Mock Exam 3, Math 241, Spring 2018

1. (a) Find the gradient vector field of $f(x,y,z) = xze^{yz} + y^2 + yz\sin x$.

   (b) State the Fundamental Theorem of Line Integrals.

   (c) $\vec{F}(x,y,z) = \langle\, yz + 3(y+z)\,,\, xz + 3(x+z)\,,\, xy + 3(x+y)\,\rangle$ is conservative. Let C be the curve parameterized by $\vec{r}(t) = \langle\, t\,,\, t^2 - 1\,,\, t^3\,\rangle$ for $0 \le t \le 1$. Evaluate $\int_C \vec{F}\cdot \mathrm{d}\vec{r}$.

   (circle one): (A) -4; (B) 4; (C) 3; (D) 9;

   (d) Let $\vec{F}(x,y,z)$ be a vector field, with $\int_C \vec{F}\cdot \mathrm{d}\vec{r} = \pi$ for the curve C parameterized by $\vec{r}(t) = \langle 6\,,\, \cos 2t\,, \sin 2t\rangle$ for $0 \le t \le \pi$. TRUE or FALSE: $\vec{F}$ is conservative.

   (circle one): (A) True; (B) False;

   (e) Which of the three vector fields:

      i. $\vec{F}(x,y) = \langle\,(1+xy)e^{xy}\,,\, x^2e^{xy}\,\rangle$

      ii. $\vec{G}(x,y) = \langle\, x^3y^2 + 2\,,\, x^2y^3 + 2\,\rangle$

      iii. $\vec{H}(x,y) = \langle\, ye^x + \sin y\,,\, e^x + x\cos y\,\rangle$

   are conservative? Find a potential function for those which are conservative.

2. Set up (but do **NOT** evaluate) a double integral to calculate the volume of the solids E and S, where:

   (a) E is the solid under the surface $z = x^2 + 2y^4$ and above the region $D = \{(x,y,0)\,|\,-1 \le x \le 1\,,\, x^2 \le y \le 2x^2\}$.

   $$\int_{\boxed{\quad}}^{\boxed{\quad}} \int_{\boxed{\quad}}^{\boxed{\quad}} \int_{\boxed{\quad}}^{\boxed{\quad}} \boxed{\quad}\, \mathrm{d}z\mathrm{d}y\mathrm{d}x$$

   (b) S is the solid under the plane $3x + 2y - z = 0$ and above the region in the xy-plane enclosed by $y = x^2$ and $x = y^2$.

   $$\int_{\boxed{\quad}}^{\boxed{\quad}} \int_{\boxed{\quad}}^{\boxed{\quad}} \int_{\boxed{\quad}}^{\boxed{\quad}} \boxed{\quad}\, \mathrm{d}z\mathrm{d}\boxed{\quad}\,\mathrm{d}\boxed{\quad}$$

3. Evaluate the following integrals by reversing the order of integration.

   (a)

$$\int_0^1 \int_{3y}^3 e^{x^2}\,\mathrm{d}x\mathrm{d}y$$

(circle one): (A) $\frac{(e^9-1)}{3}$; (B) $\frac{(e^9-1)}{6}$; (C) $\frac{e^9}{3}$; (D) $\frac{e^9}{6}$;

   (b)

$$\int_0^2 \int_{\frac{y}{2}}^1 y\cos(x^3-1)\,\mathrm{d}x\mathrm{d}y$$

(circle one): (A) $\frac{2}{3}\sin(1)$; (B) $\frac{2}{3}\sin(-1)$; (C) $\sin(1)$; (D) $\sin(-1)$;

4. Find the mass of the lamina the occupies the region R bounded by the semicircle $x=-\sqrt{4-y^2}$ and the y-axis, whose density at any point in R is $\rho(x,y)=e^{-x^2-y^2}$.

(circle one): (A) $\frac{\pi(1-e^{-4})}{2}$; (B) $\pi(1-e^{-4})$; (C) $\frac{\pi(1-e^{-4})}{4}$; (D) $\frac{\pi(1-e^{-4})}{2}$;

5. Use polar coordinates to combine the sum

$$\int_0^{\sqrt{2}} \int_0^x xy\,\mathrm{d}y\mathrm{d}x + \int_{\sqrt{2}}^2 \int_0^{\sqrt{4-x^2}} xy\,\mathrm{d}y\mathrm{d}x$$

   into one double integral. Then evaluate the double integral.

(circle one): (A) $\frac{1}{3}$; (B) $\frac{4}{3}$; (C) $\frac{2}{3}$; (D) 1;

6. Set up (but do **NOT** evaluate) the triple integral

$$\iiint_E 6xy\,\mathrm{d}V,$$

   as an iterated integral, where E lies under the plane $z=1+x+y$ and above the region in the xy-plane bounded by the curves $y=\sqrt{x}$, $y=0$, and $x=1$.

$$\int_{\square}^{\square}\int_{\square}^{\square}\int_{\square}^{\square} 6xy\,\mathrm{d}\square\,\mathrm{d}\square\,\mathrm{d}\square$$

7. Evaluate $\iiint_E (2y - x)\,\mathrm{d}V$, where E is the solid that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 16$, above the xy-plane, and below the plane $z = y + 4$.

(circle one): (A) $\frac{255\pi}{2}$; (B) $\frac{255\pi}{4}$; (C) $\frac{-255\pi}{4}$; (D) $\frac{-255\pi}{2}$;

8. Let $R = \{(x, y, z) \mid 2 \le z \le 2 + \sqrt{4 - x^2 - y^2}\}$. Set up the iterated integrals to find the volume of region R in rectangular, cylindrical, and spherical coordinates, then calculate the volume of R.

9. Let R be the region bounded by the ellipse $4x^2 + 9y^2 = 36$. Evaluate

$$\iint_R y^2\,\mathrm{d}A.$$

10. Let $S = \{(u, v) : 0 \le u \le 1, 0 \le v \le 1\}$, and $R \subset \mathbb{R}^2$ be the parallelogram with vertices $(0,0)$, $(4,-3)$, $(5,4)$ and $(9,1)$.

    (a) Find a transformation $T(u, v) = (x(u, v), y(u, v))$ which maps S to R.

    (b) Use the transformation from part (a), along with change of variables, to evaluate

$$\iint_R (x + xy)\,\mathrm{d}A.$$

Trigonometric Identities

$$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$
$$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$
$$\tan(\theta+\phi) = \frac{\tan(\theta)+\tan(\phi)}{1-\tan\theta\tan\phi}$$
$$\sin(\theta-\phi) = \sin\theta\cos\phi - \cos\theta\sin\phi$$
$$\cos(\theta-\phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$$
$$\tan(\theta-\phi) = \frac{\tan(\theta)-\tan(\phi)}{1+\tan\theta\tan\phi}$$
$$\sin(\theta)\sin(\phi) = \frac{\cos(\theta-\phi)-\cos(\theta+\phi)}{2}$$
$$\cos(\theta)\cos(\phi) = \frac{\cos(\theta-\phi)+\cos(\theta+\phi)}{2}$$
$$\sin(\theta)\cos(\phi) = \frac{\sin(\phi+\theta)-\sin(\phi-\theta)}{2}$$
$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$
$$\sin\theta = \pm\sqrt{\frac{1-\cos(2\theta)}{2}}$$
$$\cos\theta = \pm\sqrt{\frac{1+\cos(2\theta)}{2}}$$
$$\tan\theta = \frac{\sin(2\theta)}{1+\cos(2\theta)}$$