Contents

Bernoulli Distribution

- Parameter: $p \in [0, 1]$ (usually, $p \in (0, 1)$)
- Notation: $X \sim Bernoulli(p)$;
- Support: $\{0, 1\}$
- PMF:

Values of X	0	1
$\mathbb{P}(X = x)$	$1 - p$	p

Note: This can be written in the form:

$$f(x) = f(x; p) = f(x|p) = p^x \cdot (1 - p)^{1-x}, \qquad x \in \{0, 1\}.$$

- Mean and Variance: $\mathbb{E}(X) = p$, $Var(X) = p(1 - p)$.
- Models: Models binary output, "success-failure" type Experiments, a lot of examples.

- **R** name: `binom` with the parameters `size=1` and `prob`
- Example:

```
rbinom(10, size = 1, prob = 0.3)
```

```
## [1] 0 0 1 0 0 0 0 1 0 1
```

Binomial Distribution

- Parameters: $n \in \mathbb{N}$, $p \in [0, 1]$ (usually, $p \in (0, 1)$)
- Notation: $X \sim Binom(n, p)$;
- Support: $\{0, 1, 2, ..., n\}$
- PMF:

Values of X	0	1	...	k	...	n
$\mathbb{P}(X = x)$	$\binom{n}{0} p^0 (1-p)^{n-0}$	$\binom{n}{1} p^1 (1-p)^{n-1}$	...	$\binom{n}{k} p^k (1-p)^{n-k}$	...	$\binom{n}{n} p^n (1-p)^0$

- Mean and Variance: $\mathbb{E}(X) = n \cdot p$, $Var(X) = n \cdot p(1-p)$.
- Models: Models the independent repetition of the $Bernoulli(p)$ Experiment.

Geometric Distribution

- Parameter: $p \in [0,1]$ (usually, $p \in (0,1)$)
- Notation: $X \sim Geom(p)$;
- Support: $\{1, 2, 3, ...\}$ (or, sometimes, $\{0, 1, 2, 3, ...\}$)
- PMF:

Values of X	1	2	3	...
$\mathbb{P}(X = x)$	p	$p(1-p)$	$p(1-p)^2$	...

- Mean and Variance: $\mathbb{E}(X) = \frac{1}{p}$, $Var(X) = \frac{1-p}{p^2}$.
- Models: Models the independent repetition of the *Bernoulli*(p) Experiment until the *First Success*.

- **R** name: `geom` with the parameter `prob`
- **Note: R** is using the second definition of the Geometric Distribution, with the support $\{0, 1, 2, 3, ...\}$, i.e., in **R**, $X \sim Geom(p)$ shows *the number of Failures before the first Success*
- Example:

```
rgeom(10,prob = 0.3)
```

```
##  [1] 0 1 0 1 0 0 2 2 0 2
```

- **R** name: `binom` with the parameters `size` and `prob`
- Additional: If $X_1, X_2, ..., X_n \sim Bernoulli(p)$ are independent, then $X_1 + X_2 + ... + X_n \sim Binom(n, p)$.
- Example:

```
rbinom(10, size = 5, prob = 0.3)
```

```
##  [1] 2 3 0 2 1 1 1 3 1 0
```

Poisson Distribution

- Parameter: $\lambda > 0$
- Notation: $X \sim Pois(\lambda)$;
- Support: $\{0, 1, 2, 3, ...\}$
- PMF:

Values of X	0	1	2	...
$\mathbb{P}(X = x)$	$e^{-\lambda}\frac{\lambda^0}{0!}$	$e^{-\lambda}\frac{\lambda^1}{1!}$	$e^{-\lambda}\frac{\lambda^2}{2!}$	...

- Mean and Variance: $\mathbb{E}(X) = \lambda$, $Var(X) = \lambda$.
- Models: Models the number of success in a fixed interval. Say, number of calls, number of customers, number of clicks, number of page visits, ... λ is the average number of calls, customers, clicks, page visits,...

- **R** name: `pois` with the parameter `lambda`
- Example:

```
rpois(10, lambda = 2)
```

```
##  [1] 4 3 2 1 1 3 3 2 2 1
```

Uniform Distribution

- Parameters: a, b $(a < b)$
- Notation: $X \sim Unif[a, b]$;
- Support: $[a, b]$
- PDF:

$$f(x) = f(x|a, b) = \begin{cases} \dfrac{1}{b-a}, & x \in [a, b] \\ 0, & otherwise \end{cases}$$

- Mean and Variance: $\mathbb{E}(X) = \frac{a+b}{2}$, $Var(X) = \frac{(b-a)^2}{12}$.
- Models: Usually we think about the Uniform Distribution when talking about *picking a random number from an interval*

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- **R** name: `unif` with the parameters `min = 0` and `max = 1`

Exponential Distribution

- Parameter: $\lambda > 0$ (rate) (or, sometimes, $\beta = \frac{1}{\lambda}$, scale)
- Notation: $X \sim Exp(\lambda)$ (or $\widetilde{Exp}(\beta)$);
- Support: $[0, +\infty)$
- PDF:

$$f(x|\lambda) = \begin{cases} \lambda \cdot e^{-\lambda x}, & x \geq 0 \\ 0, & otherwise \end{cases}$$

- Mean and Variance: $\mathbb{E}(X) = \frac{1}{\lambda}$, $Var(X) = \frac{1}{\lambda^2}$.
- Models: time elapsed until the occurrence of certain event, or the time between events (waiting times), when that time is random. λ is the average "arrival rate", the reciprocal of the average time between the events,

$$\lambda = \frac{1}{\text{average time between events}}.$$

- **R** name: `unif` with the parameters `min = 0` and `max = 1`
- Example:

```
runif(10, min = 2, max = 5)
```

```
##  [1] 2.982517 2.011465 3.381386 4.741297 4.402699 4.911
##  [9] 3.001453 2.865652
```

- **R** name: `exp` with the parameter `rate = 1`
- Example:

```
rexp(10, rate = 2)
```

```
##  [1] 1.44393944 0.57677608 1.16237990 1.42730929 2.0337
##  [7] 1.61057840 0.02478305 0.80259162 1.77695345
```

Normal (Gaussian) Distribution

- Parameters: μ (mean) and σ^2 (variance);
- Notation: $X \sim \mathcal{N}(\mu, \sigma^2)$;
- Support: $\mathbb{R}$
- PDF:

$$f(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- Mean and Variance: $\mathbb{E}(X) = \mu$, $Var(X) = \sigma^2$.
- Models: A lot of things. The idea is the following: usually, when we are estimating something, we give that estimate in the form:

$$\text{estimate} = \text{true (unknown) value} + \text{error}$$

  And this `error` is, supposedly, close to 0. And the distribution of `error`, which is a random term, is usually taken as Normal (with the Mean $= 0$). Also it is important because of the CLT

Normal (Gaussian) Distribution

- **R** name: `norm` with the parameters `mean = 0, sd = 1`
- Example:

```
rnorm(10, mean = 2, sd = 3)
```

```
##  [1]  1.801376  3.331193  3.089171  3.242166  3.184141
##  [8]  8.128281  5.333089 -1.075895
```

Note:

- **R** is using $\mathcal{N}(mean, sd)$ format
- **In Math** we are using the $\mathcal{N}(mean, variance)$ format

So if you want to generate a sample of size 100 from $\mathcal{N}(2, 9)$, use the command `rnorm(100, mean = 2, sd = 3)`.

Normal (Gaussian) Distribution

Additional Properties:

- If $X \sim \mathcal{N}(\mu, \sigma^2)$, then $Z = \dfrac{X - \mu}{\sigma} \sim \mathcal{N}(0, 1)$ and

$$\mathbb{P}(a < X < b) = \mathbb{P}\left(\frac{a-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{b-\mu}{\sigma}\right) =$$

$$= \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right).$$

Normal (Gaussian) Distribution

- If $Z \sim \mathcal{N}(0,1)$, then $X = \mu + \sigma \cdot Z \sim \mathcal{N}(\mu, \sigma^2)$
- If $X_k \sim \mathcal{N}(\mu_k, \sigma_k^2)$ for $k = 1, ..., n$ are independent, then $X_1 + X_2 + ... + X_n \sim \mathcal{N}(\mu_1 + \mu_2 + ... + \mu_n, \sigma_1^2 + \sigma_2^2 + ... + \sigma_n^2)$.
- If $X \sim \mathcal{N}(\mu, \sigma^2)$, then

$$\mathbb{P}(\mu - \sigma < X < \mu + \sigma) \approx 0.6827,$$

$$\mathbb{P}(\mu - 2\sigma < X < \mu + 2\sigma) \approx 0.9545$$

and

$$\mathbb{P}(\mu - 3\sigma < X < \mu + 3\sigma) \approx 0.9973.$$