hw10: Product and system reliability

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**Problem 1**

The time of failure of a certain LED lamp follows an exponential distribution with mean 75,000 hours.

1. Write an expression of the pdf for the distribution of .
2. Find the probability that the item fails over the interval .
3. Find the reliability function . Find the value of the reliability function at .
4. Find the failure rate function and its value when .
5. Find the MTTF of the item.
6. Find the mean residual (or, remaining) life of the item at and at .
7. Compute the probability that the item will survive to its MTTF.

Solution:

* 1. Note that
  2. . Then,
  3. . Alternately, . Therefore, the failure rate function of an item with exponential life distribution is constant (i.e., independent of time).
  4. , which is independent of . Therefore, .

The of an item with exponential life distribution is equal to its , irrespective of the age of the item. Therefore, the item is as good as new as long as it is functioning, and we often say that the exponential distribution has no memory.

This essentially means that a used item is stochastically as good as new, so there is no reason to replace a functioning item.

(g)

**Problem 2**

Read the *reliHW10* dataset uploaded in Canvas into R.

This dataset contains three columns , and . Data vectors in one or two columns are believed to come from an exponential distribution with an unknown rate parameter . We wish identify data vectors that come from exponential distributions by estimating the rate parameters via MLE method and test if data actually fits an exponential distribution via goodness of fit test.

As a first step to proceed, we assume that , and all come from distribution, and then do the following activities:

1. Estimate the mean of the hypothesized exponential population via MLE method (invariance property of MLE applies here).
2. Estimate the rate parameter for all columns of data via the maximum likelihood method.
3. Use the estimated rate parameters to fit the data to an exponential distribution for all data vectors.
4. Test the fitted distributions if they come from exponential distributions with estimated rates in (b).
5. On the basis of the test results in (d), identify any column that comes from an exponential distribution, and make an appropriate conclusion.
6. Draw a histogram of the seemingly data vector and superimpose a true exponential distribution into the histogram.
7. Give an estimate of the survival function for the best fitted model at .

**Solution**

**#(a)**

dat=read.csv("relihw10.csv", header=T)

names(dat)

[1] "x" "y" "z"

x1=dat$x

x2=dat$y

x3=dat$z

mx1=mean(x1)

mx2=mean(x2)

mx3=mean(x3)

c(mx1,mx2,mx3)

[1] 14.910930 1.495566 12.394845

#(b)

rx1=1/mx1

rx2=1/mx2

rx3=1/mx3

c(rx1,rx2,rx3)

[1] 0.0670649 0.6686432 0.0806787

#(c) fit data to Exp(lambda) via fitdistr() function in package MASS

fit1 <- fitdistr(x1, "exponential")

fit2 <- fitdistr(x2, "exponential")

fit3 <- fitdistr(x3, "exponential")

fit1

rate

0.067064898

(0.002120778)

fit2

rate

0.66864323

(0.02114436)

fit3

rate

0.080678704

(0.002551285)

#(d) GOF test to fitted models via ks.test

ks.test(x1, "pexp", fit1$estimate)

One-sample Kolmogorov-Smirnov test

data: x1

D = 0.11557, p-value = 5.006e-12

alternative hypothesis: two-sided

ks.test(x2, "pexp", fit2$estimate)

One-sample Kolmogorov-Smirnov test

data: x2

D = 0.14147, p-value < 2.2e-16

alternative hypothesis: two-sided

ks.test(x3, "pexp", fit3$estimate)

One-sample Kolmogorov-Smirnov test

data: x3

D = 0.018746, p-value = 0.8737

alternative hypothesis: two-sided

#(e) Test results show that x1 (i.e., the first column of the given dataset ) comes from an exponential distribution with rate rx1.

#(f) plot a graph

hist(x1, freq = FALSE, breaks = 50, xlim = c(0, quantile(x1, 0.99)), main="Histogram of x1 data vector with \n sumperimposed true exponential curve", xlab="sample x1")

curve(dexp(x, rate = fit1$estimate), from = 0, col = "red", add = TRUE)

Chart, histogram

Description automatically generated

**Problem 3**

A system consists of two independent components and $C\_$ which follow exponential distributions with rates and , respectively.

Find the probability that the component fails before the component .

**Solution**

Let and be the life of failure of the two components, respectively. Then,

= 0.5517241