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Non-Regular Languages

(Part 2)

Lecture 25

Day 27/31

CS 154

Formal Languages and Computability

Fall 2019

Agenda of Day 27

- Summary of Lecture 24
- Quiz 10
- Lecture 25: Teaching ...
  - Non-Regular Languages (Part 2)
- Hints about your term project

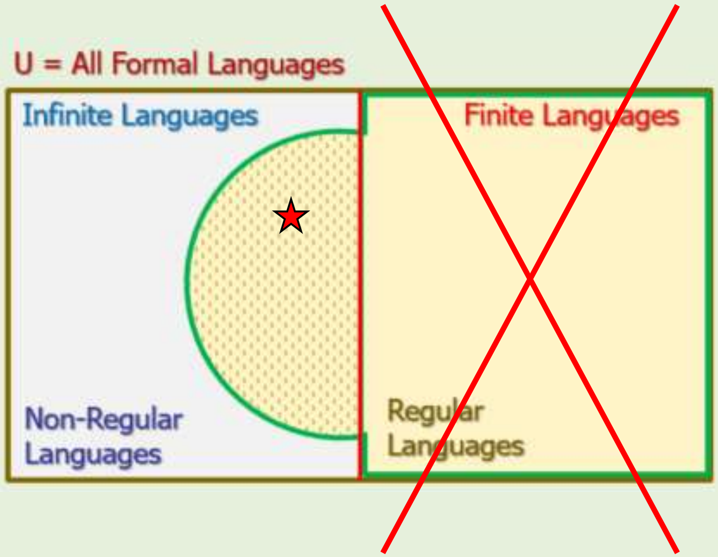
Summary of Lecture 24: We learned ...

Non-Regular Languages

- We started with this question:

  How to prove a language is NONREGULAR?

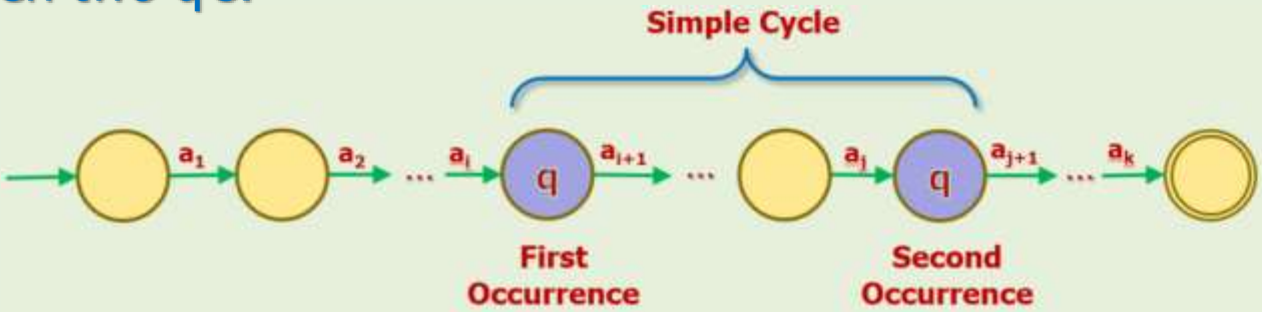
- We stated an important property of "infinite regular languages".



Summary of Lecture 24: We learned …

- We took L as a regular language.
- Since L is regular, then there is an DFA for it.
- Assume it has m states.
- Take a string $w = a_1\ a_2\ \dots\ a_k \in L$ whose size is $|w| \geq m$.
- Since $|w| \geq m$, based on pigeonhole principle, in the walk of w,

  at least one state is visited more than once.
- We called the first repeated-state as 'q'.
- We pick the q in such a way that there is no nested repeated-state between two q's.



Summary of Lecture 24: We learned ...

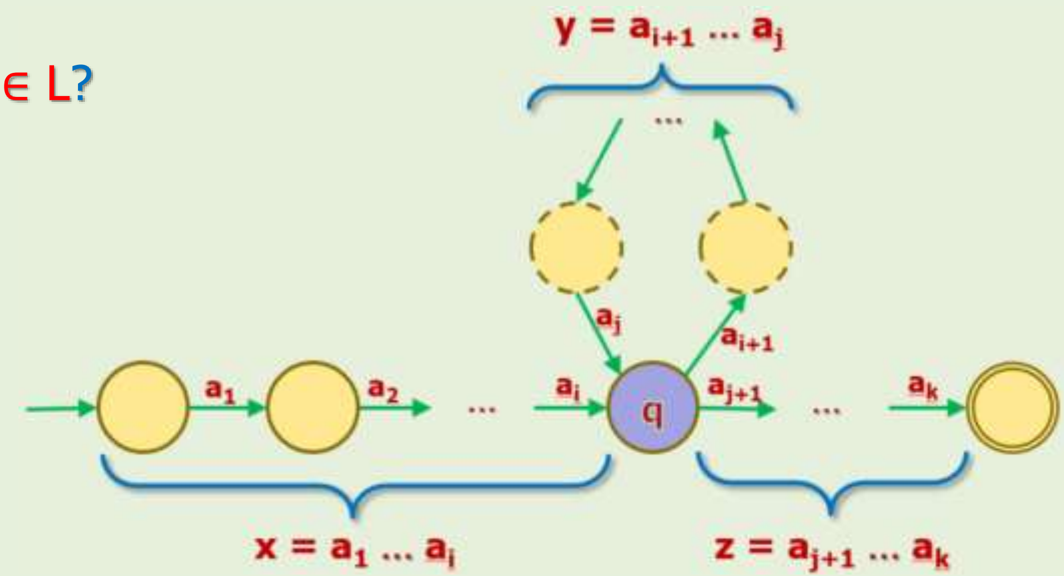
- The original DFA looks like the following figure:
- We named the 3 portions as: x y z

Questions

- $|xy| \leq m$?
- $|y| \geq 1$?
- $xz = a_1\ a_2\ \ldots\ a_i\ a_{j+1}\ \ldots\ a_k \in L$?
- How about $xyyz \in L$?
- Or, $xyyyz \in L$?
- Or in general:
  $x\ y^i\ z$, for $i = 0, 1, 2, \ldots$

- The answer is yes to all questions, so all strings $x\ y^i\ z \in L$.
- So, if some certain conditions are satisfied, we can pump any number of y's in the original string and the resulting string is still part of the language.

Any Question



Quiz 10

No Scantron

Pumping Lemma

Example 8

- Verify the pumping lemma property on the following infinite regular language.

$$L = \{bba^n : n \geq 0\}$$

Solution

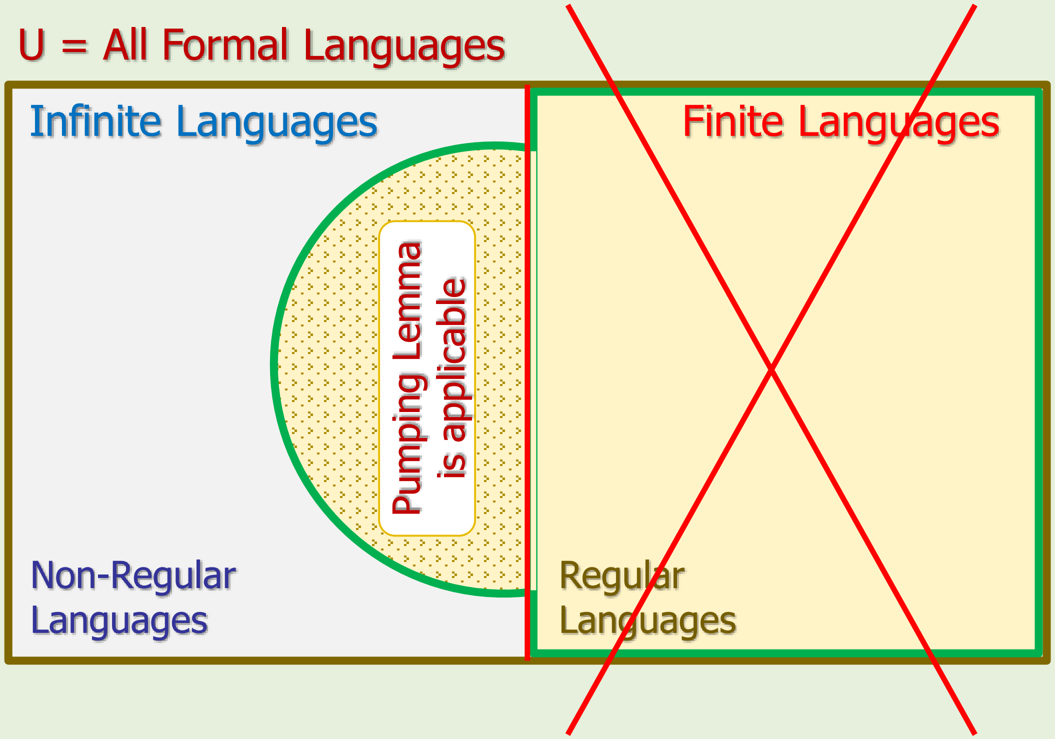
Homework

- Verify the pumping lemma property on the following infinite regular languages.

  1. $L = \{a^n b^k : n \geq 0, k \geq 0\}$
  2. $L = \{aaab^n (ab)^k : n \geq 0, k \geq 0\}$
  3. $L = \{(ab)^n : n \geq 0\}$

Conclusion

- This is a property of INFINITE REGULAR languages.



- If an "infinite language" does not have this property, it is "non-regular".

Pumping Lemma: Notes

1. A string like $a^m b$ is just one string of the language, and NOT a pattern because m is a constant.
2. In the previous example (#7), one could take w something like:
   - $a^{2m}b$ or $a^{m+100}b$
   - But, try to take it as simple as possible.
3. We should always make sure that no string gets negative power.
   - For example, if, somewhere in our proof, we have something like a^{m-3}, then we should mention "we pick $m \geq 3$".
   - Recall that pumping lemma has the power of making a boundary for 'm'.
4. But if you have something like a^{m-1}, you don't need to mention it because by default $m \geq 1$.

Application of Pumping Lemma

How to Prove a Language is Non-Regular?

- Use "proof by contradiction"

    1. Assume L is regular. So, the pumping lemma should hold for L.
    2. Apply pumping lemma
    3. Find a contradiction.
    4. Then, blame your assumption and conclude that L must be non-regular.

- Recall that all non-regular languages are infinite.

- Let's take some examples!

Applications of Pumping Lemma

Example 9

- Prove $L = \{a^n b^n : n \geq 0\}$ is non-regular language.

Proof

Applications of Pumping Lemma

Example 10

- Prove $L = \{uu : u \in \{a, b\}^*\}$ is non-regular language.

Proof

Homework

- Prove that the following languages are non-regular:
  1. $L = \{uu^R : u \in \{a, b\}^*\}$
  2. $L = \{a^n b^n c^n : n \geq 0\}$
  3. $L = \{uuu : u \in \{a, b\}^*\}$
  4. $L = \{a^n b^k c^{n+k} : n \geq 0, k \geq 0\}$

More Notes About Pumping Lemma

1. Pumping lemma is difficult to understand! [Text book, P#121]

   NOT anymore! 

2. Pumping lemma is not applicable to finite languages.

   Because we need to pump infinite y's!

3. Pumping lemma cannot prove that a languages is regular.

   Because you'd need to verify infinite strings!

Hints About Your Term Project

References

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3. Costas Busch's website: http://csc.lsu.edu/~busch/
4. Michael Sipser, "Introduction to the Theory of Computation, 3rd ed.," CENGAGE Learning, United States, 2013 ISBN-13: 978-1133187790