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Team Control Number

**9869**

Problem Chosen

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**2019**  
**HIMCM**  
**Summary Sheet**

# The Mathematical Model for The Bottle Battles

## Summary

The impact of the BAN on PET bottled water is a complex and comprehensive problem with multiple constraints. Based on microeconomic principles, we developed a market substitution model and a comprehensive evaluation model to calculate and analyze the impact of the BAN on PET bottled water.

In part 1, we have established the Single Drink Impact Evaluation Model (model 1) and Market Share Estimation Model (model 2). In model 1, we compared the differences between PET bottled water and potential substitutes and selected four key indicators of different types of drinks in four aspects, economy, per capita consumption, environment and health. Based on this, we completed a relevant evaluation index system, relevant data acquisition, data nondimensionalization and missing value processing. In model 2, we assumed that the relevant demand under the BAN would be replaced by direct drinking water, beverages, etc., and therefore we established a Market Substitution Model to analyze the impact of the BAN on beverage market share. By analyzing the impact value before and after the implementation of the BAN, we find that the implementation effect of Concord and San Francisco is neutral and optimistic respectively. At the same time, the reasons that the BAN had different effects in different cities and different dimensions in the same city were carefully analyzed.

In part 2, we compared and analyzed the differences between airports and towns in terms of population flow, portability requirements and consumption capacity, so that model 2 was improved into an incomplete replacement model of the market. Also, the marginal substitution function curve was optimized by using the least square method. The results show that the overall benefits of the BAN at San Francisco international airport outweigh the disadvantages, especially in terms of environmental protection.

In part 3, in order to make our evaluation model more universal, we adopted Entropy Weight Method to establish a comprehensive evaluation model based on model 1. We calculated and compared the comprehensive scores of several possible ways to modify the BAN. Certain policies that have been adopted in the United States historically were considered as well. Finally, we gave some detailed promotion recommendations.

The particular features of our paper lie in the establishment of Market Substitution Model based on microeconomic theory, which is closer to the actual operation law of the market and considers the positive and negative effects of potential substitutes under the background of prohibition more reasonably. In addition, we use sensitivity analysis to demonstrate the reliability of the model.

**Keywords:** Market Substitution; Entropy Weight Method; Comprehensive Evaluation

# Replace PET Bottled Water with Direct water

## DID YOU KNOW?

The PET-bottle water is cheaper than other beverages, but the production and recycling process pollutes the environment



## DID YOU KNOW?

The price of direct water is one-thousandth of that of PET bottled water, and it is safe and environmentally friendly, but the corresponding infrastructure construction is difficult.



## DID YOU KNOW?

The price of sugary drinks is high but it is loved by most Americans. Drinking a lot can lead to a series of health problems such as obesity



## DID YOU KNOW?

Alcoholic beverages, although expensive, are preferred by some adults and have a unique taste. But drinking is easy to cause safety problems



"Over the 2017-2018 holiday break 65 'hydration stations' were installed to replace water fountains on the entire main campus."

—— The Mott Community College community

The hydration stations contain NSF certified filtering to remove lead, other heavy metals, sediments and other contaminants. This filtering is indicated by a green-yellow-red indicator. Custodians are trained to alert Risk Management when a unit shows yellow so that the filter can be changed. The bottle filler unit permits filling and re-filling of both reusable and disposable bottles, greatly enhancing MCC's environmental initiatives by reducing the number of plastic bottles introduced into the waste stream. The number of bottles NOT introduced into the waste stream is indicated by the counter on the hydration station.



## How to optimize the ban?

According to our research, if the direct ban on the sale of PET bottled water will lead to a large number of sugary drinks and alcoholic beverages entering the market. This will lead to health, environmental protection and economic problems.

In order to avoid the negative effects of the previous ban and inherit the positive effects of the previous ban, we should give strategies for nationwide promotion from the following perspectives:

- Levy a tax on PET bottled water
- Distribute or encourage the purchase of reusable water bottles for free to residents
- Improve infrastructure for direct drinking water
- Carry out the whole ideological educational promotion

To put it in a nutshell, policymakers should introduce more than a few bans on specific implementation details, work to improve water direct share, and promote the infrastructure construction, ideological education, and non-mandatory reduce bottled drinks (including bottled water, sugary drinks, and alcoholic beverages). This can adjust citizens' drinking habits and give rise to a great extent of the development of the country.

2019 HIMCM

Reports by Team #9869

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# 1 Introduction

## 1.1 Restatement of the Problem

- Develop a model to analyze the impacts of a ban on the sale of single-serving water bottles in a town or city by using appropriate information and data. Use the model to discuss the impacts of the bans in the Town of Concord and the City of San Francisco.
- Analyze the impacts of a ban on the sale of single-serving water bottles within an airport. Discuss the similarity and difference between the ban in the airport and the city and possible changes of the initial model in order to fit the situation of the airport.
- Adjust the impact model according to the recommendations of possible changes to Concord's water bottle ban and analyze the promotability of the model in larger areas.

## 1.2 Problem Background

The debate about whether bottled water should be produced and sold or not has lasted for a long time. Some people, including government officers, believe that the plastic water bottles will definitely cause certain environmental and health problems. Therefore, in 2013, the town of Concord in Massachusetts firstly started to ban the bottled water. Soon, other cities started to propose similar policies, including the ban of bottled water in city property of San Francisco in 2014, and the ban in San Francisco's airport in 2019. However, some people or organizations, such as, International Bottled Water Association (IBWA), disagreed with those actions for some reasons like the availability of bottled water under emergency and some greater environmental impacts than bottled water.

## 1.3 Our Approach

The topic asks us to screen out reasonable indicators and establish a market substitution model before and after the promulgation of the bottled water ban, so that we can objectively quantify the impact of the ban on Concord and San Francisco. Based on the difference in market substitution relationships, we have correspondingly established an incomplete replacement model for the San Francisco airport ban, quantitatively analyzing the impact of the ban on the airport. In particular, we have established evaluation models for different bans and extended them nationwide.

- Analyze and process various types of data
- Establish a complete indicator evaluation system.
- Establish alternative models based on cities and airports.
- Analyze the actual impact of the ban on different regions based on the model solution results.
- Evaluate an evaluation model with different bans and promote it to a larger scope.

## 2 General Assumptions

To simplify the problem, we make the following basic assumptions, each of which is properly justified.

- **Assumption 1:** Group demand and macro market conditions are predictable.  
→ **Justification:** Although the individual's demand changes are unpredictable, according to the law of large numbers, such a group decision variable that is affected by many subtle factors is subject to a normal distribution, so we can expect changes in the group's demand and then speculate on the macro market situation.
- **Assumption 2:** It is assumed that the changes in the market share of various beverages are only related to the specific content of the ban, the behavioral preferences of consumers and beverages own features, and have nothing to do with other macroeconomic indicators, trade environment and technological breakthroughs.  
→ **Justification:** The model involves mainly static variables and does not take into account long-term effects. Therefore, in the short term, macro indicators, trade environment and technological breakthroughs are not considered in the model.
- **Assumption 3:** The products analyzed in this paper are products that comply with relevant production technical specifications. Safety and legal issues arising from violation of relevant regulations are not considered here.  
→ **Justification:** We believe that the legal constraints are the most basic conditions for a research object.
- **Assumption 4:** Assume that the data reviewed is accurate.  
→ **Justification:** We assume that the data on sites such as IWBA are not fraudulent, so that we can build a more reasonable quantitative model based on it again.
- **Assumption 5:** Assume that the drinking water demand of residents in the ban area will be replaced by other drinks after the bottled water is banned.  
→ **Justification:** Our full-text model will be based on alternative theory, which is the theoretical basis of the full text.

## 3 The Development of Models

After the introduction of the ban, the direct impact is that consumers have changed their drinking habits and changed the market share of various beverages, which ultimately led to a series of social impacts.

Therefore, it is necessary to quantitatively analyze the comprehensive impact of the ban in two parts: firstly, establish an evaluation model to evaluate the various impacts of consumers after choosing various types of drinks; secondly, analyze the resulting market share changes of the ban based on historical data and economic theory. Finally, combined with changes in market share and the impact of various types of beverages on consumers to comprehensively assess the impact of the ban on the city. In summary: models that this paper needs to establish are::

- **Model 1:** Single Drink Impact Evaluation Model. In this model, we need to solve three problems:

1. Determine the impact of the ban.
  2. What are the indicators that affect the nature of a single drink?
  3. How do the above two influence each other?
- **Model 2:** Market Share Estimation Model. In this model, we need to solve two problems:
    1. How the market structure affects the ban after its promulgation.
    2. How the actual changes are based on changes in the above pattern.

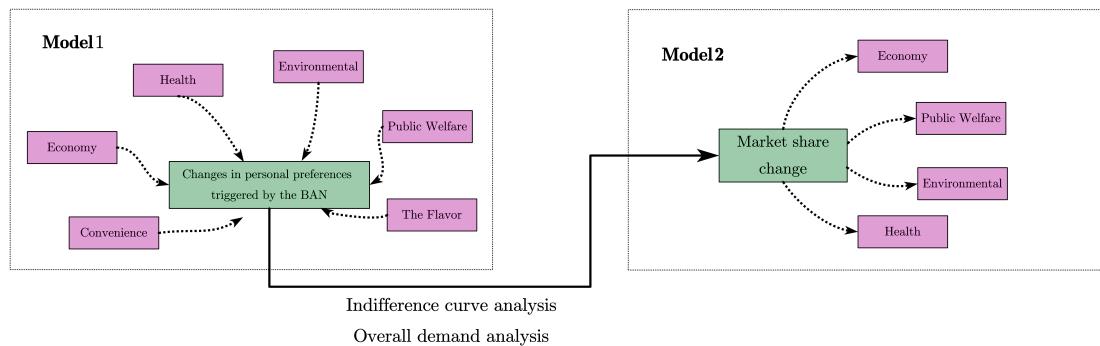


Figure 1: Problem 1 solution flow chart

**ATTENTION:** In this question, we did not calculate the combined effects of the ban, but calculated the impact of the ban on the economy, per capita expenditure, environment, and health. This is mainly because the score of the comprehensive evaluation is usually a relative concept.

- Even if we can compare the scores before and after the ban, we cannot recognize the true meaning behind this difference.
- We believe that the score cannot be simply reflected in the good or bad, and the balance of the ban is more important.

We believe that the significance of comprehensive evaluation is to compare the advantages and disadvantages of different programs, so we have established a comprehensive evaluation model in question 3 to complete this work.

### 3.1 Identification of Possible Impacts of Concord and San Francisco

For the impact of the ban on urbanization, this paper first analyzes the positive and negative effects of the ban on the macro; then constructs the evaluation indicators from the microscopic analysis of the advantages and disadvantages of various beverages, and then through the relevant data. Evaluate the implementation of the ban on the market share of the entire beverage market and determine the impact of the ban on towns by changing the situation and combining the advantages and disadvantages of the beverage itself.

### 3.1.1 Positive Impacts

- **Reduced household expenditure**

The ban on plastic bottled water may prompt residents to choose lower-cost direct drinking water for consumption. Based on this consideration, it may reduce the expenditure of families in plastic bottle areas.

- **Relieve environmental pollution**

After the plastic bottled water is banned, the corresponding manufacturer of plastic bottles reduces the production of plastic bottles, and as a white pollution product that is difficult to degrade, this will greatly contribute to environmental protection; at the same time, reduce plastics. The output of the bottle helps to reduce the emission of pollutants during the production of plastics and reduces the adverse effects on the environment.

- **May be beneficial to human health**

The micro-plastic particles contained in the plastic bottle are prone to chemical reactions harmful to the human body, and the ban can greatly alleviate this part of the harmful reaction.

- **Promote employment**

Promote the employment of convenient drinking water station workers

### 3.1.2 Negative Impacts

- **Reduce the convenience of drinking water**

For portability reasons, in order to achieve the same portability, the prohibition of plastic bottles may lead residents to choose other beverages such as beverages. From this perspective, it may increase the expenditure of families in plastic bottle areas.

- **Possible waste of resources**

For plastic bottle substitutes such as glass bottles, cans, etc., due to the complexity of the manufacturing process, more raw materials are needed, so it is more likely to cause energy waste in the production process.

- **Increase in government spending**

Advocating direct drinking water will put a lot of manpower and resources into the construction of direct drinking water infrastructure.

- **Possible health problems**

Sales of beverages (carbonated drinks, tea, etc.) will increase, which means that the amount of additives that people consume will increase, which will adversely affect human health.

## 3.2 The Data

### 3.2.1 Data Collection

The data we use mainly includes the data of various beverages, many of which can be directly reported through industry reports, such as the cost of various types of beverages, the content of various chemical substances, and some environmental indicators in American cities and towns. The data sources mainly include in Table1,

Table 1: Data source collation

Database Names	Database Websites	Data Type
IWBA	<a href="https://www.bottledwater.org/">https://www.bottledwater.org/</a>	Industry report
BMC	<a href="https://www.beveragemarketing.com/">https://www.beveragemarketing.com/</a>	Sales data
Consumer Report	<a href="https://www.consumerreports.org/">https://www.consumerreports.org/</a>	Safety indicator
Food & Water Watch	<a href="https://www.foodandwaterwatch.org/">https://www.foodandwaterwatch.org/</a>	Beverage data
Statista	<a href="https://www.statista.com/">https://www.statista.com/</a>	Other data
DataSF	<a href="https://datasf.org/opendata/">https://datasf.org/opendata/</a>	Resource data
US EPA	<a href="https://catalog.data.gov/dataset">https://catalog.data.gov/dataset</a>	Environmental data
Data world	<a href="https://data.world/">https://data.world/</a>	Macro indicator
Google Scholar	<a href="https://scholar.google.com/">https://scholar.google.com/</a>	Academic paper

The corresponding specific data will be compiled in the appendix, which will be used directly below.

### 3.2.2 Data Processing Method

- **Nondimensionalization**

For the different indicator data collected, there are differences in units and orders of magnitude, so the data needs to be dimensionless before the evaluation. For the set of data  $x_i, i = 1, 2, , N$ . The data after nondimensionalization  $x_i^*, i = 1, 2, , N$  is

$$x_i^* = \frac{x_{\max} - x_i}{x_{\max} - x_{\min}} \quad (1)$$

where,  $x_{\max}, x_{\min}$  are the maximum and minimum of the sample data.

- **Extreme Data Alignment**

Normalize the miniaturized data normalized in the indicator into the largest data according to the following formula:

$$x_i^* = 1 - x_i^* \quad (2)$$

- **Missing Value Processing**

For the social statistics mentioned in this paper, use curve fitting method to make a simple prediction and replace the missing data with the predicted value, for example:

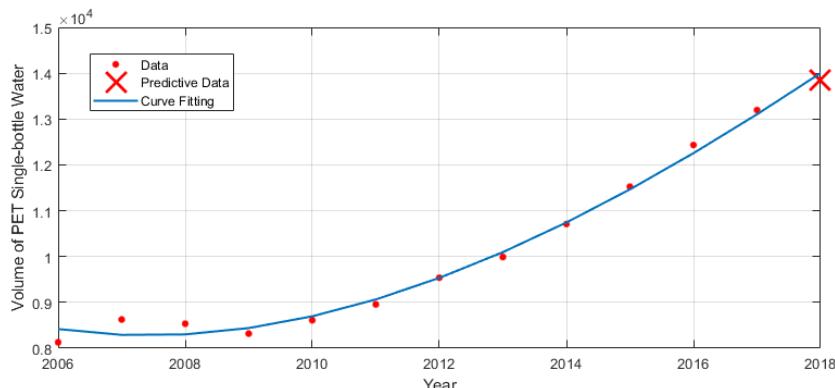


Figure 2: Missing value processing example

### 3.3 The Quantification Model of the Impacts of the Ban

Based on the above-mentioned ban on plastic bottled drinking water, refer to IBWA's research report on bottled water. We describe its own nature from the six indicators of economy, convenience, health and environmental protection, public welfare and flavor.

#### 3.3.1 Single Drink Impact Evaluation Model

##### (1) Types of Single Drinks

For alternatives in the market, IBWA's 2018 statistics report shows the market share of various beverages in the United States in 2017 as Figure3. From the Figure3, we can see that

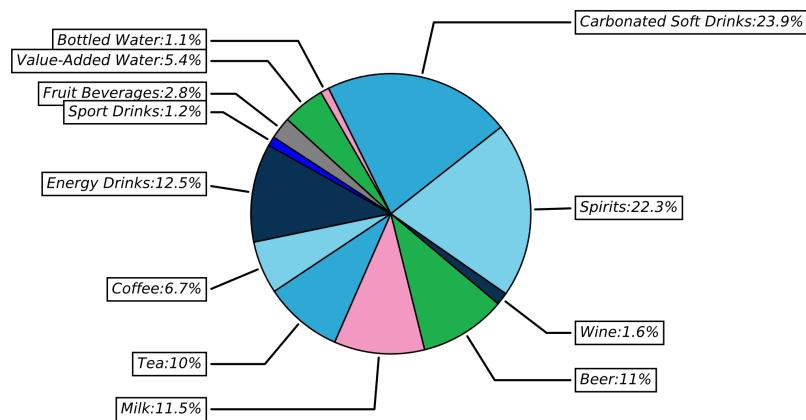


Figure 3: the market share of various beverages in the United States in 2017

the alternative products can be divided into direct drinking water, sugary drinks, health drinks and other drinks

##### (2) Principles of Indicator Selections

- **Economy**

Refers to the cost of the beverage itself, which includes the following two indicators: raw material cost, transportation cost, processing cost.

- **Convenience**

Refers to the cost of the beverage itself, which includes the following two indicators: raw material cost, transportation cost, processing cost.

- **Health**

Refers to the impact of chemicals contained in drinks on human health. The secondary indicators included are as follows: calorie content, mineral content, harmful substance content.

- **Environmental**

Refers to the various environmental impacts of beverages during the production and recycling phases. The secondary indicators included are as follows: energy consumption, carbon emissions, material recovery rate.

- **Public welfare**

Refers to the social welfare brought about by the production and sale of beverages. The secondary indicators included are as follows: job position, disaster area support.

- **The flavor**

Refers to the taste of the beverage itself. It does not contain secondary indicators.

### (3) Comprehensive Evaluation System

According to the analysis of indicator selections in Section 3.3.1(2), the final evaluation system is shown as Figure4.

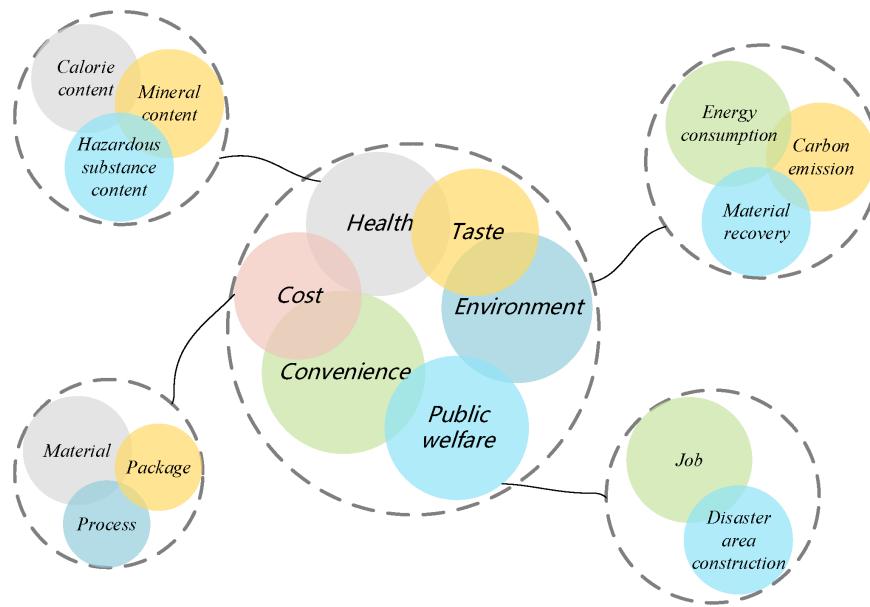


Figure 4: Comprehensive evaluation system for beverages

### (4) Evaluation results of single drink impacts

According to the data that have been processed in Section 3.2.2, the performance of several beverages on the six indicators of economy, convenience, health, environmental protection, public welfare and taste was finally obtained.

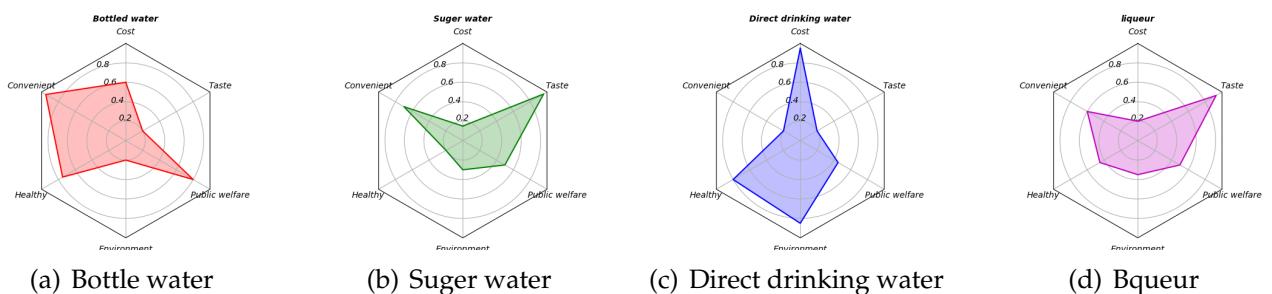


Figure 5: Radar chart for different drinks under different indicators

#### 3.3.2 Market Share Estimation Model

For consumers, according to the conclusion of Section 3.3.1, we refer to this series of drinks as a Market basket. The Indifference curve represents a combination of all the drinks that give the consumer the same level of satisfaction. Therefore, when the ban appears, in order to ensure the overall utility remains unchanged, consumers will choose to use direct

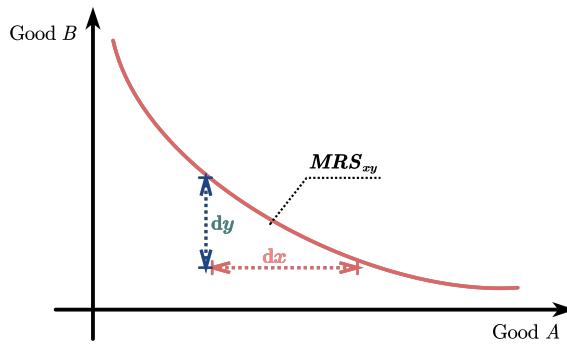


Figure 6: Utility surface and indifference curve

drinking water instead of bottled water. According to the definition of marginal replacement rate, we have,

$$MRS_{xy} = -\frac{dy}{dx} \quad (3)$$

According to the theory of microeconomics, the marginal replacement rate between two commodities is a function related to the equilibrium supply of commodities, so there is  $MRS_{xy} = g(x, y)$ . For the substitution function  $g$ , we can get a good intra-sample prediction by fitting the real sales data,

$$y_{t+1} = y_t + (x_{t+1} - x_t) \cdot \hat{g}(x_t, y_t) \quad (4)$$

where,  $\hat{g}(\cdot)$  is marginal substitution function;  $x_{t+1}$  is the bottled water sales data after the ban.

### 3.4 Application in Concord and San Francisco

For the city, the consumer's demand will hardly change, that is,  $g(x, y)$  is a constant, so the sales volume of other products after the ban can be directly estimated based on historical data, and then the market share is estimated.

#### 3.4.1 In Concord

According to the beverage sales data of all states in recent years obtained from the BMC, Therefore, based on these data, an estimate for the marginal substitution function  $\hat{g}(x, y)$  can be obtained,

Table 2: Estimated marginal rate of beverages in Concord

Succedaneum	Direct Drinking Water	Sugary drink	Alcoholic beverage
Rate	0.342	0.511	0.257

According to the results of Table2, it can be obtained that when Concord Town adopts a bottled water ban, 34.2% of the market share of bottled water on the market will be replaced by direct drinking water, 51.1% will be replaced by sugary drinks, 25.7% will be replaced by alcoholic beverages.

Changes in market share will have an impact on household expenditure, government revenue, environmental protection and public safety: To simplify the problem, we make the following basic assumptions, each of which is properly justified.

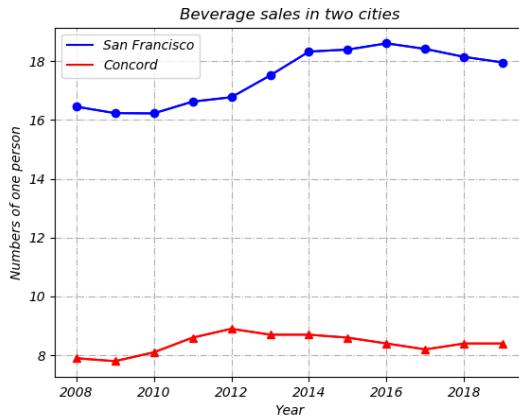


Figure 7: Sales data for Concord and San Francisco 2008-2019

- **Household expenditure**

The cost and price of different drinks are not the same. As a result, relatively high-priced sugar-sweetened beverages and alcoholic beverages account for 76.8% of the bottled water share, which will result in increased household spending on drinking water, which impose a heavier burden on family.

- **Government revenue**

Due to the emergence of the ban, the market share of sugar-sweetened beverages has increased significantly, which has led to an increase in profits of related companies, which will enable the government to obtain more taxes. At the same time, there is no increase in infrastructure investment related to direct drinking water after the ban, and it can be considered that the expenditure in this area has not changed much.

- **Environmental protection**

Based on the calculated results and the data obtained from the survey, the final environmental impact has not been significantly improved, and the recycling rate of garbage has only increased from 40.7% to 42.2%.

- **Health condition**

After the market share has changed, although the people have reduced the consumption of bottled water, excessive intake of sugary drinks and the like will lead to an increase in the overall obesity rate. On the other hand, the safety hazards from trace elements and so on have not decreased, which is a shortcoming caused by the ban.

- **Public welfare**

By referring to the social welfare brought about by the production and sale of beverages, the secondary indicators included are as follows: job position, disaster area support.

- **The flavor**

Refers to the taste of the beverage itself. It does not contain secondary indicators.

Therefore, it can be considered that the effect of this ban is not as high as the government expected: efficient, environmentally friendly and healthy. While the ban can reduce bottled water sales and increase government revenue to a certain extent, it has led to an increase in obesity rates and an increase in household spending, and has not achieved significant results in environmental protection. In general, the effect of the ban on sales is mingled with side-effects and positive externalities.

### 3.4.2 In San Francisco

Therefore, based on the data shown in Figure7, we can get the estimated value of the marginal substitution function  $g(x, y)$

Table 3: Estimated marginal rate of beverages in San Francisco

Succedaneum	Direct Drinking Water	Sugary drink	Alcoholic beverage
Rate	0.577	0.334	0.199

According to the results of Table3, it can be obtained that when Concord Town adopts a bottled water ban, 57.7% of the market share of bottled water on the market will be replaced by direct drinking water, 33.4% will be replaced by sugary drinks, 19.9% will be replaced by alcoholic beverages.

Unlike Concord, according to our model, the increase in direct drinking water sales in the San Francisco area after the ban is greater than the increase in various beverages. According to the Food&Water Watch organization's survey of community water supply in US states, San Francisco's community water supply is at 15%-25%, while Concord is only under 5%(see Figure8). Therefore, the convenience and cost of San Francisco people choosing direct drinking water will be relatively small. As a result, it can be considered that the effect

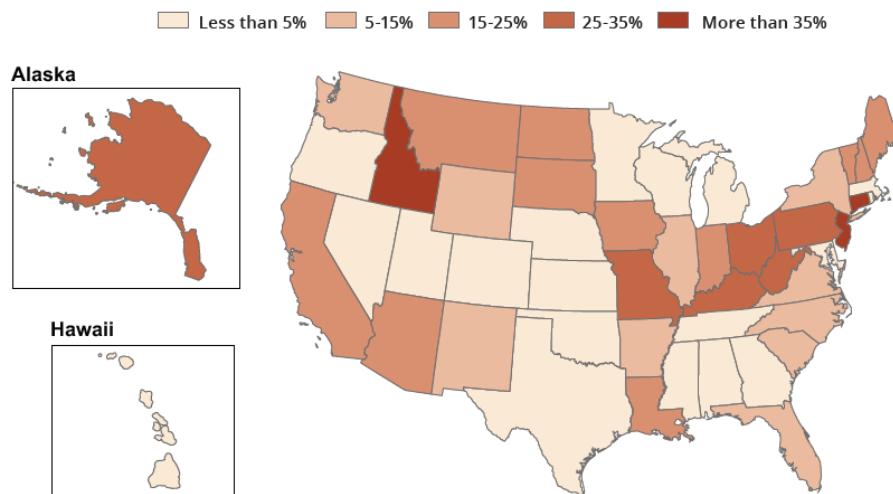


Figure 8: Community water supply in US states

of this ban on sales can basically meet the governments expectations: more people choose direct drinking water. This has led to a decline in the rate of obesity and a increased the expenditure, the effect of the ban is generally satisfactory.

## 4 Impacts of the BAN in the Airport

As for the impact of the ban on sales on the airport, this paper first analyzes some differences between the airport and the town: Concord and San Francisco are different in that Concord airport has a large flow of people, a small overall economy, a high demand for portability by passengers and a high consumption capacity of the population. Then, considering the above factors, if the corresponding ban is implemented, there may be incomplete market substitution. That is, passengers who want to buy bottled water will not buy bottled

water because there is no bottled water to buy. Finally, the actual situation of San Francisco International Airport is used for modeling and analysis.

## 4.1 Differences between Airports and Cities

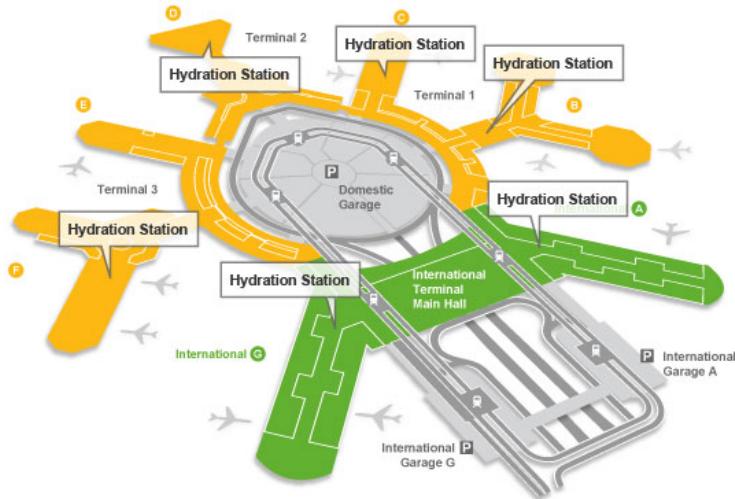


Figure 9: Distribution of drinking stations at San Francisco International Airport

As can be seen from Figure 9, the space inside San Francisco international airport is relatively small, and the average residence time of passengers in the airport will not exceed six hours. Hence for the airport, its internal beverage sales market is very different from the city's beverage sales market, mainly reflected in:

- overall economic volume is relatively small
- large population movement
- high demand for portability.
- population has higher consumption capacity.
- direct drinking water supply is not affected by water source distribution

Therefore, according to the above differences, it is necessary to make certain adjustments to the assumptions of the market share change model in subsection 4.1. As the demand for drinking water changes, the indifference curve will change from an almost straight line to a curve protruding to the origin.

## 4.2 Modification of the Market Share Estimation Model

It is necessary to make certain adjustments to the assumptions of the market share change model in Section 3.3.2. As the demand for drinking water changes, the indifference curve will change from an almost straight line to a curve protruding to the origin. Therefore, the difference is mainly that the marginal substitution function  $g(\cdot)$  becomes a curve instead of a constant. The following will use the data to estimate and obtain the impact of the ban. For fitting the specific function form of the formula, according to the knowledge of calculus,

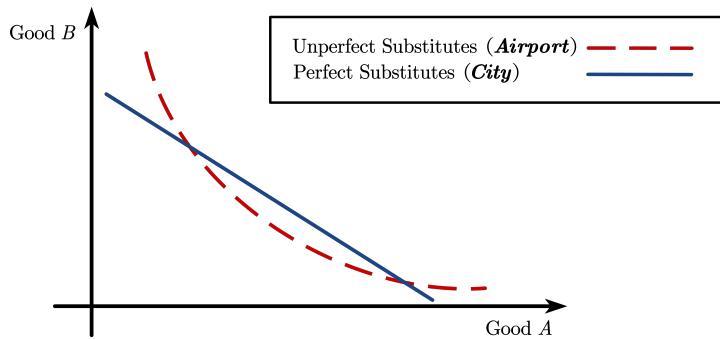


Figure 10: comparison chart of non-difference curves

any function can be better approximated by polynomial function, so we can use polynomial function to replace  $g$ ,

$$g = \mathbf{x} \mathbf{A} \mathbf{y}^T \quad (5)$$

where,  $\mathbf{x} = (1, x, x^2, \dots, x^n)^T$ ,  $\mathbf{y} = (1, y, y^2, \dots, y^n)^T$  is the vector with size  $(n + 1) \times 1$

$\mathbf{A} = \begin{pmatrix} a_{1,1} & \cdots & a_{1,n+1} \\ \vdots & \ddots & \vdots \\ a_{n,1} & \cdots & a_{n+1,n+1} \end{pmatrix}$  is the undetermined coefficient matrix with size  $(n + 1) \times (n + 1)$

### 4.3 Application in San Francisco International Airport

According to the modifications made to the model in Section 3.3.2, the estimation of the substitution function needs to be performed using a polynomial fitting method. Therefore, in order to correctly fit the utility surface, based on the least squares algorithm, the function is used to fit the data (see the appendix for the algorithm and code), and the results are as follows:

Table 4: Polynomial fitting results

The number of the highest degree	1	2	3
$R^2$	0.124	0.251	0.599

As can be seen from Table 4, when the highest degree of polynomial is 3, the fitting effect of the model is the best. Therefore, the final marginal substitution rate calculation function is,

$$\hat{g} = 0.211x + 0.354xy^2 - 0.854x^2y + 0.268x^3 - 0.124y^3 \quad (6)$$

We can use the results of equation (6) to get the final market share change.

Table 5: Estimated marginal rate of beverages in San Francisco International Airport

Succedaneum	Direct Drinking Water	Sugary drink	Alcoholic beverage
Rate	0.431	0.335	0.236

It is worth noting that the overall demand for beverages from the ban has fallen by 34.8

By using equation (6), the change trend of market share in the airport due to the ban on sales can be simulated: while the sales of bottled water decreased significantly, the share

of direct drinking water increased by 43.1%, the share of sugary beverages increased by 33.5%, and the share of alcoholic beverages increased by 23.4%. According to this trend, we can have an impact in terms of personal expenditure, convenience, airport income, environmental protection and public safety:

- **Personal expenditure**

Due to the difference between the airport and the city, after the ban, although the market share of related drinks has changed, the average personal expenditure change and the impact of expenditure on consumers can be ignored.

- **Convenience**

As the sale of bottled water is cancelled, some consumers who do not prefer beverages and do not prefer carry cups give up their drinking needs, which will lead to a decrease in overall convenience.

- **Airport revenue**

Since direct drinking water is provided free of charge, it can be seen from the changes in market share that the changes in revenue for airports are negligible.

- **Environmental**

The ban on sales has brought about a decrease in the amount of plastic waste, which has led to a greater improvement in the environmental protection level in the airport.

- **Health concern**

The increase in the share of sugary drinks and alcoholic drinks brought about by the ban on bottled water will still lead to an increase in the obesity rate, but a larger proportion of people also choose direct drinking water as a substitute, so the negative impact on health brought about by the ban on bottled water at airports is relatively low.

To put it in a nutshell, the overall benefits of the ban outweigh the disadvantages. It is believed that there will be a broader development prospect with the gradual improvement of the overall construction level of direct drinking water facilities in the airport.

## 5 Estimated and Promotion of the BAN

Question 3 requires us to propose changes to the ban and consider the impact of the ban and its impact on the larger area. First, we need to quantify the ban policy, establish an evaluation system for the ban policy, and characterize the impact scores of different bans. Next, based on the data of different regions, the model is solved in the model, and the degree of influence of the ban in different regions is obtained.

### 5.1 Evaluation System of the BAN

#### 5.1.1 Evaluation Indicators

Referring to the dimension planning evaluation system of the impact of the ban on the city in Section 3.1, we here divide the evaluation principle of the ban policy into:

- **Economics:** including changes in GDP, tax changes, and changes in household expenditure.

- **Environmental:** including carbon emissions, waste recycling rates and energy use.
- **Health:** including sugar and fine particles.
- **Public welfare:** including jobs caused by the ban and support for the disaster area.

The data preprocessing and quantification method is described in Section 3.3.1. Then we can obtain the comprehensive evaluation index system for the ban.

### 5.1.2 Comprehensive Evaluation Model

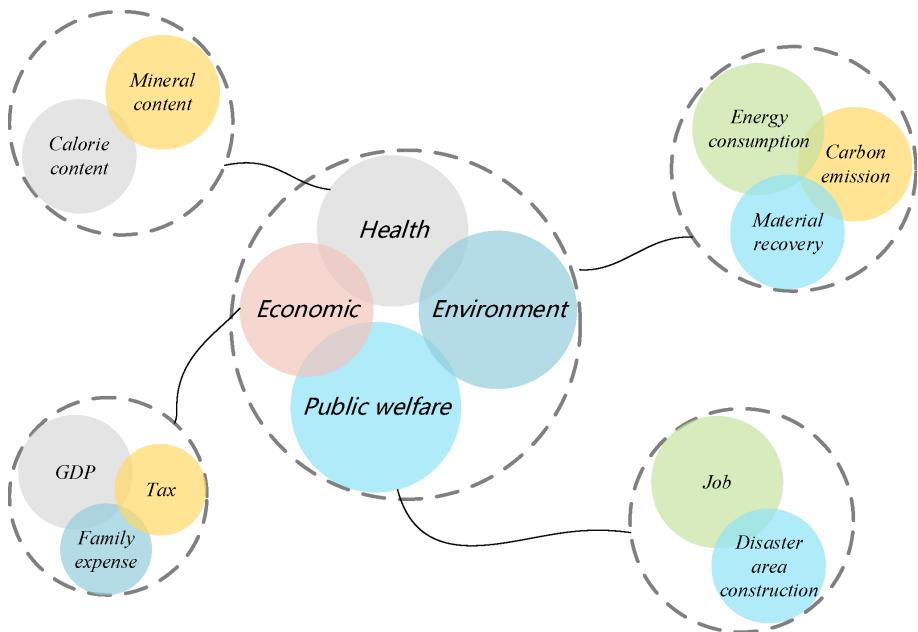


Figure 11: Comprehensive evaluation system for the BAN on sales

To comprehensively evaluate the impact of PET bottled water ban policy, it is essential to give each indicator a weight to put them together as one index. The weight of the second level index  $k$  is defined as  $W_k$ ; the weight of the third level index in the second level index is  $W_{jk}$ , and the score of indicator is  $x_{ij}$ . Based on the idea of linear weighting, the comprehensive evaluation model for quantitative evaluation of the ban policy is:

$$Q = \sum_{i \in J} \sum_{k \in K} x_{ij} w_{jk} W_k \quad (7)$$

where,  $\forall i \in J, k \in K, w_{jk}, W_k > 0$  and  $\sum_{i \in J} \sum_{k \in K} w_{jk} = 1, \sum_{k \in K} W_k = 1$ . Hence we can apply equation (7) to depict the ban effect quantitatively.

### 5.1.3 Determination of Weight Coefficients

The method of entropy control is applied. Based on the variation degree of each index, we calculated each indicators weight. The smaller the information entropy, the greater the variation and the greater the significance of the index. Conversely, the index is less important if the information entropy is large. The calculation process is as follows:

- **Step1:** Calculate the weight  $R_{ij}$  of standardized indicator  $x_{ij}$  as  $R_{ij} = \frac{x_j}{\sum_{i=1}^n x_{ij}}$

- **Step2:** Determine the entropy of the  $j^{th}$  indicator as
- **Step3:** Calculate the differential index of the  $j^{th}$  indicator as  $g_{jk} = 1 - e_{jk}$
- **Step4:** Determine the weight  $w_{jk}$  for  $x_j$ :  $w_{jk} \frac{g_{jk}}{\sum_{i=1}^n g_{ik}}$

Therefore, according to the established system of indicators in Section 5.1.2, we use MATLAB to solve to obtain the weight of each indicator.

Table 6: Estimated marginal rate of beverages in San Francisco International Airport

Second level index	Weight	Third level index	Weight
Economy	0.211	GDP	0.436
		Tax revenue	0.219
		Household payment	0.355
Environmental	0.356	Carbon emission	0.651
		Recycle rate	0.267
		Energy consumption	0.193
Health	0.271	Sugar content	0.448
		Microelement content	0.552
Public welfare	0.272	Job opportunity	0.642
		Support for disaster area	0.358

#### 5.1.4 Evaluation Results

According to the weights determined in Table6, the effects of different ban policies can be finally measured. According to the report, some policies of banning sales in the history of the United States are evaluated, and the evaluation results are presented as the follows:

Table 7: Estimated marginal rate of beverages in San Francisco International Airport

Ban policy	Factor1	Factor2	Factor3	Factor4	Total	Rank
Distribute refillable water bottles to residents	63.64	84.75	91.22	78.63	81.35	1
Levy taxes upon PET bottled water in cities	82.62	64.67	70.88	69.63	70.97	2
Ban bottled drinks	54.01	84.74	80.67	26.76	69.01	3
Forbid bottled water in city restaurant	60.17	63.64	58.79	49.27	59.48	4
Forbid bottled water in public activities	35.77	35.91	40.89	24.45	35.75	5
Forbid bottled water in vending machines	52.45	26.21	21.31	35.71	31.57	6

Analyzing these policies, we can see that the distribution of refillable water bottles score the highest because it effectively improves the replacement rate of direct drinking water. The reason for high ranking of taxing bottled water is that it improves the economics. At the same time, although the replacement rate of direct drinking water was further improved by banning bottled water and bottled drinks, it resulted in great economic losses and lower

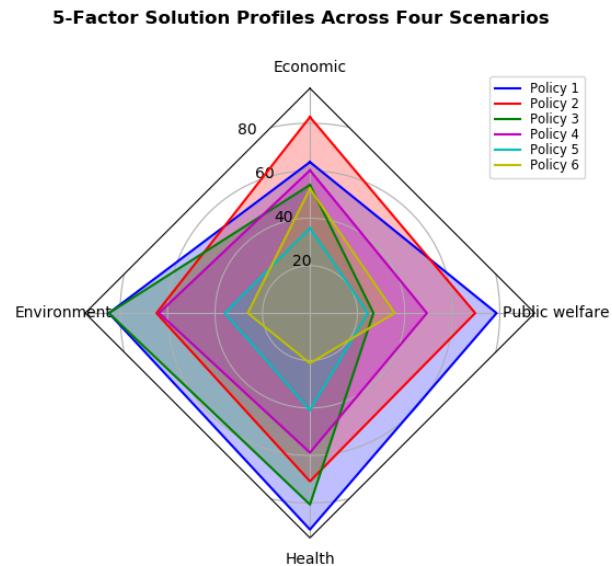


Figure 12: Radar chart for different BAN policy under different indicators

scores due to the unchanged work provided and disaster relief. At the bottom of the list was the ban on selling bottled water in vending machines, which almost entirely replaced the market share of bottled water with sugary and alcoholic drinks, without leading to significant improvements.

### 5.1.5 Sensitivity Analysis

In the part of sensitivity analysis, we define the maximum standard data as 100. Two parameter setting methods are constructed. When the data increased, the initial value was 0.01, the step length was 0.01, and the final value was -0.01; or when the data decreases, the initial value is -0.01, the step length is also 0.01, and the final value is -100.

Table 8: Estimated marginal rate of beverages in San Francisco International Airport

Indicator variable	Value Fluctuation threshold(%)	Range(%)
1	-7	3
2	-11	6
3	-3	4
4	-6	4

From the Table8, we find that the sensitivity range under health index is 3%~4%, which is the most robust index, while the sensitivity range under environmental protection is -11%~6%, which is the most sensitive index.

In order to observe the influence of the adjusted health indicators on the total score, we compared the schematic diagram of the original score and the adjusted score in the sensitivity analysis, where the red curve is the original score curve and the blue curve is the processed curve. Seen from the trend in the figure, although the curve has changed in general, the trend has not changed significantly, which indicates that our evaluation system is stable and will not excessively affect the rating result due to change of indicators.

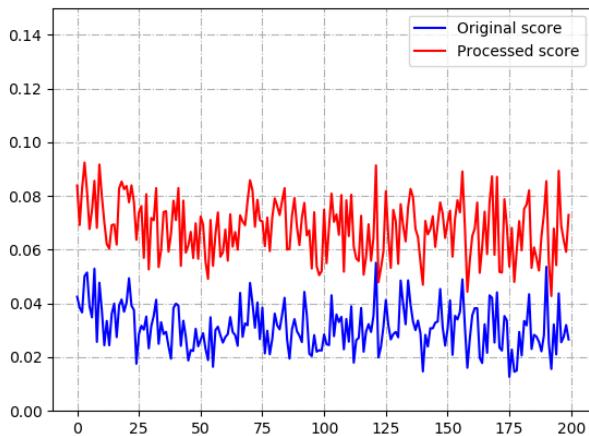


Figure 13: Sensitivity Analysis

## 5.2 Promote the BAN to the Whole Country

### 5.2.1 Promotion Principle

According to the evaluation results of Section 5.1.4 and the sensitivity analysis of Section 5.1.5, it can be seen that a ban can achieve a good effect mainly by increasing the replacement rate of direct drinking water and ensuring a certain replacement rate of bottled drinks. Therefore, in order to avoid the negative effects of the previous ban and inherit the positive effects of the previous ban, we should give strategies for nationwide promotion from the following perspectives:

- Levy a tax on PET bottled water
- Distribute or encourage the purchase of reusable water bottles for free to residents
- Improve infrastructure for direct drinking water
- Carry out the whole ideological educational promotion

### 5.2.2 Promote to Communities

For smaller places such as communities and schools, the ban is mainly promoted from the perspective of micro-infrastructure construction and other policy measures, mainly including the following points:

- Strengthen the construction of direct drinking water location: strengthening the supply and convenience of direct drinking water includes the construction of direct drinking water fountains in playgrounds, stations, schools, office buildings and other places; strengthening the number of direct drinking water stations
- Distribute or encourage the purchase of reusable direct drinking water bottles for free to residents.
- Increase the price of bottled water sold in stores

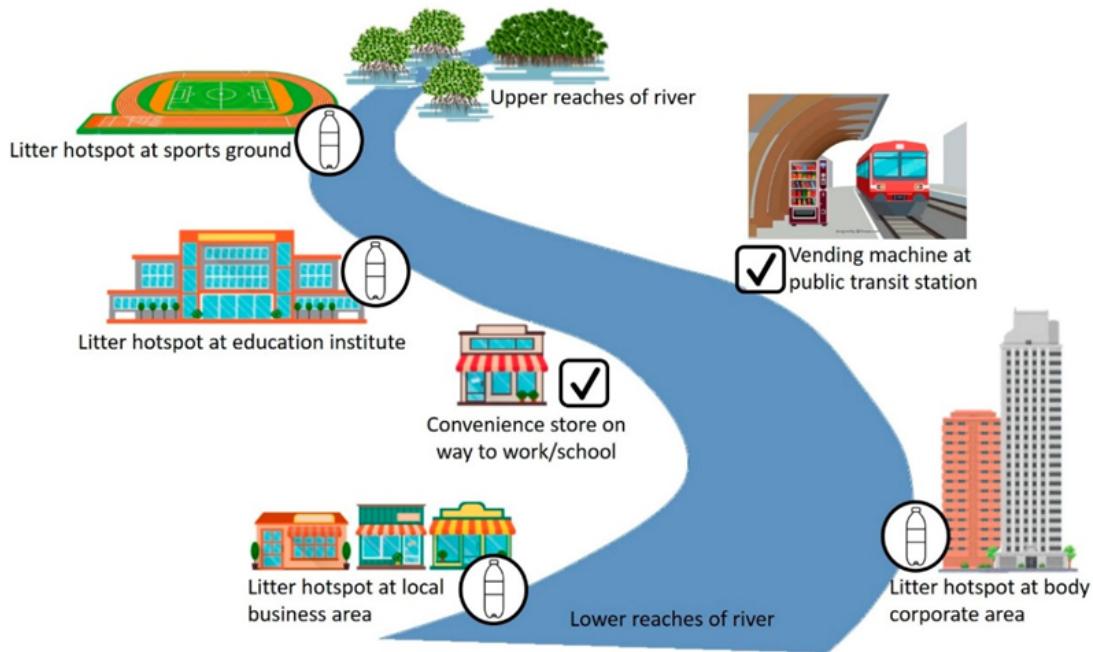


Figure 14: Infrastructure construction within the community

### 5.2.3 Promote to Cities

For large cities, the ban is mainly promoted from the perspective of macro infrastructure, policy suggestions and ideological education, mainly including the following points:

- Strengthen the concept of education: through the distribution of leaflets, production of television programs and courses in schools, strengthen the education of the dangers of sugary drinks and alcoholic drinks, and other environmental and health awareness, so as to implement the ban more effectively and win a larger market share for direct drinking water;
- Imposing tax: a nationwide tax on bottled water and bottled beverages can reduce the market share of both and make up for the economic loss caused by the promotion of direct drinking water.
- Strengthen the construction of direct drinking water infrastructure, including drinking water stations, purification plants and supporting security checks and transportation measures, so as to further improve the safety and convenience of direct drinking water and promote the market share of direct drinking water.

To put it in a nutshell, policymakers should introduce more than a few bans on specific implementation details, work to improve water direct share, and promote the infrastructure construction, ideological education, and non-mandatory reduce bottled drinks (including bottled water, sugary drinks, and alcoholic beverages). This can adjust citizens drinking habits and give rise to a great extent of the development of the country.

## 6 Advantages and Disadvantages

We analyzed the process of establishing and solving the model, sorted out the advantages and limitations of the model, and proposed possible improvement directions based on

the limitations:

## 6.1 Strength

- Our model based on competitive market substitution theory in microeconomics is objective.
- We conducted sensitivity analysis and demonstrated the stability of the model.
- We have promoted the model performance under different bans to facilitate administrators to analyze the possible impacts of different bans on bottled water and to facilitate the issuance of policies.

## 6.2 Possible Improvements

- If we have more complete data, our analysis of the impact of the ban on bottled water will be more accurate.
- Due to the limited time for completion, the promotion part of our model did not analyze all possible prohibitions, but it selected some representative prohibitions. If there is enough time, a fuller analysis can be realized in this respect.

## References

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- [2] Welle, F. (2011). Twenty years of PET bottle to bottle recyclingan overview. *Resources, Conservation and Recycling*, 55(11), 865-875.
- [3] Pindyck, R. S., & Rubinfeld, D. L. (2015). Microeconomics. Boston: Pearson,.
- [4] Willis, K., Hardesty, B. D., Vince, J., & Wilcox, C. (2019). The success of water refill stations reducing single-use plastic bottle litter. *Sustainability*, 11(19), 5232.

# Appendices

## Appendix A Tools and software

Paper written and generated via L<sup>A</sup>T<sub>E</sub>X, free distribution.

Graph generated via Microsoft<sup>C</sup> Office<sup>TM</sup> 365 Excel, institution certification and Python.

Xmind Zen, free trial license.

Calculation using Python and MATLAB R2019a.

## Appendix B The data

Since the amount of data is large and not intuitive, we directly visualize some of the data for display.

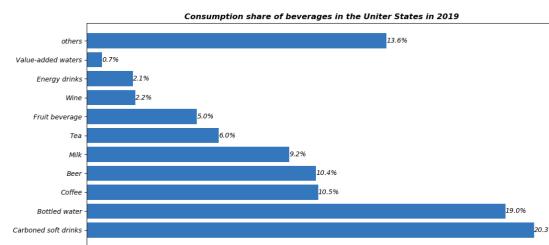


Figure 15: Consumption share of beverages in the United States in 2019

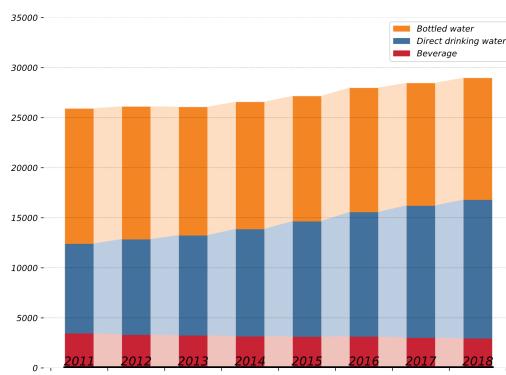


Figure 16: SBeverage market share trends

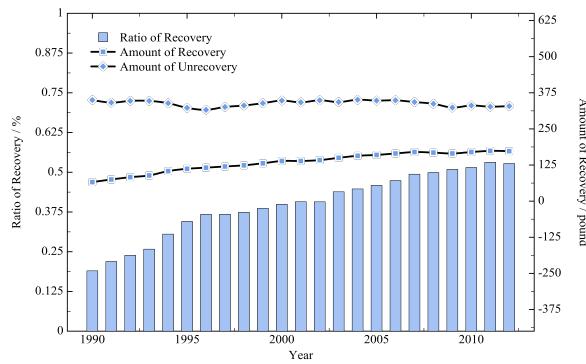


Figure 17: U.S. Municipal Solid Waste Stream

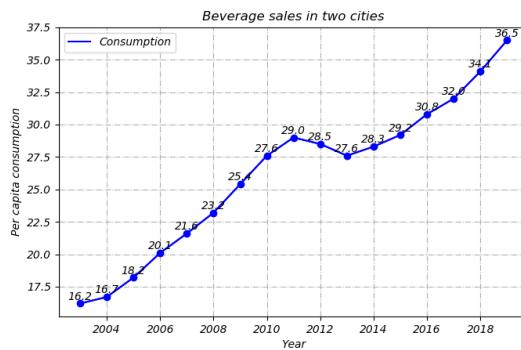


Figure 18: Beverage Consumption Per Person in the United States

## Appendix C The Codes

Here are simulation programmes we used in our model as follow.

### C.1 Polynomial Fitting Codes

---

```
% Surface fitting
% -----
clc, clear, close all
data{1} = xlsread('30.xlsx', 2, 'B3:B19');
data{2} = xlsread('30.xlsx', 2, 'C2:M2' );
data{3} = xlsread('30.xlsx', 2, 'C3:M19' );
power_x = 3;
power_y = 3;
[obj,coeff,index,name,fixdata] = datafit(data,power_x,power_y);
gaoptions = optimoptions('ga');
gaoptions.Display = 'iter';
gaoptions.PopulationSize = 200;
gaoptions.ConstraintTolerance = 0;
gaoptions.MaxGenerations = inf;
lb = min(fixdata(:,1:2));
ub = max(fixdata(:,1:2));
[minxy,minzy] = ga(@(data)minFunc(data,name,coeff),2,[],[],[],lb,ub);
```

---

```

function [obj,coeff,index,name,fixdata] = datafit(data,power_x,power_y)
    Method      = ['poly',num2str(power_x),num2str(power_y)];
    z          = data{3};
    [y,x]      = meshgrid(data{2},data{1});
    [m,n]      = size(x);
    x1         = reshape(x,m*n,1);
    y1         = reshape(y,m*n,1);
    z1         = reshape(z,m*n,1);
    options     = fitoptions('Method','LinearLeastSquares');
    [obj,index] = fit([x1,y1],z1,Method,options);
    coeff(1,:)  = coeffvalues(obj);
    coeff(2:3,:) = confint(obj);
    name        = coeffnames(obj);
    fixdata     = [x1,y1,z1];
end

```

---

## C.2 Entropy Weight Method Codes

---

```

function weights = EntropyWeight_1(R)
[rows,cols]=size(R);
k=1/log(rows);
f=zeros(rows,cols);
sumBycols=sum(R,1);
for i=1:rows
    for j=1:cols
        f(i,j)=R(i,j)./sumBycols(1,j);
    end
end
lnfij=zeros(rows,cols);
for i=1:rows
    for j=1:cols
        if f(i,j)==0
            lnfij(i,j)=0;
        else
            lnfij(i,j)=log(f(i,j));
        end
    end
end
Hj=-k*(sum(f.*lnfij,1));
weights=(1-Hj)/(cols-sum(Hj));
end

```

---