Old exam questions from Math 231

1. (i) Find the first five non-zero terms of a power series representation of

$$\int \frac{\cos x - 1}{x^2} dx.$$

(ii) Estimate $\int_0^{\frac{1}{10}} \frac{\cos x - 1}{x^2} dx$ with an error less than 10^{-9} and justify your answer.

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2. Starting with a known Maclaurin series, find the Maclaurin series of

$$\frac{x^2}{x+5}.$$

Express your answer in summation notation. What is the radius of convergence of the Maclaurin series you found?

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3. Suppose $g(x) = (\sin x)e^{-x^2} = \sum_{n=0}^{\infty} b_n x^n$ for all real x.

(i) (12 points) Find b_7, the seventh Maclaurin coefficient of $g(x)$. You must find a numerical value for b_7, but you *need not* simplify your answer.

(ii) (8 points) Evaluate $g^{(7)}(0)$. You *need not* simplify your answer.

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4. Find the exact interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(n+1)(x-2)^n}{3^n}.$$

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4. a) Use the binomial theorem to find the MacLaurin series for

$$f(x) = x(1+x^2)^{1/3}.$$

Express your answer using summation notation, but **DO NOT** expand the binomial coe cients which arise.

b) Use your answer from part (a) to evaluate the following limit. (No credit for using L'Hopital's rule.)

$$\lim_{x \to 0} \frac{f(x) - \sin(x)}{x^3}.$$

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1. a) Find the degree 2 Taylor polynomial $T_2(x)$ for the function $f(x) = e^{3x}$ at $x = 0$.
b) Use Taylor's theorem to estimate the maximum error in the approximation $e^{3x} \approx T_2(x)$ in the range $0 \leq x \leq 1/2$.
(You must give a numerical answer, but you do not need to evaluate it.)