Pavé Guillaume **Student number :** 16409823
Svoboda Liam **Student number :** 16834509

Quantitative Macroeconomics

<u>Question 2</u>

1) Which economic interpretation can you give for θ_t?

As θ_t increases, we can observe that the negative impact of labour time on households' utility function also increases. Therefore, θ_t could be likened to some sort of labour preference variable which could be used to translate labour preference shocks. We could also interpret this as a sort of labour tax (*a distortionary tax rate on labor income*) which lessens the utility of a given quantity of labour.

2) Derive the first-order conditions of the household's utility maximisation problem and the firm's profit maximisation problem.

<u>For the household:</u>

$$\max_{c_t,h_t,k_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma}\left(c_t - \theta_t \frac{h_t^{1+\chi}}{1+\chi}\right)^{1-\sigma} + \lambda_t \left[w_t h_t + (R_t - \delta)k_t - c_t - k_{t+1}\right]$$

Which gives us the following first order conditions:

<u>c_t</u> :

$$\beta^t \left(c_t - \theta_t \frac{h_t^{1+\chi}}{1+\chi}\right)^{-\sigma} = \lambda_t \qquad (1)$$

<u>h_t</u> :

$$\beta^t \left(c_t - \theta_t \frac{h_t^{1+\chi}}{1+\chi}\right)^{-\sigma} (\theta_t h_t^{\chi}) = \lambda_t w_t \qquad (2)$$

Thus, if we divide (1) by (2), we obtain

$$MRS_{hc} \equiv \frac{U_h}{U_c} = w_t$$

or the marginal rate of substitution between labour and consumption.

$\underline{k_{t+1}}$:

$$E_t \lambda_{t+1}(R_{t+1} - \delta) = \lambda_t \qquad (3)$$

From the first order conditions we can obtain two Euler equations.

From putting (1) and (3) together :

$$E_t \beta \left(\frac{c_{t+1} - \theta_{t+1} \frac{h_{t+1}^{1+\chi}}{1+\chi}}{c_t - \theta_t \frac{h_t^{1+\chi}}{1+\chi}} \right)^{-\sigma} (R_{t+1} - \delta) = 1 \qquad \text{(Euler 1)}$$

From putting (2) and (3) together:

$$E_t \beta \left(\frac{c_{t+1} - \theta_{t+1} \frac{h_{t+1}^{1+\chi}}{1+\chi}}{c_t - \theta_t \frac{h_t^{1+\chi}}{1+\chi}} \right)^{-\sigma} \left(\frac{(\theta_{t+1} * h_{t+1}^{\chi} / w_{t+1})}{(\theta_t * h_t^{\chi} / w_t)} \right) (R_{t+1} - \delta) = 1 \qquad \text{(Euler 2)}$$

<u>For the firm:</u>

$$\max_{k_t, h_t} A_t k_t^{\alpha} h_t^{1-\alpha} - (R_t - \delta) k_t - w_t h_t$$

Which gives us the following first order conditions:

$\underline{k_t}$:

$$\alpha A_t k_t^{\alpha - 1} h_t^{1-\alpha} - (R_t - 1) = 0$$

Which gives:

$$\alpha A_t \left(\frac{h_t}{k_t} \right)^{1-\alpha} = (R_t - 1) \qquad (4)$$

$\underline{h_t}$:

$$(1-\alpha)A_t k_t^{\alpha} h_t^{-\alpha} - w_t = 0$$

Which gives:

$$(1-\alpha)A_t \left(\frac{k_t}{h_t}\right)^{\alpha} = w_t \quad (5)$$

Thus, wages (w_t) equal the marginal productivity of labour and the marginal productivity of capital equals the net interest rate $(R_t - 1)$. In other words, the inputs are remunerated exactly for what they bring. No more, no less.

3) Calculate the steady state of the model. (*Recall that it can be useful to introduce auxiliary variables in doing so)*

From the shock functions of Technology (A) and labour preference (θ):

$$ln(A_t) = \rho_A ln(A_{t-1}) + \epsilon_A => ln(A) = \rho_A ln(A) => A = 1 \text{ since } \rho_A \neq 1$$

And by the exact same logic for θ.
Then, we have: $\theta = 1$

Euler 1 at steady state becomes:

$$\beta \left(\frac{c - \theta\frac{h^{1+\chi}}{1+\chi}}{c - \theta\frac{h^{1+\chi}}{1+\chi}}\right)^{-\sigma} (R-\delta) = 1$$
$$=> \beta(R-\delta) = 1$$
$$=> R = \beta^{-1} - \delta \approx 1.06$$

Equating (1) and (2):

$$\beta^t \left(c_t - \theta_t \frac{h_t^{1+\chi}}{1+\chi}\right)^{-\sigma} = \lambda_t = \beta^t \left(c_t - \theta_t \frac{h_t^{1+\chi}}{1+\chi}\right)^{-\sigma} \left(\frac{\theta_t h_t^{\chi}}{w_t}\right)$$
$$=> \beta^t \left(c - \theta \frac{h^{1+\chi}}{1+\chi}\right)^{-\sigma} = \beta^t \left(c - \theta \frac{h^{1+\chi}}{1+\chi}\right)^{-\sigma} \left(\frac{\theta h^{\chi}}{w}\right)$$

Since θ =1, we have :

$$=> \frac{\theta h^{\chi}}{w} = 1$$
$$=> h^{\chi} = w$$

In order to to find the steady-state of all endogenous variables, we define the **Capital to Labour ratio** (klr) that is equal to :

$$klr \equiv \frac{k}{h}$$

Then, From (4) and such that A=1,

$$klr = \left(\frac{R-1}{\alpha}\right)^{\frac{1}{\alpha-1}} \approx 12.703$$

From (5) , since A=1 and the value of klr:

$$w = (1-\alpha)klr^{\alpha} = h \approx 1.55$$

Thus, we can compute the level of capital at the steady-state :

$$k = klr * h \approx 19.69$$

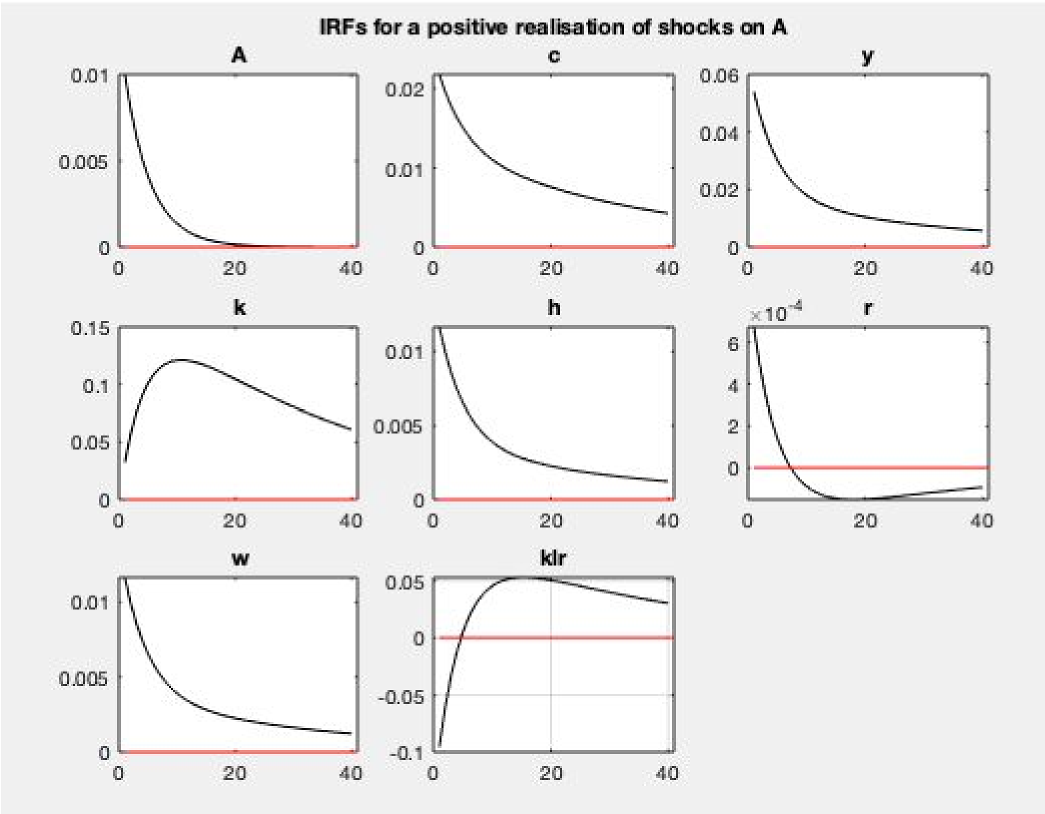
From the resource constraint function, we can compute the level of consumption at the steady-state:

$$y_t = A_t k_t^{\alpha} h_t^{1-\alpha} = c_t + k_{t+1} - k_t + \delta k_t$$
$$=> klr^{\alpha} * h = c + \delta k$$
$$=> klr^{\alpha} * h - \delta k = c \approx 2.60$$

And to finish, we compute output level at the steady-state:

$$y = klr^{\alpha} * h$$
$$y \approx 3.59$$

4) **Use Dynare to calculate the impulse response functions (IRFs) of At, ct, yt, kt, ht, Rt and wt to a positive realisation of εA,t. Employ a first-order approximation of the model and graph the IRFs for 40 periods. Give a detailed economic explanation for the shape of the IRFs.**
Save the mod-file used to generate the IRFs as Q2.mod and save a graph of the IRFs as Q2_1.pdf.



As we can see, with a positive realisation of $\epsilon A, t$, A_t increases instantaneously by construct. It then drops relatively quickly according to the fact that $ln(A_{t+1}) = \rho_A ln(A_t) + \epsilon_{A,t}$. This explains why the other variables return to their steady state.

This in turn has a positive impact on y_t because $y_t = A_t k_t^{\alpha} h_t^{1-\alpha}$. This therefore has a positive impact on k_t through the resource constraint. However the increase is small since R has increased which means that the opportunity cost of investment is therefore increased. k continues to rise for a few periods because R decreases more drastically than y does and then begins to drop when .

Since, as stated in 2), w_t and R_t depend on the marginal productivity of respectively, labour (h_t) and capital (k_t). These marginal productivities (equations (4) and (5)) depend positively on A_t and therefore increase which increases w_t and R_t.

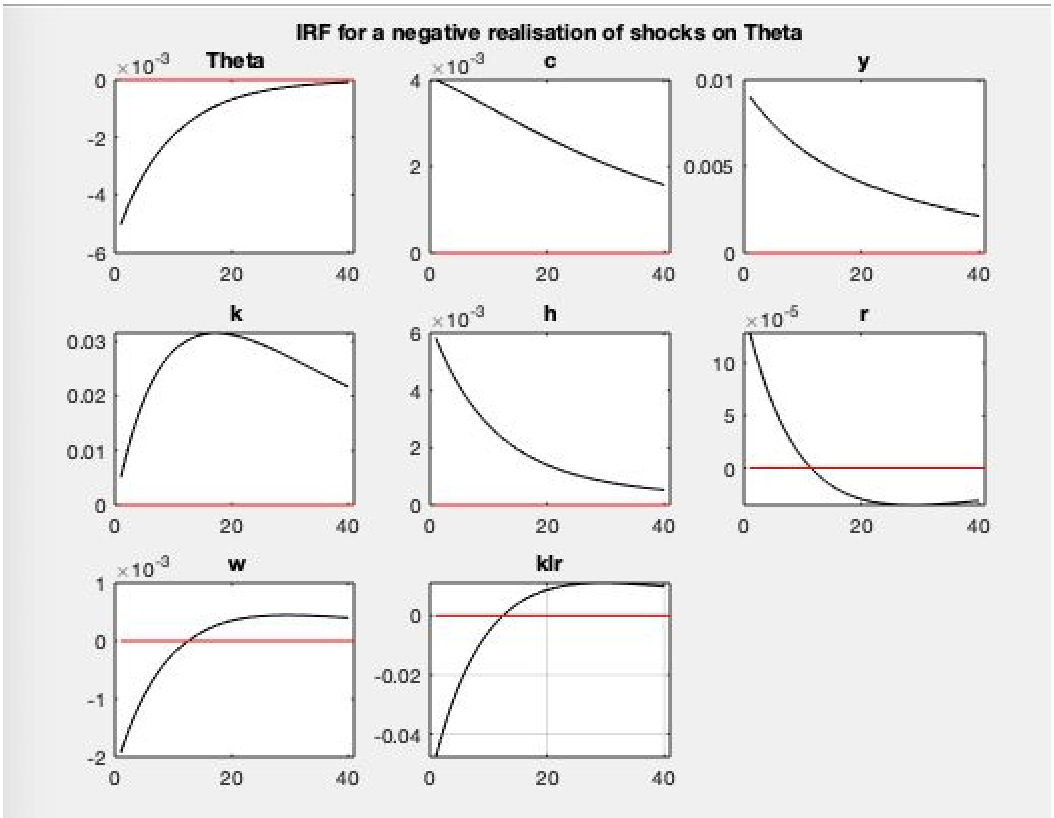
Since wages (w_t) increase, the household can increase its spending (c_t).
Higher wages, make work more attractive which is why h_t increases.

The increase of R_t can be explained through the Euler equation and by the fact that h_t increases more than c_t which means that the ratio of marginal products of consumption between t+1 and t

$$\left(\frac{c_{t+1} - \theta_{t+1} \frac{h_{t+1}^{1+\chi}}{1+\chi}}{c_t - \theta_t \frac{h_t^{1+\chi}}{1+\chi}} \right)^{-\sigma}$$

decreases (consider that the shock happens in t+1), and R_{t+1} must therefore increase for the Euler equation to hold true. Moreover, we can also tell the increasing of R_t is due to the low klr after the shock. Then, the marginal productivity of Capital is larger than his steady-state level and consequently interest rate R_t must increase for the equation (4) stay true. And decreases again because the other variables head towards their steady state but overcompensates because klr_t exceeds its steady state because k_t decreases at a much slower rate than h_t and therefore, for equation (4) to hold true, R_t must also be beneath its steady state value.

5)Calculate the IRFs of θt, ct, yt, kt, ht, Rt and wt to a negative realisation of εθ,t using a first-order approximation. Interpret the shape of the IRFs carefully again. Save a graph of the IRFs as Q2_2.pdf.



Much like before, with a negative realisation of $\epsilon\theta, t$, θ_t decreases instantaneously by construct. It then drops relatively quickly according to the equation: $ln(A_{t+1}) = \rho_\theta ln(\theta_t) + \epsilon_{\theta,t}$. This again, explains why the other variables return to their steady state.

This drop in θ_t means that the household feel less disutility from work and are therefore willing to increase their h_t which brings in more revenue which allows for more consumption(c_t).

This in turn, increases the firm's output y_t which has a positive impact on k_t through the resource constraint by investment. In addition, with an excess of labour supply, the firm can decrease the wage (w_t) on offer. The wage then increases again since all other variables tend to their steady state but it overshoots a bit due to the fact that klr (the ratio of capital to labour) exceeds its steady state because k_t decreases at a much slower rate than h_t and therefore, for equation (5) to hold true, w_t must also be above its steady state value.
R_t behaves in much the same way as in point 4).

6) Compare the response of the real wage in both sets of IRFs calculated above. Does it move in the same direction? Why or why not?

In both sets of IRFs calculated above, the responses of the real wage are different. We can explain this through Labour Market and the productivity of each labour unit or working agent.

With a positive shock on A_t, the productivity of the firm increases which elevates y. Since the productivity of inputs is ameliorated, the Labour demand of the firm increase and so, the firm is ready to offer larger real wage (w_t) than before. But with this increase of wage, the opportunity cost of not working becomes too high and agent is going to work more than at steady-state. Therefore, there are more and more agents (or simply units of labour) available in the labour market which leads to a decline in productivity (since $\frac{d^2 y_t}{dh_t^2} < 0$) and consequently, wages also begin to decrease.

On the other hand, with a negative shock on (θ) which we can interpret as a decrease of revenue tax (or as any sort of phenomenon which makes work more enjoyable or beneficial), agents want to work more. Then, initially, the labour offered by agent increases and is higher than the labour demand of the firm. Consequently, firms have more power on wages and the productivity of agent is too low and so, wages are lower than the steady-state level. But after this initial period, Capital (k_t) increases higher than labour and labour becomes more productive because the marginal productivity of labour which also equals w is proportional to klr which increases with (by equation (5)).

To sum up, no, the wage IRFs move in the opposite direction since with a shock on A, wages are boosted because labour is more productive and with a shock on θ, labour becomes more abundant because it displeases workers less wich by law of supply and demand (and of equation (5)), decreases w.