Math 1211 Discussion Assignment 4

Write an example of a function whose derivative can be found by using the following rules:

a) Product rule and special function differentiation rules

b) Power rule, quotient rule, and chain rule

c) Chain rule twice

d) Implicit differentiation and special function differentiation rule

a) Product rule

The product rule calculates the derivative of two function about x like f(x) and g(x)

$$\frac{d}{dx}(f(x)\,g(x)) = \frac{d}{dx}(f(x)) \cdot g(x) + \frac{d}{dx}(g(x)) \cdot f(x).$$

Example

To find derivative of f(x) = (x+2) (2x-1)

$$\frac{\mathrm{dy}}{\mathrm{dx}} = (x+2)'(2x-1) + (x+2)(2x-1)'$$
$$= 2x - 1 + 2x + 4 = 4x + 3$$

According to (Differentiating Special Functions – Calculus Tutorials, n.d.),

Special function differentiation rules as below

$f(x)$	$f'(x)$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \cot x$
$\cot x$	$-\csc^2 x$
$\ln x$	$\frac{1}{x}$
e^x	e^x

Image from (*Differentiating Special Functions – Calculus Tutorials*, n.d.)

Example

1) F(x) =e^x, then f'(x)= e^x
2) F(x) = csc(x)+ xtan(x)
   F'(x)=-cscxcotx + tanx + sec^2(x)
3) f(x)= lnx, f'(x)= 1/x

b) Power rule, quotient rule, and chain rule

According to (*3.3 Differentiation Rules - Calculus Volume 1 | OpenStax*, n.d.)

$$\frac{d}{dx}\left(x^2\right) = 2x \text{ and } \frac{d}{dx}\left(x^{1/2}\right) = \frac{1}{2}x^{-1/2}.$$

Power Rule Example

$$\frac{\mathbf{d}}{\mathbf{dx}}\boldsymbol{x^3} = \boldsymbol{3x^2}$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{\frac{d}{dx}(f(x)) \cdot g(x) - \frac{d}{dx}(g(x)) \cdot f(x)}{(g(x))^2}.$$

Example

F(x)= $\frac{3x^2+2}{4x+3} = \frac{6x(4x+3)-4(3x^2+2)}{(4x+3)^2} = \frac{12x^2+6x-8}{(4x+3)^2}$

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Example

H(x) = $(2x^2 + 2x)^2$

H'(x) = $2(2x^2+2x)(4x+2) = 2(8x^3+4x^2+8x^2+4x) = 16x^3 +12x^2+8x$

c) Chain rule twice

According to (3.6 The Chain Rule - Calculus Volume 1 | OpenStax, n.d.),

$$k'(x) = h'(f(g(x))\, f'(g(x))\, g'(x)).$$

$$k(x) = \cos^4(7x^2 + 1).$$

$G(x)=7x^2+1$

$H(x)=\cos x$

$F(x)=x^4$

$K'(x) = 4(\cos(7x^2+1))^3 (-\sin(7x^2+1))(14x)$

$= -56x\sin(7x^2+1) \cos^3(7x^2+1)$

d) Implicit differentiation and special function differentiation rule

Assuming that y is defined implicitly by the equation
$X^2+y^2=25$, find $\frac{dy}{dx}$.

$$\frac{d(x^2+y^2)}{dx} = \frac{d(25)}{dx}$$

$2x + 2y\frac{dy}{dx} = 0$ then $\frac{dy}{dx} = -\frac{x}{y}$

Reference

3.3 Differentiation Rules - Calculus Volume 1 | OpenStax. (n.d.). Retrieved September 18, 2022, from https://openstax.org/books/calculus-volume-1/pages/3-3-differentiation-rules

3.6 The Chain Rule - Calculus Volume 1 | OpenStax. (n.d.). Retrieved September 25, 2022, from https://openstax.org/books/calculus-volume-1/pages/3-6-the-chain-rule

Differentiating Special Functions – Calculus Tutorials. (n.d.). Retrieved September 25, 2022, from https://math.hmc.edu/calculus/hmc-mathematics-calculus-online-tutorials/single-variable-calculus/differentiating-special-functions/