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 Sections: ADJ/ADK

Alternating Series

Alternating Series Test (AST)

If the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ with $b_n > 0$ satisfies

(i) $b_{n+1} \leq b_n$ (decreasing)

(ii) $\lim_{n \rightarrow \infty} b_n = 0$

then the series is **CONVERGENT**.

With the same hypothesis as above, by the **Alternating Series Estimation Theorem** we have

$$|R_n| \leq \underline{b_{n+1}} \quad (\text{NOT } b_n !!)$$

Absolute Convergence and the Ratio and Root Tests

- Define the following notions:

a) Absolutely convergent: $\sum_{n=0}^{\infty} a_n$ is absolutely convergent if $\sum_{n=0}^{\infty} |a_n|$ is convergent.

Examples: (a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ is not absolutely convergent.

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ is absolutely convergent.

b) Conditionally convergent: $\sum_{n=0}^{\infty} a_n$ is conditionally convergent if $\sum_{n=0}^{\infty} a_n$ converges but $\sum_{n=0}^{\infty} |a_n|$ diverges.

Example: $\sum_{n=1}^{\infty} \frac{(-1)^n \ln(n)}{n}$ is conditionally convergent.

* Can you think of more examples?

The Ratio Test

$$\text{If } \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| \begin{cases} < 1 & \text{Absolutely convergent} \\ > 1 & \text{Divergent} \\ = 1 & \text{Inconclusive} \end{cases}$$

Don't forget to take the absolute value!

The Root Test

$$\text{If } \lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} \begin{cases} < 1 & \text{Absolutely convergent} \\ > 1 & \text{Divergent} \\ = 1 & \text{Inconclusive} \end{cases}$$

STEPS:

1. Use Ratio or Root Test.
2. If that is inconclusive and the series is alternating, then do BOTH of the next:
 - (a) Use AST (Alternating Series Test)
 - (b) Check whether $\sum |a_n|$ is divergent or convergent.

If the AST shows that the series is convergent, BUT $\sum |a_n|$ is divergent then $\sum a_n$ is **conditionally convergent**. However, if $\sum |a_n|$ is convergent then $\sum a_n$ is **absolutely convergent**.

(no need to use AST in this case)

Determine whether the following series converge absolutely, converge conditionally or diverge.

$$1. \sum_{n=1}^{\infty} \frac{(-1)^n \cos(n\pi)}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots = \sum_{n=1}^{\infty} \frac{1}{n} \text{ Harmonic Series!}$$

Therefore, the series DIVERGES.

! It's NOT an alternating series! We may be tempted to take $b_n = \cos(n\pi)$ and use AST. But we cannot because b_n is not always positive.

↳ CONCLUSION: Remember to check the condition $b_n > 0$.

$$2. \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+7} \text{ Ratio Test will be inconclusive.}$$

$$(a) \text{ AST: } b_n = \frac{n}{n^2+7} > 0, \quad b_{n+1} = \frac{n+1}{(n+1)^2+7} < \frac{n}{n^2+7} = b_n \text{ and } \lim_{n \rightarrow \infty} \frac{n}{n^2+7} = 0.$$

$$\hookrightarrow \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+7} \text{ is } \underline{\text{convergent}}$$

$$(b) \sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{n}{n^2+7} \sim \sum_{n=1}^{\infty} \frac{1}{n} \text{ diverges}$$

The series is

CONDITIONALLY
CONVERGENT

$$3. \sum_{n=2}^{\infty} \frac{(-1)^{n+1} (2n)!}{5^n n! n!} \text{ RATIO TEST:}$$

$$\lim_{n \rightarrow \infty} \left| \frac{[2(n+1)]!}{5^{n+1} (n+1)! (n+1)!} \cdot \frac{5^n n! n!}{(2n)!} \right| = \lim_{n \rightarrow \infty} \left| \frac{(2n+2)! \cdot \cancel{5} \cdot \cancel{n!} \cdot \cancel{n!}}{5 \cdot 5 (n+1) \cancel{n!} (n+1) \cancel{n!} (2n)!} \right|$$

$$= \lim_{n \rightarrow \infty} \left| \frac{(2n+2)(2n+1) \cancel{(2n)!}}{5(n+1)(n+1) \cancel{(2n)!}} \right| = \frac{4}{5} < 1 \Rightarrow \underline{\text{ABSOLUTELY
CONVERGENT}}$$

$$4. \sum_{n=2}^{\infty} (-1)^n \left(\frac{n+1}{3n+5} \right)^n \text{ ROOT TEST:}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{\left| (-1)^n \left(\frac{n+1}{3n+5} \right)^n \right|} = \lim_{n \rightarrow \infty} \sqrt[n]{\left(\frac{n+1}{3n+5} \right)^n} = \lim_{n \rightarrow \infty} \frac{n+1}{3n+5} = \frac{1}{3} < 1$$

⇒ ABSOLUTELY CONVERGENT

Power Series

1. Find the radius of convergence for the following power series.

$$\sum_{n=1}^{\infty} \frac{n! x^n}{5 \cdot 11 \cdot 17 \cdot \dots \cdot (6n-1)}$$

RATIO TEST :

$$\lim_{n \rightarrow \infty} \left| \frac{(n+1)! x^{n+1}}{\overbrace{5 \cdot 11 \cdot 17 \cdot \dots \cdot (6n-1)}^{(6(n+1)-1)} \cdot \frac{5 \cdot 11 \cdot 17 \cdot \dots \cdot (6n-1)}{n! x^n}} \right| =$$

$$\lim_{n \rightarrow \infty} \left| \frac{(n+1) \cancel{x} \cdot x}{(6n+5) \cancel{x} \cdot x^n} \right| = \left| \frac{n+1}{6n+5} \cdot x \right| = \frac{|x|}{6} < 1$$

Condition for Ratio Test to be abs. Convergent.

$$\frac{|x|}{6} < 1 \text{ when } |x| < 6 \Rightarrow \boxed{R = 6}$$

2. Find the interval of convergence for the following power series.

RATIO TEST : (to find R) $\sum_{n=1}^{\infty} \frac{2(x-3)^n}{3^n \cdot n}$

$$\lim_{n \rightarrow \infty} \left| \frac{2(x-3)^{n+1}}{3^{n+1} \cdot (n+1)} \cdot \frac{3^n \cdot n}{2(x-3)^n} \right| = \lim_{n \rightarrow \infty} \left| \frac{\cancel{2} \cdot (x-3)^{\cancel{n}} \cdot (x-3) \cdot \cancel{3}^n \cdot n}{\cancel{3}^n \cdot 3(n+1) \cdot \cancel{2} \cdot (x-3)^{\cancel{n}}} \right| =$$

$$= \lim_{n \rightarrow \infty} \left| \frac{(x-3) \cdot n}{3(n+1)} \right| = \frac{|x-3|}{3} < 1 \Rightarrow |x-3| < 3 \Rightarrow \boxed{R = 3}$$

→ Interval of convergence : $I = [(3-3, 3+3)]$ We need to check endpoints!!

• $x=0$: $\sum_{n=1}^{\infty} \frac{2 \cdot (0-3)^n}{3^n \cdot n} = \sum_{n=1}^{\infty} \frac{2 \cdot (-3)^n}{3^n \cdot n} = \sum_{n=1}^{\infty} \frac{2 \cdot (-1)^n \cancel{3}^n}{\cancel{3}^n \cdot n} = \sum_{n=1}^{\infty} \frac{2 \cdot (-1)^n}{n}$ CONVERG. (by AST)

• $x=6$: $\sum_{n=1}^{\infty} \frac{2(6-3)^n}{3^n \cdot n} = \sum_{n=1}^{\infty} \frac{2 \cdot \cancel{3}^n}{\cancel{3}^n \cdot n} = \sum_{n=1}^{\infty} \frac{2}{n} = 2 \cdot \sum_{n=1}^{\infty} \frac{1}{n}$ DIVERGES (Harmonic Series)

Therefore, $\boxed{I = [0, 6)}$ OR $\boxed{0 \leq x < 6}$

Representation of Functions as Power Series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \text{ for } |x| < 1$$

REMARKS:

1. If you integrate a series to find the power series for a particular function, remember to calculate +C.
2. The radius of convergence (R) doesn't change when we derivate or integrate a series. However, the interval of convergence (I) may change (i.e., you will still need to check the endpoints).

Examples: Find the power series representation of the function and determine the interval of convergence.

$$\begin{aligned}
 1. f(x) &= \frac{x}{9+x^2} = x \cdot \frac{1}{9+x^2} = \frac{x}{9} \cdot \frac{1}{1+\frac{x^2}{9}} = \frac{x}{9} \cdot \frac{1}{1-\left(-\frac{x^2}{9}\right)} = \frac{x}{9} \cdot \sum_{n=0}^{\infty} \left(-\frac{x^2}{9}\right)^n \\
 &= \sum_{n=0}^{\infty} \frac{x}{9} \cdot \frac{(-1)^n x^{2n}}{9^n} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{9^{n+1}}, \text{ for } \left|-\frac{x^2}{9}\right| < 1 \Rightarrow |x| < 3
 \end{aligned}$$

DON'T STOP THERE!

$I = (-3, 3)$

$$\begin{aligned}
 2. f(x) &= \left(\frac{x}{1+4x}\right)^2 = \frac{x^2}{(1+4x)^2} = x^2 \cdot \frac{1}{(1+4x)^2} \\
 \int \frac{1}{(1+4x)^2} dx &= \int \frac{1}{4u^2} du = -\frac{1}{4} u^{-1} + C = -\frac{1}{4(1+4x)} + C \Rightarrow \frac{1}{(1+4x)^2} = \frac{d}{dx} \left[\frac{-1}{4(1+4x)} \right] \\
 \Rightarrow \frac{1}{(1+4x)^2} &= \frac{-1}{4} \frac{d}{dx} \left(\frac{1}{1+4x} \right) = \frac{-1}{4} \frac{d}{dx} \left(\sum_{n=0}^{\infty} (-4x)^n \right) = \frac{-1}{4} \frac{d}{dx} \left(\sum_{n=0}^{\infty} (-1)^n 4^n x^n \right) \\
 &= \frac{-1}{4} \sum_{n=0}^{\infty} (-1)^n 4^n n x^{n-1} = \sum_{n=0}^{\infty} (-1)^{n+1} 4^{n-1} n x^{n-1}, \text{ for } |4x| < 1 \Rightarrow |x| < \frac{1}{4} \\
 \Rightarrow \frac{x^2}{(1+4x)^2} &= x^2 \cdot \sum_{n=0}^{\infty} (-1)^{n+1} 4^{n-1} n x^{n-1} = \sum_{n=0}^{\infty} (-1)^{n+1} 4^{n-1} n x^{n+1}
 \end{aligned}$$

$I = (-\frac{1}{4}, \frac{1}{4})$

Since we took the derivative we need to check endpoints
 even if $0 = \frac{1}{4}$. (CHECK THEM YOURSELF!)

$$3. f(x) = \ln\left(\frac{1+x}{1-x}\right) = \ln|1+x| - \ln|1-x| = \int \frac{1}{1+x} dx - \int \frac{-1}{1-x} dx =$$

$$= \int \sum_{n=0}^{\infty} (-x)^n dx + \int \sum_{n=0}^{\infty} x^n dx = \sum_{n=0}^{\infty} \int (-1)^n x^n dx + \sum_{n=0}^{\infty} \int x^n dx =$$

$$= \underbrace{\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}}_{\text{0}} + \underbrace{\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1}}_{\text{0}} =$$

$$= \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \right) + \left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots \right)$$

$$= 2 \left[x + \frac{x^3}{3} + \frac{x^5}{5} + \dots \right] = \sum_{n=0}^{\infty} \frac{2 \cdot x^{2n+1}}{2n+1} \quad \text{for } |x| < 1 = R$$

• Check endpoints: $= -1$ always!

$$\bullet x = -1 : \sum_{n=0}^{\infty} 2 \cdot \frac{(-1)^{2n+1}}{2n+1} = - \sum_{n=0}^{\infty} \frac{2}{2n+1} \quad \text{DIVERGENT (p-series, } p=1)$$

$$\bullet x = 1 \Rightarrow \sum_{n=0}^{\infty} \frac{2}{2n+1} \Rightarrow \text{DIVERGENT (p-series, } p=1)$$

$$\Rightarrow I = (-1, 1)$$

$$4. f(x) = \arctan(2x)$$

$$\arctan(2x) = \int \frac{2}{1+(2x)^2} dx = \int \frac{2}{1+4x^2} dx = \int 2 \cdot \sum_{n=0}^{\infty} (-4x^2)^n dx =$$

$$= 2 \cdot \int \sum_{n=0}^{\infty} (-1)^n 4^n x^{2n} dx = 2 \cdot \sum_{n=0}^{\infty} (-1)^n 4^n \frac{x^{2n+1}}{2n+1} =$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{2n+1}}{2n+1}, \quad \text{for } |4x^2| < 1$$

$$|x^2| < \frac{1}{4}$$

$$|x| < \frac{1}{2} \Rightarrow R = \frac{1}{2}$$

• Check endpoints:

$$\bullet x = -\frac{1}{2} : \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} \left(-\frac{1}{2}\right)^{2n+1}}{2n+1} = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} (-1)^{2n+1}}{2^{2n+1} (2n+1)} = \sum_{n=0}^{\infty} \frac{(-1)^{3n+1}}{2n+1} \Rightarrow \text{CONV. by AST}$$

$$\bullet x = \frac{1}{2} : \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} \left(\frac{1}{2}\right)^{2n+1}}{2n+1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \Rightarrow \text{CONV. by AST}$$

$$\text{Therefore, } I = \left[-\frac{1}{2}, \frac{1}{2}\right]$$

Taylor and Maclaurin Series

n^{th} derivative!

Taylor series of the function f at a :

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n, \quad |x-a| < R$$

\uparrow center

N th-degree Taylor polynomial of f at a :

$$T_N(x) = f(a) + \frac{f'(a)}{1!} (x-a) + \frac{f''(a)}{2!} (x-a)^2 + \dots + \frac{f^{(N)}(a)}{N!} (x-a)^N$$

Then $f(x) = T_N(x) + R_N(x)$ and

$$|R_N(x)| \leq \frac{|f^{(N+1)}(z)|}{(N+1)!} \cdot |x-a|^{N+1}$$

for some value z between a and x .

- How is the series called when the center is $a = 0$? **Maclaurin Series**

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots$$

$$R = 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$R = \infty$$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$R = \infty$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$R = \infty$$

$$\arctan(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$$

$$R = 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$$

$$R = 1$$

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n$$

$$R = 1$$

$$\binom{k}{n} = \frac{k(k-1)(k-2) \dots (k-n+1)}{n!}$$

← There are n terms on the top!

Practice problems

1. Find the Maclaurin series of $f(x) = 9(1-x)^{-2}$ using the definition of a Maclaurin series.

$$f(x) = 9(1-x)^{-2} \rightarrow f(0) = 9 = 9 \cdot 1!$$

$$f'(x) = 9(-2)(-1)(1-x)^{-3} \rightarrow f'(0) = 9 \cdot 2 = 9 \cdot 2!$$

$$f''(x) = 9(-2)(-1)(-3)(-1)(1-x)^{-4} \rightarrow f''(0) = 9 \cdot 3 \cdot 2 = 9 \cdot 3!$$

$$f^{(3)}(x) = 9(-2)(-1)(-3)(-1)(-4)(-1)(1-x)^{-5} \rightarrow f^{(3)}(0) = 9 \cdot 4 \cdot 3 \cdot 2 = 9 \cdot 4!$$

⋮

$$f^{(n)}(x) = 9 \cdot (n+1)! (1-x)^{-n-2} \rightarrow f^{(n)}(0) = 9 \cdot (n+1)!$$

Therefore :

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = \sum_{n=0}^{\infty} \frac{9(n+1)!}{n!} x^n = \boxed{\sum_{n=0}^{\infty} 9 \cdot (n+1) x^n}$$

2. Find the value of $f^{(8)}(0)$ given that $f(x) = \frac{1 - \cos(2x^2)}{2}$.

$$\bullet \cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \rightarrow \cos(2x^2) = \sum_{n=0}^{\infty} \frac{(-1)^n (2x^2)^{2n}}{(2n)!} \Rightarrow$$

$$\Rightarrow \cos(2x^2) = 1 - \frac{4x^2}{2!} + \frac{2^4 x^8}{4!} - \dots$$

$$\bullet 1 - \cos(2x^2) = 1 - \left(1 - \frac{4x^2}{2!} + \frac{2^4 x^8}{4!} - \dots \right)$$

$$= \frac{4x^2}{2!} - \frac{2^4 x^8}{4!} + \dots$$

$$\bullet f(x) = \frac{1 - \cos(2x^2)}{2} = \frac{4x^2}{2 \cdot 2!} - \frac{2^4 x^8}{2 \cdot 4!} + \dots$$

$\frac{2^4 x^8}{2 \cdot 4!} = \frac{f^{(8)}(0)}{8!}$

By definition of Maclaurin series, $f^{(8)}(0)$ shows up in the

coefficient of x^8 : $\frac{f^{(8)}(0)}{8!} = -\frac{2^4}{2 \cdot 4!} \Rightarrow \boxed{f^{(8)}(0) = -\frac{2^3 \cdot 8!}{4!}}$

3. Find the value of the following limit:

$$\lim_{x \rightarrow \infty} x^2 (e^{-\frac{1}{x^2}} - 1)$$

$$\bullet e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \Rightarrow e^{-\frac{1}{x^2}} = \sum_{n=0}^{\infty} \frac{(-\frac{1}{x^2})^n}{n!} = 1 - \frac{1}{x^2} + \frac{1}{x^4 \cdot 2!} - \frac{1}{x^6 \cdot 3!} + \dots$$

$$\bullet e^{-\frac{1}{x^2}} - 1 = -\frac{1}{x^2} + \frac{1}{2 \cdot x^4} - \frac{1}{6 \cdot x^6} + \dots$$

$$\bullet x^2 (e^{-\frac{1}{x^2}} - 1) = -1 + \frac{1}{2x^2} - \frac{1}{6x^4} + \dots$$

$$\bullet \lim_{x \rightarrow \infty} x^2 (e^{-\frac{1}{x^2}} - 1) = \lim_{x \rightarrow \infty} \left[-1 + \frac{1}{2x^2} - \frac{1}{6x^4} + \dots \right] = \boxed{-1}$$

4. Find the value of $\sum_{n=0}^{\infty} \frac{4^n}{5^n n!}$.

$$\text{Notice that } e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

$$\text{Then } \sum_{n=0}^{\infty} \frac{4^n}{5^n n!} = \sum_{n=0}^{\infty} \frac{(\frac{4}{5})^n}{n!} = \boxed{e^{4/5}}$$

5. Approximate $f(x) = \sqrt{x}$ by a Taylor polynomial of degree 2 at $a=9$. Then use Taylor's Inequality to estimate the accuracy of the approximation when x lies in $[9, 10]$. You don't need to simplify your answer.

$$\begin{aligned}
 f(x) &= \sqrt{x} \longrightarrow f(9) = 3 \\
 f'(x) &= \frac{1}{2} x^{-1/2} \longrightarrow f'(9) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6} \\
 f''(x) &= \frac{1}{2} \cdot \left(-\frac{1}{2}\right) x^{-3/2} \longrightarrow f''(9) = \frac{-1}{4} \cdot \frac{1}{3^3} = \frac{-1}{108}
 \end{aligned}
 \left\{ \begin{aligned}
 T_2(x) &= f(9) + f'(9)(x-9) \\
 &\quad + \frac{f''(9)}{2!} (x-9)^2 = \\
 &= 3 + \frac{1}{6}(x-9) - \frac{1}{216}(x-9)^2
 \end{aligned} \right.$$

$$|R_2(x)| \leq \frac{|f^{(3)}(z)|}{3!} |x-9|^3 \leq \frac{3/8 \cdot 9^{-5/2}}{3!} \cdot (10-9)^3 = \frac{3/8 \cdot 1}{3! \cdot 3^5} = \frac{1}{8 \cdot 3^4 \cdot 3!}$$

$f^{(3)}(z) = \frac{3}{8} z^{-5/2} \rightsquigarrow \text{max at } z=9.$

6. Give a series representation for $\int_0^1 x \arctan x dx$. Then write down enough terms to approximate $\int_0^1 x \arctan x dx$ to within $\frac{1}{100}$.

$$\begin{aligned}
 \arctan(x) &= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} \Rightarrow x \cdot \arctan(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{2n+1} \\
 \int_0^1 x \arctan(x) dx &= \int_0^1 \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{2n+1} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{(2n+1)(2n+3)} \Big|_0^1 = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)(2n+3)}
 \end{aligned}$$

It's an alternating series, so we can use the alternating series estimation theorem: $|R_n| \leq b_{n+1} < \frac{1}{100}$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)(2n+3)} = \frac{1}{3} - \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} - \frac{1}{7 \cdot 9} + \frac{1}{9 \cdot 11} - \frac{1}{11 \cdot 13}$$

we only need these terms

$\leq \frac{1}{100} \rightarrow b_5 \leq \frac{1}{100}$

$$\int_0^1 x \arctan x dx \approx S_4 = \sum_{n=0}^4 \frac{(-1)^n}{(2n+1)(2n+3)} = \frac{1}{3} - \frac{1}{15} + \frac{1}{35} - \frac{1}{63} + \frac{1}{99}$$

5 terms! (but $n=4$)