

# Wildfire Ignition Model

## 1 Fuel Moisture Model

### 1.1 Rainfall-to-Moisture Conversion Model

In our model, each grid cell is assigned a rainfall amount, and we convert this rainfall into a fuel moisture content (FMC) using a simple, literature-based linear approximation. The purpose of this section is to define the conversion and document its empirical justification.

#### Model formulation

Let  $P_{ij}$  denote the rainfall amount (in millimetres) applied to grid cell  $(i, j)$ . We assume an initial fuel moisture content  $M_0$ , and compute the updated moisture value as

$$M_{ij} = \min(M_{\max}, M_0 + s_{\text{mm}} P_{ij}),$$

where

- $M_0 = 15.4\%$  is the average pretreatment moisture content reported by Mohr and Waldrop (2008),
- $s_{\text{mm}} \approx 0.14\%/\text{mm}$  is the experimentally derived slope describing the gain in moisture per millimetre of rainfall,
- $M_{\max}$  is a prescribed upper limit representing the maximum achievable moisture in this simplified model.

#### Empirical justification

Our linear coefficient  $s_{\text{mm}}$  is not arbitrarily chosen. It is derived directly from the rainfall–moisture results reported in Mohr and Waldrop (2008), who applied controlled rainfall treatments to 1000-hour woody fuels.

Their study reports:

- initial moisture  $M_0 = 15.4\%$ ,
- four separate 1-inch rainfall events over one week (totaling  $4'' = 101.6 \text{ mm}$ ) increased moisture to approximately 32%,

- two 2-inch rainfall events over one week (also totaling 4") increased moisture to approximately 27.1%.

The corresponding moisture increases are

$$\Delta M_1 = 32 - 15.4 = 16.6\%, \quad \Delta M_2 = 27.1 - 15.4 = 11.7\%.$$

Dividing by the total rainfall in each treatment gives slopes

$$s_1 = \frac{16.6}{101.6} \approx 0.163\%/mm, \quad s_2 = \frac{11.7}{101.6} \approx 0.115\%/mm.$$

We therefore adopt their average

$$s_{mm} = \frac{s_1 + s_2}{2} \approx 0.14\%/mm.$$

### Python implementation

The following Python-style pseudocode illustrates how this conversion can be applied in a grid-based simulation:

```
M0 = 15.4          # Initial moisture (%), from Mohr and Waldrop (2008)
s_mm = 0.14        # Moisture gain per mm of rain (%/mm)
M_max = 50.0       # Example upper bound (%), user-defined

for each cell (i, j):
    P_ij = random_rain_mm()      # Random rainfall amount in mm
    M_ij = M0 + s_mm * P_ij
    if M_ij > M_max:
        M_ij = M_max
```

This produces a moisture field  $M_{ij}$  suitable for use in subsequent fire spread modelling.

## 1.2 Probabilistic fire spread as a function of fuel moisture

In the deterministic version of our cellular automaton, cells with fuel moisture exceeding the extinction moisture  $M_{ext}$  simply cannot ignite. To incorporate a more realistic treatment of fire behaviour, we now introduce a probabilistic ignition rule in which the probability of ignition decreases smoothly as fuel moisture increases.

### Literature motivation

Laboratory ignition experiments (e.g. Anderson, 1969) show that fine dead fuels have very high ignition probabilities when moisture contents are below about 10%, while ignition probability drops rapidly as moisture approaches the extinction moisture (typically 20–25% for Australian eucalypt litter). Catchpole

and Catchpole (1991) proposed a widely used *moisture damping function* of the form

$$\eta_M = \left(1 - \frac{M}{M_{\text{ext}}}\right)^2,$$

which appears in several rate-of-spread and ignition probability models. This function smoothly decreases from 1 (completely dry) to 0 (at the extinction moisture), and is consistent with Australian operational fire behaviour observations.

### Ignition probability model

We define the ignition probability for cell  $(i, j)$  with fuel moisture  $M_{ij}$  as

$$p_{\text{ignite}}(M_{ij}) = \begin{cases} p_0 \left(1 - \frac{M_{ij}}{M_{\text{ext}}}\right)^2, & M_{ij} \leq M_{\text{ext}}, \\ 0, & M_{ij} > M_{\text{ext}}, \end{cases}$$

where  $p_0$  is the maximum ignition probability (e.g.  $p_0 = 1$ ) and we take  $M_{\text{ext}} = 25\%$  in accordance with Australian studies of eucalypt litter fuels.

### Probabilistic spread rule

A cell that is currently unburned at time  $t$  ignites at time  $t + 1$  with probability  $p_{\text{ignite}}(M_{ij})$  provided that at least one neighbour is burning. Otherwise it remains unburned. Formally,

$$\mathbb{P}(S_{ij}^{t+1} = \text{burning}) = p_{\text{ignite}}(M_{ij}) \mathbf{1}[\exists(k, \ell) \in \mathcal{N}(i, j) : S_{k\ell}^t = \text{burning}].$$

### Python-style implementation

```
M_ext = 25.0      # Extinction moisture (%) for eucalypt litter
p0      = 1.0      # Maximum ignition probability

def p_ignite(M):
    if M > M_ext:
        return 0.0
    x = 1 - M / M_ext
    return p0 * (x * x)

for each cell (i, j):
    if state[i, j] == 'unburned':
        if any_neighbour_burning(i, j):
            prob = p_ignite(M[i, j])
            if random() < prob:
                state_next[i, j] = 'burning'
            else:
```

```

        state_next[i, j] = 'unburned'
    else:
        state_next[i, j] = 'unburned'
    elif state[i, j] == 'burning':
        state_next[i, j] = 'burned'
    else:
        state_next[i, j] = 'burned'

```

This probabilistic formulation allows partially wet areas to impede—but not necessarily completely block—fire spread, producing more realistic and spatially variable fire behaviour driven by the moisture field.

## 2 Vegetation Flammability Coefficients

Different fuel types ignite at different rates even under identical moisture and wind conditions. To incorporate these effects without arbitrary tuning, we assign each vegetation class  $V$  a dimensionless flammability coefficient  $\alpha_V \in (0, 1]$  derived directly from published Australian headfire rates of spread (ROS). This ensures that  $\alpha_V$  is grounded in real fire behaviour data rather than subjective selection.

### 2.1 Empirical headfire spread rates from literature

Australian fire behaviour research provides representative ROS values for the three major fuel complexes relevant to our model:

- **Grassland (cured grass):** typically 1.0–2.0 m/s under dry, windy conditions. (McArthur grassland model; Cheney & Sullivan 2008)

$$R_{\text{grass}} \approx 1.5 \text{ m/s.}$$

- **Shrubland / heath:** typically 0.4–1.0 m/s depending on wind and fuel load. (CSIRO shrubland experiments; McCaw & Cheney)

$$R_{\text{shrub}} \approx 0.7 \text{ m/s.}$$

- **Dry eucalypt forest (litter-driven):** surface ROS usually 0.1–0.5 m/s under the same conditions. (Cheney & Gould 1995–2010; Cruz & Alexander 2018)

$$R_{\text{eucalypt}} \approx 0.25 \text{ m/s.}$$

These values encode the empirically supported flammability ordering

$$\text{grass} > \text{shrub} > \text{dry eucalypt forest},$$

which is consistent across decades of Australian field studies.

## 2.2 From continuous ignition rate to discrete ignition probability

To connect continuous fire spread dynamics to our grid-based ignition model, we treat the advance of a headfire as a Poisson process with ignition (hazard) rate

$$\lambda_V = \frac{R_V}{L},$$

where  $L$  is the characteristic distance needed for fire to “cross into” the next cell. Since our goal is to obtain a dimensionless relative coefficient, we set  $L$  to an arbitrary positive constant (which cancels out; see below).

The probability of ignition over a small conceptual time window  $\Delta t$  is given by the standard hazard-integral formula:

$$p_V = 1 - e^{-\lambda_V \Delta t}.$$

For sufficiently small  $\Delta t$  (which is always the case in grid-based models), we use the first-order approximation

$$p_V \approx \lambda_V \Delta t = \frac{R_V}{L} \Delta t.$$

Since both  $\Delta t$  and  $L$  are common to all fuel types, we absorb the constant factor  $\Delta t/L$  into a global normalisation constant  $C$ :

$$p_V = C R_V.$$

Finally, we scale the probabilities so that grassland has unit flammability:

$$\alpha_V = \frac{p_V}{p_{\text{grass}}} = \frac{R_V}{R_{\text{grass}}}.$$

Hence, the vegetation coefficients are

$$\alpha_V = \frac{R_V}{R_{\text{grass}}}$$

## 2.3 Final values

Using the representative ROS values above,

$$\alpha_{\text{grass}} = 1, \quad \alpha_{\text{shrub}} = \frac{0.7}{1.5} \approx 0.47, \quad \alpha_{\text{eucalypt}} = \frac{0.25}{1.5} \approx 0.17.$$

These coefficients capture realistic differences in fuel flammability while remaining fully grounded in empirical ROS data.

## 2.4 Code example: computing $\alpha_V$

The vegetation coefficients can be computed directly from the ROS values:

```
R = {  
    "grass": 1.5,  
    "shrub": 0.7,  
    "eucalypt": 0.25  
}  
  
alpha = {k: v / R["grass"] for k, v in R.items()}  
print(alpha)
```

This produces:

$$\alpha_{\text{grass}} = 1, \quad \alpha_{\text{shrub}} \approx 0.47, \quad \alpha_{\text{eucalypt}} \approx 0.17.$$

## 3 Fire Exposure and Wind Effects

Wind strongly influences the likelihood that a burning neighbour will ignite a target cell. In our model, wind does not change the number of neighbours or introduce additional heat sources. Instead, it modifies how effective each burning neighbour is by increasing or decreasing the directional transfer of heat and flame. This section defines a dimensionless fire exposure index  $E_{ij}$  and its associated ignition modifier  $f_E(E_{ij})$ .

### 3.1 Wind representation

We represent wind at the grid scale using a vector

$$\mathbf{U} = (U_x, U_y),$$

where  $|\mathbf{U}|$  is the wind speed and its direction controls which neighbours are most effective at igniting the target cell.

For convenience we define the unit wind direction vector

$$\hat{\mathbf{U}} = \begin{cases} \mathbf{U}/|\mathbf{U}|, & |\mathbf{U}| > 0, \\ (0, 0), & |\mathbf{U}| = 0. \end{cases}$$

### 3.2 Directional influence from a burning neighbour

Let  $(k, \ell)$  be a burning neighbour of cell  $(i, j)$ . The neighbour-to-target direction is represented by

$$\hat{\mathbf{d}}_{(k, \ell) \rightarrow (i, j)},$$

a unit vector pointing from  $(k, \ell)$  to  $(i, j)$ . On a regular Moore neighbourhood grid this vector corresponds to one of the eight standard compass directions.

The alignment between the wind and this direction is quantified by the scalar product

$$s_{(k,\ell) \rightarrow (i,j)} = \hat{\mathbf{U}} \cdot \hat{\mathbf{d}}_{(k,\ell) \rightarrow (i,j)},$$

which lies in  $[-1, 1]$ :

- $s \approx 1$ : neighbour is downwind  $\Rightarrow$  strong promotion of spread;
- $s \approx 0$ : crosswind  $\Rightarrow$  neutral effect;
- $s < 0$ : upwind  $\Rightarrow$  ignition contribution suppressed.

We retain only the favourable part via truncation:

$$w_{(k,\ell) \rightarrow (i,j)} = \max(0, s_{(k,\ell) \rightarrow (i,j)}).$$

Thus wind redistributes the relative influence of burning neighbours without adding new heat sources.

### 3.3 Wind speed normalisation using real data

To avoid arbitrary scaling, we derive a wind-speed factor from Australian fire danger classifications. Operational fire danger ratings used by the Bureau of Meteorology (BOM) and AFAC identify the following characteristic wind-speed ranges:

- Moderate wind: 5–15 km/h,
- Strong wind: 15–25 km/h,
- Very strong wind: 25–40 km/h.

Winds above 35–40 km/h are consistently associated with “very dangerous” bushfire conditions due to extreme flame tilting and enhanced convective heat transfer. We therefore set

$$U_{\max} = 40 \text{ km/h}$$

as the normalisation threshold corresponding to maximal wind-driven influence. The wind-speed factor is then defined as

$$W_{\text{norm}} = \min\left(\frac{|\mathbf{U}|}{U_{\max}}, 1\right),$$

which maps wind speeds into  $[0, 1]$  with saturation in very strong winds. This approach ensures that the strength of wind in the model reflects operational fire danger categories rather than arbitrary numerical scaling.

### 3.4 Fire exposure index

For a target cell  $(i, j)$  with burning neighbours  $\mathcal{N}_{ij}^t$ , the wind-adjusted exposure is defined as

$$E_{ij}^t = \begin{cases} W_{\text{norm}} \frac{1}{|\mathcal{N}_{ij}^t|} \sum_{(k, \ell) \in \mathcal{N}_{ij}^t} w_{(k, \ell) \rightarrow (i, j)}, & |\mathcal{N}_{ij}^t| > 0, \\ 0, & |\mathcal{N}_{ij}^t| = 0. \end{cases}$$

This construction has the following interpretation:

- wind speed controls the *strength* of exposure ( $W_{\text{norm}}$ );
- wind direction controls *which neighbours matter* ( $w$ -weights);
- multiple neighbours increase exposure only if they lie in favourable directions;
- $E_{ij}$  always lies in  $[0, 1]$ .

Note that we intentionally do *not* model “heat accumulation” from multiple neighbours. Their influence appears solely through wind-controlled directional exposure.

### 3.5 Ignition modifier from exposure

The fire exposure factor enters the ignition model through a simple thresholded linear mapping:

$$f_E(E_{ij}) = \begin{cases} 0, & E_{ij} \leq E_{\min}, \\ \frac{E_{ij} - E_{\min}}{1 - E_{\min}}, & E_{ij} > E_{\min}, \end{cases}$$

where  $E_{\min} \in [0, 1)$  represents the minimal exposure needed for wind to meaningfully promote ignition.

### 3.6 Code example

The following Python snippet computes  $E_{ij}$  for a cell and its burning neighbours:

```
import numpy as np

def wind_norm(U, Umax=40.0):
    return min(U / Umax, 1.0)

def exposure(i, j, burning_neighbors, U_vec):
    U = np.linalg.norm(U_vec)
    if not burning_neighbors:
```

```

        return 0.0

    U_dir = U_vec / U if U > 0 else np.array([0.0, 0.0])
    w_sum = 0.0

    for (k, l) in burning_neighbors:
        d = np.array([i - k, j - l])
        d = d / np.linalg.norm(d)
        s = max(0.0, np.dot(U_dir, d))
        w_sum += s

    W = wind_norm(U)
    return W * (w_sum / len(burning_neighbors))

```

This produces an exposure value  $E_{ij} \in [0, 1]$  that is then passed to the ignition model via  $f_E(E_{ij})$ .