Chapter 8

8.1. $f = (x_3 \uparrow g) \uparrow ((g \uparrow g) \uparrow x_4)$ where $g = (x_1 \uparrow (x_2 \uparrow x_2)) \uparrow ((x_1 \uparrow x_1) \uparrow x_2)$

8.2. $\overline{f} = (((x_3 \downarrow x_3) \downarrow g) \downarrow ((g \downarrow g) \downarrow (x_4 \downarrow x_4))$, where
$g = ((x_1 \downarrow x_1) \downarrow x_2) \downarrow (x_1 \downarrow (x_2 \downarrow x_2))$. Then, $f = \overline{f} \downarrow \overline{f}$.

8.3. $f = (g \uparrow k) \uparrow ((g \uparrow g) \uparrow (k \uparrow k))$, where $g = (x_1 \uparrow x_1) \uparrow (x_2 \uparrow x_2) \uparrow (x_5 \uparrow x_5)$
and $k = (x_3 \uparrow (x_4 \uparrow x_4)) \uparrow ((x_3 \uparrow x_3) \uparrow x_4)$

8.4. $\overline{f} = (g \downarrow k) \downarrow ((g \downarrow g) \downarrow (k \downarrow k))$, where $g = x_1 \downarrow x_2 \downarrow x_5$
and $k = ((x_3 \downarrow x_3) \downarrow x_4) \downarrow (x_3 \downarrow (x_4 \downarrow x_4))$. Then, $f = \overline{f} \downarrow \overline{f}$.

8.5. $f = \overline{x}_1(x_2 + x_3)(x_4 + x_5) + x_1(\overline{x}_2 + x_3)(\overline{x}_4 + x_5)$

8.6. $f = x_1\overline{x}_3\overline{x}_4 + x_2\overline{x}_3\overline{x}_4 + x_1x_3x_4 + x_2x_3x_4 = (x_1 + x_2)\overline{x}_3\overline{x}_4 + (x_1 + x_2)x_3x_4$
This requires 2 OR and 2 AND gates.

8.7. $f = x_1 \cdot g + \overline{x}_1 \cdot \overline{g}$, where $g = \overline{x}_3x_4 + x_3\overline{x}_4$

8.8 $f = g \cdot h + \overline{g} \cdot \overline{h}$, where $g = x_1x_2$ and $h = x_3 + x_4$

8.9. Let $D(0, 20)$ be 0 and $D(15, 26)$ be 1. Then decomposition yields:
$g = x_5(\overline{x}_1 + x_2)$
$f = (\overline{x}_3\overline{x}_4 + x_3x_4)g + \overline{x}_3x_4\overline{g} = x_3x_4g + \overline{x}_3\overline{x}_4g + \overline{x}_3x_4\overline{g}$
Cost $= 9 + 18 = 27$

The optimal SOP form is:
$f = \overline{x}_3x_4\overline{x}_5 + \overline{x}_1x_3x_4x_5 + x_1\overline{x}_2\overline{x}_3x_4 + \overline{x}_1\overline{x}_3\overline{x}_4x_5 + x_2\overline{x}_3\overline{x}_4x_5 + x_2x_3x_4x_5$
Cost $= 7 + 29 = 36$

8.10. Let $a = x_2$ represent the subfunction in the rows where $x_3x_4 = 00$ and 11. Then, the part of f represented by a is given by $(\overline{x_3 \oplus x_4}) \cdot x_2$. Also, let $b = x_1$ represent the rows where $x_3x_4 = 01$ and 10. Then, the part of f defined by b is $(x_3 \oplus x_4) \cdot x_1$. This gives

$$f = (\overline{x_3 \oplus x_4}) \cdot x_2 + (x_3 \oplus x_4) \cdot x_1$$

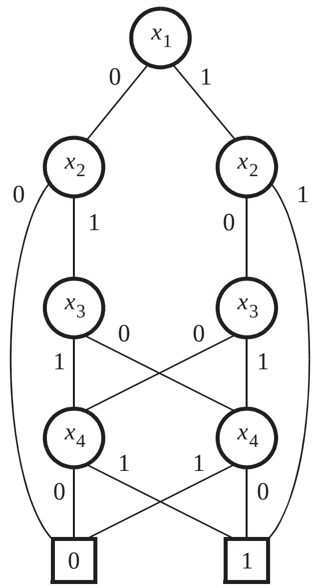
8.11. Let $\overline{k} = \overline{x_3 \oplus x_4}$ represent the subfunction in the column where $x_1 x_2 x_5 = 000$. Then, k represents the subfunction in the other columns. This gives

$$f = \overline{k} \cdot (\overline{x}_1 \overline{x}_2 \overline{x}_5) + k \cdot (x_1 + x_2 + x_5)$$

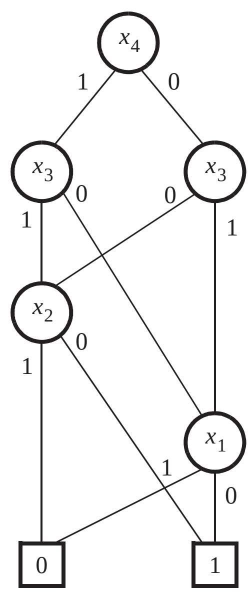
Now, letting $\overline{g} = \overline{x}_1 \overline{x}_2 \overline{x}_5$, we have

$$f = \overline{k}\overline{g} + kg = \overline{k \oplus g}$$

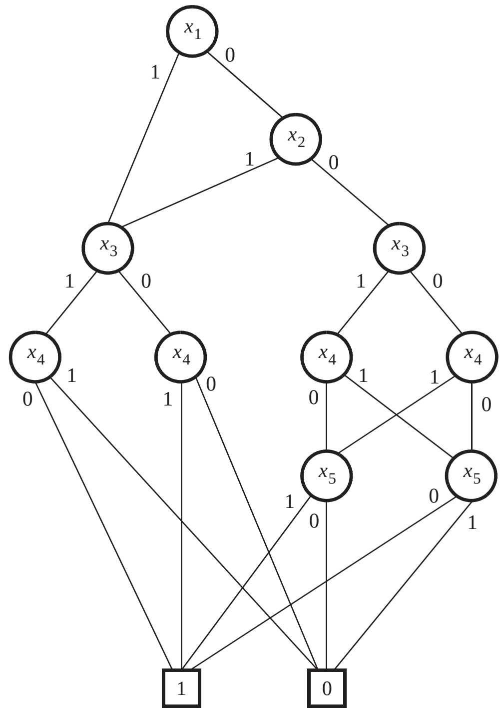
8.12. The BDD is



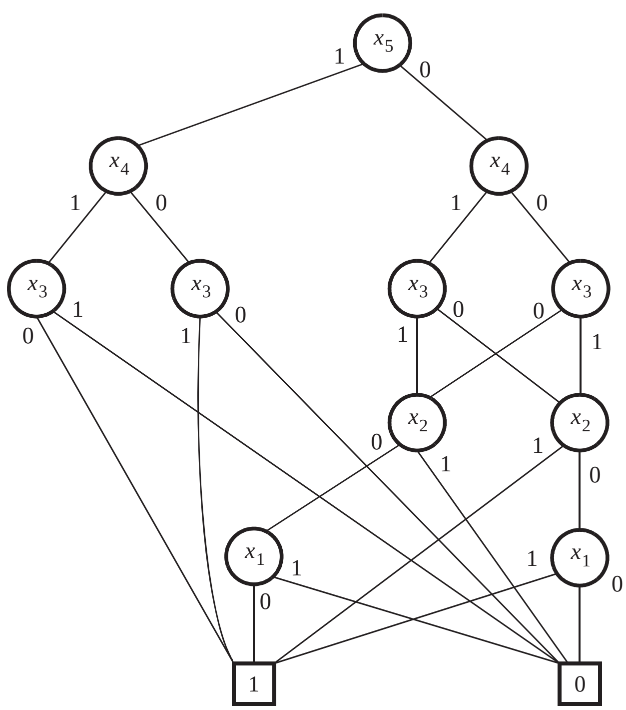
8.13. The BDD is



8.14. The BDD is



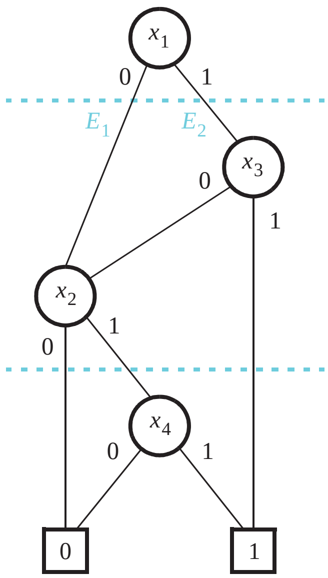
8.15. The BDD is



8.16. The BDD can be derived using Shannon's expansion as follows:

$$\begin{aligned} f &= x_1x_3 + x_2x_4 \\ &= \overline{x}_1(x_2x_4) + x_1(x_3 + x_2x_4) \\ &= \overline{x}_1(x_2x_4) + x_1(\overline{x}_3(x_2x_4) + x_3(1)) \\ &= \overline{x}_1(\overline{x}_2(0) + x_2(x_4)) + x_1(\overline{x}_3(\overline{x}_2(0) + x_2(x_4)) + x_3(1)) \end{aligned}$$

8.17. We first need to isolate nodes x_2 and x_3 from the BDD in Figure 8.37*b*, as indicated in the figure below. Since the edge E_1 depends only on node x_2, there is no reordering needed for this path. But for the edge E_2 we can create the truth table shown in the figure to enumerate the proper destination nodes for each combination of values of x_2 and x_3. Using this truth table and swapping nodes x_2 and x_3 leads directly to the BDD in Figure 8.35.



x_2	x_3	E_2
0	0	0
0	1	1
1	0	x_4
1	1	1

8.18. The prime implicants are generated as follows:

List 1

0	0 0 0 0	√
2	0 0 1 0	√
4	0 1 0 0	√
8	1 0 0 0	√
5	0 1 0 1	√
9	1 0 0 1	√
7	0 1 1 1	√
15	1 1 1 1	√

List 2

0,2	0 0 x 0
0,4	0 x 0 0
0,8	x 0 0 0
4,5	0 1 0 x
8,9	1 0 0 x
5,7	0 1 x 1
7,15	x 1 1 1

The initial prime implicant table is

Prime implicant	Minterm 0	2	4	5	7	8	9	15
p_1 = 0 0 x 0	√	√						
p_2 = 0 x 0 0	√		√					
p_3 = x 0 0 0	√					√		
p_4 = 0 1 0 x			√	√				
p_5 = 1 0 0 x						√	√	
p_6 = 0 1 x 1				√	√			
p_7 = x 1 1 1					√			√

The prime implicants p_1, p_5 and p_7 are essential. Removing these prime implicants gives

Prime implicant	Minterm 4	5
p_2	√	
p_3		
p_4	√	√
p_6		√

Since p_4 covers both minterms, the final cover is

$$\begin{aligned} C &= \{p_1, p_4, p_5, p_7\} \\ &= \{00\text{x}0, 010\text{x}, 100\text{x}, \text{x}111\} \end{aligned}$$

and the function is implemented as

$$f = \overline{x}_1\overline{x}_2\overline{x}_4 + \overline{x}_1 x_2 \overline{x}_3 + x_1\overline{x}_2\overline{x}_3 + x_2 x_3 x_4$$

8.19. The prime implicants are generated as follows:

List 1

0	0 0 0 0	✓
4	0 1 0 0	✓
8	1 0 0 0	✓
3	0 0 1 1	✓
6	0 1 1 0	✓
9	1 0 0 1	✓
7	0 1 1 1	✓
11	1 0 1 1	✓
13	1 1 0 1	✓
15	1 1 1 1	✓

List 2

0,4	0 x 0 0	
0,8	x 0 0 0	
4,6	0 1 x 0	
8,9	1 0 0 x	
3,7	0 x 1 1	✓
3,11	x 0 1 1	✓
6,7	0 1 1 x	
9,11	1 0 x 1	✓
9,13	1 x 0 1	✓
7,15	x 1 1 1	✓
11,15	1 x 1 1	✓
13,15	1 1 x 1	✓

List 3

3,7,11,15	x x 1 1
9,11,13,15	1 x x 1

The initial prime implicant table is

Prime implicant	Minterm 0	4	6	8	9	15
p_1 = 0 x 0 0	✓	✓				
p_2 = x 0 0 0	✓			✓		
p_3 = 0 1 x 0		✓	✓			
p_4 = 1 0 0 x				✓	✓	
p_5 = 0 1 1 x			✓			
p_6 = x x 1 1						✓
p_7 = 1 x x 1					✓	✓

There are no essential prime implicants. Prime implicant p_3 dominates p_5 and their costs are the same, so remove p_5. Similarly, p_7 dominates p_6, so remove p_6. This gives

Prime implicant	Minterm 0	4	6	8	9	15
p_1	✓	✓				
p_2	✓			✓		
p_3		✓	✓			
p_4				✓	✓	
p_7					✓	✓

Now, p_3 and p_7 are essential, which leaves

Prime implicant	Minterm 0	8
p_1	✓	
p_2	✓	✓
p_4		✓

Choosing p_2 results in the minimum cost cover

$$\begin{aligned} C &= \{p_2, p_3, p_7\} \\ &= \{\text{x000}, \text{01x0}, \text{1xx1}\} \end{aligned}$$

and the function is implemented as

$$f = \overline{x}_2\overline{x}_3\overline{x}_4 + \overline{x}_1 x_2 \overline{x}_4 + x_1 x_4$$

8.20. The prime implicants are generated as follows:

List 1

0	0 0 0 0	✓
4	0 1 0 0	✓
8	1 0 0 0	✓
3	0 0 1 1	✓
5	0 1 0 1	✓
9	1 0 0 1	✓
12	1 1 0 0	✓
7	0 1 1 1	✓
11	1 0 1 1	✓
13	1 1 0 1	✓
14	1 1 1 0	✓

List 2

0,4	0 x 0 0	✓
0,8	x 0 0 0	
4,5	0 1 0 x	✓
4,12	x 1 0 0	✓
8,9	1 0 0 x	✓
8,12	1 x 0 0	✓
3,7	0 x 1 1	
3,11	x 0 1 1	
5,7	0 1 x 1	
5,13	x 1 0 1	✓
9,11	1 0 x 1	
9,13	1 x 0 1	✓
12,13	1 1 0 x	✓
12,14	1 1 x 0	

List 3

0,4,8,12	x x 0 0
4,5,12,13	x 1 0 x
8,9,12,13	1 x 0 x

The initial prime implicant table is

Prime implicant	Minterm 0	3	4	5	7	9	11
p_1 = 0 x 1 1		√			√		
p_2 = x 0 1 1		√					√
p_3 = 0 1 x 1				√	√		
p_4 = 1 0 x 1						√	√
p_5 = x x 0 0	√		√				
p_6 = x 1 0 x			√	√			
p_7 = 1 x 0 x						√	
p_8 = 1 1 x 0							

Prime implicant p_5 is essential, so remove columns 0 and 4 to get

Prime implicant	Minterm 3	5	7	9	11
p_1	√		√		
p_2	√				√
p_3		√	√		
p_4				√	√
p_6		√			
p_7				√	

Here, p_3 dominates p_6, and p_4 dominates p_7; but costs of p_3 and p_4 are greater than the costs of p_6 and p_7, respectively. So, use branching. First choose p_3 to be in the final cover, which leads to

Prime implicant	Minterm 3	9	11
p_1	√		
p_2	√		√
p_4		√	√
p_6			
p_7		√	

Now, choose p_2 and p_7 (lower cost than p_4) to cover the remaining minterms. The resulting cover is

$$
\begin{aligned}
C &= \{p_2, p_3, p_5, p_7\} \\
&= \{\text{x011}, \text{01x1}, \text{xx00}, \text{1x0x}\}
\end{aligned}
$$

Next, assume that p_3 is not included in the final cover, which leads to

Prime implicant	Minterm 3	5	7	9	11
p_1	√		√		
p_2	√				√
p_4				√	√
p_6		√			
p_7				√	

Then p_6 is essential. Also, column 3 dominates 7, hence remove 3 giving

Prime implicant	Minterm 7	9	11
p_1	√		
p_2			√
p_4		√	√
p_7		√	

Choose p_1 and p_4, which results in the cover

$$\begin{aligned} C &= \{p_1, p_4, p_5, p_6\} \\ &= \{0x11, 10x1, xx00, x10x\} \end{aligned}$$

Both covers have the same cost, so choosing the first cover the function can be implemented as

$$f = \overline{x}_2 x_3 x_4 + \overline{x}_1 x_2 x_4 + \overline{x}_3 \overline{x}_4 + x_1 \overline{x}_3$$

Observe that if we had not taken the cost of prime implicants (rows) into account and pursued the dominance of p_3 over p_6 and p_4 over p_7, then we would have removed p_6 and p_7 giving the following table

Prime implicant	Minterm 3	5	7	9	11
p_1	√		√		
p_2	√				√
p_3		√	√		
p_4				√	√

Now p_3 and p_4 are essential. Also choose p_1, so that

$$\begin{aligned} C &= \{p_1, p_3, p_4, p_5\} \\ &= \{0x11, 01x1, 10x1, xx00\} \end{aligned}$$

The cost of this cover is greater by one literal compared to both covers derived above.

8.21. Note that $X \# Y = X \cdot \overline{Y}$. Therefore,

$$\begin{aligned}(A \cdot B)\#C &= A \cdot B \cdot \overline{c} \\ (A\#C) \cdot (B\#C) &= A \cdot \overline{C} \cdot B \cdot \overline{C} \\ &= A \cdot B \cdot \overline{C}\end{aligned}$$

Similarly,

$$\begin{aligned}(A + B)\#C &= (A + B) \cdot \overline{C} \\ &= A \cdot \overline{C} + B \cdot \overline{C} \\ (A\#C) + (B\#C) &= A \cdot \overline{C} + B \cdot \overline{C}\end{aligned}$$

8.22. The initial cover is $C^0 = \{0000, 0011, 0100, 0101, 0111, 1000, 1001, 1111\}$.
Using the $*$-product get the prime implicants
P = {00x0, 0x00, x000, 010x, 01x1, 100x, x111}.
The minimum cover is $C_{minimum}$ = {00x0, 010x, 100x, x111}, which corresponds to $f = \overline{x}_1\overline{x}_2\overline{x}_4 + \overline{x}_1x_2\overline{x}_3 + x_1\overline{x}_2\overline{x}_3 + x_2x_3x_4$.

8.23. The initial cover is C^0 = {0x0x0, 110xx, x1101, 1001x, 11110, 01x10, 0x011}.
Using the $*$-product get the prime implicants
P = {0x0x0, xx01x, x1x10, 110xx, x10x0, 11x01, x1101}.
The minimum cover is $C_{mimimum}$ = {0x0x0, xx01x, x1x10, 110xx, x1101}, which corresponds to $f = \overline{x}_1\overline{x}_3\overline{x}_5 + \overline{x}_3x_4 + x_2x_4\overline{x}_5 + x_1x_2\overline{x}_3 + x_2x_3\overline{x}_4x_5$.

8.24. The initial cover is C^0 = {00x0, 100x, x010, 1111, 00x1, 011x}.
Using the $*$-product get the prime implicants P = {00xx, 0x1x, x00x, x0x0, x111}.
The minimum-cost cover is $C_{minimum}$ = {x00x, x0x0, x111}, which corresponds to $f = \overline{x}_2\overline{x}_3 + \overline{x}_2\overline{x}_4 + x_2x_3x_4$.

8.25. Expansion of $\overline{x}_1\overline{x}_2\overline{x}_3$ gives $\overline{x}_1$.
Expansion of $\overline{x}_1\overline{x}_2x_3$ gives $\overline{x}_1$.
Expansion of $\overline{x}_1x_2\overline{x}_3$ gives $\overline{x}_1$.
Expansion of $x_1x_2x_3$ gives x_2x_3.
The set of prime implicants comprises $\overline{x}_1$ and x_2x_3.

8.26. Expansion of $\overline{x}_1x_2\overline{x}_3x_4$ gives $x_2\overline{x}_3x_4$ and $\overline{x}_1x_2x_4$.
Expansion of $x_1x_2\overline{x}_3x_4$ gives $x_2\overline{x}_3x_4$.
Expansion of $x_1x_2x_3\overline{x}_4$ gives $x_3\overline{x}_4$.
Expansion of $\overline{x}_1x_2x_3$ gives $\overline{x}_1x_3$.
Expansion of $\overline{x}_2x_3$ gives $\overline{x}_2x_3$.
The set of prime implicants comprises $x_2\overline{x}_3x_4$, $\overline{x}_1x_2x_4$, $x_3\overline{x}_4$, $\overline{x}_1x_3$, and $\overline{x}_2x_3$.

8.27. Implement first the complement of f as

$$\begin{aligned}\overline{f} &= x_1x_3 + x_2x_4 \\ &= (x_1 \uparrow x_3) \uparrow (x_2 \uparrow x_4)\end{aligned}$$

Then $f = \overline{f} \uparrow \overline{f}$.

8.28. Implement first the complement of f as

$$\begin{aligned}
\overline{f} &= \overline{x}_1\overline{x}_3 + x_2x_4 + x_1x_3 \\
&= (\overline{x}_1\overline{x}_3 + x_2x_4) + (x_1x_3 + x_1x_3) \\
&= ((\overline{x}_1 \uparrow \overline{x}_3) \uparrow (x_2 \uparrow x_4)) \uparrow ((x_1 \uparrow x_3) \uparrow (x_1 \uparrow x_3))
\end{aligned}$$

Then $f = \overline{f} \uparrow \overline{f}$.

8.29. Implement first the complement of f as

$$\begin{aligned}
\overline{f} &= (\overline{x}_1 + x_4)(\overline{x}_2 + \overline{x}_3) \\
&= (\overline{x}_1 \downarrow x_4) \downarrow (\overline{x}_2 \downarrow \overline{x}_3)
\end{aligned}$$

Then $f = \overline{f} \downarrow \overline{f}$.

8.30. Implement first the complement of f as

$$\begin{aligned}
\overline{f} &= (\overline{x}_1 + \overline{x}_4)(\overline{x}_2 + x_3)(x_2 + \overline{x}_3) \\
&= ((\overline{x}_1 + \overline{x}_4)(\overline{x}_2 + x_3))((x_2 + \overline{x}_3)(x_2 + \overline{x}_3)) \\
&= ((\overline{x}_1 \downarrow \overline{x}_4) \downarrow (\overline{x}_2 \downarrow x_3)) \downarrow ((x_2 \downarrow \overline{x}_3) \downarrow (x_2 \downarrow \overline{x}_3))
\end{aligned}$$

Then $f = \overline{f} \downarrow \overline{f}$.