

The "Posterior"

The "Likelihood"

The "Prior"

$$P(H|E) = \frac{P(E|H)P(H)}{P(E)}$$

The "Marginal" $P(E)$

Topic: **Bayesian VAR models**

Author: **Liana Isayan**

Bayesian Statistics, the 2nd Midterm

What is VAR? Why VAR?

$$y_t = A_1 y_{t-1} + \dots + A_p y_{t-p} + Cx_t + \epsilon_t$$

where

- $y_t = (y_{1t}, y_{2t}, \dots, y_{Kt})'$ is a $k \times 1$ vector of endogenous variables,
- $x_t = (x_{1t}, x_{2t}, \dots, x_{dt})'$ is a $d \times 1$ vector of exogenous variables,
- A_1, \dots, A_p are $k \times k$ matrices of lag coefficients to be estimated,
- C is a $k \times d$ matrix of exogenous variable coefficients to be estimated,
- $e_t = (e_{1t}, e_{2t}, \dots, e_{Kt})'$ is a $k \times 1$ white noise innovation process, with $E(e_t) = 0$, $E(e_t e_t') = \Sigma_e$, and $E(e_t e_s') = 0$ for $s \neq t$.

Suppose that we have VAR(p) model given by:

$$y_t = a_0 + \sum_{j=1}^p A_j y_{t-j} + \epsilon_t$$

where y_t for $t = 1, \dots, T$ is an vector containing observations on m different series and ϵ_t is an vector of errors where we assume ϵ_t is i.i.d. $N(0, \Sigma_\epsilon)$. For compactness we may rewrite the model as:

$$Y = XA + E,$$

or

$$y = (I_m \otimes X)\theta + e$$

where Y and E are $T \times m$ matrices and $X = (x_1, \dots, x_T)'$ is a $T \times (mp + 1)$ matrix for $x_t = (1, y'_{t-1}, \dots, y'_{t-p})$, I_m is the identity matrix of dimension m , $\theta = \text{vec}(A)$, and $e \sim N(0, \Sigma_\epsilon \otimes I_T)$. The likelihood function is:

$$l(\theta, \Sigma_\epsilon) \propto |\Sigma_\epsilon \otimes I_T|^{-1/2} \exp \left\{ -\frac{1}{2} (y - (I_m \otimes X)\theta)' (\Sigma_\epsilon \otimes I_T)^{-1} (y - (I_m \otimes X)\theta) \right\}$$

To illustrate how to derive the posterior moments, let us assume Σ_ϵ is known and a multivariate normal prior for θ :

$$\Pi(\theta) \propto |V_0|^{-1/2} \exp \left\{ -\frac{1}{2} (\theta - \theta_0)' V_0^{-1} (\theta - \theta_0) \right\}$$

where θ_0 is the prior mean and V_0 is the prior covariance.

Suppose that we have VAR(p) model given by:

When we combine this prior with the likelihood function given above, the posterior density can be written as:

$$\begin{aligned} \Pi(\theta | y) = & \exp \left\{ -\frac{1}{2} \cdot ((V_0^{-1/2}(\theta - \theta_0))' (V_0^{-1/2}(\theta - \theta_0)) \right. \\ & \left. + \{(\Sigma_\epsilon^{-1/2} \otimes I_T)y - (\Sigma_\epsilon^{-1/2} \otimes X)\theta\}' \{(\Sigma_\epsilon^{-1/2} \otimes I_T)y - (\Sigma_\epsilon^{-1/2} \otimes X)\theta\} \right\} \end{aligned}$$

which is a multivariate normal pdf. For simplicity, define

$$\begin{aligned} w &\equiv \begin{bmatrix} V_0^{-1/2}\theta_0 \\ (\Sigma_\epsilon^{-1/2} \otimes I_T)y \end{bmatrix} \\ W &\equiv \begin{bmatrix} V_0^{-1/2} \\ (\Sigma_\epsilon^{-1/2} \otimes X) \end{bmatrix} \end{aligned}$$

Then the exponent in above equation can be written as

$$\begin{aligned} \Pi(\theta | y) &\propto \exp \left\{ -\frac{1}{2} (w - W\theta)' (w - W\theta) \right\} \propto \\ &\exp \left\{ -\frac{1}{2} (\theta - \bar{\theta})' W' W (\theta - \bar{\theta}) + (w - W\bar{\theta})' (w - W\bar{\theta}) \right\} \end{aligned}$$

where the posterior mean $\bar{\theta}$ is

$$\bar{\theta} = (W' W)^{-1} W' w = [V_0^{-1} + (\Sigma_\epsilon^{-1} \otimes X' X)]^{-1} [V_0^{-1}\theta_0 + (\Sigma_\epsilon^{-1} \otimes X)' y]$$

Since is known, the last part in exponent equation has no randomness about $\bar{\theta}$. The posterior therefore may be summarized as:

$$\begin{aligned} \pi(\theta | y) &\propto \exp \left\{ -\frac{1}{2} (\theta - \bar{\theta})' W' W (\theta - \bar{\theta}) \right\} \\ &= \exp \left\{ -\frac{1}{2} (\theta - \bar{\theta})' \bar{V}^{-1} (\theta - \bar{\theta}) \right\} \end{aligned}$$

and the posterior covariance \bar{V} is given as:

$$\bar{V} = [V_0^{-1} + (\Sigma_\epsilon^{-1} \otimes X' X)]^{-1}.$$

Prior Selection

1. The Litterman/Minnesota prior: A normal prior on β with fixed Σ .
2. The normal-flat prior: A normal prior on β that is independent of the distribution for Σ .
3. The normal-Wishart prior: A normal prior on β and a Wishart prior on Σ .
4. The independent normal-Wishart prior. A normal prior on β and a Wishart prior on Σ , where each endogenous equation's coefficients' distributions may be independent from each other.
5. The Sims-Zha normal-flat. A structural VAR equivalent of the normal-flat prior.
6. The Sims-Zha normal-Wishart prior. A structural VAR equivalent of the normal-Wishart prior.
7. The Giannone, Lenza and Primiceri prior. A prior that treats the hyper-parameters as parameters that can be selected through an optimization procedure, etc.

Often a prior is specified that simplifies the posterior analysis. In particular, it is convenient to specify the prior such that the posterior is from a known family of distributions.

The Litterman/Minnesota prior:

$$\theta \sim N(\theta_0, V_0)$$

$\theta_0 = 0$ (where the hyper-parameter $\mu_1 = 0$, which indicates a zero mean model) and nonzero prior covariance $V_0 \neq 0$. Note that although the choice of zero mean could lessen the risk of over-fitting, theoretically any value for μ_1 is possible.

V_0 is assumed to be a diagonal matrix. The diagonal elements corresponding to endogenous variables, $v_{i,j}$ at lag l are specified by:

$$v_{ij}^l = \begin{cases} \left(\frac{\lambda_1}{l^{\lambda_3}}\right)^2 & \text{for } (i = j) \\ \left(\frac{\lambda_1 \lambda_2 \sigma_i}{l^{\lambda_3} \sigma_j}\right)^2 & \text{for } (i \neq j) \end{cases}$$

Given this choice of prior, the posterior for θ takes the form

$$\theta \sim N(\bar{\theta}, \bar{V})$$

where

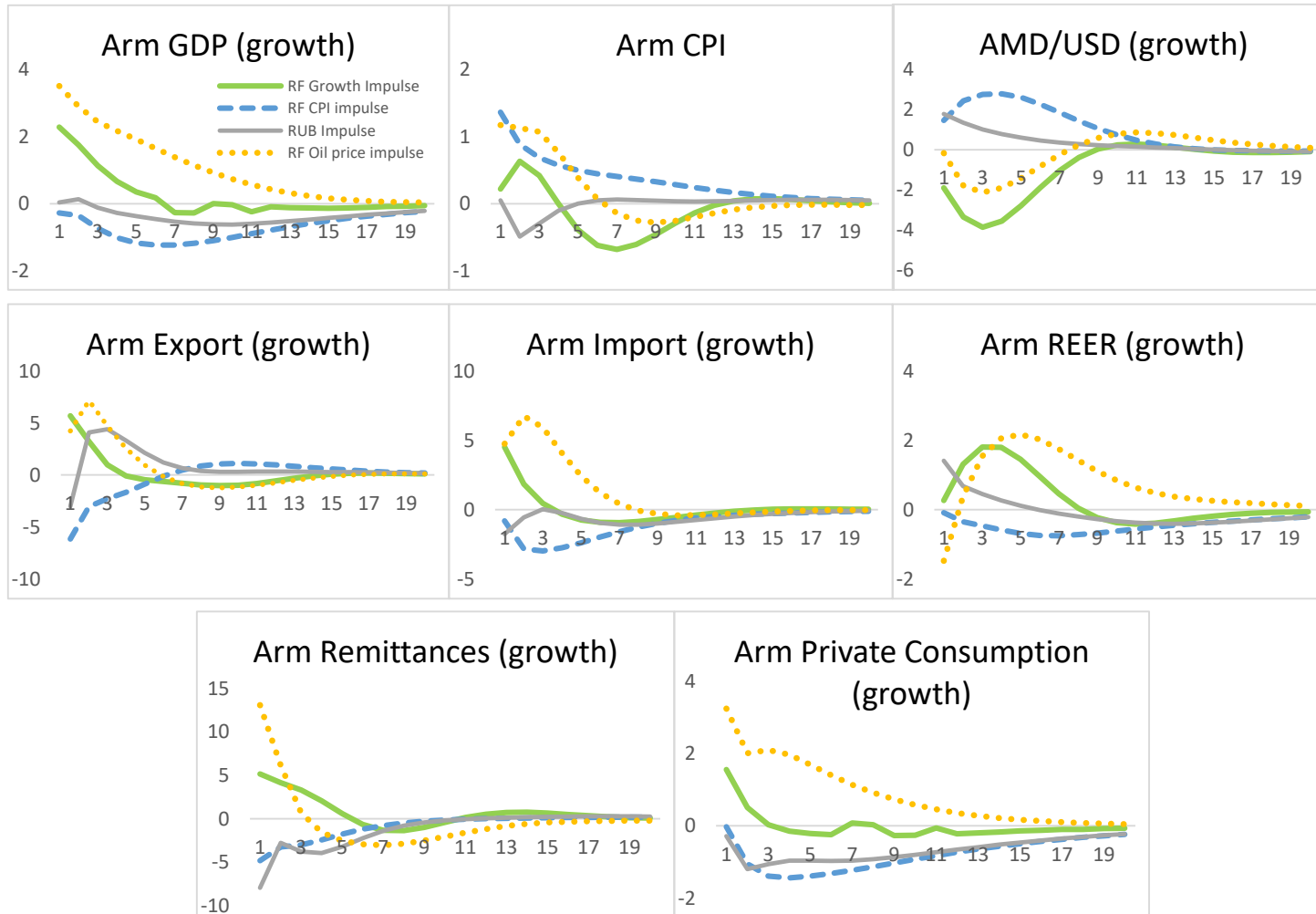
$$\bar{V} = [V_0^{-1} + (\hat{\Sigma}_\epsilon^{-1} \otimes X'X)]^{-1}$$

and

$$\bar{\theta} = \bar{V}[V_0^{-1}\theta_0 + (\hat{\Sigma}_\epsilon^{-1} \otimes X)'y]$$

A numerical comparison: SVAR vs BVAR

The assessment of the impact of the economic shock in Russia by a SVAR auto-regression model



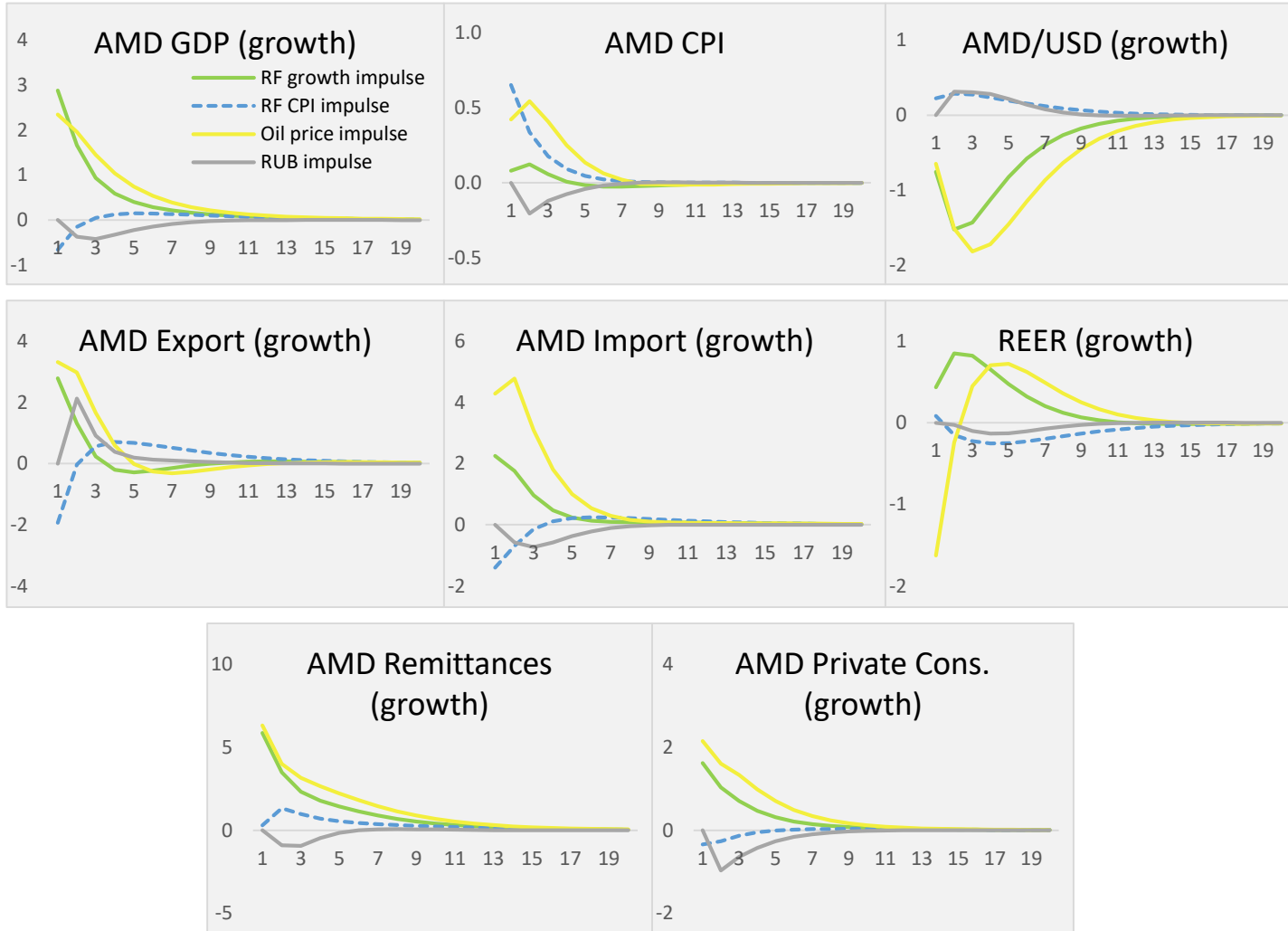
Highlight

- According to the results of the analysis, the one unit standard deviation change (i.e. 3.1pp change) of the economic growth of the Russian Federation in the short run leads to a change of the RA GDP growth by up to 2.28 percentage points. The impact of the unit sd change (25.8pp) in oil prices (3.5pp) on the GDP growth of the Republic of Armenia is also transmitted by influencing other variables. However, it fades quickly after 1-3 quarters.
- Meanwhile, the effects of the RUB/USD exchange rate and the GDP growth are more stable. Among the mechanisms of transmission of the economic shock in Russia, the impact of the growth rates of remittances, as well as imports and exports is significant. In the case of the former, 1 unit sd shock effect reaches up to 13.1 percentage points. The impact on the growth of private consumption in Armenia is similar to that of GDP growth.
- In a short run (1 quarter) the one unit standard deviation increase (9.8pp) of the RUB/USD exchange rate shocks the CPI of the Republic of Armenia positively (-0.5 percentage points), while in a two year period it leads to a growth of up to 0.06 percentage points.
- The most significant impact on RA CPI has the inflation shock in Russia (up to 1.36pp), while the most significant impact on the AMD / USD exchange rate has the inflation shock in Russia (up to 2.8 percentage points). In a 1-1.5 year period the REER is also significantly affected by the oil price and RF GDP growth, while the impact of RUB / USD exchange rate shock is seen instantly.

Source: Authors' calculations.

A numerical comparison: SVAR vs BVAR

The Bayesian VAR model



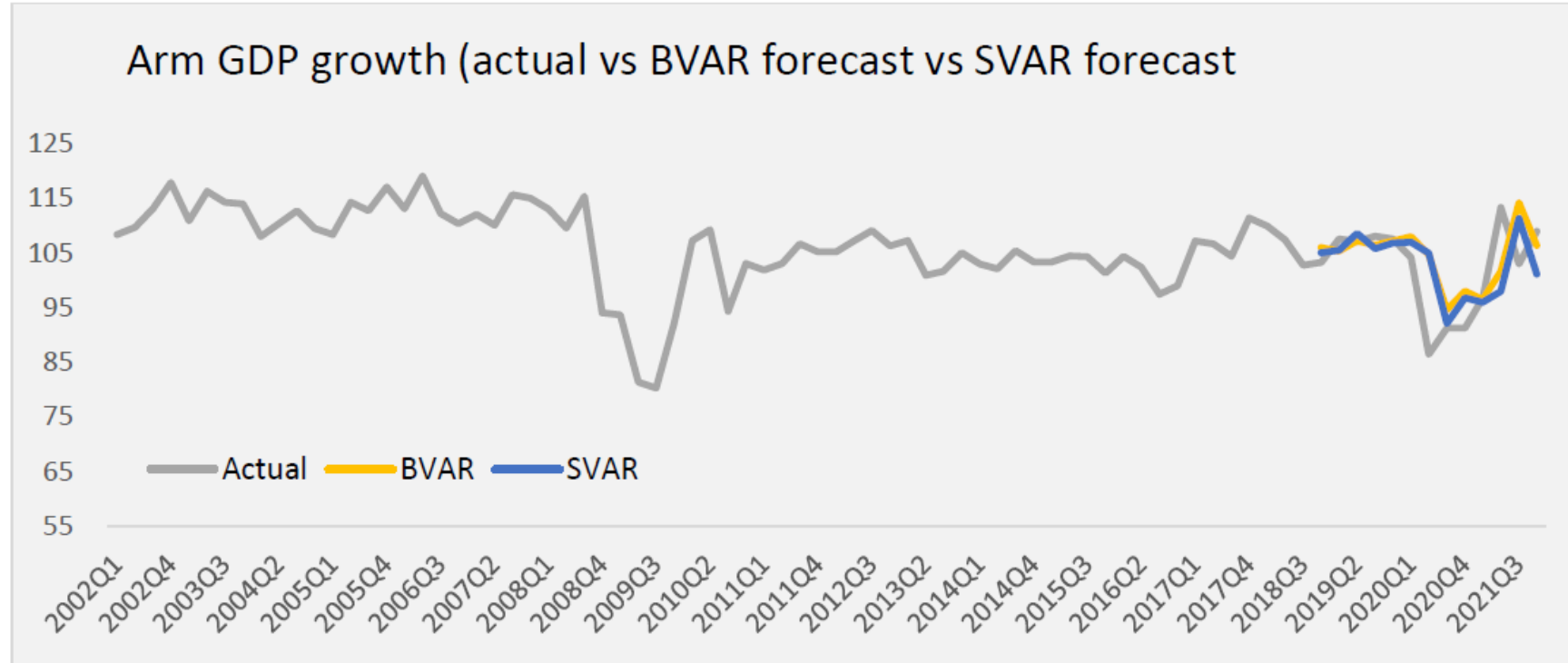
Highlight

- An one lag Bayesian VAR model was developed with Minnesota/Litterman prior and with the following parameters: μ_1 (AR(1) coefficient for prior) = 0, $\lambda_1 = 0.1$, $\lambda_2 = 0.99$, and $\lambda_3 = 1$.
- Initial residual covariance option is selected to be Univariate AR estimate.
- The directions and amplitudes of the effects in the two models are almost the same.
- To compare the performance of the two models, a well-known method in time series analysis of out of sample forecasting is applied.

Source: Authors' calculations.

A numerical comparison: SVAR vs BVAR

Out of sample forecasts

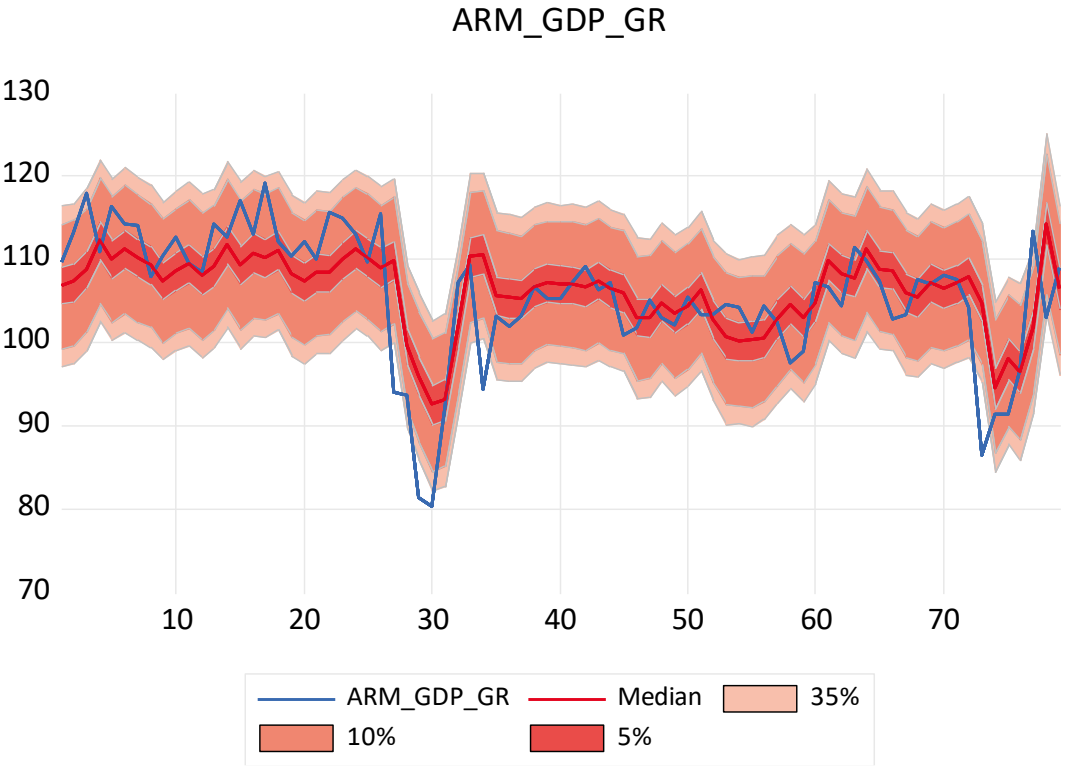


RMSE indicator for the BVAR model out of sample forecasts (7.26) is smaller than the RMSE indicator (7.64) for the SVAR model out of sample forecasts

References

- Bock M., Feldkircher M., Siklos P. (2021), International Effects of Euro Area Forward Guidance, *OXFORD BULLETIN OF ECONOMICS AND STATISTICS*, 0305-9049, doi: 10.1111/obes.12438.
- Bock M., Feldkircher M., Huber F. (2020), BGVAR: Bayesian Global Vector Autoregressions with Shrinkage Priors in R. *Globalization Institute Working Paper No. 395*, Available at SSRN: <https://ssrn.com/abstract=3682355> or <http://dx.doi.org/10.24149/gwp395>.
- Chan J., (2021): “Minnesota-type adaptive hierarchical priors for large Bayesian VARs,” *International Journal of Forecasting*, 37(3), 1212–1226.
- Chan J. (2021), Asymmetric Conjugate Priors for Large Bayesian VARs, Draft, <https://arxiv.org/pdf/2111.07170v1.pdf>.
- Chan J., Koop G., Yu X. (2021), Large Order-Invariant Bayesian VARs with Stochastic Volatility, Draft, <https://arxiv.org/pdf/2111.07225v1.pdf>.
- Doan, T., R. Litterman, and C. Sims (1984): “Forecasting and conditional projection using realistic prior distributions,” *Econometric reviews*, 3(1), 1–100.
- Enders W. (2015), *Applied econometric time series*, John Wiley & Sons, Inc., USA (pp 285-305).
- Hamilton J. (1994), *Time Series Analysis*, Princeton University Press, Princeton, New Jersey (pp 360-362).
- IHS Markit (2020), *EViews 12 User’s Guide II*, Seal Beach, CA, 1411 pages.
- Kook J., Vaughn K., DeMaster D., Ewing-Cobbs L., Vannucci M. (2020), BVAR-Connect: A Variational Bayes Approach to Multi-Subject Vector Autoregressive Models for Inference on Brain Connectivity Networks, *Neuroinformatics*, <https://arxiv.org/pdf/2006.04608v1.pdf>.
- Koop, G., Korobilis, D. Pettenuzzo, D. (2019). ‘Bayesian compressed vector autoregressions’, *Journal of Econometrics*, Vol. 210, pp. 135–154.
- Kotzé K. (2017), Bayesian vector autoregression models, <https://kevinkotze.github.io/ts-9-bvar/>.
- Litterman, R. B. (1986). Forecasting with Bayesian Vector Autoregressions: Five Years of Experience. *Journal of Business & Economic Statistics*, 4(1), 25–38. <https://doi.org/10.2307/1391384>.
- Nurfalah I., Aam Slamet Rusydiana A., Laila N., Cahyono E. (2018), Early Warning to Banking Crises in the Dual Financial System in Indonesia: The Markov Switching Approach, *JKAU: Islamic Econ.*, Vol. 31 No. 2, pp. 133-156, DOI:10.4197/Islec. 31-2.10.
- Schorfheide F., Song D. (2013), *Real-time forecasting with a mixed-frequency VAR*, Working Paper 19712, Cambridge, MA 02138, <http://www.nber.org/papers/w19712>.
- Sims, C. A., and T. Zha (1998): “Bayesian methods for dynamic multivariate models,” *International Economic Review*, 39(4), 949–968.

Appendix 1



Forecast Evaluation
Date: 05/22/22 Time: 20:45
Sample: 2002Q1 2021Q4
Included observations: 80

Variable	Inc. obs.	RMSE	MAE	MAPE	Theil
OIL_GR	80	26.68002	20.26156	20.24664	0.116186
USD_RUB_GR	80	10.81261	7.438419	6.737471	0.050401
RUS_CPI_INF	80	2.562801	1.949276	82.52901	0.389733
RUS_GDP_GR	80	3.791897	2.217284	401.3456	0.427244
USD_AMD_GR	80	5.833716	4.523339	160.7803	0.293975
REER_GR	80	4.697310	3.873658	3.848860	0.023063
EXPORT_GR	80	24.32010	16.67623	102.7956	0.750270
IMPORT_GR	80	11.54012	8.430824	8.470199	0.054583
REM_GR	80	26.91111	19.72932	108.9811	0.507154
CPI_INF_ARM	80	3.318230	2.792331	202.7482	0.500309
PCONS_GR	80	6.114204	4.295627	4.232168	0.029192
ARM_GDP_GR	80	5.530471	3.907050	3.839054	0.026038

RMSE: Root Mean Square Error
MAE: Mean Absolute Error
MAPE: Mean Absolute Percentage Error
Theil: Theil inequality coefficient

Appendix 2

ADF tests for stationarity

Null Hypothesis: ARM_GDP_GR has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on AIC, maxlag=11)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-3.928795	0.0029
Test critical values:		
1% level	-3.511262	
5% level	-2.896779	
10% level	-2.585626	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: IMPORT_GR has a unit root
Exogenous: Constant
Lag Length: 7 (Automatic - based on AIC, maxlag=11)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-3.653729	0.0068
Test critical values:		
1% level	-3.519050	
5% level	-2.900137	
10% level	-2.587409	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: REER_GR has a unit root
Exogenous: Constant
Lag Length: 9 (Automatic - based on AIC, maxlag=11)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-3.430760	0.0129
Test critical values:		
1% level	-3.521579	
5% level	-2.901217	
10% level	-2.587981	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: CPI_INF_ARM has a unit root
Exogenous: Constant
Lag Length: 4 (Automatic - based on AIC, maxlag=11)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-2.747058	0.0708
Test critical values:		
1% level	-3.515536	
5% level	-2.898623	
10% level	-2.586605	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: OIL_GR has a unit root
Exogenous: Constant
Lag Length: 6 (Automatic - based on AIC, maxlag=11)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-3.292129	0.0186
Test critical values:		
1% level	-3.517847	
5% level	-2.899619	
10% level	-2.587134	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: REM_GR has a unit root
Exogenous: Constant
Lag Length: 4 (Automatic - based on AIC, maxlag=11)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-2.332564	0.1645
Test critical values:		
1% level	-3.515536	
5% level	-2.898623	
10% level	-2.586605	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: EXPORT_GR has a unit root
Exogenous: Constant
Lag Length: 5 (Automatic - based on AIC, maxlag=11)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-2.072311	0.2563
Test critical values:		
1% level	-3.516676	
5% level	-2.899115	
10% level	-2.586866	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: PCONS_GR has a unit root
Exogenous: Constant
Lag Length: 4 (Automatic - based on AIC, maxlag=11)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-2.981995	0.0410
Test critical values:		
1% level	-3.515536	
5% level	-2.898623	
10% level	-2.586605	

*MacKinnon (1996) one-sided p-values.

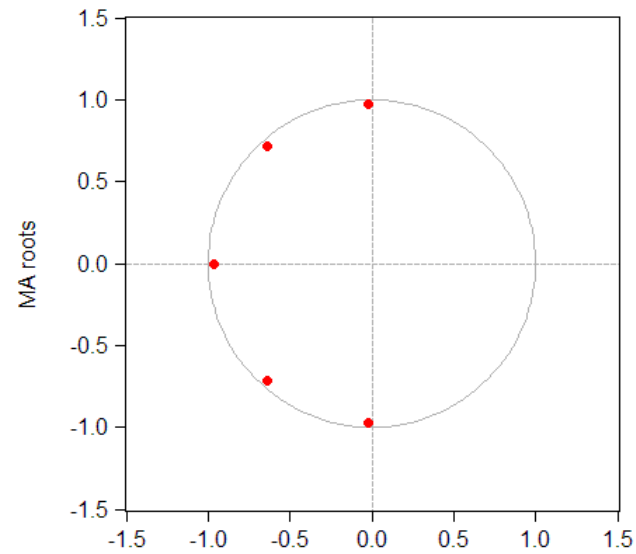
Null Hypothesis: RUS_CPI_INF has a unit root
Exogenous: Constant
Lag Length: 8 (Automatic - based on AIC, maxlag=11)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-2.208425	0.2051
Test critical values:		
1% level	-3.520307	
5% level	-2.900670	
10% level	-2.587691	

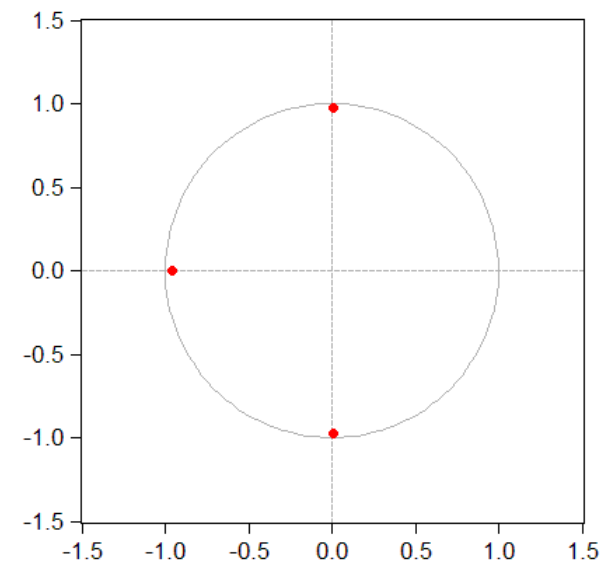
*MacKinnon (1996) one-sided p-values.

Stability conditions: characteristic root test

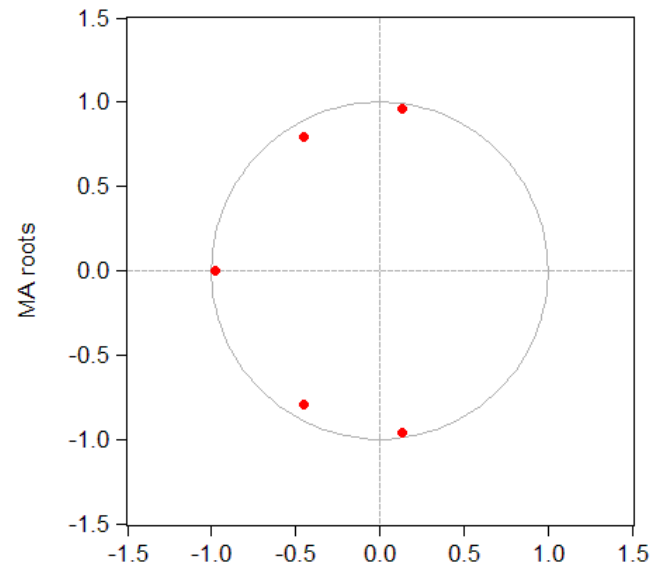
RUS_GDP_GR: Inverse Roots of AR/MA Polynomial(s)



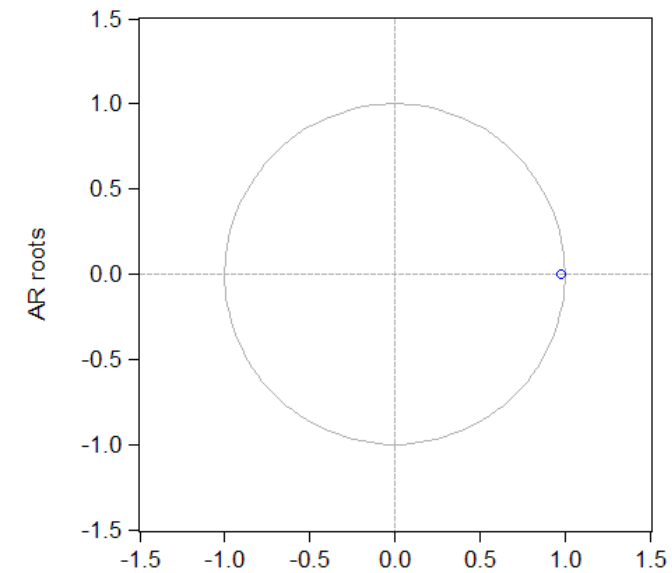
IMPORT_GR: Inverse Roots of AR/MA Polynomial(s)

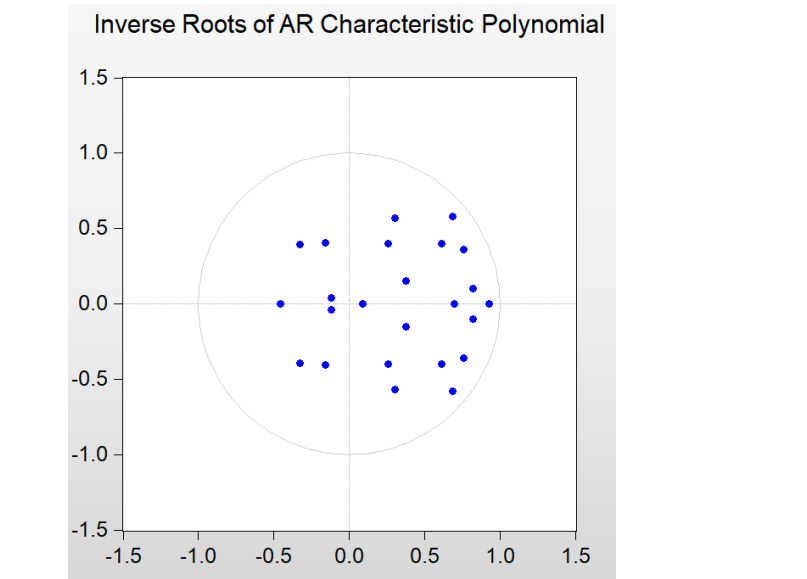


PCONS_GR: Inverse Roots of AR/MA Polynomial(s)



REM_GR: Inverse Roots of AR/MA Polynomial(s)



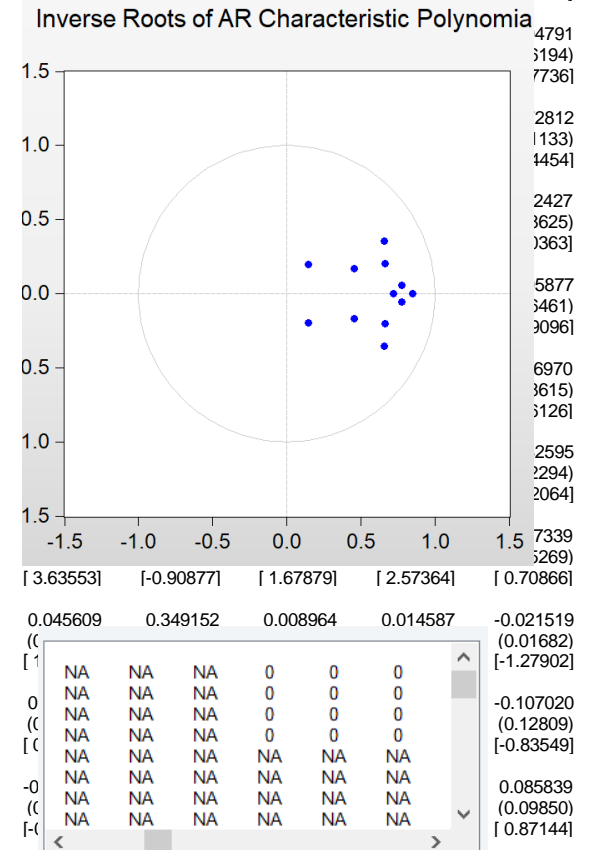


VAR Lag Order Selection Criteria
Endogenous variables: OIL_GR USD_RUB_GR RUS_CPI_INF RUS_GDP_GR U...
Exogenous variables: C
Date: 03/29/22 Time: 22:16
Sample: 2002Q1 2021Q4
Included observations: 77
*Note: selection calculation does not impose restricted VAR coefficient restrictions

Lag	LogL**	LR	FPE	AIC	SC	HQ
0	-3308.727	NA	4.67e+22	86.25264	86.61791	86.39874
1	-2879.546	713.4435	2.95e+19	78.84535	83.59383*	80.74470*
2	-2708.768	230.6612	1.89e+19	78.14981	87.28152	81.80242
3	-2474.580	243.3121*	3.83e+18*	75.80727*	89.32220	81.21312

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

	OIL_GR	USD_RUB...	RUS_CPI_INF	RUS_GDP...	USD_AMD...	REER_GR	EXPORT_GR	IMPORT_GR	REM_GR	CPI_INF_ARM	PCONS_GR	ARM_GDP...
OIL_GR(-1)	0.862695 (0.12705) [6.79022]	-0.094086 (0.05534) [-1.70009]	0.009626 (0.01280) [0.75217]	0.040879 (0.02045) [1.99894]	0.019439 (0.02911) [0.66769]	-0.001769 (0.02205) [-0.08025]	0.257135 (0.11344) [2.26672]	0.189621 (0.06401) [2.96230]	-0.039455 (0.14303) [-0.27585]	-0.021164 (0.01709) [-1.23809]	0.030845 (0.03462) [0.89086]	0.025363 (0.03108) [0.81601]
USD_RUB_GR(-1)	0.074828 (0.23209) [0.32241]	0.626706 (0.10110) [6.19908]	0.007424 (0.02338) [0.31753]	0.059872 (0.03736) [1.60268]	-0.061327 (0.05959) [-1.02917]	-0.047560 (0.04537) [-1.04834]	0.725994 (0.23436) [3.09775]					
RUS_CPI_INF(-1)	-1.543173 (0.80140) [-1.92559]	0.741801 (0.34909) [2.12498]	0.806875 (0.08073) [9.99491]	-0.297732 (0.12900) [-2.30808]	0.497768 (0.20264) [2.45645]	-0.062755 (0.15417) [-0.40704]	0.183926 (0.79607) [0.23104]					
RUS_GDP_GR(-1)	-0.973906 (0.62625) [-1.55513]	0.104122 (0.27279) [0.38169]	0.049642 (0.06309) [0.78690]	0.591557 (0.10080) [5.86843]	-0.549293 (0.18390) [-2.98685]	0.292996 (0.14071) [2.08232]	0.218614 (0.72945) [0.29970]					
USD_AMD_GR(-1)	0.000000 ---	0.000000 ---	0.000000 ---	0.000000 ---	0.680461 (0.07016) [9.69923]	0.097158 (0.05453) [1.78182]	-0.141098 (0.28577) [-0.49376]					
REER_GR(-1)	0.000000 ---	0.000000 ---	0.000000 ---	0.000000 ---	0.023191 (0.09355) [0.24789]	0.871843 (0.07271) [11.9907]	0.072911 (0.38106) [0.19134]					
EXPORT_GR(-1)	0.000000 ---	0.000000 ---	0.000000 ---	0.000000 ---	-0.049283 (0.02491) [-1.97813]	0.016018 (0.01936) [0.82723]	0.467444 (0.10148) [4.60623]					
IMPORT_GR(-1)	0.000000 ---	0.000000 ---	0.000000 ---	0.000000 ---	0.121662 (0.05721) [2.12647]	-0.028456 (0.04447) [-0.63991]	-0.119674 (0.23305) [-0.51352]					
REM_GR(-1)	0.000000 ---	0.000000 ---	0.000000 ---	0.000000 ---	-0.027448 (0.01827) [-1.50242]	0.026756 (0.01420) [1.88430]	0.180934 (0.07442) [2.43140]					
CPI_INF_ARM(-1)	0.000000 ---	0.000000 ---	0.000000 ---	0.000000 ---	-0.536171 (0.13909) [-3.85484]	0.229451 (0.10810) [2.12249]	0.211374 (0.56655) [0.37309]					
PCONS_GR(-1)	0.000000 ---	0.000000 ---	0.000000 ---	0.000000 ---	-0.119523 (0.10696) [-1.11746]	-0.102328 (0.08313) [-1.23092]	1.423155 (0.43567) [3.26657]					
ARM_GDP_GR(-1)	0.000000 ---	0.000000 ---	0.000000 ---	0.000000 ---	-0.077829 (0.11034) [-0.70534]	0.155314 (0.08576) [1.81101]	-1.343934 (0.44946) [-2.99014]					
C	7.151472 (35.9386) [0.19899]	50.45731 (15.6546) [3.22316]	-1.898176 (3.62023) [-0.52432]	-11.06179 (5.78474) [-1.91224]	10.26415 (15.1999) [0.67528]	15.58141 (11.7254) [1.32886]	-106.1520 (61.1363) [-1.73632]	-24.36057 (33.0345) [-0.73743]	67.61116 (71.6013) [0.94427]	18.66452 (8.83630) [2.11226]	23.86645 (17.9795) [1.32742]	20.15908 (14.6877) [1.37252]
R-squared	0.562692	0.659894	0.656453	0.540163	0.815904	0.765264	0.483588	0.572080	0.511285	0.566918	0.438782	0.497126
Sum sq. resids	43872.11	8324.359	445.1839	1136.671	1991.301	1132.198	29790.13	9674.696	48918.90	689.4776	2822.923	2360.734
Mean dependent	111.9310	105.9113	-0.697553	-0.006524	0.027317	101.6277	-0.496203	104.3785	-0.058780	0.231481	104.4136	105.8241
S.D. dependent	35.86355	17.71417	4.075954	5.629475	11.77604	7.863646	27.19509	17.02512	35.82310	4.517809	8.030388	7.757944
Determinant resid covariance		7.66E+18										
Log likelihood		-2977.504										
Akaike information criterion		78.51909										
Schwarz criterion		82.23822										
Number of coefficients		124										
Number of restrictions		32										



oil_gr usd_rub_gr d_rus_cpi_inf d_rus_gdp_gr d_usd_amd_gr reer_gr d_export_gr import_gr d_rem_gr
d_cpi_inf_arm pcons_gr arm_gdp_gr

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
NA	NA	NA	NA	NA	NA
NA	NA	NA	NA	NA	NA
NA	NA	NA	NA	NA	NA
NA	NA	NA	NA	NA	NA

VAR Lag Order Selection Criteria

Endogenous variables: OIL_GR USD_RUB_GR D_RUS_CPI_INF D_RUS_GDP...

Exogenous variables: C

Date: 03/23/22 Time: 23:53

Sample: 2001Q1 2021Q4

Included observations: 78

*Note: selection calculation does not impose restricted VAR coefficient restrictions

Lag	LogL**	LR	FPE	AIC	SC	HQ
0	-3077.016	NA	4.06e+19	79.20553	79.56810	79.35067
1	-2712.508	607.5134	1.48e+17	73.55148	78.26489*	75.43834
2	-2551.128	219.3110	1.21e+17	73.10584	82.17010	76.73443
3	-2360.926	199.9558	7.52e+16	71.92118	85.33629	77.29149
4	-2079.391	209.3466*	1.16e+16	68.39464	86.16060	75.50668
5	-1719.711	156.7836	2.05e+15*	62.86438*	84.98118	71.71814*

* indicates lag order selected by the criterion

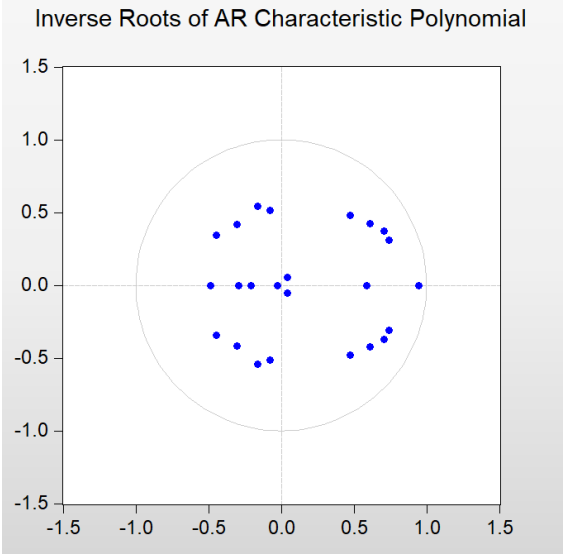
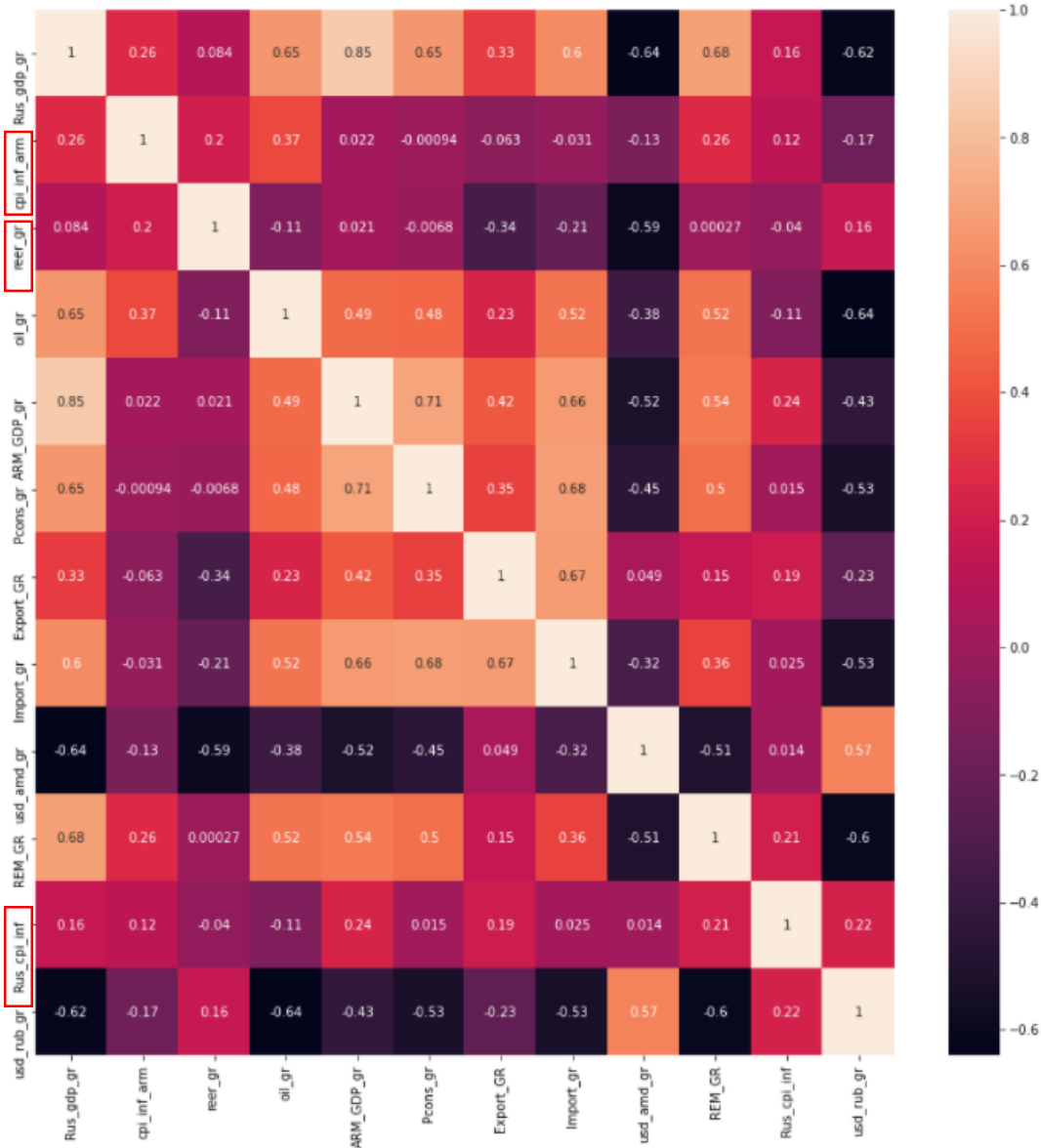
LR: sequential modified LR test statistic (each test at 5% level)

FPE: Final prediction error

AIC: Akaike information criterion

SC: Schwarz information criterion

HQ: Hannan-Quinn information criterion



VAR Residuals

OIL_G VAR Residual Serial Correlation LM Tests

Date: 03/29/22 Time: 22:35

Sample: 2002Q1 2021Q4

Included observations: 79

Null hypothesis: No serial correlation at lag h

Lag	LRE* stat	df	Prob.	Rao F-stat	df	Prob.
1	76.83570	144	1.0000	0.490881	(144, 386.9)	1.0000

Null hypothesis: No serial correlation at lags 1 to h

Lag	LRE* stat	df	Prob.	Rao F-stat	df	Prob.
1	76.83570	144	1.0000	0.490881	(144, 386.9)	1.0000

*Edgeworth expansion corrected likelihood ratio statistic.

Is

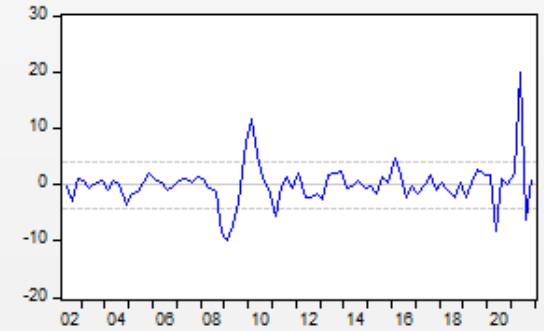


Component	Jarque-B...	df	Prob.
1	61.98173	2	0.0000
2	80.66896	2	0.0000
3	68.81653	2	0.0000
4	6.188836	2	0.0453
5	0.558819	2	0.7562
6	4.283849	2	0.1174
7	0.044684	2	0.9779
8	3.184349	2	0.2035
9	4.375126	2	0.1122
10	0.373495	2	0.8297
11	4.232884	2	0.1205
12	30.76145	2	0.0000
Joint	265.4707	24	0.0000

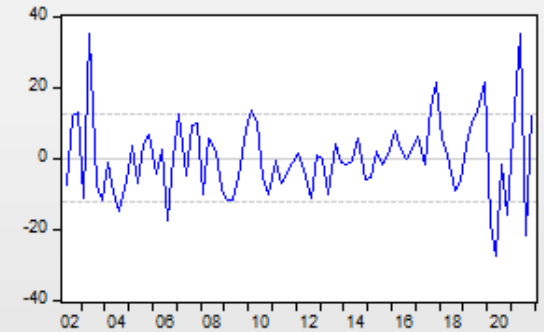
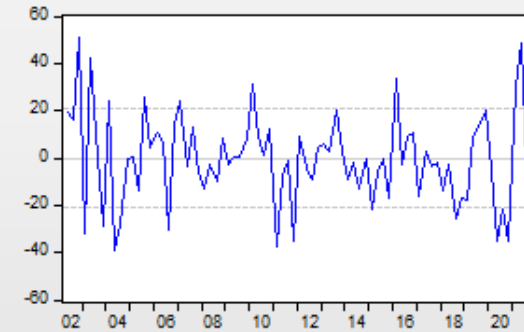
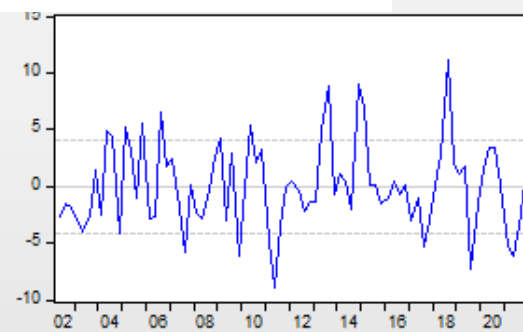
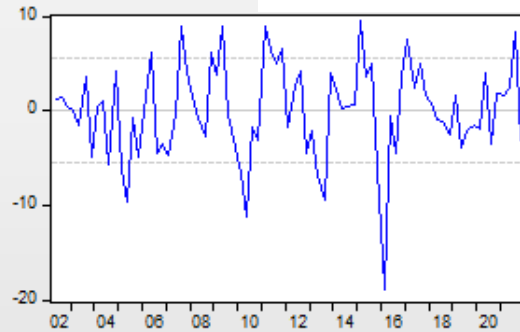
*Approximate p-values do not account for coefficient estimation



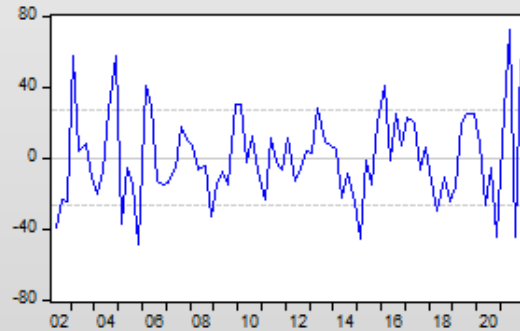
RUS_GDP_GR Residuals



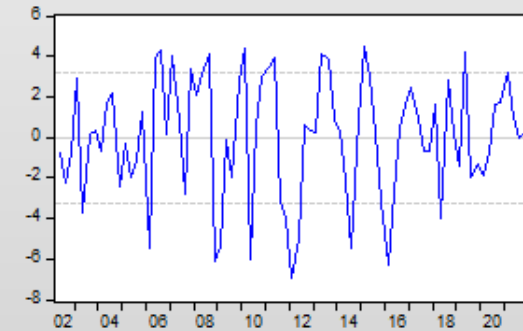
USD_ARM



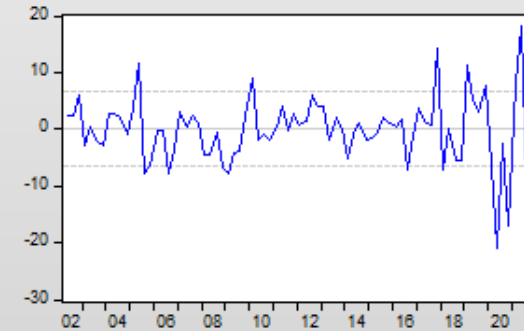
REM_GR Residuals



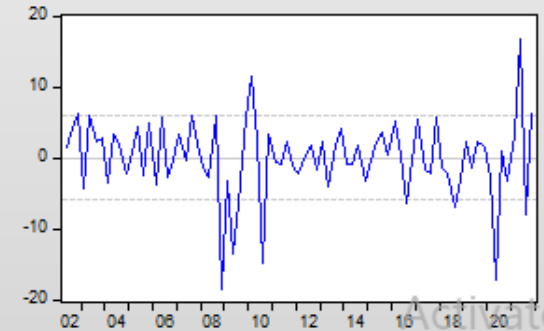
CPI_INF_ARM Residuals



PCONS_GR Residuals



ARM_GDP_GR Residuals



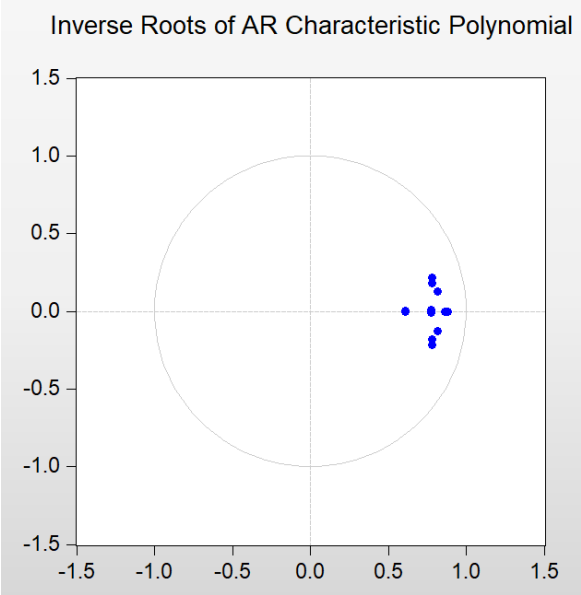
VAR Residual Cross-Correlations
Ordered by variables
Date: 03/29/22 Time: 22:38
Sample: 2002Q1 2021Q4
Included observations: 79

	OIL_GR	USD_RUB_GR	RUS_CPI_INF	RUS_GDP_GR	USD_AMD_GR	REER_GR	EXPORT_GR	IMPORT_GR	REM_GR	CPI_INF_ARM	PCONS_GR	ARM_GDP_GR
OIL_GR	1.000000	-0.479793	0.137369	0.598651	-0.030391	-0.356869	0.198438	0.392353	0.480466	0.363486	0.495655	0.587781
OIL_GR(-1)	0.217935	-0.172486	-0.078645	0.250844	-0.030880	-0.153771	0.029636	0.025654	0.082006	0.153598	0.012220	0.161625
USD_RUB_GR	-0.479793	1.000000	0.278993	-0.304882	0.296556	0.470214	-0.219383	-0.316501	-0.487401	-0.161256	-0.276340	-0.277017
USD_RUB_GR(-1)	-0.101114	0.118989	0.327436	-0.328996	0.245852	0.134079	-0.223676	-0.145989	-0.179335	0.103159	-0.097290	-0.250354
RUS_CPI_INF	0.137369	0.278993	1.000000	-0.176775	0.362283	0.065354	-0.291816	-0.065522	-0.210130	0.439202	0.047471	0.040911
RUS_CPI_INF(-1)	0.231223	-0.210055	0.362899	0.066517	0.060028	-0.134329	-0.022060	0.045390	0.054006	0.377522	0.115548	0.092435
RUS_GDP_GR	0.598651	-0.304882	-0.176775	1.000000	-0.355931	-0.167430	0.402680	0.536381	0.484870	0.152054	0.477786	0.651638
RUS_GDP_GR(-1)	0.021352	-0.038510	-0.196015	0.205803	-0.050801	-0.031893	0.034025	0.010998	-0.024594	-0.080096	-0.079228	0.024030
USD_AMD_GR	-0.030391	0.296556	0.362283	-0.355931	1.000000	-0.378141	-0.237142	-0.153639	-0.189160	0.058586	-0.105244	-0.211342
USD_AMD_GR(-1)	0.106349	-0.042767	0.207722	-0.134880	0.263729	-0.092475	-0.169939	-0.014267	-0.004098	0.118179	0.051650	0.061165
REER_GR	-0.356869	0.470214	0.065354	-0.167430	-0.378141	1.000000	-0.271650	-0.339641	-0.377276	0.105223	-0.297921	-0.238932
REER_GR(-1)	-0.225647	0.207827	0.171387	-0.179364	-0.037754	0.248146	-0.178528	-0.271387	-0.147921	0.044754	-0.215431	-0.243315
EXPORT_GR	0.198438	-0.219383	-0.291816	0.402680	-0.237142	-0.271650	1.000000	0.641697	0.292425	-0.163731	0.401555	0.359465
EXPORT_GR(-1)	0.132075	0.018872	-0.089094	0.143374	0.081258	0.004510	-0.017969	-0.012394	0.026342	0.043777	-0.002745	0.102097
IMPORT_GR	0.392353	-0.316501	-0.065522	0.536381	-0.153639	-0.339641	0.641697	1.000000	0.333404	-0.066453	0.628255	0.618181
IMPORT_GR(-1)	0.072753	-0.020146	-0.161621	0.239662	0.033699	-0.105441	0.050088	0.017357	0.051020	-0.026550	0.033892	0.058650
REM_GR	0.480466	-0.487401	-0.210130	0.484870	-0.189160	-0.377276	0.292425	0.333404	1.000000	0.064573	0.421201	0.321781
REM_GR(-1)	-0.056495	-0.079646	-0.206215	0.157750	-0.054597	-0.069741	0.136492	0.094253	0.035571	-0.027226	-0.112918	0.053567
CPI_INF_ARM	0.363486	-0.161256	0.439202	0.152054	0.058586	0.105223	-0.163731	-0.066453	0.064573	1.000000	0.024028	0.151288
CPI_INF_ARM(-1)	0.113121	-0.216396	0.191170	0.191454	-0.027693	-0.160262	0.045857	0.121516	0.046279	0.200005	0.048767	0.194736
PCONS_GR	0.495655	-0.276340	0.047471	0.477786	-0.105244	-0.297921	0.401555	0.628255	0.421201	0.024028	1.000000	0.661120
PCONS_GR(-1)	0.037235	-0.093856	-0.129430	0.172175	-0.028649	-0.118731	0.045128	-0.001637	0.109936	-0.041803	0.038400	0.043338
ARM_GDP_GR	0.587781	-0.277017	0.040911	0.651638	-0.211342	-0.238932	0.359465	0.618181	0.321781	0.151288	0.661120	1.000000
ARM_GDP_GR(-1)	0.025108	-0.123701	-0.158350	0.160239	0.040141	-0.166430	-0.009339	-0.032356	0.058025	-0.031353	-0.009956	-0.077853

Bayesian VAR Estimates
Date: 05/20/22 Time: 00:32
Sample (adjusted): 2002Q3 2021Q4
Included observations: 78 after adjustments
Prior type: Litterman / Minnesota
Initial residual covariance: Univariate AR
Constant included in covariance calculation
Hyper-parameters: Mu1: 1, L1: 0.1, L2: 0.99, L3: 1, L4: inf
Standard errors in ()

With AR (myu) = 1

	OIL_GR	USD_RUB...	RUS_CPI_INF	RUS_GDP...	USD_AMD...	REER_GR	EXPORT_GR	IMPORT_GR	REM_GR	CPI_INF_ARM	PCONS_GR	ARM_GDP...
OIL_GR(-1)	0.936620 (0.07874)	-0.007390 (0.03591)	0.002791 (0.00685)	0.005174 (0.01250)	-0.001906 (0.01782)	0.009294 (0.01442)	0.058745 (0.07750)	0.038547 (0.04148)	-0.029685 (0.09883)	0.000591 (0.01048)	0.005023 (0.02229)	0.008947 (0.01888)
OIL_GR(-2)	-0.039718 (0.04683)	0.012890 (0.02130)	0.000138 (0.00406)	-0.004446 (0.00742)	0.002454 (0.01057)	0.003522 (0.00855)	-0.001223 (0.04598)	-0.002856 (0.02460)	-0.005107 (0.05863)	-0.002538 (0.00622)	0.004628 (0.01322)	-0.002053 (0.01120)
USD_RUB_GR(-1)	0.022612 (0.16886)	0.894129 (0.07794)	0.004358 (0.01478)	-0.012398 (0.02697)	-0.015910 (0.03845)	-0.016862 (0.03110)	0.073412 (0.16721)	0.003060 (0.08948)	0.042647 (0.21323)	-0.002802 (0.02261)	-0.028539 (0.04810)	-0.018628 (0.04073)
USD_RUB_GR(-2)	0.019985 (0.10015)	-0.031435 (0.04635)	-0.006602 (0.00877)	0.008636 (0.01600)	-0.021629 (0.02281)	-0.004991 (0.01845)	0.042476 (0.09920)	0.026855 (0.05309)	0.026868 (0.12650)	-0.001312 (0.01342)	0.008052 (0.02853)	0.010914 (0.02416)
RUS_CPI_INF(-1)	-0.554366 (0.75976)	0.072975 (0.34860)	0.903007 (0.06685)	-0.033450 (0.12139)	0.147493 (0.17304)	-0.062516 (0.14000)	0.377410 (0.75253)	-0.160792 (0.40272)	-0.497631 (0.95963)	-0.010866 (0.10179)	-0.155649 (0.21645)	-0.034310 (0.18330)
RUS_CPI_INF(-2)	-0.445465 (0.50611)	0.182147 (0.23221)	-0.083504 (0.04468)	-0.027615 (0.08087)	0.004102 (0.11528)	0.031725 (0.09325)	-0.212295 (0.50136)	-0.071755 (0.26829)	-0.298627 (0.63930)	-0.062083 (0.06780)	-0.044919 (0.14421)	-0.066794 (0.12212)
RUS_GDP_GR(-1)	-0.405576 (0.49778)	-0.023860 (0.22833)	0.025069 (0.04357)	0.912341 (0.08005)	-0.206965 (0.11337)	0.145225 (0.09170)	0.032104 (0.49304)	-0.028653 (0.26384)	0.045616 (0.62873)	0.031134 (0.06668)	-0.032570 (0.14180)	-0.000799 (0.12009)
RUS_GDP_GR(-2)	-0.041547 (0.29216)	0.021610 (0.13404)	0.012649 (0.02558)	-0.031855 (0.04711)	-0.005230 (0.06654)	0.017056 (0.05383)	-0.037345 (0.28940)	-0.031827 (0.15487)	0.085172 (0.36903)	0.008863 (0.03914)	0.026149 (0.08324)	0.001920 (0.07049)
USD_AMD_GR(-1)	-0.136621 (0.29603)	-0.013881 (0.13583)	-0.010832 (0.02592)	0.036045 (0.04732)	0.884908 (0.06780)	0.028786 (0.05456)	0.125198 (0.29325)	0.017138 (0.15693)	0.155917 (0.37398)	0.043452 (0.03966)	-0.017871 (0.08435)	0.004414 (0.07143)
USD_AMD_GR(-2)	-0.082333 (0.19147)	-0.009766 (0.08785)	-0.012238 (0.01676)	0.012230 (0.03060)	-0.104829 (0.04397)	0.028583 (0.03528)	0.114594 (0.18967)	0.027258 (0.10150)	0.079999 (0.24186)	0.000358 (0.02565)	0.013652 (0.05455)	-0.010402 (0.04620)
REER_GR(-1)	-0.250336 (0.38156)	0.095507 (0.17506)	0.029439 (0.03341)	-0.074515 (0.06097)	0.102748 (0.08693)	0.923052 (0.07069)	0.034164 (0.37796)	-0.161287 (0.20227)	-0.203717 (0.48197)	-0.066041 (0.05112)	0.001778 (0.10871)	-0.046584 (0.09206)
REER_GR(-2)	0.076237 (0.24569)	-0.019178 (0.11272)	0.000405 (0.02151)	0.001341 (0.03926)	0.057785 (0.05596)	-0.052241 (0.04566)	0.042570 (0.24338)	0.059366 (0.13024)	-0.143301 (0.31035)	-0.002808 (0.03292)	0.007930 (0.07000)	0.021286 (0.05928)
EXPORT_GR(-1)	0.011898 (0.08148)	-0.010602 (0.03738)	-0.002381 (0.00713)	-0.002277 (0.01302)	-0.010318 (0.01856)	0.002674 (0.01501)	0.846303 (0.08125)	-0.004464 (0.04320)	-0.129643 (0.10292)	0.003654 (0.01092)	-0.005337 (0.02321)	-0.002176 (0.01966)
EXPORT_GR(-2)	-0.013318 (0.04749)	-0.006918 (0.02179)	-0.001849 (0.00416)	-0.001687 (0.00759)	-0.001364 (0.01082)	-0.001597 (0.00875)	-0.014005 (0.04747)	-0.007118 (0.02517)	0.031143 (0.05999)	-0.000972 (0.00636)	-0.004699 (0.01353)	-0.002852 (0.01146)
IMPORT_GR(-1)	-0.089623 (0.15714)	0.022182 (0.07209)	-0.004181 (0.01376)	-0.010486 (0.02511)	0.011214 (0.03579)	0.006933 (0.02895)	-0.137542 (0.15568)	0.862383 (0.08390)	-0.118968 (0.19849)	0.005262 (0.02105)	-0.011035 (0.04478)	-0.016042 (0.03792)
IMPORT_GR(-2)	-0.041151 (0.08783)	0.014024 (0.04030)	0.000620 (0.00769)	-0.011645 (0.01403)	0.007511 (0.02000)	0.002984 (0.01618)	-0.047172 (0.08700)	-0.033896 (0.04698)	0.000643 (0.11095)	-0.000644 (0.01177)	-0.013419 (0.02503)	-0.008675 (0.02119)
REM_GR(-1)	-0.002002 (0.00429)	-0.004290 (0.00183)	0.001835 (0.00108)	-0.001083 (0.00179)	-0.017923 (0.00122)	0.012122 (0.00759)	0.075939 (0.02320)	0.023207 (0.01819)	0.818195 (0.00182)	-0.002295 (0.00022)	-0.001387 (0.00013)	



X

Basics

Prior type

Hyper-parameters

Options

Prior type

Litterman / Minnesota

Dummy observations

☐ Sum-of-coefficients (unit roots)

☐ Dummy-initial-observation (cointegration)

Initial residual covariance options

Univariate AR

Exogenous variables:

☐ None

☒ Constant only

☐ All exogenous

☐ Include dummy observations in calculation

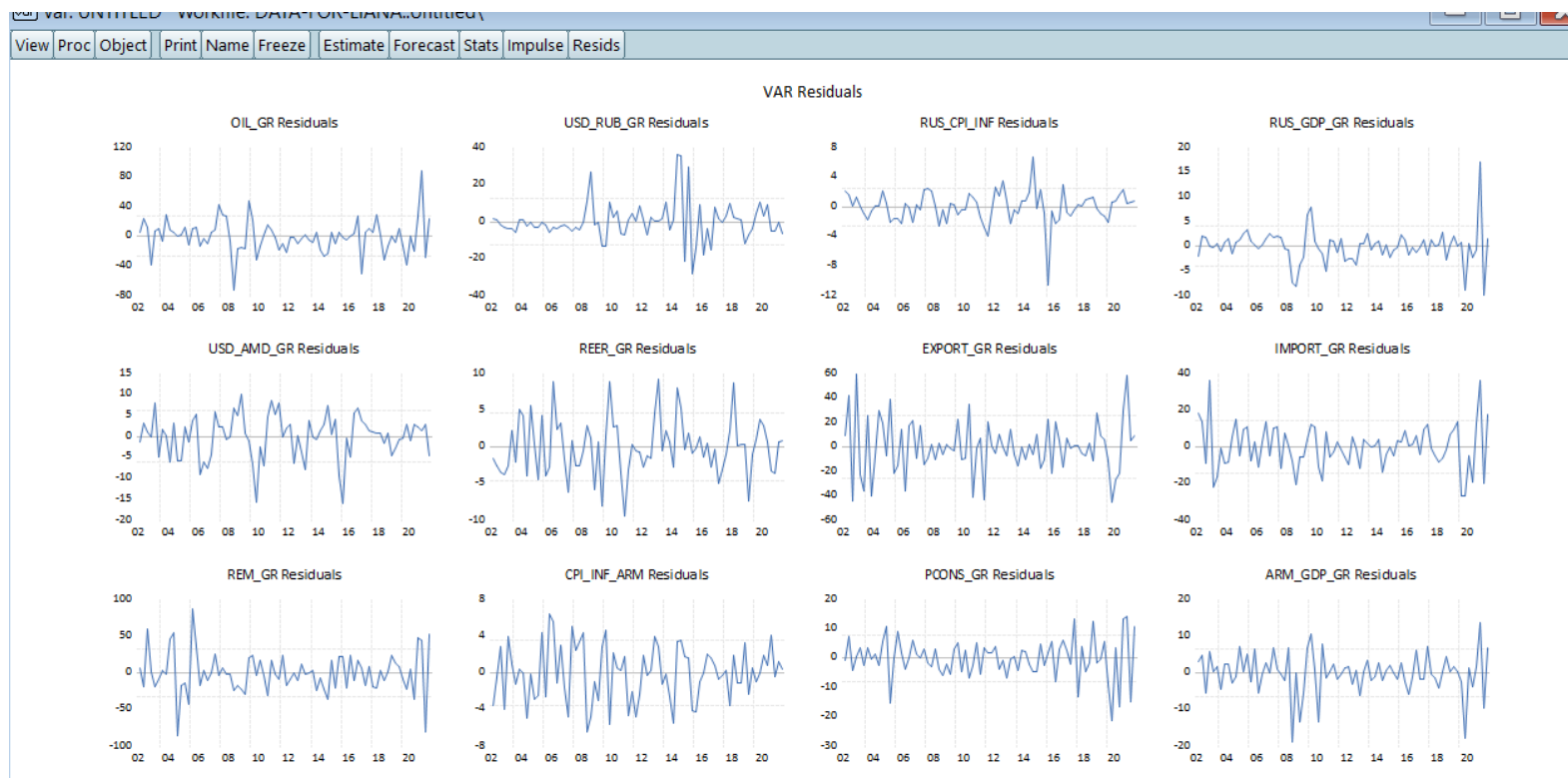
☒ Degrees of freedom correction

Sample:

(leave blank for estimation sample)

OK

Cancel

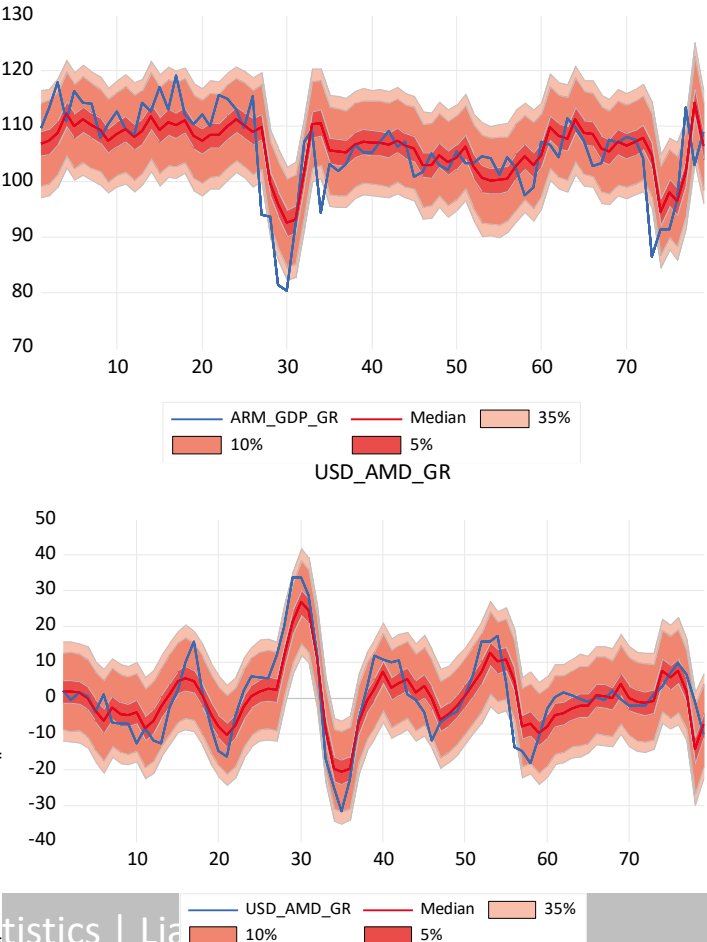
[illegible]

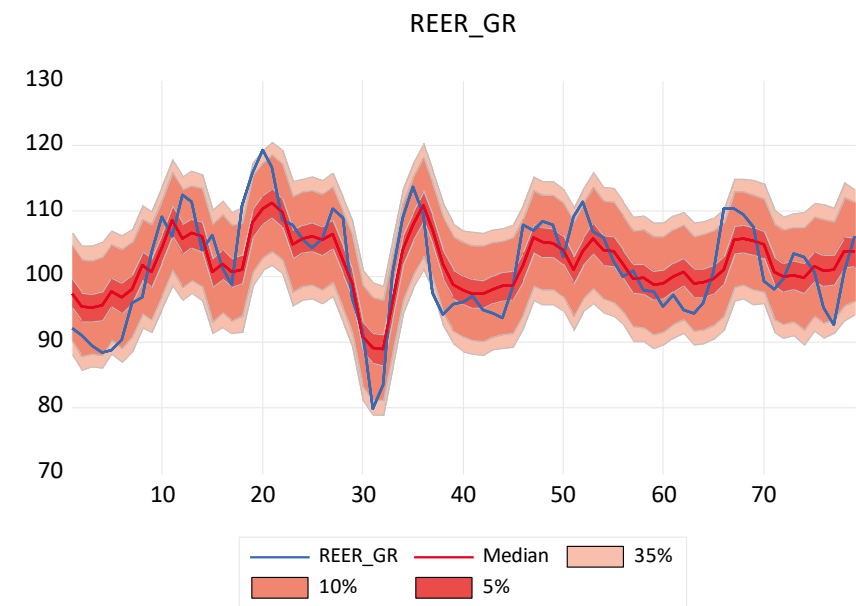
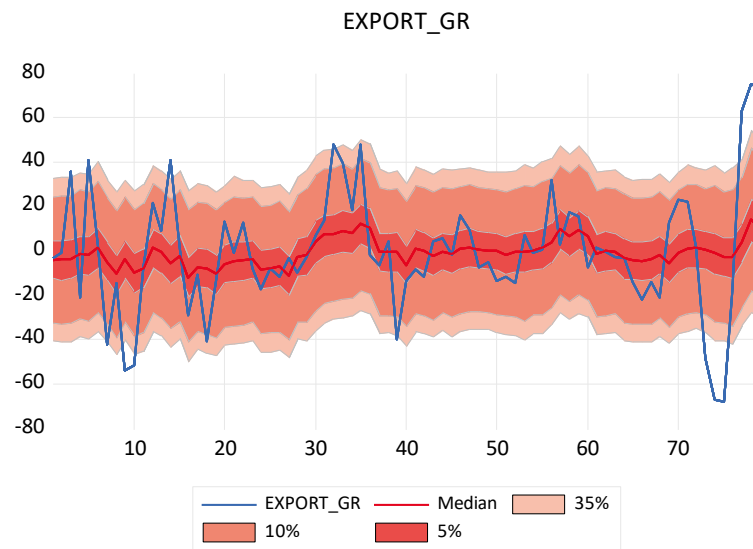
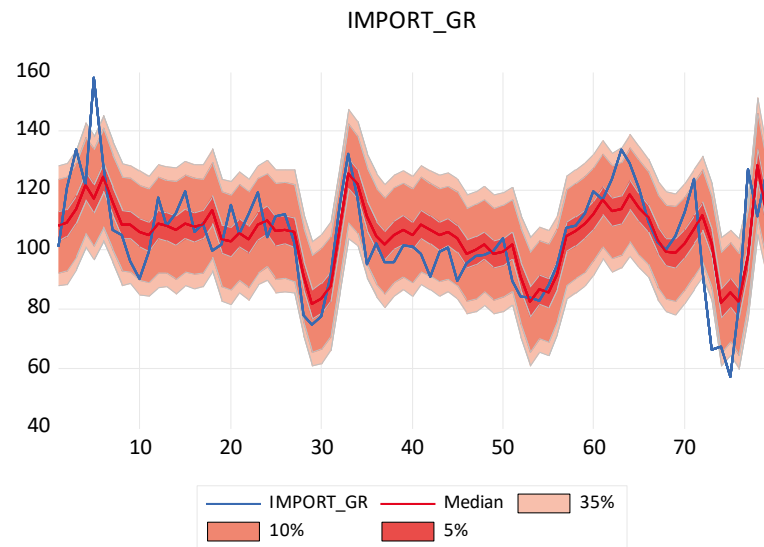
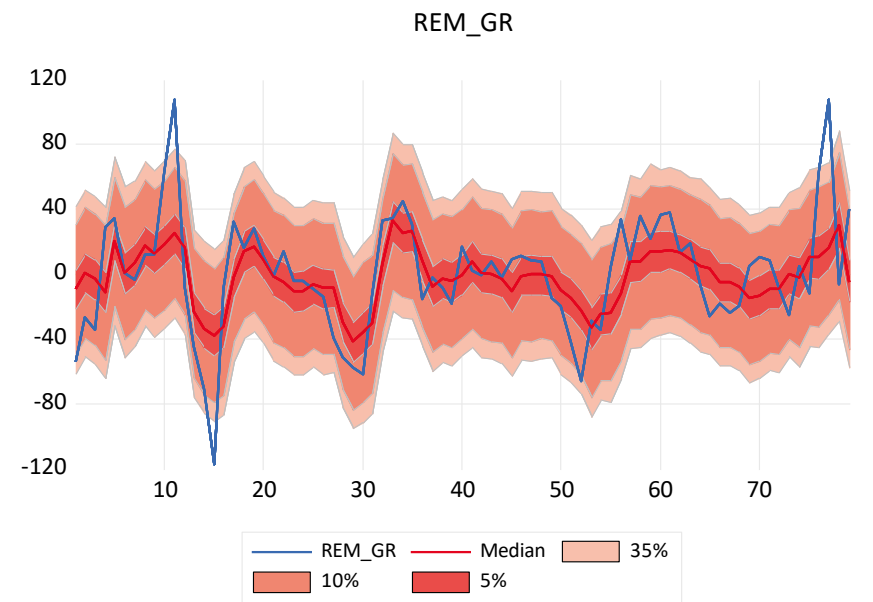
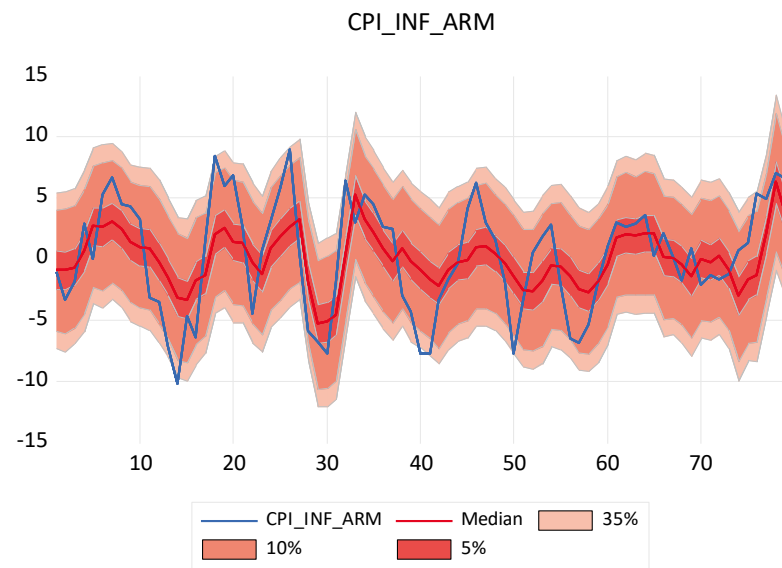
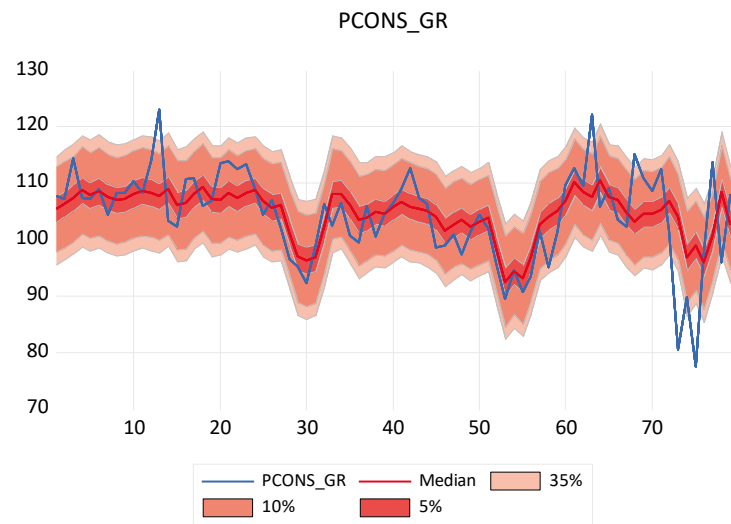
Out of sample – 15%: BVAR

Bayesian VAR Estimates
Date: 05/22/22 Time: 20:43
Sample (adjusted): 2002Q2 2018Q4
Included observations: 67 after adjustments
Prior type: Litterman / Minnesota
Initial residual covariance: Univariate AR
Constant included in covariance calculation
Hyper-parameters: Mu1: 0, L1: 0.1, L2: 0.99, L3: 1, L4: inf
Standard errors in ()

	OIL_GR	USD_RUB...	RUS_CPI_INF	RUS_GDP...	USD_AMD...	REER_GR	EXPORT_GR	IMPORT_GR	REM_GR	CPI_INF_ARM	PCONS_GR	ARM_GDP...
OIL_GR(-1)	0.293524 (0.07770)	-0.091134 (0.04001)	0.010843 (0.00911)	0.019188 (0.01154)	0.003210 (0.02614)	0.004986 (0.01760)	0.004689 (0.06912)	0.058012 (0.03827)	0.010464 (0.09735)	0.003113 (0.01212)	0.019023 (0.01839)	0.019969 (0.01813)
USD_RUB_GR(-1)	-0.315466 (0.14811)	0.290465 (0.07711)	0.021763 (0.01745)	-0.000197 (0.02212)	0.022209 (0.05011)	-0.001670 (0.03374)	0.090004 (0.13248)	-0.052254 (0.07335)	-0.046148 (0.18661)	-0.014309 (0.02323)	-0.081712 (0.03525)	-0.028996 (0.03475)
RUS_CPI_INF(-1)	-0.361335 (0.59459)	0.614903 (0.30790)	0.397810 (0.07044)	-0.158084 (0.08883)	0.274279 (0.20125)	0.194118 (0.13552)	-0.005298 (0.53207)	-0.419528 (0.29460)	-1.344309 (0.74949)	0.050623 (0.09330)	-0.230836 (0.14155)	0.037112 (0.13956)
RUS_GDP_GR(-1)	0.146589 (0.50067)	0.002253 (0.25921)	-0.019551 (0.05902)	0.370173 (0.07522)	-0.537032 (0.16948)	0.153811 (0.11408)	0.317181 (0.44803)	0.264157 (0.24807)	1.188631 (0.63110)	0.102985 (0.07856)	-0.011769 (0.11919)	0.183256 (0.11752)
USD_AMD_GR(-1)	-0.154104 (0.21280)	0.124176 (0.11020)	-0.001361 (0.02509)	-0.003530 (0.03180)	0.324649 (0.07241)	-0.057724 (0.04852)	-0.068362 (0.19045)	-0.028820 (0.10544)	-0.071965 (0.26827)	0.028856 (0.03339)	-0.006923 (0.05066)	-0.015163 (0.04995)
REER_GR(-1)	-0.254568 (0.30451)	0.203189 (0.15767)	0.088948 (0.03591)	-0.093913 (0.04549)	-0.062291 (0.10312)	0.424107 (0.06973)	-0.122358 (0.27251)	-0.296431 (0.15089)	-0.445514 (0.38383)	-0.047563 (0.04778)	0.020682 (0.07250)	-0.029814 (0.07147)
EXPORT_GR(-1)	0.018059 (0.08990)	-0.003606 (0.04655)	0.001090 (0.01060)	0.014456 (0.01343)	-0.029975 (0.03043)	-0.007442 (0.02049)	0.121440 (0.08098)	0.043489 (0.04455)	-0.164157 (0.11332)	0.001521 (0.01411)	-0.013500 (0.02140)	-0.011553 (0.02110)
IMPORT_GR(-1)	0.028157 (0.16032)	-0.030197 (0.08302)	-0.007401 (0.01890)	0.009415 (0.02395)	0.000580 (0.05427)	-0.022917 (0.03654)	-0.027546 (0.14349)	0.180664 (0.07995)	-0.150208 (0.20209)	0.020562 (0.02516)	0.044737 (0.03817)	0.030006 (0.03764)
REM_GR(-1)	-0.005394 (0.06204)	-0.030339 (0.03213)	-0.001197 (0.00731)	0.005667 (0.00927)	-0.045570 (0.02100)	0.015293 (0.01414)	0.047123 (0.05552)	0.018198 (0.03074)	0.195442 (0.07870)	0.010152 (0.00974)	0.007310 (0.01477)	-0.004916 (0.01456)
CPI_INF_ARM(-1)	-0.260478 (0.47284)	-0.163260 (0.24484)	0.048840 (0.05575)	0.005061 (0.07065)	-0.281118 (0.16005)	0.174095 (0.10776)	-0.168691 (0.42317)	-0.002092 (0.23429)	0.921662 (0.59607)	0.295914 (0.07461)	-0.065726 (0.11257)	-0.048682 (0.11099)
PCONS_GR(-1)	0.348757 (0.33863)	-0.191161 (0.17536)	0.038380 (0.03992)	-0.026808 (0.05059)	0.025176 (0.11461)	0.004718 (0.07717)	0.021125 (0.30303)	0.131490 (0.16780)	-0.144272 (0.42682)	-0.012431 (0.05313)	0.134722 (0.08115)	0.123081 (0.07950)
ARM_GDP_GR(-1)	-0.013610 (0.32687)	-0.035543 (0.16924)	0.020483 (0.03853)	0.014642 (0.04884)	-0.042601 (0.11063)	0.070306 (0.07449)	-0.467232 (0.29252)	0.192707 (0.16198)	-0.010348 (0.41200)	0.025133 (0.05129)	0.102919 (0.07784)	0.257854 (0.07718)
C	100.1233 (58.0293)	92.44797 (30.0816)	-18.57556 (6.84187)	7.454350 (8.66960)	5.389637 (19.6430)	52.98164 (13.2405)	51.85818 (51.9297)	80.91392 (28.7573)	77.58084 (73.1423)	2.560156 (9.10495)	79.33779 (13.8338)	66.80055 (13.6336)
R-squared	0.523672	0.636889	0.609220	0.633067	0.763426	0.659882	0.205802	0.594259	0.495951	0.452737	0.577332	0.522331
Adj. R-squared	0.417821	0.556198	0.522380	0.551526	0.710854	0.584301	0.029314	0.504094	0.383940	0.331123	0.483406	0.416182
Sum sq. resids	33406.17	8606.423	463.6288	692.2432	2483.024	1548.296	23231.31	6167.545	40905.95	784.6563	1278.437	1765.977
S.E. equation	24.87233	12.62451	2.930140	3.580408	6.780997	5.354639	20.74149	10.68709	27.52304	3.811912	4.865672	5.718678
F-statistic	4.947272	7.892914	7.015419	7.763818	14.52156	8.730717	1.166094	6.590815	4.427704	3.722732	6.146664	4.920749
Mean dependent	112.3512	105.9168	-1.049787	-0.122579	-0.149283	101.6753	-1.294030	105.6015	-2.575302	-0.006210	105.0340	106.4836
S.D. dependent	32.59780	18.95046	4.239815	5.346431	12.61059	8.305017	21.05235	15.17608	35.06590	4.660898	6.769682	7.484400

Forecast Evaluation					
Date: 05/22/22 Time: 20:45					
Sample: 2002Q1 2021Q4					
Included observations: 80					
Variable	Inc. obs.	RMSE	MAE	MAPE	Theil
OIL_GR	80	26.68002	20.26156	20.24664	0.116186
USD_RUB_GR	80	10.81261	7.438419	6.737471	0.050401
RUS_CPI_INF	80	2.562801	1.949276	82.52901	0.389733
RUS_GDP_GR	80	3.791897	2.217284	401.3456	0.427244
USD_AMD_GR	80	5.833716	4.523339	160.7803	0.293975
REER_GR	80	4.697310	3.873658	3.848860	0.023063
EXPORT_GR	80	24.32010	16.67623	102.7956	0.750270
IMPORT_GR	80	11.54012	8.430824	8.470199	0.054583
REM_GR	80	26.91111	19.72932	108.9811	0.507154
CPI_INF_ARM	80	3.318230	2.792331	202.7482	0.500309
PCONS_GR	80	6.114204	4.295627	4.232168	0.029192
ARM_GDP_GR	80	5.530471	3.907050	3.839054	0.026038
RMSE: Root Mean Square Error					
MAE: Mean Absolute Error					
MAPE: Mean Absolute Percentage Error					
Theil: Theil inequality coefficient					





Standard errors in () & t-statistics in []