# Chapter 2

2.1. The proof is as follows:

$$\begin{aligned}
(x+y)\cdot(x+z) &= xx + xz + xy + yz \\
&= x + xz + xy + yz \\
&= x(1 + z + y) + yz \\
&= x \cdot 1 + yz \\
&= x + yz
\end{aligned}$$

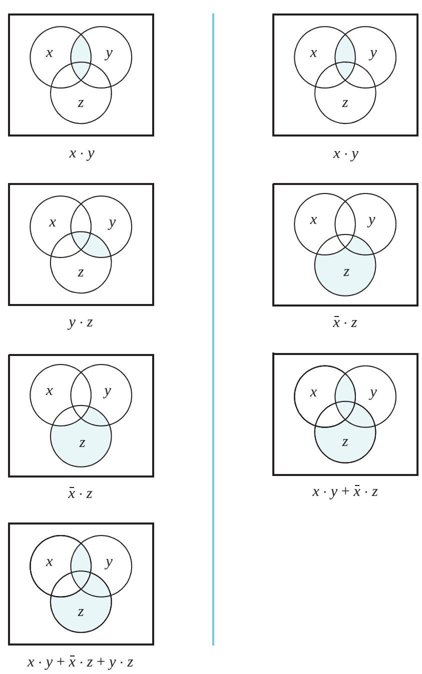
2.2. The proof is as follows:

$$\begin{aligned}
(x+y)\cdot(x+\overline{y}) &= xx + xy + x\overline{y} + y\overline{y} \\
&= x + xy + x\overline{y} + 0 \\
&= x(1 + y + \overline{y}) \\
&= x \cdot 1 \\
&= x
\end{aligned}$$

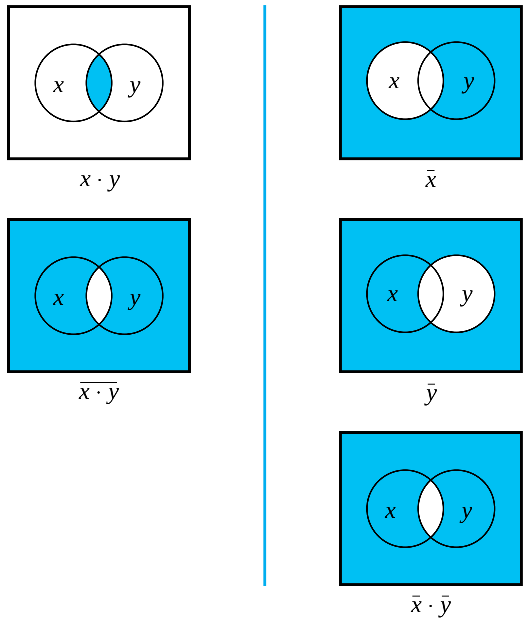
2.3. Manipulate the left hand side as follows:

$$\begin{aligned}
xy + yz + \overline{x}z &= xy + (x+\overline{x})yz + \overline{x}z \\
&= xy + xyz + \overline{x}yz + \overline{x}z \\
&= xy(1+z) + \overline{x}(y+1)z \\
&= xy \cdot 1 + \overline{x} \cdot 1 \cdot z \\
&= xy + \overline{x}z
\end{aligned}$$

2.4. Proof using Venn diagrams:

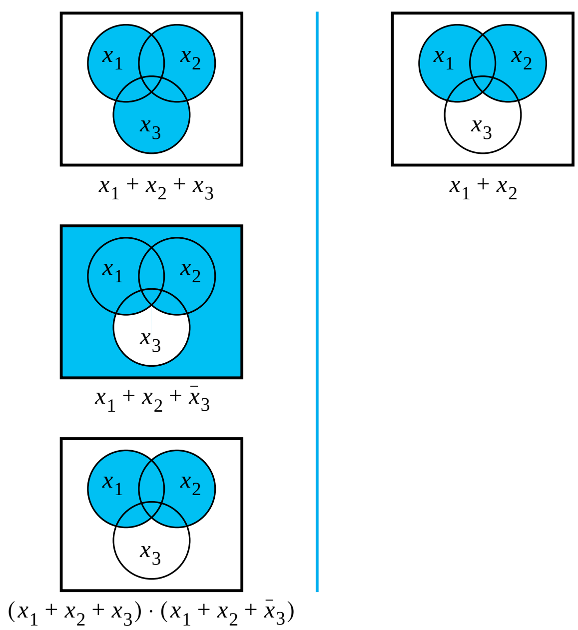


2.5. Proof of 15$a$ using Venn diagrams:




A similar proof is constructed for 15$b$.

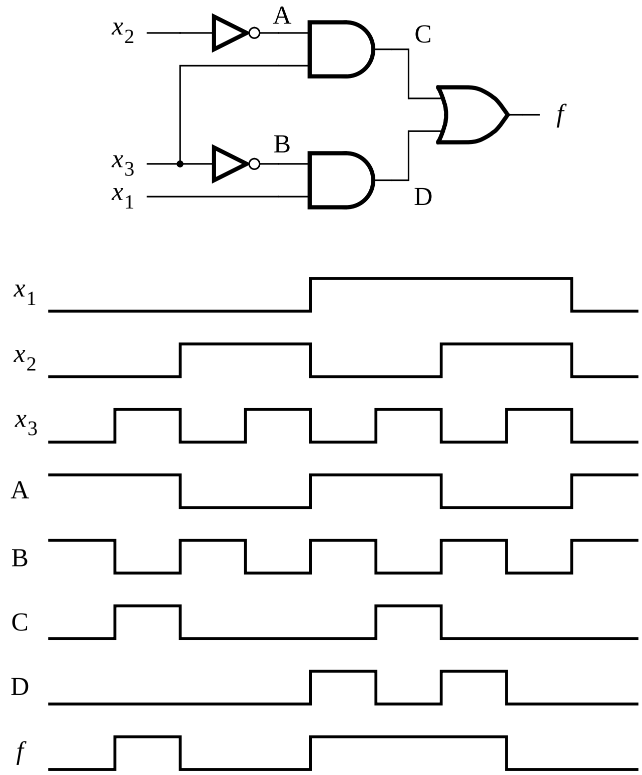
2.6. Proof using Venn diagrams:



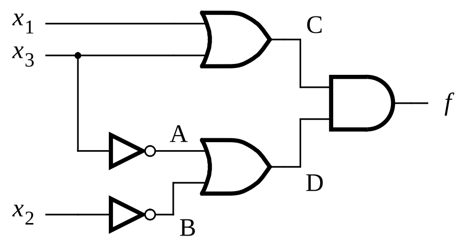
2.7. A possible approach for determining whether or not the expressions are valid is to try to manipulate the left and right sides of an expression into the same form, using the theorems and properties presented in section 2.5. While this may seem simple, it is an awkward approach, because it is not obvious what target form one should try to reach. A much simpler approach is to construct a truth table for each side of an expression. If the truth tables are identical, then the expression is valid. Using this approach, we can show that the answers are:

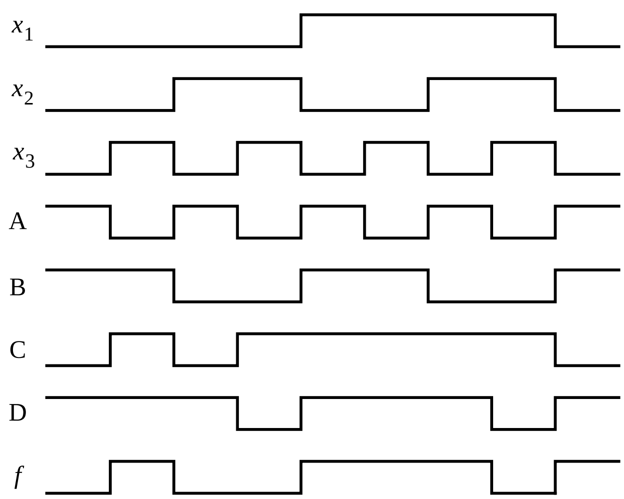
(a) Yes
(b) Yes
(c) No

2.8. Timing diagram of the waveforms that can be observed on all wires of the circuit:



2.9. Timing diagram of the waveforms that can be observed on all wires of the circuit:




2.10. Starting with the canonical sum-of-products for $f$ get

$$\begin{aligned}
f &= \overline{x}_1\overline{x}_2x_3 + \overline{x}_1x_2\overline{x}_3 + \overline{x}_1x_2x_3 + x_1\overline{x}_2\overline{x}_3 + x_1\overline{x}_2x_3 + x_1x_2\overline{x}_3 + x_1x_2x_3 \\
&= x_1(\overline{x}_2\overline{x}_3 + \overline{x}_2x_3 + x_2\overline{x}_3 + x_2x_3) + x_2(\overline{x}_1\overline{x}_3 + \overline{x}_1x_3 + x_1\overline{x}_3 + x_1x_3) \\
&\quad + x_3(\overline{x}_1\overline{x}_2 + \overline{x}_1x_2 + x_1\overline{x}_2 + x_1x_2) \\
&= x_1(\overline{x}_2(\overline{x}_3 + x_3) + x_2(\overline{x}_3 + x_3)) + x_2(\overline{x}_1(\overline{x}_3 + x_3) + x_1(\overline{x}_3 + x_3)) \\
&\quad + x_3(\overline{x}_1(\overline{x}_2 + x_2) + x_1(\overline{x}_2 + x_2)) \\
&= x_1(\overline{x}_2 \cdot 1 + x_2 \cdot 1) + x_2(\overline{x}_1 \cdot 1 + x_1 \cdot 1) + x_3(\overline{x}_1 \cdot 1 + x_1 \cdot 1) \\
&= x_1(\overline{x}_2 + x_2) + x_2(\overline{x}_1 + x_1) + x_3(\overline{x}_1 + x_1) \\
&= x_1 \cdot 1 + x_2 \cdot 1 + x_3 \cdot 1 \\
&= x_1 + x_2 + x_3
\end{aligned}$$

2.11. Starting with the canonical product-of-sums for $f$ can derive:

$$\begin{aligned}
f &= (x_1 + x_2 + x_3)(x_1 + x_2 + \overline{x}_3)(x_1 + \overline{x}_2 + x_3)(x_1 + \overline{x}_2 + \overline{x}_3) \cdot \\
&\quad (\overline{x}_1 + x_2 + x_3)(\overline{x}_1 + x_2 + \overline{x}_3)(\overline{x}_1 + \overline{x}_2 + x_3) \\
&= ((x_1 + x_2 + x_3)(x_1 + x_2 + \overline{x}_3))((x_1 + \overline{x}_2 + x_3)(x_1 + \overline{x}_2 + \overline{x}_3)) \cdot \\
&\quad ((\overline{x}_1 + x_2 + x_3)(\overline{x}_1 + x_2 + \overline{x}_3))((\overline{x}_1 + \overline{x}_2 + x_3)(\overline{x}_1 + x_2 + x_3)) \\
&= (x_1 + x_2 + x_3\overline{x}_3)(x_1 + \overline{x}_2 + x_3\overline{x}_3) \cdot \\
&\quad (\overline{x}_1 + x_2 + x_3\overline{x}_3)(\overline{x}_1 + \overline{x}_2x_2 + x_3) \\
&= (x_1 + x_2)(x_1 + \overline{x}_2)(\overline{x}_1 + x_2)(\overline{x}_1 + x_3)
\end{aligned}$$

$$\begin{aligned}
&= (x_1 + x_2\overline{x}_2)(\overline{x}_1 + x_2x_3) \\
&= x_1(\overline{x}_1 + x_2x_3) \\
&= x_1\overline{x}_1 + x_1x_2x_3 \\
&= x_1x_2x_3
\end{aligned}$$

2.12. Derivation of the minimum sum-of-products expression:

$$\begin{aligned}
f &= x_1x_3 + x_1\overline{x}_2 + \overline{x}_1x_2x_3 + \overline{x}_1\overline{x}_2\overline{x}_3 \\
&= x_1(\overline{x}_2 + x_2)x_3 + x_1\overline{x}_2(\overline{x}_3 + x_3) + \overline{x}_1x_2x_3 + \overline{x}_1\overline{x}_2\overline{x}_3 \\
&= x_1\overline{x}_2x_3 + x_1x_2x_3 + x_1\overline{x}_2\overline{x}_3 + \overline{x}_1x_2x_3 + \overline{x}_1\overline{x}_2\overline{x}_3 \\
&= x_1x_3 + (x_1 + \overline{x}_1)x_2x_3 + (x_1 + \overline{x}_1)\overline{x}_2\overline{x}_3 \\
&= x_1x_3 + x_2x_3 + \overline{x}_2\overline{x}_3
\end{aligned}$$

2.13. Derivation of the minimum sum-of-products expression:

$$\begin{aligned}
f &= x_1\overline{x}_2\overline{x}_3 + x_1x_2x_4 + x_1\overline{x}_2x_3\overline{x}_4 \\
&= x_1\overline{x}_2\overline{x}_3(\overline{x}_4 + x_4) + x_1x_2x_4 + x_1\overline{x}_2x_3\overline{x}_4 \\
&= x_1\overline{x}_2\overline{x}_3\overline{x}_4 + x_1\overline{x}_2\overline{x}_3x_4 + x_1x_2x_4 + x_1\overline{x}_2x_3\overline{x}_4 \\
&= x_1\overline{x}_2\overline{x}_3 + x_1\overline{x}_2(\overline{x}_3 + x_3)\overline{x}_4 + x_1x_2x_4 \\
&= x_1\overline{x}_2\overline{x}_3 + x_1\overline{x}_2\overline{x}_4 + x_1x_2x_4
\end{aligned}$$

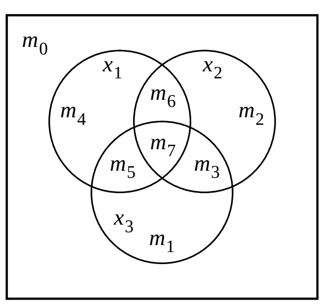
2.14. The simplest POS expression is derived as

$$\begin{aligned}
f &= (x_1 + x_3 + x_4)(x_1 + \overline{x}_2 + x_3)(x_1 + \overline{x}_2 + \overline{x}_3 + x_4) \\
&= (x_1 + x_3 + x_4)(x_1 + \overline{x}_2 + x_3)(x_1 + \overline{x}_2 + x_3 + x_4)(x_1 + \overline{x}_2 + \overline{x}_3 + x_4) \\
&= (x_1 + x_3 + x_4)(x_1 + \overline{x}_2 + x_3)((x_1 + \overline{x}_2 + x_4)(x_3 + \overline{x}_3)) \\
&= (x_1 + x_3 + x_4)(x_1 + \overline{x}_2 + x_3)(x_1 + \overline{x}_2 + x_4) \cdot 1 \\
&= (x_1 + x_3 + x_4)(x_1 + \overline{x}_2 + x_3)(x_1 + \overline{x}_2 + x_4)
\end{aligned}$$

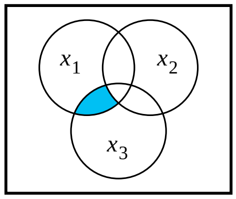
2.15. Derivation of the minimum product-of-sums expression:

$$\begin{aligned}
f &= (x_1 + x_2 + x_3)(x_1 + \overline{x}_2 + x_3)(\overline{x}_1 + \overline{x}_2 + x_3)(x_1 + x_2 + \overline{x}_3) \\
&= ((x_1 + x_2) + x_3)((x_1 + x_2) + \overline{x}_3)(x_1 + (\overline{x}_2 + x_3))(\overline{x}_1 + (\overline{x}_2 + x_3)) \\
&= (x_1 + x_2)(\overline{x}_2 + x_3)
\end{aligned}$$

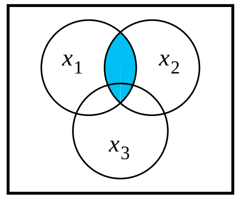
2.16. $(a)$ Location of all minterms in a 3-variable Venn diagram:



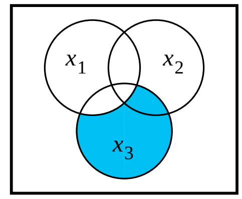
($b$) For $f = x_1\overline{x}_2x_3 + x_1x_2 + \overline{x}_1x_3$ have:




$x_1 \cdot \bar{x}_2 \cdot x_3$

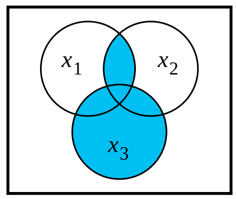



$x_1 \cdot x_2$



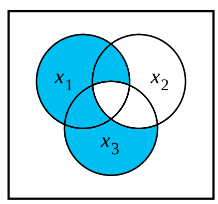

$\bar{x}_1 \cdot x_3$

Therefore, $f$ is represented as:



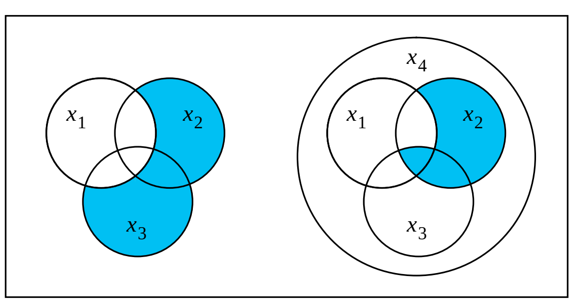

$$f = x_3 + x_1x_2$$

2.17. The function in Figure 2.23 in Venn diagram form is:




2.18. In Figure P2.1$a$ it is possible to represent only 14 minterms. It is impossible to represent the minterms $\overline{x}_1\overline{x}_2x_3x_4$ and $x_1x_2\overline{x}_3\overline{x}_4$.
In Figure P2.1$b$, it is impossible to represent the minterms $x_1x_2\overline{x}_3\overline{x}_4$ and $x_1x_2x_3\overline{x}_4$.

2.19. Venn diagram for $f = \overline{x}_1\overline{x}_2x_3\overline{x}_4 + x_1x_2x_3x_4 + \overline{x}_1x_2$ is



2.20. The simplest SOP implementation of the function is

$$\begin{aligned} f &= \overline{x}_1 x_2 x_3 + x_1 \overline{x}_2 \overline{x}_3 + x_1 x_2 \overline{x}_3 + x_1 x_2 x_3 \\ &= (\overline{x}_1 + x_1) x_2 x_3 + x_1 (\overline{x}_2 + x_2) \overline{x}_3 \\ &= x_2 x_3 + x_1 \overline{x}_3 \end{aligned}$$

2.21. The simplest SOP implementation of the function is

$$\begin{aligned} f &= \overline{x}_1 \overline{x}_2 x_3 + \overline{x}_1 x_2 x_3 + x_1 \overline{x}_2 \overline{x}_3 + x_1 x_2 \overline{x}_3 + x_1 x_2 x_3 \\ &= \overline{x}_1 (\overline{x}_2 + x_2) x_3 + x_1 (\overline{x}_2 + x_2) \overline{x}_3 + (\overline{x}_1 + x_1) x_2 x_3 \\ &= \overline{x}_1 x_3 + x_1 \overline{x}_3 + x_2 x_3 \end{aligned}$$

Another possibility is

$$f = \overline{x}_1 x_3 + x_1 \overline{x}_3 + x_1 x_2$$

2.22. The simplest POS implementation of the function is

$$\begin{aligned} f &= (x_1 + x_2 + x_3)(x_1 + \overline{x}_2 + x_3)(\overline{x}_1 + x_2 + \overline{x}_3) \\ &= ((x_1 + x_3) + x_2)((x_1 + x_3) + \overline{x}_2)(\overline{x}_1 + x_2 + \overline{x}_3) \\ &= (x_1 + x_3)(\overline{x}_1 + x_2 + \overline{x}_3) \end{aligned}$$

2.23. The simplest POS implementation of the function is

$$\begin{aligned} f &= (x_1 + x_2 + x_3)(x_1 + x_2 + \overline{x}_3)(\overline{x}_1 + x_2 + \overline{x}_3)(\overline{x}_1 + \overline{x}_2 + \overline{x}_3) \\ &= ((x_1 + x_2) + x_3)((x_1 + x_2) + \overline{x}_3)((\overline{x}_1 + x_3) + x_2)((\overline{x}_1 + x_3) + \overline{x}_2) \\ &= (x_1 + x_2)(\overline{x}_1 + \overline{x}_3) \end{aligned}$$

2.24. The simplest SOP expression for the function is

$$\begin{aligned} f &= x_1 \overline{x}_3 \overline{x}_4 + x_2 \overline{x}_3 x_4 + x_1 \overline{x}_2 \overline{x}_3 \\ &= x_1 \overline{x}_3 \overline{x}_4 + x_2 \overline{x}_3 x_4 + x_1 x_2 \overline{x}_3 + x_1 \overline{x}_2 \overline{x}_3 \\ &= x_1 \overline{x}_3 \overline{x}_4 + x_2 \overline{x}_3 x_4 + x_1 \overline{x}_3 \\ &= x_2 \overline{x}_3 x_4 + x_1 \overline{x}_3 \end{aligned}$$

2.25. The simplest SOP expression for the function is

$$\begin{aligned} f &= \overline{x}_1\overline{x}_3\overline{x}_5 + \overline{x}_1\overline{x}_3\overline{x}_4 + \overline{x}_1 x_4 x_5 + x_1\overline{x}_2\overline{x}_3 x_5 \\ &= \overline{x}_1\overline{x}_3\overline{x}_5 + \overline{x}_1\overline{x}_3\overline{x}_4 + \overline{x}_1 x_4 x_5 + \overline{x}_1\overline{x}_3 x_5 + x_1\overline{x}_2\overline{x}_3 x_5 \\ &= \overline{x}_1\overline{x}_3 + \overline{x}_1\overline{x}_3\overline{x}_4 + \overline{x}_1 x_4 x_5 + x_1\overline{x}_2\overline{x}_3 x_5 \\ &= \overline{x}_1\overline{x}_3 + \overline{x}_1 x_4 x_5 + x_1\overline{x}_2\overline{x}_3 x_5 \\ &= \overline{x}_1\overline{x}_3 + \overline{x}_1 x_4 x_5 + \overline{x}_2\overline{x}_3 x_5 \end{aligned}$$

2.26. The simplest POS expression for the function is

$$\begin{aligned} f &= (\overline{x}_1 + \overline{x}_3 + \overline{x}_4)(\overline{x}_2 + \overline{x}_3 + x_4)(x_1 + \overline{x}_2 + \overline{x}_3) \\ &= (\overline{x}_1 + \overline{x}_3 + \overline{x}_4)(\overline{x}_2 + \overline{x}_3 + x_4)(\overline{x}_1 + \overline{x}_2 + \overline{x}_3)(x_1 + \overline{x}_2 + \overline{x}_3) \\ &= (\overline{x}_1 + \overline{x}_3 + \overline{x}_4)(\overline{x}_2 + \overline{x}_3 + x_4)(\overline{x}_2 + \overline{x}_3) \\ &= (\overline{x}_1 + \overline{x}_3 + \overline{x}_4)(\overline{x}_2 + \overline{x}_3) \end{aligned}$$

2.27. The simplest POS expression for the function is

$$\begin{aligned} f &= (\overline{x}_2 + x_3 + x_5)(x_1 + \overline{x}_3 + x_5)(x_1 + x_2 + x_5)(x_1 + \overline{x}_4 + \overline{x}_5) \\ &= (\overline{x}_2 + x_3 + x_5)(x_1 + \overline{x}_3 + x_5)(x_1 + \overline{x}_2 + x_5)(x_1 + x_2 + x_5)(x_1 + \overline{x}_4 + \overline{x}_5) \\ &= (\overline{x}_2 + x_3 + x_5)(x_1 + \overline{x}_3 + x_5)(x_1 + x_5)(x_1 + \overline{x}_4 + \overline{x}_5) \\ &= (\overline{x}_2 + x_3 + x_5)(x_1 + x_5)(x_1 + x_5(\overline{x}_4 + \overline{x}_5)) \\ &= (\overline{x}_2 + x_3 + x_5)(x_1 + x_5)(x_1 + x_5\overline{x}_4) \\ &= (\overline{x}_2 + x_3 + x_5)(x_1 + x_5)(x_1 + \overline{x}_4) \end{aligned}$$

2.28. The lowest-cost circuit is defined by

$$f(x_1, x_2, x_3) = x_1x_2 + x_1x_3 + x_2x_3$$

2.29. The function, *f*, of this circuit is equal to 0 when either none of the inputs or all three inputs are equal to 0; otherwise, *f* is equal to 1. Therefore, using the POS form, the desired circuit can be realized as

$$\begin{aligned} f(x_1, x_2, x_3) &= \Pi\mathrm{M}(0, 3) \\ &= (x_1 + x_2 + x_3)(\overline{x}_1 + \overline{x}_2 + \overline{x}_3) \end{aligned}$$

2.30. The circuit can be implemented as

$$\begin{aligned} f &= x_1x_2x_3\overline{x}_4 + x_1x_2\overline{x}_3x_4 + x_1\overline{x}_2x_3x_4 + \overline{x}_1x_2x_3x_4 + x_1x_2x_3x_4 \\ &= x_1x_2x_3(\overline{x}_4 + x_4) + x_1x_2(\overline{x}_3 + x_3)x_4 + x_1(\overline{x}_2 + x_2)x_3x_4 + (\overline{x}_1 + x_1)x_2x_3x_4 \\ &= x_1x_2x_3 + x_1x_2x_4 + x_1x_3x_4 + x_2x_3x_4 \end{aligned}$$

2.31. The truth table that corresponds to the timing diagram in Figure P2.3 is

| $x_1$ | $x_2$ | $x_3$ | $f$ |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

The simplest SOP expression is $f = \overline{x}_1\overline{x}_2\overline{x}_3 + \overline{x}_1 x_2 x_3 + x_1\overline{x}_2 x_3 + x_1 x_2\overline{x}_3$.

2.32. The truth table that corresponds to the timing diagram in Figure P2.3 is

| $x_1$ | $x_2$ | $x_3$ | $f$ |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

The simplest POS expression is $f = (x_1 + x_2 + \overline{x}_3)(x_1 + \overline{x}_2 + x_3)(\overline{x}_1 + x_2 + x_3)(\overline{x}_1 + \overline{x}_2 + \overline{x}_3)$.

2.33. The truth table that corresponds to the timing diagram in Figure P2.4 is

| $x_1$ | $x_2$ | $x_3$ | $f$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

The simplest SOP expression is derived as follows:

$$
\begin{aligned}
f &= \overline{x}_1\overline{x}_2x_3 + \overline{x}_1x_2\overline{x}_3 + \overline{x}_1x_2x_3 + x_1\overline{x}_2\overline{x}_3 + x_1x_2x_3 \\
&= \overline{x}_1(\overline{x}_2 + x_2)x_3 + \overline{x}_1\overline{x}_2(\overline{x}_3 + x_3) + (\overline{x}_1 + x_1)x_2x_3 + x_1\overline{x}_2\overline{x}_3 \\
&= \overline{x}_1 \cdot 1 \cdot x_3 + \overline{x}_1x_2 \cdot 1 + 1 \cdot x_2x_3 + x_1\overline{x}_2\overline{x}_3 \\
&= \overline{x}_1x_3 + \overline{x}_1x_2 + x_2x_3 + x_1\overline{x}_2\overline{x}_3
\end{aligned}
$$

2.34. The truth table that corresponds to the timing diagram in Figure P2.4 is

| $x_1$ | $x_2$ | $x_3$ | $f$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

The simplest POS expression is $f = (x_1 + x_2 + x_3)(\overline{x}_1 + x_2 + \overline{x}_3)(\overline{x}_1 + \overline{x}_2 + x_3)$.

2.35. ($a$)

| $x_1$ | $x_0$ | $y_1$ | $y_0$ | $f$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |

($b$) The simplest POS expression is $f = (x_1 + \overline{y}_1)(\overline{x}_1 + y_1)(x_0 + \overline{y}_0)(\overline{x}_0 + y_0)$.

2.36. ($a$)

| $x_1$ | $x_0$ | $y_1$ | $y_0$ | $f$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

($b$) The canonical SOP expression is

$$\begin{aligned} f \;=\;& \overline{x}_1\overline{x}_0\overline{y}_1\overline{y}_0 + \overline{x}_1 x_0\overline{y}_1\overline{y}_0 + \overline{x}_1 x_0\overline{y}_1 y_0 + x_1\overline{x}_0\overline{y}_1\overline{y}_0 + x_1\overline{x}_0\overline{y}_1 y_0 + x_1\overline{x}_0 y_1\overline{y}_0 \\ &+ x_1 x_0\overline{y}_1\overline{y}_0 + x_1 x_0\overline{y}_1 y_0 + x_1 x_0 y_1\overline{y}_0 + x_1 x_0 y_1 y_0 \end{aligned}$$

(c) The simplest SOP expression is

$$f = x_1 x_0 + \overline{y}_1\overline{y}_0 + x_1\overline{y}_0 + x_0\overline{y}_1$$

2.37. SOP form: $f = \bar{x}_1 x_2 + \bar{x}_2 x_3$
POS form: $f = (\bar{x}_1 + \bar{x}_2)(x_2 + x_3)$

2.38. SOP form: $f = x_1\bar{x}_2 + x_1 x_3 + \bar{x}_2 x_3$
POS form: $f = (x_1 + x_3)(x_1 + \bar{x}_2)(\bar{x}_2 + x_3)$

2.39. SOP form: $f = \bar{x}_1 x_2 x_3 \bar{x}_4 + x_1 x_2 \bar{x}_3 x_4 + \bar{x}_2 x_3 x_4$
POS form: $f = (\bar{x}_1 + x_4)(x_2 + x_3)(\bar{x}_2 + \bar{x}_3 + \bar{x}_4)(x_2 + x_4)(x_1 + x_3)$

2.40. SOP form: $f = \bar{x}_2\bar{x}_3 + \bar{x}_2\bar{x}_4 + x_2 x_3 x_4$
POS form: $f = (\bar{x}_2 + x_3)(x_2 + \bar{x}_3 + \bar{x}_4)(\bar{x}_2 + x_4)$

2.41. SOP form: $f = \bar{x}_3\bar{x}_5 + \bar{x}_3 x_4 + x_2 x_4 \bar{x}_5 + \bar{x}_1 x_3 \bar{x}_4 x_5 + x_1 x_2 \bar{x}_4 x_5$
POS form: $f = (\bar{x}_3 + x_4 + x_5)(\bar{x}_3 + \bar{x}_4 + \bar{x}_5)(x_2 + \bar{x}_3 + \bar{x}_4)(x_1 + x_3 + x_4 + \bar{x}_5)(\bar{x}_1 + x_2 + x_4 + \bar{x}_5)$

2.42. SOP form: $f = \bar{x}_2x_3 + \bar{x}_1x_5 + \bar{x}_1x_3 + \bar{x}_3\bar{x}_4 + \bar{x}_2x_5$
POS form: $f = (\bar{x}_1 + \bar{x}_2 + \bar{x}_3)(\bar{x}_1 + \bar{x}_2 + \bar{x}_4)(x_3 + \bar{x}_4 + x_5)$

2.43. SOP form: $f = x_3\bar{x}_4\bar{x}_5 + \bar{x}_3\bar{x}_4x_5 + x_1x_4x_5 + x_1x_2x_4 + x_3x_4x_5 + \bar{x}_2x_3x_4 + x_2\bar{x}_3x_4\bar{x}_5$
POS form: $f = (x_3 + x_4 + x_5)(\bar{x}_3 + x_4 + \bar{x}_5)(x_1 + \bar{x}_2 + \bar{x}_3 + \bar{x}_4 + x_5)$

2.44. $f = \sum m(0,7)$
$f = \sum m(1,6)$
$f = \sum m(2,5)$
$f = \sum m(0,1,6)$
$f = \sum m(0,2,5)$
etc.

2.45. $f = x_1x_2x_3 + x_1x_2x_4 + x_1x_3x_4 + x_2x_3x_4$

2.46. SOP form: $f = x_1x_2\bar{x}_3 + x_1\bar{x}_2x_4 + x_1x_3\bar{x}_4 + \bar{x}_1x_2x_3 + \bar{x}_1x_3x_4 + x_2\bar{x}_3x_4$
POS form: $f = (x_1 + x_2 + x_3)(x_1 + x_2 + x_4)(x_1 + x_3 + x_4)(x_2 + x_3 + x_4)(\bar{x}_1 + \bar{x}_2 + \bar{x}_3 + \bar{x}_4)$
The POS form has lower cost.

2.47. The statement is false. As a counter example consider $f(x_1, x_2, x_3) = \sum m(0,5,7)$.
Then, the minimum-cost SOP form $f = x_1x_3 + \bar{x}_1\bar{x}_2\bar{x}_3$ is unique.
But, there are two minimum-cost POS forms:
$f = (x_1 + \bar{x}_3)(\bar{x}_1 + x_3)(x_1 + \bar{x}_2)$ and
$f = (x_1 + \bar{x}_3)(\bar{x}_1 + x_3)(\bar{x}_2 + x_3)$

2.48. If each circuit is implemented separately:
$f = \bar{x}_1\bar{x}_4 + \bar{x}_1x_2x_3 + x_1\bar{x}_2x_4$ Cost$= 15$
$g = \bar{x}_1\bar{x}_3\bar{x}_4 + \bar{x}_2x_3\bar{x}_4 + x_1\bar{x}_3x_4 + x_1x_2x_4$ Cost $= 21$

In a combined circuit:
$f = \bar{x}_2x_3\bar{x}_4 + \bar{x}_1\bar{x}_3\bar{x}_4 + x_1\bar{x}_2\bar{x}_3x_4 + \bar{x}_1x_2x_3$
$g = \bar{x}_2x_3\bar{x}_4 + \bar{x}_1\bar{x}_3\bar{x}_4 + x_1\bar{x}_2\bar{x}_3x_4 + x_1x_2x_4$
The first 3 product terms are shared, hence the total cost is 31.

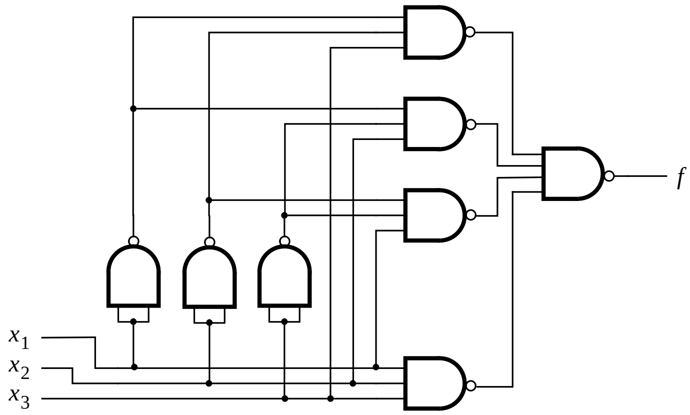
2.49. If each circuit is implemented separately:
$f = \bar{x}_1x_2x_4 + x_2x_4x_5 + x_3\bar{x}_4\bar{x}_5 + \bar{x}_1\bar{x}_2\bar{x}_4x_5$ Cost $= 22$
$g = \bar{x}_3\bar{x}_5 + \bar{x}_4\bar{x}_5 + \bar{x}_1\bar{x}_2\bar{x}_4 + \bar{x}_1x_2x_4 + x_2x_4x_5$ Cost $= 24$

In a combined circuit:
$f = \bar{x}_1x_2x_4 + x_2x_4x_5 + x_3\bar{x}_4\bar{x}_5 + \bar{x}_1\bar{x}_2\bar{x}_4x_5$
$g = \bar{x}_1x_2x_4 + x_2x_4x_5 + x_3\bar{x}_4\bar{x}_5 + \bar{x}_1\bar{x}_2\bar{x}_4x_5 + \bar{x}_3\bar{x}_5$

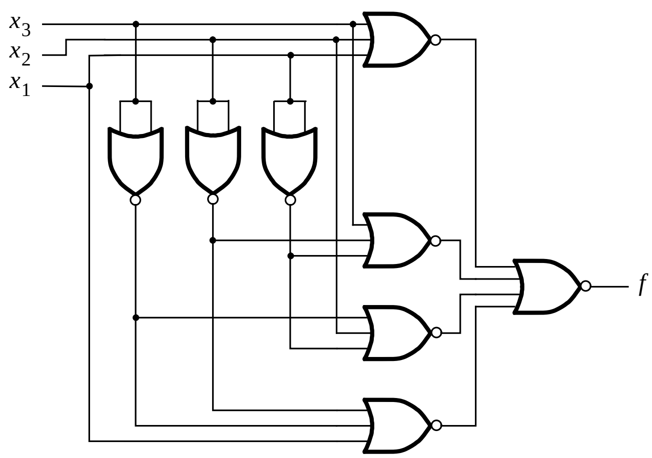
The first 4 product terms are shared, hence the total cost is 31. Note that in this implementation $f \subseteq g$, thus $g$ can be realized as $g = f + \bar{x}_3\bar{x}_5$, in which case the total cost is lowered to 28.

2.50. $f = (x_1 + x_4 + x_5)(x_1 + x_2 + x_3)(\bar{x}_1 + \bar{x}_2 + x_3)(\bar{x}_1 + \bar{x}_4 + x_5)$

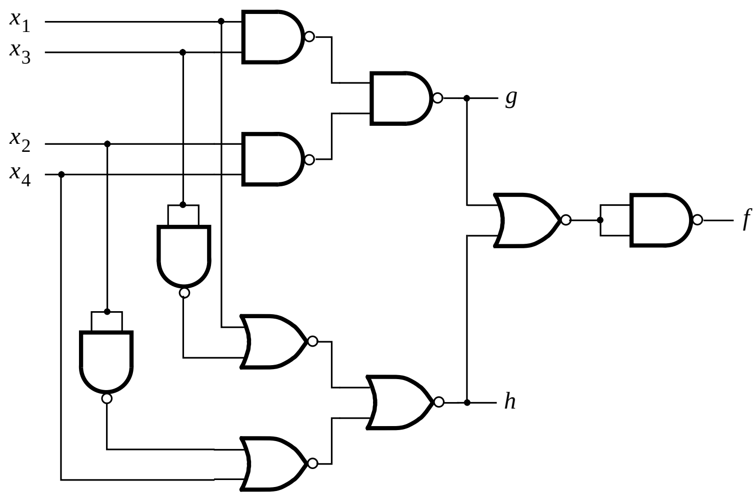
2.51 (etc). Using the ciruit in Figure 2.32$a$ as a starting point, the function in Figure 2.31 can be implemented using NAND gates as follows:




2.52. Using the ciruit in Figure 2.32$b$ as a starting point, the function in Figure 2.31 can be implemented using NOR gates as follows:



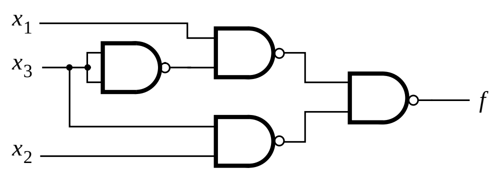

2.53. The circuit in Figure 2.39 can be implemented using NAND and NOR gates as follows:




2.54. The minimum-cost SOP expression for the function $f(x_1, x_2, x_3) = \sum m(3, 4, 6, 7)$ is

$$f = x_1\overline{x}_3 + x_2x_3$$

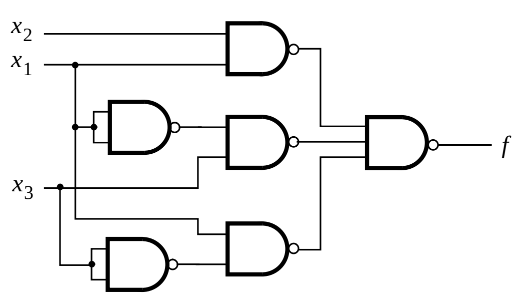
The corresponding circuit implemented using NAND gates is




2.55. A minimum-cost SOP expression for the function $f(x_1, x_2, x_3) = \sum m(1, 3, 4, 6, 7)$ is

$$f = x_1x_2 + x_1\overline{x}_3 + \overline{x}_1x_3$$

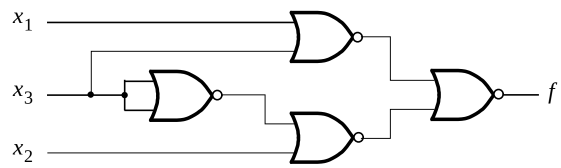
The corresponding circuit implemented using NAND gates is



2.56. The minimum-cost POS expression for the function $f(x_1, x_2, x_3) = \sum m(3, 4, 6, 7)$ is

$$f = (x_1 + x_3)(x_2 + \overline{x}_3)$$

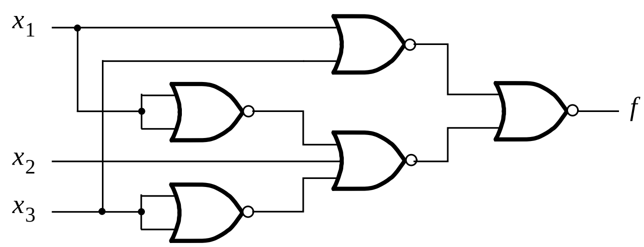
The corresponding circuit implemented using NOR gates is




2.57. The minimum-cost POS expression for the function $f(x_1, x_2, x_3) = \sum m(1, 3, 4, 6, 7)$ is

$$f = (x_1 + x_3)(\overline{x}_1 + x_2 + \overline{x}_3)$$

The corresponding circuit implemented using NOR gates is




2.60. The circuit in Figure 2.32$a$ can be implemented using;

```
module prob2_46 (x1, x2, x3, f);
    input x1, x2, x3;
    output f;

    not (notx1, x1);
    not (notx2, x2);
    not (notx3, x3);
    and (a, notx1, notx2, x3);
    and (b, notx1, x2, notx3);
    and (c, x1, notx2, notx3);
    and (d, x1, x2, x3);
    or (f, a, b, c, d);

endmodule
```

2.61. The circuit in Figure 2.32$b$ can be implemented using;

```
module prob2_47 (x1, x2, x3, f);
    input x1, x2, x3;
    output f;

    not (notx1, x1);
    not (notx2, x2);
    not (notx3, x3);
    or (a, x1, x2, x3);
    or (b, notx1, notx2, x3);
    or (c, notx1, x2, notx3);
    or (d, x1, notx2, notx3);
    and (f, a, b, c, d);

endmodule
```

2.62. The simplest circuit is obtained in the POS form as

$$f = (x_1 + x_2 + x_3)(\overline{x}_1 + \overline{x}_2 + \overline{x}_3)$$

Verilog code that implements the circuit is

```
module prob2_48 (x1, x2, x3, f);
    input x1, x2, x3;
    output f;

    or (g, x1, x2, x3);
    or (h, ~x1, ~x2, ~x3);
    and (f, g, h);

endmodule
```

2.63. The simplest circuit is obtained in the SOP form as

$$f = \overline{x}_2 + \overline{x}_1 x_3 + x_1 \overline{x}_3$$

Verilog code that implements the circuit is

```
module prob2_49 (x1, x2, x3, f);
    input x1, x2, x3;
    output f;

    assign f = ~x2 | (~x1 & x3) | (x1 & ~x3);
endmodule
```

2.64. The Verilog code is

```
module prob2_50 (x1, x2, x3, x4, f1, f2);
   input x1, x2, x3, x4;
   output f1, f2;

   assign f1 = (x1 & ~x3) | (x2 & ~x3) | (~x3 & ~x4) | (x1 & x2) | (x1 & ~x4);
   assign f2 = (x1 | ~x3) & (x1 | x2 | ~x4) & (x2 | ~x3 | ~x4);

endmodule
```

2.65. The Verilog code is

```
module prob2_51 (x1, x2, x3, x4, f1, f2);
   input x1, x2, x3, x4;
   output f1, f2;

   assign f1 = (x1 & x3) | (~x1 & ~x3) | (x2 & x4) | (~x2 & ~x4);
   assign f2 = (x1 & x2 & ~x3 & ~x4) | (~x1 & ~x2 & x3 & x4) |
         (x1 & ~x2 & ~x3 & x4) | (~x1 & x2 & x3 & ~x4);

endmodule
```

2.66. Representing both functions in the form of Karnaugh map, it is easy to show that $f = g$. The minimum cost SOP expression is
$f = g = \bar{x}_2\bar{x}_3\bar{x}_5 + \bar{x}_2x_3\bar{x}_4 + x_1x_3x_4 + x_1x_2x_4x_5$.

2.67. Representing both functions in the form of Karnaugh map, it is easy to show that $f = g$. The minimum cost SOP expression is
$f = g = x_2x_4 + x_1\overline{x}_2\overline{x}_4 + \overline{x}_1x_2x_3 + \overline{x}_2\overline{x}_3\overline{x}_4$.

2.68. Representing both functions in the form of Karnaugh map, it is easy to show that $f$ and $g$ do not represent the same function. In particular: $f(1,1,0,1,0) = 1$ while $g(1,1,0,1,0) = 0$ and $f(1,1,1,1,1) = 0$ while $g(1,1,1,1,1) = 1$.

2.69. Implementing the circuit as

$$f = \overline{x}_2x_3\overline{x}_4 + x_1x_2x_3x_4 + \overline{x}_2\overline{x}_3$$

$$g = \overline{x}_2x_3\overline{x}_4 + x_1x_2x_3x_4 + x_1\overline{x}_3x_4 + \overline{x}_1x_3x_4$$

there are 7 gates and 22 inputs for a cost of 29.

2.70. Implementing the circuit as

$$
\begin{aligned}
f &= (x_1 + \overline{x}_2 + x_3 + x_4)(x_1 + \overline{x}_2 + \overline{x}_3 + \overline{x}_4)(x_1 + x_3 + \overline{x}_4)(\overline{x}_1 + x_2 + \overline{x}_3)(\overline{x}_1 + x_2 + x_4) \\
g &= (x_1 + \overline{x}_2 + x_3 + x_4)(x_1 + \overline{x}_2 + \overline{x}_3 + \overline{x}_4)(\overline{x}_1 + \overline{x}_2 + \overline{x}_4)(\overline{x}_1 + x_2 + \overline{x}_3)
\end{aligned}
$$

there are 9 gates and 32 inputs for a cost of 41.

2.71. Assuming that the condition where all sensors produce the output of 0 is a don't care, the complement of the desired function is

$$\overline{f} = \overline{x}_1\overline{x}_2\overline{x}_3 + \overline{x}_1\overline{x}_2\overline{x}_4 + \overline{x}_1\overline{x}_3\overline{x}_4 + \overline{x}_2\overline{x}_3\overline{x}_4$$

Then, $f = \overline{\overline{f}}$.

2.72. Assuming that the condition where all sensors produce the output of 0 is a don't care, the complement of the desired function is

$$
\begin{aligned}
\overline{f} &= \overline{x}_1\overline{x}_2\overline{x}_3\overline{x}_4\overline{x}_5\overline{x}_6 + \overline{x}_1\overline{x}_2\overline{x}_3\overline{x}_4\overline{x}_5\overline{x}_7 + \overline{x}_1\overline{x}_2\overline{x}_3\overline{x}_4\overline{x}_6\overline{x}_7 + \overline{x}_1\overline{x}_2\overline{x}_3\overline{x}_5\overline{x}_6\overline{x}_7 + \\
&\quad \overline{x}_1\overline{x}_2\overline{x}_4\overline{x}_5\overline{x}_6\overline{x}_7 + \overline{x}_1\overline{x}_3\overline{x}_4\overline{x}_5\overline{x}_6\overline{x}_7 + \overline{x}_2\overline{x}_3\overline{x}_4\overline{x}_5\overline{x}_6\overline{x}_7
\end{aligned}
$$

Then, $f = \overline{\overline{f}}$.

2.73. Implement first the complement of $f$ as

$$
\begin{aligned}
\overline{f} &= x_1x_3 + x_2x_4 \\
&= (x_1 \uparrow x_3) \uparrow (x_2 \uparrow x_4)
\end{aligned}
$$

Then $f = \overline{f} \uparrow \overline{f}$.

2.74. Implement first the complement of $f$ as

$$
\begin{aligned}
\overline{f} &= \overline{x}_1\overline{x}_3 + x_2x_4 + x_1x_3 \\
&= (\overline{x}_1\overline{x}_3 + x_2x_4) + (x_1x_3 + x_1x_3) \\
&= ((\overline{x}_1 \uparrow \overline{x}_3) \uparrow (x_2 \uparrow x_4)) \uparrow ((x_1 \uparrow x_3) \uparrow (x_1 \uparrow x_3))
\end{aligned}
$$

Then $f = \overline{f} \uparrow \overline{f}$.

2.75. Implement first the complement of $f$ as

$$
\begin{aligned}
\overline{f} &= (\overline{x}_1 + x_4)(\overline{x}_2 + \overline{x}_3) \\
&= (\overline{x}_1 \downarrow x_4) \downarrow (\overline{x}_2 \downarrow \overline{x}_3)
\end{aligned}
$$

Then $f = \overline{f} \downarrow \overline{f}$.

2.76. Implement first the complement of $f$ as

$$
\begin{aligned}
\overline{f} &= (\overline{x}_1 + \overline{x}_4)(\overline{x}_2 + x_3)(x_2 + \overline{x}_3) \\
&= ((\overline{x}_1 + \overline{x}_4)(\overline{x}_2 + x_3))((x_2 + \overline{x}_3)(x_2 + \overline{x}_3)) \\
&= ((\overline{x}_1 \downarrow \overline{x}_4) \downarrow (\overline{x}_2 \downarrow x_3)) \downarrow ((x_2 \downarrow \overline{x}_3) \downarrow (x_2 \downarrow \overline{x}_3))
\end{aligned}
$$

Then $f = \overline{f} \downarrow \overline{f}$.

2.77. The cost of the circuit in Figure P2.5 is 11 gates and 30 inputs, for a total of 41. The functions implemented by the circuit can also be realized as

$$\begin{aligned} f &= \overline{x}_1\overline{x}_2\overline{x}_4 + x_2\overline{x}_3\overline{x}_4 + \overline{x}_1x_3x_4 + x_1x_4 \\ g &= \overline{x}_1\overline{x}_2\overline{x}_4 + x_2\overline{x}_3\overline{x}_4 + \overline{x}_1x_3x_4 + \overline{x}_2x_4 + x_3\overline{x}_4 \end{aligned}$$

The first three product terms in $f$ and $g$ are the same; therefore, they can be shared. Then, the cost of implementing $f$ and $g$ is 8 gates and 24 inputs, for a total of 32.

2.78. The cost of the circuit in Figure P2.6 is 11 gates and 26 inputs, for a total of 37. The functions implemented by the circuit can also be realized as

$$\begin{aligned} f &= (\overline{x}_2 \uparrow x_4) \uparrow (\overline{x}_1 \uparrow x_2 \uparrow x_3) \uparrow (x_1 \uparrow \overline{x}_2 \uparrow x_3) \uparrow (\overline{x}_2 \uparrow \overline{x}_3) \\ g &= (\overline{x}_2 \uparrow x_4) \uparrow (\overline{x}_1 \uparrow x_2 \uparrow x_3) \uparrow (x_1 \uparrow \overline{x}_2 \uparrow x_3) \uparrow (\overline{x}_1 \uparrow \overline{x}_1) \end{aligned}$$

The first three NAND terms in $f$ and $g$ are the same; therefore, they can be shared. Then, the cost of implementing $f$ and $g$ is 7 gates and 20 inputs, for a total of 27.