Mock Exam 3, Math 241, Spring 2018

- 1. (a) Find the gradient vector field of $f(x, y, z) = xze^{yz} + y^2 + yz\sin x$.
 - (b) State the Fundamental Theorem of Line Integrals.
 - (c) $\vec{F}(x, y, z) = \langle yz + 3(y + z), xz + 3(x + z), xy + 3(x + y) \rangle$ is conservative. Let C be the curve parameterized by $\vec{r}(t) = \langle t, t^2 1, t^3 \rangle$ for $0 \le t \le 1$. Evaluate $\int_C \vec{F} \cdot d\vec{r}$.

(circle one): (A) -4; (B) 4; (C) 3; (D) 9;

(d) Let $\vec{F}(x, y, z)$ be a vector field, with $\int_C \vec{F} \cdot d\vec{r} = \pi$ for the curve C parameterized by $\vec{r}(t) = \langle 6, \cos 2t, \sin 2t \rangle$ for $0 \le t \le \pi$. TRUE or FALSE: \vec{F} is conservative.

(circle one): (A) True; (B) False;

(e) Which of the three vector fields:

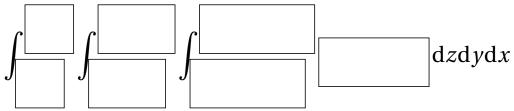
i.
$$\vec{F}(x, y) = \langle (1 + xy)e^{xy}, x^2e^{xy} \rangle$$

ii.
$$\vec{G}(x, y) = \langle x^3 y^2 + 2, x^2 y^3 + 2 \rangle$$

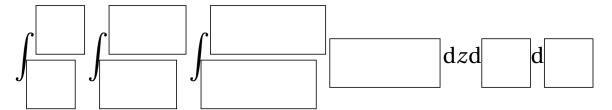
iii.
$$\vec{H}(x, y) = \langle ye^x + \sin y, e^x + x \cos y \rangle$$

are conservative? Find a potential function for those which are conservative.

- 2. Set up (but do **NOT** evaluate) a double integral to calculate the volume of the solids *E* and *S*, where:
 - (a) E is the solid under the surface $z = x^2 + 2y^4$ and above the region $D = \{(x, y, 0) \mid -1 \le x \le 1, x^2 \le y \le 2x^2\}$.



(b) *S* is the solid under the plane 3x + 2y - z = 0 and above the region in the *xy*-plane enclosed by $y = x^2$ and $x = y^2$.



3. Evaluate the following integrals by reversing the order of integration.

(a)

$$\int_0^1 \int_{3y}^3 e^{x^2} \mathrm{d}x \mathrm{d}y$$

(A)
$$\frac{(e^9-1)}{3}$$
;

(circle one): (A)
$$\frac{(e^9-1)}{3}$$
; (B) $\frac{(e^9-1)}{6}$; (C) $\frac{e^9}{3}$; (D) $\frac{e^9}{6}$;

(C)
$$\frac{e^9}{3}$$
;

(D)
$$\frac{e^9}{6}$$
;

(b)

$$\int_0^2 \int_{\frac{y}{2}}^1 y \cos(x^3 - 1) \, \mathrm{d}x \, \mathrm{d}y$$

(circle one): (A)
$$\frac{2}{3}\sin(1)$$
; (B) $\frac{2}{3}\sin(-1)$; (C) $\sin(1)$; (D) $\sin(-1)$;

4. Find the mass of the lamina the occupies the region R bounded by the semicircle $x = -\sqrt{4 - y^2}$ and the y-axis, whose density at any point in R is $\rho(x, y) = e^{-x^2 - y^2}$.

(A)
$$\frac{\pi(1-e^{-4})}{2}$$

(circle one): (A)
$$\frac{\pi(1-e^{-4})}{2}$$
; (B) $\pi(1-e^{-4})$; (C) $\frac{\pi(1-e^{-4})}{4}$; (D) $\frac{\pi(1-e^{-4})}{2}$;

(C)
$$\frac{\pi(1-e^{-4})}{4}$$

(D)
$$\frac{\pi(1-e^{-4})}{2}$$

5. Use polar coordinates to combine the sum

$$\int_0^{\sqrt{2}} \int_0^x xy \, dy dx + \int_{\sqrt{2}}^2 \int_0^{\sqrt{4-x^2}} xy \, dy dx$$

into one double integral. Then evaluate the double integral.

(circle one): (A) $\frac{1}{3}$; (B) $\frac{4}{3}$; (C) $\frac{2}{3}$; (D) 1;

(A)
$$\frac{1}{2}$$
;

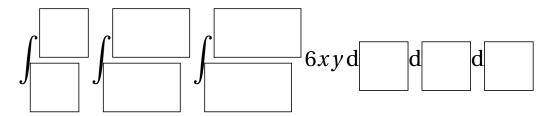
(B)
$$\frac{4}{2}$$
;

(C)
$$\frac{2}{3}$$
;

6. Set up (but do **NOT** evaluate) the triple integral

$$\iiint_E 6xy\,\mathrm{d}V,$$

as an iterated integral, where E lies under the plane z = 1 + x + y and above the region in the xy-plane bounded by the curves $y = \sqrt{x}$, y = 0, and x = 1.



7. Evaluate $\iiint_E (2y - x) \, dV$, where E is the solid that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 16$, above the xy-plane, and below the plane z = y + 4.

(circle one): (A) $\frac{255\pi}{2}$; (B) $\frac{255\pi}{4}$; (C) $\frac{-255\pi}{4}$; (D) $\frac{-255\pi}{2}$;

- 8. Let $R = \{(x, y, z) | 2 \le z \le 2 + \sqrt{4 x^2 y^2}\}$. Set up the iterated integrals to find the volume of region R in rectangular, cylindrical, and spherical coordinates, then calculate the volume of R.
- 9. Let *R* be the region bounded by the ellipse $4x^2 + 9y^2 = 36$. Evaluate

$$\iint_R y^2 \, \mathrm{d}A.$$

- 10. Let $S = \{(u, v) : 0 \le u \le 1, 0 \le v \le 1\}$, and $R \subset \mathbb{R}^2$ be the parallelogram with vertices (0, 0), (4, -3), (5, 4) and (9, 1).
 - (a) Find a transformation T(u, v) = (x(u, v), y(u, v)) which maps S to R.
 - (b) Use the transformation from part (a), along with change of variables, to evaluate

$$\iint_{R} (x + xy) \, \mathrm{d}A.$$

TRIGONOMETRIC IDENTITIES

$$\sin(\theta + \phi) = \sin\theta \cos\phi + \cos\theta \sin\phi$$

$$\cos(\theta + \phi) = \cos\theta \cos\phi - \sin\theta \sin\phi$$

$$\tan(\theta + \phi) = \frac{\tan(\theta) + \tan(\phi)}{1 - \tan\theta \tan\phi}$$

$$\sin(\theta - \phi) = \sin\theta \cos\phi - \cos\theta \sin\phi$$

$$\cos(\theta - \phi) = \cos\theta \cos\phi + \sin\theta \sin\phi$$

$$\tan(\theta - \phi) = \frac{\tan(\theta) - \tan(\phi)}{1 + \tan\theta \tan\phi}$$

$$\sin(\theta) \sin(\phi) = \frac{\cos(\theta - \phi) - \cos(\theta + \phi)}{2}$$

$$\cos(\theta) \cos(\phi) = \frac{\cos(\theta - \phi) + \cos(\theta + \phi)}{2}$$

$$\sin(\theta) \cos(\phi) = \frac{\sin(\phi + \theta) - \sin(\phi - \theta)}{2}$$

$$\sin(2\theta) = 2\sin\theta \cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$\sin\theta = \pm\sqrt{\frac{1 - \cos(2\theta)}{2}}$$

$$\cos\theta = \pm\sqrt{\frac{1 + \cos(2\theta)}{2}}$$

$$\tan\theta = \frac{\sin(2\theta)}{1 + \cos(2\theta)}$$