act 10: Product and system reliability

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# Objectives

Estimate various measures of reliability of a product or system.

# Introduction

**Definition of reliability**

The reliability of a product or system is the probability that the system operates under the specifications or stated conditions.

# Measures of interest

Given a sample and specified model, we wish to measure the following important characteristics in a reliability study:

* Estimate the reliability (survivor) function
* Estimate the failure rate function
* Determine the mean time to failure (MTTF)
* Evaluate the mean remaining or residual life (MRL)

# Probability Models for Reliability

The most popular models for the study of reliability of life data are

* Exponential Distribution
* Gamma Distribution
* Weibull Distribution

Due to the popularity of the Weibull model, the analysis of life data is often referred to as a Weibull Analysis.

# Basic concepts

## Time to Failure

Let be the time-to-failure of a product or system, measured in hours, and be its probability density function.

The probability that the product fails within the time interval is the cumulative density function (CDF) defined by

## Reliability Function

The reliability function, , is the probability that the product keeps operating beyond the time interval , and is defined by

In other words, is the measure of probability that a time old product is still functioning.

The reliability function is also called the survivor function.

# Failure Rate Function

The failure rate function is the rate at which a time old item fails in the next instant of time, and is given by

Because , we have

Then,

which implies that

The above equation can be used to compute reliability function given the failure rate function.

# Mean time to failure (MTTF)

The mean time to failure (MTTF) of a product is defined by

By the fact that , it follows that

By partial integration formula

it follows

It can be shown that when , then , of course, then

# Mean Residual Life

The mean residual (or, remaining) life, , of the item at age is defined by

When , then the item is new, and we have

# Applications

**Example 1**

The time to failure of an item (put into operation at time ) has probability density function given by

This distribution of is called an exponential distribution with parameter .

1. Find the probability that the item fails over the interval .
2. Find the reliability function . Find the value of the reliability function at .
3. Find the failure rate function .
4. Find the MTTF of the item.
5. Find the mean residual (or, remaining) life of the item at and at .
6. Compute the probability that the item will survive to its MTTF.

Solution

1. Note that
2. . Then,
3. . Alternately, . Therefore, the failure rate function of an item with exponential life distribution is constant (i.e., independent of time).
4. , which is independent of . Therefore, .

The of an item with exponential life distribution is equal to its , irrespective of the age of the item. Therefore, the item is as good as new as long as it is functioning, and we often say that the exponential distribution has no memory.

This essentially means that a used item is stochastically as good as new, so there is no reason to replace a functioning item.

# Estimating parameter of

Given a sample from an exponential distribution with an unknown rate parameter , we wish to estimate the parameter via the maximum likelihood method (MLE). The likelihood function

Taking log on both sides:

Take the derivative of w.r.t. and set equal to zero:

**Example 2**

Read the *reliAct10* dataset uploaded in Canvas into R.

This dataset contains two columns and . Data vector in one of the columns is believed to come from an exponential distribution with an unknown rate parameter . We wish to test which data vector is a potential candidate to come from an exponential distribution by estimating the rate parameter via MLE method and testing if data actually fits an exponential distribution via goodness of fit test.

To proceed for the task, assume that both and comes from distribution, and then do the following activities:

1. Estimate the mean of the hypothesized exponential population via MLE method (invariance property of MLE applies here).
2. Estimate the rate parameter for both and columns of data via the maximum likelihood method.
3. Use the estimated rate parameters to fit the data to an exponential distribution for both data vectors.
4. Test the fitted distributions if they come from exponential distributions with estimated rates in (a).
5. On the basis of the test results in (d), which column do you recommend coming from an exponential distribution?
6. Draw a histogram of the seemingly data vector and superimpose a true exponential distribution into the histogram.
7. Give an estimate of the survival function for the best fitted model at .

#require(vcd)  
require(MASS)

Loading required package: MASS

# reading data  
dat=read.csv("reliAct10.csv", header=T)  
names(dat)

[1] "x" "y"

x1=dat$x  
x2=dat$y  
#(a)  
mx1=mean(x1)  
mx2=mean(x2)  
c(mx1,mx2)

[1] 1.245357 1.458794

#(b)  
rx1=1/mx1  
rx2=1/mx2  
c(rx1,rx2)

[1] 0.8029823 0.6854978

#(c) fit data to Exp(lambda) via fitdistr() function in package MASS  
fit1 <- fitdistr(x1, "exponential")   
fit2 <- fitdistr(x2, "exponential")  
fit1

rate   
 0.80298233   
 (0.02539253)

fit2

rate   
 0.68549784   
 (0.02167735)

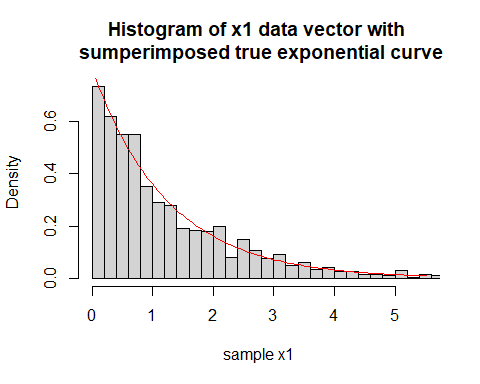
# note that rates from (b) and (c) are very close!  
  
#(d) GOF test to fitted models via ks.test  
ks.test(x1, "pexp", fit1$estimate)

One-sample Kolmogorov-Smirnov test  
  
data: x1  
D = 0.022327, p-value = 0.7011  
alternative hypothesis: two-sided

ks.test(x2, "pexp", fit2$estimate)

One-sample Kolmogorov-Smirnov test  
  
data: x2  
D = 0.11484, p-value = 7e-12  
alternative hypothesis: two-sided

#(e) Test results show that x1 (i.e., the first column of the given dataset ) comes from an exponential distribution with rate rx1.   
  
#(f) plot a graph  
hist(x1, freq = FALSE, breaks = 50, xlim = c(0, quantile(x1, 0.99)), main="Histogram of x1 data vector with \n sumperimposed true exponential curve", xlab="sample x1")  
curve(dexp(x, rate = fit1$estimate), from = 0, col = "red", add = TRUE)



# R basic functions

The true and of distribution can be computed directly using **pexp(t, rate=lambda)** and **pexp(t, rate=lambda, lower.tail=F)** functions, respectively:

**Remarks:**

An exponential distribution is a popular life distribution in applied reliability analysis because of its mathematical simplicity and interpretation.

# Useful results

Consider a system of two independent components with failure rates and , respectively. The probability that component 1 fails before component 2 is

This result can easily be generalized to a system of independent components with failure rates . The probability that component is the first component to fail is

**Example 3**

A system consists of two independent components with failure rates and , respectively.

Find the probability that the component 1 fails before the component 2.

**Solution**

Let and be the life of failure of the two components, respectively. Then,

**Example 4**

Time to failure, , of LED lamps follow an exponential distribution with mean 50,000 hours.

1. Find the reliability of lamps over a year, assuming that 1 year= 365 day or 8,760 hours.
2. Find the reliability of lamps at hours 50,000.

**Solution**

Let of LED lamps follows an distribution with pdf given by

.

Given since .

1. Reliability of at age is

Thus, the reliability over a year is 83.9%.

1. The reliability at hours 50,000 is

Thus, the reliability at hours 50,000 is 36.8%.

#F(t) and R(t) for exp(lamda) can be computed directly using pexp(t, rate=lambda) and pexp(t, rate=lambda, lower.tail=F), functions respectively:  
  
#(a)  
pexp(8760, rate=1/50000, lower.tail=F)

[1] 0.8392891

#(b)  
pexp(50000, rate=1/50000, lower.tail=F)

[1] 0.3678794