Math 231 - Midterm 2 Review

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Sections: ADJ/ADK

Approximate Integration

$\Delta x =$

$\bar{x}_i =$

$x_i =$

Midpoint Rule

$$\int_a^b f(x)dx = \Delta x[f(\bar{x}_1) + f(\bar{x}_2) + \cdots + f(\bar{x}_n)]$$

Trapezoidal Rule

$$\int_a^b f(x)dx = \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

Simpsons Rule

$$\int_a^b f(x)dx = \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 4f(x_{n-1}) + f(x_n)]$$

Arc Length

- Write the formula **in terms of x**: $y = f(x), a \leq x \leq b$

$$\boxed{L = \int_a^b \sqrt{\qquad\qquad\qquad} dx}$$

- Write the formula **in terms of y**: $x = g(y), c \leq y \leq d$

$$\boxed{L = \int_c^d \sqrt{\qquad\qquad\qquad} dy}$$

Area of a Surface of Revolution

General formula:

$$\boxed{\text{S}=\int 2\pi R ds}$$

	integral in terms of x	integral in term of y
rotate about x-axis (R=)		
rotate about y-axis (R=)		

Example:
Set up an integral for the area of the surface obtained by rotation the curve $y = \tan(x)$, $0 \leq x \leq \pi/3$

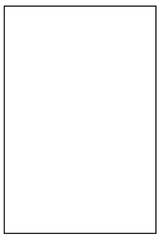
- about the x-axis in terms of x:
- about the x-axis in terms of y:
- about the y-axis in terms of x:
- about the y-axis in terms of y:

Hydrostatic force

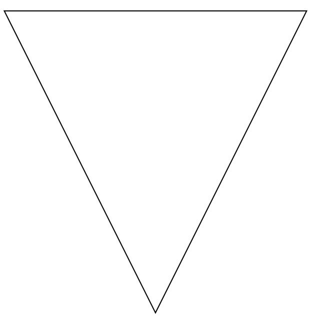
General formula:

$$\boxed{F = \int_a^b \rho g (\text{depth})(\text{width})\, dy}$$

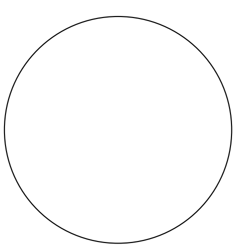
1. Rectangle:



2. Triangle:



3. Parabola, circle, elipse etc:



Moments and Centers of Mass

$$\boxed{M_y = \rho \int_a^b \qquad\qquad\qquad dx}$$

$$\boxed{M_x = \rho \int_a^b \qquad\qquad\qquad dx}$$

$$\boxed{\bar{x} = \qquad\qquad\qquad}$$

$$\boxed{\bar{y} = \qquad\qquad\qquad}$$

What if the region lies between two curves $y = f(x)$ and $y = g(x)$ where $f(x) \geq g(x)$?

$$\boxed{M_y = \qquad\qquad\qquad\qquad}$$

$$\boxed{M_x = \qquad\qquad\qquad\qquad}$$

Example : A lamina of density ρ kg/m^2has the shape of the half circle defined by

$$x^2 + y^2 = 9, \qquad y \geq 0.$$

Set up but **do not evaluate** an integral to compute the moment M_x about the x-axis.

Sequences

If $\lim_{n\to\infty} a_n$ exists (as a finite number), we say the the sequence $\{a_n\}$. Otherwise we say that the sequence is .

Examples: Are the following sequences convergent or divergent?

a. $a_n = \dfrac{\sqrt{9n^2 - 2n}}{2n+3}$

b. $a_n = \ln(n+6) - \ln(n)$

c. $a_n = \dfrac{\cos^2 n}{4^n}$

Series

Given a series $\sum_{n=1}^{\infty} a_n$, let s_n denote its nth partian sum:

$$\boxed{s_n = \qquad\qquad\qquad\qquad}$$

If the sequence $\{s_n\}$ is convergent, then the series $\sum_{n=1}^{\infty} a_n$ is called and we write

$$\boxed{\sum_{n=1}^{\infty} a_n = \qquad\qquad\qquad\qquad}$$

- How can we find a_n if s_n is given?

$$\boxed{a_n = \qquad\qquad\qquad\qquad}$$

- Examples of series we know well:

The geometric series

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \dots$$

is convergent/divergent if $|r| < 1$ and its sum is:

If , the geometric series is **divergent**.

p-series

- Tests we can use to find convergence or divergence:

Test for Divergence

The Integral Test

- What are the hypothesis for f?
- What is the conclusion?

The Comparison Test

Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms.

- If $\sum b_n$ is convergent and $\quad \leq \quad$ for all n, then $\sum a_n$ is also convergent.
- If $\sum b_n$ is divergent and $\quad \leq \quad$ for all n, then $\sum a_n$ is also divergent.

The Limit Comparison Test

Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms.

If $\lim_{n\to\infty} \frac{a_n}{b_n} =$, then either both series converge or both diverge.

Remarks:

- Careful when using the divergence test: If $\lim_{n\to\infty} a_n = 0$, we **cannot** conclude anything from the divergence test.

  Example: Look at $\sum_{n=1}^{\infty} \frac{1}{n}$ and $\sum_{n=1}^{\infty} \frac{1}{n^2}$. Both limits are 0, but the first one diverges and the second one converges.
- If using the Integral test make sure you check the hypothesis.

Example: Use the integral test to show whether $\sum_{n=1}^{\infty} e^{-n}$ converges:

Examples: Are the following series convergent or divergent?

a. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2(n)}{2n^2+3}$

b. $\displaystyle\sum_{n=1}^{\infty} \arctan(n)$

c. $\displaystyle\sum_{n=1}^{\infty} \frac{2^{3n+1}}{3^n}$

d. $\displaystyle\sum_{n=1}^{\infty} \sin(\frac{4}{n})$

e. $\displaystyle\sum_{n=1}^{\infty} \frac{n^3+5n}{e^n}$

Remember: $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + \cdots + a_n + \ldots$ and we define

$$S_n = a_1 + \cdots + a_n \qquad \text{and} \qquad R_n = a_{n+1} + a_{n+2} + \ldots$$

When we approximate $\sum_{n=1}^{\infty} a_n$ by S_n we make and "error" R_n and we want to know how big this error is.

Reminder Estimate for the Integral Test

$$\leq R_n \leq$$

$$\leq S = \sum_{n=1}^{\infty} a_n \leq$$

- How many terms of the series $\sum_{n=1}^{\infty} \frac{5}{n^3}$ would we need to add to estimate the sum to within 0.1?

- Approximate $\sum_{n=1}^{\infty} \frac{5}{n^3}$ within 0.1.