Thursday, February 15 $**$ *Taylor series, the* 2^{nd} *derivative test, and changing coordinates.*

1. Consider $f(x, y) = 2\cos x - y^2 + e^{xy}$.

   (a) Show that $(0,0)$ is a critical point for f.

   (b) Calculate each of f_{xx}, f_{xy}, f_{yy} at $(0,0)$ and use this to write out the 2^{nd}-order Taylor approximation for f at $(0,0)$.

   (c) To make sure the next two problems go smoothly, check your answer to (b) with the instructor.

2. Let $g(x, y)$ be the approximation you obtained for $f(x, y)$ near $(0,0)$ in 1(b). It's not clear from the formula whether g, and hence f, has a min, max, or a saddle at $(0,0)$. Test along several lines until you are convinced you've determined which type it is. In the next problem, you'll confirm your answer in two ways.

3. Consider alternate coordinates (u, v) on $\mathbb{R}^2$ given by $(x, y) = (u - v, u + v)$.

   (a) Sketch the u- and v-axes relative to the usual x- and y-axes, and draw the points whose (u, v)-coordinates are: $(-1,2)$, $(1,1)$, $(1,-1)$.

   (b) Express g as a function of u and v, and expand and simplify the resulting expression.

   (c) Explain why your answer in 3(b) confirms your answer in 2.

   (d) Sketch a few level sets for g. What do the level sets of f look like near $(0,0)$?

   (e) It turns out that there is always a similar change of coordinates so that the Taylor series of a function f which has a critical point at $(0,0)$ looks like $f(u, v) \approx f(0,0) + au^2 + bv^2$. In fact this is why the 2^{nd} derivative test works.

   Double check your answer in 2 by applying the 2^{nd}-derivative test directly to f.

4. Consider the function $f(x, y) = 3xe^y - x^3 - e^{3y}$.

   (a) Check that f has only one critical point, which is a local maximum.

   (b) Does f have an absolute maxima? Why or why not? Check your answer with the instructor.