Tuesday, April 10 $**$ *Surface Parameterpalooza*

1. Let S be the portion of the plane $x + y + z = 1$ which lies in the positive octant.

   (a) Draw a picture of S.

   (b) Find a parameterization $\mathbf{r}\colon D \to S$, being sure to clearly indicate the domain D. Check your answer with the instructor.

   (c) Use your answer in (b) to compute the area of S via an integral over D.

   (d) Check your answer in (c) using only things you learned in the first few weeks of this class.

2. Consider the surface S which is the part of $z + x^2 + y^2 = 1$ where $z \geq 0$.

   (a) Draw a picture of S.

   (b) Find a parameterization $\mathbf{r}\colon D \to S$. Check your answer with the instructor.

3. Let S be the surface given by the following parameterization. Let $D = [-1, 1] \times [0, 2\pi]$ and define

$$\mathbf{r}(u, v) = (u \cos v, u \sin v, v).$$

   (a) Consider the vertical line segment $L = \{u = 0\}$ in D. Describe geometrically the image of L under $\mathbf{r}$.

   (b) Repeat for the vertical segments where $u = -1$ and $u = 1$.

   (c) Use your answers in (a) and (b) to make a sketch of S.

4. Consider the ellipsoid E given by $\dfrac{x^2}{9} + \dfrac{y^2}{4} + z^2 = 1$.

   (a) Draw a picture of E.

   (b) Find a parameterization of E. Hint: Find a transformation $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ which takes the unit sphere S to E, and combine that with our existing parameterization of the plain sphere S.