Math 241: Exam 1, Sept 25th, 2017

Name: NetID:

Circle your discussion section:

Professor Fernandez:

- ADA: 8am Kim
- ADB: 9am Kim
- ADC: 10am Butler
- ADD: 11am Butler
- ADE: Noon Kelm
- ADF: 1pm Penciak
- ADG: 2pm Kelm
- ADH: 3pm Romney
- ADI: 4pm Carmody
- ADK: 9am Yadav
- ADL: 10am Yadav
- ADM: 11am O'Neill
- ADN: Noon Donepudi
- ADO: 1pm Carmody
- AD1: 11am Rennie
- AD2: 9am Karve
- AD3: 3pm Michiels

Professor Robles:

- DDA: 8am Park
- DDB: 9am Nam
- DDC: 10am Robles
- DDD: 11am Penciak
- DDE: Noon Christenson
- DDF: 1pm Donepudi

Professor Bergvelt:

- BDA: 8am Zhao
- BDB: 9am Zhao
- BDC: 10am Christenson
- BDD: 11am Block Gorman
- BDE: Noon Block Gorman
- BDF: 1pm Bhardwaj
- BDG: 2pm Bhardwaj
- BDH: 3pm O'Neill
- BDI: 4pm Okano
- BDJ: 8am Linz
- BDK: 9am Linz
- BDL: 10am Shakan
- BDM: 11am Okano
- BDN: 2pm Wester
- BDO: 3pm Wester

Professor Anema:

- CDA: 8am Tatum
- CDB: 9am Park
- CDC: 10am Kirkoryan
- CD@: 11am Kosar
- CDE: Noon Kirkoryan
- CDF: 1pm Tatum
- CDG: 2pm Dunn
- CDH: 3pm Dunn
- CDI: 4pm Field
- CDJ 8am Nam
- CDK: 9am Shakan
- CDL: 10am Ellis
- CDM: 11am Ellis
- CDO: 1pm Li
- CDP: 2pm LI

Write and bubble in your UIN:

0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9

Instructions: You have **75 minutes** to complete this exam. There are **80 points** available and not all problems are weighted equally. Calculators, books, notes, and suchlike aids are **not** permitted. **When space is provided, show work that justifies your answer** as in those problems **credit will not be given** for correct answers without proper justification. Work written outside of the space provided for a problem will **not** be graded.

Do not open exam until instructed.

Question:	1	2	3	4	5	6	7	8	Total
Points:	6	8	10	12	12	8	12	12	80
Score:									

1. (a) **(3 points)** Let A, B be two points in $\mathbb{R}^4$ with displacement vector $\overrightarrow{AB} = \langle 2, 2, 0, -2 \rangle$. What is the distance between the points A and B?

Solution The distance is just the magnitude $\sqrt{2^2 + 2^2 + 0^2 + 2^2} = \sqrt{12}$.

Distance =

(b) **(3 points)** Let S be the sphere with center $C(0, 1, -1)$ and radius 2. Does the point $P(2, 0, 0)$ lie on S? What about $Q(2, 1, -1)$?

Solution The equation of the sphere is $x^2 + (y-1)^2 + (z+1)^2 = 4$. Then P is *not* on the sphere, because $2^2 + (0-1)^2 + (0+1)^2 \neq 4$, but Q *is* on the sphere, as $2^2 + (1-1)^2 + (-1+1)^2 = 4$.

P is on/not on S	Q is on/not on S

2. Let $\vec{u} = \langle 1,1,1 \rangle$.

(a) (**2 points**) The vector projection of $\vec{u}$ on the vector $\vec{y} = \langle 2,-2,0 \rangle$ is the vector $\langle 2,-2,0 \rangle$.

True | False

Solution False

(b) (**2 points**) The vector projection of $\vec{u}$ on the vector $\vec{z} = \langle 0,2,-2 \rangle$ is the vector $\langle 0,0,0 \rangle$.

True | False

Solution True

(c) (**2 points**) The vector projection of $\vec{u}$ on the vector $\vec{w} = \langle 2,2,0 \rangle$ is the vector $\langle 1,1,0 \rangle$.

True | False

Solution True

(d) (**2 points**) The vector projection of $\vec{u}$ on the vector $\vec{x} = \langle 2,2,2 \rangle$ is the vector $\langle 2,2,2 \rangle$.

True | False

Solution False

3. **(10 points)** Consider two lines L_1, L_2 with symmetric equations

$$L_1: \quad \frac{x-3}{1} = \frac{y-1}{-1} = \frac{z+2}{6},$$
$$L_2: \quad \frac{x}{2} = \frac{y-2}{-1} = \frac{z-2}{1}.$$

(a) (4 points) Find a parametrization $\vec{r}_1(t)$ and $\vec{r}_2(s)$ for the lines L_1 and L_2, respectively.

$\vec{r}_1(t) = \langle 3+t, 1-t, -2+6t \rangle$, $\vec{r}_2(s) = \langle 2s, 2-s, 2+s \rangle$

$$\boxed{\vec{r}_1(t) = \langle \quad , \quad , \quad \rangle, \quad \vec{r}_2(s) = \langle \quad , \quad , \quad \rangle}$$

(b) (6 points) The lines L_1 and L_2 intersect in exactly one point. Determine this point of intersection.

Solution Setting the vector representations of the two lines equal to each other we get a system of equations for s, t, with solution $s = 2, t = 1$, corresponding to the point of intersection $(4, 0, 4)$,

$$\boxed{\text{Point of Intersection} = (\quad , \quad , \quad)}$$

4. (a) **(6 points)** Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be the function given by

$$f(x,y) = \begin{cases} \frac{x^3}{x^2+y^2}, & (x,y) \neq (0,0), \\ 0, & (x,y) = (0,0). \end{cases}$$

Find $\lim_{(x,y)\to(0,0)} f(x,y)$ or show that the limit does not exist. Justify your answer.

Solution Since, $0 \leq \left|\frac{x^3}{x^2+y^2}\right| \leq |x|$, the squeeze theorem gives this limit is zero.

(**Alt. solution**) Since $0 \leq \frac{x^2}{x^2+y^2} \leq 1$, we have:

$$|f(x,y)| = \frac{|x|^3}{x^2+y^2} \leq \frac{x^2|x|}{x^2+y^2} \leq |x| \leq \sqrt{x^2+y^2}.$$

Hence, we conclude that for every $\epsilon > 0$, by setting $\delta = \epsilon$, we have:

$$\text{If } 0 < \sqrt{x^2+y^2} < \delta, \text{ then } |f(x,y) - 0| < \epsilon.$$

giving us that $\lim_{(x,y)\to(0,0)} f(x,y) = 0$.

(**Alt. solution**) If we use polar coordinates the function becomes $f(r,\theta) = \frac{r^3\cos^3(\theta)}{r^2} = r\cos^3(\theta)$, which has the limit 0.

$\lim_{(x,y)\to(0,0)} f(x,y) =$, **or** the limit does not exist

(b) **(6 points)** Let $g\colon \mathbb{R}^2 \to \mathbb{R}$ be the function given by

$$g(x,y) = \begin{cases} \frac{x^2}{x^2+y^2}, & (x,y) \neq (0,0), \\ 0, & (x,y) = (0,0). \end{cases}$$

Is $g(x,y)$ continuous at $(0,0)$? Explain!

Solution Take $y = ax$, then along this line the function becomes $g(x,y) = \frac{x^2}{(1+a^2)x^2} = \frac{1}{1+a^2}$. Since the function takes a different value for every line through the origin, the limit of g does not exist and is therefore not continuous at $(0,0)$.

$g(x,y)$ **is** continuous	$g(x,y)$ **is not** continuous

5. (a) **(6 points)** Find the distance between the point $P(3,4,3)$ and the plane $7x+5y-3z+6=0$.

Solution The plane is $7x+5y-3z+6=0$, so set $A=7, B=5, C=-3, D=6$ and from P set $x_1=3, y_1=4, z_1=3$. The formula for the distance d is given by

$$d=\frac{Ax_1+By_1+Cz_1+D}{\sqrt{A^2+B^2+C^2}}=\frac{(7)(3)+(5)(4)+(-3)(3)+6}{\sqrt{7^2+5^2+(-3)^2}}=\frac{38}{\sqrt{83}}.$$

Distance =

(b) **(6 points)** Determine the equation of the plane that contains the points $P(1,-2,0)$, $Q(3,1,4)$, and $R(0,-1,2)$.

Solution We start by forming the vectors

$$\overrightarrow{PQ}=\langle 2,-3,4\rangle, \quad \overrightarrow{PR}=\langle -1,1,2\rangle.$$

We know that the cross product of two vectors will be orthogonal to both of these vectors. Since both of these are in the plane any vector that is orthogonal to both of these will also be orthogonal to the plane. Therefore, we can use the cross product as the normal vector

$$\mathbf{n}=\overrightarrow{PQ}\times\overrightarrow{PR}=2\vec{i}-8\vec{j}+5\vec{k}.$$

The equation of the plane is then,

$$2(x-1)-8(y+2)+5(z-0)=0,$$

or

$$2x-8y+5z=18.$$

The plane has equation ____ $x+$ ____ $y+$ ____ $z=$ ____

6. (a) **(4 points)** Determine whether the plane in $\mathbb{R}^3$ given by $-x+2z=10$ and the line with parametrization $\vec{r}(t) = \langle 5, 2-t, 10+4t\rangle$ are orthogonal, parallel or neither.

Solution The vector normal to the plane is $\mathbf{n} = \langle -1, 0, 2\rangle$. The vector parallel to the line is $\mathbf{v} = \langle 0, -1, 4\rangle$. If these two vectors are parallel then the line and the plane will be orthogonal. If the two vectors are parallel the line and plane will be orthogonal. The cross product is

$$\overrightarrow{n} \times \overrightarrow{v} = 2\overrightarrow{i} + 4\overrightarrow{j} + \overrightarrow{k} \neq \overrightarrow{0}.$$

So, the vectors are not parallel and so the plane and the line are not orthogonal. Let us check to see if the plane and line are parallel. If the line is parallel to the plane then any vector parallel to the line will be orthogonal to the normal vector of the plane, i.e. if **n** and **v** are orthogonal, then the line and the plane will be parallel. But

$$\overrightarrow{n} \cdot \overrightarrow{v} = 8 \neq 0.$$

So, the line and the plane are neither orthogonal nor parallel.

Circle one: Orthogonal, Parallel, Neither

(b) **(4 points)** Compute the volume of the parallelepiped determined by the vectors $\langle 1, 1, 2\rangle$, $\langle 2, 1, 0\rangle$, and $\langle 0, 0, -1\rangle$.

Solution The volume is the absolute value of

$$\det\begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = 1$$

Volume =

7. Let $f(x,y,z) = \dfrac{y}{x+y+z}$. Determine $\dfrac{\partial f}{\partial x}$, $\dfrac{\partial f}{\partial y}$, and $\dfrac{\partial^2 f}{\partial x \partial y}$, for $x+y+z \neq 0$.

(a) **(4 points)** $\dfrac{\partial f}{\partial x} = \boxed{}$

Solution

$$\frac{\partial f}{\partial x} = -\frac{y}{(x+y+z)^2}$$

(b) **(4 points)** $\dfrac{\partial f}{\partial y} = \boxed{}$

Solution

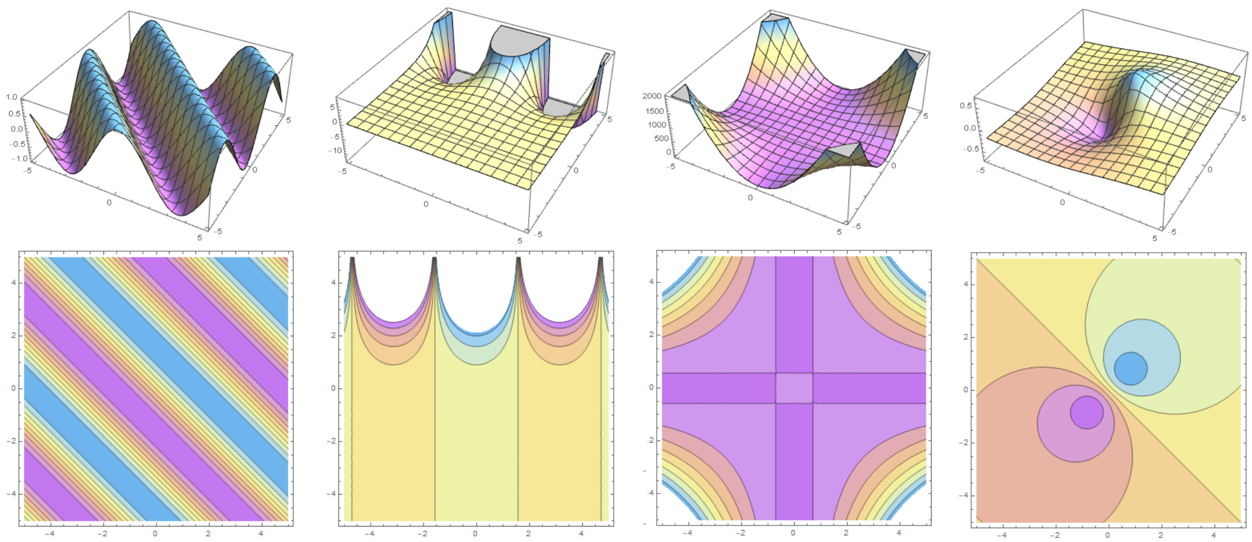
$$\frac{\partial f}{\partial y} = -\frac{y}{(x+y+z)^2} + \frac{1}{x+y+z}$$

(c) **(4 points)** $\dfrac{\partial^2 f}{\partial x \partial y} = \boxed{}$

Solution

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{2y}{(x+y+z)^3} - \frac{1}{(x+y+z)^2}$$

8. **(12 points)** Below you will find the graphs of four surfaces. Below each surface is the contour plot for that surface.



A B C D

Match each of the following equations with the corresponding graph/contour plot above.

(1) $f(x, y) = (1 - 2x^2)(1 - 3y^2)$ C

(2) $f(x, y) = \sin(x + y)$ A

(3) $f(x, y) = e^y \cos x$ B

(4) $f(x, y) = \dfrac{x + y}{1 + x^2 + y^2}$ D