Fueling Our Economy:
A Prediction of Gas Prices

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2012

15th Annual High School Mathematical Contest in Modeling (HiMCM) Summary Sheet

(Please attach a copy of this page to each copy of your Solution Paper.)

Team Control Number: 3594

Problem Chosen: B

Please type a summary of your results on this page. Please remember not to include the name of your school, advisor, or team members on this page.

Executive Summary:

Given the constantly fluctuating price of gas, as consumers, we face a challenging decision every time we pull up to the pump: to fill or not to the fill. Without knowledge of what the gas prices will do in the coming weeks, we risk spending our money only to watch the gas prices drop later. Our task was to solve this problem by developing a model to instruct a consumer on when to buy gas so as to minimize spending. Specifically, we looked at consumers who use a half tank in a week and those who use a quarter tank in a week. We developed a model that could predict future gas prices and determine how much gas a consumer should buy in a given week based on whether the price would go up or down in the next couple of weeks. Our future gas price predictions were based on the correlation between crude oil price and gas price. Since the process of refining and transporting crude oil to produce gas takes time, the current trend of the crude oil prices forecasts the trend in future gas prices.

Once we had a model for predicting prices, we used a decision tree to develop an algorithm for how much gas a consumer should buy on a given week in order to be cost efficient. Therefore, the consumer only needs to input the crude oil prices for the past three weeks, the current gas price, and the current amount of gas in their tank, and our model will tell them whether to buy a full tank, half tank, or no gas. Our model is applicable to large cities across the United States, predicts future gas prices with only 1.63% error, and recommends a buying pattern that differs only 1% from the ideal spending that would occur if consumers actually knew future gas prices. Therefore, our model is useful as a way for drivers to save money.

Letter to Local Paper:

Team #: 3594
November 7, 2012

Dear Reader:

Unsure how much gas to buy at the pump? Feel you have wasted money buying gas at its highest prices? What if you could base your buying patterns on next week's prices? We have developed a way to predict future gasoline prices based on the change in the previous weeks' crude oil prices. With this information, you can fill your tank before the gas prices rise, thus avoiding paying at the pump during the peak prices. When prices are going to decrease, you can wait until the prices hit their low point to fill up again, thus saving the maximum amount of money at the pump.

Our system's method of saving you money is simple. We just make sure you are buying gas when the prices are lowest. All you have to do is plug in the crude oil prices from the past three weeks and the current gas price; our model will calculate whether the price of gas is going to trend up or down. It can even tell you how much gas to buy (none, half tank, or full tank) based on how much gas you have in your tank now (¾, ½, ¼, or empty) and how much gas you use up every week (¼ tank or ½ tank). Once the calculations are done, it will tell you how much to buy today or if you should wait for better prices.

This model is incredibly useful for the average commuter; at the beginning of every week, you can just plug in the information and know within seconds whether you should fill up your car on the way in to work or whether you should expect to see gas prices drop and should fill up next week. The idea behind the model is that it takes a trend in the crude oil prices and applies it to the price of gas right now in order to predict gas prices two weeks out. Once it has that information and has determined which week out of the three weeks it predicts has the lowest gas prices, then it takes into account how much gas you have at the moment. The idea behind this is that you want to purchase as much gas as you can when the price is lowest, so our model will tell you how much to fill up (none, half, or full) in order to minimize the amount of gas you have to buy on expensive weeks. Would you rather be spending more money and worrying about when to fill up your car, or follow our plan and save money easily? Don't worry, be happy now.

Sincerely,
Team 3594

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Introduction:

Restatement of the Problem:

The problem we faced was how to predict what gas prices will look like in the coming weeks so that consumers could make an educated decision about whether or not to buy gas now. Since our model would be used by everyday drivers, we wanted it to be user-friendly and simple while still maintaining a high level of accuracy. We looked at two separate types of commuters, ones who drove 100 miles per week, and those who drove 200 miles per week. To eliminate excessive variables, we chose to set a single car that had a range of 400 miles per tank, which means that the driver will empty their tank in either four weeks (100 mile per week commuter), or two weeks (200 mile per week commuter). Also, we assumed that a consumer would only buy either a half tank, full tank, or no gas and only buy gas once a week. Using these parameters, we needed to determine when a consumer should buy gas based on whether prices of gas in the coming weeks would trend up or down. Therefore, we needed to predict future gas prices by using factors that correlate with gas prices.

Assumptions and Justifications:

Customers will only buy gas in half and full tank increments.

This assumption simplifies our model and was given in the problem statement. If a buyer could choose any amount of gas, the amount which the gas price varies would become more significant; he would want to buy proportionally to how good of a deal the gas is. By restricting his choices to a few values, the model will not vary greatly based on small changes in the miles driven by the customer in a given week. This makes our model for 200 miles per week and 100 miles per week relevant with only minor modifications to drivers who drive from 100 to 200 miles per week. Indeed, the actual number of miles driven is less important than the ratio between the amount of miles driven in a week and the number of potential miles in a tank. In our case, the tank holds 16 gallons and the car gets 25 mpg allowing the driver to travel 400 miles on a single tank. However, any consumer who drives either a ½ or a ¼ of their car's potential ratios every week could use our model without modification.

The historical correlation of crude oil and gas prices will continue.

The assumption was necessary given the nature of our model. In using prior crude oil prices to model future gas prices, we needed to assume that the correlation between crude oil and gas would continue. This assumption allows us to assume that data from 2011 and prior, which we based our model off of, will still correlate with the data from 2012. This assumption is grounded in the basis that since 2011, the method for the refinement of oil to gasoline has not changed dramatically, which allows for the correlation that we based our model off of to remain relevant. Moreover, we will later test our model to make sure it applies to 2012.

It takes time for the gas market to respond to changes in crude oil prices.
Since the production of gas is dependent on the production of crude oil, the price of gas depends on the price of crude oil. Because of the time lag from the transportation and the refinement process, it takes time, one to two weeks according to our calculations, for gas prices to respond to changing oil prices.

People would like to minimize their expenditure on gas while still maintaining their desired amount of driving.
A consumer will make a decision on whether or not to buy gas based on trying to pay the least amount possible. However, they do not want to ever run out of gas, so the idea is to buy the same total amount of gas and run your car at the same level of operation that it is now, but buy the gas at more optimal times, thus saving money by purchasing the gas when the price is optimal.

The trend in the average of the Brent and WTI crude oil prices is representative of the trends in the price of all crude oil relevant to U.S. gasoline.
From the data that was given to us at the beginning of the problem, we assumed that the trends in the prices of Brent and WTI crude oil are representative of the trends for all sources of crude oil used in major U.S. cities. Moreover, when we compared the graphs of the Brent and WTI, we found that these two had very similar trends in terms of price changes. Therefore, we felt justified in saying that crude oil price trends are fairly consistent across the industry and do not depend heavily on the specific source. We used an average of the Brent and WTI prices as our crude oil price.

The Model:

Model Building:

In order to determine how much gas a consumer should buy each week, we had to find a method of effectively predicting future gas prices. Our model is constructed to predict gas prices based on the past weeks' crude oil prices. We discovered this correlation by investigating the relationships between current gas prices and those factors in the world that would affect the price of gas. When we investigated these relationships, we found that time of year, temperature, and economic stability all had very low correlation coefficients with gas price (see appendices 2a and 2b). In contrast, the price of crude oil was very closely related to the price of gas since crude oil is refined into gasoline (see appendices 2c and 3a). Additionally, its price takes into account economic and natural factors because a recession or natural disaster would affect the price of crude oil as well as the price of gas. By using recent oil prices, we can accurately predict what the price of gas will be for up two weeks in the future. We knew that due to transporting and refining time requirements, there would be a lag between when the crude oil price changed and when the gas price experienced the same change. Therefore, we could use crude oil prices from past weeks to predict future gas price changes. Next we needed to determine the amount of lag between the crude oil and gas prices.

When we graphed scatter plots of crude oil, shifted 1-6 weeks, with the unshifted price of gas, we found that the strongest correlation between the gas and crude oil occurred when the shift was only 1 week. Since this correlation was so strong, it provided a convenient way to predict what the future gas prices would be, given only the knowledge of what the price of crude oil had done over the past several weeks. However, a 1 week shift only allowed us to predict 1 week into the future and we needed 2 weeks worth of predictions for reasons explained later (see paragraph 6). Since a 2 week shift also produced a very strong correlation, we decided to use this relationship as the basis for our model. The recursive equation for finding our new price point for gas then followed logically from this shift: $P_1=(C_{-2}-C_{-1})+P_0$ where P_1 is next week's gas price, $(C_{-2}-C_{-1})$ is how much the crude oil price changed between one week ago and two weeks ago, and P_0 is the current gas price. We modelled this relationship in Excel and then graphed our predicted values with the actual gas values. We found that since the price of crude oil had more noise than the price of gas, our predicted values also jumped around more, while still following the correct overall trend.

We discovered that we could smooth out the graph by using a system of averaging. Instead of using the actual crude oil price (C_0) for a given week, we averaged the real values of the week before that week (C_{-1}), the week itself (C), and the week following it (C_1) to obtain a new value (C_a) for the week ($C_a=(C_{-1}+C_0+C_1)/3$). This adjustment of values greatly diminished the noise and allowed us to accurately model gas prices with an average percent error of only 1.63 % (see appendices 4a-b).

We then moved on to the real question: how to determine whether or not someone should buy gas today or wait and buy it later. We started by creating two different option trees for a 200 miles per week driver (see appendix 1a) and a 100 miles per week driver (see appendix 1b) based on the assumption that they could buy a full tank, half tank, or no gas in any given week. This assumption reduces the number of options a buyer can choose from to two or less for any given week. There are three options total: buy none, buy a half-tank, and buy a full tank. However, a customer will only be able to buy a full tank if they have no gas. If they have no gas, they would not be able to get away with buying no gas. Thus, a customer can never have both the option of buying both a full tank and no gas, restricting his options to two at the most.

First we will investigate our model for the 200 miles per week driver. We used the equation $g_t=g_{t-1}+g_{b-1}-1/2$, where g_t is the fraction filled for the tank at the beginning of the current week, g_{t-1} is the fraction filled at the beginning of the previous week, g_{b-1} is the fraction of a tank bought in the previous week, and a half tank of gas is used each week. We then input a series of nested if/then statements, based on whether or not our predicted gas price for the next week would be lower or higher than the current price, to determine how much gas a driver should buy each week (g_b). If the price of gas is increasing, gas should be purchased in the maximum quantity as soon as possible so as to avoid paying the peak price for the gas. If the gas prices are decreasing, as little gas as possible should be bought so as to be able to save the maximum amount when the gas reaches its low point. This means that if the price was expected to rise and the driver had no gas, they should buy a full tank, and if they had half a tank, they should buy another half tank. If the price was expected to drop and the driver had a half tank, they should buy no gas, and if they had no gas they should buy a half tank(see appendix 1a). Once we had determined how much gas the driver should buy each week, we could multiply the number of gallons by the known gas price for that week to determine how much the driver would spend, which should ideally be the lowest price possible with the driving lifestyle they want to maintain.

Next we will investigate our model for the 100 miles per week driver. We used the equation $g_t=g_{t-1}+g_{b-1}-1/4$, where g_t is the fraction filled for the tank at the beginning of the current week, g_{t-1} is the fraction filled at the beginning of the previous week, g_{b-1} is the fraction of a tank bought in the previous week, and a quarter tank of gas is used each week. We used the same approach of nested if/then statements to determine g_b for a given week. If the driver had no gas and the current price was the lowest within the next two weeks, they should buy a full tank, but if the current price was not the lowest within the next two weeks, they should buy a half tank. If the driver had a quarter tank and the current price was lower than next week's price, they should buy a half tank, but if the current price was not lower, they should not buy gas. If the driver had a half tank and the current price was the lowest within the next two weeks, they should buy a half tank, but if the current price was not the lowest, they should buy no gas. If the driver had a three quarters full tank, they could not buy gas(see appendix 1b). We could then calculate the amount a driver spent on gas in the same method as described above.

Our model was then essentially complete except for one detail. Since the 100 miles per week driver model had to predict gas prices two weeks into the future, it used crude oil prices from the current week. That meant that using our averaging system, the averaged value for the current week would include a future crude oil value that would not be available yet. We solved this problem by using the actual crude oil price to predict two weeks out. Our one week prediction followed the equation $P_1=(C_{a-1}-C_{a-2})+P_0$ and our two week prediction followed the equation $P_2=(C_0-C_{a-1})+P_1$. Another difference is that the one week prediction is based on the the difference in crude oil price added to the current gas price whereas the two week prediction is based on the difference in crude oil price added to the predicted one week gas price.

We originally developed our model using Boston data, and then we applied it to data from other major US cities(see appendices 5a-5f and 6a-6f).

Model Testing:

The first level of testing for our model was a calculation of the average percent error between our predicted gas prices and the actual gas prices. Our graphs help to visualize this error (see appendices 7a-b). We first calculated the percent error for data from Boston 2003-2011, which was the data we had built our model from, and then calculated percent error for 2012. The very small errors in both cases demonstrated the accuracy of our model as well as the fact that it could be applied to new data.

Boston 2003-2011: 1.63 % average error between predicted and actual gas prices
Boston 2012: 0.88% average error between predicted and actual gas prices

The next level of testing for our model was a comparison between how much a consumer would spend on gas if they followed our model, how much they would spend if they knew future gas prices in advance, and how much they would spend if they did the opposite of what our model suggested. This 3-way comparison allowed us to see how our predicted spending fit into a spectrum from the lowest possible spending to the highest possible spending. In all cases, our model instructed the consumer to exercise very similar buying habits to the ideal spending pattern. Our model also saved the consumer a significant amount of money from the worst case scenario which could occur if a consumer was buying gas on a whim. The usefulness of our model was verified by its ability to save a consumer money.

100 miles per week driver-
Boston 2003-2011: 0.46% difference between model spending and omniscient spending
2.48 % saved from worst case spending
Boston 2012: 0.51 % difference between model spending and omniscient spending
6.08 % saved from worst case spending

200 miles per week driver-
Boston 2003-2011: 0.36% difference between model spending and omniscient spending
1.54% saved from worst case spending
Boston 2012: 0.26 % difference between model spending and omniscient spending
3.60 % saved from worst case spending

The final level of testing we performed was to see how well our model could be applied to other large United States cities: Houston, Los Angeles, San Francisco, New York, Chicago, and Denver. We took the model that we had created from our 2004-2011 Boston data and applied the same formulas to the new sets. With each set, we created summary graphs that plotted our predicted gas prices against the recorded ones in the data(see appendices 5a-5f and 6a-6f). For almost all the cities, there was less than a 1% difference between our recommended spending and the ideal spending, so our model was clearly effective across the United States.

Reflections on the Model:

Our model predicts the change in gas prices based the previous weeks' crude oil prices. We account for local gas prices in order to reset our model to the local prices before each prediction, in order to not let our model run away from reality. However our recommendation on how much gas to buy is only dependent on the predicted change in the prices and not the prices themselves. This comes a double edged sword to our model, because our model uses international crude oil prices and no local information to predict the change in the prices. The advantage of this is that our recommendation on how much gas to buy can be used internationally. The disadvantage to this is that any local issues that change in the price of gas will go unpredicted.

Another double edged sword that applies to the data that we would be using to generate the recommendation of whether to buy or not, is the volatility of the gas prices. If the price of gas was very volatile than it would be possible to save a lot of money by skipping the nadir of the gas price, by buying early. However, given our current model for generating the predictions, if the change in price of gas prices became very volatile, our ability to predict the trend would decrease dramatically.

STRENGTHS:

- **The cost of buying gas using our Model closely is very similar to the most efficient mode of buying gas.** Since one of our goals was to generate a model that would predict future gas prices in an effort to reduce excessive spending on gas, and this model generates a buying plan that closely resembles the ideal pattern, we can conclude that this model has succeeded in achieving this aspect of our project.

- **Our model can be used to predict prices throughout the United States.** When testing this model on data sets from other cities, we saw that our model accurately predicts gas prices in other areas as well as Boston.
- **Our model predicts gas prices with only a 1.63 % average percent error.** This high amount of precision allowed us to very accurately determine how much gas the consumer should buy in a given week.
- **Our model is easy to use.** A user only needs to enter crude oil prices from the last three weeks and the current gas price, and our model will not only predict the prices for the next two weeks, but also tell the user exactly how much gas to buy for the week and how much it will cost.

AREAS FOR IMPROVEMENT:

- **For some cities, our predicted values were slightly shifted from the actual values when graphed over a shorter time frame.** Longer transportation times and other factors may mean that the time lag between crude oil prices and gas prices varies from city to city. With more time to investigate individual cities thoroughly, we could adjust our model specifically to each city and correct this minor inaccuracy.
- **Separate models are needed for consumers who drive different amounts during the week.** The more weeks a driver can go without buying gas, the more weeks they need to predict in order to buy gas at the best price. While both of our models are accurate, it would be more convenient if a user could access both models in one interface.
- **Our model does not accommodate drivers who do not fit the profile given in the problem.** If someone uses some value other than a quarter or a half of their tank each week, the model will not apply to them.
- **If the gas continued at a constant pace, there would be no opportunity for our model to save money.** The only opportunities to save money are when the gas price reaches its local maximum or minimum. Unfortunately, these are the points that are hardest to predict using past data. Past data repeatedly predicts the continuation of a trend whether it be positive or negative when the gas price rebounds in the other directions. Furthermore, the price of crude oil is more volatile than the price of gas often making the model to predict more static of noise than is really present. However the model is accurate enough to save a consumer money. For example, if our consumer driving 200 miles a week in Chicago was to follow our model from the beginning of January 2012 to the end of October 2012 they would have saved an estimated $52.50 compared to someone who filled up their tank during the most expensive weeks.

Conclusion:

•Results:

Our model accurately informs the consumer on how much gas they should purchase at the pumps on a weekly basis. Even in freak events and sudden changes in the price of gas (such as the economic recession of 2008, the largest drop in gas prices in our data sets), our model remained consistent with real data. The results compare the information that we have to both a model where the knowledge of future gas prices was omniscient, and a model where the most amount of money possible was spent on gas. Our model consistently ended up very close to the ideal purchasing pattern. The reason why %Saved for the 100 mile per week is almost twice that of the 200 mile per week, is due to the number of times you have to fill up in that time frame. The 200 mile per week model simply uses more gas and as a result has to fill up more often, meaning that the amount of money that is able to be saved is proportional to the time frame that you are buying gas with. If you buy gas in 2 week increments, the amount of money saved is lower than if you buy gas in 4 week increments. The same is true on larger scales; if you were buying in 2 month increments, you would save even more than if you were buying in 4 week increments. The amount of money is heavily weighted towards the frequency with which you have to fill your tank.

1/2/12 - 10/22/12 for the 200 mile a week consumer

City	Total spent according to our model	Total spent ideally	Difference between predicted and ideal	% Difference predicted and ideal	Total spent with worst case scenario	Saved (compared to worst case scenario)	% Saved (compared to worst case scenario)
Boston	$1203.15	$1200.11	$3.04	0.26%	$1243.91	$40.76	3.28%
Houston	$1119.42	$1116.82	$2.60	0.23%	$1160.43	$41.02	3.54%
Los Angeles	$1332.53	$1323.10	$9.43	0.71%	$1384.06	$51.53	3.72%
San Francisco	$1323.20	$1315.13	$8.07	0.61%	$1374.14	$50.94	3.85%
New York	$1199.10	$1195.78	$3.32	0.28%	$1240.37	$41.27	3.33%
Chicago	$1259.82	$1252.46	$7.35	0.59%	$1312.40	$52.58	4.01%
Denver	$1133.24	$1128.38	$4.86	0.43%	$1171.55	$38.31	3.27%

1/2/12 - 10/22/12 for the 100 mile a week consumer

City	Total spent according to our model	Total spent ideally	Difference between predicted and ideal	% Difference predicted and ideal	Total spent with worst case scenario	Saved (compared to worst case scenario)	% Saved (compared to worst case scenario)
Boston	$587.38	$584.42	$2.96	0.51%	$622.23	$34.85	5.60%
Houston	$557.65	$555.84	$1.81	0.33%	$594.65	$37.00	6.22%
Los Angeles	$666.78	$660.01	$6.77	1.03%	$709.74	$42.96	6.05%
San Francisco	$659.42	$652.56	$6.86	1.05%	$702.64	$43.22	6.15%
New York	$599.25	$596.56	$2.69	0.45%	$635.15	$35.90	5.65%
Chicago	$627.54	$623.19	$4.35	0.70%	$668.55	$41.01	6.13%
Denver	$569.14	$564.84	$4.30	0.76%	$602.16	$33.02	5.48%

We applied our model to the cases where a driver wanted to travel 100 miles a week or 200 miles a week. With minor modifications, this model could be applied to distances in between the two. Our model could also be applied to values below 100 miles but in this case, we would need to be able to predict gas prices even further in advance, since a consumer could choose to drive for longer on a single tank. However, if you drive more miles than 200 miles a week, or, more generally, travel far enough to use up more than a half tank of gas, the problem changes significantly. For example, let's say you travel 201 miles a week. The first week you must fill up a full tank in order to be able to complete the week's travel because if you put in a half tank, you would only be able to travel 200 miles. The next week, you would have enough gas left to travel 199 miles. You would need to add more gas to make it through the next week and would have to buy another half tank. This example illustrates that people who drive more than 200 miles a week will need to refill their gas tank every week. Even with a full tank that would allow them to drive 400 miles, they would need more gas because they would have less than 200 miles left after the first week. On the other hand, almost every week, they would have some gas left in the tank (just never more than ½), which would preclude completely filling the tank. Therefore, customers would buy a ½ tank almost every week. They would only have the additional option of buying a full tank on the weeks where they had nothing in the tank left going into the week. (For example, every 4th week a consumer who drove 250 miles a week would have the option of buying a full tank.) When they need more than half a tank to complete their weekly driving, the customer's choices become so restricted that they have much less need of a model like ours.

•Extensions:

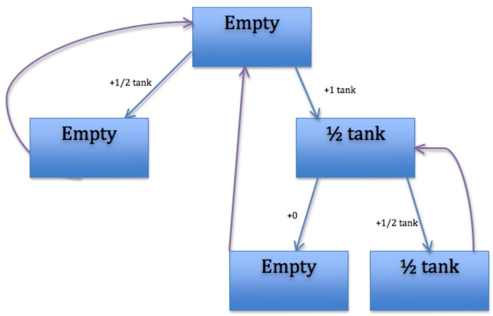
Several extensions that could elaborate on the areas for improvement were identified over the course of this project. We concluded that if one were to create an Excel or Stella program that would take the input of whatever car someone was using (mpg, miles per week, and gas tank size), we could accurately describe the choices that the said person would have to face in order to minimize his/her spending. Similarly if we had a larger pool of data to model from, we could have included some more factors that are not already affecting the crude oil.

While we were looking at the amount of money that our model would save, we uncovered an interesting fact about the modeling system that we had created. If for instance we were not a commuter buying gas for a single car, but instead a company that was buying gas in bulk to resell for profit. The realization that we uncovered was that buying in bulk increases the amount one can save. This is true because of the way the price of gas fluctuates. There are peaks, and valleys, and if a company that consumes a large quantity of gas were to buy all of their gas in one of the valleys then they would be able to save quite a bit of money. We would have to expand our model to explore this option, since our prediction window does not extend farther than a few weeks with reasonable accuracy.

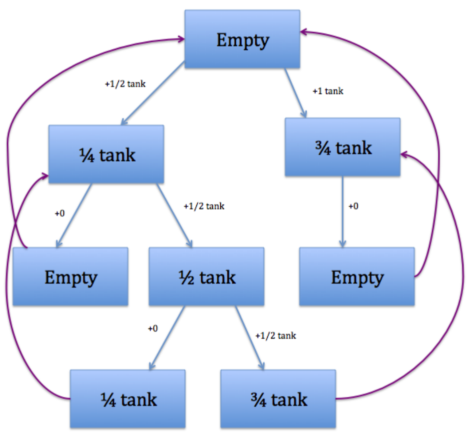
We set out with a simple goal of telling the commuter whether or not they should buy gas this week or wait until later. With the results that support our model, and show that in fact our model saves money, we are not only taking away the decision of whether to buy gas, and the stress associated with it, but also saving you money every time you fill up using our model.

Appendices:

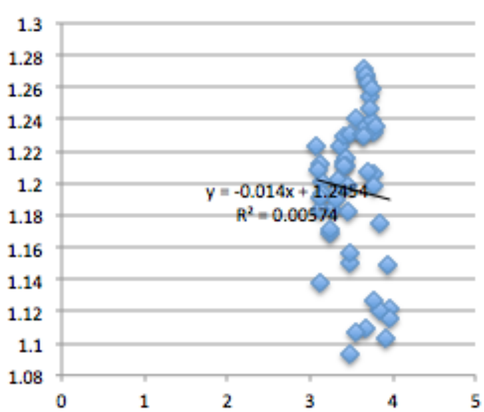
1a: 200 mile per week decision map



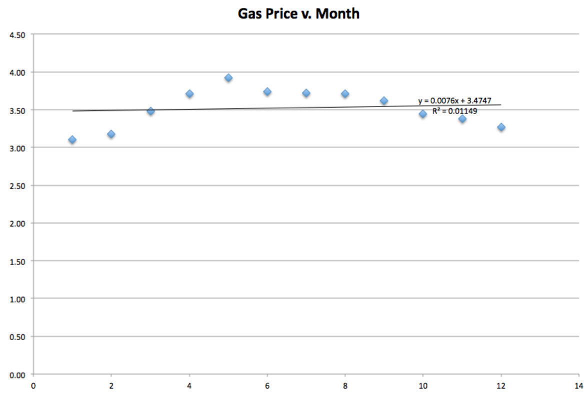
1b: 100 mile per week decision map



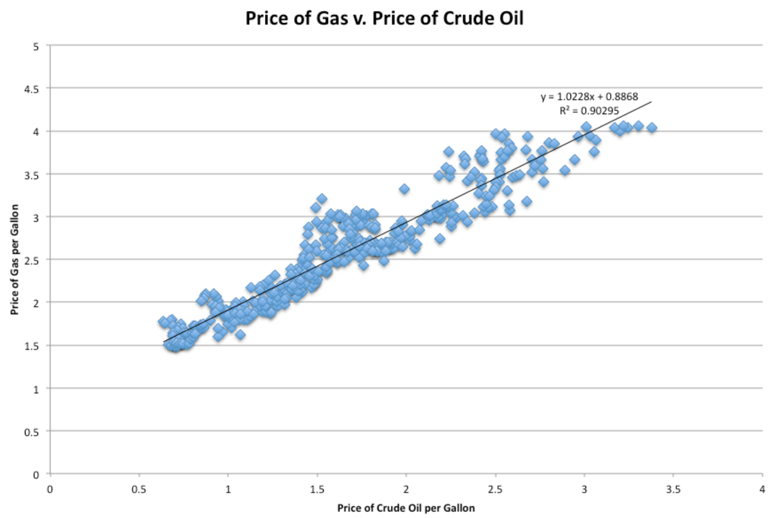
2a: Regression of DJIA v. Gas Prices (y-axis is in thousands)



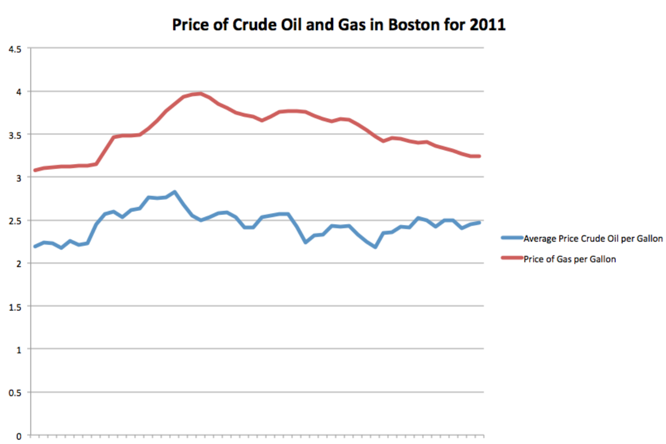
2b: Regression of Season v. Gas Prices



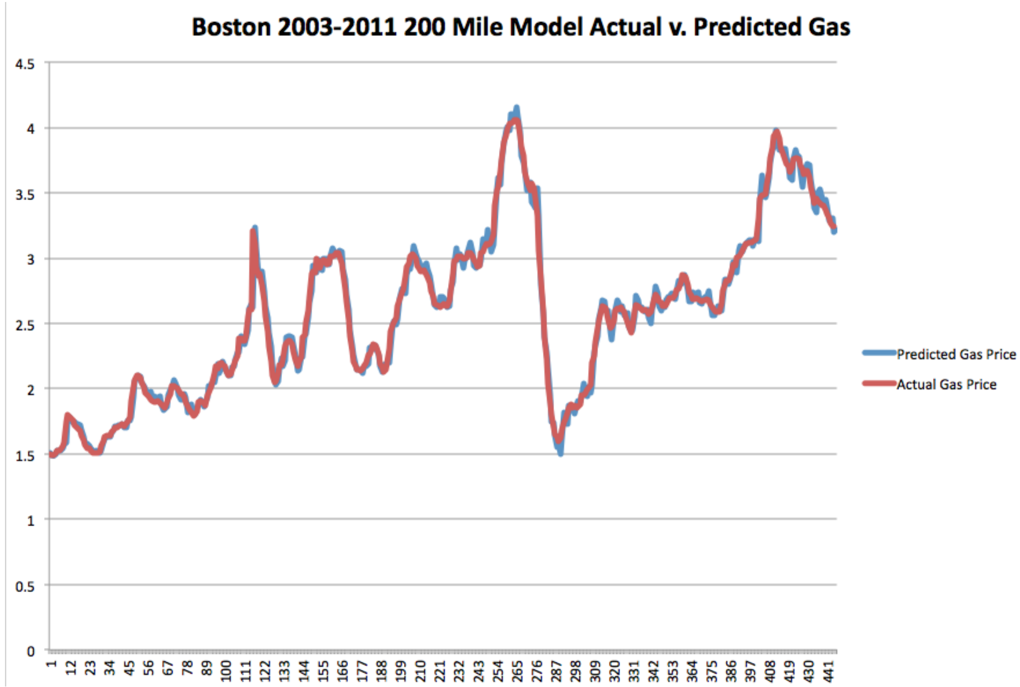
2c: Boston (2003-2011)



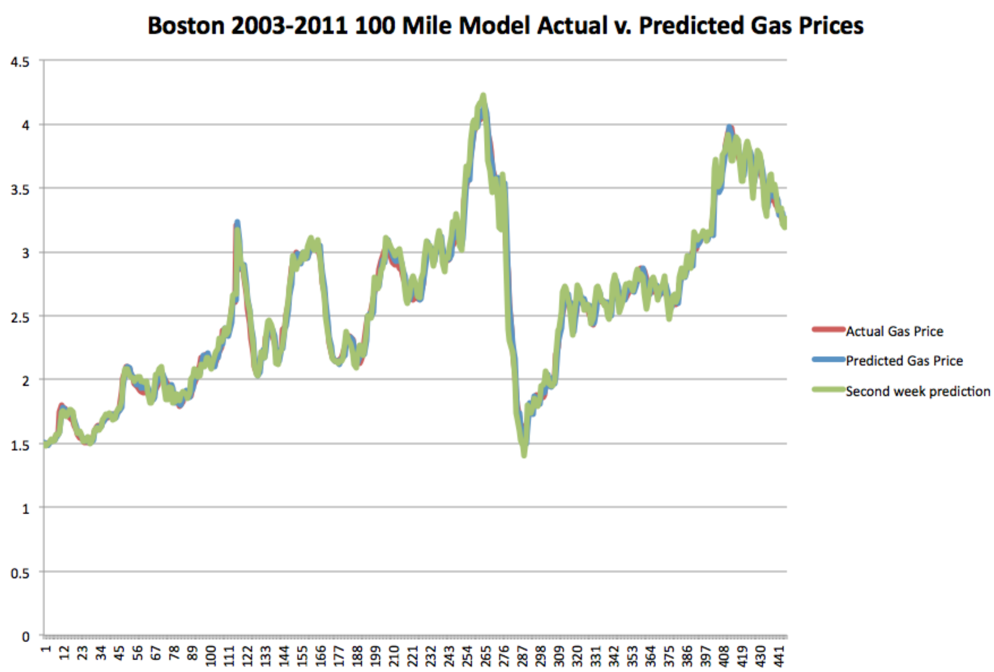
3a:



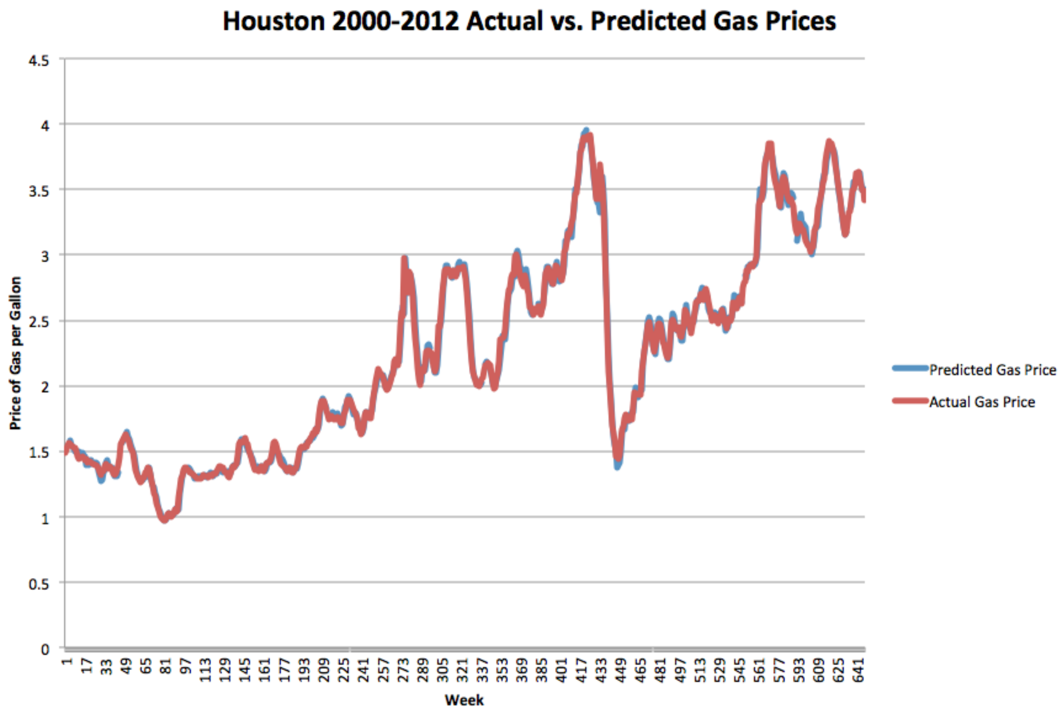
4a:


Boston 2003-2011 200 Mile Model Actual v. Predicted Gas
Predicted Gas Price
Actual Gas Price

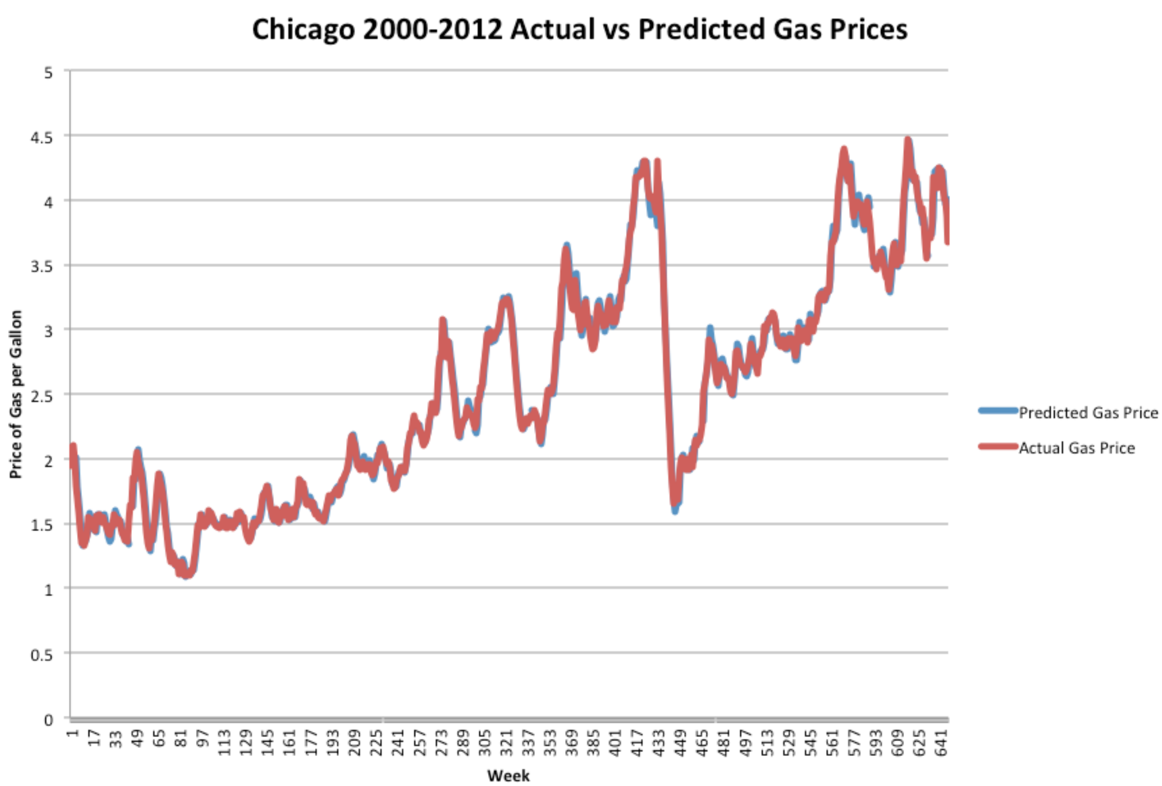
4b:


Boston 2003-2011 100 Mile Model Actual v. Predicted Gas Prices
Actual Gas Price
Predicted Gas Price
Second week prediction

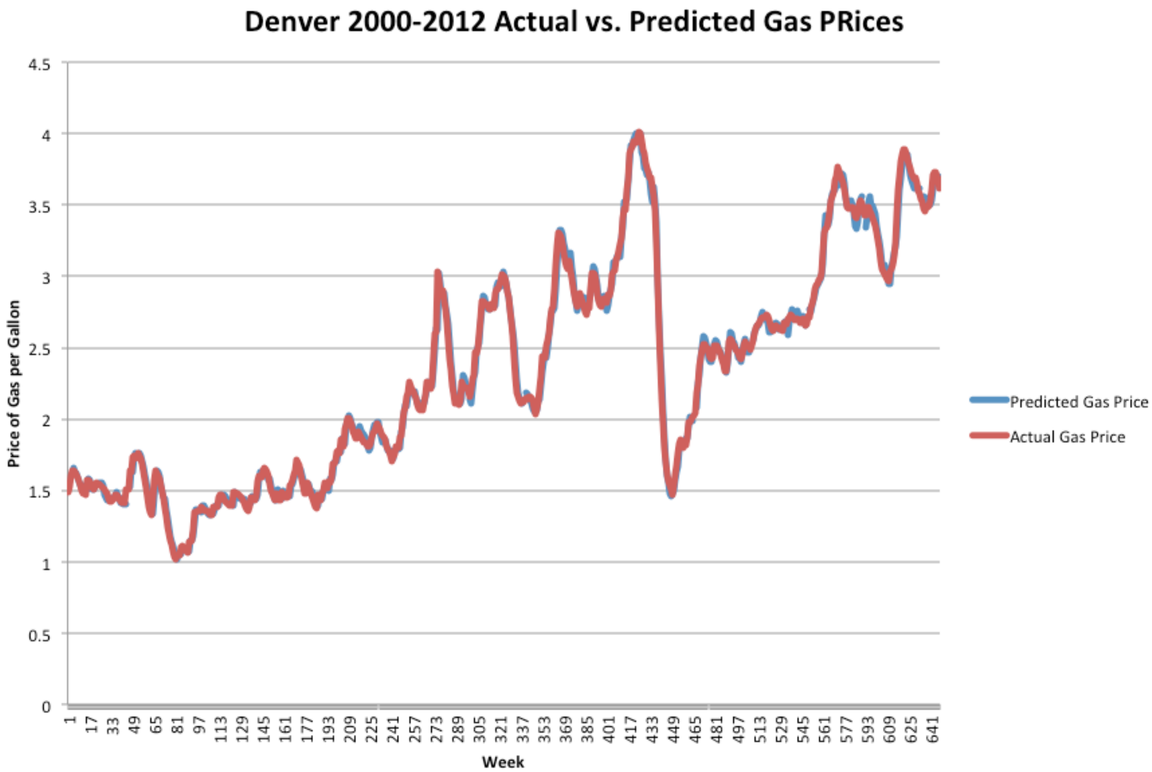
5a: 200 mile per week model



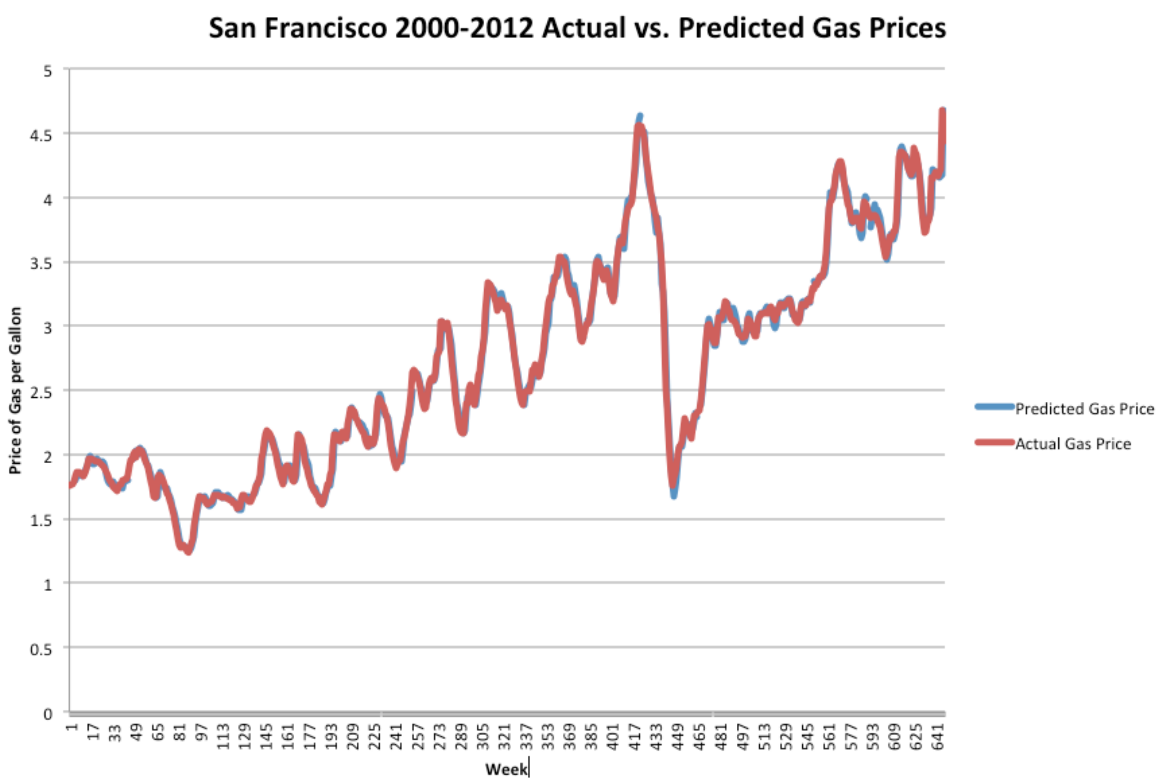
5b: 200 mile per week model



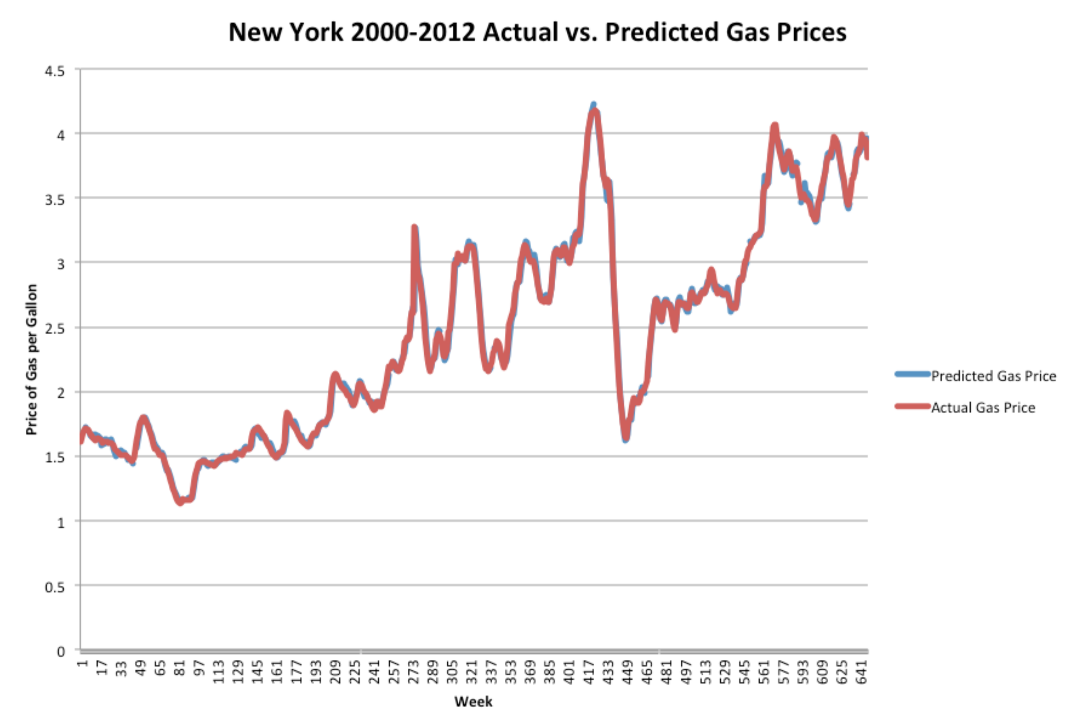
5c: 200 mile per week model



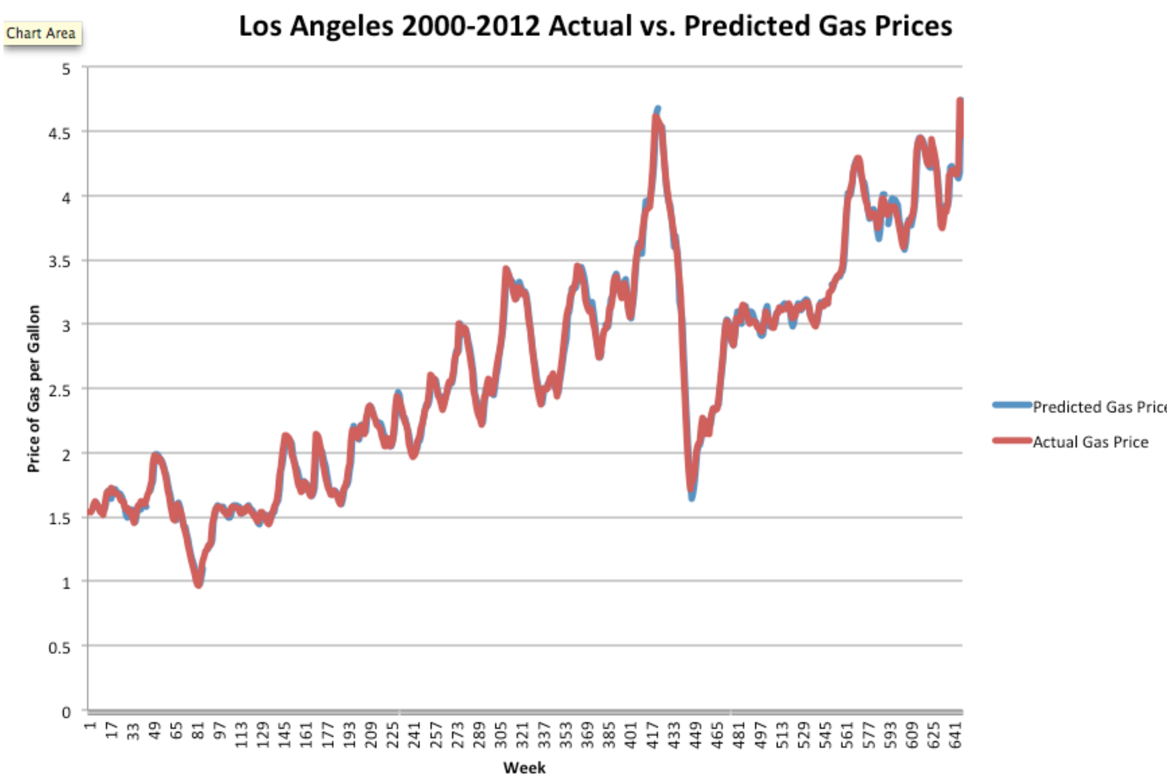
5d: 200 mile per week model



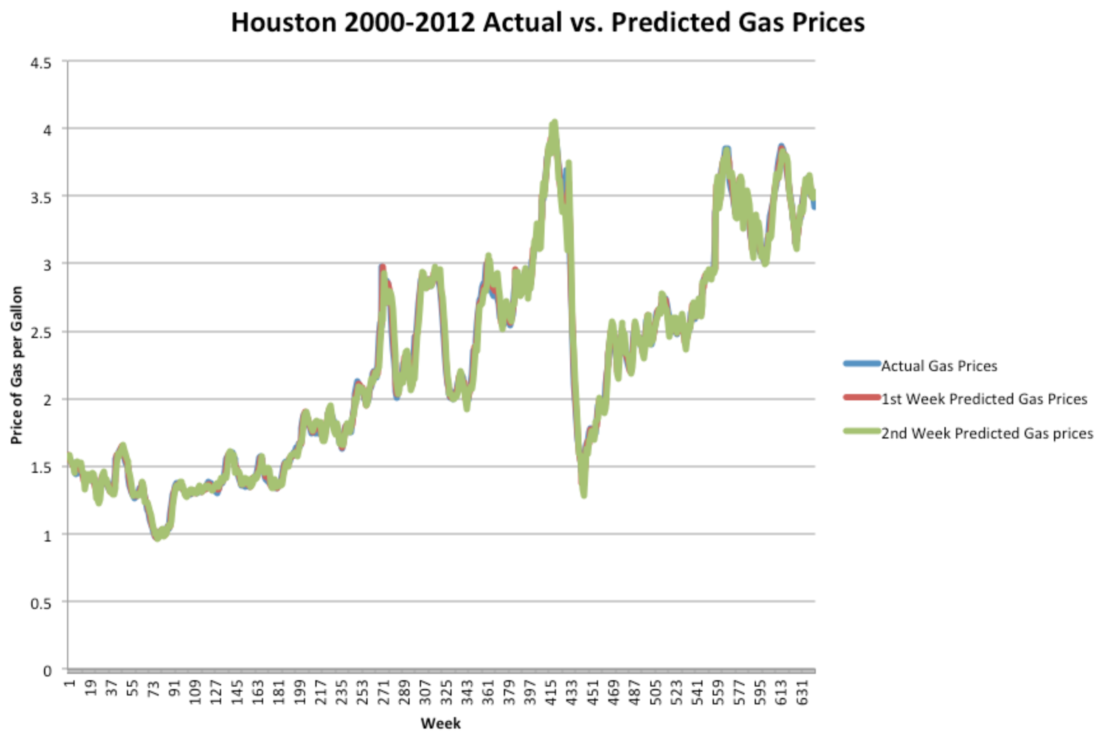
5e: 200 mile per week model



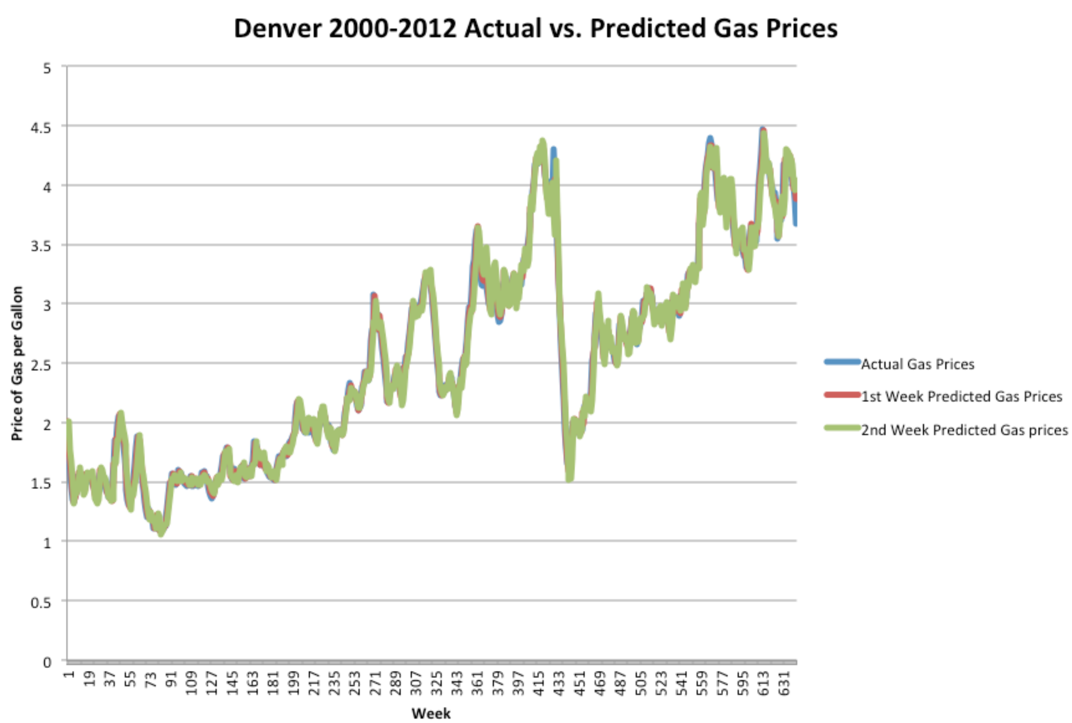
5f: 200 mile per week model



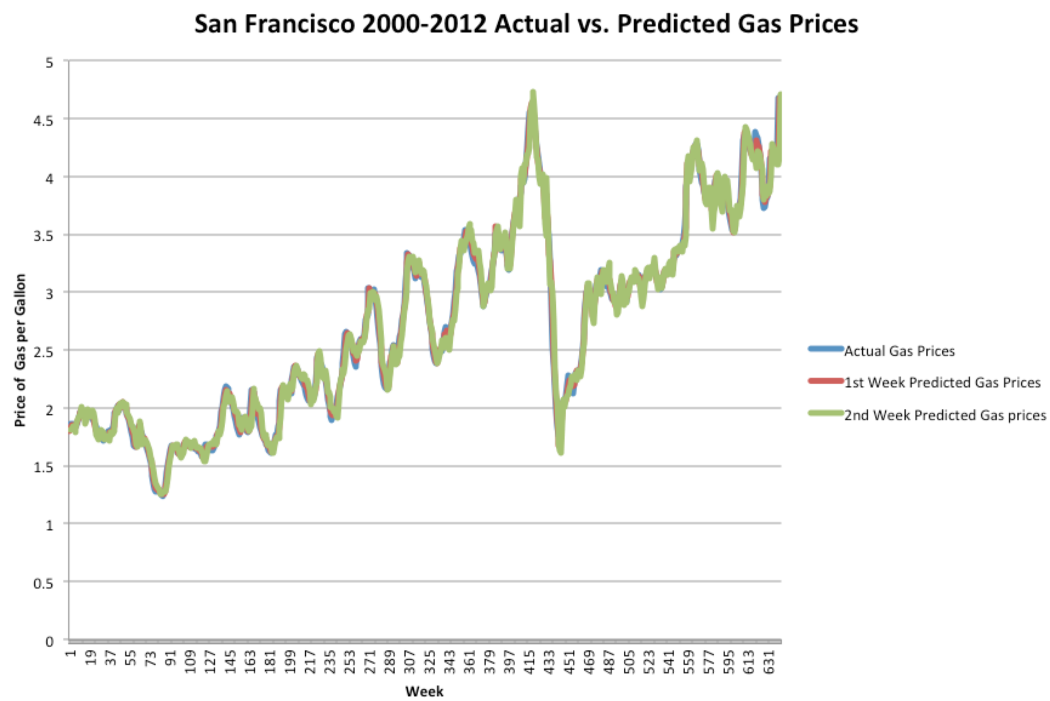
6a: 100 mile per week model



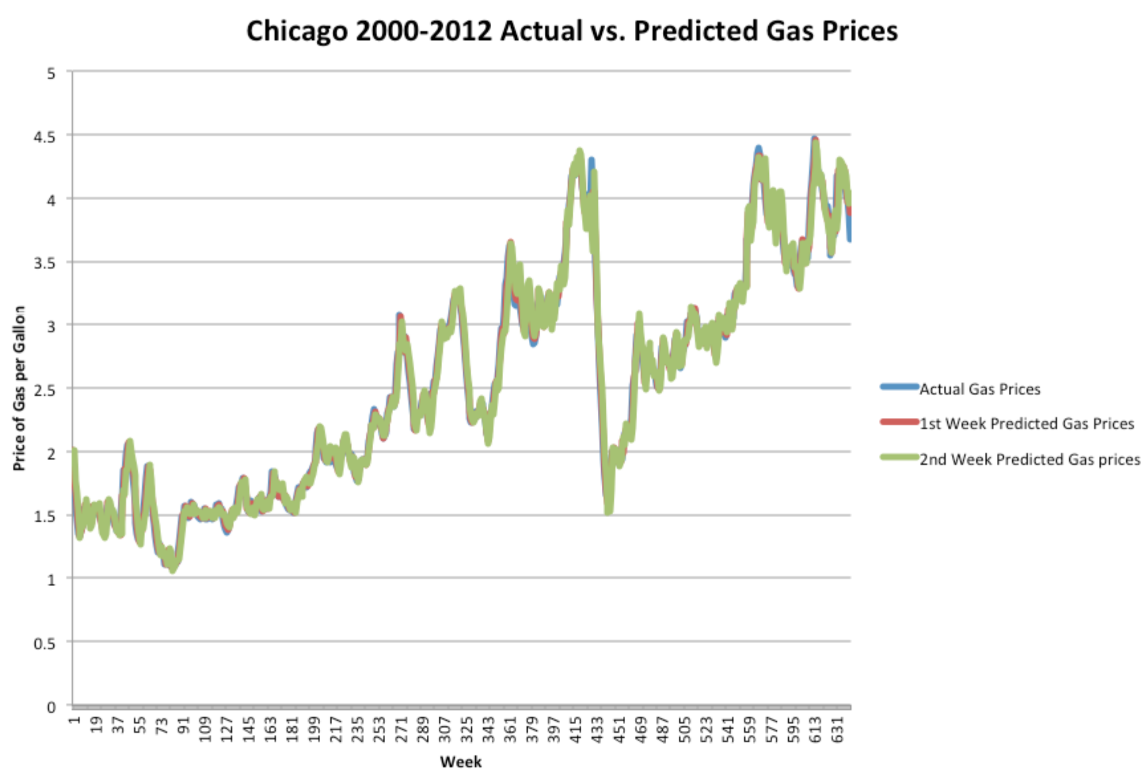
6b: 100 mile per week model



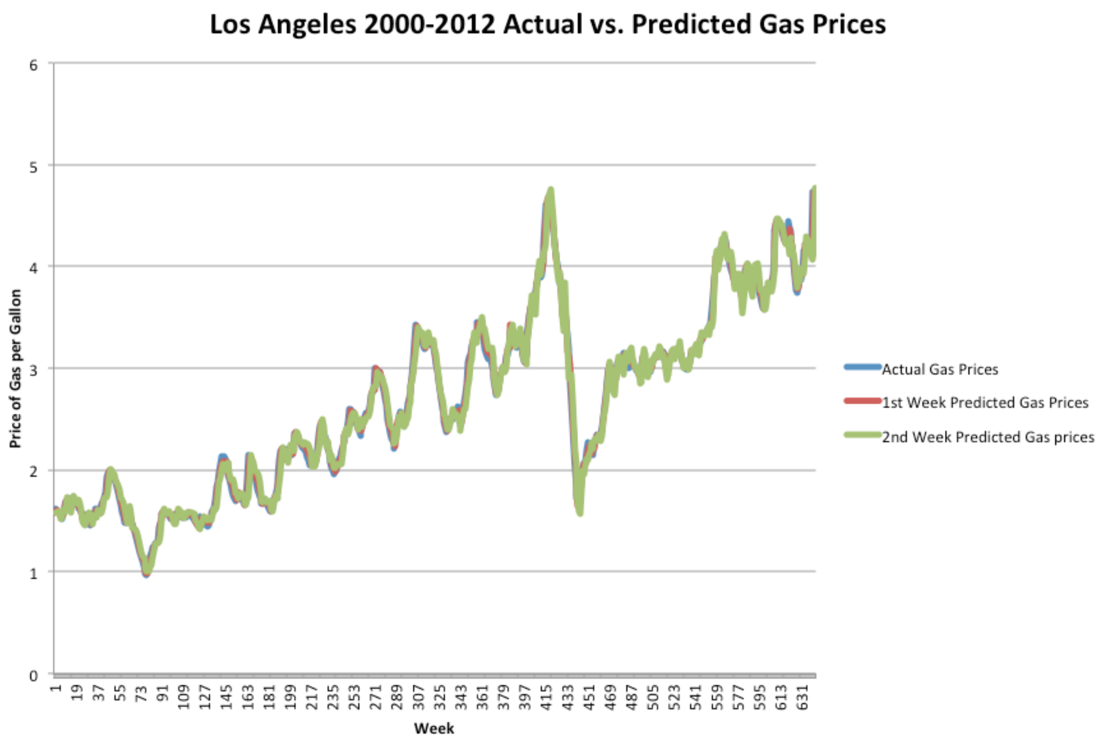
6c: 100 mile per week model



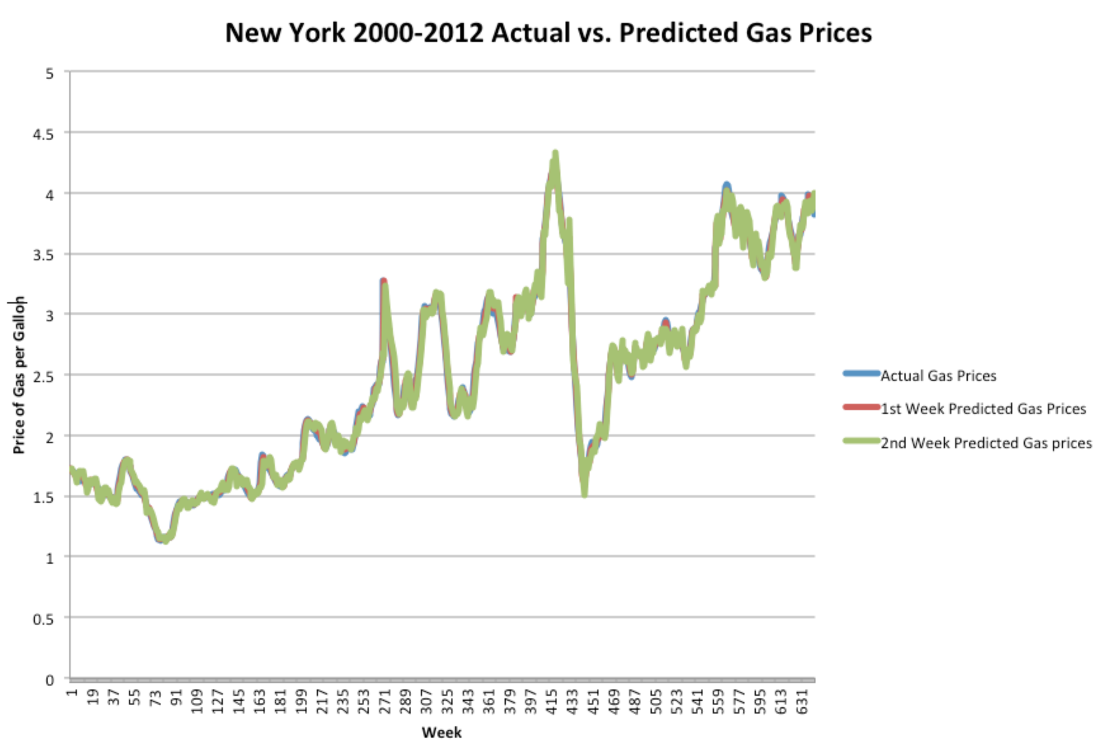
6d: 100 mile per week model



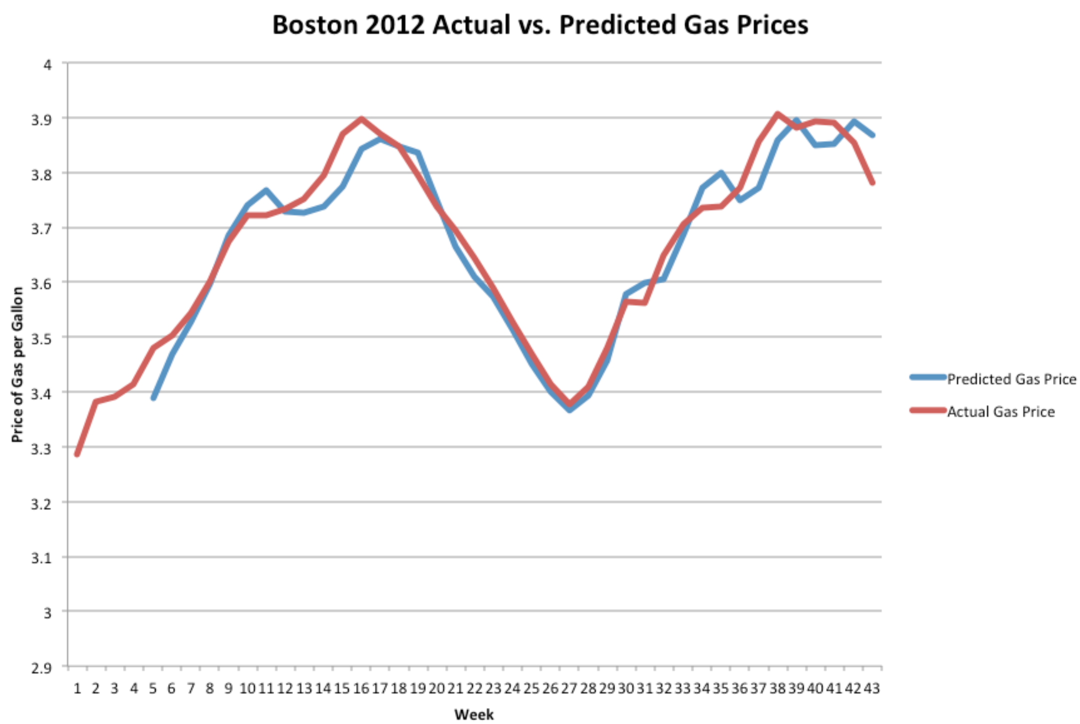
6e: 100 mile per week model



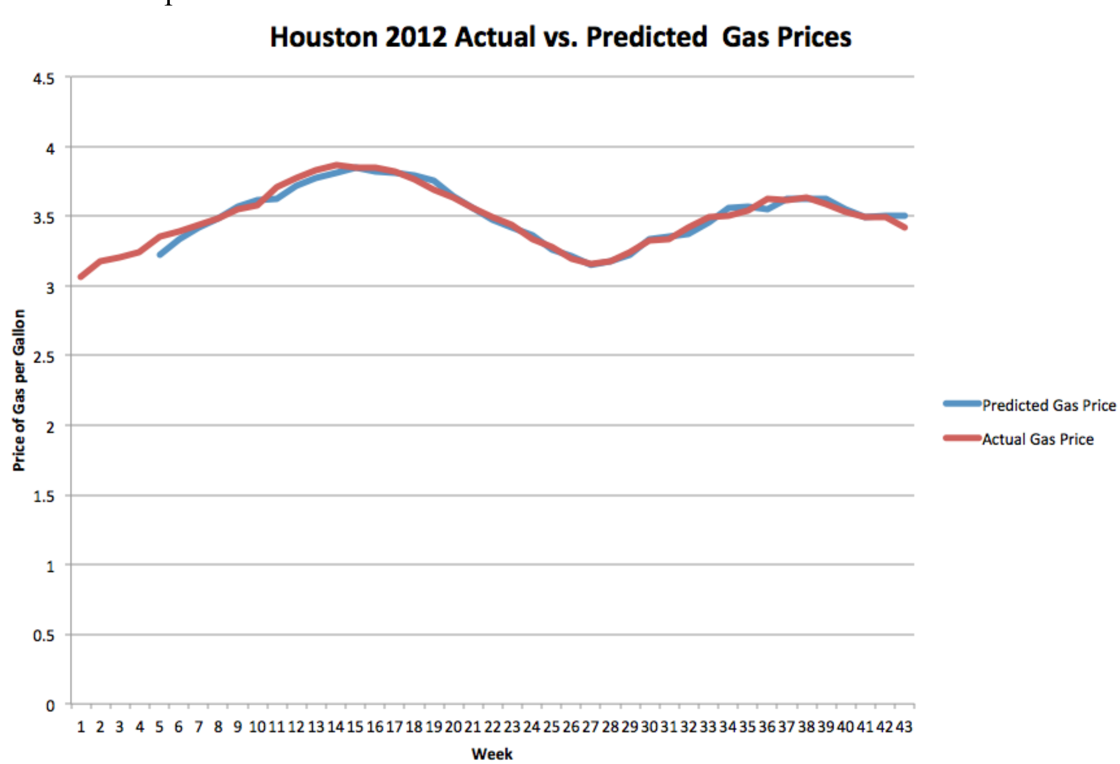
6f: 100 mile per week model



7a:



7b: 200 mile per week model



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