Group: ____________ ____________ ____________ Name: ____________

Math 231 A. Worksheet 15.

1. a) Use the Maclaurin series for $\cos x$ to find the Maclaurin series for $f(x) = x^3 \cos(x^2)$.

b) Use part a) and the definition of Maclaurin series to find the value of $f^{(11)}(0)$.

2. Find the Taylor series (centered at the given point) for each of the following functions by differentiating the given functions and finding the pattern.

(a) $f(x) = \sin x$ centered at $a = \frac{\pi}{2}$.

(b) $f(x) = \ln x$ centered at $a = 2$.

3. a) Write down the first three terms of the Maclaurin series for $f(x) = \dfrac{\sin(x^2) - x^2 \cos x}{x^4}$.

b) Use this to evaluate $\lim_{x \to 0} f(x)$.

4. Recall the binomial series:

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n, \qquad |x| < 1,$$

where

$$\binom{k}{0} = 1, \qquad \binom{k}{n} = \frac{k(k-1)\cdots(k-n+1)}{n!} \quad \text{for } n \geq 1.$$

a) Use this to write down the Maclaurin series for $\sqrt{1+x^2}$. **No simplification necessary– leave the binomial coefficients in your answer.**

b) Write out the first four terms ($n = 0$ through $n = 3$) in this series and simplify the coefficients as much as you can.

c) Write out the first four terms of the Maclaurin series for $\dfrac{1}{\sqrt[4]{1+x}}$ and simplify the coefficients.

5. Use the Maclaurin series for e^x, $\sin x$, or $\cos x$ to find the sum of each series.

a) $1 - \dfrac{\pi^2}{2!} + \dfrac{\pi^4}{4!} - \dfrac{\pi^6}{6!} + \dots.$

b) $1 - \dfrac{\pi^2}{2^2 2!} + \dfrac{\pi^4}{2^4 4!} - \dfrac{\pi^6}{2^6 6!} + \dots$

c) $\dfrac{1}{2!} - \dfrac{1}{3!} + \dfrac{1}{4!} - \dfrac{1}{5!} + \dots$