University of Illinois at Urbana-Champaign Dept. of Electrical and Computer Engineering

ECE 120: Introduction to Computing

Extra Slides

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I Need Your Help

Do you ever watch movies?

My friends want to have a movie club.

There are three coming out that we're considering...

Jackie Chan ... funny

Wild Life ... animation, also funny

Beatles documentary

The problem: we're all pretty picky. If we can't agree in advance which ones to watch, we won't do the club.

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Which Movies Should We Watch?

Jackie Chan ... funny Wild Life ... animation, also funny **Beatles documentary**

Me: Jackie Chan came out in Asia LAST month, so I won't watch again. I do want to watch at least one movie.

Jan: Three is too many.

Alice: Let's watch exactly one comedy. Beatles or no Beatles is fine.

Bob: I love the Beatles, so we have to see that

one.

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Can You Translate My Needs To Boolean?

J = Jackie Chan ... funny

W = Wild Life ... animation, also funny

B = Beatles documentary

Me: Jackie Chan came out in Asia LAST month, so I won't watch again. I do want to watch at least one movie.

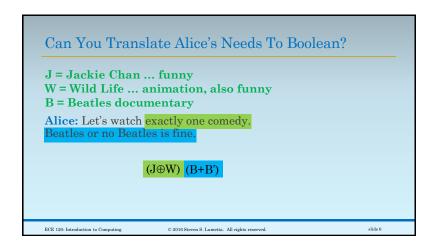
J'(J + W + B)

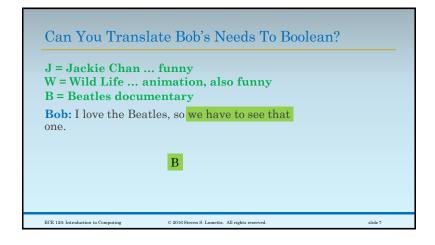
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Can You Translate Jan's Needs To Boolean? J = Jackie Chan ... funny W = Wild Life ... animation, also funny B = Beatles documentary Jan: Three is too many. (J' + W' + B')





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Me: J'(J+W+B)	J	W	В	Movie	$\mathbf{s}_{\underline{}}$
Jan: (J'+W'+B')	0	0	0	0	
Alice: (J⊕W)	0	0	1	0	
Bob: B	0	1	0	0	
Бор. Б	0	1	1	1	
We need to satisfy ALL	1	0	0	0	
four people AND!	1	0	1	0	
So: Wild Life and the	1	1	0	0	
Beatlesthanks!	1	1	1	0	

Long Definition for Overflow of 2's Complement Addition

Recall the overflow condition **V** for **2's complement** addition.

Add two N-bit 2's complement patterns.

We can calculate

$$V = ABS' + A'B'S$$

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Another Way to Define 2's Complement Overflow?

Other lectures saw a different condition. Let's first name two of the carry bits.

$$\begin{array}{c} \textbf{C}_{\textbf{N}} \ \textbf{C}_{\textbf{N-1}} \\ \textbf{A} \ \textbf{a}_{\textbf{N-2}} \ \dots \ \textbf{a}_{\textbf{0}} \ (\text{sign bit is A}) \\ \textbf{+} \ \textbf{B} \ \textbf{b}_{\textbf{N-2}} \ \dots \ \textbf{b}_{\textbf{0}} \ (\text{sign bit is B}) \\ \hline \textbf{S} \ \textbf{s}_{\textbf{N-2}} \ \dots \ \textbf{s}_{\textbf{0}} \ (\text{sign bit is S}) \end{array}$$

The other lectures were then told that

$$V = C_N \oplus C_{N-1}$$

Are these two expressions the same?

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1:1.10

One Proof Strategy: Algebra

We can use Boolean algebra to prove that

$$V = ABS' + A'B'S$$
 equals $C_N \oplus C_{N-1}$

But it's not really so fun.

Trust me, I did it.

What about brute force? (a truth table)

We can calculate S and C_N from A, B, and C_{N-1} , so we only have 3 variables as "inputs."

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Proof by Exhaustion / Brute Force

A	В	C_{N-1}	$\mathbf{C}_{\mathbf{N}}$	\mathbf{S}	\mathbf{V}	$C_N \oplus C_{N-1}$
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	0	1	0	0
0	1	1	1	0	0	0
1	0	0	0	1	0	0
1	0	1	1	0	0	0
1	1	0	1	0	1	1
1	1	1	1	1	0	0

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Always Choose the Right Proof Strategy

Always choose the **clearest and fastest proof strategy** (usually those two metrics correlate).

Using brute force for proofs doesn't make you a brute!

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