Tuesday, March 6 $**$ *Integrating vector fields.*

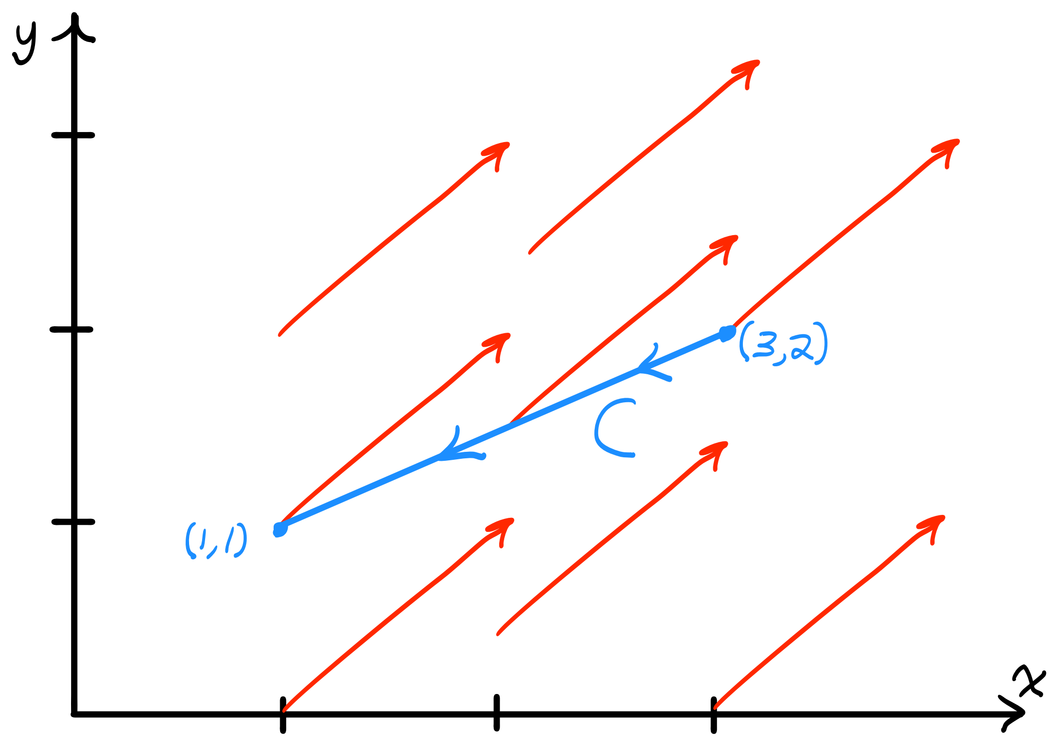
1. Consider the vector field $\mathbf{F} = (y, 0)$ on $\mathbb{R}^2$.

   (a) Draw a sketch of $\mathbf{F}$ on the region where $-2 \le x \le 2$ and $-2 \le y \le 2$.

   (b) Consider the following two curves which *start* at $A = (-2, 0)$ and *end* at $B = (2, 0)$, namely the line segment C_1 and upper semicircle C_2.

   Add these curves to your sketch, and compute both $\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$ and $\int_{C_2} \mathbf{F} \cdot d\mathbf{r}$.

2. Consider the curve C and vector field $\mathbf{F}$ shown below.



   (a) Calculate $\mathbf{F} \cdot \mathbf{T}$, where here $\mathbf{T}$ is the unit tangent vector along C. Without parameterizing C, evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ by using the fact that it is equal to $\int_C \mathbf{F} \cdot \mathbf{T}\, ds$.

   (b) Find a parameterization of C and a formula for $\mathbf{F}$. Use them to check your answer in (a) by computing $\int_C \mathbf{F} \cdot d\mathbf{r}$ explicitly.

3. Consider the points $A = (0, 0)$ and $B = (\pi, -2)$. Suppose an object of mass m moves from A to B and experiences the constant force $\mathbf{F} = -mg\mathbf{j}$, where g is the gravitational constant.

   (a) If the object follows the straight line from A to B, calculate the work W done by gravity using the formula: Work $= F \cdot D$, where F is a constant force vector, and D is a displacement vector.

(b) Now suppose the object follows half of an inverted cycloid C as shown below. A parametrization for the inverted cycloid C is $\mathbf{r}(t) = (t - \sin t, \cos t - 1), 0 \leq t \leq \pi$, use this parametrization to calculate the work done via a line integral.