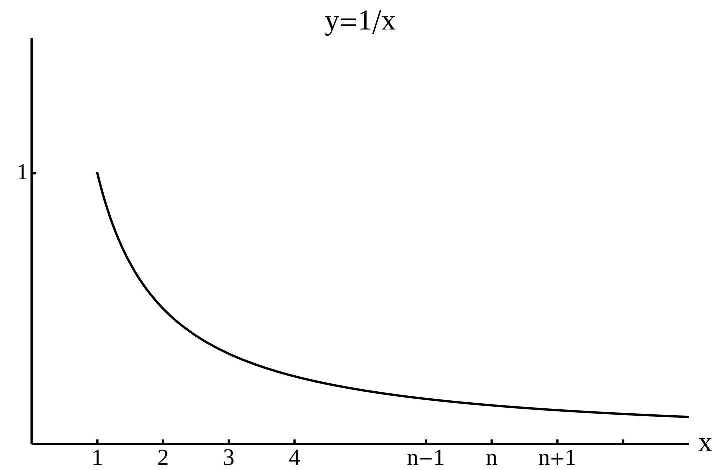
Group: ______________ ______________ ______________ Name: ______________

Math 231 A. Worksheet 11.

1. Recall that the harmonic series is $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots$. Let $s_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ be the nth partial sum.

a) What is $\lim_{n\to\infty} s_n$?



b) Draw a careful picture on the graph to the right which illustrates that $s_n \leq 1 + \ln n$. Be sure that your reasoning is explained.

c) Suppose that you were to add $3,000,000,000$ terms of the harmonic series. Show that the sum would be less than 23.

Recall that if the integral test proves that a series converges, then

$$\int_{n+1}^{\infty} f(x)\,dx < R_n < \int_{n}^{\infty} f(x)\,dx \qquad \text{and} \qquad S_n + \int_{n+1}^{\infty} f(x)\,dx < S < S_n + \int_{n}^{\infty} f(x)\,dx$$

where $a_n = f(n)$, $S = \sum_{n=1}^{\infty} a_n$, $S_n = a_1 + a_2 + \cdots + a_n$, and $R_n = S - S_n$.

2. How many terms of the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ would you need to add to estimate the sum to within 0.01? (Hint: you want $R_n < 0.01$.)

3. a) Estimate the maximum possible error when the 20th partial sum $\sum_{n=1}^{20} \frac{1}{n^3}$ is used to estimate the sum $s = \sum_{n=1}^{\infty} \frac{1}{n^3}$.

b) The 20th partial sum is $s_{20} \approx 1.200867842...$ Find a short interval (a, b) which contains s.

4. For which p does a p-series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ converge?

For which r does a geometric series $\displaystyle\sum_{n=0}^{\infty} ar^n$ converge?

5. Use the Comparison Test to determine if the following series converge.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{4^n + 3}{5^n + n}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^2 + 1}}{n^2 + 3}$

c) $\displaystyle\sum_{n=3}^{\infty} \frac{\ln(n) + \sin(n)}{n^2}$