PHYS 20323/60323: Fall 2024 - LaTeX Example

1. An electron is found to be in the spin state (in the z-basis): $\chi = A \begin{pmatrix} 3i \\ 4 \end{pmatrix}$

   (a) (5 points) Determine the possible values of A such that the state is normalized.

   (b) (5 points) Find the expectation values of the operators S_x, S_y, S_z, and $\vec{S}^2$.

   The matrix representations in the z-basis for the components of electron spin operators are given by:

   $$\mathbf{S_x} = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \qquad \mathbf{S_y} = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \qquad \mathbf{S_y} = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

2. The average electrostatic field in the earth's atmosphere in fair weather is approximately given:

   $$\vec{E} = E_0(Ae^{-\alpha z} + Be^{-\beta z})\hat{z} \tag{1}$$

   where A, B, α, β are positive constants and z is the height above the (locally flat) earth surface.

   (a) (5 points) Find the average charge density in the atmosphere as a function of height

   (b) (5 points) Find the electric potential as a function height above the earth.

3. **The following questions refer to stars in the Table below.**
   Note: There may be multiple answers.

Name	Mass	Luminosity	Lifetime	Temperature	Radius
β Cyg.	1.3 $M_\odot$	3.5 $L_\odot$			
α Cen.	1.0 $M_\odot$				1 $R_\odot$
η Car.	60. $M_\odot$	10^6 $L_\odot$	8.0×10^5 years		
ε Eri.	6.0 $M_\odot$	10^3 $L_\odot$		20,000 K	
δ Scu.	2.0 $M_\odot$		5.0×10^8 years		2 $R_\odot$
γ Del.	0.7 $M_\odot$		4.5×10^{10} years	5000 K	

   (a) (4 points) Which of these stars will produce a planetary nebula.

   (b) (4 points) Elements heavier than Carbon will be produced in which stars.