Math 231 - Midterm 1 Review

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Sections: BDJ/BDK

Table of integrals

$\int x^n dx=$	$\int \frac{1}{x} dx=$
$\int e^x dx=$	$\int a^x dx=$
$\int \sin x dx=$	$\int \cos x dx=$
$\int \sec^2 x dx=$	$\int \csc^2 x dx=$
$\int \sec x \tan x dx=$	$\int \csc x \cot x dx=$
$\int \sec x dx=$	$\int cscx dx=$
$\int \tan x dx=$	$\int \cot x dx=$
$\int \frac{1}{x^2+a^2} dx=$	$\int \frac{1}{\sqrt{1-x^2}} dx=$

Integration by parts

$$\boxed{\int_a^b udv =}$$

What does LIATE stand for? It may help you choose u.

Trigonometric Integrals

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)]$$

$$\sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)]$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

Case 1: $\int \sin^m x \cos^n x dx$

(a) If n odd:

- save a copy of:
- use itentity:
- u- substitution:

(b) If m odd:

- save a copy of:
- use itentity:
- u- substitution:

(c) If both n and m are even:

- use itentities:
- Sometimes useful to use: $\sin(2\theta) = 2\sin\theta\cos\theta$

Case 2: $\int \tan^m x \sec^n x dx$

(a) If n even

- save a copy of:
- use itentity:
- u- substitution:

(b) If m odd:

- save a copy of:
- use itentity:
- u- substitution:

Trigonometric Substitutions

Expression	Substitution	Identity	Restriction
$\sqrt{a^2 - x^2}$			
$\sqrt{a^2 + x^2}$			
$\sqrt{x^2 - a^2}$			

What substitution would you use in the following examples?

1. $\sqrt{9 - x^2}$

2. $\sqrt{x^2 + 4x}$

Common mistakes Comments:

1. Don't forget to replace dx by the corresponding expression. It is not just $d\theta$.
2. Give the solution in term of x, not θ. Use the triangle to do that.
3. Remember to use the expression $\sin(2\theta) = 2\sin\theta\cos\theta$ when necessary.
4. Complete the square if necessary!

Integration of Rational Functions by Partial Fractions

1. **CASE I: The denominator is a product of distinct linear factors**

   Example: $\dfrac{x^2 + 2x - 1}{2x^3 - 5x^2 + 2x} =$

2. **CASE II: The denominator is a product of linear factors, some of which are repeated**

   Example: $\dfrac{x^2 + 2x - 1}{2x^4 - 5x^3 + 2x^2} =$

3. **CASE III: The denominator contains *irreducible* quadratic factors, none of which is repeated**

   Example: $\dfrac{x^2 + x - 5}{x^3 + 5x} =$

4. **CASE IV: The denominator contains a repeated *irreducible* quadratic factor**

   Example: $\dfrac{x^2 - 3}{x(x^2 + 3)^2} =$

Improper Integrals

1. **Type 1: Infinite Intervals**

   (a) If $\int_a^t f(x)dx$ exists for every number $t \geq a$, then

   $$\int_a^{\infty} f(x)dx =$$

   provided this limit exists (as a finite number).

   (b) If $\int_t^b f(x)dx$ exists for every number $t \leq b$, then

   $$\int_{-\infty}^{b} f(x)dx =$$

   provided this limit exists (as a finite number).

   The improper integrals and are called **convergent** if the corresponding limit exists and **divergent** if the limit does not exist.

   (a) If both and are convergent, then

   $$\int_{-\infty}^{\infty} f(x)dx =$$

2. **Type 2: Discontinuous Integrands**

   (a) If f is continuoun on $[a, b)$ and is discontinuoun at b, then

   $$\int_a^b f(x)dx =$$

   if this limit exists (as a finite number).

   (b) If f is continuoun on $(a, b]$ and is discontinuoun at a, then

   $$\int_a^b f(x)dx =$$

   if this limit exists (as a finite number).

   The improper integral is called **convergent** if the corresponding limit exists and **divergent** if the limit does not exist.

   (a) If f has a discontinuity at c, where $a < c < b$, and both and are convergent, then

   $$\int_a^b f(x)dx =$$

Comparison theorem

Suppose that f and g are continuous functions with $\quad \geq \quad \geq 0$ for $x \geq a$.

(a) If $\qquad\qquad$ is convergent, then

(b) If $\qquad\qquad$ is divergent, then

Remark: The comparison theorem only tells you whether the integral diverges or converges. If it converges, it does not tell you to which number!

p-test

(a) $\displaystyle\int_1^\infty \frac{1}{x^p}dx$ converges/diverges for $p > 1$ and converges/diverges for $p \leq 1$.

(b) $\displaystyle\int_0^1 \frac{1}{x^p}dx$ converges/diverges for $p \geq 1$ and converges/diverges for $p < 1$.