Mock Exam 2, Math 241, Spring 2018

1. Consider the function $f(x,y) = \cos(xy)e^x$.

   (a) Use the linear approximation to $f(x,y)$ at the point $(0,\pi)$ to find the approximate value of $f(0.1,\pi)$.

   (circle one): (A) 1.3; (B) 1.1; (C) 1.0; (D) 1.2;

   (b) Find the tangent line to the level curve $f(x,y) = 1$ at the point $(0,\pi)$.

   (circle one): (A) $x+y=\pi$; (B) $y=0$; (C) $2x+2y=2\pi$; (D) $x=0$;

2. Let $f(x,y,z) = y\ln(xyz)+x$ and consider a function $y = y(x,z)$ defined around the point $(1,1)$, satisfying:

$$f(x,y(x,z),z) = 1, \quad y(1,1) = 1,$$

   Find the partial derivative $\frac{\partial y}{\partial z}(1,1)$.

   (circle one): (A) $\frac{1}{2}$; (B) 1; (C) $\frac{-1}{2}$; (D) -1;

3. (a) Consider the function $f(x,y,z) = ye^{x+z}$. Find the unit vector $\vec{u}$ which *maximizes* the value of the directional derivative $D_{\vec{u}}f(1,0,1)$.

   (circle one): (A) $\langle 0,e^2,0\rangle$; (B) $\langle 1,0,0\rangle$; (C) $\langle 0,1,0\rangle$; (D) $\langle 0,0,e^2\rangle$;

   (b) Consider a function $f\colon \mathbb{R}^2 \to \mathbb{R}$, differentiable at $(1,3)$, with directional derivatives along the unit vectors $\vec{u}_1 = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\rangle$ and $\vec{u}_2 = \langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\rangle$ given by:

$$D_{\vec{u}_1}f(1,3) = D_{\vec{u}_2}f(1,3) = \sqrt{2}.$$

   Find $\nabla f(1,3)$.

   (circle one): (A) $\langle 2,0\rangle$; (B) $\langle 0,1\rangle$; (C) $\langle 1,0\rangle$; (D) $\langle 0,2\rangle$;

4. A function $f\colon \mathbb{R}^2 \to \mathbb{R}$ has all second order partial derivatives continuous. At the points $(1,1)$, $(1,-1)$, $(-1,1)$ and $(-1,-1)$, one knows the following values of f and its derivatives:

(x,y)	f	f_x	f_y	f_{xx}	f_{xy}	f_{yy}
$(1,\ 1)$	-20	0	1	2	1	2
$(1,-1)$	0	0	0	-4	3	-4
$(-1,\ 1)$	0	0	0	0	3	-1
$(-1,-1)$	-20	0	0	4	-3	4

(a) How many of these points are critical points of $f(x,y)$?

(circle one): (A) 1; (B) 2; (C) 3; (D) 4;

(b) At how many of these critical points does $f(x,y)$ have a local minimum?

(circle one): (A) 0; (B) 1; (C) 2; (D) 3;

(c) At how many of these critical points does $f(x,y)$ have a local maximum?

(circle one): (A) 0; (B) 1; (C) 2; (D) 3;

(d) At how many of these critical points does $f(x,y)$ have a saddle point?

(circle one): (A) 0; (B) 1; (C) 2; (D) 3;

5. (a) Find a parameterization $\vec{r}(t)$ for the intersection of the cylinder $x^2 + y^2 = 4$ with the surface $z = y^2 - x^2$.

(circle one):

(A) $\langle 4\sin t, 4\cos t, 16\cos 2t\rangle$; (B) $\langle 2\sin t, 2\cos t, 4\cos 2t\rangle$;

(C) $\langle 2\sin t, 2\cos t, -4\cos 2t\rangle$; (D) $\langle 2\sin t, 2\cos t, 2\cos 2t\rangle$;

(b) Find the tangent line, $\vec{l}(t)$ to the curve $\vec{r}(t) = \langle e^t \cos t, e^t, e^t \sin t\rangle$ at the point $(1,1,0)$.

(circle one):

$(A)\quad \vec{l}(t) = \langle 1+t, 1+t, t\rangle;\qquad (B)\quad \vec{l}(t) = \langle 1+2t, 1+t, -2t\rangle;$

$(C)\quad \vec{l}(t) = \langle 1, 1+t, 0\rangle;\qquad (D)\quad \vec{l}(t) = \langle 1+t, 1-t, -3t\rangle;$

6. (a) Find the length of the cycloid parameterized by

$$\vec{r}(t) = \langle 5, 1+\cos(2t), 2t+\sin(2t)\rangle, \text{ for } 0 \le t \le \frac{\pi}{2}.$$

(circle one): (A) 0; (B) 1; (C) 2; (D) 4;

(b) Evaluate the integral $\int_C xy \, \mathrm{d}s$ where the curve C is the boundary of the half disk $R = \{(x,y) : x^2+y^2 = 3, y \ge 0\}$ travelled counter-clockwise.

(circle one): (A) 0; (B) $\frac{3\sqrt{3}}{4}$; (C) $\frac{3\sqrt{3}}{2}$; (D) $\sqrt{3}$;

7. Evaluate the integral $\int_C \vec{F} \cdot \mathrm{d}\vec{r}$, where $\vec{F}(x,y) = \langle -y, x\rangle$ and C is the path given by the ellipse $9x^2+4y^2 = 36$ travelled clockwise.

$\int_C \vec{F} \cdot \mathrm{d}\vec{r} =$

8. Find the absolute maximum and absolute minimum values of $f(x,y) = 3x^3+4y^3$ on the region $D = \{(x,y) \,|\, x^2+y^2 \le 4\}$.

absolute max value = ; absolute min value =

9. Find the smallest distance between points in the ellipsoid $x^2+2y^2+z^2 = 16$ and the point $(0,0,1)$.

Smallest distance =

TRIGONOMETRIC IDENTITIES

$$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$
$$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$
$$\tan(\theta+\phi) = \frac{\tan(\theta)+\tan(\phi)}{1-\tan\theta\tan\phi}$$
$$\sin(\theta-\phi) = \sin\theta\cos\phi - \cos\theta\sin\phi$$
$$\cos(\theta-\phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$$
$$\tan(\theta-\phi) = \frac{\tan(\theta)-\tan(\phi)}{1+\tan\theta\tan\phi}$$
$$\sin(\theta)\sin(\phi) = \frac{\cos(\theta-\phi)-\cos(\theta+\phi)}{2}$$
$$\cos(\theta)\cos(\phi) = \frac{\cos(\theta-\phi)+\cos(\theta+\phi)}{2}$$
$$\sin(\theta)\cos(\phi) = \frac{\sin(\phi+\theta)-\sin(\phi-\theta)}{2}$$
$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$
$$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$$
$$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$$
$$\tan\theta = \frac{\sin(2\theta)}{1+\cos(2\theta)}$$