Group: ____________ ____________ ____________ Name: solutions

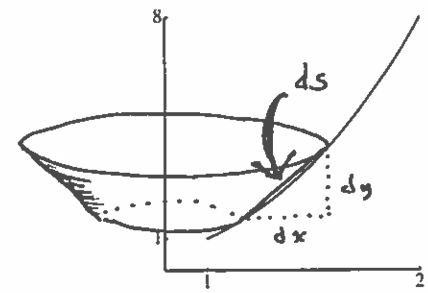
Math 231 A. Fall, 2015. Worksheet 8. 9/29/15

We will work with the formulas $A = \int 2\pi y\, ds$ for the surface area of rotation about the x axis and $A = \int 2\pi x\, ds$ for the surface area of rotation about the y axis (see your lecture notes). These formulas must be correctly interpreted in each case to produce an expression which is ready to be evaluated.

1. The curve $y = x^3$ between the points $(1, 1)$ and $(2, 8)$ is rotated about the y-axis.

a) Indicate the meaning of dx and dy and the arclength differential ds on the curve.

b) Sketch the frustum which is created by the rotation of ds.



c) Set up but do not evaluate an integral **with respect to** x which represents the surface area. All quantities involved must refer to x.

$$S = \int 2\pi x\, ds = \int_1^2 2\pi x \sqrt{1 + 9x^4}\, dx$$

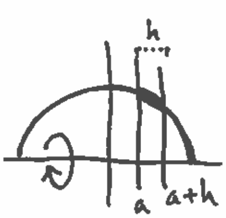
d) Set up but do not evaluate an integral **with respect to** y which represents the surface area. All quantities involved must refer to y.

$$S = \int 2\pi x\, ds = \int_1^8 2\pi y^{1/3} \sqrt{1 + \frac{1}{9} y^{-4/3}}\, dy$$

2. A hollow sphere of radius r is the surface formed by rotating a semi-circle of radius r about the x-axis. Show that if the sphere is cut by two parallel planes at $x = a$ and $x = a+h$, then the surface area of the sphere between the planes is given by the simple formula $S = 2\pi rh$.

In particular, the surface area is the same no matter where the sphere is cut. (If this does not seem interesting, think about taking a 10 foot slice of the earth (a) at the north pole, and (b) at the equator). **Hint:** The final integral is **easy** to evaluate. Have faith and keep simplifying.

$y = \sqrt{r^2 - x^2}$, $\frac{dy}{dx} = \frac{-x}{\sqrt{r^2-x^2}}$, $\left(\frac{dy}{dx}\right)^2 = \frac{x^2}{r^2-x^2}$.

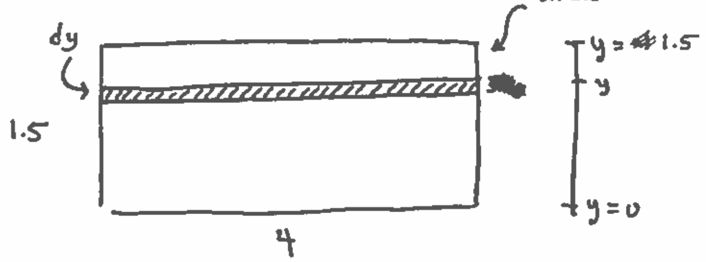


$$S = \int 2\pi y\, ds = \int_a^{a+h} 2\pi \sqrt{r^2-x^2} \cdot \sqrt{1 + \frac{x^2}{r^2-x^2}}\, dx = 2\pi \int_a^{a+h} \sqrt{r^2 - x^2 + x^2}\, dx$$

$$= 2\pi \int_a^{a+h} r\, dx = 2\pi r \int_a^{a+h} dx = 2\pi r\,(a+h) - 2\pi r(a) = 2\pi r h.$$

3. At the zoo, the underwater window to view the penguins has the shape of a rectangle 1.5 meters high and 4 meters wide. The top of the window is level with the surface of the water. Find the total hydrostatic force on the window. Use $\rho = 1000$ kg/m^3 for the density of water, and $g = 9.8$m/s^2 for the acceleration due to gravity. Please show your units.

area of strip $= 4\,dy$, depth $= 1.5 - y$,



force on strip: $\rho \cdot g\,(1.5-y) \cdot 4\,dy$

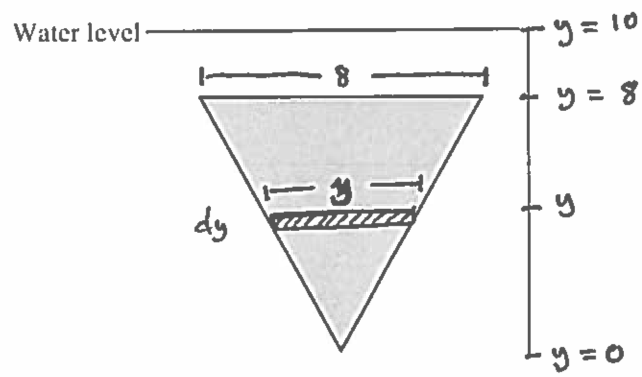
total force:

$$\int_0^{1.5} 1000 \cdot 9.8 \cdot (1.5-y) \cdot 4\,dy$$

$$= 39{,}200 \int_0^{1.5} (1.5-y)\,dy = 44{,}100 \text{ N.}$$

4. An underwater window has the shape of a triangle whose top edge is 2 meters below the surface. The height of the triangle is 8 meters and the length of the top edge is also 8 meters. Set up **but do not evaluate** an integral for the hydrostatic force on the window. Use ρ kg/m^3 for the density of water, and g m/s^2 for the gravitational constant.

Hint: Clearly label your coordinates on the "ruler" to the right of the diagram. Put $y = 0$ at the bottom point of the triangle.



strip has width $x = y$ (similar triangles) and height dy, so area $y\,dy$.

the depth of the strip is $10 - y$. with pressure $\rho \cdot g \cdot (10-y)$,

the force on the strip is $\rho \cdot g\,(10-y) \cdot y\,dy$, so the force on the triangle is

$$\int_0^8 \rho \cdot g \cdot (10-y)\,y\,dy.$$

5. Set up the integral if the top edge of the triangle is 5 meters below the surface.

$$\int_0^8 \rho \cdot g \cdot (13-y)\,y\,dy.$$