CSE 250A: Assignment 2

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2.1 Probabilistic inference

(a)

$$\begin{aligned}
P(A=1|E=1) &= P(A=1|E=1,B=0)P(B=0)+P(A=1|E=1,B=1)P(B=1)=0.2907 \\
P(A=1|E=0) &= P(A=1|E=0,B=0)P(B=0)+P(A=1|E=0,B=1)P(B=1)=0.0019 \\
P(A=1) &= P(A=1|E=1)P(E=1)+P(A=1|E=0)P(E=0)=0.0025 \\
P(E=1|A=1) &= \frac{P(A=1|E=1)P(E=1)}{P(A=1)}=0.2326
\end{aligned}$$

(b)

$$\begin{aligned}
P(A=1|B=1) &= P(A=1|B=1,E=0)P(E=0)+P(A=1|B=1,E=1)P(E=1)=0.94 \\
P(E=1|A=1,B=1) &= \frac{P(A=1|E=1,B=1)P(E=1|B=1)}{P(A=1|B=1)}=0.002
\end{aligned}$$

(c)

$$\begin{aligned}
P(J=1) &= P(J=1|A=1)P(A=1)+P(J=1|A=0)P(A=0)=0.0521 \\
P(A=1|J=0) &= \frac{P(J=0|A=1)P(A=1)}{P(J=0)}=0.0003
\end{aligned}$$

(d)

$$\begin{aligned}
P(A=1|J=0,M=0) &= \frac{P(J=0,M=0|A=1)P(A=1)}{P(J=0,M=0)} \\
&= \frac{P(J=0|A=1)P(M=0|A=1)P(A=1)}{\sum_a P(J=0|A=a)P(M=0|A=a)P(A=a)} \quad (marginalization\&d-separationIII) \\
&= 0.00008
\end{aligned}$$

(e)

$$\begin{aligned}
P(M=1) &= P(M=1|A=1)P(A=1)+P(M=1|A=0)P(A=0)=0.0117 \\
P(A=1|M=1) &= \frac{P(M=1|A=1)P(A=1)}{P(M=1)}=0.1500
\end{aligned}$$

(f)

$$
\begin{aligned}
P(A=1|M=1,E=0) &= \frac{P(M=1,E=0|A=1)P(A=1)}{P(M=1,E=0)} \\
&= \frac{P(M=1|A=1)P(E=0|A=1)P(A=1)}{aP(M=1|A=a)P(E=0|A=a)P(A=a)} \quad (marginalization\&d-separation III) \\
&= 0.1197
\end{aligned}
$$

Results seem consistent with commonsense patterns of reasoning by one factor explaining away another one.

2.2 Probabilistic reasoning

(a)

First, we compute the $P(D|S_1=1,...,S_k=1)$ as a function of k.

$$
\begin{aligned}
P(D|S_1=1,S_2=1...,S_k=1) &= \frac{P(S_1=1,S_2=1...,S_k=1|D)P(D)}{P(S_1=1,S_2=1...,S_k=1)} \\
&= \frac{\prod_{i=1}^{k} P(S_i|D)P(D)}{P(S_1=1,S_2=1...,S_k=1)} \quad (d-separation\ \ III)
\end{aligned}
$$

then we have

$$
\begin{aligned}
r_k &= \frac{P(D=0|S_1=1,S_2=1...,S_k=1)}{P(D=1|S_1=1,S_2=1...,S_k=1)} \\
&= \frac{\prod_{i=1}^{k} P(S_i|D=0)}{\prod_{i=1}^{k} P(S_i|D=1)} \\
&= \frac{2^k}{2^k+(-1)^k}
\end{aligned}
$$

for k as odd numbers, $r_k = \frac{2^k}{2^k-1} > 1$, the first form of the disease will be diagnosed; for k as odd numbers, $r_k = \frac{2^k}{2^k+1} < 1$, the second form of the disease will be diagnosed.

(b)

As showed in (a) above, two form of the disease will be diagnosed alternatively according to the parity of k. Thus, the diagnosis dosen't become more or less certain as more symptoms are observed. The form of disease D are uniformly likely to be observed instead.

Sigmoid function

(a)

$$
\begin{aligned}
\sigma'(z) &= -\frac{1}{(1+e^{-z})^2} \times (-e^{-z}) \\
&= \frac{1}{1+e^{-z}} \times \frac{e^{-z}}{1+e^{-z}} \\
&= \frac{1}{1+e^{-z}} \times \frac{1}{1+e^{z}} \\
&= \sigma(z)\sigma(-z)
\end{aligned}
$$

(b)

$$\begin{aligned}\sigma(-z)+\sigma(z) &= \frac{1}{1+e^{z}}+\frac{1}{1+e^{-z}} \\ &= \frac{1}{1+e^{z}}+\frac{e^{z}}{1+e^{z}} \\ &= 1\end{aligned}$$

(c)

$$\begin{aligned}L(\sigma(z)) &= \log(\frac{\sigma(z)}{1-\sigma(z)}) \\ &= \log(\frac{\frac{1}{1+e^{-z}}}{1-\frac{1}{1+e^{-z}}}) \\ &= \log(e^{z}) \\ &= z\end{aligned}$$

2.4 Conditional independence

X	Y	E
month	water	sprinkler, rain
month	water	sprinkler, rain, fall
month	fall	sprinkler, rain
month	fall	water
month	fall	water, sprinkler
month	fall	water, rain
month	fall	water, sprinkler, rain
sprinkler	rain	month
sprinkler	fall	water
sprinkler	fall	water, month
sprinkler	fall	water, rain
sprinkler	fall	water, month, rain
rain	fall	water
rain	fall	water, month
rain	fall	water, sprinkler
rain	fall	water, month, sprinkler

Table 1: Question 2.4

2.5 Markov blanket

For any node Y, satisfying $Y \notin B_X$ and $Y \notin X$, there only exist five different kinds of paths from Y to X:

1. Through the parents of its parents (denoted as node 1 in Fig.1); Then this path is 'd-separated' due to rule I;
2. Through the parents of its children's parents (denoted as node 2 in Fig.1); Then this path is 'd-separated' due to rule I;
3. Through the children of its parents (denoted as node 3 in Fig.1); Then this path is 'd-separated' due to rule II;
4. Through the children of its children's parents (denoted as node 4 in Fig.1); Then this path is 'd-separated' due to rule II;

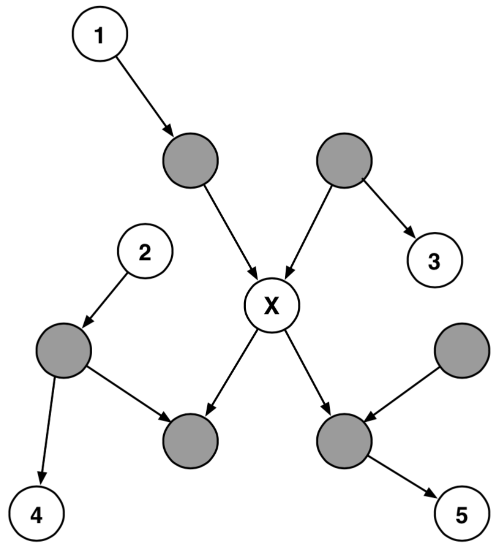

Figure 1: Five Possible Paths

5. Through the children of its children (denoted as node 5 in Fig.1); Then this path is 'd-separated' due to rule I.

Above all, we can say that

$$P(X, Y|B_X) = P(X|B_X)P(Y|B_X)$$

2.6 Noisy-OR

$$\begin{aligned}
P(Z=1|X=0,Y=0) &< P(Z=1|X=0,Y=1) \\
P(Z=1|X=1,Y=0) &< P(Z=1|X=0,Y=1) \\
P(Z=1|X=1,Y=0) &< P(Z=1|X=1,Y=1) \\
P(X=1) &< P(X=1|Z=1) \\
P(X=1) &= P(X=1|Y=1) \\
P(X=1|Z=1) &> P(X=1|Y=1,Z=1) \\
P(X=1)P(Y=1)P(Z=1) &< P(X=1,Y=1,Z=1)
\end{aligned}$$

2.7 More conditional independence

statements	independence?
$P(E,F\|D) = P(E\|D)P(F\|D)$	False
$P(E,F\|C,D) = P(E\|C,D)P(F\|C,D)$	False
$P(E,F\|A,B,D) = P(E\|A,B,D)P(F\|A,B,D)$	True
$P(D\|C) = P(D)$	False
$P(D\|A,B) = P(D\|A,B,C)$	True
$P(A,B) = P(A)P(B)$	True
$P(A\|C,D) = P(A\|C,D,F)$	False
$P(A\|B,C,D) = P(A\|B,C,D,F)$	True
$P(B\|A,C,D,F) = P(B\|A,C,D,F,E)$	True
$P(B,F,A,E\|C,D) = P(B,F\|C,D)P(A,E\|C,D)$	False

Table 2: Question 2.7

2.8 Even more conditional independence

$P(B\|D) = P(B\|\mathcal{S})$	$\mathcal{S} = \{A, C, D, E, G\}$
$P(B\|D, F) = P(B\|\mathcal{S})$	$\mathcal{S} = \{D, F, G\}$
$P(C\|D) = P(C\|\mathcal{S})$	$\mathcal{S} = \{B, D, G\}$
$P(C\|F, G) = P(C\|\mathcal{S})$	$\mathcal{S} = \{F, G\}$
$P(C\|A, E, F) = P(C\|\mathcal{S})$	$\mathcal{S} = \{A, E, F\}$
$P(E\|F) = P(E\|\mathcal{S})$	$\mathcal{S} = \{F\}$
$P(E\|C) = P(E\|\mathcal{S})$	$\mathcal{S} = \{A, B, C, D, F, G\}$
$P(F) = P(F\|\mathcal{S})$	$\mathcal{S} = \emptyset$
$P(F\|C, D) = P(F\|\mathcal{S})$	$\mathcal{S} = \{C, D, E, G\}$
$P(A, B) = P(A, B\|\mathcal{S})$	$\mathcal{S} = \emptyset$

Table 3: Question 2.8