Group: ______________ ______________ ______________ Name: ______________

Math 231 A. Worksheet 10.

1. Recall the fundamental geometric series

$$1 + r + r^2 + r^3 + \cdots = \begin{cases} \frac{1}{1-r} & \text{if } |r| < 1 \\ \text{Divergent} & \text{if } |r| \geq 1 \end{cases}$$

Write each of the following series in the form $a(1 + r + r^2 + r^3 + \dots)$. Identify the value of r in each case. Find the sum of the series, or write "Diverges".

a) $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots$

b) $\frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \frac{1}{24} + \dots + \frac{1}{768} + \dots$

c) $\displaystyle\sum_{n=1}^{\infty} 5(-2)^{n-1}$

d) $\displaystyle\sum_{n=2}^{\infty} \frac{2^{2n-1}}{7^n}$

2. Show that the following series all diverge:

$$\sum_{n=0}^{\infty} \frac{n^2}{n^2+1}$$

$$\sum_{n=0}^{\infty} e^{-\frac{n}{n^2+1}}$$

$$\sum_{n=0}^{\infty} \cos\left(\frac{n\pi}{2}\right)$$

3. Given the partial sum $S_n = \frac{n}{n+1}$, find a_n and $\displaystyle\sum_{n=1}^{\infty} a_n$. (It's easy if you know the definitions.)

To use the integral test, you should check that the function $f(x)$ in question is positive and decreasing. Remember that you only need to check whether or not $\int_1^\infty f(x)\,dx$ converges or diverges. **You do not have to evaluate the integral.**

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4. Use the integral test to determine if $\sum_{n=1}^{\infty} \frac{n^4}{e^n}$ converges or diverges.

5. Use the integral test to determine if $\sum_{n=1}^{\infty} \frac{n}{n^3+1}$ converges or diverges.

6. Use the integral test to determine if $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)}$ converges or diverges.

7. Use the integral test to show that $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ converges for $p > 1$. What does problem (6) tell you about what happens when $p \leq 1$?