\,d\theta$

B. $\frac{1}{8}\int \frac{d\theta}{\tan^2\theta\sec\theta}$

C. $\int \frac{2\tan\theta}{\sec\theta}\,d\theta$

D. $\frac{1}{16}\int \frac{d\theta}{\tan^2\theta\sec\theta}$

E. $\int \frac{\tan^2\theta}{\sec^2\theta}\,d\theta$

$x = 2\tan\theta \qquad dx = 2\sec^2\theta\,d\theta$

$$\int \frac{x^2}{(x^2+4)^{3/2}}\,dx = \int \frac{4\tan^2\theta\, 2\sec^2\theta}{8\sec^3\theta}\,d\theta$$

$$= \int \frac{\tan^2\theta}{\sec\theta}\,d\theta$$

(6) Evaluate $\int_0^1 \frac{1}{x^2+6x+13}\,dx$.

A. $\ln(20/13)$

B. $\frac{1}{3}\arctan(1) - \frac{1}{3}\arctan(\frac{3}{2})$

C. $\ln(20/13) + \arctan(\frac{2}{3})$

D. $\frac{1}{2}\arctan(2) - \frac{1}{2}\arctan(\frac{3}{2})$

E. $\ln(20/13) + \frac{1}{2}\arctan(2)$

Complete the square:

$x^2+6x+13 = (x+3)^2+4$

Sub: $u = x+3 \qquad du = dx$

$$\int_0^1 \frac{1}{x^2+6x+13} = \int_3^4 \frac{du}{u^2+4}$$

Sub: $u = 2v \qquad du = 2dv$

$$\int_3^4 \frac{du}{u^2+4} = \int_{3/2}^2 \frac{2dv}{4v^2+4} = \frac{1}{2}\int_{3/2}^2 \frac{dv}{v^2+1} = \frac{1}{2}\left(\arctan(2) - \arctan(\tfrac{3}{2})\right)$$

For the next five question determine if the improper integral converges or diverges. If the integral converges then mark A (True) on the Scantron form. If it diverges mark B (False). Each question here is worth 3 points.

(7) $\displaystyle\int_1^\infty \frac{100x^2+1}{x^4+20x^2+1}\,dx.$

$$\frac{100x^2+1}{x^4+20x^2+1} < \frac{101\,x^2}{x^4} = 101\,\frac{1}{x^2}$$

Conv.

Since $\int_1^\infty \frac{1}{x^2}\,dx$ converges, so does the integral by Comparison Thm.

(8) $\displaystyle\int_1^\infty e^{-3x}(x^{200}+2)\,dx.$

$$e^{-3x}(x^{200}+1) < \frac{2x^{200}}{e^{3x}} < \frac{1}{e^x}$$

for large x.

Conv.

So integral is convergent by Comp. Thm.

(9) $\displaystyle\int_1^\infty \left(\frac{1}{x^2}+2\right)dx.$

$$\left(\frac{1}{x^2}+1\right) > 1$$

Div.

Since $\int_1^\infty 1\,dx$ diverges, so does integral

(10) $\displaystyle\int_1^\infty \frac{\sqrt{x+2}}{x^2}\,dx.$

$$\frac{\sqrt{x+2}}{x^2} < \frac{\sqrt{3x}}{x^2} = \sqrt{3}\,\frac{1}{x^{3/2}}$$

Conv.

Since $\int_1^\infty \frac{1}{x^{3/2}}\,dx$ converges, so does integral.

(11) $\displaystyle\int_0^1 \frac{\sqrt{x+2}}{x^2}\,dx.$

$$\frac{\sqrt{x+2}}{x^2} > \frac{\sqrt{x}}{x^2} = \frac{1}{x^{3/2}}$$

Div.

Since $\int_0^1 \frac{1}{x^{3/2}}\,dx$ diverges, so does integral

The next three questions are free response. You must write complete solutions and show all your work for full credit. Please write your final answer on the answer line.

(12) (11 points) Evaluate $\displaystyle\int_0^4 \frac{dx}{(x^2+16)^{3/2}}$.

Trig Sub: $x = 4\tan\theta \quad dx = 4\sec^2\theta\, d\theta$

$$\int_0^4 \frac{dx}{(x^2+16)^{3/2}} = \int_0^{\pi/4} \frac{4\sec^2\theta}{64\sec^3\theta}\, d\theta$$

$$= \frac{1}{16}\int_0^{\pi/4} \cos\theta\, d\theta$$

$$= \frac{1}{16}\Big[\sin\theta\Big]_0^{\pi/4}$$

$$= \frac{1}{16}\left[\frac{1}{\sqrt{2}} - 0\right]$$

$$= \frac{1}{16\sqrt{2}}$$

Answer: $\frac{1}{16\sqrt{2}}$

(13) (11 points) Evaluate $\displaystyle\int_0^{16} \frac{\sqrt{x}}{x+1}\,dx$.

Sub: $u = \sqrt{x}$, $2u\,du = dx$

$$\int_0^{16} \frac{\sqrt{x}}{x+1}\,dx = \int_0^4 \frac{2u^2}{u^2+1}\,du$$

$$= \int_0^4 \left(2 - \frac{2}{u^2+1}\right) du$$

$$= \Big[2u - 2\arctan(u)\Big]_0^4$$

$$= 8 - 2\arctan(4) - 0 + 2\arctan(0)$$

$$= 8 - 2\arctan(4)$$

Answer: $8 - 2\arctan(4)$

(14) Part A . (9 points) Evaluate the improper integral $\int_e^{\infty} \frac{dx}{x(\ln x)^4}$.

$$\int_e^{\infty} \frac{dx}{x(\ln x)^4} = \lim_{t\to\infty} \int_e^{t} \frac{dx}{x(\ln x)^4}$$

Sub $u = \ln x \quad du = \frac{1}{x}dx$

$$\int_e^{t} \frac{dx}{x(\ln x)^4} = \int_1^{\ln t} \frac{du}{u^4} = \left[\frac{-1}{3u^3}\right]_1^{\ln t} = -\frac{1}{3(\ln t)^3} + \frac{1}{3}$$

$$\int_e^{\infty} \frac{dx}{x(\ln x)^4} = \lim_{t\to\infty}\left(-\frac{1}{3(\ln t)^3} + \frac{1}{3}\right) = 1/3$$

Answer: $1/3$

Part B. (3 points) Suppose that $h(x) \geq \frac{1}{x^2}$ for all $x \geq 1$. Since we know how $\int_1^{\infty} \frac{1}{x^2}\,dx$ behaves, what does the Comparison Theorem tell us about $\int_1^{\infty} h(x)\,dx$?

Circle one:

a) That $\int_1^{\infty} h(x)\,dx$ diverges.

b) That $\int_1^{\infty} h(x)\,dx$ converges.

c) The Comparison Theorem says nothing in this case.

$\int_1^{\infty} \frac{1}{x^2}dx$ converges, so would need $h(x) \leq \frac{1}{x^2}$ to use to use Comparison Thm.