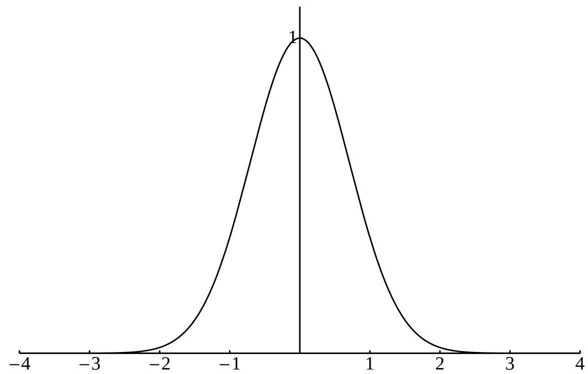
Group: ______________ ______________ ______________ Name: ______________

Math 231 A. Worksheet 7.

1. We consider the famous "Bell Curve," defined by $y = e^{-x^2}$. In particular, we will try to determine the area under the curve (from $-\infty$ to ∞). By symmetry, this area is $2\int_0^\infty e^{-x^2}\,dx$. Remember that there is no elementary way to express the antiderivative $\int e^{-x^2}\,dx$.



(a) Use a simple comparison to prove that the integral $I = \int_0^\infty e^{-x^2}\,dx$ converges.

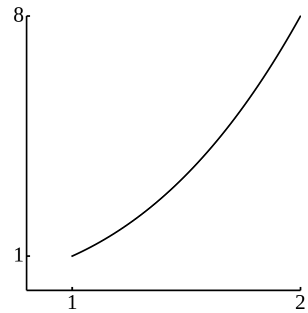
(b) Write $I = I_1 + I_2$, where $I_1 = \int_0^4 e^{-x^2}\,dx$ and $I_2 = \int_4^\infty e^{-x^2}\,dx$. Estimate I_1 using Simpson's rule with $n = 8$. Keep six decimal places of accuracy in your calculations.

(c) Notice that $e^{-x^2} \leq e^{-4x}$ if $x \geq 4$. Use this fact to show that $I_2 \leq 0.0000001$.

(d) Make an educated guess as to the area under the Bell Curve. Hint: You have approximated I to high accuracy. Do you recognize the value of I? I^2? $2I^2$? etc.?

We work with the arclength differential $ds = \sqrt{(dx)^2 + (dy)^2}$ and the formula $S = \int ds$. See your lecture notes from Wednesday. This formula must be correctly interpreted in each case to produce an expression which is ready to be evaluated.

2. The curve $y = x^3$ between the points $(1, 1)$ and $(2, 8)$ is shown.



a) Indicate the meaning of the arclength differential ds on the curve.

b) Set up but do not evaluate an integral **with respect to** x for the length. All quantities involved must refer to x.

c) Set up but do not evaluate an integral **with respect to** y which represents the length. All quantities involved must refer to y.

3. Find the length of the curve $y = \ln(\cos x)$, $0 \leq x \leq \pi/3$.