1. (a) Find the gradient vector field of $f(x,y,z) = xye^yz + x^2 + yz$.

   (b) State the Fundamental Theorem of Line Integrals.

   (c) $\vec{F}(x,y,z) = \langle yz + 2(y+z)\,,\, xz + 2(x+z)\,,\, xy + 2(x+y) \rangle$ is conservative. Let C be the curve parameterized by $\vec{r}(t) = \langle t\,,\, t^2\,,\, t^3 \rangle$ for $0 \le t \le 1$. Evaluate $\int_C \vec{F} \cdot \mathrm{d}\vec{r}$.

   (d) Let $\vec{F}(x,y,z)$ be a vector field, with $\int_C \vec{F} \cdot \mathrm{d}\vec{r} = \pi$ for the curve C parameterized by $\vec{r}(t) = \langle 6, \cos 2t, \sin 2t \rangle$ for $0 \le t \le \pi$. TRUE or FALSE: $\vec{F}$ is conservative.

   (e) Which of the three vector fields:

      i. $\vec{F}(x,y) = \langle (1+xy)e^{xy}\,,\, x^2e^{xy} \rangle$

      ii. $\vec{G}(x,y) = \langle x^3y^2 + 2\,,\, x^2y^3 + 2 \rangle$

      iii. $\vec{H}(x,y) = \langle ye^x + \sin y\,,\, e^x + x\cos y \rangle$

   are conservative? Find a potential function for those which are conservative.

2. Set up (but do **NOT** evaluate) a double integral to calculate the volume of the solids E and S, where:

   (a) E is the solid under the surface $z = x^2 + 2y^4$ and above the region $D = \{(x,y,0) \mid -1 \le x \le 1\,,\, x^2 \le y \le 2x^2\}$.

   (b) S is the solid under the plane $3x + 2y - z = 0$ and above the region enclosed by $y = x^2$ and $x = y^2$.

3. Evaluate the following integrals by reversing the order of integration.

   (a)
$$\int_0^1 \int_{3y}^3 e^{x^2}\, \mathrm{d}x\mathrm{d}y$$

   (b)
$$\int_0^2 \int_{\frac{y}{2}}^1 y\cos(x^3 - 1)\, \mathrm{d}x\mathrm{d}y$$

4. Find the mass of the lamina the occupies the region R bounded by the semicircle $x = -\sqrt{4 - y^2}$ and the y-axis, whose density at any point in R is $\rho(x,y) = e^{-x^2-y^2}$.

5. Use polar coordinates to combine the sum

$$\int_0^{\sqrt{2}} \int_0^{x} xy \, \mathrm{d}y \mathrm{d}x + \int_{\sqrt{2}}^{2} \int_0^{\sqrt{4-x^2}} xy \, \mathrm{d}y \mathrm{d}x$$

into one double integral. Then evaluate the double integral.

6. Set up (but do **NOT** evaluate) the triple integral

$$\iiint_E 6xy \, \mathrm{d}V,$$

as an iterated integral, where E lies under the plane $z = 1 + x + y$ and above the region in the xy-plane bounded by the curves $y = \sqrt{x}$, $y = 0$, and $x = 1$.

7. Evaluate $\iiint_E (x - y) \, \mathrm{d}V$, where E is the solid that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 16$, above the xy-plane, and below the plane $z = y + 4$.

8. Let $R = \{(x, y, z) \,|\, 2 \le z \le 2 + \sqrt{4 - x^2 - y^2}\}$. Set up the iterated integrals to find the volume of region R in rectangular, cylindrical, and spherical coordinates, then calculate the volume of R.

Trigonometric Identities

$$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$
$$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$
$$\tan(\theta+\phi) = \frac{\tan(\theta)+\tan(\phi)}{1-\tan\theta\tan\phi}$$
$$\sin(\theta-\phi) = \sin\theta\cos\phi - \cos\theta\sin\phi$$
$$\cos(\theta-\phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$$
$$\tan(\theta-\phi) = \frac{\tan(\theta)-\tan(\phi)}{1+\tan\theta\tan\phi}$$
$$\sin(\theta)\sin(\phi) = \frac{\cos(\theta-\phi)-\cos(\theta+\phi)}{2}$$
$$\cos(\theta)\cos(\phi) = \frac{\cos(\theta-\phi)+\cos(\theta+\phi)}{2}$$
$$\sin(\theta)\cos(\phi) = \frac{\sin(\phi+\theta)-\sin(\phi-\theta)}{2}$$
$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$
$$\sin\theta = \pm\sqrt{\frac{1-\cos(2\theta)}{2}}$$
$$\cos\theta = \pm\sqrt{\frac{1+\cos(2\theta)}{2}}$$
$$\tan\theta = \frac{\sin(2\theta)}{1+\cos(2\theta)}$$