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Quantitative Asset & Risk Management

Assignement 2

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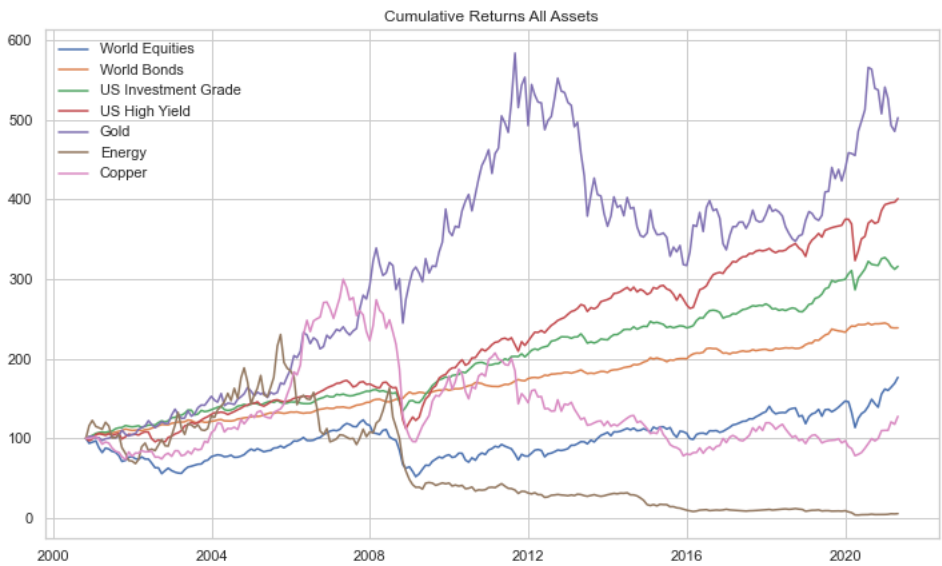
1 Introduction

This report aims to develop a portfolio by designing an investment process that will combine a *Strategic Asset Allocation* (SAA) component with a *Tactical Asset Allocation* (TAA) component. The paper is separated into three parts. The first part will aim to create an SAA component through various types of optimizations. The second part will aim to create a TAA component, using long/short value and momentum factors, and developing a strategy using the VIX Index. The third and final part will aim to combine both models to create a model portfolio. As we will be restricted on our position on some assets, we will have to create a replication portfolio where its performance will be as close as possible to the model portfolio. We will present various performance measures such as the Sharpe ratio, maximum drawdown, hit ratio, tracking error, and information ratio throughout the parts. Both code and data set are available in our Github **here**.

2 Description of the Data Set

Our data set includes 7 indices representing the monthly prices of world equities, world bonds, US investment-grade bonds, US high yield bonds, gold, energy and copper, from 2000 to 2021 with a total of 247 observations per asset. The following graph depicts the cumulative returns of each asset over our time frame.

Figure 1 – Cumulative Returns of All Assets



The data processing has been exclusively conducted on Python, using various libraries. The codes will be included in a separate file as guidance for the reader.

3 First Steps of Data Processing

Before starting the report, we will need to define some essential elements for our analysis. Let P_t be the stock price at time t. Then the one-period simple return from $t-1$ to t is:

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{1}$$

The cumulative return can be defined as the process of holding an asset for n period from t to $t+n$, which yields the n-period simple return:

$$r_t[n] = \frac{P_{t+n} - P_t}{P_t} \tag{2}$$

For this report, the original data is divided into two sub-parts:

1. **In-Sample:** Includes data from October 2000 to December 2010.
2. **Out-of-Sample:** Includes data from January 2011 to the end of the sample (April 2021).

The rationale for dividing our data is that the in-sample part is used to calibrate our asset allocations, and the out-of-sample part allows us to see how our hypothetical portfolio would have performed.

For this report, we used the following libraries on Python 3.8:

- Pandas (Version: 1.2.2)
- Scipy (Version: 1.6.1)
- Seaborn (Version: 0.11.1)
- Statsmodels (Version: 0.12.2)
- Matplotlib (Version: 3.3.4)
- Numpy (Version: 1.20.2)

4 Strategic Asset Allocation (SAA)

4.1 Strategy

As a general guideline for our portfolio allocation, we first outline a Strategic Asset Allocation (SAA). This long-only allocation across our investment universe of 7 assets will remain constant throughout the investment period in our case. To find our SAA, we take 3 different approaches from which we will choose the one we deem most suitable. Note that there is no real all-encompassing right answer since our client's risk preference and objectives are unclear.

The different types of SAAs we chose to use are the following:

Equal Risk Contribution Portfolio:

In the first approach, we allocate our capital between the 7 assets in a way that Equalizes the Risk Contribution (ERC) of each asset. In mathematical terms, this is expressed by:

$$\alpha_i MCR_i = \alpha_j MCR_j \tag{3}$$

With the vector of Marginal Contributions to Risk (MCR) defined in the following manner:

$$MCR = \frac{\Sigma\alpha}{\sigma_p} \tag{4}$$

Where:

- Σ is the covariance matrix of asset returns;
- α is the vector containing the portfolio weights;
- σ_p is the portfolio volatility.

We also define the risk contribution:

$$RC = \sigma_i(\alpha) = \alpha' MCR \tag{5}$$

Which is vector composed of the seven $\sigma_i(\alpha)$, the risk contributions of each asset.

Finally, to satisfy equation (3), we solve the optimization problem given by equation (6), which yields the results given in figure 2.

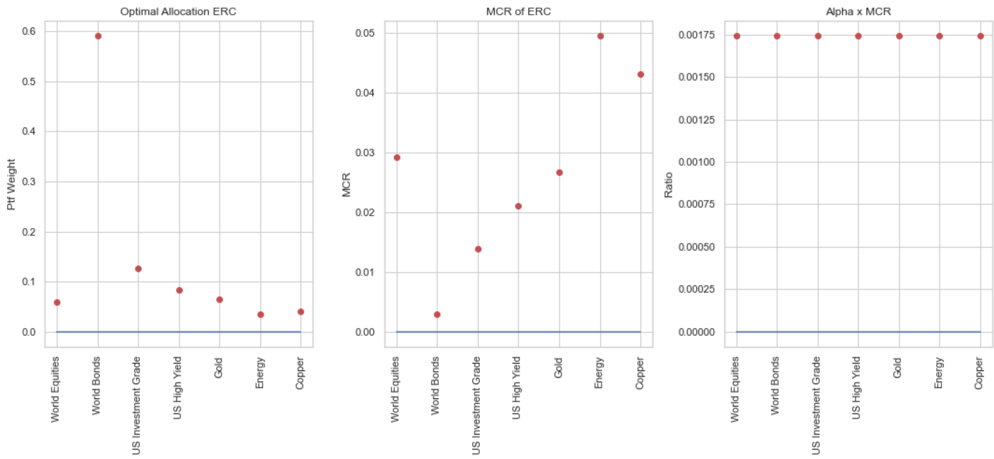
$$\begin{cases} \min_{\alpha} f(\alpha) = \sum_{i=1}^{7} (\sigma_i(\alpha) - \sigma_p(\alpha))^2 \\ s.t. \sum_{i=1}^{7} \alpha_i = 1, 0.01 < \alpha_j \leq 1, 0 \leq \alpha_i \leq 1 \end{cases} \tag{6}$$

With the constraint that we need to have at least 1% of allocation to world bonds and US investment grades:

- $j \in$[World Bonds, US Investment Grade]
- $i \notin$[World Bonds, US Investment Grade]

After minimizing the aforementioned criterion using a Sequential Least-Squares Programming (SLSQP) method, we obtained the following results:

Figure 2 – Result of the ERC SAA



Given that in the third graph, all points are aligned, we can say that the client's requirements to hold World Bonds and US Investment Grade have not impacted our optimisation problem since this SAA method would make us invest in all the assets in any case. Our capital is spread across the 7 assets with weights ranging from 3.5% to 59.1%.

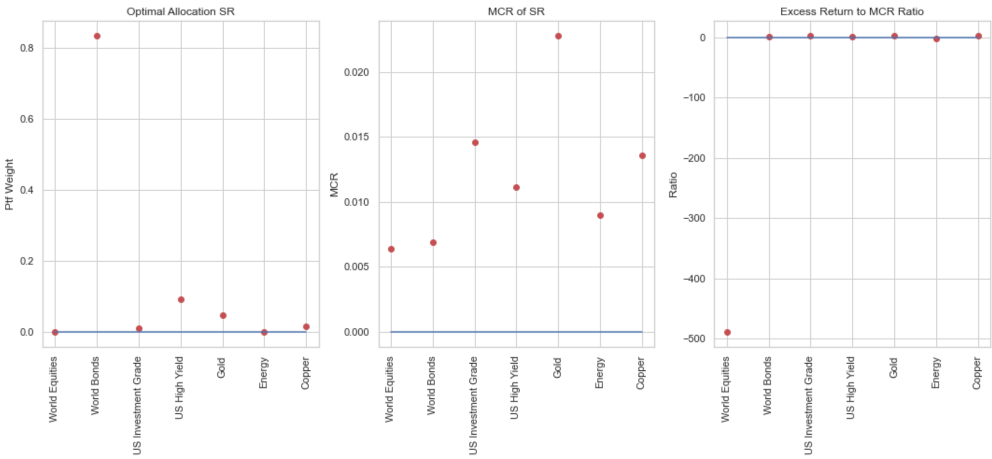
Maximum Sharpe Ratio Portfolio:

The second method seeks to construct a portfolio with the highest possible average Sharpe Ratio (SR) across the period. In mathematical terms, assuming the risk-free rate is 0, we want to solve:

$$\max_{\alpha} SR = \frac{\bar{r_p}}{\sigma_p} \tag{7}$$

After maximizing the aforementioned Sharpe Ratio (which is equivalent to minimizing the negative Sharpe Ratio) using a Sequential Least-Squares Programming (SLSQP) method, we obtained the following results given in figure 3.

Figure 3 – Result of the Max SR SAA



Here, unlike with the ERC method, the points are not aligned in the last graph. This is because the constraints we have set limit our optimisation. Looking at the values, it seems that the assets causing issues are World Equities, US Investment Grade, and Energy since they are right on the lower bound given by the constraints taking values of 0, 0.01 and 0, respectively. The 4 other assets are considered investment worthy by our optimizer, but World Bonds receive a disproportionate amount of attention. If we had allowed for short-selling and were not forced to own US Investment Grade bonds, the outcome would be very different, and we could reach the optimal solution to (7).

Most Diversified Portfolio:

The last method we use to find our SAA aims to seek the highest possible level of diversification. This means solving equation (8).

$$\max_{\alpha} D(\alpha) = \frac{\alpha' \sigma}{\sigma_p} \tag{8}$$

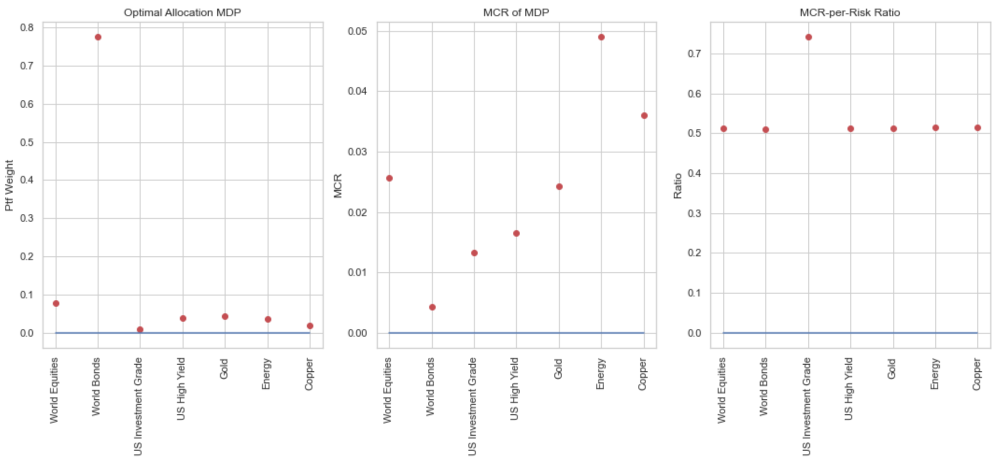
Where:

$D(\alpha)$ = The Diversification Ratio

σ = The vector of asset volatilities $(\sigma_i(\alpha))$

Solving this equation yields the results displayed in figure 4.

Figure 4 – Result of the MDP SAA



Much like in the SR maximising SAA, the constraints imposed by our clients forbid us from reaching the optimal allocation, which solves (8). We can notice that US Investment Grade bonds receive the minimum possible amount with only 1% of the total wealth.

4.2 In-Sample Performances

Now that we have computed our SAA, we can observe the hypothetical performance we would have achieved had we used the SAA. This is best illustrated with figure 5 to figure 7:

Figure 5 – In-Sample Performance with ERC

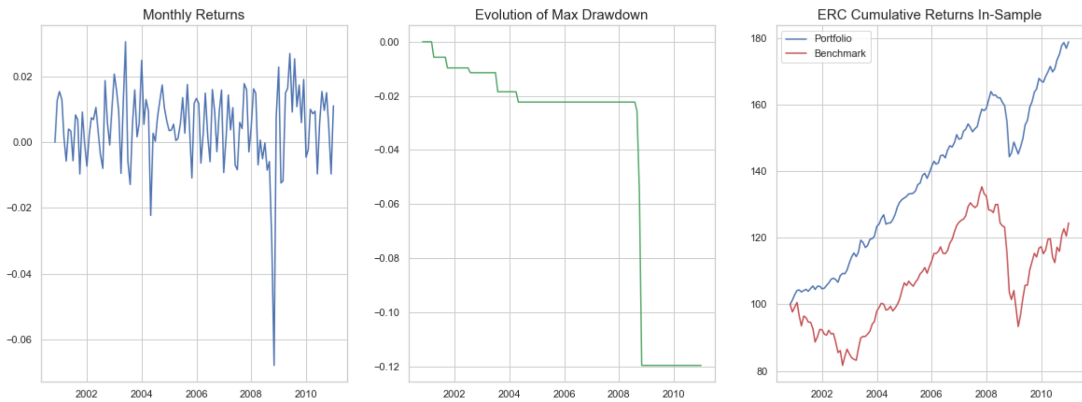


Figure 6 – In-Sample Performance with SR

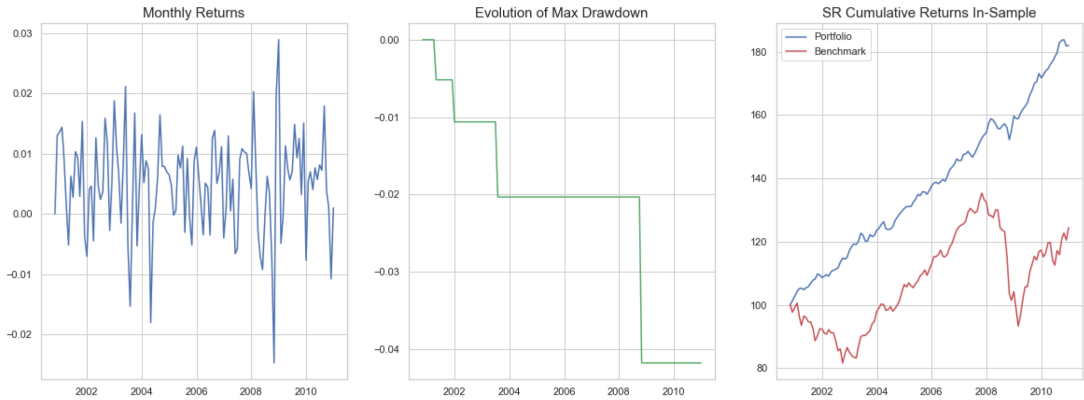
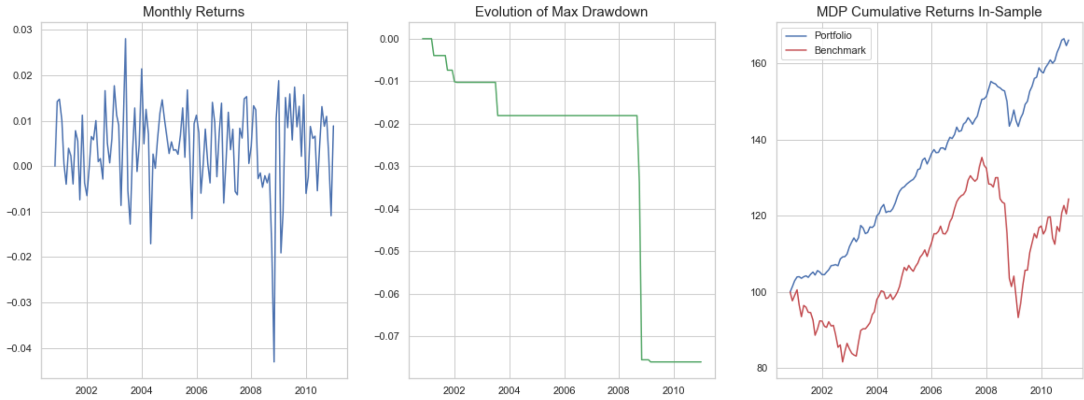


Figure 7 – In-Sample Performance with MDP



The following table also displays some interesting summary statistics.

Table 1 – In-Sample Performances SAA

	Benchmark IS	ERC	SR	MDP
Mean	0.024888	0.057761	0.058972	0.050150
Volatility	0.083199	0.041931	0.028272	0.032743
SharpeRatio	0.299132	1.377531	2.085923	1.531633
MaxDrawdown	-0.310457	-0.119611	-0.041801	-0.076081
HitRatio	0.585366	0.731707	0.756098	0.723577

The two previously unseen measures are the Maximum Drawdown and the Hit Ratio, defined by equations (9) and equation (10), respectively.

$$MaxDrawdown = min[\frac{P_s - P_t}{P_t}] \tag{9}$$

with $t < s$

$$HitRatio = \frac{n_{pos}}{N} \tag{10}$$

Where n_{pos} is the number of times the portfolio witnessed a positive return and N is the total number of observations.

Obviously, since we knew how the assets would behave over this period, it comes as no surprise that we have outdone the benchmark. Although all three methodologies yield close results, the SR maximizing one appears to be the most attractive given that it has the best results across every single measure in the aforementioned table.

The real test however, is to see how our SAA fares out of sample.

4.3 Out-Sample Performances

Plotting the same graph as before but over the out-sample yields much less spectacular results, to say the least.

Figure 8 – Out-Sample Performance with ERC

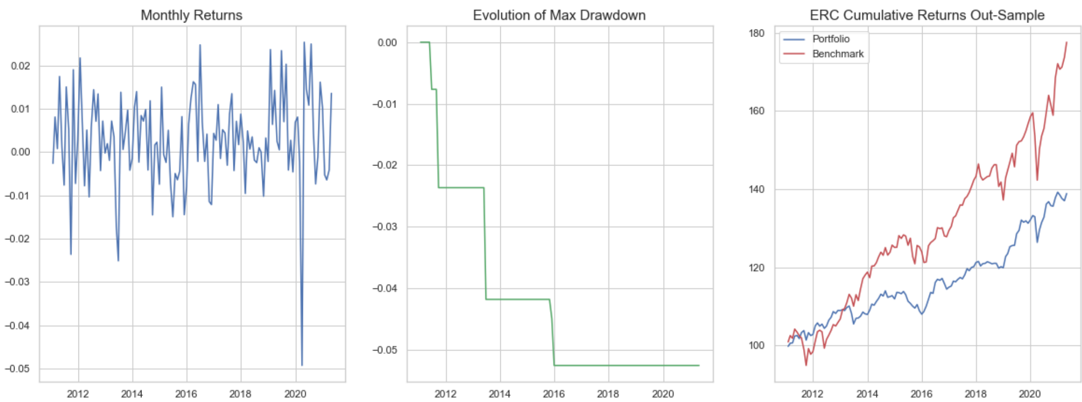


Figure 9 – Out-Sample Performance with SR

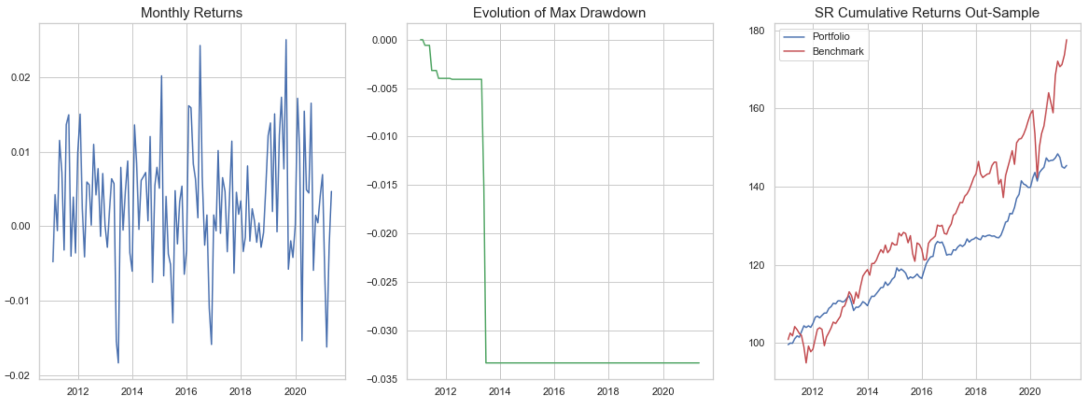
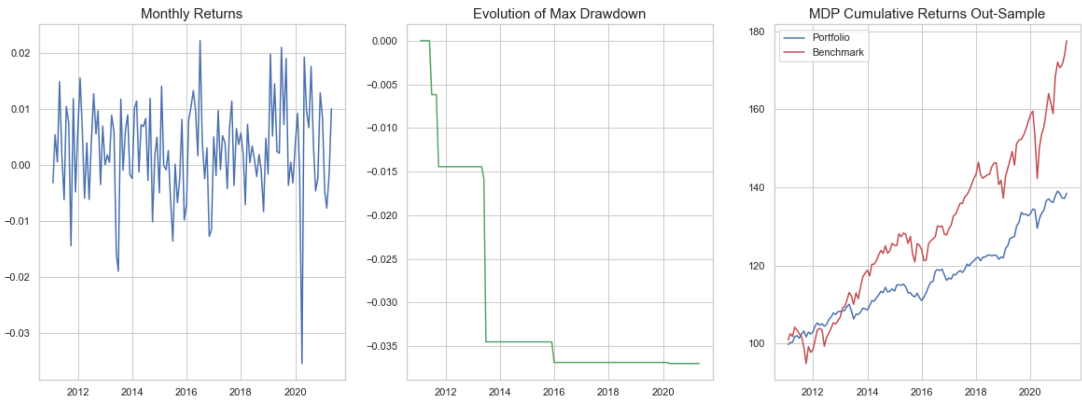


Figure 10 – Out-Sample Performance with MDP



Again, displaying the same summary statistics, we find:

Table 2 – Out-Sample Performances SAA

	Benchmark OS	ERC	SR	MDP
Mean	0.058004	0.032523	0.036678	0.032014
Volatility	0.067447	0.037449	0.027977	0.030000
SharpeRatio	0.859999	0.868476	1.310978	1.067135
MaxDrawdown	-0.108245	-0.052549	-0.033378	-0.036985
HitRatio	0.685484	0.612903	0.645161	0.637097

Indeed, now none of the allocations beat the benchmark. Something that remained true, however, is the superiority of the SAA resulting from the SR maximization. Hence, had we chosen the SAA method, which performed best in-sample, we would have also had the best out-sample results. However, it might also be due in part to luck.

Regarding the performance relative to the benchmark, this is a prime example of the industry's catchphrase: "Past performance is no guarantee of future results."

4.4 Tracking-Error & Information Ratio between SAA & Benchmark

Before continuing with the next section, we will first have to outline some metrics describing the performance of our portfolio allocation.

- **Tracking Error (TE)**: It is the standard deviation between the returns of our portfolio compared to the returns of a benchmark. This metric can be viewed as an indicator to determine how actively a portfolio is managed and its corresponding risk level. Formally, we defined it as:

$$TE = \sqrt{w^T \Sigma w} \tag{11}$$

  Where $w = w_p - w_b$ is the difference between the weight of our portfolio and the benchmark, and Σ is the covariance matrix.

- **Information Ratio (IR)** : It measures the difference between the returns of our portfolio and the benchmark regarding the volatility of those returns. This ratio is used to assess the capacity of the portfolio manager to yield better returns than the benchmark, similar to having a positive alpha. Formally, we define it as:

$$IR = \frac{r_p - r_b}{Var(r_p - r_b)} = \frac{r_p - r_b}{TE} \tag{12}$$

  Where r_p and r_b are the returns of the portfolio and benchmark, respectively.

  Computing these measures between the SAA and the benchmark yields the following results:

Table 3 – Annualized Tracking Error

	In-Sample	Out-Sample
SAA vs. Benchmark	0.080494	0.080494

Figure 11 – Monthly TE: SAA vs. Benchmark

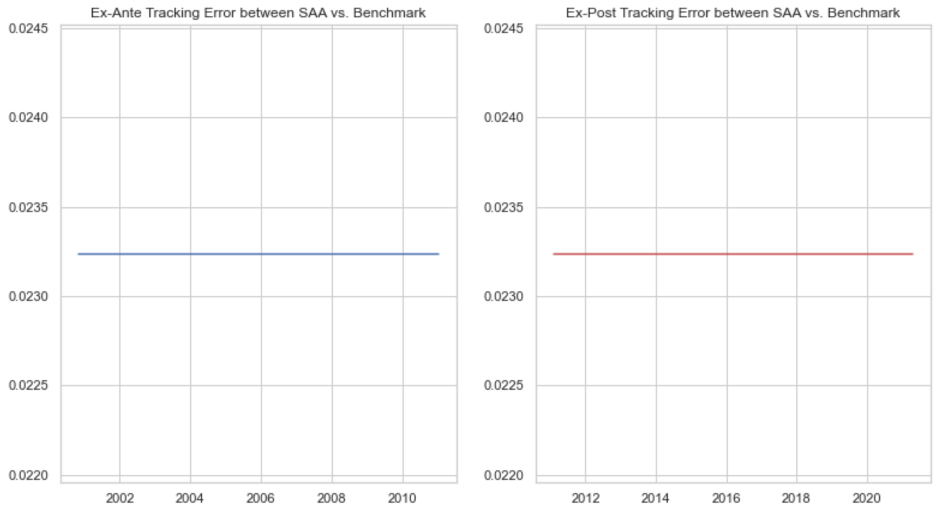


Table 4 – Information Ratio

	In-Sample	Out-Sample
SAA vs. Benchmark	0.423443	-0.323786

From the results of the TE, we first notice that it remains constant across the entire time frame. This is due to the fact that we are comparing 2 static portfolios using the same In-sample covariance matrix for the In-Sample as for the Out-Sample, which makes the gap between the two stay constant. Its level, around 8% on an annualized basis is not negligible but seems like an adequate trade-off for the supplementary returns in the In-Sample period. This 8% in the Out-Sample however, is a bad sign. Indeed, we have more volatility but for less returns.

Regarding the IR, the In-Sample result is quite good, which is normal as our optimization uses all information in the In-Sample to build the best portfolio. The Out-Sample performance, however, is awful since it is well below 0.

5 Tactical Asset Allocation (TAA)

5.1 Long-Short Value Factor

5.1.1 Strategy

In this section, we have created a long-short value portfolio based on the carry values of our seven categories of assets. A carry value is the expected performance of a security when prices are not changing. A value portfolio is commonly described as a portfolio in which undervalued assets are longed, and overvalued assets are shorted. To determine which assets are overvalued and undervalued, the first step consisted of normalizing the carry values of our assets to compare these between each other. We used the Z-Score of each carry value, which is the underlying method of the Central Limit Theorem. The Z-Score is computed by subtracting the mean of the asset from the monthly value, which is then divided by the standard deviation of the asset.

$$Z_{it} = \frac{x_{it} - \mu_i}{\sigma_i} \tag{13}$$

With the Z-Scores computed for every month t of our seven assets i, we computed the median of the Z-Scores for every month.Given that we have the carry values of our assets, each month, we had to long assets with a Z-Score higher than the median and short assets with a Z-Score lower than the median. Therefore, we will allocate the same weights for all long positions and short positions distinctly. Ultimately, we got a portfolio in which long assets are entirely funded by short assets.

The next step consisted of multiplying our weights matrix with our returns matrix computed earlier. This gave us the monthly return of our value portfolio.

The final step of the value strategy consisted of scaling our portfolio to match the volatility budget of the client, who required a 2% ex-ante maximum volatility. To obtain the new scaling of our portfolio, we divided the volatility budget with the volatility (standard deviation) of our in-sample portfolio, thus obtaining the ratio to which our portfolio had to be expanded or shrunk. This computation gave us a value of 0.53, which meant that our portfolio had to be scaled down to 53% of its initial value to match the volatility budget of 2%. Indeed, with the initial weights, our portfolio had an average monthly volatility of 3.76%, a number beyond the constraint and needed, therefore, to be scaled down.

The latter strategy has been applied to get our value portfolio. The same strategy was applied to the in-sample and the out-sample case. We decided to scale our out-sample value portfolio using the same ex-ante risk budget of 2% and the in-sample volatility. We will now look at the performance of these portfolios compared to the benchmark requested by the client.

5.1.2 In-Sample Performances

The table below summarizes the performance of our In-Sample value portfolio compared to the benchmark.

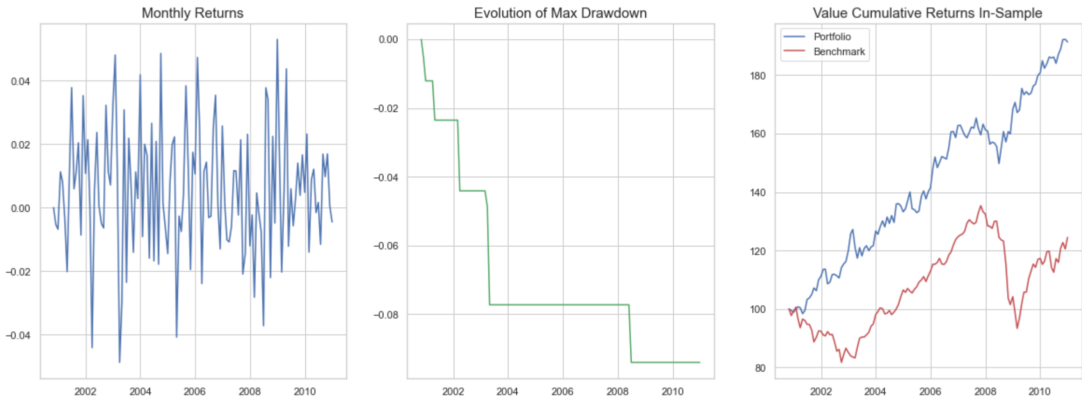
Table 5 – In-Sample Performances Value Factor

	Benchmark IS	Value
Mean	0.024888	0.065875
Volatility	0.083199	0.069000
SharpeRatio	0.299132	0.954719
MaxDrawdown	-0.310457	-0.094119
HitRatio	0.585366	0.617886

Our value portfolio outperformed the benchmark on all aspects of our performances criteria. It achieved a higher annualized return with a lower annualized volatility, thus increasing the Sharpe Ratio. Furthermore, the maximum drawdown was lower while the hit ratio was slightly higher, showing higher average positive returns.

Figure 12 illustrates the monthly returns, the maximum drawdown, and the overall return of our value portfolio and the benchmark when standardized to 100 at the first period.

Figure 12 – Value In-Sample Portfolio Performance



Our value portfolio nearly doubled (up approx. 90%) within the 10 years of the in-sample period, while the benchmark achieved only an approximately 25% increase. More striking is the consistency of the value portfolio, being far less affected by the 2008 crisis than the benchmark. Indeed, this strength in the 2008 crisis can be explained by the short position in gold, energy and copper, which were one of the classes of assets that took the largest hit during the drop. Overall, the monthly rebalancing of the value portfolio gave it a higher level of flexibility, which was adapted to the additional information brought by the carry values each month.

5.1.3 Out-Sample Performances

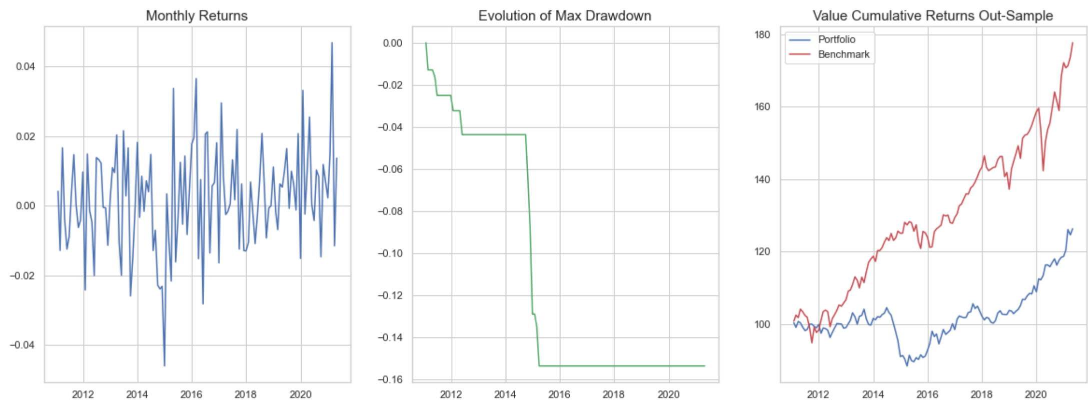
Moving on to the 2010-2020 decade, the table below shows the overall performance of our out-sample value portfolio against the benchmark. Unlike in the in-sample case, our value portfolio was this time outperformed by the benchmark on all aspects. It achieved a lower average annualized returns with a higher annualized volatility, which dropped the Sharpe Ratio, increased the maximum drawdown and reduced the hit ratio. This may suggest that our model would over-fit the in-sample data set, explaining the poor performances in the out-sample.

Table 6 – Out-Sample Performances Value Factor

	Benchmark OS	Value
Mean	0.058004	0.023931
Volatility	0.067447	0.050973
SharpeRatio	0.859999	0.469478
MaxDrawdown	-0.108245	-0.153731
HitRatio	0.685484	0.540323

When looking at figure 13, we can see the clear underperformance of our value portfolio.

Figure 13 – Value Out-Sample Portfolio Performance



The most striking decline happened in the second half of 2014. This decline is mainly explained by a long position in the Energy commodities, which saw a large drop at the end of 2014. Indeed, the Energy commodities experienced a 9% drop in October 2014, an 11% drop in November

2014 and a major 25% drop in December 2014. This sharp decline in the energy sector can in part be attributed to the decline in oil prices in late 2014, which was caused by an excessive supply of oil compared to demand [1]. Our portfolio was longing Energy commodities, as well as Gold and Copper, which are commodities that also experienced declines in late 2014. This shows a potential weakness of the value portfolio, which induced a sharp decline in performance, continuing to long commodities when these were dropping fast. Our maximum drawdown, in turn fell to -20%. Our value portfolio failed to beat the benchmark in the following years, performing only better in the 2017 recession and the 2020 Covid-19 related drop, during which the largest decliner, Energy commodities, with a peak 43% decline in March 2020 was shorted. Another reason could be explained by the general under-performances of the value factor since 2010 and simultaneously a better performing momentum factor due partly to the rise of tech companies[2].

5.2 Long-Short Momentum Factor

5.2.1 Strategy

In this second part, we defined a portfolio based on a momentum strategy. This strategy takes into account the continued strength of an asset in a trend. For example, if the value of an asset has been consistently increasing, it will be longed to extract the value of a potential continuing uptrend. On the other hand, an asset will be shorted if it entered a period of a continuous downtrend.

There are many different definitions and ways to create a momentum strategy. We decided to mathematically define the momentum strategy through a method described in a video by Algovibes.[3] This method used the monthly rolling returns of an asset over the past year, excluding the most recent month to avoid mean-reversion effects, which gave a total of 11 period rolling returns for each value. Applied to our case, the rolling returns were computed for every month in our in- and out-sample cases. Thus, these 11 period rolling returns could show a trend created over the last year on which we would base our strategy to take advantage of uptrends, downtrends, or neutral trends. Naturally, the first 12 months of the samples did not have a rolling return, given that there were not 11 months before to compute these returns.

[1] https://blogs.worldbank.org/developmenttalk/what-triggered-oil-price-plunge-2014-2016-and-why-it-failed-deliver-economic-impetus-eight-charts

[2] https://www.msci.com/www/blog-posts/factors-behind-value-s/01647315543

[3] https://www.youtube.com/watch?v=dnrJ4zwCADMab$_channel = Algovibes$

Once computed, we created 5 quantiles to split the rolling returns of our seven assets every month. We decided to long the 2 assets in the highest quantile and short the 2 assets in the lowest quantile while not taking any position in the 3 other assets. This process was repeated every month, which can be viewed as rebalancing. The longed and shorted assets were equally weighted at 50% each for the longed ones and -50% each for the shorted ones. Other allocations (such as a 40/40/20/0/-20/-40/-40) were tested but were not as efficient in terms of annualized returns as the allocation shown above (50/50/0/0/0/-50/-50).

The same reasoning as for the value portfolio was then applied. The matrix of our monthly weights was multiplied with the monthly returns of our assets, which gave us the monthly return of our momentum portfolio.

Then, the volatility budget constraint was taken into account. By dividing the 2% constraint by the portfolio's standard deviation, we obtained a coefficient of 0.356. This meant that our portfolio adjusted for the constraint had to be scaled down to 35.6% of its initial value. Indeed, our momentum portfolio's average monthly volatility is much higher and far beyond the constraint, with a value of 5.61%. We decided to scale our out-sample momentum portfolio using the same ex-ante risk budget of 2% and the in-sample volatility.

5.2.2 In-Sample Performances

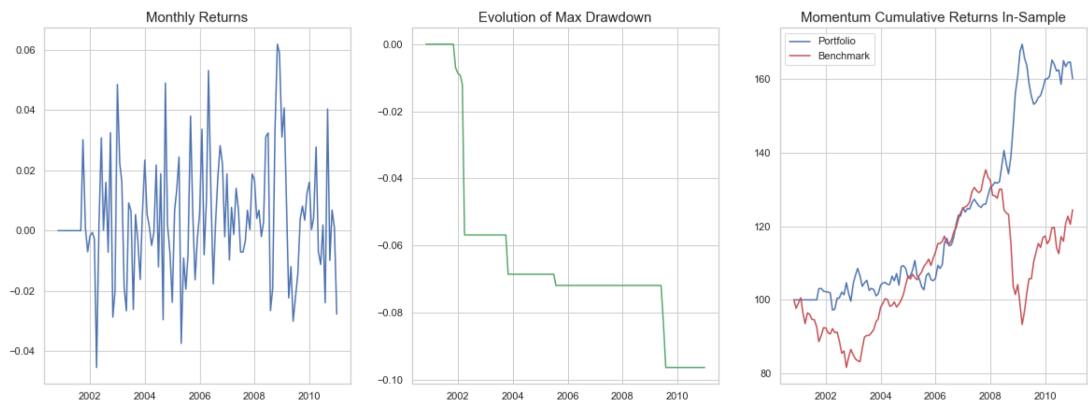
We will now go through the performance of our in-sample momentum portfolio when compared to the benchmark.

Table 7 – In-Sample Performances Momentum Factor

	Benchmark IS	Momentum
Mean	0.024888	0.048307
Volatility	0.083199	0.069000
SharpeRatio	0.299132	0.700108
MaxDrawdown	-0.310457	-0.096352
HitRatio	0.585366	0.626016

Our momentum portfolio beat the benchmark on all aspects during the 2000-2010 decade. Annualized returns were higher while annualized volatility was lower, causing an increase in the Sharpe Ratio. The maximum drawdown was lower, and the hit ratio was higher. Figure 14 depict what happened. The benchmark saw sharp declines at the start of the decade following the recession caused by the DotCom bubble and during the 2008 housing crisis.

Figure 14 – Momentum In-Sample Portfolio Performance



The momentum portfolio showed high strength during the decade, hardly ever experiencing major declines. This strength was shown during the start of the decade and, more importantly, during the 2008 crisis, during which the benchmark suffered a major decline while the momentum portfolio was experiencing large gains.

In the second half of 2008, our momentum portfolio was shorting Copper and World Equities while longing Gold and World Bonds. Our strategy took profit from the continuous downtrend in World Equities and Copper, which both saw large negative returns in the second half of 2008. Meanwhile, the benchmark was longing World Bonds and World Equities at a 50% weight each, which caused a major decline in performance. World Equities saw an impressive -12% and -21% returns in respectively September and October of 2008. Overall, the momentum strategy taking advantage of continuous downtrends, such as the ones suffered by World Equities and Copper in 2008, gave this strategy strong returns in times of continued market weakness. Indeed, a momentum strategy performs best when instability trends persist. We believe that we will be able to benefit from major downturns and/or possible financial crisis during the out-sample time frame.

Overall, both value and momentum portfolios performed better than the benchmark between 2001 and 2010. The benchmark saw a large decline in value following the 2008 housing crisis, which led to a large drop in the World Equities prices, contributing to 50% of the benchmark. Indeed, as stated in the previous paragraph, World Equities took a major hit in September and October of 2008, dropping respectively by -12% and -21% in those months. While the value and momentum strategies performed well, it must be said that the large difference in performance with the benchmark can be attributed to the extreme negative returns of it during the 2008 crisis.

5.2.3 Out-Sample Performances

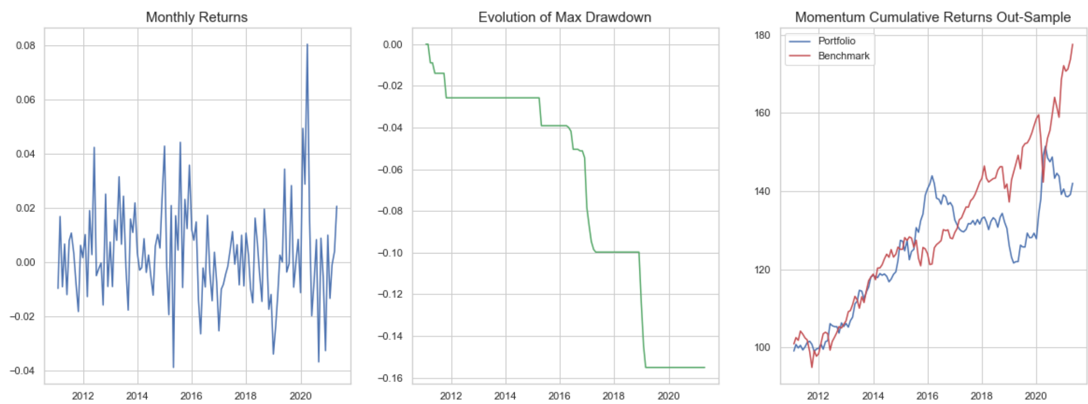
Moving on to the 2010-2020 decade, the momentum portfolio underperformed compared to the benchmark. The same observation was made for the value portfolio: it performed better in the in-sample than in the out-sample. In the out-sample, the benchmark outperformed the momentum portfolio, beating it on all aspects: higher average annualized returns, lower average annualized volatility, higher Sharpe Ratio, lower max drawdown and a higher hit ratio.

Table 8 – Out-Sample Performances Momentum Factor

	Benchmark OS	Momentum
Mean	0.058004	0.035894
Volatility	0.067447	0.061797
SharpeRatio	0.859999	0.580839
MaxDrawdown	-0.108245	-0.155123
HitRatio	0.685484	0.532258

When looking at the performance chart of both portfolios, we can once again see the strength of the momentum portfolio.

Figure 15 – Momentum Out-Sample Portfolio Performance



Indeed, between 2011 and 2016, the momentum portfolio showed consistent positive returns, taking advantage of the strength in World Equities and High Yield Investment Grade Bonds. Indeed, in the years following the subprime crisis, equities and bonds showed increased strength setting up an uptrend, while commodities such as Oil and Gold saw large downtrends with prices dropping at the start and the middle of the decade. However, the momentum portfolio declined between 2016 and 2020, which were years where our momentum portfolio struggled to generate positive returns out of the seven assets. When looking at the data, our allocation often switched between assets, showing a notable weakness to find consistent up or downtrends in these years. As in the 2008 crisis, the momentum portfolio again performed well in the 2020 Covid-19 related recession by shorting Copper and Energy commodities, which already saw large drops in 2008. On the other hand, safer assets such as Gold and Investment Grade Bonds were longed to profit from their continued strength through the pandemic recession.

Overall, the benchmark performed better than the value and momentum portfolios between 2010 and 2020. Such strength can be attributed to the 50% weight given to World Equities in the benchmark, with these equities more than doubling within the last 10 years. MSCI World, the main index for World Equities, went from a level of approx. 90 units in January 2011 to approx. 240 units in December 2020, almost tripling its value. Such strength could not be found in the value and momentum portfolios. The value portfolio struggled to generate positive returns in late 2014 when the price of commodities was declining fast, and the momentum portfolio underperformed between 2016 and 2020 when momentum was low, dramatically switching its allocation almost every month.

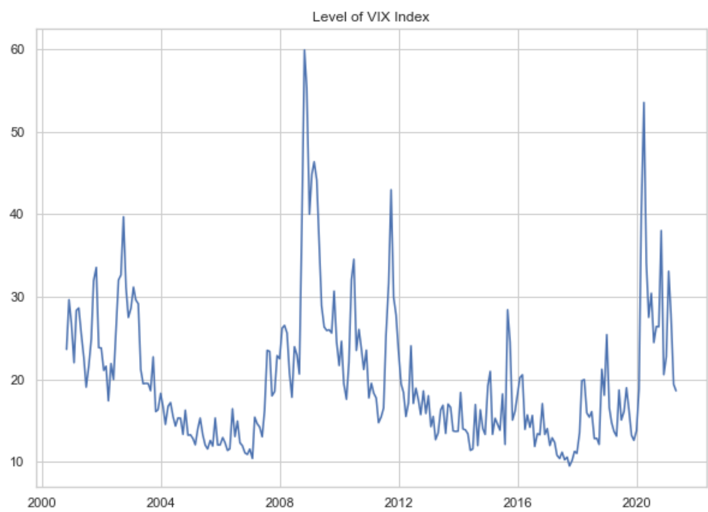
To close out this part, we will explain **why we are computing long-short factors rather than long-only factors (in the case of a TAA)**. Long-short portfolios give the possibility to adapt monthly to new information that is provided by new data, such as the carry value in the value strategy or the rolling returns in the momentum strategy. With this additional data, every month, one can long the undervalued and short the overvalued assets in the value case and take advantage of continuing uptrends and downtrends in the momentum case. This strategy enables a high degree of flexibility, giving the portfolio the possibility to take advantage of additional data through monthly rebalancing. A long-only portfolio would not be able to take advantage of overvalued assets by shorting them in the value case and downtrends in the momentum case, also by shorting them, giving the long-only portfolio less flexibility to operate. Moreover, a long-short strategy is nearly entirely self-funded. The money received from shorting can be used to long assets, which is a process that cannot be done in a long-only case.

5.3 TAA Portfolio

5.3.1 Deriving a Strategy using VIX & Parametric Weights

Now that we have determined the performances of the Value and Momentum factors, both in-sample and out-sample, we will have to determine a strategy to determine whenever to long or short the factors and by how much. To do so, we first want to determine a relationship between the returns of these two long-short factors and standardized VIX levels. As a small reminder; the VIX is an index that measures the implied volatility embedded in the market prices of S&P500 (SPX) index options. The Chicago Board Options Exchange (CBOE) is responsible for computing, publishing, and trading the futures and options on the VIX. The level of the VIX index in the following year was as follow:

Figure 16 – Level of VIX Index



Unsurprisingly, the VIX level is extremely high during periods of crisis (e.g. Great Financial Crisis of 2008, Covid Pandemic in 2020), and it is low during calm periods. This would indicate the high level of implied volatility during a period of economic downturn. Consequently, to determine the performance of our two long-short factors as a function of the standardized VIX level, we computed the covariance for the full period of the in-sample and out-sample, respectively, as well as the 10 months rolling-covariance.

Figure 17 – Relationship Between Factors and VIX (In-sample)

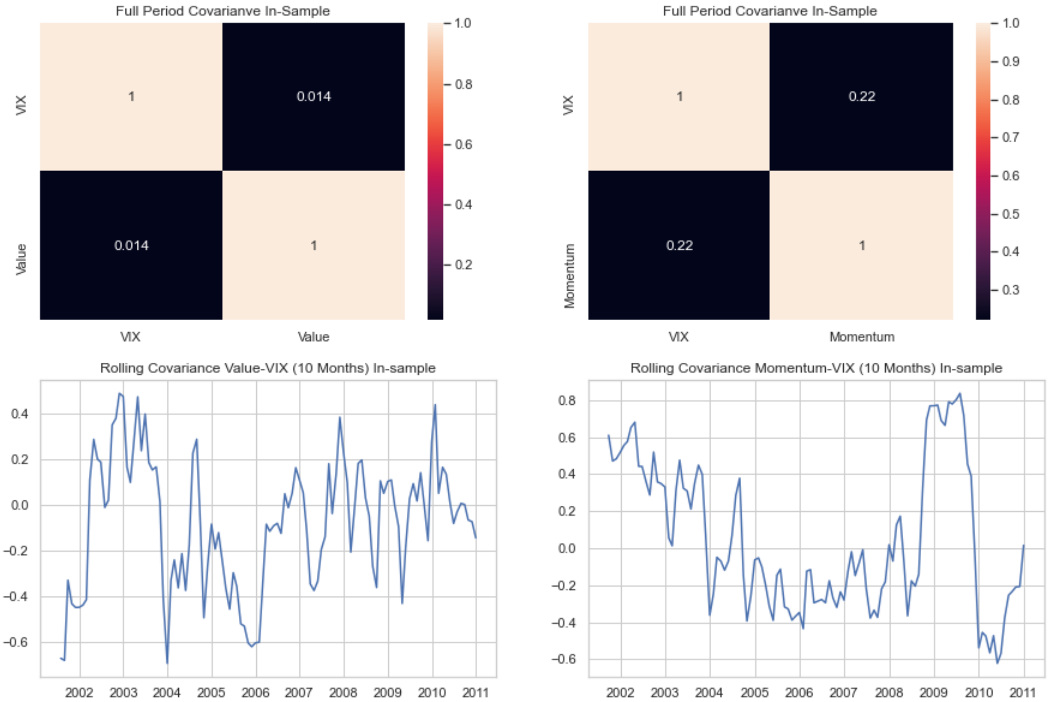
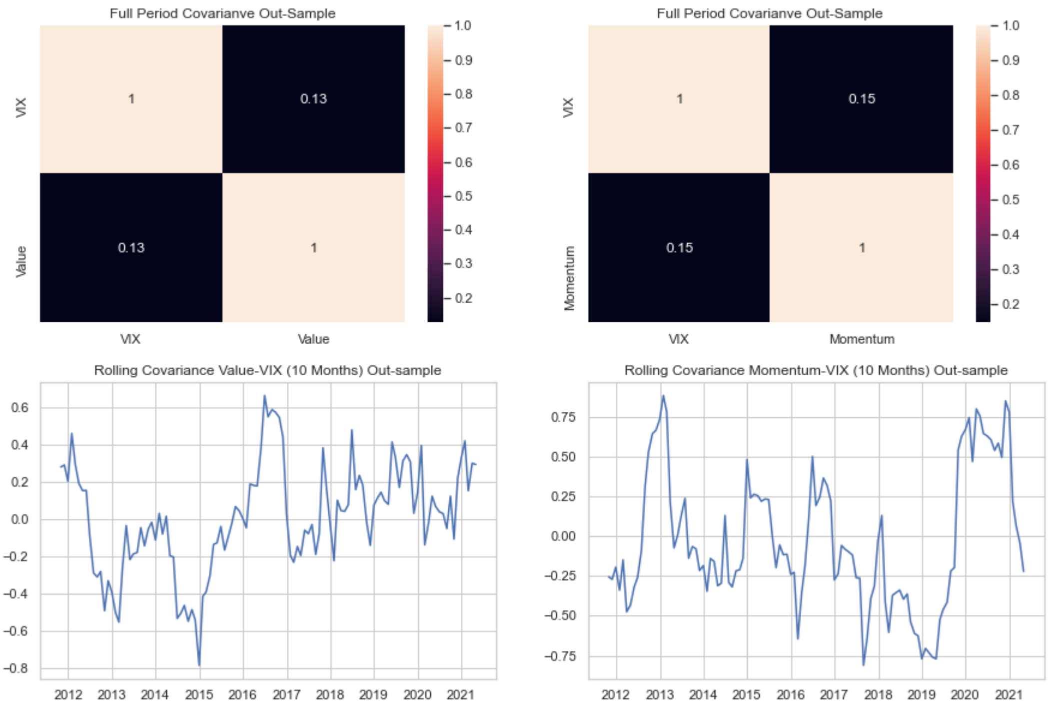


Figure 18 – Relationship Between Factors and VIX (Out-sample)



To determine a dynamic tilt using the above results, we decided to parametrize the weights as a function of the VIX level (i.e. parametric weights), then adjust the weights in case of extreme volatility in order to have a strategy that captures the dynamic of our factors as a function of economic stability/instability (i.e. volatility). Thus, we deviate from our parametric weights to take some short-bets during highly volatile periods to create value for our client.

As we are in December 2010, our dynamic tilt needs to be based on in-sample observations. Overall, we notice that the momentum factor is more correlated with the standardized VIX index than the value factor (the latter is practically un-correlated). Nevertheless, when observing the 10 months rolling-covariance, we notice some interesting results. Indeed, the value factor does not perform well in periods of high volatility: its covariance has been negative and/or close to zero. Nevertheless, on the other hand, the momentum factor yields better results in the same periods: its rolling correlation with the standardized VIX index can be extremely high, close to 80%, for example, in 2008. This would indicate that the momentum factor should be considered as an investment element during a recession. Consequently, to derive a dynamic tilt, we need to consider (1) the level of the VIX index and (2) the direction in which the VIX index is heading. For simplicity, we assumed that the total weight allocated to each factor is equal to one. Therefore, we decided that:

- **When the standardized VIX is lower than its 90% quantile**, allocate the parametric weights to each factor respectively.
- **When the standardized VIX is higher than its 90% quantile and its percentage change is positive (VIX increases)** (i.e. the variation of the standardized VIX index from a month to another), allocate 100% to the momentum factor, as we expected the performance of the latter to increase, hence deciding to allocate everything to it.
- **When the standardized VIX is higher than its 90% quantile and its percentage change is negative (VIX decreases)** (i.e. the variation of the standardized VIX index from a month to another), allocate 200% to the value factor and -100% to the momentum factor, as we expected the performance of the latter to decrease, hence deciding to short-sell it.

Figure 19 – Standardized VIX Index & Percentage Change (In-sample)

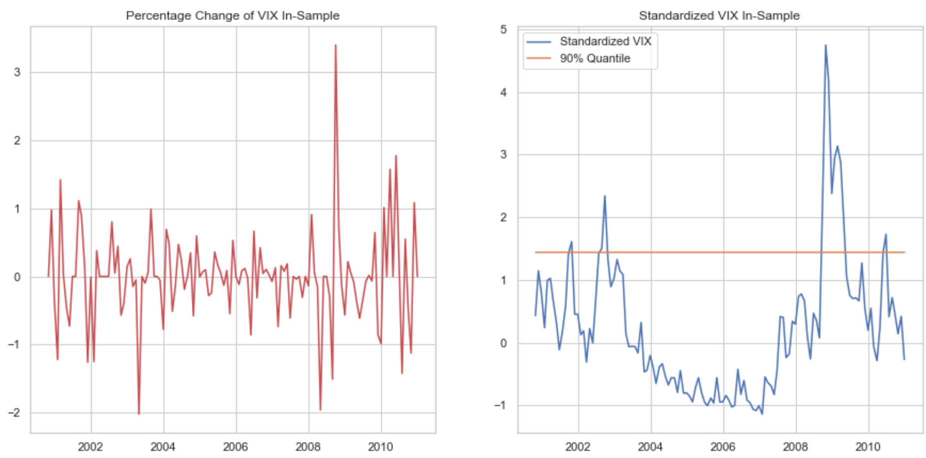
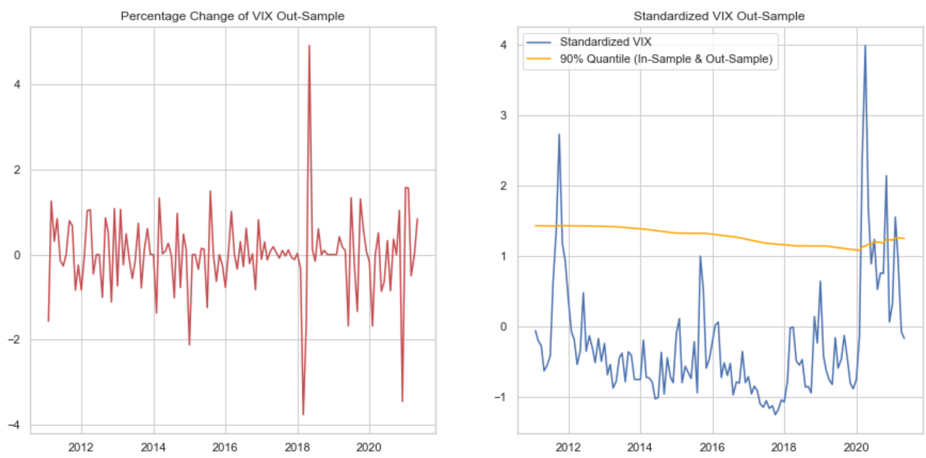


Figure 20 – Standardized VIX Index & Percentage Change (Out-sample)



Since this dynamic tilt will have to be implemented on out-sample data, the 90% quantile previously computed with the in-sample data will be updated each month after new observations have been received in the out-sample time series.

To determine the parametric weights (i.e. the weights during "calm periods" of our strategy), we decided to use the model derived in the paper of Brandt & Santa-Clara (2006) as well as Brandt, Santa-Clara & Valkanov (2009). The idea is to parametrize the portfolio weights as functions of observable quantities (i.e. macroeconomic variables, firm's characteristics, etc.), which in our case is the level of the standardized VIX Index. Then we solve for the parameters that maximize the expected utility. Let z_t be the set of explanatory variables. Then, the parameterized portfolio weights can be determined as follows:

$$\alpha_t = \theta z_t \tag{14}$$

Let's consider a single-period mean-variance problem. Assume that the optimal portfolio weights are a linear function of K state variables z_t. Then, the investor's conditional optimization problem, with a risk aversion λ is:

$$\max_{\{\theta\}} E_t[(\theta z_t)' r_{t+q}] - \frac{\lambda}{2} V_t[(\theta z_t)' r_{t+q}] \tag{15}$$

Let's define $\tilde{\alpha}_t = vec(\theta)$ and $\tilde{r}_{t+1} = z_t \otimes r_{t+1}$. Thus, we obtain:

$$\max_{\{\tilde{\alpha}\}} \mathbb{E}[\tilde{\alpha}' \tilde{r}_{t+1}] - \frac{\lambda}{2} \mathbb{V}[\tilde{\alpha}' \tilde{r}_{t+1}] \tag{16}$$

Since the same $\tilde{\alpha}$ maximizes the conditional expected utility at all dates t, it also maximizes the unconditional expected utility, which corresponds to the problem of finding the unconditional portfolio weights. Thus, the optimal solution is:

$$\tilde{\alpha} = \frac{1}{\lambda} \mathbb{E}[\tilde{r}_{t+1} \tilde{r}'_{t+1}]^{-1} \mathbb{E}[\tilde{r}_{t+1}] = \frac{1}{\lambda} \mathbb{E}[z_t z_t' \otimes r_{t+1} r'_{t+1}]' \, \mathbb{E}[z_t \otimes r_{t+1}] \tag{17}$$

The optimal solution can also be computed using sample averages (as we did in our strategy):

$$\tilde{\alpha} = \frac{1}{\lambda} \left[\sum_{t=1}^{T} z_t z_t' \otimes r_{t+1} r'_{t+1} \right]^{-1} \left[\sum_{t=1}^{T} z_t \otimes r_{t+1} \right] \tag{18}$$

In our case, the only binary explanatory variable was determined according to the level of VIX Index in the in-sample:

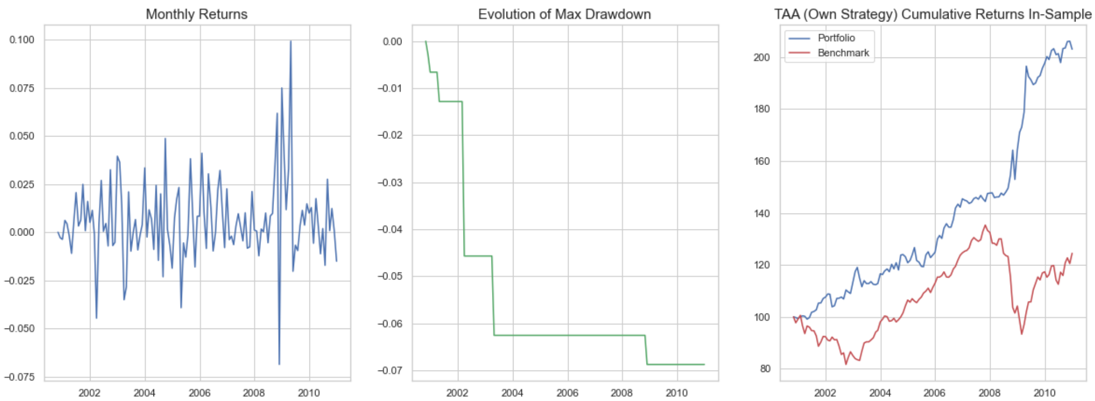
- If the VIX Index is **lower** than its 90% quantile, the variable is equal to 1 (i.e. non-crisis period).
- If the VIX Index is **higher** than its 90% quantile, the variable is equal to -1 (i.e. crisis period).

Consequently, after implementing the In-sample returns of our value and momentum factors into the above formula, using the aforementioned binary explanatory variable, we obtained that the allocations of the value and momentum factors are 54.45% and 45.55% respectively. The reason we decided to change these allocations during periods of high volatility, as explained above, is because we believe this model does not capture dynamically well the periods of high uncertainties in the markets. We will observe that this adaptation of the parametric weights will allow us to capture our factors' short-term upwards and downwards movements, thus obtaining better performances.

5.3.2 In-Sample Performances

After implementing the aforementioned strategy among the value and momentum factors, we obtained the following performances for our Tactical Asset Allocations (TAA):

Figure 21 – TAA (Own Strategy) Cumulative Returns In-Sample

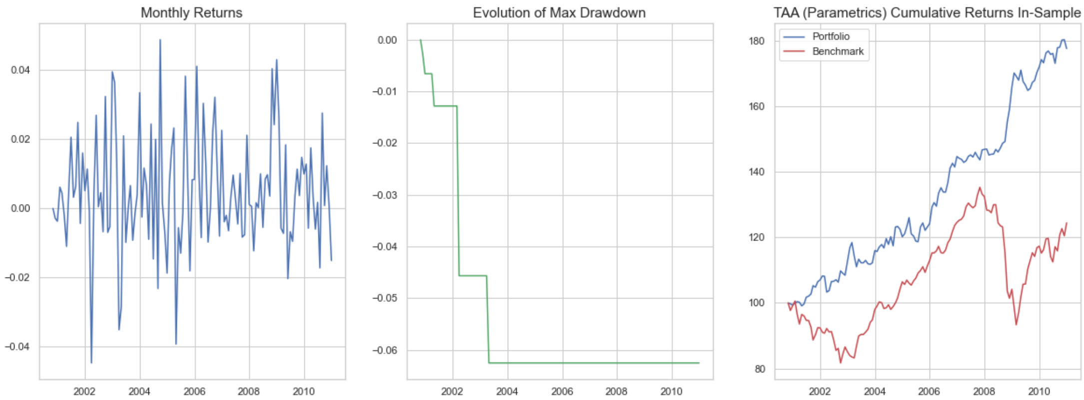


In addition, we also computed the performance of the TAA portfolio if we were to use only the parametric allocation (i.e. excluding the adjustment during periods of high volatility) as a mean of comparison with our own strategy. The performance summary can also be found in the table table 9.

Table 9 – In-Sample Performances TAA

	Benchmark	Value	Momentum	TAA	Parametrics
Mean	0.024888	0.065875	0.048307	0.071942	0.057873
Volatility	0.083199	0.069000	0.069000	0.073493	0.057539
Sharpe Ratio	0.299132	0.954719	0.700108	0.978891	1.005798
Max Drawdown	-0.310457	-0.094119	-0.096352	-0.068739	-0.062549
HitRatio	0.585366	0.617886	0.626016	0.634146	0.617886

Figure 22 – TAA (Parametrics) Cumulative Returns In-Sample



Standalone Basis

We notice that overall, the performances of our strategy has been particularly good. Indeed, with a Sharpe Ratio of approximately 1 and a reasonable max drawdown of -6.9%, our TAA portfolio has been on an overall upward movement during the entire period. Its long position on momentum when the market was tanking in 2008 and the short position on the same factor when the global economy was recovering allowed us to increase our portfolio's cumulative returns substantially. One of the reasons our portfolio did so well during the GFC was because we had a short position on commodities, which is usually one of the most negatively impacted asset during periods of crisis as global demand shrinks. Ultimately, our TAA portfolio reached approximately a +18% cumulative return by the end of the period. We therefore believe that our TAA component will be particularly fruitful during periods of recessions in the out-sample. Another reason for these great performances is mainly due to a greater long position in the value factor in-sample, except during recessions as explained previously. Indeed, the value factor has been able to deliver great performances, as value/undervalued companies has been fruitful, compared to growth companies.

Benchmark Comparison

Compared to our benchmark, our TAA portfolio was able to outperform it by a significant margin. There was almost no single period where the cumulative returns of the benchmark were higher than our portfolio. We clearly notice that our TAA portfolio has outperformed the client's benchmark on all performance measures (i.e. annualized returns & volatility, Sharpe ratio, max drawdown and hit ratio).

Strategy vs. Parametrics

On the last two columns of the table table 9, we determined the performances of the TAA portfolio using our selected strategy explained above, as well as the performances of the TAA portfolio if we were to use only the parametric weights. We notice that the strategy of adapting the weights between the value and momentum factors during periods of high volatility has been fruitful since we have been able to generate better annualized returns. Adapting our views during period of uncertainties has been therefore fruitful. Nevertheless, this strategy does not come without cost, as our annualized volatility has also increased from 5.7% to 7.3%, thus lowering slightly the Sharpe Ratio. Although the risk has increased, we believe this strategy will be more beneficial in the forthcoming future, as our strategy was able to adapt accurately during periods of crisis which are inevitably going to occur again some time in the future.

5.3.3 Out-Sample Performances

Standalone basis

As mentioned previously, we proposed a strategy using VIX as a trigger function for our TAA portfolio. This strategy delivered an annualized return roughly equal to the momentum portfolio but with lower volatility, thus obtaining a better sharpe ratio. Moreover, this strategy has the lowest max drawdown of our different strategies (approximately -7% in one month). It is indeed a significant improvement compared to the max drawdown of the value and & momentum portfolio. In addition, it allowed us to have a lower significant loss than the previous strategy. During the covid crisis, our strategy was able to yield outstanding returns, thanks to a long position in momentum when the VIX was particularly high and increasing, and a short position in momentum when the VIX was still higher than its 90% quantile but decreasing.

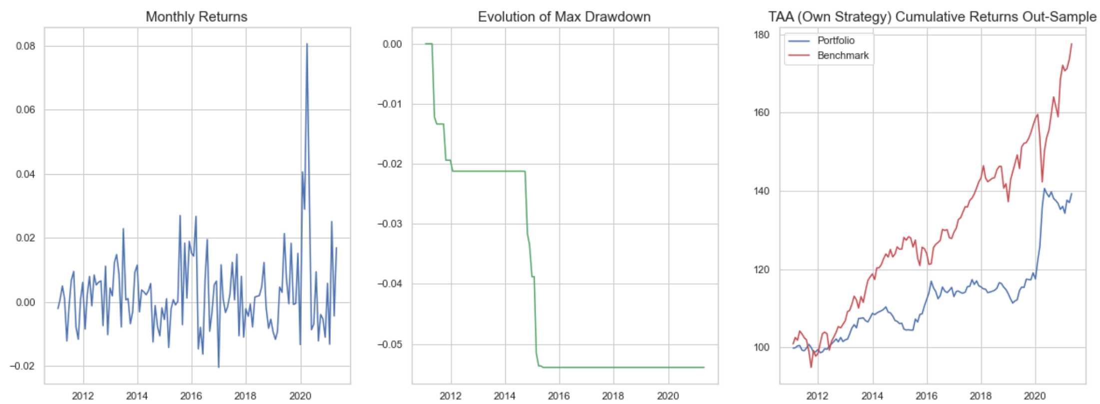
Table 10 – Out-Sample Performances TAA

	Benchmark	Value	Momentum	TAA	Parametrics
Mean	0.058004	0.023931	0.035894	0.033128	0.029380
Volatility	0.067447	0.050973	0.061797	0.045109	0.037870
Sharpe Ratio	0.859999	0.469478	0.580839	0.734396	0.775815
Max Drawdown	-0.108245	-0.153731	-0.155123	-0.053907	-0.053907
Hit Ratio	0.685484	0.540323	0.532258	0.556452	0.572581

Benchmark comparison

Compared to the benchmark, this strategy does not perform well as we obtain a lower Sharpe Ratio. Nevertheless, our TAA portfolio was able to yield a lower volatility than the Benchmark, which could be beneficial for a particularly risk-adverse investor. During COVID crisis, our strategy was able to catch up with the performance of the benchmark. However, the market's recovery creates a new divergence between our performance and that of our benchmark. Therefore, we notice that our strategy over-performed the benchmark when the VIX index is particularly high (e.g. COVID crisis):

Figure 23 – TAA (Own Strategy) Cumulative Returns Out-Sample (Complex Strategy)

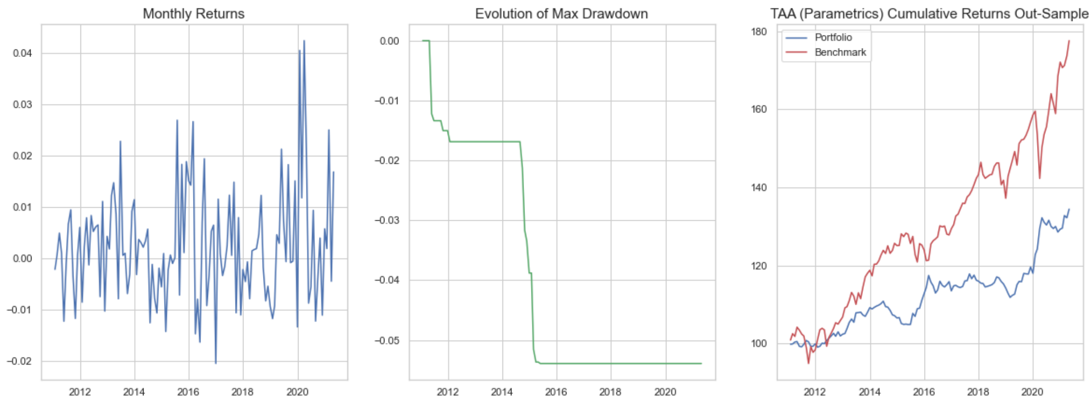


A reason that could explain the poor performances of our portfolio is mainly due to a greater long position in the value factor out-sample expected during period of recession as explained previously. Indeed, the value factor has delivered poor performances since 2010 approximately. This can be justified by the fact that value/undervalued companies have been overshadowed by trends and companies operating in the tech sector. The latter having been particularly fruitful.

Strategy vs. Parametrics

On the last two columns of the table section 5.3.3, we determined the performances of the TAA portfolio using our selected strategy explained above, as well as the performances of the TAA portfolio if we were to use only the parametric weights. We notice that during periods of recession, our own strategy has been able to generate greater returns compared to a parametric-only portfolio. Adapting our views during period of uncertainties has been indeed fruitful. Nevertheless, during calm periods, as our own strategy will set the parametric weights between the value and momentum portfolio, we will have the same performances as a parametric-only portfolio. Nevertheless, this strategy does not come without cost, as our annualized volatility has also increased from 3.7% to 4.5%, thus lowering slightly the Sharpe Ratio. Nevertheless, if the client is low risk-adverse, we would suggest to implement our own strategy if its ultimate goal is to maximize returns. If the client were to be concerned about a lower Sharpe Ratio, we would shift quickly the TAA allocation from our strategy to the parametric-only portfolio, as the client is always king.

Figure 24 – TAA (Parametrics) Cumulative Returns Out-Sample



5.3.4 Tracking-Error & Information Ratio between TAA & Benchmark

To further analyse the performance of our portfolio, we determined the tracking error (TE) and information ratio (IR) between the TAA and the client's benchmark. In the In-Sample data set, we obtained a rather large TE since our TAA portfolio had reasonable long/short positions in all assets, and the benchmark only had 50%/50% allocated to world equities and world bonds. The same can be said about the ex-post TE. Regarding the information ratio, prior to December 2010, we have been able to outperform the benchmark by a significant margin, which may indicate our level of skill and ability to generate excess returns relative to the benchmark (our in-sample views were fruitful). Nevertheless, such performances cannot be said in the out-sample since our TAA portfolio under-performed the benchmark by a significant margin. Ultimately, we notice that during periods of crisis (e.g. GFC and COVID), the tracking error is particularly high, indicating how dynamic our TAA component is during these particular periods, compared to the static benchmark portfolio.

Table 11 – Annualized Tracking Error

	In-Sample	Out-Sample
SAA vs. Benchmark	0.080494	0.080494
TAA vs. Benchmark	0.152105	0.140315

Figure 25 – Monthly TE: TAA vs. Benchmark

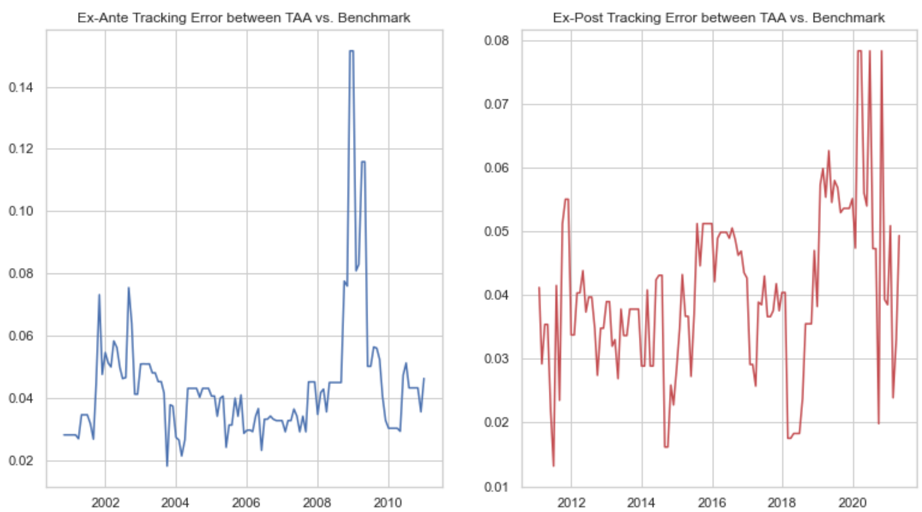


Table 12 – Information Ratio

	In-Sample	Out-Sample
SAA vs. Benchmark	0.423443	-0.323786
TAA vs. Benchmark	0.399976	-0.274740

5.3.5 Tracking-Error & Information Ratio between TAA & SAA

For this part, we also determined the tracking error (TE) and information ratio (IR) between the TAA and SAA components. Similarly to the previous section, the ex-ante TE is fairly significant since the TAA portfolio had reasonable long/short positions in all assets, and the SAA portfolio mainly consisted of world bonds. Although slightly lower, the ex-post TE still remains significant for the same reasons. Additionally, the TAA portfolio has been able to outperform the SAA portfolio, thanks to its dynamic allocation, prior to December 2010. Nevertheless, on the out-sample, we notice that the SAA component delivered better performance, partly explained by a significant exposure to world bonds, where the latter performed well in this time frame. Once again, we notice that during periods of crisis (e.g. GFC and COVID), the tracking error is particularly high, which may indicate how dynamic our TAA component is during these particular periods (i.e. our views are getting more aggressive), compared to the static SAA portfolio, and/or the securities in our portfolio are also more volatile.

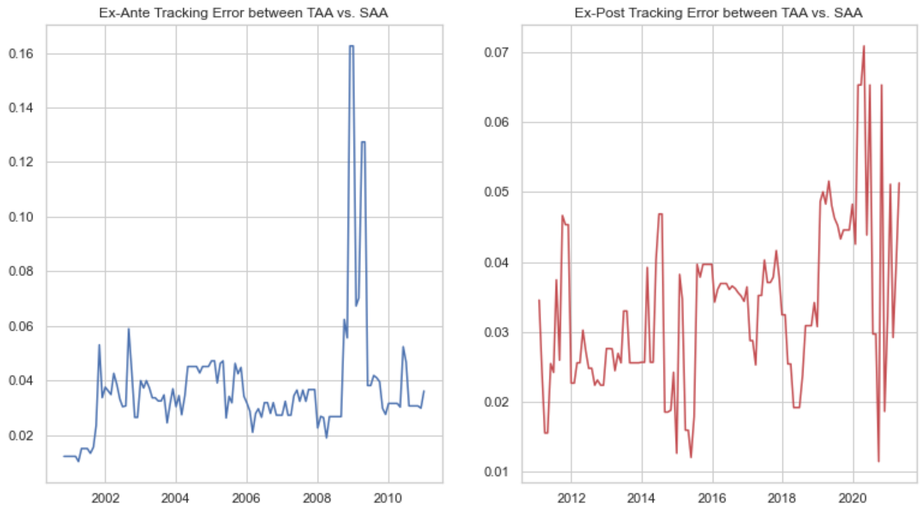
Table 13 – Annualized Tracking Error

	In-Sample	Out-Sample
SAA vs. Benchmark	0.080494	0.080494
TAA vs. Benchmark	0.152105	0.140315
TAA vs. SAA	0.128970	0.115942

Table 14 – Information Ratio

	In-Sample	Out-Sample
SAA vs. Benchmark	0.423443	-0.323786
TAA vs. Benchmark	0.399976	-0.274740
TAA vs. SAA	0.165900	-0.067405

Figure 26 – Monthly TE: TAA vs. SAA



6 Implementation

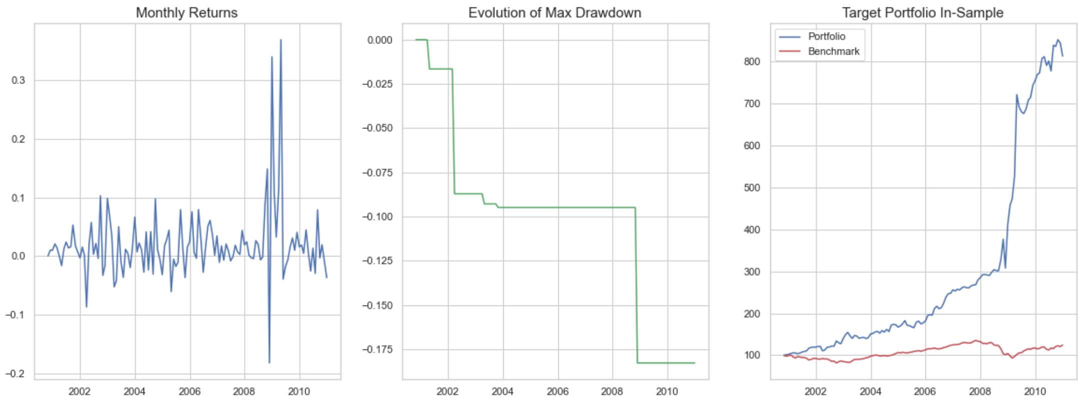
6.1 Target Portfolio

The target portfolio has been determined as a combination of our SAA and TAA strategy. In the previous parts, we explain how these two strategies have been determined. The Strategic Asset Allocation (SAA) described in part 4 allows us to build a static long-term vision. The Tactical Asset Allocation (TAA) allows us to update our portfolio to short-term dynamics. This allocation exploits medium / short-term expected returns with some forecasting model, using the value and momentum factors and the VIX index as a trigger function. In the tactical allocation, we were also able to deviate from our parametrics weights to take some short-bets during highly volatile periods to create value for our client. Therefore, we could tell that these strategies are complementary. Consequently, the target portfolio offers a complete investment solution (Long and short-term horizon) for our client.

6.1.1 In-Sample Performance

Table 17 illustrate key metrics of the target portfolio. When compared to our SAA & TAA, one can note that our portfolio held much more volatility returns (between -15% and 30%) when the returns of the SAA & TAA allocations were between -4% and +3%. As expected, the Target Portfolio consistently beats the benchmark during the In-Sample period by a significant margin.

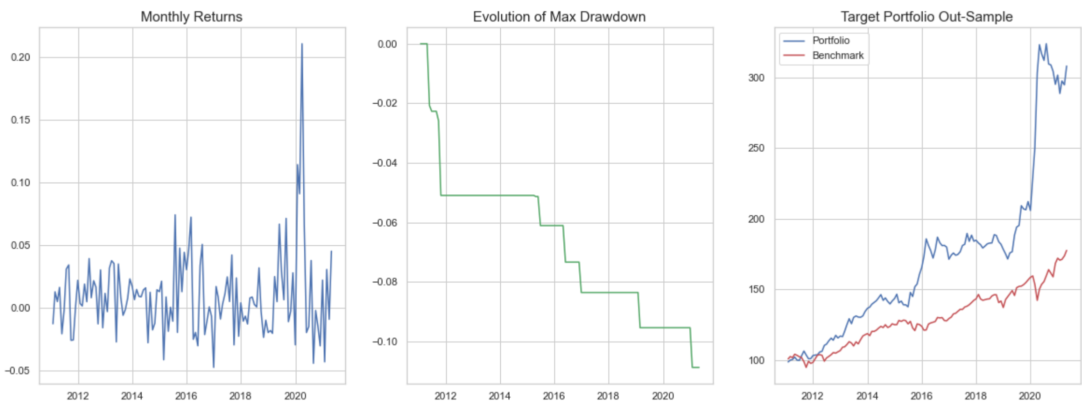
Figure 27 – Target Portfolio In-Sample



6.1.2 Out-Sample Performances

However, the most interesting results appear when applying the Target Portfolio to the out-sample period. Table 18 illustrate key metrics of the target porfolio performance out-sample. Again, the Target portfolio consistently outperforms the Benchmark. Moreover, in contrast to the in-sample values, the returns remain constant between -5% and 7%, except for the post-2020 period. Indeed, the target portfolio has made outstanding results due to its overall long position in the momentum factor, and full-in during the COVID crisis, which was induced by a rise of volatility and the substantial growth of tech companies, which can explain the over-performances of the momentum factor.

Figure 28 – Target Portfolio Out-Sample



6.1.3 Tracking Error and information Ratio between Target Portfolio and SAA

Finally, as for the previous sections, we computed the Information Ratio (IR) and Tracking Error (TE) between the Target Portfolio (TAA + SAA) and the SAA (see table 15 & table 16).

One can notice that the addition of the TAA to the SAA has no effect on both the TE in- and out-sample. Indeed, as the TAA portfolio is a long-short allocation and a dynamic deviation of the long-run allocation (i.e. SAA), the volatility of the TAA is indeed the TE between the SAA and SAA+TAA. The difference probably comes from rounding error in the Python implementation. The table 16 illustrates strong IR for the target portfolio when compared to the SAA for both in and out of sample, respectively 81.67% & 70%. This strong performance can be explained by the large returns made by the Target portfolio during the Covid crisis (see figure 27 & figure 28). When comparing the IR of the target portfolio to the SAA table 16, one can notice that the IR In and out sample are relatively similar, indicating that our target portfolio has be able to deliver consistent performance outperforming the benchmark over time.

Table 15 – Annualized Tracking Error of Target Portfolio

	In-Sample	Out-Sample
SAA vs. Benchmark	0.080494	0.080494
TAA vs. Benchmark	0.152105	0.140315
TAA vs. SAA	0.128970	0.115942
TAA+SAA vs. SAA	0.126539	0.111716

Figure 29 – Monthly TE: Replication vs. Target Portfolio

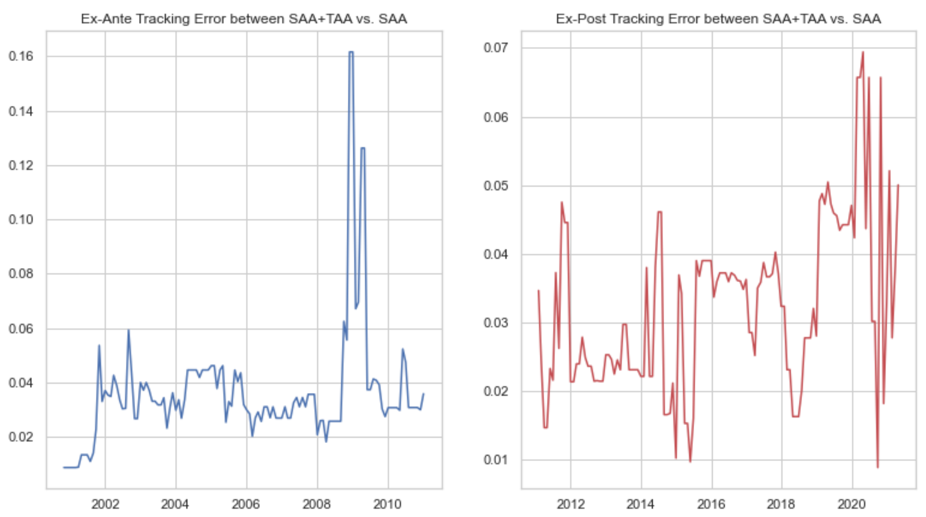


Table 16 – Information Ratio

	In-Sample	Out-Sample
SAA vs. Benchmark	0.423443	-0.323786
TAA & SAA vs. SAA	0.399976	-0.274740
TAA vs. SAA	0.165900	-0.067405
TAA+SAA vs. SAA	0.816689	0.700314

6.2 Replication Portfolio

Now, the client requires a new constraint: no investment in the US Investment-Grade asset is allowed. Moreover, we need to offer the same performance with this constraint as with the target portfolio. This new portfolio could be constructed by minimizing the tracking error between the target and replication portfolio. We launched an optimizer that finds the optimal weights of the different assets (except US investment grade) to minimize tracking errors for each months. By minimizing the TE, we will be able to find another allocations, while being as close as possible to the target portfolio in terms of performances:

$$\min_{w_p} TE = \sqrt{w^T \Sigma w} = \sqrt{(w_p - w_b)^T \Sigma (w_p - w_b)} \tag{19}$$

Our constraint was to have no position in US Investment Grade in the replication portfolio $w_{p,USInvest.Grade} = 0$, and the sum of weighs of this portfolio to equal to one $\sum_{i=1}^{N} w_{p,i} = 1$. We define $w = w_p - w_b$ as the difference between the weight of our portfolio and the benchmark, and Σ is the covariance matrix. Our optimizer has been created with the help of the python library Scipy. We used the method Sequential Least Squares Programming (SLSQP) to find optimal weights that respect all our different constraints.

6.2.1 In-Sample

In our in-sample period, one can observe that our replication portfolio matches the target portfolio fairly well. Moreover, the two performances are very close to each other, and thus the tracking error is very low. The two portfolios' returns increase rapidly during the subprime crisis because of the volatility of the period and our strategy combining the parametric weights and the VIX. The interesting observations lie in the fact that generally, with volatile assets, the tracking error should have increased. Hence, the US investment grade does not explain our performance in a highly volatile period. During the global financial crisis between 2008 and 2010, one can notice that the tracking error between the SAA+TAA portfolio and our actual portfolio increased slightly compared to the rest of the period, indicating the difficulty of replicating the target portfolio during periods of high intensity (i.e. our portfolio is more dynamic during period of crisis). The following figures illustrate that the difference between the two portfolios increases during periods of uncertainty:

Figure 30 – Target vs. Replication Portfolio In-Sample

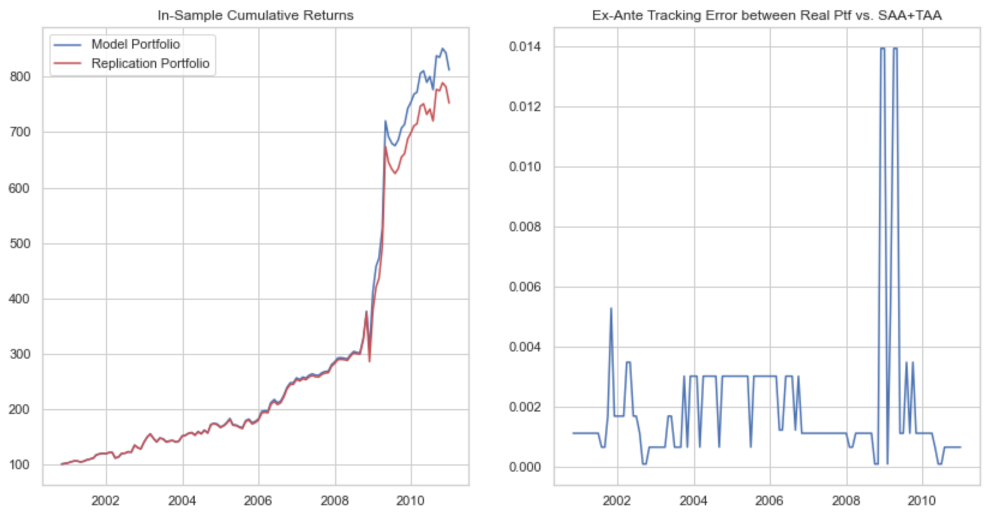


Table 17 – Performances In-Sample

	Model	Replication
Mean	0.225231	0.218964
Volatility	0.205980	0.210198
SharpeRatio	1.093461	1.041707
MaxDrawdown	-0.182738	-0.236723
HitRatio	0.666667	0.666667

6.2.2 Out-Sample

In our out-sample period, the value of tracking error is also relatively low. However, its value is very close to the one in the in-sample period (see table 19). Nevertheless, we could see that some divergence between the two performances exists. The first divergence was created during a period of low-volatility, the second with the Covid crisis that brought extreme volatility. Our replication portfolio has caught up with our target portfolio. Our position with a momentum portfolio where US investment grade does not play a significant role in this period could be explained. Nevertheless, the performances of the target and replication portfolio are fairly close.

Figure 31 – Target vs. Replication Portfolio Out-Sample

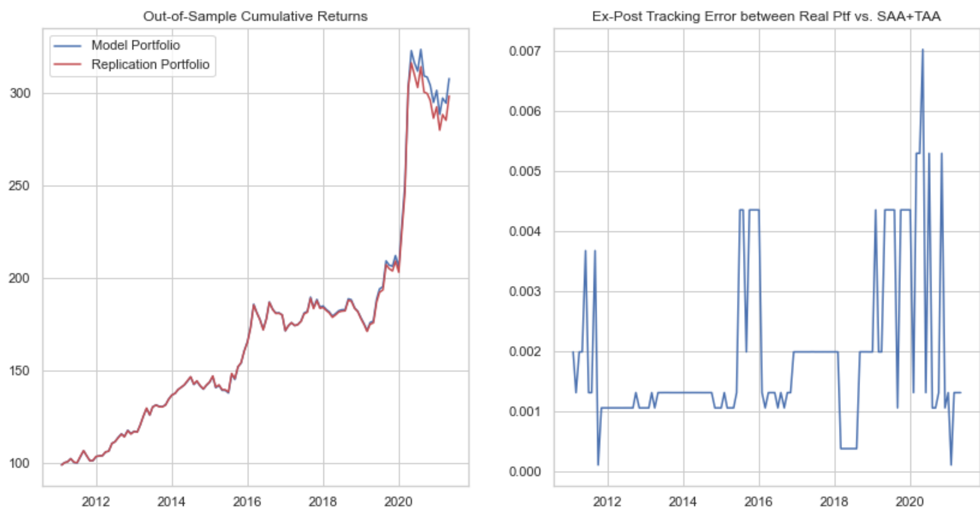


Table 18 – Performances Out-Sample

	Model	Replication
Mean	0.115748	0.112873
Volatility	0.115864	0.117739
SharpeRatio	0.998995	0.958671
MaxDrawdown	-0.108660	-0.115049
HitRatio	0.612903	0.604839

6.2.3 Tracking Error and information Ratio between Replication and Target Portfolio

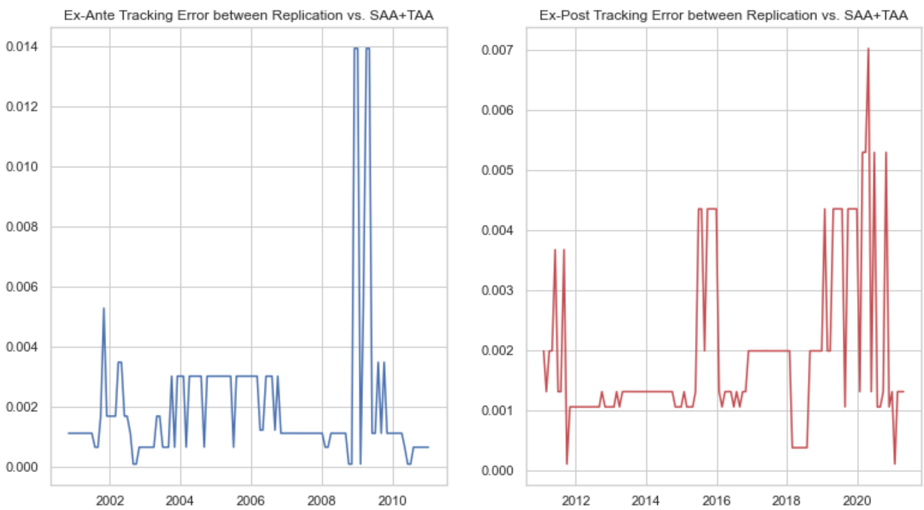
The Tracking Error for the replication portfolio vs. TAA+SAA of less than 1% in both in and out sample illustrates how effective our optimisation is (see table 19). Indeed, the above-mentioned numbers are expected as our optimiser is set to minimise the TE. These close to 1% can be explained by numerical errors occurring during the minimisation process.

However, the negative information ratios of 19 highlight the failure of our replication portfolio to obtain the identical performances the target portfolio. When looking at table 17, one can observe that the main lag occurs after the surge in returns from our model portfolio during the Global Financial Crisis between 2008 and 2010. The lack of liquidity of assets can explain this during this period. Additionally, table 18 illustrate that the largest lag between our model and the replication portfolio appears after a considerable surge in returns right after the Covid Crisis in 2020. Once again, this emphasizes that our replication portfolio seems unable to cope with our model in times of high volatility.

Table 19 – IR & TE for Replication Portfolio

	Anualized Tracking Error		Information Ratio	
	In-Sample	Out-Sample	In-Sample	Out-Sample
SAA vs. Benchmark	0.080494	0.080494	0.423443	-0.323786
TAA vs. Benchmark	0.152105	0.140315	0.399976	-0.274740
TAA vs. SAA	0.128970	0.115942	0.165900	-0.067405
TAA+SAA vs. SAA	0.126539	0.111716	0.816689	0.700314
Replication vs. TAA+SAA	0.007099	0.006610	-0.327434	-0.283394

Figure 32 – Monthly TE: Replication vs. Target Portfolio



6.3 Performance attribution: Allocation vs Selection

At this point of our portfolio analysis, it is essential to determine a performance attribution of our replication errors on our available sectors (i.e. bonds, equities, commodities). To do so, we apply the Brinson-Hood-Beebower (1986) performance attribution model, as their model makes a difference between **allocation** (i.e. effect of sector misalignment) and **selection** (i.e. effect of security level choices among a given sector). Let us denote:

- Portfolio return: $R = \sum_{k=1}^{K} w_k r_k$
- Target portfolio return: $B = \sum_{k=1}^{K} w_k^* b_k$

w_k and w_k^* are portfolio and target portfolio weights for a given sector, and K is the number of available sectors. To determine the total difference selection, we will have to define some more additional elements:

- The **allocation contribution** is defined as.

$$B_s - B = \sum_{k=1}^{K} (w_k - w_k^*) B_k \tag{20}$$

  Where the *allocation notional fund*, where its performance is defined as $B_s = \sum_{k=1}^{K} w_k B_k$ is a fund such that we hold the target portfolio allocation within sectors but potentially different section allocations.

- The **selection contribution** is defined as:

$$R_s - B = \sum_{k=1}^{K} w_k^* (R_k - B_k) \tag{21}$$

  Where the *selection notional fund*, where its performance is defined $R_s = \sum_{k=1}^{K} w_k^* R_k$, is a fund such that the portfolio holds exactly the target portfolio's weight in terms of sectors, but with potential differences in terms of the security selected.

- The **interaction effect** is defined as:

$$I = R - R_s - B_s + B \tag{22}$$

  It is the interaction effect between the allocation and selection effect.

Consequently, we can define the total performance difference as:

$$R - B = \underbrace{R_s - B}_{\text{Selection}} + \underbrace{B_s - B}_{\text{Allocation}} + \underbrace{R - R_s - B_s + B}_{\text{Interaction Effect}} \tag{23}$$

As we want to replicate the target portfolio, the top priority is to minimize the allocation effect in order to allocate the same weights to each sector as the target portfolio. Therefore, the second priority would be to minimize the selection effect. Using the data available for our replication and target portfolio, we obtained the following annualized results:

Table 20 – Performance Attribution

	In-Sample	Out-of-Sample
R	0.218964	0.112873
B	0.225231	0.115748
R_S	0.222033	0.110130
B_S	0.222024	0.117579
Interaction	0.000140	0.000912
Allocation Effect	-0.003208	0.001831
Selection Effect	-0.003199	-0.005618

6.3.1 Ex-Ante Replication Quality Analysis (In-Sample)

As the tracking error between the replication and target portfolio was fairly low on average, determining the cause of our deviation for the model portfolio could be more challenging. Indeed, we observe that the allocation and selection effect are fairly low, indicating that the allocations of the replication portfolio among bonds, equities and commodities, as well as among the securities in each type of asset of our replication close genuinely close to the target portfolio. As expected, the interaction effect between the allocation and selection effect is also relatively low. Ultimately, the total performance difference between the replication and benchmark is fairly low.

To understand when these effects are high or low, we plotted the following monthly results of each effect:

Figure 33 – Performance Attribution In-Sample

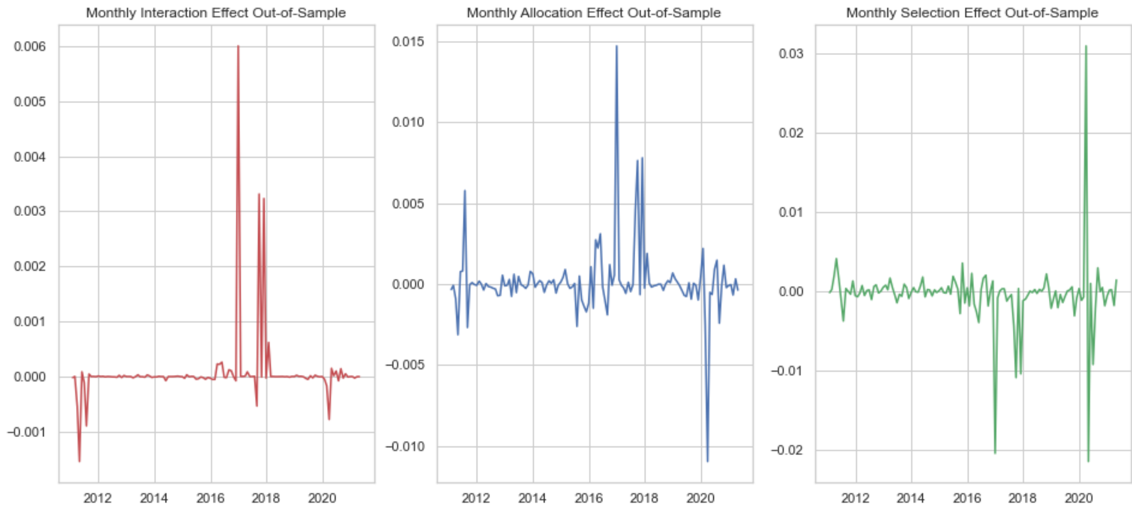
Although minimal, we notice that the effects are higher (both positive and negative) mainly during periods of uncertainties (e.g. GFC of 2008), which we believe is due to the fact that our portfolio is more dynamic (i.e. our views are getting more aggressive) during these periods.

6.3.2 Ex-Post Replication Quality Analysis (Out-Sample)

Although the tracking error was fairly small in the out-sample, the results regarding the performance attribution were fairly different compared to the in-sample. Again, the allocation effect was relative low, reaching approximately 0.2%, which is an optimal result since the top priority when determining a replication portfolio is to minimize the allocation effect, as we want the same exposure to each asset classes. This means that the weight allocated between each sector is, on average, genuinely close to the same weights in the target portfolio. Nevertheless, the deviations of the replication portfolio from our model can be mainly explained by the selection effect, reaching almost −0.5%, but still very small in absolute comparison. This result means that the security selection among each asset class (i.e. bonds, equities, commodities) is legitimately different between the target and real portfolio: our optimizer has not been able to replicate the model as well as we wished. Nevertheless, the interaction effect between the allocation and selection effect, which is quite large, counter-weight the high selection effect, explaining our low total performance difference.

To understand when these effects are high or low, we plotted the following monthly results of each effect again:

Figure 34 – Performance Attribution Out-Sample



Once again, we notice that the effects are higher/lower mainly during periods of uncertainties, for example, the European debt crisis in 2010, or the COVID-19 pandemic in 2020, this to a higher magnitude compared to in-sample.

7 Conclusion

We showed that a strategy optimised to outperform the benchmark in-sample does not necessarily yield out-performance when applied to the out-sample. Indeed, the out-sample performance depends on whether the assumptions based on in-sample metrics hold on the whole time frame. In this sense, luck appears to be an essential determinant of the out-sample performance.
Indeed, we saw that the TAA and SAA standalone were able to outperform the benchmark in-sample but failed to maintain this performance out-sample. Hence illustrating, once again, that historical performances do not assure future performance.
Nevertheless, when combining both long-term views (SAA) and short-term dynamics(TAA) in the target portfolio, we have been able to outperform the benchmark both in and out sample. As the client is always king, we had to replicate the same performance of the target portfolio (i.e. the addition of the SAA and TAA), while considering its constraint; excluding US investments grade.
We showed that the most challenging periods in terms of replication accuracy are the ones with higher dynamics and thus higher volatility. Indeed, during these times, our views are becoming more aggressive while securities are also becoming more volatile.

After determining a replication portfolio (yielding similar performances to the target one), we noticed that the allocation contribution was fairly low, both in and out sample. These results were pleasing, given that the top priority when determining a replication portfolio is to minimise the allocation effect. Finally, we found a selection effect slightly more significant than the allocation effect out-sample, although also minor in absolute comparison.

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