CSE 250A: Assignment 5

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5.1 EM algorithm

(a) Posterior probability

$$
\begin{aligned}
P(a, c | b, d) &= \frac{P(a, c, b, d)}{P(b, d)} \\
&= \frac{P(a, c, b, d)}{\sum_{a', c'} P(a', b, c', d)} \\
&= \frac{P(a)P(b|a)P(c|a, b)P(d|b, c)}{\sum_{a', c'} P(a')P(b|a')P(c'|a', b)P(d|b, c')}
\end{aligned}
$$

(b) Posterior probability

$$
\begin{aligned}
P(a | b, d) &= \sum_{c'} P(a, c' | b, d) \\
P(c | b, d) &= \sum_{a'} P(a', c | b, d)
\end{aligned}
$$

(c) Log-likelihood

$$
\begin{aligned}
\mathcal{L} &= \sum_t \log P(B = b_t, D = d_t) \\
&= \sum_t \log \sum_{a', c'} P(A = a', B = b_t, C = c', D = d_t) \\
&= \sum_t \log \sum_{a', c'} P(A = a')P(B = b_t | A = a')P(C = c' | A = a', B = b_t)P(D = d_t | B = b_t, C = c')
\end{aligned}
$$

(d) EM algorithm

E-step

$$
\begin{aligned}
P(b, a | b_t, d_t) &= I(b, b_t)P(a | b_t, d_t) \\
P(c, a, b | b_t, d_t) &= I(b, b_t)P(a, c | b_t, d_t) \\
P(d, b, c | b_t, d_t) &= I(b, b_t)I(d, d_t)P(c | b_t, d_t)
\end{aligned}
$$

M-step

$$P(b|a) = \frac{\sum_t P(b, a|b_t, d_t)}{\sum_t P(a|b_t, d_t)}$$

$$= \frac{\sum_t I(b, b_t) P(a|b_t, d_t)}{\sum_t P(a|b_t, d_t)}$$

$$P(c, a, b|b_t, d_t) = \frac{\sum_t P(c, b, a|b_t, d_t)}{\sum_t P(a, b|b_t, d_t)}$$

$$= \frac{\sum_t I(b, b_t) P(a, c|b_t, d_t)}{\sum_t I(b, b_t) P(a|b_t, d_t)}$$

$$P(d, b, c|b_t, d_t) = \frac{\sum_t P(d, c, b|b_t, d_t)}{\sum_t P(b, c|b_t, d_t)}$$

$$= \frac{\sum_t I(b, b_t) I(d, d_t) P(c|b_t, d_t)}{\sum_t I(b, b_t) P(c|b_t, d_t)}$$

5.2 EM algorithm for noisy-OR

(a)

$$P(Y = 1|X) = \sum_{Z \in \{0,1\}^n} P(Y = 1, Z|X)$$

$$= \sum_{Z \in \{0,1\}^n} P(Y = 1|Z, X) P(Z|X)$$