# Mock Exam 3, Math 241, Spring 2018

1. (a) Find the gradient vector field of $f(x,y,z) = xze^{yz} + y^2 + yz\sin x$.

   (b) State the Fundamental Theorem of Line Integrals.

   (c) $\vec{F}(x,y,z) = \langle yz + 3(y+z),\, xz + 3(x+z),\, xy + 3(x+y)\rangle$ is conservative. Let $C$ be the curve parameterized by $\vec{r}(t) = \langle t,\, t^2 - 1,\, t^3\rangle$ for $0 \le t \le 1$. Evaluate $\int_C \vec{F}\cdot d\vec{r}$.

   > (circle one):   (A) $-4$;    (B) 4;    (C) 3;    (D) 9;

   (d) Let $\vec{F}(x,y,z)$ be a vector field, with $\int_C \vec{F}\cdot d\vec{r} = \pi$ for the curve $C$ parameterized by $\vec{r}(t) = \langle 6,\, \cos 2t,\, \sin 2t\rangle$ for $0 \le t \le \pi$. TRUE or FALSE: $\vec{F}$ is conservative.

   > (circle one):   (A) True;    (B) False;

   (e) Which of the three vector fields:

   i. $\vec{F}(x,y) = \langle (1+xy)e^{xy},\, x^2 e^{xy}\rangle$

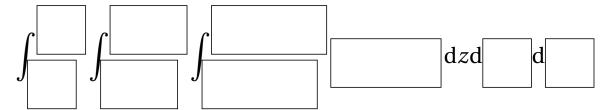   ii. $\vec{G}(x,y) = \langle x^3 y^2 + 2,\, x^2 y^3 + 2\rangle$

   iii. $\vec{H}(x,y) = \langle ye^x + \sin y,\, e^x + x\cos y\rangle$

   are conservative? Find a potential function for those which are conservative.

2. Set up (but do **NOT** evaluate) a triple integral to calculate the volume of the solids $E$ and $S$, where:

   (a) $E$ is the solid under the surface $z = x^2 + 2y^4$ and above the region $D = \{(x,y,0)\,|\, -1 \le x \le 1,\, x^2 \le y \le 2x^2\}$.

   $$\int_{\boxed{\phantom{x}}}^{\boxed{\phantom{x}}} \int_{\boxed{\phantom{x}}}^{\boxed{\phantom{x}}} \int_{\boxed{\phantom{x}}}^{\boxed{\phantom{x}}} \boxed{\phantom{xxx}}\, dz\,dy\,dx$$

   (b) $S$ is the solid under the plane $3x + 2y - z = 0$ and above the region in the $xy$-plane enclosed by $y = x^2$ and $x = y^2$.

   $$\int_{\boxed{\phantom{x}}}^{\boxed{\phantom{x}}} \int_{\boxed{\phantom{x}}}^{\boxed{\phantom{x}}} \int_{\boxed{\phantom{x}}}^{\boxed{\phantom{x}}} \boxed{\phantom{xxx}}\, dz\,d\boxed{\phantom{x}}\,d\boxed{\phantom{x}}$$

3. Evaluate the following integrals by reversing the order of integration.

(a)
$$\int_0^1 \int_{3y}^3 e^{x^2}\,dx\,dy$$

(circle one):   (A) $\frac{(e^9-1)}{3}$;   (B) $\frac{(e^9-1)}{6}$;   (C) $\frac{e^9}{3}$;   (D) $\frac{e^9}{6}$;

(b)
$$\int_0^2 \int_{\frac{y}{2}}^1 y\cos(x^3-1)\,dx\,dy$$

(circle one):   (A) $\frac{2}{3}\sin(1)$; (B) $\frac{2}{3}\sin(-1)$; (C) $\sin(1)$; (D) $\sin(-1)$;

4. Find the mass of the lamina the occupies the region $R$ bounded by the semicircle $x = -\sqrt{4-y^2}$ and the $y$-axis, whose density at any point in $R$ is $\rho(x,y) = e^{-x^2-y^2}$.

(circle one):   (A) $\frac{\pi(1-e^{-4})}{2}$;   (B) $\pi(1-e^{-4})$;   (C) $\frac{\pi(1-e^{-4})}{4}$;   (D) $\frac{\pi(e^{-4}-1)}{2}$;

5. Use polar coordinates to combine the sum
$$\int_0^{\sqrt{2}} \int_0^x xy\,dy\,dx + \int_{\sqrt{2}}^2 \int_0^{\sqrt{4-x^2}} xy\,dy\,dx$$

into one double integral. Then evaluate the double integral.
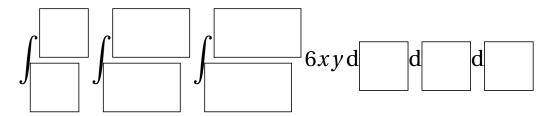
(circle one):   (A) $\frac{1}{3}$;   (B) $\frac{4}{3}$;   (C) $\frac{2}{3}$;   (D) 1;

6. Set up (but do **NOT** evaluate) the triple integral
$$\iiint_E 6xy\,dV,$$

as an iterated integral, where $E$ lies under the plane $z = 1 + x + y$ and above the region in the $xy$-plane bounded by the curves $y = \sqrt{x}$, $y = 0$, and $x = 1$.

$$\int_{\square}^{\square} \int_{\square}^{\square} \int_{\square}^{\square} 6xy\, d\square\, d\square\, d\square$$

7. Evaluate $\iiint_E (2y - x)\, dV$, where E is the solid that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 16$, above the $xy$-plane, and below the plane $z = y + 4$.

(circle one):   (A) $\frac{255\pi}{2}$;   (B) $\frac{255\pi}{4}$;   (C) $\frac{-255\pi}{4}$;   (D) $\frac{-255\pi}{2}$;

8. Let $R = \{(x, y, z)\,|\,2 \le z \le 2 + \sqrt{4 - x^2 - y^2}\}$. Set up the iterated integrals to find the volume of region $R$ in rectangular, cylindrical, and spherical coordinates, then calculate the volume of $R$.

9. Let $R$ be the region bounded by the ellipse $4x^2 + 9y^2 = 36$. Evaluate

$$\iint_R y^2\, dA.$$

10. Let $S = \{(u, v) : 0 \le u \le 1, 0 \le v \le 1\}$, and $R \subset \mathbb{R}^2$ be the parallelogram with vertices $(0,0)$, $(4, -3)$, $(5, 4)$ and $(9, 1)$.

   (a)  Find a transformation $T(u, v) = (x(u, v), y(u, v))$ which maps $S$ to $R$.

   (b)  Use the transformation from part (a), along with change of variables, to evaluate

$$\iint_R (x + xy)\, dA.$$

# TRIGONOMETRIC IDENTITIES

$$\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$

$$\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$

$$\tan(\theta + \phi) = \frac{\tan(\theta) + \tan(\phi)}{1 - \tan\theta\tan\phi}$$

$$\sin(\theta - \phi) = \sin\theta\cos\phi - \cos\theta\sin\phi$$

$$\cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$$

$$\tan(\theta - \phi) = \frac{\tan(\theta) - \tan(\phi)}{1 + \tan\theta\tan\phi}$$

$$\sin(\theta)\sin(\phi) = \frac{\cos(\theta - \phi) - \cos(\theta + \phi)}{2}$$

$$\cos(\theta)\cos(\phi) = \frac{\cos(\theta - \phi) + \cos(\theta + \phi)}{2}$$

$$\sin(\theta)\cos(\phi) = \frac{\sin(\phi + \theta) - \sin(\phi - \theta)}{2}$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$\sin\theta = \pm\sqrt{\frac{1 - \cos(2\theta)}{2}}$$

$$\cos\theta = \pm\sqrt{\frac{1 + \cos(2\theta)}{2}}$$

$$\tan\theta = \frac{\sin(2\theta)}{1 + \cos(2\theta)}$$