**Stat 468/568 Biostatistics**

**Total points 15**

**Due 01/25/2022**

**Last Name: First Name:**

**Student class:** Graduate/Undergrad (select one as applies)

***Special Instruction: No extension of the due date is allowed to any particular student. However, if an extension is required, whole class will be allowed an extension. Start doing any homework early to allow significant amount of time to work on it depending on your need.***

1) Two events A and B are independent if P(AB)=P(A)\*P(B) [definition provided in text and discussed in the class). **Prove that if two events A and B are independent then**

1. P(A|B)=P(A)

Proof: P(A|B)=P(AB)/P(B)=P(A)\*P(B)/P(B)=P(A)

1. P(B|A)=P(B)

Proof: P(B|A)=P(AB)/P(A)=P(A)\*P(B)/P(A)=P(B)

2) [**Only graduate students need to prove this problem, BUT others should know the conclusions of the problem to solve problems using these properties**].

If two events A and B are independent, then prove that

1. Events and B are independent.
2. Events A and are independent.
3. Events and are independent.

Proofs:

Because and are independent, . Also, , and similarly, .

Then,

1. .
2. .
3. .

**Alternative proofs:**

If two events A and B are independent, then

1. by the **Law of Total Probability (LTP**)

1. by the **LTP**

1. By the LTP:

**Alternative:**

3) From the conclusion of problems 1 and 2 above, note that if two events A and B are **independent** then, we have the following six results in (i)-(vi):

1. P(AB)=P(A)\*P(B)
2. P(A|B)=P(A)
3. P(B|A)=P(B)
4. Events and B are independent.
5. Events A and are independent.
6. Events and are independent.

Use facts in (i)-(vi) as required **to compute probabilities in (a)-(k),** wheremultiple problems might have the same answer when they refer to identical events.

Two events A and B are independent with probabilities P(A)=0.31 and P(B)=0.30. **Compute the following probabilities**:

1. P( )=1-P(A)=1-0.31=0.69
2. P( ) =1-P(B)=1-0.30=0.70
3. P(A and B)=P(A)\*P(B)=0.31\*0.30=0.093
4. P()= P(A)\*P(B)=0.31\*0.30=0.093
5. P(A|B)=P(A)=0.31
6. P(B|A)=P(B)=0.30
7. P( and )= P( ) \*P( )=0.69\*0.70=0.483
8. P()= P( ) \*P( )=0.69\*0.70=0.483
9. P(A or B) =P(A)+P(B)-P(A and B)=0.31+0.30-0.093= 0.517
10. P(|B)= P( )=0.69
11. P()=P(B)=0.3

4) Among 1820 subjects in a study, 30 suffered from tuberculosis and 1790 did not. Chest x-rays were administered to all individuals; 73 had a positive x-ray, whereas other 1747 were negative. The results of the study are presented in the table below:

|  |  |  |  |
| --- | --- | --- | --- |
|  | Tuberculosis | |  |
| X-ray | No: | Yes: | Total |
| Negative: | 1739 | 8 | 1747 |
| Positive: | 51 | 22 | 73 |
| Total | 1790 | 30 | 1820 |

If we let = {event that individual is suffering from tuberculosis) and = {event that individual does not have tuberculosis}. Similarly, and refer to events with positive and negative x-ray results. Given these facts, compute the following probabilities:

Solutions

1. False positive of x-ray
2. False negative of x-ray
3. Sensitivity of x-ray
4. Specificity of x-ray