Group: _____ _____ _____ Name: _____

Math 231 A. Worksheet 6.

1. Evaluate each of the improper integrals or show that it diverges.

a) $\displaystyle\int_1^\infty \frac{dx}{\sqrt{x}}$

b) $\displaystyle\int_1^\infty \frac{dx}{1+x^2}$

c) $\displaystyle\int_e^\infty \frac{1}{x \ln x}\, dx$

2. As we saw in class, the region $R = \{(x, y) : x \geq 1, 0 \leq y \leq 1/x\}$ has infinite area. The *Horn of Gabriel* is formed by rotating this region about the x-axis. Make a careful sketch of R and of the Horn. Then find the volume of the Horn of Gabriel.

3. Which of the following statements shows a correct use of the Comparison Theorem?

a) Since $\displaystyle\int_2^\infty \frac{dx}{\sqrt{x}}$ diverges, and $\displaystyle\frac{1}{\sqrt{x}} < \frac{1}{\sqrt{x-1}}$ for all $x > 2$, $\displaystyle\int_2^\infty \frac{dx}{\sqrt{x-1}}$ must diverge.

b) Since $\displaystyle\int_2^\infty \frac{dx}{\sqrt{x}}$ diverges, and $\displaystyle\frac{1}{\sqrt{x}} > \frac{1}{\sqrt{x+1}}$ for all $x > 2$, $\displaystyle\int_2^\infty \frac{dx}{\sqrt{x+1}}$ must diverge.

4. Use a comparison to a known integral to determine if these improper integrals converge or diverge.

a) $\displaystyle\int_0^\infty \frac{1}{x^4+27}\,dx.$ Is the integral improper at 0 or at ∞?

b) $\displaystyle\int_0^\infty e^{-x}\sin^2(x)\,dx$ Hint: How big can $\sin^2 x$ be?

c) $\displaystyle\int_3^\infty \frac{x}{x^3+e^x+\cos^2 x}\,dx$