

**You have 20 minutes. Please show all your work.**

**No notes, calculators or electronic devices of any kind are permitted.**

1. Consider the series  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2}{1+n}$ .

a) Use the Alternating Series Test to confirm that this series converges.

b) If the value of the series is  $S$ , find the smallest number  $n$  such that the  $n^{\text{th}}$  partial sum  $S_n$  satisfies  $|S - S_n| < 1/5$ .

2. Find the interval of convergence of the power series  $\sum_{n=0}^{\infty} \frac{(-3)^n}{n^2 + 1} (x - 2)^n$ .

3. Consider the function  $f(x) = \frac{x^2}{2 + 4x^2}$ .

- a) Express the function  $f(x)$  as a power series centered at  $a = 0$ . Write your answer using  $\sum$  notation AND write the first three terms.

$\sum$	$=$	$+ \dots$
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- b) Find the radius of convergence of this power series.

- c) Find the power series centered at  $a = 0$  for  $f'(x)$ . If you are not confident that your answer in part a) is correct, find the power series for  $g'(x)$  instead, where

$$g(x) = \sum_{n=0}^{\infty} 3^n x^{2n+3}.$$