Group: solutions. ____________ ____________ ____________ Name: ____________

Math 231 A. Fall 2015. Worksheet 2. 8/27/15

1. Evaluate using integration by parts

(a) $\int \arctan x \, dx$ $= x \arctan x - \int \frac{x}{1+x^2}\, dx$

parts: $\left[\begin{array}{ll} u = \arctan x & v = x \\ du = \frac{1}{1+x^2}\, dx & dv = dx \end{array} \right]$

$[u = x^2, \ \frac{1}{2} du = x\, dx]$ sub.

$= x \arctan x - \frac{1}{2} \int \frac{1}{1+u}\, du$

$= x \arctan x - \frac{1}{2} \ln(1+u) + c$

$= x \arctan x - \frac{1}{2} \ln(1+x^2) + c$.

(b) $\int \frac{\ln x}{x^2}\, dx$ $= \frac{-\ln(x)}{x} - \int \frac{-1}{x^2}\, dx$

parts: $\left[\begin{array}{ll} u = \ln(x) & v = \frac{-1}{x} \\ du = \frac{1}{x}\, dx & dv = \frac{1}{x^2}\, dx \end{array} \right]$

$= \frac{-\ln(x)}{x} + \int \frac{1}{x^2}\, dx$

$= \frac{-\ln(x)}{x} - \frac{1}{x} + c$.

(c) $\int x \tan^2 x \, dx$. (**Hint: Start by using $\tan^2 x + 1 = \sec^2 x$.**)

$= \int x\,(\sec^2 x - 1)\, dx = \int x \sec^2 x\, dx - \int x\, dx = x \tan x - \int \tan x\, dx - \int x\, dx$

parts: $\left[\begin{array}{ll} u = x & v = \tan x \\ du = dx & dv = \sec^2 x\, dx \end{array} \right]$

$= x \tan x + \int \frac{-\sin x}{\cos x}\, dx - \frac{1}{2} x^2$

$[u = \cos x, \ du = -\sin x\, dx]$ sub.

$= x \tan x + \int \frac{1}{u}\, du - \frac{1}{2} x^2 = x \tan x + \ln|u| - \frac{1}{2} x^2 + C$

$= x \tan x + \ln|\cos(x)| - \frac{1}{2} x^2 + C$.

2. Use "process of elimination" to evaluate the integral $\int \frac{\ln(x^2+1)}{x^2}\,dx$.

(a) Try substitution. Write down two possible choices of u. For each choice, either solve the problem or explain clearly why the choice does not work.

1. $u = x^2$, $du = 2x\,dx$, $\frac{du}{2\sqrt{u}} = dx$ $\Big\}$ $\int \frac{\ln(u+1)}{2u^{3/2}}\,du$

Not. simpler. X

2. $u = \ln(x^2+1)$

$du = \frac{2x}{x^2+1}\,dx \longrightarrow$ can't make this expression show up in the integral.

X

(a) Try parts. Write down two possible choices of u and dv. For each choice, either solve the problem or explain clearly why the choice does not work.

$$\begin{bmatrix} u = \ln(x^2+1) & v = \frac{-1}{x} \\ du = \frac{2x}{x^2+1}\,dx & dv = \frac{1}{x^2}\,dx \end{bmatrix}$$

$$\int \frac{\ln(x^2+1)}{x^2}\,dx = \frac{-\ln(x^2+1)}{x} - \int \frac{-2x}{x^3+x}\,dx = \frac{-\ln(x^2+1)}{x} + 2\int \frac{1}{x^2+1}\,dx$$

$$= \frac{-\ln(x^2+1)}{x} + 2\tan^{-1}(x) + C.$$

3. Use "process of elimination" to evaluate the integrals.

(a) $\int x^3\sqrt{x^2+1}\,dx$ sub: $[u = x^2+1,\ du = 2x\,dx]$

so: $x^2 = u-1$ $\quad \frac{1}{2}du = x\,dx$

$$= \int x^2 \cdot x \cdot \sqrt{x^2+1}\,dx = \frac{1}{2}\int (u-1)\sqrt{u}\,du = \frac{1}{2}\int u^{3/2} - u^{1/2}\,du$$

$$= \frac{1}{2}\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right) + c = \frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2} + C$$

[probably can do parts, too].

(b) $\int e^{\sqrt{x}}\,dx$

sub: $\left[u = \sqrt{x} \quad du = \frac{1}{2\sqrt{x}}\,dx\right]$

so $2u\,du = dx$

$$= 2\int u e^u\,du = 2\int t e^t\,dt = 2\left(te^t - \int e^t\,dt\right) = 2(te^t - e^t) + c$$

(to use u, v for parts) $\begin{bmatrix} u = t & v = e^t \\ du = dt & dv = e^t\,dt \end{bmatrix}$ $= 2\sqrt{x}\,e^{\sqrt{x}} - 2e^{\sqrt{x}} + C$