Chapter 7

hypothesis Testing:

One-sample inference

**7.1 Introduction**

**Objectives:**

1. In this chapter we will perform one sample inference for

* mean,
* proportion and
* variance

via one sample tests of hypotheses.

1. Implement SAS for performing one-sample inference given **summary statistics** and **dataset**.

**Problem specifications**

Given a sample, we wish to test hypothesis regarding any parameter of the form:

The form of hypothesis in relation to the unknown mean (), variance () and proportion () can be specified as follows:

against three possible alternative hypotheses. For example, for , the three possible alternatives are

They are called right-tailed (upper-tailed), left-tailed (lower-tailed) and two-tailed tests, respectively.

Similar alternative hypotheses can be set for variance () and proportion ().

The null hypothesis is a statement of equality such as , or no difference such as .

An ***alternative hypothesis*** *is any of the three sensible hypotheses specified above, which contradicts the* .

**Objectives**

We wish to either accept or reject the null hypothesis against the alternative given the sample evidence.

**Example 7.1**

We wish to test whether the population of patients under a certain condition has the mean diastolic blood pressure of (DBP) of 81 mmHg or not by setting

versus the tree alternatives:

**Example 7.2**

As per CDC, 14% of all cancer diagnoses cases are lung cancer patients. Is this true in a population with a particular ethnic group? To test that, a researcher may set up the null and alternative hypotheses as follows:

versus

**Example 7.3**

Given the previous knowledge, the variance of DBP of a population is 13 mmHg. A researcher having been doubtful about variance, wises to test if it is different from 13 mmHg. Therefore, the researcher may set up the null and alternative hypotheses for the test as follows:

**Errors while making the decision**

While making the decision as to whether accept or reject the null hypothesis, one may commit two types of error.

**Type I error and level of significance**

To **reject** when it is actually **true** iscalled theType I error. The probability of Type I error is called the level of significance, and is denoted by , i.e.,

How often do we wish to make this error? Of course, very rarely, and therefore, we set to be small, may be 10%, 5% and 1%, by specifying, and , respectively.

**Type II error and power of a test**

To accept when it is actually false is called the Type II error. The probability of Type II error is denoted by .

The power of a test is the probability of rejection of the null hypothesis when it is actually false. In other words, the power is probability of the complement of Type II error:

**What is an inference in a TOH?**

An inference in any TOH means to either **reject or accept the null hypothesis** given the sample evidence and a specified significance level .

**One sample inference for mean, proportion and variance**

Sections 7.3/7.4/7.9 covers tests for unknown mean and proportion by implementing Z and T tests, whereas section 3.8 is about the test for variance.

[Sections 7.3/7.4] Given sample of size , from a population with mean unknown mean and standard deviation , we wish to test .

The test statistic for the test is

* when is known, or
* when is unknown.

where and are the sample mean and sample standard deviation, respectively.

[Section 7.9] The test of is given by

For a better approximation, with a continuity correction, the test given by

It also follows that or (see section 7.9 page, 249 for more details).

[Section 7.8] The test statistic to test is given by

**Assumptions of the Z/T test:**

1. The Z test assumes that the sample comes from the normal population with standard deviation is known.
2. The T test assumes that either the population the sample comes from is normal or the sample is large (i.e., ) and is unknown.
3. The Z-test for proportion assumes that

**P-value and relation with**

For an inference of acceptance or rejection of the null hypothesis, we use p-value computed using the test statistic and the direction of the alternative.

**What is a p-value?**

Given an observed value or t of the test statistic or T, the ***-*value** is the probability that the test statistic takes values beyond or t in the ***direction of alternative***, assuming that the null hypothesis is true. Therefore, the p-value computed as follows:

1. -value or if (or for proportion test)
2. -value or if (or for proportion test)
3. -valueor if (or for proportion test)

Given the observed value of the test statistic , the p-value for is computed by

1. if
2. if

**Decision Rule**

The lower the p-value, the stronger is the evidence against the null hypothesis.

How low is the low? The cut-off value is what is specified by .

For a given ,

1. Reject if -value.
2. Accept or fail to reject if -value).

**Computing p-value using SAS**

Let z be the observed value of the Z statistic. Then, the p-value for Z-test is computed in SAS as follows:

pL=**cdf**(“normal”,z) if

pR=**sdf**(“normal”,z) if

pT=2\***sdf**(“normal”,abs(z)) if

Let t be the observed value of the T statistic. Then, the p-value for Z-test is computed in SAS as follows:

pL=**cdf**(“t”,t, df) if

pR=**sdf**(“t”, t, df) if

pT=2\***sdf**(“t”,abs(t),df) if

**/\*sdf=1-cdf=survival distribution function gives right area or right-tailed probability\*/**

Let chi2obs be the observed value of the statistic. Then the p-value is computed as follows:

pL=**cdf**(“chisq”, chiobs, n-1) if

pR=**sdf**(“chisq”, chiobs, n-1) if

pT=**2**\*min(pL,pR)if

**/\*sdf=1-cdf=survival distribution function gives right area or right-tailed probability\*/**

Notes:

* **cdf(“f”,f, df1,df2)** or **sdf(“f”,f, df1,df2)** computes left- or right-tailed probability of F(df,df2) distribution.
* **cdf(“chisq”,chi2, df)** or **sdf(“chisq”,chi2, df)** computes left- or right-tailed probability of chisq(df) distribution.
* **cdf(“poisson”,x, meanx)** or **sdf(“poisson”,x, means)** computes left- or right-tailed probability of poisson(meanx) distribution.

**Example 7.4 (Activity 7.1)**

Do mothers with low socio-economic status (SES) deliver babies with lower birthweight than the average birth weight in the United States? To test it, birthweights of a sample of 100 full-term babies delivered in a hospital for mothers with a low-SES status is considered. The mean birthweight () is found to be **115** oz. It is known that the mean birthweight in the United States is 120 oz. Does the sample provide any evidence that the mean birthweight () in this hospital is lower than the mean in the United States?

Assume that the birthweights in the specified hospital is normally distributed with unknown mean and standard deviation () of 24 oz.

1. Set up the null and alternative hypotheses for this problem.
2. What test statistic should you use for this test and why?
3. Find the value of the test statistic and p-value using SAS coding.
4. Make a conclusion about the test at 5% level of significance.

Solution

We have oz, oz and

1. We wish to test vs .
2. The test statistic is .
3. z=? -value
4. Conclusion?

**data** exp4;

input xbar mu0 sigma n;

z=(xbar-mu0)/(sigma/sqrt(n));

pval=cdf("normal",z,**0**,**1**);

drop xbar mu0 sigma n;

cards;

115 120 24 100

;

**run**;

**proc** **print** data=exp4 noobs;

title"Value of the TS and p-value for exp4";

**run**;

|  |
| --- |
| Value of the TS and p-value for exp4 |

| **z** | **pval** |
| --- | --- |
| -2.08333 | 0.018610 |

**Example 7.5 (Act 7.2)**

Given a sample of 45 payments, the average payment for a cesarean childbirth is $16,500, with a standard deviation of $3,600. Assume that population of payment for a cesarean childbirth is normal with unknown mean and unknown standard deviation. Does the given information provide any statistical evidence that the average payment for all cesarean childbirth is $16,000?

1. What test statistic would be reasonable for this test?
2. Report value of the test statistic and compute p-values for three possible alternatives (i) , (ii) and (iii)
3. Make conclusions about the test against the three given alternatives of the test at .

**Solution**

1. A t-test given by
2. t=?, pL=?, pR=? and pB=?
3. Conclusion ?

**data** exp5;

input xbar mu0 s n;

t=(xbar-mu0)/(s/sqrt(n));

pL=cdf("t",t,n-**1**);

pR=**1**-pL;

pB=**2**\*sdf("t",abs(t),n-**1**);

drop xbar mu0 s n;

cards;

16500 16000 3600 45

;

**run**;

**proc** **print**;

title"Value of the TS and three p-values for exp5";

**run**;

|  |
| --- |
| Value of the TS and three p-values for exp5 |

| **Obs** | **t** | **pL** | **pR** | **pB** |
| --- | --- | --- | --- | --- |
| **1** | 0.93169 | 0.82171 | 0.17829 | 0.35658 |

**Example 7.6 (Act 7.3)**

It is believed that the rate of positivity of Covid-19 at a given city is below 6%. Given a sample of 742 people tested for Covid-19, 47 people have been tested positive in the city on a given day. Does the sample provide any evidence to support the belief at 5% level of significance? Answer the following question in regard to this test.

1. What test statistic should you use?
2. Report value of the test statistic and compute p-values for three possible alternatives (i) , (ii) and (iii)
3. Make conclusions about the test against the three given alternatives of the test at .

**data** exp6;

input n x p0;

phat=x/n;

zc=(abs(phat-p0)-**1**/(**2**\*n))/sqrt((p0\*(**1**-p0)/n));

pLzc=cdf("normal",zc);

pRzc =**1**-pLzc;

pBzc =**2**\*sdf("normal",abs(zc));

chi2=zc\*\***2**;

pLchi=cdf("chisq",chi2, **1**);

pRchi=sdf("chisq",chi2, **1**);

pBchi =**2**\*min(pLchi, pRchi);

drop n x p0;

cards;

742 47 0.06

;

**run**;

**proc** **print** data=exp6 noobs;

title "Results of Prop Test via Z and Chisq test";

**run**;

|  |
| --- |
| Results of Prop Test via Z and Chisq test |

| **phat** | **zc** | **pLzc** | **pRzc** | **pBzc** | **chi2** | **pLchi** | **pRchi** | **pBchi** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0.063342 | 0.30607 | 0.62023 | 0.37977 | 0.75955 | 0.093680 | 0.24045 | 0.75955 | 0.48090 |

Notes:

* **cdf(“f”,f, df1,df2)** or **sdf(“f”,f, df1,df2)** computes left- or right-tailed probability of F(df,df2) distribution.
* **cdf(“chisq”,chi2, df)** or **sdf(“chisq”,chi2, df)** computes left- or right-tailed probability of chisq(df) distribution.
* **cdf(“poisson”,x, meanx)** or **sdf(“poisson”,x, means)** computes left- or right-tailed probability of poisson(meanx) distribution.

**\*The SDF(y) function returns the probability that a discrete distribution takes value greater than**y**.**

**Section 7.5: Relation between hypothesis testing and CI**

1. Reject a null hypothesis at significance level if CI [] does not contain , otherwise accept the null hypothesis.
2. Similar, approaches apply to test for any parameter. (See page

See section 7.5 for details.

**SAS Implementation for TOH using CIs given a dataset**

SAS proc univariate can be used for implementing tests of mean, variance and standard deviation via confidence interval estimates. It follows from the fact that if confidence interval of any parameter captures the hypothesized value specified in the null statement, then the test is insignificant, i.e., we fail to reject the null hypothesis.

**Example 7.7 (Act 7.4)**

Given a sample of males with their ages, in years, rounded to the nearest integer, at the diagnosis of diabetes in a health care center for a specified ethnic group of people.

39 29 41 41 42 49 40 33 37 40 39 38 45 43 37

Assume that the age of diagnosis of diabetes of males in the population follows a normal distribution with unknown mean and unknown standard deviation. A previous study concluded that the mean age, , of diagnosis of diabetes is 39 years. Does the sample provide any evidence to believe that the mean age of diagnosis of diabetes is indeed 30 years? Answer the following questions in regard to the test.

1. We wish to test . Construct a 95% confidence interval for and make a conclusion about the hypothesis against the two-sided alternative at significance level . What confidence interval have you used and why?
2. Find the 95% confidence interval for andWhat conclusion can you make in regard to the hypotheses and on the basis of your 95% confidence interval against the two-sided alternative and ?

**Solution**

**data** exp7;

Input age @@;

Cards;

39 29 41 41 42 49 40 33 37 40 39 38 45 43 37

;

**Run**;

title "CIs for mean, variance and std towards TOH";

**proc** **univariate** data= exp7 cibasic alpha=**0.05** mu0=**39**;

ods select BasicIntervals;

var age;

title "CIs for mean, variance and std towards TOH";

**run**;

|  |
| --- |
| CIs for mean, variance and std towards TOH |

Variable: age

| **Basic Confidence Limits Assuming Normality** | | | |
| --- | --- | --- | --- |
| **Parameter** | **Estimate** | **95% Confidence Limits** | |
| **Mean** | 39.53333 | 36.92017 | 42.14649 |
| **Std Deviation** | 4.71876 | 3.45473 | 7.44195 |
| **Variance** | 22.26667 | 11.93514 | 55.38257 |

Since , and are contained in the corresponding 95% Cis, we fail to reject the null hypotheses of associated parameters at 5% level of significance.

**Example 7.8 (Act 7.5. Ref: Example 7.47 page247)**

It appears that n=10, . We wish to test against three alternatives (i) , (ii) and (iii .

1. What test statistic should you use?
2. Find the value of the test statistic.
3. Find p-values for the test for alternatives (i)-(iii).
4. What are the conclusions from (c) regarding rejection or acceptance of the null hypothesis at 5% level of significance?

Solution

1. The observed value of the test statistic is

pL=Pr(

pR=Pr(

pT=2\*min{pL, pR)=2\*0.0103=0.0206

1. (i) Reject H0, (ii) Accept H0, and (iii) Reject H0 at 5% level of significance.

**SAS Implementation**

**data** exp8;

input n s2 sigma02;

chi2=(n-**1**)\*s2/sigma02;

pL=cdf("chisq",chi2,n-**1**);

pR=sdf("chisq",chi2,n-**1**);

pT=**2**\*min(pL,pR);

cards;

10 8.178 35

;

**run**;

**proc** **print** data=exp8;

title "Results of Chisq test for specified variance";

**run**;

|  |
| --- |
| Results of Chisq test for specified variance |

| **Obs** | **n** | **s2** | **sigma02** | **chi2** | **pL** | **pR** | **pT** |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **1** | 10 | 8.178 | 35 | 2.10291 | 0.010266 | 0.98973 | 0.020533 |