$y_1 = 3\cos\left(3t + \frac{\pi}{4}\right)$ und $y_2 = 2\sin\left(3t - \frac{\pi}{5}\right)$

Mathe-Test 3

Aufgabe I

$y_1 = 3\sin\left(3t + \frac{\pi}{4} + \frac{\pi}{2}\right) = 3\sin\left(3t + \frac{3\pi}{4}\right)$

1) $\underline{A_1} = 3 \cdot e^{i\frac{3\pi}{4}}$, $\underline{A_2} = 2 \cdot e^{-i\frac{\pi}{5}}$.

2) $\underline{A_1} = 3 \cdot \left(\cos\left(\frac{3\pi}{4}\right) + i \cdot \sin\left(\frac{3\pi}{4}\right)\right) = 3 \cdot \left(-\frac{\sqrt{2}}{2} + i \cdot \frac{\sqrt{2}}{2}\right)$

$= -\frac{3\sqrt{2}}{2} + i\,\frac{3\sqrt{2}}{2}$

$\underline{A_2} = 2\left(\cos\left(-\frac{\pi}{5}\right) + i \cdot \sin\left(-\frac{\pi}{5}\right)\right) = 2 \cdot (0{,}809 + i \cdot -0{,}588)$

$= 1{,}618 + i \cdot 1{,}176$

3) $\underline{A} = \underline{A_1} + \underline{A_2} = \left(-\frac{3\sqrt{2}}{2} + i \cdot \frac{3\sqrt{2}}{2}\right) + (1{,}618 + i \cdot 1{,}176) =$

$= \left(-\frac{3\sqrt{2}}{2} + 1{,}618\right) + i\left(\frac{3\sqrt{2}}{2} + 1{,}176\right) = -0{,}503 + i\,3{,}297$

4) $\underline{A} = A \cdot e^{i\varphi}$

$\underline{A} = x + iy$

$\Rightarrow A = \sqrt{x^2 + y^2} \approx \sqrt{-0{,}503^2 + 3{,}297^2}$

$\approx 3{,}258 \,|\, 3{,}336$

$\varphi = \arctan\left(\frac{y}{x}\right) + \pi \approx \arctan(-6{,}555) + \pi$

$\approx 1{,}722$

$\Rightarrow y(t) = 3{,}258 \cdot \sin(3t + 1{,}722)$

Aufgabe 2. Bestimmen Sie die Orthogonalprojektion von

$\vec{u}\begin{pmatrix}5\\-1\\3\end{pmatrix}, \vec{v}\begin{pmatrix}-3\\2\\-2\end{pmatrix}$ | $\vec{u}_{\vec{v}} = \frac{\vec{u}^T\vec{v}}{|\vec{v}|^2}\vec{v}$

$$\vec{u}_{\vec{v}} = \frac{-15-2-6}{17}\begin{pmatrix}-3\\2\\-2\end{pmatrix} = \frac{-23}{17}\begin{pmatrix}-3\\2\\-2\end{pmatrix} = \begin{pmatrix}\frac{69}{17}\\-\frac{46}{17}\\\frac{46}{17}\end{pmatrix}$$

$$\vec{v}_{\vec{u}} = \frac{-23}{35}\begin{pmatrix}5\\-1\\3\end{pmatrix} = \begin{pmatrix}-\frac{23}{7}\\\frac{23}{35}\\-\frac{69}{35}\end{pmatrix}$$

$\vec{u} = \begin{pmatrix}2\\-2\\3\end{pmatrix}, \vec{v} = \begin{pmatrix}-4\\-3\\1\end{pmatrix}$

$\vec{w} = \begin{pmatrix}-4\\3\\-2\end{pmatrix}$

a) $\vec{u}\cdot\vec{v} = 2\cdot -4 - 2\cdot -3 + 3\cdot 1 = 1$

c) $2\cdot\vec{v}\times(-1)\cdot\vec{w} = \begin{pmatrix}-8\\-6\\2\end{pmatrix}\times\begin{pmatrix}4\\-3\\2\end{pmatrix}$

$$= \begin{pmatrix}-6\cdot 2 - 2\cdot(-3)\\2\cdot 4-(-8)\cdot 2\\-8\cdot(-3)-(-6)\cdot 4\end{pmatrix} = \begin{pmatrix}-6\\24\\48\end{pmatrix}$$

d) $-3\vec{u} - \vec{v} + 4\vec{w}$

$-3\vec{u} = \begin{pmatrix}-6\\6\\-9\end{pmatrix}, -\vec{v} = \begin{pmatrix}4\\3\\-1\end{pmatrix}, 4\vec{w} = \begin{pmatrix}-16\\12\\-8\end{pmatrix}$

$$= \begin{pmatrix}-6+4-16\\6+3+12\\-9-1-8\end{pmatrix} = \begin{pmatrix}-18\\21\\-18\end{pmatrix}$$

b) $\|\vec{w}-\vec{u}\|^2$ $\quad \vec{w}-\vec{u} = \begin{pmatrix}-4-2\\3+2\\-2-3\end{pmatrix} = \begin{pmatrix}-6\\5\\-5\end{pmatrix}$

$\|\vec{w}-\vec{u}\|^2 = (-6)^2+5^2+(-5)^2 = 86$

$= ((-4)^2+3^2+(-2)^2) - ((-2)^2+2^2+(-3)^2) = 12$

$\vec{u} = \begin{pmatrix}-2\\-3\\3\end{pmatrix}, \vec{v} = \begin{pmatrix}8\\-8\\6\end{pmatrix}, \vec{w} = \begin{pmatrix}0\\0\\3\end{pmatrix}$

$(\vec{u}-\vec{v})\circ(\vec{v}-\vec{w})$

$\vec{uv} = \begin{pmatrix}-10\\5\\-3\end{pmatrix}, \vec{vw} = \begin{pmatrix}10\\-5\\3\end{pmatrix}, \vec{uv}\circ\vec{vw} = -10\cdot 10+5\cdot(-5)-3\cdot 3 = -134$

$\|\vec{w}\| = \sqrt{0^2+0^2+3^2} = \sqrt{3^2} = 3$

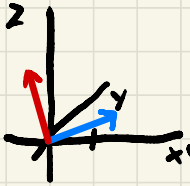
$-2\vec{u} - \frac{1}{2}\vec{v} + 2\vec{w}$ $\quad -2\vec{u} = \begin{pmatrix}4\\6\\-6\end{pmatrix}, -\frac{1}{2}\vec{v} = \begin{pmatrix}-4\\4\\-3\end{pmatrix}, 2\vec{w} = \begin{pmatrix}0\\0\\6\end{pmatrix}$

$$= \begin{pmatrix}4-4+0\\6+4+0\\-6-3+6\end{pmatrix} = \begin{pmatrix}0\\10\\-3\end{pmatrix}$$

$$-\vec{v} \times 4\vec{w} = \begin{pmatrix} 8\cdot -12 - 6\cdot 0 \\ -6\cdot 0 + 8\cdot(-12) \\ -8\cdot 0 - 8\cdot 0 \end{pmatrix} = \begin{pmatrix} -96 \\ -96 \\ 0 \end{pmatrix}$$

$$-\vec{v} = \begin{pmatrix} -8 \\ 8 \\ -6 \end{pmatrix} \times 4\vec{w} = \begin{pmatrix} 0 \\ 0 \\ -12 \end{pmatrix} \checkmark$$

$$\vec{u} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} \circ \begin{pmatrix} 2 \\ -3 \\ 0 \end{pmatrix} = 3\cdot 2 + 2\cdot(-3) + 0\cdot 0 = \underline{\underline{0}}$$



0,5
0,5
0,5

$$\vec{v} = \begin{pmatrix} 2 \\ -3 \\ 0 \end{pmatrix}, \vec{w} = \begin{pmatrix} 0 \\ 0 \\ -8 \end{pmatrix}$$

$(a,b,0), (b,-a,0), (0,0,c)$

$$v_1 = \begin{pmatrix} 5 \\ 3 \\ 7 \end{pmatrix}; v_2 = \begin{pmatrix} -5 \\ 3 \\ 0 \end{pmatrix}; v_3 \checkmark$$

1. In neuen Vektor, x und y vertauscht einsetzen und dann im neuen y negieren. Dann z = 0.
2. Kreuzprodukt aus beiden Vektoren = dritter ⊥ Vektor.

$$\begin{pmatrix} -18 \\ 12 \\ 2 \end{pmatrix} = \lambda_1 \begin{pmatrix} 4 \\ -4 \\ -2 \end{pmatrix} + \lambda_2 \begin{pmatrix} -5 \\ 2 \\ -1 \end{pmatrix}$$

$$-18 = \lambda_1 \cdot 4 + \lambda_2 \cdot (-5) \quad | +18$$
$$12 = \lambda_1 \cdot (-4) + \lambda_2 \cdot 2 \quad | -12$$
$$2 = \lambda_1 \cdot (-2) + \lambda_2 \cdot (-1) \quad | -2$$

I + II Additionsverfahren

$$4\lambda_1 - 5\lambda_2 + 18 + (-4)\lambda_1 + 2\lambda_2 - 12 = 0$$

$$-3\lambda_2 + 6 = 0 \quad | -6; :(-3)$$

$$\lambda_2 = \frac{-6}{-3} = \frac{-2}{-1} = 2$$

$$4\lambda_1 - 5\lambda_2 + 18 = 0 \quad (I)$$
$$-4\lambda_1 + 2\lambda_2 - 12 = 0 \quad (II)$$
$$-2\lambda_1 + -1\lambda_2 - 2 = 0 \quad (III)$$

λ_2 in (III) einsetzen: $-2\lambda_1 + (-1)\cdot 2 - 2 = 0 \quad | +2\lambda_1; :2$

$$\frac{-1\cdot 2 - 2}{2} = \lambda_1$$

$$-2 = \lambda_1$$

$$\begin{pmatrix} -8 \\ 8 \\ 4 \end{pmatrix} + \begin{pmatrix} -10 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} -18 \\ 12 \\ 2 \end{pmatrix} \checkmark$$

14. Übungsblatt

$\vec{a} = \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix}, \vec{b} = \begin{pmatrix} -1 \\ 5 \\ 8 \end{pmatrix}, \vec{c} = \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix}$ | $\underline{x} = \begin{pmatrix} 10 \\ 25 \\ 15 \end{pmatrix}$ $m = \begin{pmatrix} 10 \\ 0 \\ 2 \end{pmatrix}$ $y = \begin{pmatrix} 9 \\ 30 \\ 23 \end{pmatrix}$ $\underline{u} = \begin{pmatrix} -27 \\ -90 \\ -69 \end{pmatrix}$

1.

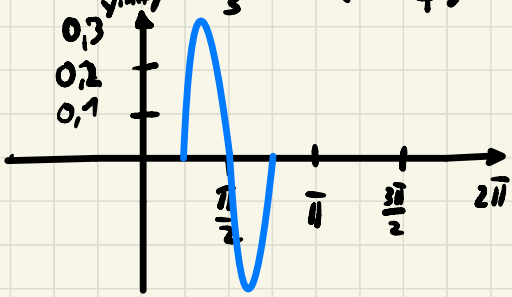
a) $\vec{v}_1 = \underbrace{5\vec{a}}_{x} - 3\underbrace{(\underbrace{\vec{b}}_{u} + \underbrace{2\vec{c}}_{m})}_{y} + 3\underbrace{(\underbrace{2\vec{a}}_{z} - \underbrace{\vec{c}}_{k})}_{j}$ | $z = \begin{pmatrix} 4 \\ 10 \\ 6 \end{pmatrix}$ $k = \begin{pmatrix} -1 \\ 10 \\ 5 \end{pmatrix}$ $\underline{j} = \begin{pmatrix} -3 \\ 30 \\ 15 \end{pmatrix}$ | $|\vec{v}_1| = \sqrt{6^2 + 65^2 + 35^2} \approx 74$

$x + u + j = \begin{pmatrix} 10 - 1 - 3 \\ 25 + 10 + 30 \\ 15 + 5 + 15 \end{pmatrix} = \begin{pmatrix} 6 \\ 65 \\ 35 \end{pmatrix} = \vec{v}_1$ | $\cos\alpha = \frac{6}{74} =$

Klausurvorbereitungsaufgaben:

11) ges.: A, T, φ $\qquad y = \frac{1}{3}\sin\left(3x - \frac{\pi}{4}\right)$

$A = \frac{1}{3} \qquad T = 3 \qquad \varphi = -\frac{\pi}{4}$



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12) $\underbrace{\sin(2x-5) = \frac{\sqrt{3}}{2}}$ $\quad |\arcsin();\ +5;\ :2| = x = \frac{\arcsin\left(\frac{\sqrt{3}}{2}\right)+5}{2} \approx 3{,}0236$

Photomath: $\Leftrightarrow 2x - 5 = \frac{\pi}{3}$ & $2x - 5 = \frac{2\pi}{3}$ // wegen Einheitenkreis, die Winkel finden wo $\sin(x) = \frac{\sqrt{3}}{2}$

$5\cos(2x) = a \ (a \in [1,-1]) \quad |\arccos();\ :5;\ :2| = x = \frac{\arccos\left(\frac{a}{5}\right)}{2} \ (a \in [1,-1])$

$\sin\left(\frac{x}{2}+3\right) = \frac{1}{5} \quad |\arcsin();\ \cdot 2;\ -3| = x = \left(\arcsin\left(\frac{1}{5}\right) - 3\right)\cdot 2 = -5{,}537234158$

$\tan(3x) = 15 \quad |\arctan();\ :3| = x = \frac{\arctan(15)}{3} = 0{,}5014093877$

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10) $P(x) = 1 + x^2 - 4x^5$ & $Q(x) = 1 + x + x^2 - x^3$

a) $P(x) + Q(x) = -4x^5 - x^3 + 2x^2 + x + 2$

b) $P(x) \times Q(x) = 1 + x + x^2 - x^3 + x^2 + x^3 + x^4 - x^5 - 4x^5 - 4x^6 - 4x^7 + 4x^8$
$= 4x^8 - 4x^7 - 4x^6 - 5x^5 + x^4 + 2x^2 + x + 1$

c) $\frac{P(x)}{Q(x)}$

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13) Beträge; Argumente; konjugiert komplex; von Normalform in trigonometrische & Exponentielle

a) $2 - 3i \quad |r = |z| = \sqrt{x^2 + y^2} = \sqrt{2^2 + (-3)^2} = \sqrt{13} \approx 3{,}61 \quad |\bar{z} = x - iy \Rightarrow \bar{a} = 2 + 3i$

$\arg(z) = \varphi = \{$look in big table $\Rightarrow \arg(a) = \arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{-3}{2}\right) = -0{,}98 = \varphi = \arg$

Exponentielle Form von $\mathbb{C}$: $r \cdot e^{i\varphi} = 3{,}61\, e^{-0{,}98i}$

Trigonometrische Form von $\mathbb{C}$: $3{,}61\left(\cos(-0{,}98) + i\sin(-0{,}98)\right)$

b) $\frac{3}{5}+\frac{4}{5}i$ | $|b| = \sqrt{\frac{3^2}{5}+\frac{4^2}{5}} = 1 = r$ | $\bar{b} = \frac{3}{5}-\frac{4}{5}i$ | Exp.Form: $1e^{0{,}927i}$

$\arg(b) = \varphi = \arctan\left(\frac{4/5}{3/5}\right) \approx 0{,}927$ | Trigo.Form: $1(\cos(0{,}927)+i\sin(0{,}927))$

14) a) $z_1 = 3;\ z_2 = 4i$ | $\bar{z}_1 = 3$ | $\varphi_{z_1} = \arctan\left(\frac{0}{3}\right) = 0$ | $3e^{i}$ oder $3e^{0}$?

$\bar{z}_2 = 4$ | $\varphi_{z_2} = \frac{\pi}{2}$ | $4e^{\frac{\pi}{2}i}$ | $z_1 \cdot z_2 = 12e^{\frac{\pi}{2}i} \Rightarrow \left.\begin{matrix} r\cdot\cos(\varphi) = x = 12\cdot\cos\left(\frac{\pi}{2}\right) = 0 \\ r\cdot\sin(\varphi) = y = 12\cdot\sin\left(\frac{\pi}{2}\right) = 12 \end{matrix}\right\} = \underline{\underline{12i}}$

$z_1/z_2 = \frac{3}{4}e^{-\frac{\pi}{2}i} \underset{s.o.}{\Rightarrow} \left.\begin{matrix} x=0 \\ y=-\frac{3}{4} \end{matrix}\right\} = -\frac{3}{4}i$ | $z_1^5 (r^n, n\cdot\varphi) = 243e^{0i} \underset{s.o.}{\Rightarrow} \left.\begin{matrix} x=243 \\ y=0 \end{matrix}\right\} = \underline{\underline{243}}$

$z_2^{10} = 1048576e^{5\pi i} \Rightarrow \left.\begin{matrix} x=-1048576 \\ y=0 \end{matrix}\right\} = -1048576$ | b) $z_1 = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$ | $z_2 = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

$\bar{z}_1 = 1$ | $\varphi_{z_1} = \arctan\left(\frac{-\sqrt{2}/2}{\sqrt{2}/2}\right) = -\frac{1}{4}\pi$ | $1e^{-\frac{1}{4}\pi i}$ | $\bar{z}_2 = 1$ | $\varphi_{z_2} = \frac{1}{3}\pi$ | $1e^{\frac{1}{3}\pi i}$

$z_1 \cdot z_2 = 1e^{\frac{1}{12}\pi i} \Rightarrow \left.\begin{matrix} x = \frac{\sqrt{6}+\sqrt{2}}{4} \\ y = \frac{\sqrt{6}-4\sqrt{2}}{4} \end{matrix}\right\} = \frac{\sqrt{6}+\sqrt{2}}{4} + \frac{\sqrt{6}-\sqrt{2}}{4}i$

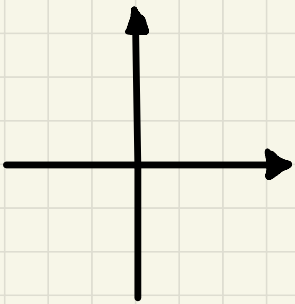
$z_1/z_2 = 1e^{-\frac{3}{12}\pi i} \Rightarrow \left.\begin{matrix} x = \frac{-\sqrt{6}+\sqrt{2}}{4} \\ y = -\frac{\sqrt{6}+\sqrt{2}}{4} \end{matrix}\right\} = \frac{-\sqrt{6}+\sqrt{2}}{4} - \frac{\sqrt{6}+\sqrt{2}}{4}i$

$z_1^5 = 1e^{-\frac{5}{4}\pi i} \Rightarrow \left.\begin{matrix} x = -\frac{\sqrt{2}}{2} \\ y = \frac{\sqrt{2}}{2} \end{matrix}\right\} = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ | $z_2^{10} = 1e^{\frac{10}{3}\pi i} \Rightarrow \left.\begin{matrix} x = -\frac{1}{2} \\ y = -\frac{\sqrt{3}}{2} \end{matrix}\right\} -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

15) a) Komplexer Zeiger | Amplitude | Komplexe Amplitude | Grafisch

$y(t) = 3\sin(2t)$ | $3e^{i(2t+\varphi)} \Leftrightarrow 3e^{2ti}$ | $3e^{0i}$

komplexer Zeiger? | komplexe A?

$y(t) = 2\sin(3t+\pi)$ | $2e^{3t+\pi i}$ | $2e^{\pi i}$

$y(t) = \cos\left(t+\frac{\pi}{4}\right) = \sin\left(t+\frac{\pi}{4}+\frac{\pi}{2}\right) = \sin\left(t+\frac{3}{4}\pi\right)$ | $e^{t+\frac{3}{4}\pi i}$ | $e^{\frac{3}{4}\pi i}$