Math 231 - Midterm 3 Review

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Sections: ADJ/ADK

Alternating Series

Alternating Series Test (AST)

If the alternating series $\sum_{n=1}^{\infty}(-1)^{n-1}b_n$ with $b_n > 0$ satisfies

(i)

(ii)

then the series is .

With the same hypothesis as above, by the **Alternating Series Estimation Theorem** we have

$$\boxed{|R_n| \leq \qquad}$$

Absolute Convergence and the Ratio and Root Tests

- Define the following notions:

a) Absolutely convergent:

b) Conditionally convergent:

The Ratio Test

If $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}|$

The Root Test

If $\lim_{n\to\infty} \sqrt[n]{|a_n|}$

STEPS:

1. Use Ratio or Root Test.
2. If that is inconclusive and the series is alternating, then do BOTH of the next:
   (a) Use AST
   (b) Check whether $\sum |a_n|$ is divergent or convergent.

If the AST shows that the series in convergent, BUT $\sum |a_n|$ is divergent then $\sum a_n$ is **conditionally convergent**. However, if $\sum |a_n|$ is convergent then $\sum a_n$ is **absolutely convergent**.

Determine whether the following series converge absolutely, converge conditionally or diverge.

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1. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \cos(n\pi)}{n}$

2. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+7}$

3. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n+1}\,(2n)!}{5^n\, n!\, n!}$

4. $\displaystyle\sum_{n=2}^{\infty} (-1)^n \left(\frac{n+1}{3n+5}\right)^n$

Power Series

1. Find the radius of convergence for the following power series.

$$\sum_{n=1}^{\infty} \frac{n!x^n}{5 \cdot 11 \cdot 17 \cdot ... \cdot (6n-1)}$$

2. Find the interval of convergence for the following power series.

$$\sum_{n=1}^{\infty} \frac{2(x-3)^n}{3^n \cdot n}$$

Representation of Functions as Power Series

$$\boxed{\frac{1}{1-x} = \qquad\qquad\qquad , \text{for } |x| < \qquad}$$

REMARKS:

1. If you integrate a series to find the power series for a particular function, remember to calculate +C.
2. The radius of convergence (R) doesn't change when we derivate or integrate a series. However, the interval of convergence (I) may change (i.e., you will still need to check the endpoints).

Examples: Find the power series representation of the function and determine the interval of convergence.

1. $f(x) = \dfrac{x}{9+x^2}$

2. $f(x) = \left(\dfrac{x}{1+4x}\right)^2$

3. $f(x) = \ln\left(\frac{1+x}{1-x}\right)$

4. $f(x) = \arctan(2x)$

Taylor and Maclaurin Series

Taylor series of the function f at a:

$$f(x) = \sum_{n=0}^{\infty} \qquad\qquad , |x - a| < R$$

Nth-degree Taylor polynomial of f at a:

$$T_N(x) =$$

Then $f(x) = T_N(x) + R_N(x)$ and

$$|R_N(x)| \leq$$

for some value z between a and x.

- How is the series called when the center is $a = 0$?

$$\frac{1}{1-x} = \qquad\qquad R =$$

$$e^x = \qquad\qquad R =$$

$$\sin(x) = \qquad\qquad R =$$

$$\cos(x) = \qquad\qquad R =$$

$$\arctan(x) = \qquad\qquad R =$$

$$\ln(1+x) = \qquad\qquad R =$$

$$(1+x)^k = \qquad\qquad R =$$

Practice problems

1. Find the Maclaurin series of $f(x) = 9(1-x)^{-2}$ using the definition of a Maclaurin series.

2. Find the value of $f^{(8)}(0)$ given that $f(x) = \dfrac{1-\cos(2x^2)}{2}$.

3. Find the value of the following limit:

$$\lim_{x\to\infty} x^2(e^{\frac{-1}{x^2}} - 1)$$

4. Find the value of $\displaystyle\sum_{n=0}^{\infty} \frac{4^n}{5^n n!}$.

5. Approximate $f(x) = \sqrt{x}$ by a Taylor polynomial of degree 2 at a=9. Then use Taylor's Inequality to estimate the accuracy of the approximation when x lies in $[9, 10]$. You don't need to simplify your answer.

6. Give a series representation for $\int_0^1 x \arctan x \, dx$. Then write down enough terms to approximate $\int_0^1 x \arctan x \, dx$ to within $\frac{1}{100}$.