Problem Set

Quantitative Macroeconomics

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Please submit your answers no later than ***Monday, 16 December, 11.00 (am)****. All computer code (Q1.m, Q2.mod) and figures (Q1_1.pdf, Q2_1.pdf, Q2_2.pdf) must be sent by email to seda.basihos@unil.ch and andreas.tischbirek@unil.ch. The written part of the responses can also be submitted by email, or as a hard copy (on Monday, 17 December, 9.30am - 11.00am, Internef 506). The problem set may be completed in a group of no more than two. Please state clearly who is part of your group when submitting work.*

1 Stochastic Booms and Beliefs

Consider an economy in which occasionally large booms of stochastic length occur, for example, due to sudden technological advancements. Let the indicator $s_t \in \{0, 1\}$ describe the state of the economy in period $t = 1, 2, \ldots, T$. If the economy is in a boom phase at time t, then $s_t = 1$, and $s_t = 0$ otherwise. Assume that $T = 500,000$.

In period $t = 1$, the economy is not in a boom, i.e. $s_1 = 0$. For all periods $t > 1$, $\Pr(s_t = 0|s_{t-1} = 0) = 0.9$ and $\Pr(s_t = 1|s_{t-1} = 1) = 0.2$, i.e. the economy remains out of a boom with probability 0.9 if it was not in a boom in the previous period and it stays in a boom with probability 0.2 if it was in a boom in the last period. Consequently, the probabilities of state changes are $\Pr(s_t = 1|s_{t-1} = 0) = 0.1$ and $\Pr(s_t = 0|s_{t-1} = 1) = 0.8$.

Write an m-file in Matlab to answer the questions below and save it as Q1.m.

1. Generate a $T \times 1$-vector u where each element is drawn from the standard uniform distribution.

2. Calculate the mean of the vector u.

3. Simulate the stochastic process $\{s_1, s_2, \ldots, s_T\}$. *(Hint—You can use u_t to determine whether the state switches in t or not.)*

4. Calculate the share of time periods in which the economy is in a boom, i.e. the mean of $\{s_1, s_2, \ldots, s_T\}$.

Suppose now that there is a large number of households in the economy. In each period, only a share θ of the households observes the current state s_t. All remaining households simply assume that the probability of being in a boom is given by the aggregate belief in the last period. Hence, the aggregate belief about being in a boom in period t, denoted e_t, evolves according to

$$e_t = \theta s_t + (1 - \theta)e_{t-1}$$

5. Using your result from 3., simulate the process $\{e_1, e_2, \ldots, e_T\}$ for $\theta = 0.5$ and calculate its mean. Plot the first 80 periods of both $\{s_1, s_2, \ldots, s_T\}$ and $\{e_1, e_2, \ldots, e_T\}$ in the same figure. Label the figure appropriately. *Save the figure as Q1_1.pdf.*

2 Labour and Business Cycles

Let an economy contain two representative agents, a household and a firm.

The profit-maximising firm is a price taker in the markets for its inputs and sells the goods that it produces in a fully competitive market. In each period t, output y_t depends on the prevailing level of technology A_t as well as the amount of capital k_t and labour h_t used as inputs. The production technology is described by

$$y_t = A_t k_t^{\alpha} h_t^{1-\alpha}$$

with $0 < \alpha < 1$.

The household maximises expected utility

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} \left(c_t - \theta_t \frac{h_t^{1+\chi}}{1+\chi} \right)^{1-\sigma}$$

subject to the budget constraint

$$c_t + k_{t+1} = w_t h_t + (R_t - \delta) k_t$$

Above, c_t is consumption, w_t is the real wage, and R_t is the gross real interest rate. $0 < \beta < 1$, $\sigma > 1$, $\chi \geq 0$, and $0 < \delta < 1$ are parameters.

The evolution of A_t and θ_t is given by

$$\begin{aligned} \ln A_t &= \rho_A \ln A_{t-1} + \varepsilon_{A,t} \\ \ln \theta_t &= \rho_\theta \ln \theta_{t-1} + \varepsilon_{\theta,t} \end{aligned}$$

with $\varepsilon_{A,t} \sim N(0, \sigma_A^2)$, $\varepsilon_{\theta,t} \sim N(0, \sigma_\theta^2)$, $0 < \rho_A < 1$, and $0 < \rho_\theta < 1$.

Employ the following parameterisation in your model simulations.

$$\begin{aligned} \beta &= 0.99 \\ \sigma &= 2 \\ \chi &= 1 \\ \alpha &= 0.33 \\ \delta &= 0.05 \\ \rho_A &= 0.8 \\ \sigma_A &= 0.01 \\ \rho_\theta &= 0.9 \\ \sigma_\theta &= 0.005 \end{aligned}$$

1. Which economic interpretation can you give for θ_t?

2. Derive the first-order conditions of the household's utility maximisation problem and the firm's profit maximisation problem.

3. Calculate the steady state of the model. *(Recall that it can be useful to introduce auxiliary variables in doing so)*

4. Use Dynare to calculate the impulse response functions (IRFs) of A_t, c_t, y_t, k_t, h_t, R_t and w_t to a positive realisation of $\varepsilon_{A,t}$. Employ a first-order approximation of the model and graph the IRFs for 40 periods. Give a detailed economic explanation for the shape of the IRFs.
   Save the mod-file used to generate the IRFs as Q2.mod and save a graph of the IRFs as Q2_1.pdf.

5. Calculate the IRFs of θ_t, c_t, y_t, k_t, h_t, R_t and w_t to a negative realisation of $\varepsilon_{\theta,t}$ using a first-order approximation. Interpret the shape of the IRFs carefully again.
   Save a graph of the IRFs as Q2_2.pdf.

6. Compare the response of the real wage in both sets of IRFs calculated above. Does it move in the same direction? Why or why not?