Group: ____________ ____________ ____________ Name: SOLUTIONS

1. Consider the parametric curve $x = \sin^2 t$, $y = \sin 3t$, $0 \le t \le \pi/3$. Set up but do not evaluate integrals which represent the following:

a) The area under the curve.

$$\int_0^{\pi/3} y\,dx = \int_0^{\pi/3} \sin(3t)\, 2\sin t \cos t \, dt$$

b) The surface area created by rotating the curve about the x-axis.

$$\int_0^{\pi/3} 2\pi y\, ds = \int_0^{\pi/3} 2\pi \sin(3t) \sqrt{(2\sin t\cos t)^2 + (3\cos(3t))^2}\, dt$$

c) The surface area created by rotating the curve about the line $y = 5$.

$$\int_0^{\pi/3} 2\pi(5-y)\, ds = \int_0^{\pi/3} 2\pi(5 - \sin 3t) \sqrt{(2\sin t\cos t)^2 + (3\cos 3t)^2}\, dt$$

d) The surface area created by rotating the curve about the y-axis.

$$\int_0^{\pi/3} 2\pi x\, ds = \int_0^{\pi/3} 2\pi \sin^2 t \sqrt{(2\sin t\cos t)^2 + (3\cos(3t))^2}\, dt$$

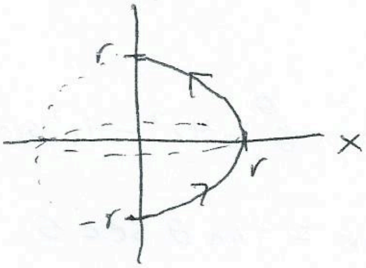
2. A sphere of radius r is formed by rotating the semicircle

$$x = r\cos\theta, \quad y = r\sin\theta, \quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

about the y axis. Sketch a graph. Then compute the surface area of the sphere.

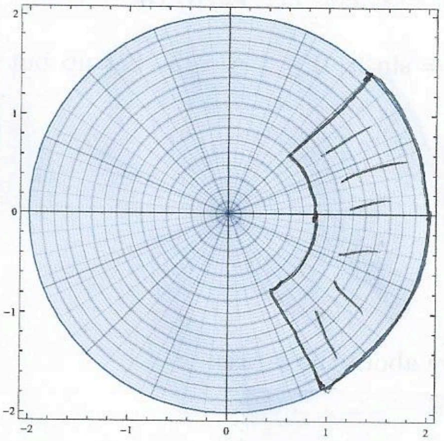
$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\pi x\, ds = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\pi r\cos\theta \sqrt{(-r\sin\theta)^2 + (r\cos\theta)^2}\, d\theta$$

$$= \int_{-\frac{\pi}{2}}^{\pi/2} 2\pi r^2 \cos\theta\, d\theta = 2\pi r^2 \sin\theta \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 4\pi r^2$$

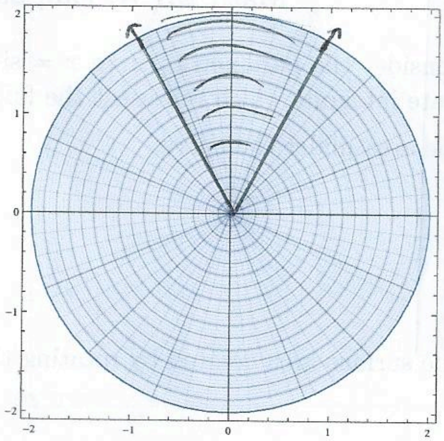


3. Sketch the regions

a) $1 \le r \le 2,\ -\frac{\pi}{3} \le \theta \le \frac{\pi}{4}$.



b) $r \le 0,\ \frac{4\pi}{3} \le \theta \le \frac{5\pi}{3}$.



Recall: To convert from polar to rectangular (Cartesian) coordinates, we use

$$x = r\cos\theta, \qquad y = r\sin\theta.$$

To convert from rectangular to polar coordinates, we use

$$y/x = \tan\theta, \qquad x^2 + y^2 = r^2.$$

4. Identify each polar curve by finding a Cartesian equation.

a) $\theta = \frac{\pi}{3}$

$\frac{y}{x} = \tan\frac{\pi}{3} = \sqrt{3} \Rightarrow y = \sqrt{3}\,x$ (line)

b) $r = 2\sin\theta$

$r = 2\left(\frac{y}{r}\right) \Rightarrow r^2 = 2y \Rightarrow x^2 + y^2 = 2y \Rightarrow x^2 + (y-1)^2 = 1$

(circle of radius 1 centered at $(0,1)$)

5. Find a simple polar equation which represents each of the following.

a) $y = 3x$

$\frac{y}{x} = 3 \Rightarrow \tan\theta = 3 \Rightarrow \theta = \arctan(3)$

b) $y = 4x^2$

$r\sin\theta = 4r^2\cos^2\theta \Rightarrow \frac{\sin\theta}{\cos^2\theta} = 4r$

$\Rightarrow 4r = \tan\theta\sec\theta$