Group: __________ __________ __________ Name: solutions.

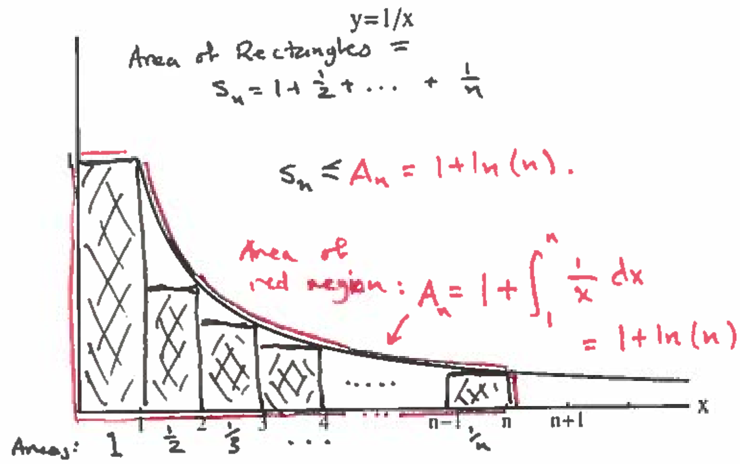
Math 231 A. Fall, 2015. Worksheet 11. 10/13/15

1. Recall that the harmonic series is $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$. Let $s_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$ be the nth partial sum.

a) What is $\lim_{n\to\infty} s_n$?

limit diverges to infinity.

b) Draw a careful picture on the graph to the right which illustrates that $s_n \le 1 + \ln n$. Be sure that your reasoning is explained.



c) Suppose that you were to add 3,000,000,000 terms of the harmonic series. Show that the sum would be less than 23.

$$s_{3000000000} \le 1 + \ln(3000000000) = 22.822 < 23.$$

Recall that if the integral test proves that a series converges, then

$$\int_{n+1}^{\infty} f(x)\,dx < R_n < \int_n^{\infty} f(x)\,dx \qquad \text{and} \qquad S_n + \int_{n+1}^{\infty} f(x)\,dx < S < S_n + \int_n^{\infty} f(x)\,dx$$

where $a_n = f(n)$, $S = \sum_{n=1}^{\infty} a_n$, $S_n = a_1 + a_2 + \dots + a_n$, and $R_n = S - S_n$.

2. How many terms of the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ would you need to add to estimate the sum to within 0.01? (Hint: you want $R_n < 0.01$.)

Need $R_n < 0.01$. so we need $\int_n^{\infty} \frac{1}{x(\ln(x))^2}\,dx < 0.01$

$u = \ln(x)$, $du = \frac{1}{x}dx$

$$\int_{\ln(n)}^{\infty} \frac{1}{u^2}\,du = \lim_{b\to\infty} -u^{-1}\Big|_{\ln(n)}^{b} = \frac{1}{\ln(n)}.$$

when is:

$R_n < \frac{1}{\ln(n)} < 0.01$? when $\ln(n) > 100$

so when $n > e^{100}$

roughly 2.7×10^{43} terms are needed for the partial sum to be within $\frac{1}{100}$ of the actual sum

3. a) Estimate the maximum possible error when the 20th partial sum $\sum_{n=1}^{20} \frac{1}{n^3}$ is used to estimate the sum $s = \sum_{n=1}^{\infty} \frac{1}{n^3}$.

max error: $R_n < \int_n^\infty \frac{1}{x^3}\,dx = \lim_{b\to\infty} \left.\frac{-x^{-2}}{2}\right|_n^\infty = \frac{1}{2n^2}$

so the max. possible error for the 20^{th} partial sum is $\frac{1}{800}$

b) The 20th partial sum is $s_{20} \approx 1.200867842\ldots$ Find a short interval (a, b) which contains s.

$$s_{20} + \int_{21}^\infty \frac{1}{x^3}\,dx < s < s_{20} + \int_{20}^\infty \frac{1}{x^3}\,dx$$

$$s_{20} + \frac{1}{2(21)^2} < s < s_{20} + \frac{1}{2(20)^2}$$, so s is in the interval

$(1.202001 , 1.202118)$

4. For which p does a p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converge? it converges for $p>1$

For which r does a geometric series $\sum_{n=0}^{\infty} ar^n$ converge? it converges for $|r|<1$

5. Use the Comparison Test to determine if the following series converge.

a) $\sum_{n=1}^{\infty} \frac{4^n+3}{5^n+n} = \sum_{n=1}^{\infty} \frac{4^n}{5^n+n} + \sum_{n=1}^{\infty} \frac{3}{5^n+n} \leq \sum_{n=1}^{\infty} \frac{4^n}{5^n} + \sum_{n=1}^{\infty} \frac{3}{5^n}$

$= \underbrace{\sum_{n=1}^{\infty} \left(\frac{4}{5}\right)^n}_{\text{converges, } |\frac{4}{5}|<1} + \underbrace{3\sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^n}_{\text{converges, } |\frac{1}{5}|<1}$ so the series converges

b) $\sum_{n=1}^{\infty} \frac{\sqrt{n^2+1}}{n^2+3} \geq \sum_{n=1}^{\infty} \frac{\sqrt{n^2}}{n^2+3} \geq \sum_{n=1}^{\infty} \frac{n}{n^2+3n^2} = \sum_{n=1}^{\infty} \frac{n}{4n^2}$

$= \underbrace{\frac{1}{4}\sum_{n=1}^{\infty} \frac{1}{n}}_{\text{harmonic or p test, } p=1}$. this diverges, so the series diverges.

c) $\sum_{n=3}^{\infty} \frac{\ln(n)+\sin(n)}{n^2} \leq \sum_{n=3}^{\infty} \frac{\ln(n)+1}{n^2}$ } pos, dec, cont.

look at: $\int_3^\infty \frac{\ln(x)+1}{x^2}\,dx$, $\left[\begin{array}{l} u=\ln(x) \\ du=\frac{1}{x}dx \\ x=e^u \end{array}\right]$ $\int_{\ln(3)}^\infty \frac{u+1}{e^u}\,du = \int_{\ln(3)}^\infty \frac{u}{e^u}\,du + \int_{\ln(3)}^\infty \frac{1}{e^u}\,du$

$= \lim_{b\to\infty} -(u+1)e^{-u}\Big|_{\ln(3)}^{b} + \lim_{b\to\infty} -e^{-u}\Big|_{\ln(3)}^{b} = \frac{1}{3}(\ln(3)+1) + \frac{1}{3}$ converges,

so the series converges.