Group: solutions ____________ ____________ ____________ Name: ____________

Math 231 A. Fall, 2015. Worksheet 1. 8/25/15

These problems review some material from Calculus I. You will work on them in groups today, and turn the papers in at the end of the section. Do them in order, and do as many as you can. Don't be concerned if you don't finish all of the problems.

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1. Suppose that water flows into a tank at a rate of $200-4t$ liters per minute, for $0 \le t \le 50$. If the tank starts with 1000 liters at time $t=0$, determine how much liquid is in the tank at time $t = 30$ minutes.

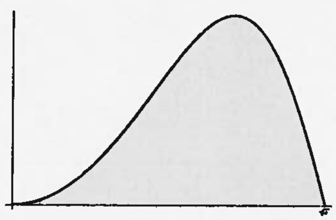
$\frac{dV}{dt} = -4t + 200$, so $V(t) = -2t^2 + 200t + C$.

$V(0) = -2(0)^2 + 200(0) + C = 1000$, so $C = 1000$ and

$V(t) = -2t^2 + 200t + 1000$.

$V(30) = -2(30)^2 + 200(30) + 1000 = 5200$ liters.

2. The region under the graph of $y = \sin(x^2)$ between $x = 0$ and $x = \sqrt{\pi}$ is shown below. Set up, **but do not evaluate**, integrals which represent the volume of the solid formed when



(a) The region is rotated about the x-axis.

$$\int_0^{\sqrt{\pi}} \pi \left(\sin(x^2)\right)^2 dx \quad \text{(washer method)}$$

(b) The region is rotated about the y-axis.

$$\int_0^{\sqrt{\pi}} 2\pi x \sin(x^2)\, dx \quad \text{(shell method)}$$

3. Suppose f is continuous on $(-\infty, \infty)$ and the function g is defined by $g(x) = \int_{-10}^{x} f(t)\, dt$. What is $g'(x)$?

$g'(x) = f(x)$ (FTC)

4. Evaluate $\int_{-2}^{2}(x+3)\sqrt{4-x^2}\,dx$ by writing it as a sum of two integrals. Use symmetry to evaluate one of the two, and evaluate the other by interpreting it in terms of area. You should be evaluate the integrals without any difficult computation.

$$\int_{-2}^{2}(x+3)\sqrt{4-x^2}\,dx = \underbrace{\int_{-2}^{2} x\sqrt{4-x^2}\,dx}_{①} + 3\underbrace{\int_{-2}^{2}\sqrt{4-x^2}\,dx}_{②} = 0 + 3(2\pi) = 6\pi$$

① the integral of an odd function over a symmetric domain is 0

② half the area of a circle of radius 2 is 2π

5. Evaluate the following integrals

(a) $\int x\cos(x^2)\,dx$

$u = x^2$, $\frac{1}{2}du = x\,dx$

$$= \frac{1}{2}\int \cos(u)\,du = \frac{1}{2}\sin(u) + C$$

$$= \frac{1}{2}\sin(x^2) + C.$$

(b) $\int_0^{\pi} \sec^2(t/4)dt$

$u = t/4$, $4du = dt$, $\begin{bmatrix} t=\pi \\ t=0 \end{bmatrix} \Rightarrow \begin{bmatrix} u=\pi/4 \\ u=0 \end{bmatrix}$

$$= \int_0^{\pi/4} \sec^2(u)\,du = \tan(u)\Big|_{u=0}^{u=\pi/4} = \tan(\pi/4) - \tan(0) = 1$$

(c) $\int_0^1 \frac{1}{x^2+4}\,dx = \frac{1}{4}\int_0^1 \frac{1}{\left(\frac{x}{2}\right)^2+1}\,dx$.

$u = x/2$, $2du = dx$, $\begin{bmatrix} x=1 \\ x=0 \end{bmatrix} \Rightarrow \begin{bmatrix} u=\frac{1}{2} \\ u=0 \end{bmatrix}$

$$= \frac{1}{4}\int_0^{1/2} \frac{2}{u^2+1}\,du = \frac{1}{2}\int_0^{1/2} \frac{1}{u^2+1}\,du = \frac{1}{2}\tan^{-1}(u)\Big|_0^{1/2}$$

$$= \frac{1}{2}\left(\tan^{-1}\left(\tfrac{1}{2}\right) - \tan^{-1}(0)\right) = \frac{1}{2}\tan^{-1}\left(\tfrac{1}{2}\right).$$