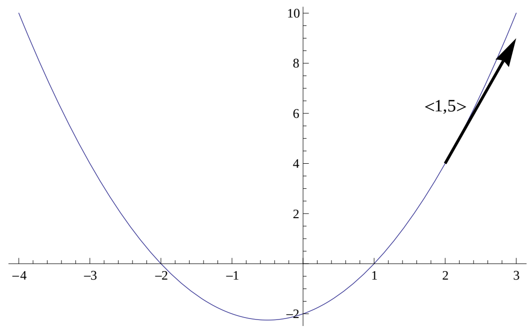
Thursday, January 18 $*$ **Solutions** $*$ *Parametric Curves Defined Using Vector Arithmetic*

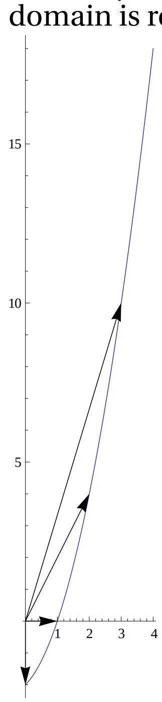
1. (a) Plot of $f(x) = x^2 + x - 2$



(b) $f'(x) = 2x + 1$, so the equation for the tangent line to $f(x)$ at $x = 2$ is $T(x) = f(2) + f'(2)(x - 2) = 4 + 5(x - 2) = 5x - 6$.

(c) A vector in the direction of the tangent line has a slope of 5, so the vector $\langle 1, 5 \rangle$ is a good choice. It is shown on the graph above based at $(2, 4)$.

2. (a) Plot of $\begin{cases} x = t \\ y = t^2 + t - 2 \end{cases}$ for $0 \le t < 4$. This is different from the graph above because the domain is restricted.

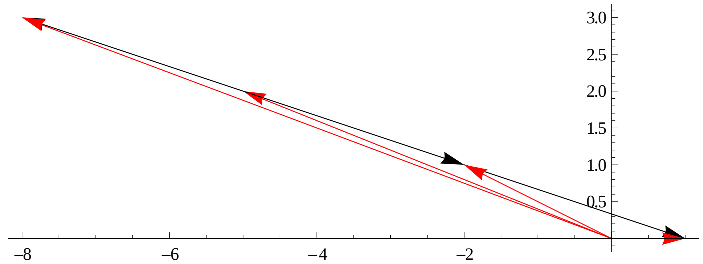


(b) The vectors based at $(0, 0)$ and ending at $(x(t), y(t))$ for $t = 0, 1, 2, 3$ are shown on the graph above.

(c) $\left\langle x'(2), y'(2) \right\rangle = \langle 1, 5 \rangle$. This represents velocity - this vector is shown on the curve in the graph below 1.a.

(d) The speed of the particle is the magnitude of the velocity, or $\sqrt{1^2 + 5^2} = \sqrt{26}$.

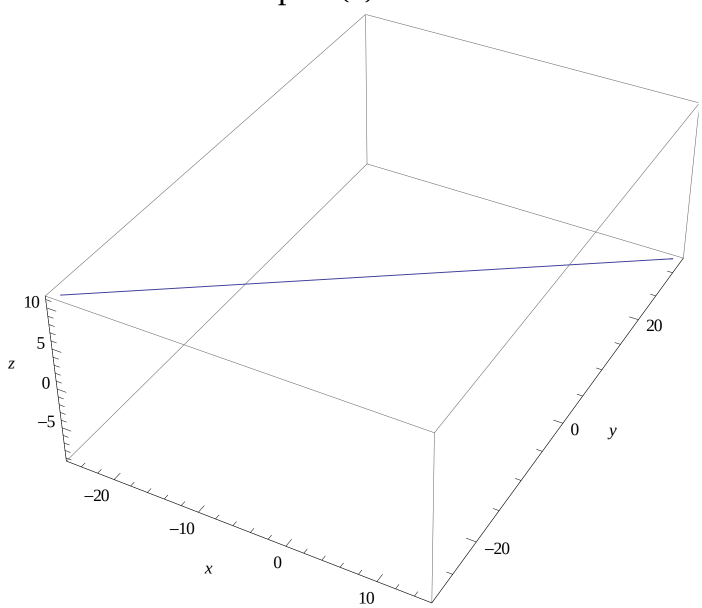
3. • (a)-(d) shown below. The red arrows (from left to right) are the vectors $\langle -8, 3\rangle$, $\langle -5, 2\rangle$, $\langle -2, -1\rangle$, and $\langle 1, 0\rangle$. The black arrows show how these are obtained by adding the multiples $-\mathbf{v}, 0, \mathbf{v}$, and $2\mathbf{v}$ of the vector $\mathbf{v} = \langle 3, -1\rangle$ to the vector $\langle -5, 2\rangle$.



• (e) If we allow the scalar t to vary in the parametric equation $\langle -5, 2\rangle + t\langle 3, -1\rangle$ we get a line through the point $(-5, 2)$ in the direction of the vector $\langle 3, -1\rangle$.

4. (a) $\mathbf{l}(t) = \langle -5+2t, 2+3t, 1-t\rangle = \langle -5, 2, 1\rangle + t\langle 2, 3, -1\rangle$, so $\mathbf{p} = \langle -5, 2, 1\rangle$ and $\mathbf{v} = \langle 2, 3, -1\rangle$.

(b) Plot of the line from part (a)



(c) $\mathbf{v}$ is called the direction vector because it points in the direction of the line.

5. Let $\mathbf{a} = \left\langle -\sqrt{3}, 0, -1, 0\right\rangle$ and $\mathbf{b} = \langle 1, 1, 0, 1\rangle$ be vectors in $\mathbb{R}^4$.

(a) The distance between $(-\sqrt{3}, 0, -1, 0)$ and $(1, 1, 0, 1)$ is $\sqrt{(1+\sqrt{3})^2 + 1^2 + 1^2 + 1^2} = \sqrt{7+2\sqrt{3}}$.

(b) The angle between $\mathbf{a}$ and $\mathbf{b}$ is found by:

$$\arccos\left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}\right) = \arccos\left(\frac{-\sqrt{3}}{2\sqrt{3}}\right) = \arccos(-1/2) = 2\pi/3$$