Group: __________ __________ __________ Name: __________

Math 231 A. Fall, 2015. Worksheet 12. 10/22/15

1. a) Use the alternating series test to prove that that the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$ converges.

b) Using a calculator, find the partial sum $s_6 = \sum_{n=0}^{6} \frac{(-1)^n}{n!}$ to four decimal places. What is the maximum value of $|R_6|$?

c) Soon we will prove that $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = 1/e$. Compute $1/e$ to four decimal places to check that your answer agrees with this.

2. Consider the series

$$1 - \frac{1}{10^1} + \frac{1}{2} - \frac{1}{10^2} + \frac{1}{3} - \frac{1}{10^3} + \frac{1}{4} - \frac{1}{10^4} + \dots. \qquad (\star)$$

a) Show that the series diverges.

Hint: we know that the series

$$0 + \frac{1}{10^1} + 0 + \frac{1}{10^2} + 0 + \frac{1}{10^3} + 0 + \frac{1}{10^4} + \dots$$

converges. If the series (**??**) converged as well, what would happen?

b) Why doesn't the Alternating Series Test apply to the series (**??**)?

3. Around 1910, the mathematician Ramanujan discovered the formula

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{n=0}^{\infty} \frac{(4n)!(1103 + 26390n)}{(n!)^4 396^{4n}}.$$

William Gosper used this series in 1985 to compute π to 17 million digits.

(a) Verify that the series is convergent.

(b) How many correct decimal places of π do you get if you use just the first term of the series? What if you use the first 2 terms? (Use a calculator.)

In each problem, determine if the series converges absolutely, converges conditionally, or diverges. Show work and state explicitly which test or tests you are using (Ratio, Root, Alternating Series, etc.).

4. $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^n n^2}{n!}.$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^{2n}}{n^n}$

6. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (\ln n)^2}{n}$