A Low-Cost High-Accuracy Laser Rangefinder Using Frequency-Modulated Continuous-Wave

Final Report

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DECLARATION

I have read and understood the College and Department's statements and guidelines concerning plagiarism.

I declare that all material described in this report is all my own work except where explicitly and individually indicated in the text. This includes ideas described in the text, figures and computer programs.

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Abstract

The aim of this project is to build a low-cost, high-accuracy laser rangefinder using a simple laser pointer. A laser rangefinder can measure the distance between two objects by measuring the time difference between the transmitted signal and the reflected signal given the speed of light. This project proposes to modulate a Frequency-Modulated Continuous-Wave onto a laser to measure the distance. A frequency-modulated signal will introduce frequency difference when there is a time delay, measuring this frequency difference can lead us to the distance between two objects. Digital signal processing techniques are used to acquire the frequency difference and improve accuracy as well as resolution. The constructed laser rangefinder has an outstanding performance: an accuracy of more than 99% and a resolution of 1mm for distance measurements up to 12m. The distance measurements are limited by the size of the laboratory, and the maximum measuring distance is predicted to be 36m.

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1. Introduction

In daily life, there are scenarios when people want to know the distance between two objects, such as measuring the size of a room or the depth of a hole. There are many methods available, such as using a measuring tape, a ruler or a laser rangefinder if a more accurate reading is required. The laser rangefinder operates on the time-of-flight (TOF) principle: a narrow beam is emitted from the laser and reflected by the object, by measuring the round-trip time taken by this narrow beam and multiplying it with the speed of light, the distance between the rangefinder and the target can be found [1].

Laser rangefinders are widely used in civilian, industrial and military applications, and the measuring range varies in different products. Typically, military applications have longer measuring range than civilian and industrial products. For example, when measuring the yardage of a shot on a golf course or measuring the level in a container, the measuring range is about tens of meters; while in circumstances such as providing distance information for a sniper, the range is relatively long (several kilometres) [2]. Despite high accuracy of laser rangefinders in the market, the price of them is rather expensive. The price of a laser rangefinder increases with the measuring range, resolution as well as accuracy, and can vary from tens of dollars to hundreds of dollars. In the literature, there are many previous successful attempts at making a laser rangefinder. However, the trade-off between measuring range and resolution is usually present. In addition, the experimental setup is complicated. Therefore, the objective of this project is to design and build a low-cost laser rangefinder with high accuracy using a simple laser pointer.

There are two main parts of this project, one is to build the transmitter and receiver, another part is to develop a signal processing algorithm for comparing the transmitted and received signals in order to produce a precise distance reading. Fig. 1.1 demonstrates basic elements of the laser rangefinder system which have been developed in this project. MATLAB is used to synthesise the required Frequency-Modulated Continuous-Wave (FMCW) to be generated using an arbitrary waveform generator which in turn is sent to the transmitter. FMCW is then modulated onto a laser intensity output and is sent from the transmitter. The receiver collects the light reflected by the object, which is converted to an electrical signal with a photodiode. The received signal is sent to an oscilloscope, and data is acquired by MATLAB for further signal processing.

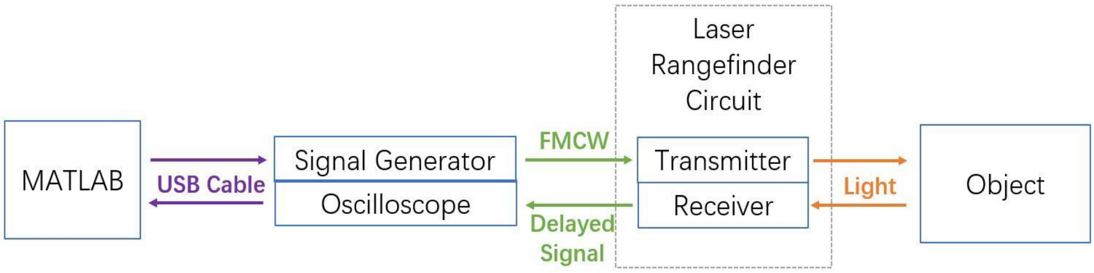

Fig. 1.1. Block diagram of the proposed laser rangefinder system.

In the following sections, I will first review some previous work. Then I will explain the theory of proposed methods and also the hardware configuration. Next, I will demonstrate the results and relevant parameter specification. I will conclude by analysing the system performance and discussing limitations of this work as well as potential future improvements.

2. Literature Review

2.1 Phase-Shifted Method

Journet and Poujouly used a phase-shift approach to measure the time delay [3]. A laser is modulated by a sinusoidal signal, and the time difference is measured by the phase difference using a phase meter. The phase difference is given by:

$$\Delta\varphi = \omega_0\tau_d = 2\pi f_0\tau_d \tag{2.1}$$

where f_0 is the frequency of the transmitted signal, τ_d is the time of flight. If we use distance (d) and speed of light (c) to express τ_d, we can have:

$$\tau_d = \frac{2d}{c} \tag{2.2}$$

Therefore, the distance between objects can be expressed using the phase difference:

$$d = \Delta\varphi \frac{c}{4\pi f_0} \tag{2.3}$$

and the resolution of the system is given by:

$$\delta d = \delta\varphi \frac{c}{4\pi f_0} \tag{2.4}$$

If a good resolution is required, then the modulating frequency f_0 needs to be high. However, phase meters are not able to measure a phase difference between such high-frequency signals. Journet *et al.* [4] proposed the following method: define signal at emission $s_e = A_{m1}\cos(2\pi f_0 t)$, received signal $s_r = A_{m2}\cos(2\pi f_0 t + \Delta\varphi)$, and another reference signal $s_{ref} = A_{m0} cos(2\pi f_1 t + \Psi)$. Multiply the emitted signal and received with reference signal respectively, we can get:

$$X_e = s_e s_{ref} = A_{m1}A_{m0}\cos(2\pi f_0 t)\, cos(2\pi f_1 t + \Psi)$$

$$= \frac{1}{2}A_{m1}A_{m0}\{\cos[2\pi(f_0 - f_1)t - \Psi] + \cos[2\pi(f_0 + f_1)t + \Psi]\} \tag{2.5}$$

$$X_r = s_r s_{ref} = A_{m2}\, A_{m0}(2\pi f_0 t + \Delta\varphi)\cos(2\pi f_1 t + \Psi)$$

$$= \frac{1}{2}A_{m2}A_{m0}\{\cos[2\pi(f_0 - f_1)t + (\Delta\varphi - \Psi)]$$

$$+ \cos[2\pi(f_0 + f_1)t + (\Delta\varphi + \Psi)]\,\} \tag{2.6}$$

If a low-pass filter is used to get rid of the high-frequency components from above signals, we can have:

$$X_e{}' = \frac{1}{2}A_{m1}A_{m0}\cos[2\pi(f_0 - f_1)t - \Psi] \tag{2.7}$$

$$X_r{}' = \frac{1}{2}A_{m2}A_{m0}\cos[2\pi(f_0 - f_1)t - \Psi + \Delta\varphi] \tag{2.8}$$

If we choose the value of f_0 and f_1 carefully to ensure $f_0 - f_1$ is a relatively small value, then the phase meter is capable of measuring the phase difference $\Delta\varphi$ between two signals $X_e{}'$ and $X_r{}'$.

Fig. 2.1 demonstrates the laser rangefinder system using this phase-shift method. There are three main parts: an optical head, an electronic measurement system and a PC. The electronic measurement system measures the phase difference between the transmitted and received signals, and a PC is functioned as a user interface. In the electric measuring system, active mixers, phase-locked loops (PLL) and voltage-controlled oscillators (VCO) are used to generate stable signals with modulating frequency f_0 and reference frequency f_1.

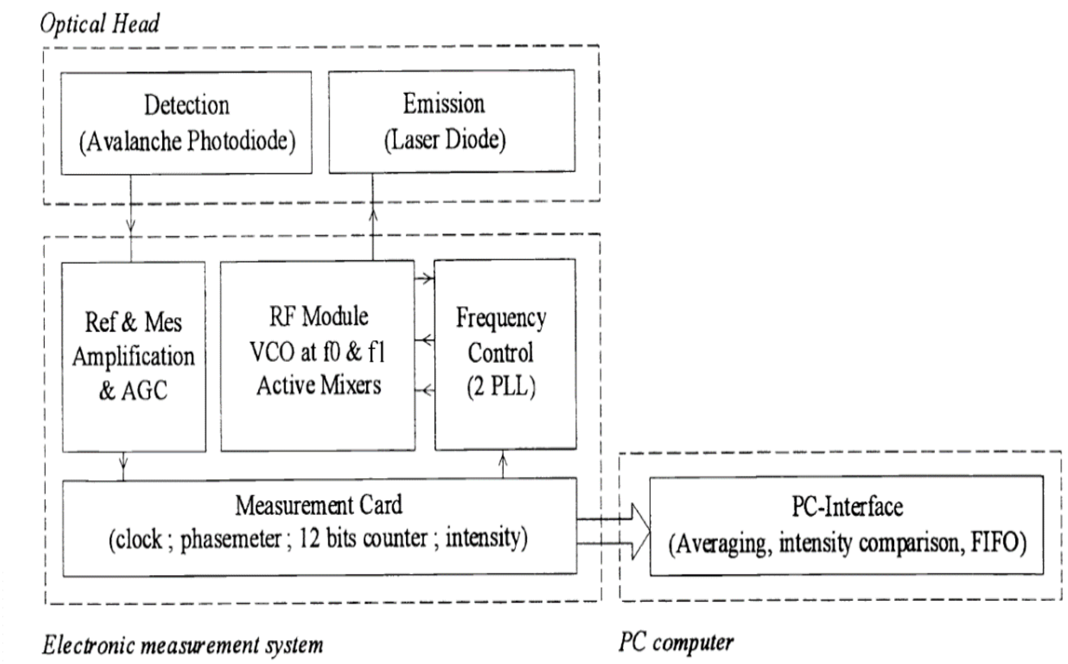

Fig. 2.1. Block diagram of the laser rangefinder system using phase-shift method [3].

As shown in Fig. 2.1, amplification is needed as phase detection requires signals ideally with $1V_{pp}$. Automatic Gain Control (AGC) is used to control the amplitude of the received signal which will be multiplied by the reference signal. The reason for using an AGC is that the intensity of the received signal depends on the measuring range, so the output of AGC can maintain a constant voltage level despite the changing amplitude of AGC input. Therefore, AGC can generate a constant amplitude for received signal which will be multiplied by the reference signal. A measurement card is used to measure the phase difference and to communicate with the computer.

The optical head is shown in Fig. 2.2, which consists of an emitter (Laser Diode or LD) and a detector (Avalanche Photodiode or APD). A focusing lens is used to converge the signal reflected by the object so that the receiver can detect it.

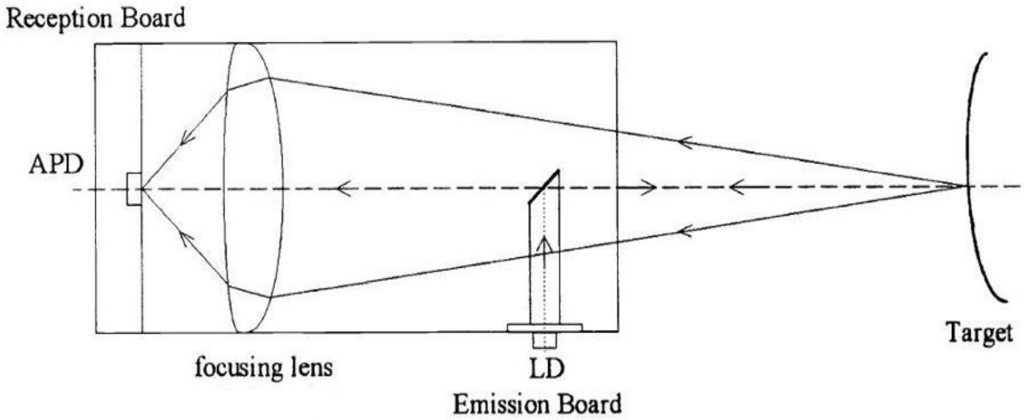

Fig. 2.2. The optical head of a phase-shifted laser rangefinder [3].

Yoon and Park [5] also designed a laser rangefinder system using the phase-shifted method. Instead of making the difference between modulating frequency f_0 and reference frequency f_1 very small, they let $f_0 = f_1$, equation (2.8) then becomes:

$$X_r'' = \frac{1}{2} A_{m1} A_{m2} \cos(\Delta\varphi - \Psi) \tag{2.9}$$

The phase difference can be determined by the amplitude of X_r'' given A_{m1}, A_{m2} and Ψ are known parameters. One disadvantage of this approach is that the system is vulnerable to noise since noise can easily affect the amplitude of X_r''.

However, the drawback of the phase-shifted theory itself limits the system performance. Using sinusoidal signals, the phase-shifted method is not able to tell if the distance exceeds the no-ambiguity-range (NAR, the distance when the phase difference is 2π), as phase difference $\Delta\varphi$ is the same as $\Delta\varphi + 2k\pi$ for sinusoidal waves (k is an integer). The expression for the no-ambiguity-range can be derived from equation (2.3):

$$d_{nar} = \Delta\varphi \frac{c}{4\pi f_0} = 2\pi \frac{c}{4\pi f_0} = \frac{c}{2f_0} \tag{2.10}$$

Therefore, f_0 need to be small to get a longer measuring range for the system. However, from equation (2.4) we know the resolution is: $\delta d = \delta\varphi \frac{c}{4\pi f_0}$, f_0 need to be large to obtain a good resolution, and this is the trade-off needs to be balanced. The results from the work done by Journet and Poujouly [3][4][6] demonstrated that either the resolution can reach as good as several millimetres but at a cost of short measuring range (several meters), or the measuring range is tens of meters while the resolution is several centimetres.

2.2 Time-to-Digital Converter Method

Besides Journet's approach, Nissinen *et al.* used a time-to-digital converter (TDC) for building a laser rangefinder [7]. As mentioned before, the core of TOF principle is to measure the time difference between the transmitted signal and received signal. The time difference between two signals can be easily measured by counting the number of cycles of the reference clock. However, the resolution is limited by the clock frequency, and the measurement is not precise enough given the available clock frequency. Therefore, in order to improve resolution, multiphase ring-oscillator-based TDC is introduced.

Fig. 2.3 demonstrates a timing diagram for an 8-phase ring-oscillator-based TDC. One clock cycle is divided into 8 sections with different phases. The operation principle of such TDC is to record the phase of the signal. The resolution improves by 8 times as there are 8 phase slots in one clock period.

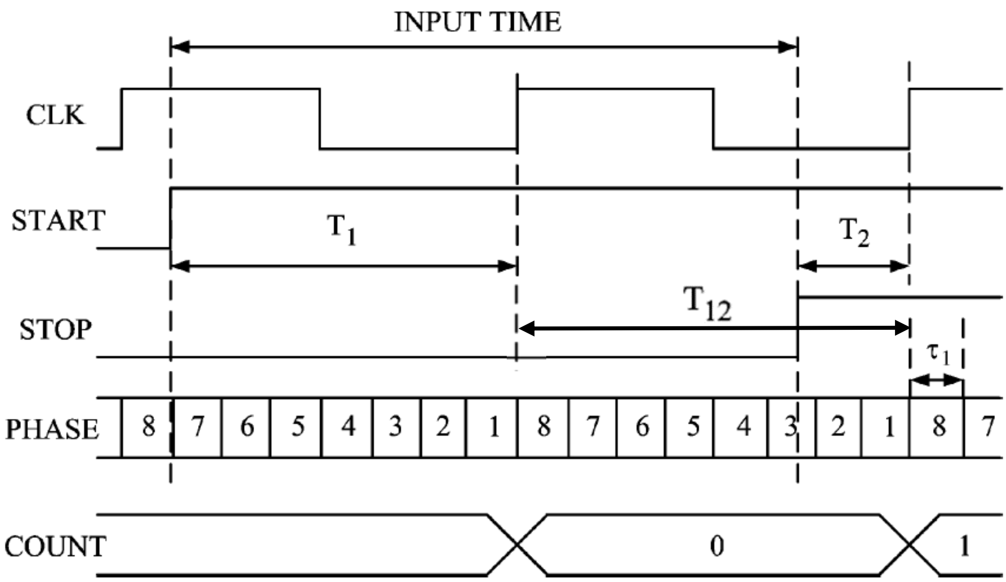

Fig. 2.3. The timing diagram of an 8-phase ring-oscillator-based TDC [7].

In Fig. 2.3, CLK represents the reference clock, START stands for the transmitted signal and STOP is received signal. A counter (COUNT) starts from the next rising edge of the CLK after START signal and terminates at next rising edge of the CLK after STOP signal, and the time interval is denoted as T_{12}. T_1 and T_2 are the time between the beginning of the signal and the next rising edge of CLK for START and STOP respectively. From Fig. 2.3, the time difference T_d can be expressed as:

$$T_d = T_1 + T_{12} - T_2 \tag{2.11}$$

Fig. 2.4 shows the structure of an 8-phase ring-oscillator-based TDC. This TDC consists of a counter (to measure T_{12}), an 8 ring oscillators (OSC) to generate signals with different phases, two registers (START and STOP) to store the state of the oscillator (for measuring T_1 and T_2 respectively), and a calibration counter to compensate for supply voltage and temperature drifts. Buffers are also used to control the output.

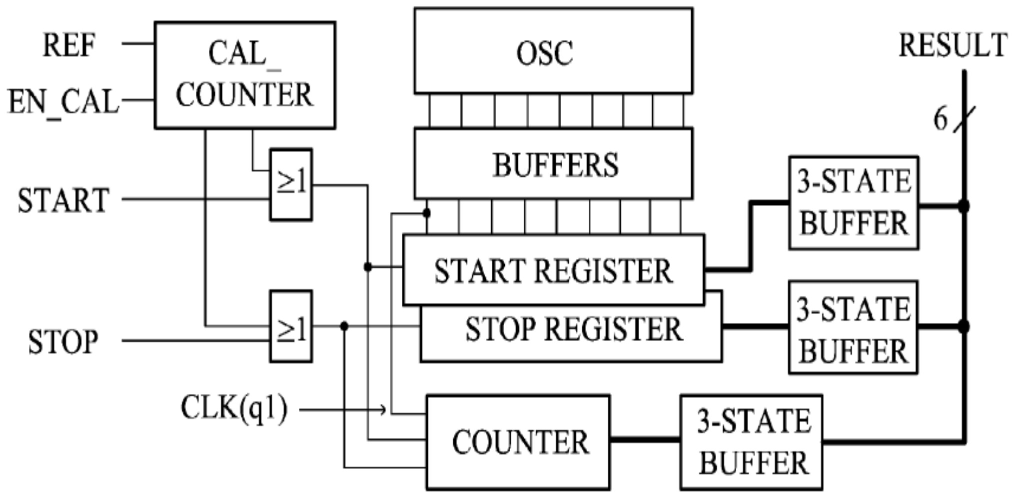

Fig. 2.4. Structure of an 8-phase ring-oscillator-based TDC [7].

Another issue is how to mark the time when the reflected signal arrives at the receiver. The use of a leading-edge time discriminator can realize this function. Fig. 2.5 demonstrates how a time discriminator detects the arrival of both large and small signals. A threshold voltage needs to be defined, and the time when the threshold voltage is achieved is defined as the signal arrival time. In Fig. 2.5, we can notice that there is a time difference between the actual arrival time and the measured arrival time, and this time error is called walk error. In Fig. 2.5, Δt_1 and Δt_2 are introduced by the finite bandwidth of the receiver, and t_g is caused by the finite rise of the laser when the signal amplitude is small.

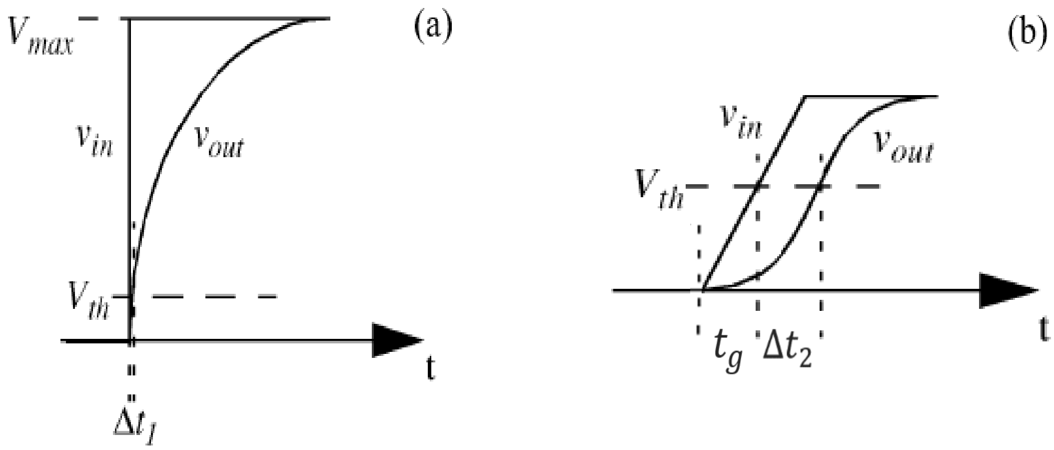

Fig. 2.5. Leading-edge time detection for (a) large signals (b) small signals [8].

The walk error needs to be compensated in order to get an accurate measurement. Fig. 2.6 demonstrates the principle of compensation. The waveforms in Fig. 2.6 are idealized (assume linearity), which is an approximation of the true signal. In order to compensate for the walk error, two time-discriminators are used, one with threshold voltage V_{thr} and another one with threshold voltage $C \times V_{thr}$, where C is a constant ($C \times V_{thr}$ is smaller than the peak voltage V_p of small signals).

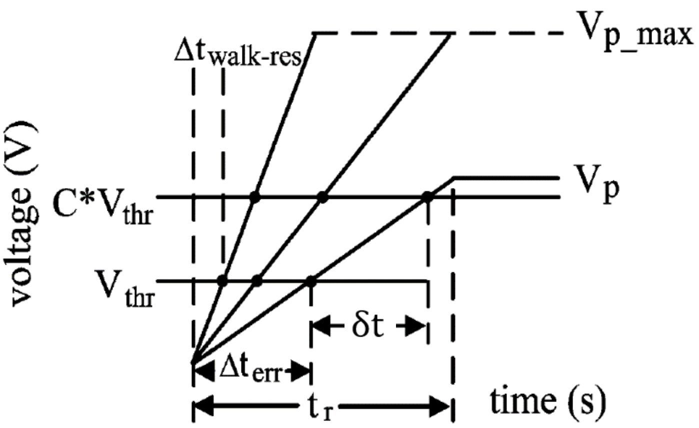

Fig. 2.6. The principle of compensating for a walk error [7].

Walk error is denoted as Δt_{err}, and δt is the time for a signal to achieve $C \times V_{thr}$ from V_{thr}. Using geometrical analysis, Δt_{err} can be expressed as:

$$\Delta t_{err} = \frac{\delta t}{C-1} \tag{2.12}$$

As C is a constant and δt can be measured, Δt_{err} can be calculated and then compensated. The time resolution of this laser rangefinder is therefore defined as δt, using geometrical analysis, δt can be written as:

$$\delta t = \frac{(C-1)\cdot t_r \cdot V_{thr}}{V_p} \tag{2.13}$$

If we assume $(C-1)\cdot t_r \cdot V_{thr}$ is a constant, then $\delta t \cdot V_p$ is also a constant. V_p is the amplitude of the received signal, and the longer the measuring distance, the smaller the V_p. The resolution and measuring range trade-off appears. If the resolution is good (small δt), then the amplitude of the received signal need to be large (measuring range is small) and vice versa.

Nissinen *et al.* integrated the components of the receiver onto a chip and fabricated it using 130nm CMOS process. Fig. 2.7 shows a block diagram of the receiver. The optical signal is converted to electrical signal using an APD; capacitors and amplifiers are used to get a less-noise and lager-amplitude signal. Then the signal is sent into two time-discriminators to compensate for the walk error. A TDC is used to measure the time difference between START (transmitted signal) and STOP (received signal).

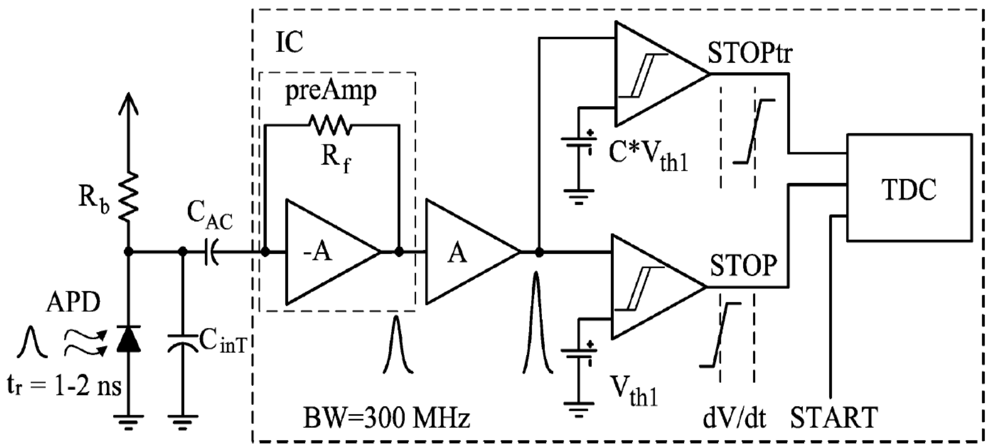

Fig. 2.7. Block diagram of the TDC laser rangefinder receiver [7].

The resolution of this product is high (better than 3.9mm), however, measuring range is comparability low (around 10m). Using a similar technique, Palojärvi *et al.* were able to achieve 30m measuring range, but at a cost of relatively poor resolution (35mm) [8]. There are other disadvantages of this TDC approach. First, although the current CMOS technology has already evolved to 10nm process, many academic institutes are not capable of manufacturing 130nm process circuit which Palojärvi *et al.* had built. Another disadvantage is the pulse carving technique. The optical signals are pulses with high power and narrow pulse width (only a few nanoseconds). In order to a get a very narrow pulse, mode-locked laser or other bulky and expensive devices are needed, which can be costly and difficult to implement if these items of equipment are not available in the laboratory.

3. Theory

3.1 Proposed Technique: Frequency-Modulated Continuous-Wave

As mentioned in the literature review, there are several disadvantages of previous approaches, especially the trade-off between the measuring range and resolution. Therefore, in this project, a new technique for building a laser rangefinder will be investigated. The new method should be easy to implement, the major equipment should be handy, low-cost and ought to be found easily in an undergraduate laboratory. The trade-off between the measuring range and resolution needs to be avoided or at least well balanced. In terms of performance, the new technique should have a relatively long measuring range (0-20m). The resolution is expected to be better than 5mm and the accuracy is expected to achieve more than 95%.

Based on the expected outcome, FMCW is proposed to modulate directly the intensity of a laser pointer. There are many types of FMCW, in this project, a linear form of FMCW which is a chirp signal is used. The frequency of a chirp signal increases linearly (up-chirp) or decreases linearly (down-chirp) with time. FMCW is widely used for target detection, especially in radar and sonar systems. The changing frequency widens the bandwidth and narrows the time-domain signal after pulse compression. More details regarding the reasons for choosing a chirp signal will be presented at the end of this section.

The frequency and time relationship of a chirp signal is shown in Fig. 3.1. The chirp rate k is defined as the slope of the frequency-time plot: $k = \frac{f_{max}-f_{min}}{T}$, where T is the time duration of the chirp signal. The time domain representation of chirp signal is given by:

$$y(t) = sin[\phi_0 + 2\pi(f_{min}t + \frac{k}{2}t^2)] \tag{3.1}$$

where ϕ_0 is the initial phase at time t = *0*.

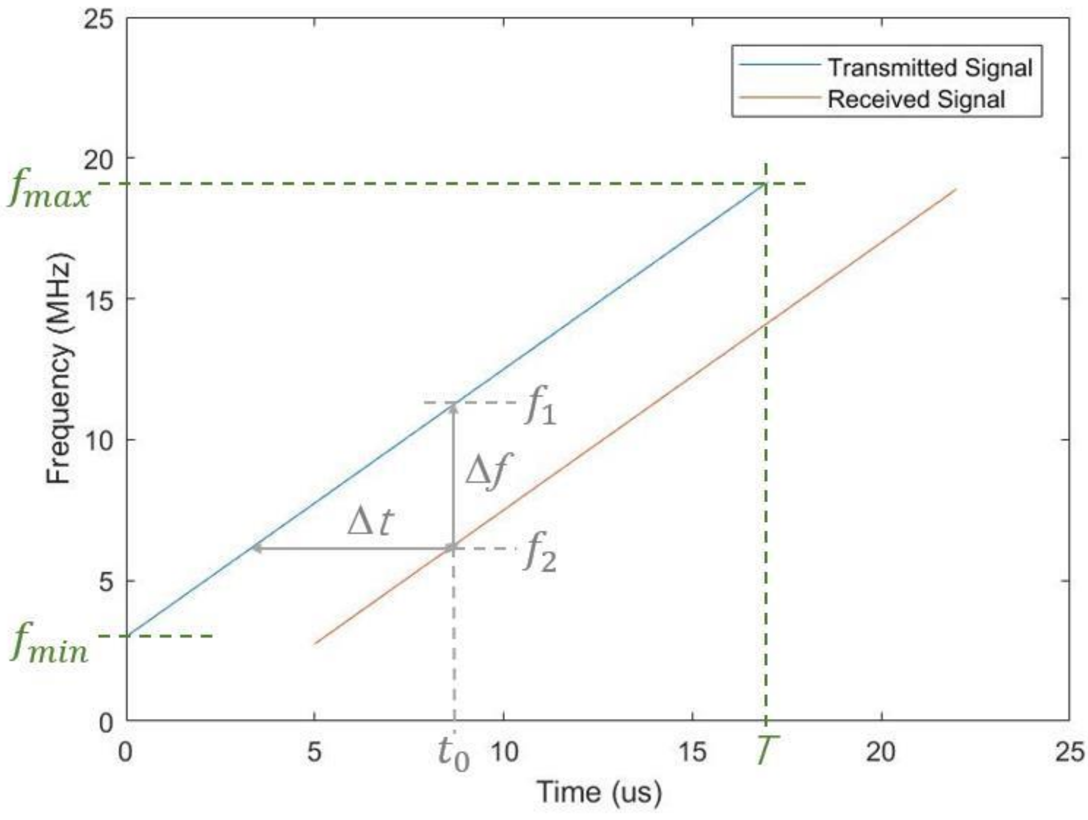

Fig. 3.1. Chirp signal technique used in the project.

Fig. 3.1 also illustrates the technique using a chirp signal which is proposed for this project. In Fig. 3.1, the blue curve represents the transmitted signal and orange curve represents the received signal. Two signals differ by Δt in the time domain, which can be measured using the chirp rate and the frequency difference:

$$\Delta t = \frac{\Delta f}{k} \tag{3.2}$$

Now the question is how to get the frequency difference Δf. The method used is called frequency deramping, which means multiplying the transmitted signal and the received signal together. As shown in Fig. 3.1, for a specific time t_0, frequency deramping yields:

$$c(t_0) = \sin(2\pi f_1 t_0) \times \sin(2\pi f_2 t_0) \tag{3.3}$$

Equation (3.3) can be further expanded using trigonometric identities:

$$c(t_0) = \frac{1}{2}\{cos[2\pi(f_1 - f_2)t_0] - cos[2\pi(f_1 + f_2)t_0]\}$$

$$c(t_0) = \frac{1}{2}\{cos[2\pi\Delta f t_0] - cos[2\pi(f_1 + f_2)t_0]\} \tag{3.4}$$

In equation (3.4), two components are present at any given point in time: one is the frequency difference (Δf), the other one is the sum of two instantaneous frequencies ($f_1 + f_2$). By performing Fourier Transform of equation (3.4), we can get the frequency spectrum as demonstrated in Fig. 3.2.

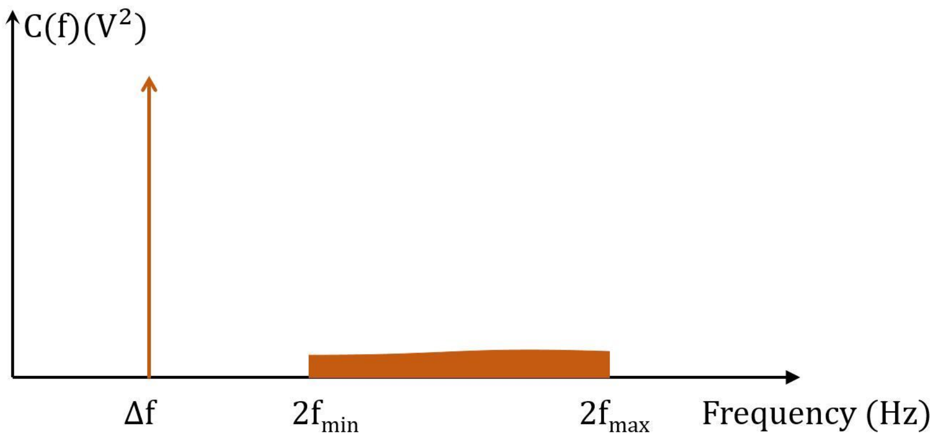

Fig. 3.2. The frequency spectrum of the de-ramped signal.

Two frequency components have the same energy. Therefore, in Fig. 3.2 there is a spike in Δf components (frequency difference is the same at any point in time). Frequency sum component $f_1 + f_2$ has a much smaller amplitude compared to Δf as the energy of them are the same while $f_1 + f_2$ has a wider bandwidth. The frequency sum is different at different points in time and ranges from $2f_{min}$ to $2f_{max}$ ($2f_{min}$ and $2f_{max}$ is the boundary case when there is no delay). Therefore, if we can find the frequency that corresponds to the maximum value in the frequency spectrum, Δf will be found; and by using equation (3.2): $\Delta t = \frac{\Delta f}{k}$, we can get time delay Δt as well as the distance between two objects.

The proposed method is actually inspired by the matched filter theory, whose main feature is to maximise the signal-to-noise ratio (SNR). The impulse response of a matched filter is determined by a specific signal which will achieve the maximum SNR possible [9]. Fig. 3.3 shows a block diagram of a matched filtering system. The input of the matched filter is the sum of signal $s(t)$ and Additive White Gaussian Noise (AWGN, power spectral density of it is $\frac{N_0}{2}$). The output consists of both signal $s_o(t)$ and noise $n_o(t)$.

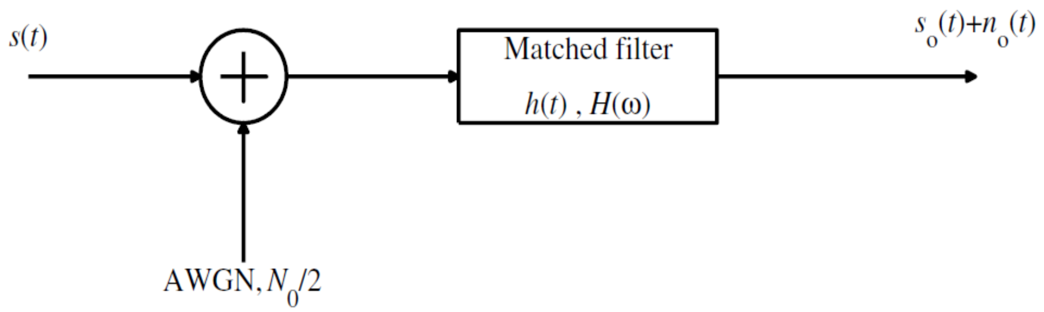

Fig. 3.3. Block diagram of a matched filtering system [9].

The matched filter with an impulse response $h(t)$ or a transfer function $H(\omega)$ will maximise the SNR at the output of the filter given a predetermined delay t_0, which means we need to maximise the following expression:

$$SNR = \frac{|s_o(t_0)|^2}{\overline{n_o{}^2(t)}} \tag{3.5}$$

The output signal $s_o(t_0)$ can be written as:

$$s_o(t_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)S(\omega)\exp(j\omega t_0)\,d\omega \tag{3.6}$$

where $S(\omega)$ is the Fourier Transform of $s(t)$. The average or mean-squared value of noise is independent of *t*, and can be expressed as:

$$\overline{n_o{}^2(t)} = \frac{N_0}{2}\cdot\frac{1}{2\pi}\int_{-\infty}^{\infty}|H(\omega)|^2\,d\omega = \frac{N_0}{4\pi}\int_{-\infty}^{\infty}|H(\omega)|^2\,d\omega \tag{3.7}$$

Using equation (3.6) and (3.7), substitute $s_o(t_0)$ and $\overline{n_o{}^2(t)}$ into (3.5) we can have:

$$SNR = \frac{\left|\int_{-\infty}^{\infty} H(\omega)S(\omega)\exp(j\omega t_0)d\omega\right|^2}{\pi N_0\int_{-\infty}^{\infty}|H(\omega)|^2 d\omega} \tag{3.8}$$

In order to get the maximum SNR, Schwarz inequality is used. Schwarz stated that for two complex signals $M(\omega)$ and $N(\omega)$, the following inequality is true:

$$\left|\int_{-\infty}^{\infty} M(\omega)N(\omega)\,d\omega\right|^2 \leq \int_{-\infty}^{\infty}|M(\omega)|^2\,d\omega\int_{-\infty}^{\infty}|N(\omega)|^2\,d\omega \tag{3.9}$$

The equality holds if and only if $M(\omega) = KN^*(\omega)$, where K is a constant and $N^*(\omega)$ is the conjugate of $N(\omega)$. By comparing equation (3.9) with the nominator of (3.8), choosing $M(\omega) = H(\omega)$ and $N(\omega) = S(\omega)\exp(j\omega t_0)$ will yield the maximum SNR:

$$SNR_{max} = \frac{\int_{-\infty}^{\infty}|H(\omega)|^2 d\omega\int_{-\infty}^{\infty}|S(\omega)\exp(j\omega t_0)|^2 d\omega}{\pi N_0\int_{-\infty}^{\infty}|H(\omega)|^2\,d\omega} = \frac{1}{\pi N_0}\int_{-\infty}^{\infty}|S(\omega)|^2 d\omega$$

$$= \frac{2}{N_0}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty}|S(\omega)|^2\,d\omega\right] = \frac{2}{N_0}\int_{-\infty}^{\infty}|s(t)|^2 dt = \frac{2E_s}{N_0} \tag{3.10}$$

where E_s is the energy of the signal ($E_s = \int_{-\infty}^{\infty}|s(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}|S(\omega)|^2\,d\omega$, which can be proved using Parseval's theorem). Equation (3.10) is true if and only if:

$$H(\omega) = KS^*(\omega)\exp(-j\omega t_0) \tag{3.11}$$

where K is a constant. Equation (3.11) is then defined as the transfer function of the matched filter. By performing inverse Fourier Transform of equation (3.11), we can get the impulse response of the matched filter:

$$H(t) = Ks^*(t_0 - t) \tag{3.12}$$

Essentially, the impulse response of a matched filter is the delayed and mirrored version of the conjugate of the input signal. This is the reason for naming it matched filter as it matches the input signal. The matched filter output at other delays can be derived by convolving the input signal and the matched filter impulse response given $t_0 = 0$:

$$s_o(t) = s(t) * h(t) = \int_{-\infty}^{\infty} s(\tau)\, h(t-\tau) d\tau$$

$$= \int_{-\infty}^{\infty} s(\tau)\, Ks^*[t_0 - (t-\tau)] d\tau = \int_{-\infty}^{\infty} s(\tau)\, Ks^*(\tau - t) d\tau \qquad (3.13)$$

Equation (3.13) is also known as the autocorrelation function. The first step of the above-proposed technique for this project is to multiply the received and transmitted signal together (deramping), which is a mimic of matched filtering. In equation (3.13), $s(\tau)$ can be seen as the transmitted signal and $Ks^*(\tau - t)$ can be regarded as the received signal (a delayed version of the transmitted signal, K is the attenuation when the optical signal propagates in the air). Instead of using a matched filter, deramping behaves the same way as a matched filter.

Inspired by the matched filter, SNR of laser rangefinder can be maximised by using the deramping method. Fig. 3.4 demonstrates the effectiveness of the deramping method. In Fig. 3.4, the transmitted signal is a chirp signal; after adding time delay, channel attenuation and AWGN to the transmitted signal in MATLAB, the received signal is heavily degraded: the chirp signal waveform is not visible and is hidden under the noise floor of the oscilloscope. However, after deramping and other signal processing steps, the frequency spectrum still yields a peak which indicates Δf and Δt as well. Therefore, by using the concept of the matched filter, SNR can be maximised and the system is more tolerant to noise. Such a deramping process can be compared to the other spread spectrum systems used in communications.

As mentioned in the literature review, the main issue of the previous approach is their theoretical fault, namely, the theory itself introduces the trade-off between the measuring range and resolution. In the phase-shifted method, the period of the sinusoidal signal is the cause of the trade-off; in the proposed method, the time period of a chirp signal is also related to the measuring range, and the time duration can be set to a relatively long period. In the TDC method, the received signal intensity leads to the trade-off; in the chirp-signal method, this can be solved by deramping, as this mimic version of matched filtering is able to maximise the SNR. The resolution of this laser rangefinder can be improved by performing digital signal processing (DSP) techniques to the de-ramped signal (more on this later in section 4.4).

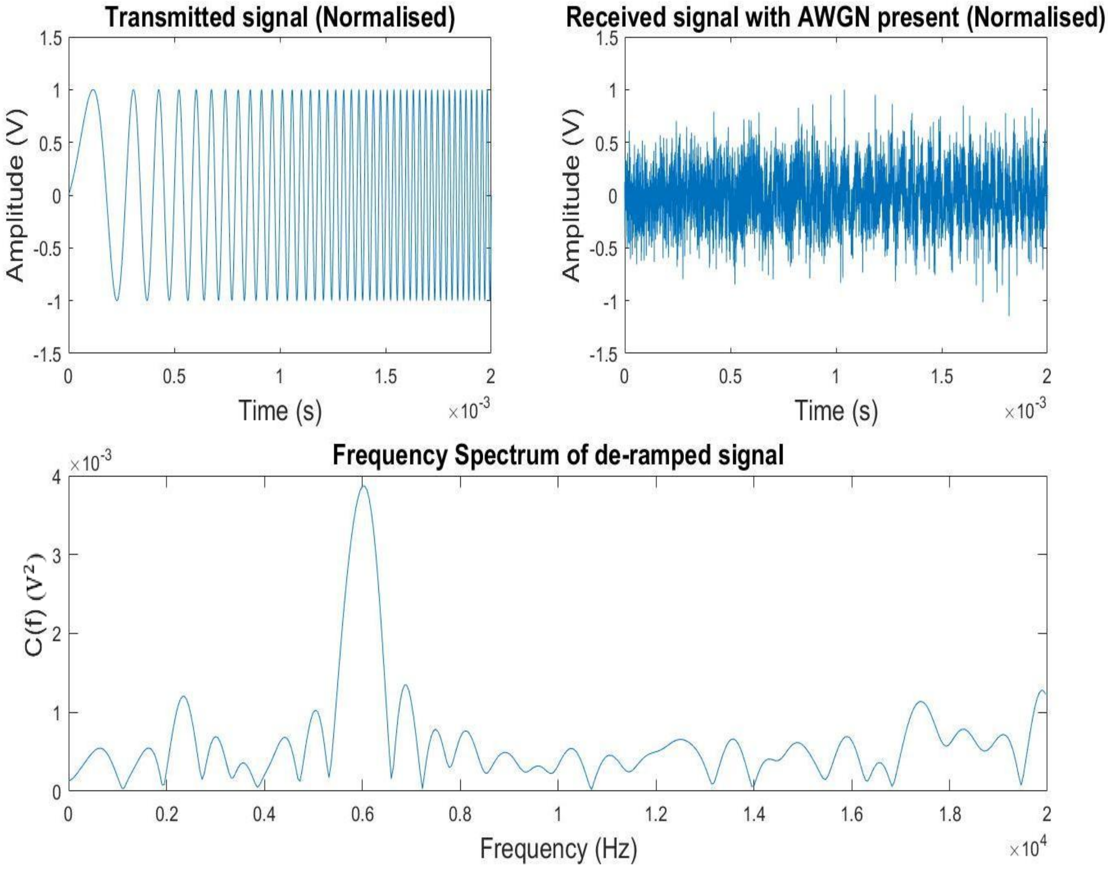

Fig. 3.4. The effectiveness of signal deramping to maximise SNR.

Therefore, the trade-off between the measuring range and resolution, which is caused by the previous theories themselves is no longer present. By using FMCW, the measuring range are determined by the time duration of the chirp signal; and by performing deramping, SNR and the measuring range can be maximised. The resolution is not determined by carrier signal anymore and can be improved by using DSP techniques.

3.2 *Laser Transmitter and Photodetector Circuits*

Besides the algorithm for processing the chirp signal, electrical circuits also need to be built. Fig. 3.5 (a)(b) demonstrates the laser transmitter circuit and photodetector circuit respectively. In Fig. 3.5 (a), the chirp signal directly modulates a laser diode. Since a laser diode is similar to a P-N junction, a forward bias needs to be applied. To achieve this, a chirp signal is added on top of the DC bias. An inductor is used as well for providing pure DC bias since inductors behave as a wire at low frequencies and an open-circuit at high frequencies. The coupling capacitor ensures that only AC signals can pass, and the resistor is used to convert the voltage into a current as laser diodes convert electrical current to optical power.

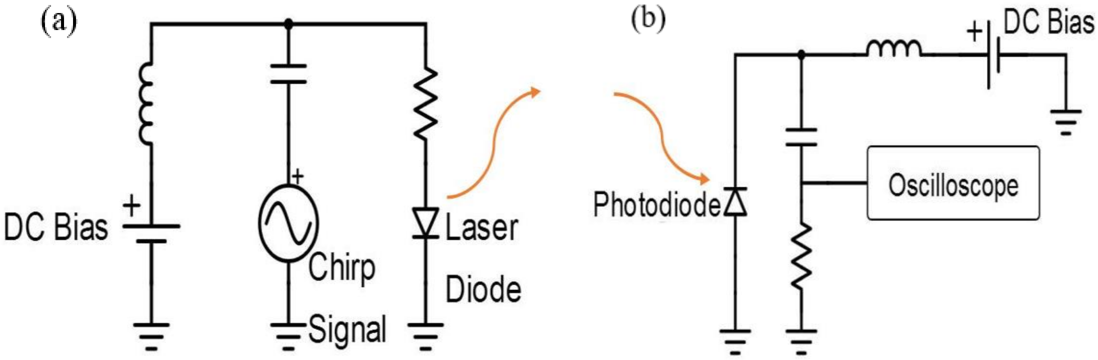

Fig. 3.5. Circuit diagram for (a) Laser transmitter and (b) Photodetector.

In the photodetector circuit shown in Fig. 3.5 (b), a photodiode is used to convert the optical power to an electrical current. The photodiode needs to be reverse biased so that its depletion capacitance is reduced, and a wider bandwidth is achieved. Therefore, a DC bias is used to provide the reverse bias. A capacitor and an inductor are also used to obtain pure AC signal and DC current respectively. In addition, a resistor is used to convert the current to a voltage as oscilloscopes measure voltage.

4. MATLAB Algorithm

4.1 Overview

In this section, detailed DSP algorithm will be presented. MATLAB is used for signal processing, namely multiplying the transmitted and received signal together and determining the time delay as presented in section 3.1. Fig. 4.1 is a block diagram illustrating the MATLAB algorithm, which will be explained in the following sub-sections.

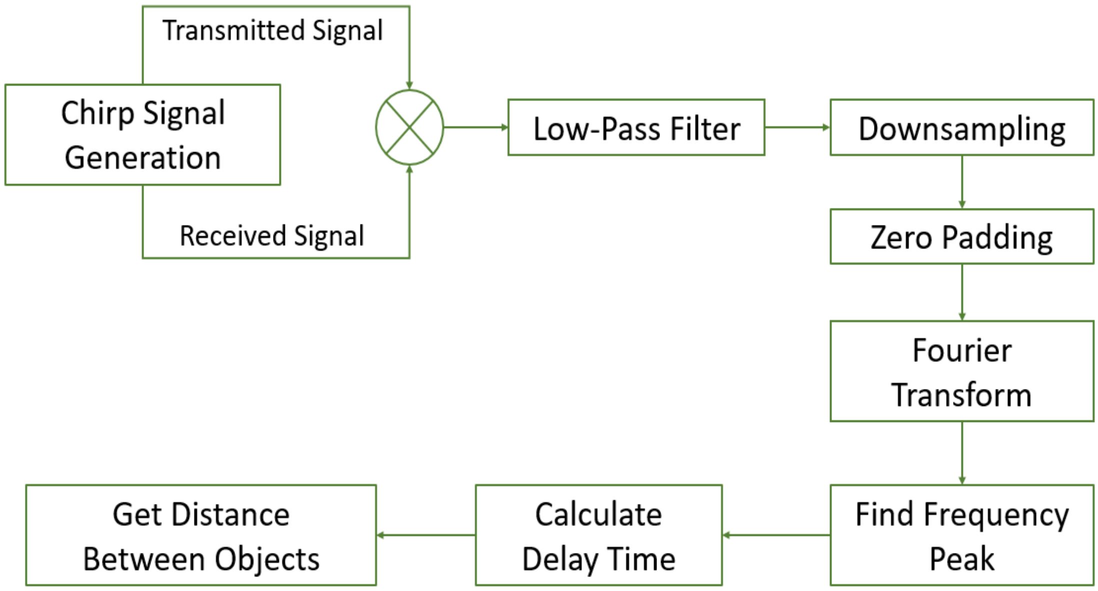

Fig. 4.1. MATLAB algorithm block diagram.

I define relevant parameters as follows: time duration of chirp signal $T = 800\ \mu s$, maximum and minimum frequency of chirp signal: $f_{min} = 2MHz, f_{max} = 102MHz$, chirp rate $k = \frac{f_{max}-f_{min}}{T} = \frac{102MHz-2MHz}{800\mu s} = 1.25 \times 10^{11} Hz/s$. I also add zeros (dead zone) after the chirp signal so that we can distinguish one period of chirp signal from another. The length of the dead zone is 200 µs, therefore the period of the periodic chirp is $1ms$. If the round-trip distance is between 0 to 60m, divided by the speed of light, then the longest time delay is $200ns$, $\Delta f = k \times \Delta t$ should lie in the range from 0 to 25kHz. I will justify my choice of time duration and bandwidth of the chirp signal in the following sub-sections. In this section, only a few lines of MATLAB programme will be presented and explained; for full MATLAB code, please refer to Appendix B.

4.2 Frequency Deramping

Frequency deramping simply means multiplying transmitted and received signal together. When we multiply transmitted and received signals together, the envelope of the resulting waveform has a frequency of Δf. From equation (3.2), we know:

$$\Delta f = \Delta t \cdot k = \Delta t \cdot \frac{BW}{T} \tag{4.1}$$

where BW is bandwidth, T is the time duration of the chirp signal and k is chirp rate. In order to get the frequency (Δf) of the envelope, the envelope must exist for more than one cycle, which means the chirp signal period T needs to be larger than the period of the envelope $\frac{1}{\Delta f}$:

$$T > \frac{1}{\Delta f} \rightarrow T \cdot \Delta f > 1 \tag{4.2}$$

From equation (4.1) we know $T \cdot \Delta f = \Delta t \cdot BW$, then equation (4.2) becomes:

$$\Delta t \cdot BW > 1 \rightarrow \Delta t > \frac{1}{BW} \tag{4.3}$$

If the speed of light is c and the distance between the rangefinder and the object is d, we can have: $\Delta t = \frac{2d}{c}$, from equation (4.3), we can get:

$$\frac{2d}{c} > \frac{1}{BW} \rightarrow d > \frac{c}{2 \times BW} \tag{4.4}$$

From equation (4.4) we can know that the minimum distance which the laser rangefinder can measure is $\frac{c}{2 \times BW}$. Therefore, choosing a wide bandwidth is vital as wider bandwidth will lead to higher accuracy. Given the specification of the signal generator, the bandwidth I define is 100MHz (more on bandwidth later in section 7.1). The minimum distance which the laser rangefinder is able to measure is: $d_{min} = \frac{3 \times 10^8 m/s}{2 \times 1 \times 10^6 Hz} = 1.5m$. This is still not good enough as we are also interested in distances below 1.5m. Thus, I add another fixed time delay to the received signal, manually adding some distance to make sure small distances can also be measured. This time delay will be subtracted when doing the final calculation. Relevant MATLAB code is shown below:

```
anotherdelay = 10e-9; %introduce another fixed delay of 10ns

y21 = [zeros(1,fs*anotherdelay),y2]; %add another delay, y2 is received signal

y22 = y21(1:length(y1));  %make sure the length of arrays are the same, y1 is transmitted signal. This will not affect multiplication as omitted part will be multiplied by zero if it’s not omitted, the result is zero anyway.

y3 = y1.*y22;  %frequency deramping
```

4.3 Low-Pass Filter & Downsampling

The first step after deramping is low-pass filtering. As presented in section 3.1 and Fig. 3.2, in frequency spectrum we are only interested in the Δf component ($2f_{min}$ to $2f_{max}$ component is not useful). As calculated in section 4.1, Δf is in the range from 0 to 25kHz and $2f_{min}$ is $4MHz$. Therefore, we need a low-pass filter to get rid of high-frequency components ($2f_{min}$ to $2f_{max}$). After the high-frequency components have been filtered out, we can use a lower sampling rate in order to save resources and increase the running speed of the MATLAB programme, this process is called downsampling. Relevant MATLAB code is shown below:

```
b = fir1(10,fcut/(0.5*fs),'low');  %10th order low-pass filter, fcut is the cut-off frequency

y4 = filter(b,1,y3);  %y3 is filter input, y4 is filter output

y5 = downsample(y4,1000); %downsampling by 1000 times
```

4.4 Zero Padding

Zero-padding is the key step in improving resolution. The resolution of the frequency spectrum in MATLAB using FFT (Fast Fourier Transform) is: $\frac{1}{T} = \frac{1}{1ms} = 1kHz$, where T is the time duration of the signal (including chirp signal and dead zone zeros). As calculated above in section 4.1, Δf lies in the range from 0 to 25kHz, a resolution of 1kHz is clearly not good enough. We can improve the resolution by putting zeros after the end of the signal so as to increase time duration of the signal and get a better resolution, this process is called zero padding. One may argue that increasing the time duration T of the chirp signal can also improve resolution, but in order to achieve the desired resolution, the time duration would be a few seconds. Using a signal with a few seconds of duration is not realistic as there would be too many data points to process and also the target may move during this time period. Therefore, I chose an appropriate time duration of $1ms$ (800μs chirp signal plus $200\mu s$ dead zone).

If a 1mm distance resolution is needed, then the time resolution is $\frac{1\times10^{-3}m}{3\times10^{8}\ m/s} = 3.33\times10^{-12}\ s$, the frequency resolution is $3.33\times10^{-12}\ s\times1.25\times10^{11}Hz/s = 0.4167\text{Hz}$ and the time duration of the signal should be $\frac{1}{0.4167\text{Hz}} = 2.4s$, which is $\frac{2.4s}{1\text{ms}} = 2400$ times of the original signal length. Therefore, the signal length needs to be increased by 2400 times to get 1mm resolution; to achieve this, zeros need to be padded after the original signal. Relevant MATLAB code is shown below:

```
zeropad=-1+2400; %increase the signal length by 2400 times in order to get a resolution of 1mm (-1 is just to the exclude the signal length)

y6 = [y5,zeros(1,zeropad*length(y5))]; %zero padding
```

The reason for setting the desired resolution to be 1mm is that when determining the actual distance using a measuring tape, the minimum scale of the tape is 1mm. Therefore, there is no point setting a resolution below 1mm at the moment as we won't know the precision of the measurement anyway. However, if the laser rangefinder turns out to be high-performance then we can set a resolution smaller than 1mm for more precise measurements. As mentioned in previous sections, the trade-off between measuring range and resolution is no longer present as the resolution is determined by the zero-padding technique using DSP while the measuring range is determined by the period of the periodic chirp signal. In addition, adjusting resolution is fairly easy by changing the value of "zeropad" in MATLAB.

4.5 Fourier Transform and Mathematical Calculation

After downsampling and zero padding, we can perform Fourier Transform to the time domain signal and find Δf (which is the frequency with peak amplitude), then using equation $\Delta t = \frac{\Delta f}{k}$ to find Δt.

5. Hardware Configuration and Equipment Setup

5.1 Crosstalk Suppression

This sub-section will demonstrate how crosstalk is suppressed. The amplitude of the transmitted signal I set is 500mV, while the amplitude of the received signal is only 0-20mV. As the transmitter and the receiver need to be placed next to each other to form the laser rangefinder system, there will be a significant amount of cross-talk between them and the received signal would be obscured. Therefore, I put transmitter and receiver circuits into two metal boxes to suppress crosstalk. For the AC/DC inputs/outputs, I use BNC connector to transmit and receive signals between circuits and the equipment. By doing this, the signal would be enclosed within metal boxes, no leakage would occur and therefore no crosstalk. For details regarding box drilling please refer to Appendix C; for a picture of the constructed laser rangefinder please refer to section 6 - Results.

5.2 Impedance Matching

In this sub-section, I will discuss input/output impedance matching. As shown in section 3.2 and Fig. 3.5, the resistor in the transmitter circuit is used to convert the voltage to a laser current and the resistor in the receiver circuit is used to convert the current to a voltage as oscilloscopes measure voltage. In practice, these two external resistors are not used because of the presence of internal resistors. For the receiver circuit, there are internal resistors in the oscilloscope, and $50\,\Omega$ is chosen as the frequency of the transmitted signal is relatively high (2 to 102MHz).

For the transmitter circuit, there is an internal resistor integrated inside the laser. The IV curve for a laser is similar to the P-N junction IV curve: when above threshold voltage, the current increases dramatically while the voltage remains constant; when below the threshold voltage (still forward-biased), the current is nearly zero. The IV curve of a resistor is a straight line. Therefore, the IV curve for a laser in series with a resistor is just the sum of those two IV curves. Fig. 5.1 shows the laser IV curve based on laser current measurements at different bias voltages. The resistance is then given by: $\frac{1}{14.824\ mS} = 67.5\Omega$. The resistance of the signal generator is 50Ω, ideally, a 50Ω input resistor is needed to match the impedance. As the laser resistance 67.5Ω is close to 50Ω, no external resistor is added for the simplicity of the system.

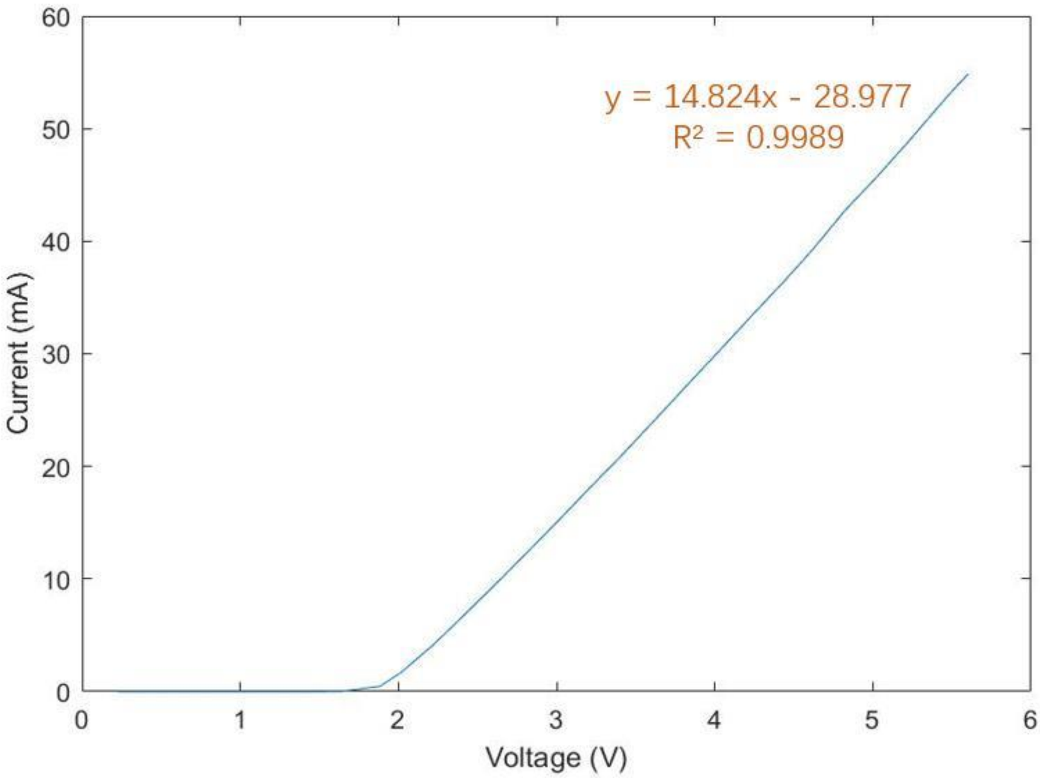

Fig. 5.1. Laser pointer IV curve.

5.3 Signal Generation and Acquisition

For the chirp signal generation, I use the R&S SMBV-100A Vector Signal Generator as the sampling frequency of this equipment is high (up to 150MHz) and signals with bandwidths up to 120MHz can be generated. This equipment generates vector signals using I/Q modulation (In-phase and Quadrature). A chirp signal can be rewritten as:

$$y(t) = \sin\left[\phi_0 + 2\pi\left(f_0 t + \frac{k}{2}t^2\right)\right]$$

$$y(t) = \cos(2\pi f_m t)\cdot\sin\{\phi_0 + 2\pi[(f_0 - f_m)t + \frac{k}{2}t^2]\}$$

$$+\sin(2\pi f_m t)\cdot\cos\{\phi_0 + 2\pi[(f_0 - f_m)t + \frac{k}{2}t^2]\}$$

$$y(t) = \cos(2\pi f_m t)\cdot\sin[g(t)] + \sin(2\pi f_m t)\cdot\cos[g(t)] \quad (5.1)$$

where f_m is modulation frequency, f_0 is minimum chirp frequency, k is chirp rate and $g(t) = \phi_0 + 2\pi[(f_0 - f_m)t + \frac{k}{2}t^2]$. The frequency of the chirp signal is from 2MHz to 102MHz, so I set f_m to be 52MHz. I use MATLAB to generate $sin[g(t)] + jcos[g(t)]$ signal and send it to generator, the generator would then perform I/Q modulation (multiply $\cos(2\pi f_m t)$ and $\sin(2\pi f_m t)$ with $\sin[g(t)]$ and $\cos[g(t)]$ respectively); the product is the required chirp signal which will be sent to the transmitter.

The voltage for normal laser operation is 4.5V. The laser reaches its threshold when a 2V DC bias is applied across its series resistor. In order to operate the system safely, I set the bias voltage of laser to be 4V and the signal amplitude $V_p = 500mV$. The bias voltage of the photodiode is set as 15V (25V is the maximum). In addition, I use MATLAB to communicate with the oscilloscope and acquire data from it. Relevant MATLAB code is shown below:

```
myScope = oscilloscope();
myScope.Resource = 'USB0::0x0699::0x040C::C011482::INSTR';
connect(myScope);
enableChannel(myScope,{'CH1','CH2'});
[y1, y2] = readWaveform(myScope, 'acquisition', false); %y1
is transmitted signal and y2 is received signal
```

6. Results

Fig. 6.1 is a picture of the constructed laser rangefinder. Metal boxes are used to supress the crosstalk as discussed in section 5.1.

Fig. 6.1. Picture of the constructed laser rangefinder.

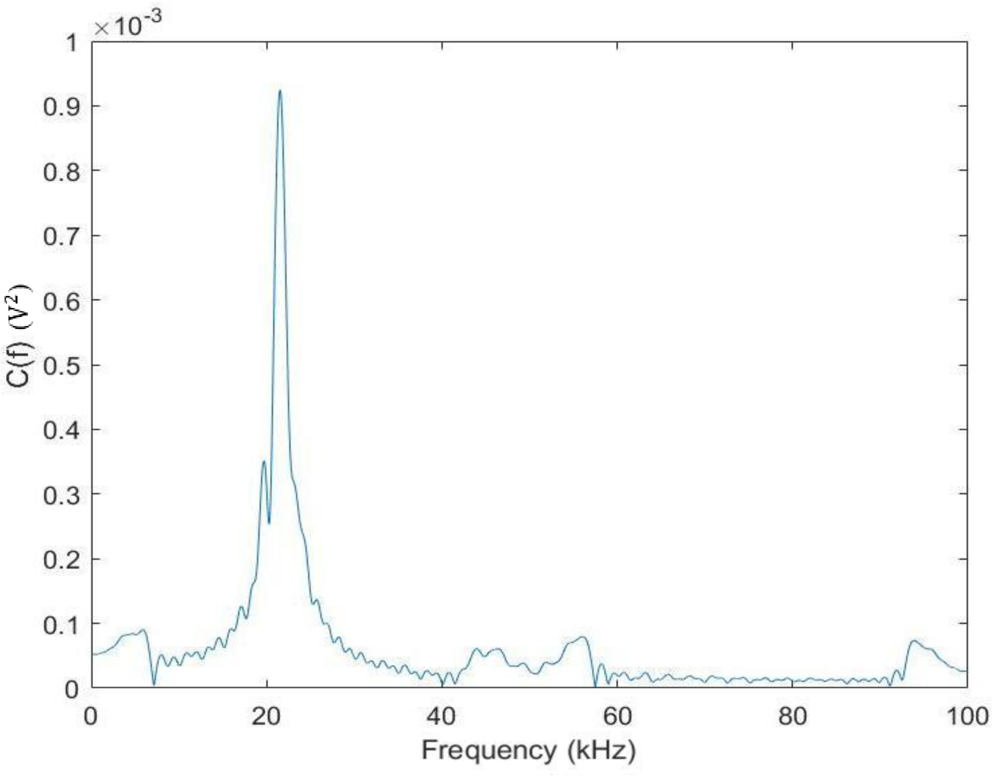

Fig. 6.2. The frequency spectrum of the deramped signal for one of the measurements.

Fig. 6.2 is one of the frequency spectra I got when measuring different distances. As shown in the figure, there is a frequency peak which corresponds to Δf, and the time delay can be found using equation (3.2): $\Delta t = \frac{\Delta f}{k}$.

I took measurements at different distances, Fig. 6.3 shows the relationship between measured data using the laser rangefinder and the actual distance (the actual distance is determined by a measuring tape). As shown in the figure, the slope of the best fit line is 0.9976, which indicates the laser rangefinder can achieve an accuracy of more than 99%. The coefficient of determination (R^2) is 1, which means the data points are well fitted onto the best-fit line, the laser rangefinder works properly over the distance range I've measured.

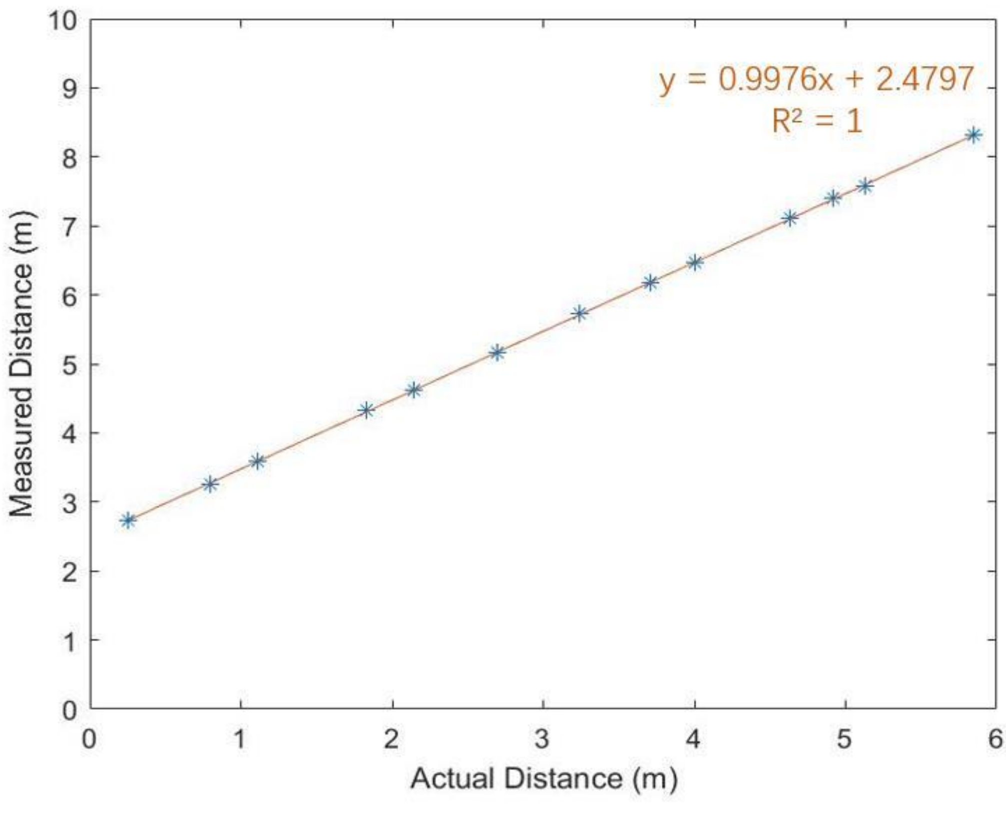

Fig. 6.3. Measured distance using laser rangefinder versus actual distance (without system calibration).

However, the y-axis intercept in Fig. 6.3 is not close to zero. This is due to the time delay caused by cables which are used to connect different items of equipment. I calibrated this system error by subtracting the y-axis intercept value (2.4797) from the MATLAB reading. After system calibration, I take measurements from 0 to 12 meters (12m is the maximum range I am able to measure due to the size of the lab, the maximum measuring distance will be discussed later in section 7.3). Fig. 6.4 shows new measurements with system calibration. As shown in the figure, the slope of the best-fit line is extremely close to 1 (0.9999), y-axis intercept is almost zero (0.0037) and R^2 is 1. Therefore, the result is satisfying and has met the requirements.

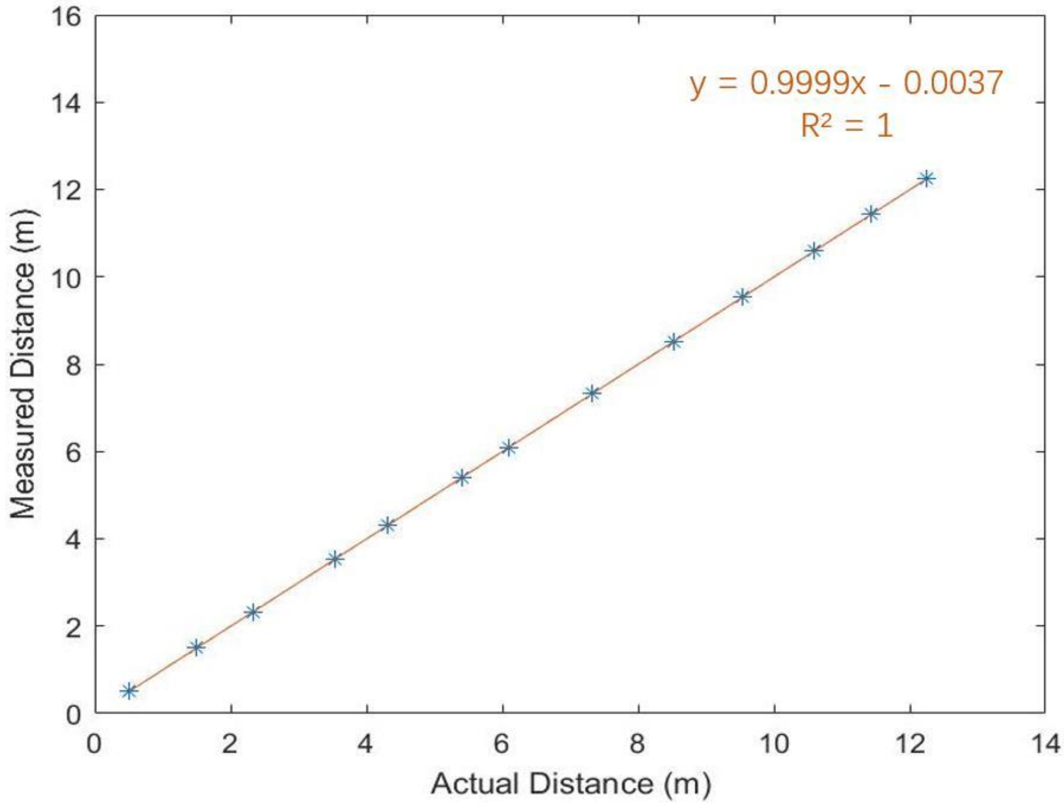

Fig. 6.4. Measured distance using laser rangefinder versus actual distance (with system calibration).

One more thing needs to be noticed is that even if $R^2 = 1$, it doesn't mean that the measured value y_{data} is strictly equal to the dependent variable y_0 of the best-fit line given independent variable x_0; $R^2 = 1$ indicates y_{data} is equal to y_0 or they are extremely close to each other. In the measured data above, y_{data} is very close (not strictly equal) to y_0. I calculated the root-mean-square (RMS) error, which is defined as the standard deviation of $y_{data} - y_0$, the result is 0.01m. As 10mm RMS error is much smaller than the distances of a few metres, data analytics yield $R^2 = 1$. In section 4.4, I define the theoretical resolution to be 1mm. However, since 1mm is a small value, in practice it's difficult to have two distances which are differed by 1mm due to potential human error. Therefore, we wouldn't know whether the laser rangefinder can generate correct reading for two distances differed by 1mm and we are not able to validate whether the resolution is indeed 1mm either. In addition, RMS error would also influence the justification of resolution as RMS error (10mm) is larger than theoretical resolution (1mm). Therefore, strictly speaking, 1mm is the theoretical resolution; the practical resolution is hard to justify, the RMS error is 10mm.

7. Specifications

In this section, several parameters will be specified: bandwidth, signal-to-noise ratio (SNR) and measuring range. These parameters are vital to the performance of laser rangefinder.

7.1 Bandwidth

In order to determine the bandwidth, I measured both the transmitted and received signals at 1m distance using sinusoidal signals as the carrier, with frequency varying from 2 to 102MHz. I divide the received signal amplitude by transmitted signal amplitude and plot this result versus corresponding frequency. The blue curve in Fig. 7.1 demonstrates the modulation frequency response of the laser rangefinder system (with a normal operation: laser bias voltage is 4V, corresponding bias current is 26mA). As we can see from Fig. 7.1, 3dB bandwidth is around 60MHz. The bandwidth of the laser or photodiode could be the factor limiting the laser performance. The bandwidth of the photodiode is given by:

$$f_{BW} = \frac{1}{2\pi \cdot R_L \cdot C_j} \tag{7.1}$$

where R_L is load resistance (50Ω) and C_j is junction capacitance ($24pF$ according to the spec sheet of the photodiode). Then the bandwidth can be calculated: $f_{BW} = 133\ MHz$. As the frequency of the chirp signal is from 2 to 102MHz, the photodiode is not the limiting factor and the laser is. One parameter that relates to the laser bandwidth is relaxation resonance frequency: bandwidth increases with relaxation resonance frequency, and relaxation resonance frequency increases with laser bias current. Thus, in order to verify the bandwidth of my rangefinder system is limited by the laser, I increased the laser current to 36mA by increasing the bias voltage and then measure the frequency response. Fig. 7.1 shows the comparison of frequency response with laser current of 26mA and 36mA respectively. As shown in the figure, the 3dB bandwidth is near 72MHz when the laser current is increased to 36mA. Hence, the bandwidth is mainly limited by laser's frequency response.

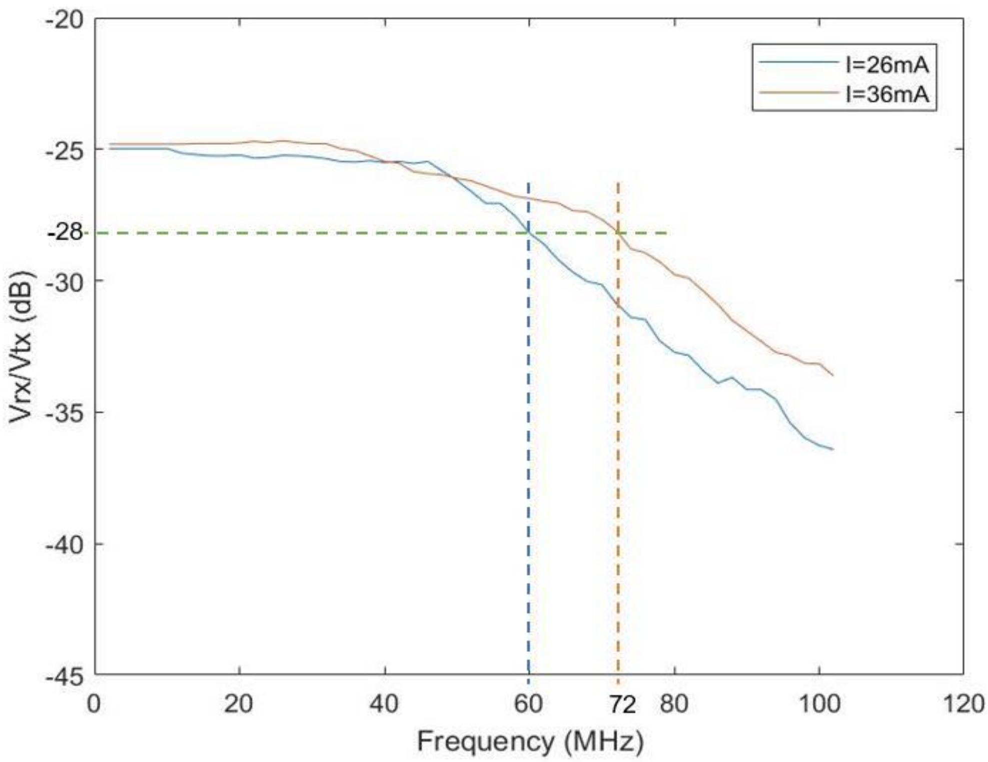

Fig. 7.1. Frequency responses of the laser rangefinder when laser current is 26mA and 36mA.

One may argue that if using a narrower chirp bandwidth (i.e. below 60MHz), the signal will not be attenuated as much. However, the bandwidth is crucial to the accuracy and consistency of the measurement. As mentioned before, deramping makes the signal more tolerant to noise as this mimic of matched filter can maximise the SNR. Therefore, the attenuation at high frequency doesn't significantly degrade the performance.

7.2 Signal-to-Noise Ratio

Another important parameter is the SNR. I used MATLAB to calculate RMS value of the signal and the noise based on the data array which represents the received waveform. The noise may originate from the oscilloscope, power supply and laser. The laser noise consists of shot noise, thermal noise and relative intensity noise. I measured the noise first with the oscilloscope, power supply and laser on, and then I turned off the power supply and only measured the noise from the oscilloscope. Two measurements are very close to each other, which indicates the noise is mainly from the oscilloscope.

I measured the intensity of the received signal at 1m measuring distance. I used sinusoidal signals with different frequencies as the input signal to examine SNR. I measured RMS voltage of the received sinusoidal signal (including noise) at 2MHz (minimum frequency), 102MHz (maximum frequency) and 52MHz (centre frequency). I also measured the RMS value of the chirp signal. I define SNR as follows:

$$SNR = 20\,log\left(\frac{V_{rx\_rms} - V_{noise\_rms}}{V_{noise\_rms}}\right)\,dB \tag{7.2}$$

Table 1 shows SNR when the measuring distance is 1m. In Table 1, when frequency increases, SNR decreases. This is due to the fact that 3dB bandwidth of the laser is 60MHz and the maximum passband frequency is around 40MHz as shown in section 7.1 and Fig. 7.1, at high frequencies the signal is attenuated. The SNR of the chirp signal is 17.33dB, which is in between with SNR at 2MHz (22.72dB) and at 102 MHz (6.30dB).

Signal Type		**V_{rms}(V)**	**SNR(dB)**
Noise		0.001275	N/A
Received Signal (including noise)	Sinusoidal signal @ 2MHz	0.018720	22.72
	Sinusoidal signal @ 52MHz	0.009433	16.12
	Sinusoidal signal @ 102MHz	0.003907	6.30
	Chirp Signal	0.010656	17.33

Table 1: SNR of the laser rangefinder system.

7.3 Measuring Range

There are two ways to define the measuring range, one is based on proposed theory, and another one is based on experimental measurements. Theoretically, if I define a valid measurement as the overlap of the transmitted signal and the received signal is at least half of the time duration of the chirp signal, then the maximum measuring range is: $0.5 \times \frac{800\mu s}{2} \times 3 \times 10^8 m/s = 60km$. Due to the limited power of the simple laser pointer used in this project and the imperfect collimation of the laser beam, no optical signal will be detected at 60km distance. Therefore, defining the maximum measuring distance as 60km is not a valid argument.

Regarding justifying the measuring range from a practical point of view, it's actually difficult to determine it by purely relying on experimental measurements as we don't know the approximate interval of the maximum measuring distance and the space in the lab is limited as well. Therefore, I will make several assumptions and justify the measuring range based on them.

The first step is to find out the minimum received power which can generate an accurate reading as the minimum received power corresponds to the maximum measuring distance. I measured the RMS value of the received signal at a certain distance ($V_{rms} = 0.0093V$), and got the distance reading using the laser rangefinder. Then I connected a microwave attenuator between the receiver and the oscilloscope (distance between the object and the laser rangefinder is still the same), and I adjusted the attenuator to find the maximum attenuation for the received signals which can yield the same measurement. The maximum attenuation is 23dB, which is $10^{-23 \div 10} = 5.012 \times 10^{-3}$ times of power attenuation. Power is defined as the square of the RMS voltage divided by the resistance, which is 50Ω load resistance of oscilloscope. Then the minimum received power which can generate an accurate reading is: $P_{min} = \frac{0.0093V^2}{50\Omega} \times (5.012 \times 10^{-3}) = 8.67 \times 10^{-9}W$.

The second step is to the find the relationship between the laser power and distance. When considering a point source, the light would propagate in different directions and the intensity at a given point in space is inversely proportional to the square of the distance. The laser beam, on the other hand, is collimated [10]. If the laser is perfectly collimated, then the beam is always parallel. The simple laser pointer used in the project is not perfectly collimated, so the beam size changes with distance. Fig. 7.2 demonstrates the effect of an imperfect collimating lens. As shown in the figure, the collimating lens essentially moves the original light source *S* to a virtual point *S'*. Therefore, I assume the laser power at a given point in space is inversely proportional to the square of the distance with respect to *S'*, this mathematical model is then defined as:

$$P = \frac{x_1}{(2\mathrm{d}+x_2)^2} \tag{7.3}$$

where x_1 and x_2 are constants, *d* is the distance between the transceiver and the object.

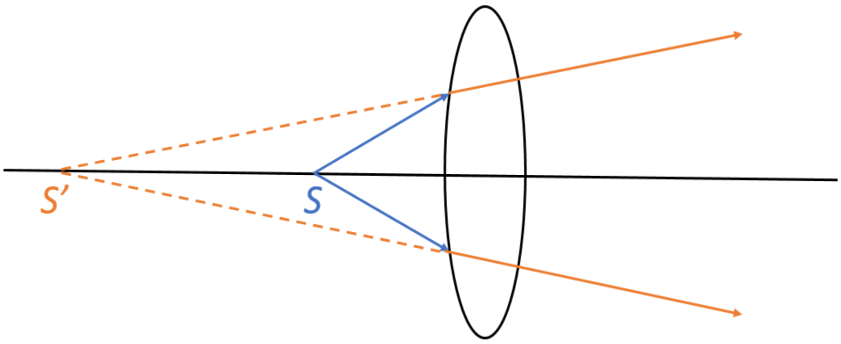

Fig. 7.2. The effect of an imperfect collimating lens.

In equation (7.3), *2d* is used for round-trip distance, as the signal needs to be reflected to the receiver. I also assume that there is no loss during the reflection process since I use an optical mirror which has minimum reflection-loss. The maximum measurement distance can be expressed as:

$$d_{max} = \frac{1}{2}(-x_2 + \sqrt{\frac{x_1}{P_{min}}}) \tag{7.4}$$

In order to develop this model, I measured the RMS value of the received signal at different distances (the received signal was directly from the transmitter, no optical mirror was used for this specific measurement). Then I use MATLAB to find x_1 and x_2, and calculate the maximum measuring range. Relevant MATLAB code is shown below:

```
fun = @(x,xdata)x(1)./((x(2)+xdata).^2); %fun is the
mathematical model which will fit xdata (distance) and
ydata(optical power).x(1) and x(2) are unknown constants
x0 = [1,10]; %make an initial guess of x1 and x2
x = lsqcurvefit(fun,x0,xdata,ydata); %nonlinear least
squares curve-fitting
```

Fig. 7.3 demonstrates the fitted curve based on measured data. Using MATLAB, I get $x_1 = 0.0541$, $x_2 = 6.9324$. Using equation (7.4), the maximum measuring distance can be calculated: $d_{max} = 36m$.

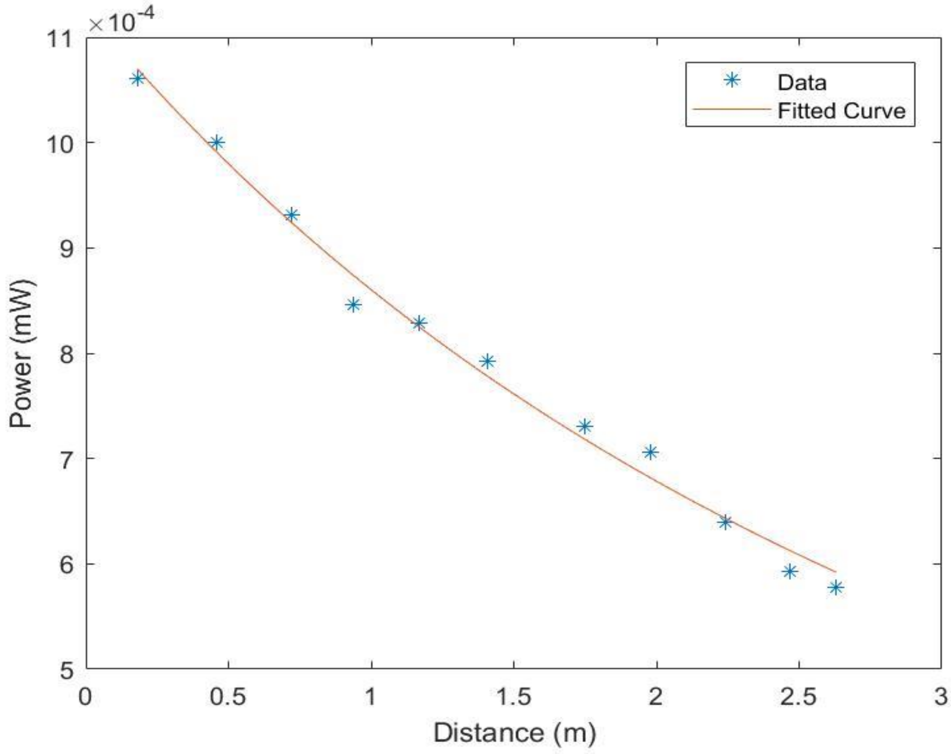

Fig. 7.3. The mathematical model for the laser power and distance relationship.

The minimum measuring distance is 15cm. When testing the laser rangefinder, I assume the optical path from the transmitter to the receiver is two times of the distance. In practice, I use an optical mirror as a reflector, the laser and photodiode are separated by approximately 3cm due to the width of the metal box. Fig. 7.4 demonstrates the relationship between the actual distance and the optical path, the transmitted path and the reflected path are not the same as the actual distance. The transmitter, receiver and mirror form a triangle with a tiny vertex angle. Using Pythagorean Theorem: $d = \sqrt{d_{tx/rx}^2 - 0.015^2}$, it can be shown that at long distances, two legs of the triangle (transmitted path and reflected path) are extremely close to the height of the triangle (actual distance); while at short distances (shorter than 15 cm), this approximation is not valid anymore and the accuracy drops. Therefore, I define the minimum measuring distance as 15cm.

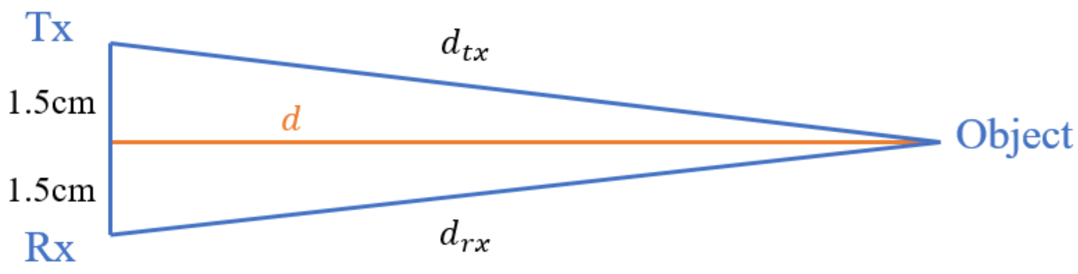

Fig. 7.4. The relationship between the optical path and the actual distance.

7.4 Datasheet

Based on measurements I've taken, relevant calculations and the spec sheets of the laser and photodiode, the datasheet for the laser rangefinder system is shown in Table 2.

Parameter	Notation	MIN	TYP	MAX	UNIT
Measuring Distance	d	0.15	/	36	m
Resolution	δd	/	1	/	mm
Laser Diode Bias Voltage	V_{bias_LD}	3	4	5	V
Laser Diode Bias Current	I_{bias_LD}	13	26	40	mA
Input Signal Amplitude	V_{p_sig}	400	500	600	mV
Input Signal Frequency	f_{sig}	2	/	102	MHz
Laser Wavelength	λ	/	650	/	nm
3dB Bandwidth	BW	/	60	/	MHz
Photodiode Bias Voltage	V_{bias_PD}	5	15	25	V
Photodiode Current ($\boldsymbol{d = 0.1m}$ $\boldsymbol{V_{p\_sig} = 500mV}$)	I_{PD}	870	890	910	mA
Transmitter Load Resistance	R_{L_TX}	/	67.5	/	Ω
Receiver Load Resistance	R_{L_RX}	/	50	/	Ω
Signal-to-Noise Ratio ($\boldsymbol{d = 1m}$)	SNR	/	17.33	/	dB

Table 2: Datasheet of the laser rangefinder.

8. Conclusions and Future Work

This report has first reviewed some previous work on laser rangefinders and pointed out main disadvantages: trade-off between measuring range and resolution is hard to balance and previous methods are difficult to implement. Therefore, the Frequency-Modulated Continuous-Wave (chirp signal to be specific) technique is proposed for this laser rangefinder project. The core issue is to measure the time difference Δt between the transmitted signal and received signal, then the distance between two objects can be calculated by multiplying the speed of light. By using a chirp signal, measuring Δt becomes measuring frequency difference Δf given the relationship $\Delta t = \frac{\Delta f}{k}$. Frequency difference can be found using DSP techniques: first deramping (a mimic of the matched filter, SNR can be maximised), secondly is low-pass filtering and down-sampling (get rid of undesired high-frequency components), then is zero-padding (increase resolution), finally is to using Fourier Transform to find Δf. Using the proposed method, the trade-off between measuring range and resolution, which is caused by the previous theories themselves is no longer present.

The total cost of purchasing different components including two metal boxes, a photodiode and a laser pointer is about £24. Using these components, I have built the laser transmitter and photoreceiver, and signals are processed using the proposed DSP algorithm. The laser rangefinder has a measuring range of 0.15 to 36m with an accuracy of more than 99% and a theoretical resolution of 1 mm (the RMS error is 10mm). I have also produced a datasheet for the laser rangefinder based on different measurements I've taken. The resolution can be improved by simply adding more zeros into the digital signal array if a precise reading is needed. The maximum measuring distance may not be very accurate since this is an approximation based on several assumptions I've made. Besides the possible error introduced by assumptions, other interference such as the laser scattering in the air or other light sources in the environment may also affect the justification of the maximum measuring distance. These systematic errors are not avoidable, and would slightly affect the accuracy as well as my justification of the measuring range.

Given the limited time period we have for the project, although the result is satisfying, there is still room for improvement in the future. For example, the circuit I built is on stripboard. Although it's easy to amend and the final measurement is accurate, PCB would be a better choice as it could reduce potential crosstalk between different strips on stripboard. Furthermore, PCBs allows the use of more compact components and smaller boxes can be used for the transmitter and receiver. This can reduce both the cost of metal boxes and optical path error.

The cost of the laser rangefinder is low; however, if we also take into account the price of the vector signal generator and oscilloscope, the total cost is relatively high. Alternatives could be using a direct digital synthesiser (DDS) to generate the chirp waveform and using Arduino to processing signal. The price of a DDS increases with its available bandwidth, and a DDS with similar performances costs about £450 (AD9956 model, manufactured by Analog Devices). If we use a less expensive one, the bandwidth would be narrower; and as I mentioned earlier the bandwidth is crucial to the laser rangefinder performance, using a low-cost device with narrower bandwidth would degrade the performance, and this is the new trade-off that can be improved in the future. Compared with other laser rangefinders available in the market, £24 cost of my device is not a very competitive price. Device with a slightly better performance (BEVA CJY040 FBA model, measuring range 50m, accuracy $\pm$2cm) has a price of £40. However, I only built one single device; if the rangefinder can be mass-produced in a factory, the component cost would significantly decrease but other labour costs are also needed to be considered. As the price of the laser rangefinder is not the main part of this project, I would assume that the unit price of a mass-produced rangefinder would decrease as that's normally the case.

In summary, the performance of laser rangefinder has met the requirement: 99% accuracy, 1mm resolution and £24 component cost. A copy of datasheet with specifications of different parameters is also produced. To conclude, a low-cost, high-accuracy laser rangefinder using Frequency-Modulated Continuous-Wave has been successfully built.

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Appendix

A. Abbreviations

AGC: Automatic Gain Control.

APD: Avalanche Photodiode.

AWGN: Additive White Gaussian Noise.

CMOS: Complementary Metal Oxide Semiconductor.

DDS: Direct Digital Synthesiser.

DSP: Digital Signal Processing.

FFT: Fast Fourier Transform.

FM: Frequency Modulation.

FMCW: Frequency-Modulated Continuous-Wave.

LD: Laser Diode.

NAR: No-Ambiguity-Range.

PCB: Printed Circuit Board.

PLL: Phase-Locked Loop.

RMS: Root Mean Square.

SMD: Surface-Mount Device.

SNR: Signal-to-Noise Ratio.

TDC: Time-to-Digital Converter.

TOF: Time-of-Flight.

VCO: Voltage-Controlled Oscillator

B. *MATLAB Programmes*

➢ **Programme for simulating the effectiveness of maximising SNR.**

```
fs=2e6; %sampling frequency
T=2e-3; % period of signal
f0=1e3; %f0 is the instantaneous frequency at time 0
f1=41e3; %f1 is the instantaneous frequency at time T
slope=(f1-f0)/T; %chirp rate
nosample=fs*T; %number of samples in one period
t1=linspace(0,T,nosample); %time domain array

y1=sin(2*pi*(f0*t1+0.5*slope*t1.^2)); %chirp signal
t2=t1-0.3e-3; %add time delay
y2=sin(2*pi*(f0*t2+0.5*slope*t2.^2)); %generate delayed signal
y2(1:1000)=zeros(1,1000);
snr=15; %signal to noise ratio
y2noise=awgn(0.1*y2,snr); %add some noise


y3 = y1.*y2noise; %frequency deramping
b = fir1(10,20/(0.5*fs),'low');  %10th order low-pass filter,b
is filter coefficient
y4 = filter(b,1,y3);  %filter output

y5 = downsample(y4,10); %dowm sampling
y6 = [y5,zeros(1,10*length(y5))]; %zero padding


fft6 = abs(fft(y6)); %Fast Fourier Transform
fft6h = fft6(1:round(0.5*length(y6)));
f2 = linspace(0,0.5*fs/10,round(0.5*length(y6)));
f2 = f2(1:0.2*length(f2));
fft6h = fft6h(1:0.2*length(fft6h));

figure(1)
ax1 = subplot(2,2,1);
plot(ax1,t1,y1)
title(ax1,'Transmitted signal (Normalised)','fontsize',14)
ylim([-1.5,1.5]);
xlabel(ax1,'Time (s)','fontsize',14)
ylabel(ax1,'Amplitude (V)','fontsize',14)
```

```
ax2 = subplot(2,2,2);
plot(ax2,t1,y2noise/max(y2noise))
title(ax2,'Received signal with AWGN present
(Normalised)','fontsize',14)
xlabel(ax2,'Time (s)','fontsize',14)
ylabel(ax2,'Amplitude (V)','fontsize',14)
ylim([-1.5,1.5]);

ax3 = subplot(2,1,2);
plot(ax3,f2,fft6h/(round(0.5*length(y6))));
title(ax3,'Frequency Spectrum of de-ramped
signal','fontsize',14)
xlabel(ax3,'Frequency (Hz)','fontsize',14)
ylabel(ax3,'C(f)','fontsize',14)
```

- **Programme for signal generation.**

```
fs=200e6; %sampling frequency
T=800e-6; %period
BW=100e6; %bandwidth
f0=-BW/2; %f0 is the instantaneous frequency at time 0
f1=BW/2;  %f1 is the instantaneous frequency at time T
slope=(f1-f0)/T; %chirp rate
nosample=fs*T; %number of samples in one period


t1=linspace(0,T,nosample);
i0=sin(2*pi*(f0*t1+0.5*slope*t1.^2));
q0=cos(2*pi*(f0*t1+0.5*slope*t1.^2));
p0=i0-1i*q0; %chirp signal


z1=zeros(1,0.25*length(i0)); %dead zone
p1=z1-1i*z1;
p2=[p0,p1]; %signal sent to the generator


save('vectgenDATA.mat','p2'); %.mat file which will be sent to
the generator
A=[slope,T,nosample,f0,f1,fs]; %save parameters
fileID=fopen('fsetting.txt','w'); fprintf(fileID,'%12.9f
\n',A); fclose(fileID);
```

➢ Programme for measuring the distance.

```
%% Basic parameters
B = importdata('fsetting.txt');
slope = B(1); %import chirp rate
fstek = 5e9; %sampling frequency of Tektronix oscilloscope
p = 1000; %downsampling rate
zeropad = -1+2400; %zero padding rate:
anotherdelay = 10e-9; %time period of another delay
fcut = slope*(anotherdelay+200e-9); %set cut-off frequency


%% Get data from oscilloscope
myScope = oscilloscope();
myScope.Resource = 'USB0::0x0699::0x040C::C011482::INSTR';
connect(myScope);
enableChannel(myScope,{'CH1','CH2'});
[y1, y2] = readWaveform(myScope, 'acquisition', false);
y1 = y1/max(y1); %y1: Transmitted signal
y2 = y2/max(y2); %y2: Received signal
disconnect(myScope);
fprintf('Data acquired. ');

%% introduce another delay
y21 = [zeros(1,fstek*anotherdelay),y2]; %add another delay
y22 = y21(1:length(y1));  %make sure length of arrays are the
same


%% Signal Processing
%frequency deramping
y3 = y1.*y22;

%LP filter
b = fir1(10,fcut/(0.5*fstek),'low');  %10th order low-pass
filter,b is filter coefficient
y4 = filter(b,1,y3);  %filter output

%dowmsampling and zero padding
y5 = downsample(y4,p);
y6 = [y5,zeros(1,zeropad*length(y5))];
%FFT
fft6 = abs(fft(y6)); %Fast Fourier Transform
```

```
fft6h = fft6(1:round(0.5*length(y6))); %take half of the
frequency spectrum for signal processing as the other half is
the same
[~,idx] = max(fft6h);   %find the position of the frequency
with peak amplitude
ffind = (fstek/p)*(idx/length(fft6));  %find the frequency
with peak amplitude
tfind1 = ffind/slope;
tfind = tfind1-anotherdelay; %find time delay
fprintf('diatsnce: %f m  \n',-2.48+0.5*tfind*3e8); %-2.48 is
the system calibration, 0.5 is to measure one-way trip

figure(1)
f2 = linspace(0,0.5*fstek/p,round(0.5*length(y6)));
f2 = f2(1:0.04*length(f2));
fft6h = fft6h(1:0.04*length(fft6h));
plot(f2,fft6h/(round(0.5*length(y6))));
xlabel('Frequency (kHz)');ylabel('C(f)');
```

- **Programme for determining the maximum measuring distance.**

```
xdata = importdata('x52.txt'); xdata = xdata'; %import data
ydata = importdata('y52.txt'); ydata = ydata';
fun = @(x,xdata)x(1)./((x(2)+xdata).^2); %fun is the
mathematical model which will fit xdata(distance) and
ydata(optical power).x(1) and x(2) are unknown constants
x0 = [1,10]; %make an innitial guess of x1 and x2
x = lsqcurvefit(fun,x0,xdata,ydata); %nonlinear least squares
curve-fitting function
time = linspace(xdata(1),xdata(end));
Pmin=8.67e-6;%mW
Dmax=0.5*(-x(2)+sqrt(x(1)/Pmin));
plot(xdata,ydata,'*',time,fun(x,time),'-')
legend('Data','Fitted Curve')
xlabel('Distance (m)');
ylabel('Power (mW)');
fprintf('Dmax: %f \n',Dmax);
```

C. *Metal Box Hole-Drilling Design*

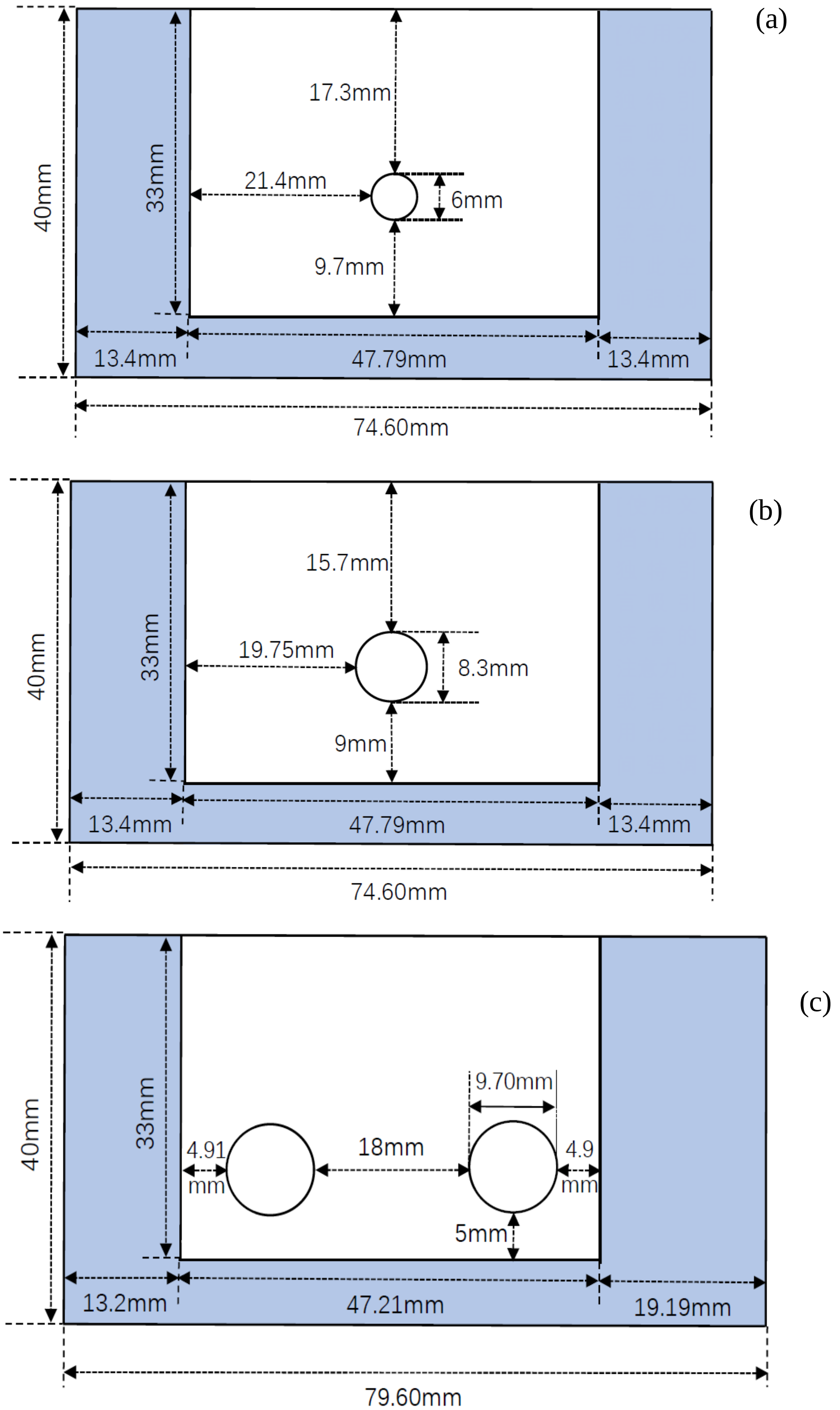

Fig. 9.1. Components placement on metal box: (a) Laser placement (b) Photodiode placement (c) BNC-connector placement (blue area in the figure means not available for drilling).

D. Experimental Data

Table 3: Voltage and current measurement of the laser diode (for Fig. 5.1).

Voltage (V)	Current (mA)	Voltage (V)	Current (mA)
0.223	0	3.012	15.199
0.389	0	3.224	18.299
0.656	0.000058	3.401	20.826
0.821	0.000073	3.647	24.51
1.023	0.000091	3.812	27.015
1.255	0.000143	4.023	30.176
1.445	0.001064	4.221	33.196
1.641	0.01366	4.435	36.384
1.885	0.477	4.614	39.171
2.011	1.629	4.821	42.686
2.223	4.18	5.043	45.993
2.397	6.543	5.228	48.902
2.608	9.445	5.465	52.775

Table 4: Distance measurements without system calibration (for Fig. 6.3).

Measurements (m)							Maxi value of 1-7	Min value of 1-7	Average value (exclude max&min)	Actual Distance (m)
1	2	3	4	5	6	7				
2.743	2.739	2.748	2.743	2.743	2.743	2.744	2.748	2.739	2.743	0.256
3.262	3.262	3.264	3.259	3.264	3.261	3.257	3.264	3.257	3.262	0.798
3.581	3.584	3.607	3.581	3.587	3.615	3.584	3.615	3.581	3.589	1.112
4.317	4.322	4.323	4.311	4.315	4.324	4.323	4.324	4.311	4.320	1.832
4.611	4.601	4.613	4.613	4.612	4.607	4.610	4.613	4.601	4.611	2.141
5.161	5.164	5.161	5.161	5.163	5.162	5.162	5.164	5.161	5.162	2.692
5.723	5.727	5.726	5.733	5.727	5.725	5.728	5.733	5.723	5.727	3.242
6.174	6.176	6.160	6.175	6.165	6.174	6.178	6.178	6.160	6.173	3.708
6.471	6.472	6.500	6.468	6.457	6.467	6.461	6.500	6.457	6.468	4.009
7.116	7.110	7.117	7.116	7.117	7.114	7.112	7.117	7.110	7.115	4.632
7.476	7.402	7.382	7.378	7.402	7.398	7.380	7.476	7.378	7.393	4.914
7.574	7.558	7.602	7.567	7.576	7.578	7.576	7.602	7.558	7.574	5.132
8.321	8.324	8.287	8.322	8.349	8.319	8.322	8.349	8.287	8.322	5.848

Table 5: Distance measurements with system calibration (for Fig. 6.4).

Measurements (m)							Max value of 1-7	Min value of 1-7	Average value (exclude max&min)	Actual Distance (m)
1	2	3	4	5	6	7				
0.511	0.518	0.515	0.517	0.513	0.513	0.514	0.518	0.511	0.514	0.509
1.485	1.506	1.504	1.509	1.506	1.507	1.505	1.509	1.485	1.506	1.492
2.311	2.311	2.315	2.316	2.309	2.314	2.314	2.316	2.309	2.313	2.329
3.525	3.519	3.508	3.525	3.511	3.518	3.518	3.525	3.508	3.518	3.529
4.300	4.303	4.325	4.314	4.306	4.303	4.302	4.325	4.300	4.306	4.312
5.409	5.391	5.376	5.379	5.374	5.411	5.376	5.411	5.374	5.386	5.408
6.093	6.092	6.090	6.084	6.092	6.091	6.090	6.093	6.084	6.091	6.087
5.317	7.318	7.324	7.331	7.322	7.320	7.314	7.331	5.317	7.320	7.332
8.517	8.520	8.518	8.521	8.521	8.521	8.516	8.521	8.516	8.519	8.531
9.533	9.540	9.539	9.550	9.547	9.563	9.537	9.563	9.533	9.543	9.544
10.581	10.592	10.591	10.590	10.578	10.599	10.596	10.599	10.578	10.590	10.581
11.419	11.428	11.429	11.427	11.415	11.429	11.421	11.429	11.415	11.425	11.428
12.243	12.243	12.241	12.247	12.245	12.253	12.249	12.253	12.241	12.245	12.251

Table 6: Frequency response for laser current of 26mA (for Fig. 7.1).

Frequency (MHz)	Vin (mVpp)	Vout (mVpp)	Vout/Vin (V/V)	Vout/Vin (dB)
2	702	39.6	0.05642	-24.9713
4	702	39.6	0.05642	-24.9713
6	702	39.6	0.05642	-24.9713
8	702	39.6	0.05642	-24.9713
10	702	39.6	0.05642	-24.9713
12	702	38.8	0.05528	-25.1486
14	692	38.0	0.05492	-25.2054
16	681	37.2	0.05463	-25.2514
18	681	37.2	0.05463	-25.2514
20	671	36.8	0.05485	-25.2165
22	662	35.8	0.05408	-25.3393
24	660	35.8	0.05425	-25.312
26	650	35.6	0.05477	-25.2291
28	648	35.4	0.05463	-25.2514
30	629	34.2	0.05438	-25.2912
32	597	32.2	0.05394	-25.3618
34	597	31.8	0.05327	-25.4703
36	587	31.2	0.05316	-25.4883
38	576	30.8	0.05348	-25.4362
40	558	29.6	0.05305	-25.5063
42	548	29.2	0.05329	-25.4671
44	534	28.2	0.05281	-25.5457
46	503	26.8	0.05329	-25.4671
48	493	25.2	0.05112	-25.8282
50	514	25.2	0.04903	-26.1908
52	514	24.0	0.04670	-26.6137
54	514	22.8	0.04436	-27.0602
56	514	22.8	0.04436	-27.0602

58	514	21.6	0.04203	-27.5288
60	534	20.8	0.03896	-28.1876
62	566	21.0	0.03711	-28.6102
64	566	19.6	0.03463	-29.211
66	597	19.6	0.03284	-29.6719
68	597	18.8	0.03150	-30.0338
70	618	19.2	0.03107	-30.1532
72	629	18.0	0.02862	-30.8666
74	639	17.2	0.02692	-31.3985
76	660	17.6	0.02667	-31.4795
78	692	16.8	0.02428	-32.295
80	692	16.0	0.02313	-32.7165
82	702	16.0	0.02280	-32.8413
84	713	15.2	0.02132	-33.4243
86	713	14.4	0.02020	-33.893
88	734	15.2	0.02071	-33.6764
90	744	14.6	0.01963	-34.1416
92	744	14.6	0.01963	-34.1416
94	755	14.2	0.01881	-34.5122
96	755	12.8	0.01696	-35.4115
98	755	12.0	0.01590	-35.9721
100	755	11.6	0.01537	-36.2665
102	755	11.4	0.01510	-36.4205

Table 7: Frequency response for laser current of 36mA (for Fig. 7.1).

Frequency (MHz)	Vin (mVpp)	Vout (mVpp)	Vout/Vin (V/V)	Vout/Vin (dB)
2	480	27.6	0.05750	-24.8066
4	480	27.6	0.05750	-24.8066
6	480	27.6	0.05750	-24.8066
8	480	27.6	0.05750	-24.8066
10	480	27.6	0.05750	-24.8066
12	480	27.6	0.05750	-24.8066
14	472	27.2	0.05763	-24.787
16	472	27.2	0.05763	-24.787
18	472	27.2	0.05763	-24.787
20	464	26.8	0.05776	-24.7675
22	464	27.0	0.05819	-24.703
24	456	26.4	0.05790	-24.7464
26	456	26.6	0.05834	-24.6807
28	456	26.4	0.05790	-24.7464
30	448	25.8	0.05759	-24.7931
32	448	25.8	0.05759	-24.7931
34	440	24.8	0.05637	-24.979
36	440	24.6	0.05591	-25.0502
38	440	24.0	0.05455	-25.2641
40	424	22.6	0.05331	-25.4638
42	416	22.0	0.05289	-25.5325
44	400	20.4	0.05100	-25.8486
46	384	19.4	0.05053	-25.929
48	366	18.4	0.05028	-25.9721
50	376	18.6	0.04947	-26.1132
52	376	18.4	0.04894	-26.2067
54	376	18.0	0.04788	-26.3969
56	376	17.6	0.04681	-26.5932

58	376	17.2	0.04575	-26.7922
60	384	17.4	0.04532	-26.8742
62	384	17.2	0.04480	-26.9744
64	392	17.4	0.04439	-27.0543
66	400	17.2	0.04300	-27.3306
68	416	17.8	0.04279	-27.3732
70	416	17.2	0.04135	-27.6705
72	432	17.0	0.03936	-28.0989
74	440	16.0	0.03637	-28.7851
76	448	16.0	0.03572	-28.9418
78	472	16.2	0.03433	-29.2865
80	480	15.6	0.03250	-29.7623
82	488	15.6	0.03197	-29.9051
84	496	15.0	0.03025	-30.3855
86	512	14.6	0.02852	-30.897
88	512	13.6	0.02657	-31.5122
90	520	13.2	0.02539	-31.9067
92	528	12.8	0.02425	-32.3058
94	528	12.2	0.02311	-32.724
96	536	12.2	0.02277	-32.8527
98	536	11.8	0.02202	-33.1437
100	528	11.6	0.02197	-33.1634
102	528	11.0	0.02084	-33.622

Table 8: Received signal intensity and corresponding distance (for Fig. 7.3).

Distance (m)	Vrms (mV)	Power (mW)
0.18	7.283	0.0010609
0.46	7.071	0.001
0.72	6.824	0.000931225
0.94	6.505	0.0008464
1.17	6.435	0.0008281
1.41	6.293	0.0007921
1.75	6.046	0.000731025
1.98	5.940	0.0007056
2.24	5.657	0.00064
2.47	5.445	0.0005929
2.63	5.374	0.0005776