Mock Exam 1, Math 241, Spring 2018

1. Let $A = (-3, 6, 4)$ and $B = (-1, 2, 8)$.

   (a) Which of the following points is on the sphere that passes through the point B and whose center is the point A.

   (circle one): (A) $(1, -2, 2)$; (B) $(2, 1, -2)$; (C) $(-2, 1, 2)$; (D) $(1, 2, 2)$;

   (b) Find an equation of the set of all points equidistant from the points A and B.

   (circle one):

   (A) $2x - 4y + 4z = 8$; (B) $2x + 4y + 4z = 8$;

   (C) $4x - 8y + 8z = 8$; (D) $4x + 8y + 8z = 8$;

2. Let $\vec{a} = \langle 1, -2, 2\rangle$, $\vec{b} = \langle 2, 5, -5\rangle$, $\vec{c} = \langle 3, 1, 4\rangle$

   (a) Find the vector projection of $\vec{b}$ onto $\vec{a}$.

   $\text{Proj}_{\vec{a}}\vec{b}$ is (circle one):

   (A) $\langle 1, -2, 2\rangle$; (B) $\langle -1, 2, -2\rangle$;

   (C) $\langle 2, -4, 4\rangle$; (D) $\langle -2, 4, -4\rangle$;

   (b) Find the vector projection of $\vec{c}$ onto $\vec{a}$.

   $\text{Proj}_{\vec{a}}\vec{c}$ is (circle one):

   (A) $\langle 1, -2, 2\rangle$; (B) $\langle -1, 2, -2\rangle$;

   (C) $\langle 2, -4, 4\rangle$; (D) $\langle -2, 4, -4\rangle$;

(c) Find $\sin(\theta)$, where θ is the angle between the vectors $\text{Proj}_{\vec{a}}\vec{b}$ and $\text{Proj}_{\vec{a}}\vec{c}$.

$\sin(\theta)$ is (circle one): (A) $\frac{1}{\sqrt{2}}$; (B) $\frac{1}{3}$; (C) $\frac{1}{\sqrt{3}}$; (D) 0 ;

3. Let $\vec{a} = \langle -2, 1, -3\rangle$, $\vec{b} = \langle 2, -1, -3\rangle$, and $\vec{c} = \langle 1, 1, -2\rangle$.

(a) Find the Area of the parallelogram determined by the vectors $\vec{b}$ and $\vec{c}$.

Area is (circle one): (A) $\frac{\sqrt{42}}{2}$; (B) $\frac{\sqrt{35}}{2}$; (C) $\sqrt{42}$; (D) $\sqrt{35}$;

(b) Find the volume, V, of the parallelepiped determined by the vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$.

V is (circle one): (A) 18; (B) $\frac{21}{2}$; (C) $\sqrt{42}$; (D) 9 ;

(c) Find the length of the vector projection of $\vec{a}$ onto vector $\vec{b} \times \vec{c}$.

(circle one): (A) $\frac{9}{\sqrt{42}}$; (B) $\frac{21}{\sqrt{42}}$; (C) $\frac{18}{\sqrt{35}}$; (D) $\frac{9}{\sqrt{35}}$;

4. Let P_1 be the plane of $x + 2y - 2z = 1$, and P_2 be the plane of $-4x + 2y - 4z = -6$

(a) Find $\cos(\theta)$, where θ is the angle between the planes P_1 and P_2.

$\cos(\theta)$ is (circle one): (A) $\frac{4}{9}$; (B) $\frac{2}{9}$; (C) $\frac{1}{2}$; (D) 0 ;

(b) The line $\vec{r}(t) = \langle 2 + 2t, 3 + t, -1 + 2t\rangle$ is parallel one of the planes P_1 or P_2. Find the distance from the line to the plane (P_1 or P_2) which it is parallel to.

(circle one): (A) $\sqrt{6}$; (B) $\sqrt{3}$; (C) 6; (D) 3 ;

(c) The line $\vec{r}(t) = \langle 2 + 2t, 3 + t, -1 + 2t\rangle$ intersects one of the planes P_1 or P_2 at a point labelled as (x_0, y_0, z_0). Find y_0.

y_0 is (circle one): (A) $\frac{2}{7}$; (B) $\frac{25}{7}$; (C) $\frac{22}{7}$; (D) $\frac{24}{7}$;

5. Let $P = (0,0,-2)$, $Q = (2,-3,0)$, and $R = (0,-2,-1)$

(a) Find and equation of the plane which contains points P, Q, and R.

(circle one):

$(A)\quad 2x-4y-4z=8;$ $\qquad (B)\quad x-2y-4z=8;$

$(C)\quad 4x-8y+8z=-16;$ $\qquad (D)\quad 8x+8y+2z=-8;$

(b) Find the shortest vector, $\vec{v}$, from the plane of part (a) to the point $(2,-4,-10)$.

$\vec{v}$ is (circle one):

$(A)\quad \langle 2,-2,-6\rangle;$ $\qquad (B)\quad \langle 2,-4,-8\rangle;$

$(C)\quad \langle 4,-4,2\rangle;$ $\qquad (D)\quad \langle -6,2,3\rangle;$

(c) Find the distance, D, from the point $(2,-4,-10)$ to the plane from part (a).

D is (circle one): (A) $\sqrt{36}$; (B) $\sqrt{44}$; (C) $\sqrt{49}$; (D) $\sqrt{84}$;

6. Which of the following properties hold for all vectors $\vec{u}$ and $\vec{v}$ and scalars c and d? For each property, circle either True or False.

(a) $\vec{u}+\vec{v}=\vec{v}+\vec{u}$ True / False

(b) $\vec{u}\times\vec{v}=\vec{v}\times\vec{u}$ True / False

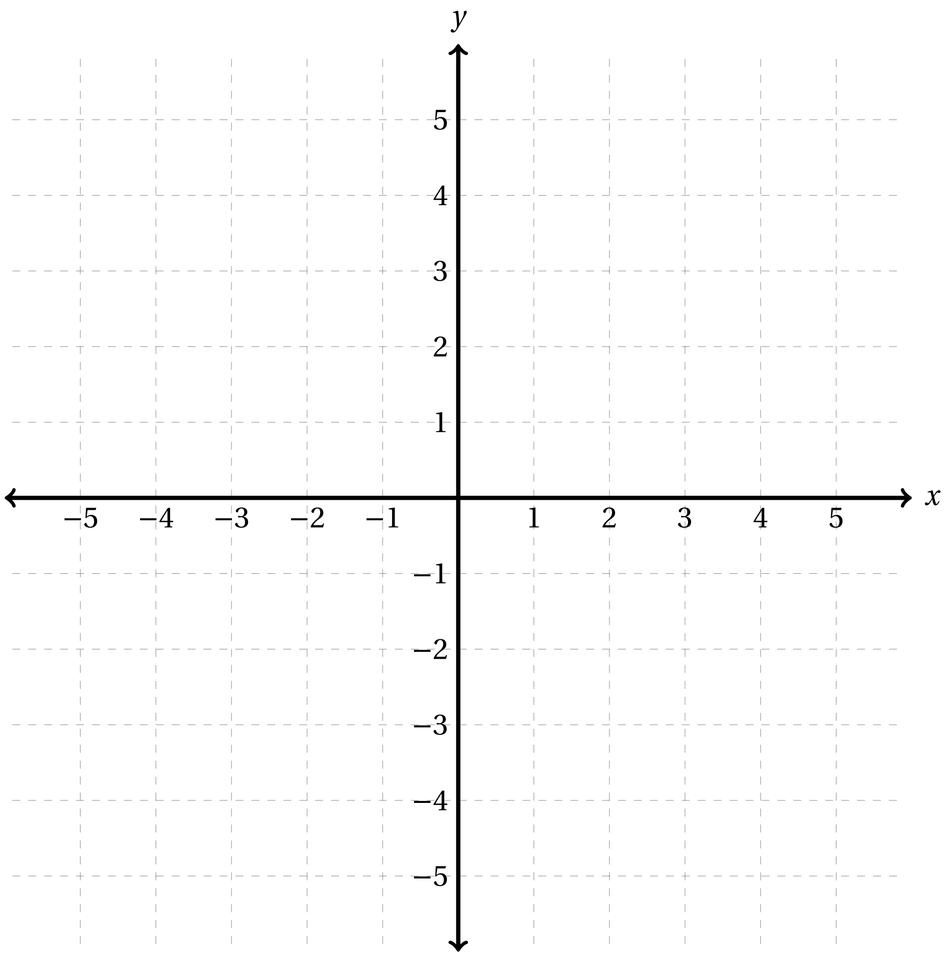
(c) $\vec{u}+\vec{v}=\vec{u}\times\vec{v}$ True / False

(d) $\vec{u}\cdot\vec{u}=|\vec{u}|$ True / False

(e) $\vec{u}\times\vec{u}=\vec{0}$ True / False

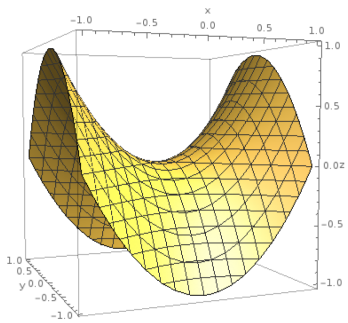
(f) $(c+d)(\vec{u}+\vec{v})=c\vec{u}+d\vec{v}$ True / False

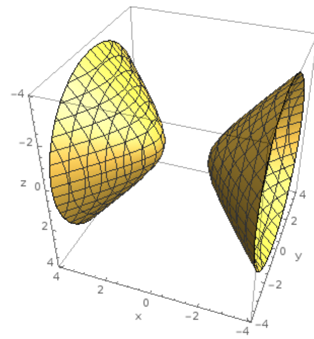
7. Sketch a contour map of $f(x, y) = x^2 - 4x + y^2 + 5$ for level curves corresponding to $z = 2, 5$ and 10.

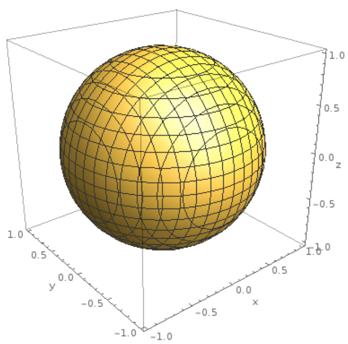


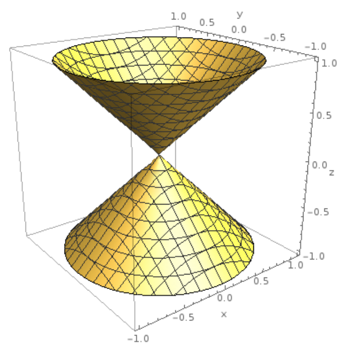
8. Identify the equations of each of the following surfaces (write the letter of your selection in the box below each surface):

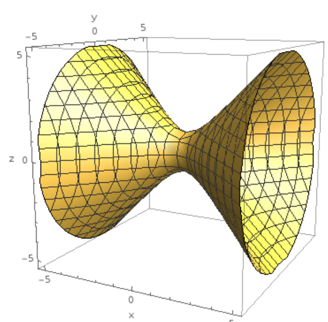
(A) $x^2 + y^2 + z^2 = 1$ (B) $-x^2 + y^2 + z^2 = 1$ (C) $x^2 + y^2 + z^2 = 0$ (D) $x^2 + y^2 - z^2 = 0$
(E) $x^2 - y^2 - z = 0$ (F) $x^2 + y^2 - z = 0$ (G) $x^2 + y^2 + z^2 = -1$ (H) $x^2 - y^2 - z^2 = 1$

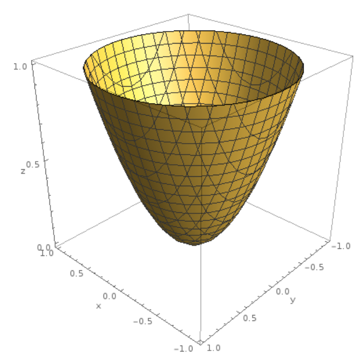












9. (a) Find the limit, if it exists, or show that the limit does not exist (show work that justifies your answer)

$$L = \lim_{(x,y)\to(0,0)} \frac{y^2 \sin^2 x}{x^4 + y^4}.$$

$L =$

(b) Find the limit, if it exists, or show that the limit does not exist (show work that justifies your answer)

$$L = \lim_{(x,y)\to(0,0)} \frac{x^2 y^3}{2x^2 + y^2}.$$

$L =$

(c) Find the limit, if it exists, or show that the limit does not exist (show work that justifies your answer)

$$L = \lim_{(x,y)\to(0,0)} \frac{x^2 + y^2}{\sqrt{1 + x^2 + y^2} - 1}.$$

$L =$

10. (a) Let $f(x, y) = 3\sin(x^2)ye^{xy}$. Compute $f_x(x, y)$ and $f_y(x, y)$.

$f_x(x, y) =$	; $f_y(x, y) =$

(b) Let $g(x, y) = 3\ln(x^2 + y^2 + 1) + x^2 + y^2 + \sin(xy)$. Compute $g_{xx}(x, y), g_{xy}(x, y)$, and $g_{yy}(x, y)$.

$g_{xx}(x, y) =$	; $g_{xy}(x, y) =$	; $g_{yy}(x, y) =$

Trigonometric Identities

$$
\begin{aligned}
\sin(\theta+\phi) &= \sin\theta\cos\phi+\cos\theta\sin\phi \\
\cos(\theta+\phi) &= \cos\theta\cos\phi-\sin\theta\sin\phi \\
\tan(\theta+\phi) &= \frac{\tan(\theta)+\tan(\phi)}{1-\tan\theta\tan\phi} \\
\sin(\theta-\phi) &= \sin\theta\cos\phi-\cos\theta\sin\phi \\
\cos(\theta-\phi) &= \cos\theta\cos\phi+\sin\theta\sin\phi \\
\tan(\theta-\phi) &= \frac{\tan(\theta)-\tan(\phi)}{1+\tan\theta\tan\phi} \\
\sin(\theta)\sin(\phi) &= \frac{\cos(\theta-\phi)-\cos(\theta+\phi)}{2} \\
\cos(\theta)\cos(\phi) &= \frac{\cos(\theta-\phi)+\cos(\theta+\phi)}{2} \\
\sin(\theta)\cos(\phi) &= \frac{\sin(\phi+\theta)-\sin(\phi-\theta)}{2} \\
\sin(2\theta) &= 2\sin\theta\cos\theta \\
\cos(2\theta) &= \cos^2\theta-\sin^2\theta \\
\tan(2\theta) &= \frac{2\tan\theta}{1-\tan^2\theta} \\
\sin^2\theta &= \frac{1-\cos(2\theta)}{2} \\
\cos^2\theta &= \frac{1+\cos(2\theta)}{2} \\
\tan\theta &= \frac{\sin(2\theta)}{1+\cos(2\theta)}
\end{aligned}
$$