Group: ______________ ______________ ______________ Name: Key

Math 231 A. Fall, 2014. Worksheet 18. 11/20/14

1. Parametric equations for a curve are given by $x = \cos t$, $y = \sin^2 t$.

$1 = \cos^2 t + \sin^2 t$
$= x^2 + y$

a) Eliminate the parameter t to find an equation which involves only x and y.

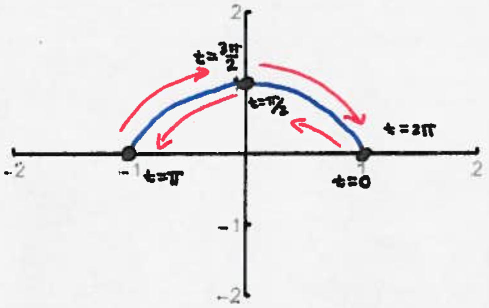
So, $y = 1 - x^2$

b) What range of values of x arise? What range of values of y arise? Make a **neat, careful** sketch of this portion of the graph.

t	x	y
0	1	0
$\pi/2$	0	1
π	-1	0
$\frac{3\pi}{2}$	0	1
2π	1	0

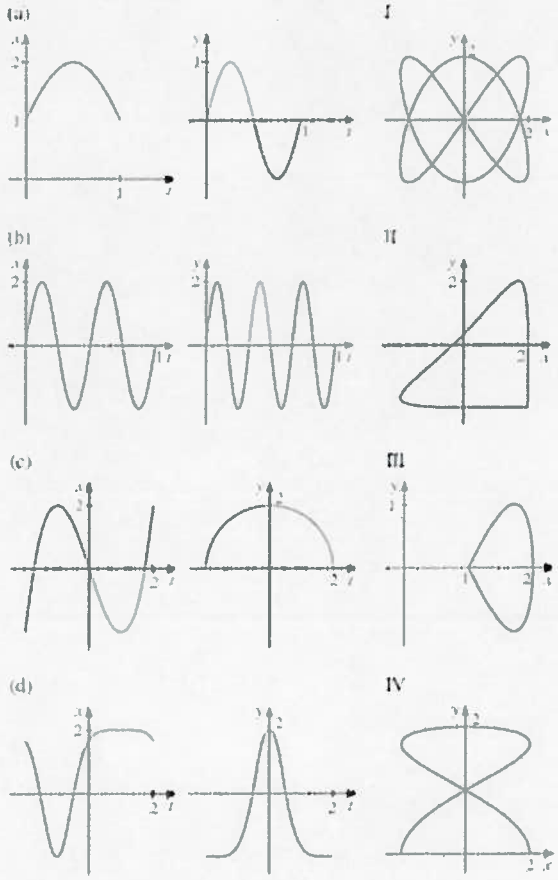
$x = \cos t$
So $-1 \leq x \leq 1$
$y = \sin^2 t$
So $0 \leq y \leq 1$

So, not all of $y = 1 - x^2$ shows up!



c) Now think of t as representing time, and suppose that the equations describe the motion of a particle. Describe the motion of the particle as t moves through the range $0 \leq t \leq 2\pi$. Sketch the motion **carefully** on the graph above using arrows to indicate direction.

24. Match the graphs of the parametric equations $x = f(t)$ and $y = g(t)$ in (a)–(d) with the parametric curves labeled I–IV. Give reasons for your choices.



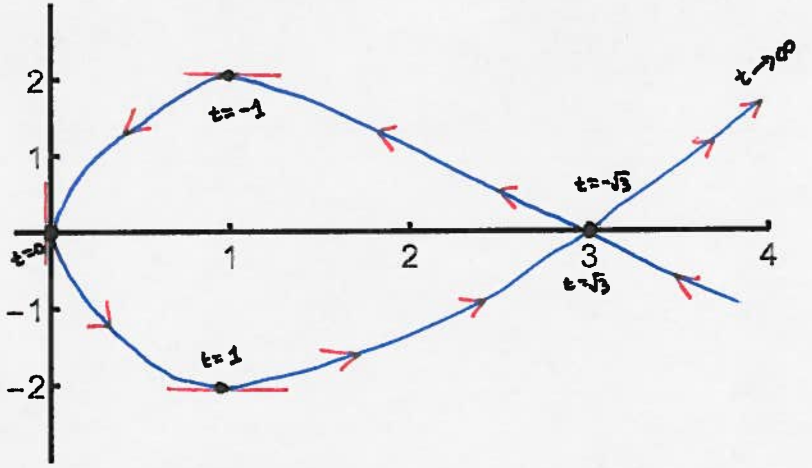
a) x has to lie between 1 and 2. So III only choice.

b) x and y both begin at 0, so I only choice. Also, x and y cycle many times as t goes from 0 to 1, so we expect many loops. Again I only choice.

c) y is always positive, so IV only choice.

d) x drops from 2 to -2, then comes back up and stays at 2 for a while. On the other hand, y loops from -2 to 2 and back. So II only choice.

2. Consider the curve $x(t) = t^2$, $y(t) = t^3 - 3t$, for $-\infty < t < \infty$.



(a) Find all t that give x intercepts and y-intercepts, and plot them.

x-intercepts : $0 = y(t) = t^3 - 3t = t(t^2-3) \Rightarrow t = 0, \pm\sqrt{3}$

y-intercepts : $0 = x(t) = t^2 \Rightarrow t = 0$

$t = 0 \quad (x(0), y(0)) = (0,0)$

$t = \pm\sqrt{3} \quad (x(\pm\sqrt{3}), y(\pm\sqrt{3})) = (3,0)$

(b) Find all t which give horizontal or vertical tangents, and plot the corresponding points, with a short horizontal or vertical segment to indicate the tangent line.

$\frac{dx}{dt} = 2t = 0 \Rightarrow t = 0$ (vertical tangent at $(x(0), y(0)) = (0,0)$)

$\frac{dy}{dt} = 3t^2 - 3t = 3(t^2-1) = 0 \Rightarrow t = \pm 1$ (horizontal tangents at $(x(1), y(1)) = (1,-2)$
$(x(-1), y(-1)) = (1, 2)$

(c) Find the values of t for which $x(t)$ is increasing and those for which it is decreasing. Do the same for y.

$$\frac{dx}{dt} = 2t : \quad \begin{array}{ccc} - & & + \\ \hline & 0 & \end{array}\ t$$

x(t) increasing for $t > 0$
& decreasing for $t < 0$

$$\frac{dy}{dt} = 3(t^2-1) : \quad \begin{array}{ccccc} + & & - & & + \\ \hline & -1 & & 1 & \end{array}\ t$$

y(t) increasing for $|t| > 1$
& decreasing for $|t| < 1$

(d) Determine what happens to $x(t)$ and $y(t)$ as $t \to \infty$.

$$\lim_{t\to\infty} x(t) = \lim_{t\to\infty} t^2 = \infty \qquad \lim_{t\to\infty} y(t) = \lim_{t\to\infty} (t^3 - 3t) = \infty$$

(e) Make a **neat, careful sketch** of the graph on the axes above. Use arrows which indicate the direction, and label all important values of t. Your graph must be consistent with the information in parts (a)-(d).