10 Persistence

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L Basic

1.1 compile

```
# presetting before coding
echo "cd ~/Desktop" >> ~/.bashrc
gedit -> preference -> tab width: 4
# Editor
gedit a.cpp
# Compile
g++ a.cpp -std=c++11
**All file will be compiled to a.out unless you use -o(
    not recommanded, just use a.out)**
# Run
./a.out
# Run with file input
./a.out < input.txt</pre>
# Run with file input and output
./a.out < input.txt > output.txt
# Python Run
python3 a.py < input.txt > output.txt
# Copy Paste In Ubuntu
* copy: ctrl+insert
* paste: shift+insert
```

1.2 default code

```
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef pair<int,int> pii;
#ifdef ONLINE_JUDGE
#define cerr if(false) cerr
#endif

int main(){
#ifndef ONLINE_JUDGE
    //freopen("input.txt","r",stdin);
    freopen("output.txt","w",stdout);
    freopen("debug.txt","w",stdcerr);
#else
    ios_base::sync_with_stdio(0);
    cin.tie(false);
#endif
}
```

1.3 debug list

```
模板要記得 init priority_queue 要清空 把邊界條件都加入測資 邊界條件都加入測資 邊界條件 (過程溢位,題目數據範圍),會不會爆 long long 是否讀錯題目,想不到時可以自己讀一次題目 環狀or凸包問題一定要每種都算n次 比較容易有問題的地方換人寫注意公式有沒有推錯或抄錯 精度誤差 sqrt(大大的東西) + EPS 測試 %11d or %164d 喇分 random_suffle 隨機演算法
```

2 Dark Code

2.1 IO optimization

```
*if output to much, consider put all output in array
     first, then output the array.
getchar() -> getchar_unlocked()
fread() -> fread_unlocked()
inline char readchar() {
 const int S = 1<<20; // buffer size
static char buf[S], *p = buf, *q = buf;</pre>
  if(p == q \&\& (q = (p=buf)+fread(buf,1,S,stdin)) ==
       buf) return EOF;
  return *p++;
inline int nxtint() {
 // if readchar can't use, change readchar() to
      getchar()
  int x = 0;
  int c = readchar(), neg = false;
  if (c == EOF) return -1;
  while (('0' > c | | c > '9') \&\& c != '-' \&\& c != EOF)
       c = readchar();
  if (c == '-')neg = true, c = readchar();
while ('0' <= c && c <= '9') x = x * 10 + (c ^ '0'),</pre>
       c = readchar();
  if (neg) x = -x;
  return x;
```

3 Flow

3.1 Dinic

```
(a) Bounded Maxflow Construction:
1. add two node ss, tt
2. add_edge(ss, tt, INF)
3. for each edge u -> v with capacity [1, r]:
        add_edge(u, tt, 1)
        add_edge(ss, v, 1)
        add_edge(u, v, r-1)
4. see (b), check if it is possible.
5. answer is maxflow(ss, tt) + maxflow(s, t)
(b) Bounded Possible Flow:
1. same construction method as (a)
2. run maxflow(ss, tt)
3. for every edge connected with ss or tt:
        rule: check if their rest flow is exactly 0
4. answer is possible if every edge do satisfy the rule
5. otherwise, it is NOT possible.
(c) Bounded Minimum Flow:
1. same construction method as (a)
2. answer is maxflow(ss, tt)
(d) Bounded Minimum Cost Flow:
* the concept is somewhat like bounded possible flow.
1. same construction method as (a)
2. answer is maxflow(ss, tt) + (\Sigma 1 * cost for every
(e) Minimum Cut:

 run maxflow(s, t)

2. run cut(s)
3. ss[i] = 1: node i is at the same side with s.
const long long INF = 1LL<<60;</pre>
struct Dinic { //O(VVE), with minimum cut
```

```
static const int MAXN = 5003;
struct Edge{
    int u, v;
    long long cap, rest;
int n, m, s, t, d[MAXN], cur[MAXN];
vector<Edge> edges;
vector<int> G[MAXN];
void init(){
    edges.clear();
    for ( int i = 0 ; i < MAXN ; i++ ) G[i].clear()</pre>
// min cut start
bool side[MAXN];
void cut(int u) {
    side[u] = 1;
    for ( int i : G[u] ) {
        if ( !side[ edges[i].v ] && edges[i].rest )
              cut(edges[i].v);
    }
// min cut end
void add_edge(int u, int v, long long cap){
    edges.push_back( {u, v, cap, cap} );
edges.push_back( {v, u, 0, 0LL} );
    m = edges.size();
    G[u].push_back(m-2);
    G[v].push_back(m-1);
}
bool bfs(){
    memset(d, -1, sizeof(d));
    queue<int> que;
    que.push(s); d[s]=0;
    while (!que.empty()){
        int u = que.front(); que.pop();
        for (int ei : G[u]){
            Edge &e = edges[ei];
             if (d[e.v] < 0 && e.rest > 0){
                 d[e.v] = d[u] + 1;
                 que.push(e.v);
        }
    return d[t] >= 0;
long long dfs(int u, long long a){
    if ( u == t || a == 0 ) return a;
    long long flow = 0, f;
    for ( int &i=cur[u]; i < (int)G[u].size() ; i++</pre>
         ) {
        Edge &e = edges[ G[u][i] ];
        if ( d[u] + 1 != d[e.v] ) continue;
        f = dfs(e.v, min(a, e.rest) );
        if (f > 0) {
            e.rest -= f;
            edges[ G[u][i]^1 ].rest += f;
            flow += f;
            a -= f;
            if ( a == 0 )break;
        }
    return flow;
long long maxflow(int s, int t){
    this->s = s, this->t = t;
    long long flow = 0, mf;
    while ( bfs() ){
        memset(cur, 0, sizeof(cur));
        while ( (mf = dfs(s, INF)) ) flow += mf;
```

3.2 Gomory Hu

```
Construct of Gomory Hu Tree
1. make sure the whole graph is clear
2. set node 0 as root, also be the parent of other
3. for every node i > 0, we run maxflow from i to
    parent[i]

 hense we know the weight between i and parent[i]

5. for each node j > i, if j is at the same side with i
   make the parent of j as i
int e[MAXN][MAXN];
int p[MAXN];
Dinic D; // original graph
void gomory_hu() {
    fill(p, p+n, 0);
    fill(e[0], e[n], INF);
    for ( int s = 1 ; s < n ; s++ ) {</pre>
        int t = p[s];
        Dinic F = D;
        int tmp = F.max_flow(s, t);
        for ( int i = 1 ; i < s ; i++ )
            e[s][i] = e[i][s] = min(tmp, e[t][i]);
        for ( int i = s+1 ; i <= n ; i++ )</pre>
            if ( p[i] == t && F.side[i] ) p[i] = s;
    }
}
```

3.3 min cost flow

```
// long long version
typedef pair<long long, long long> pll;
struct CostFlow {
    static const int MAXN = 350;
    static const long long INF = 1LL<<60;</pre>
    struct Edge {
        int to, r;
        long long rest, c;
    int n, pre[MAXN], preL[MAXN]; bool inq[MAXN];
    long long dis[MAXN], fl, cost;
    vector<Edge> G[MAXN];
    void init() {
        for ( int i = 0 ; i < MAXN ; i++) G[i].clear();</pre>
    void add_edge(int u, int v, long long rest, long
        long c) {
        G[u].push_back({v, (int)G[v].size() , rest, c
        G[v].push_back({u, (int)G[u].size()-1, 0, -c});
    pll flow(int s, int t) {
        fl = cost = 0;
        while (true) {
            fill(dis, dis+MAXN, INF);
            fill(inq, inq+MAXN, 0);
            dis[s] = 0;
            queue<int> que;
            que.push(s);
```

```
while ( !que.empty() ) {
                 int u = que.front(); que.pop();
                 inq[u] = 0;
                 for ( int i = 0 ; i < (int)G[u].size()</pre>
                     ; i++) {
                     int v = G[u][i].to;
                     long long w = G[u][i].c;
                     if ( G[u][i].rest > 0 && dis[v] >
                         dis[u] + w) {
                         pre[v] = u; preL[v] = i;
                         dis[v] = dis[u] + w;
                         if (!inq[v]) {
                             inq[v] = 1;
                             que.push(v);
                     }
                }
            }
            if (dis[t] == INF) break;
            long long tf = INF;
             for (int v = t, u, 1; v != s; v = u) {
                 u = pre[v]; l = preL[v];
                 tf = min(tf, G[u][1].rest);
             for (int v = t, u, 1; v != s; v = u) {
                u = pre[v]; 1 = preL[v];
                G[u][1].rest -= tf;
                G[v][G[u][1].r].rest += tf;
            cost += tf * dis[t];
            fl += tf;
        return {fl, cost};
    }
} flow;
```

4 Geometry

4.1 2D Point Template

```
typedef double Double;
struct Point {
 Double x,y;
  bool operator < (const Point &b)const{</pre>
    //return tie(x,y) < tie(b.x,b.y);</pre>
    //return atan2(y,x) < atan2(b.y,b.x);
    assert(0 && "choose compare");
  Point operator + (const Point &b)const{
   return (Point){x+b.x,y+b.y};
  Point operator - (const Point &b)const{
   return (Point){x-b.x,y-b.y};
  Point operator * (const Double &d)const{
   return Point(d*x,d*y);
  Double operator * (const Point &b)const{
    return x*b.x + y*b.y;
  Double operator % (const Point &b)const{
   return x*b.y - y*b.x;
  friend Double abs2(const Point &p){
   return p.x*p.x + p.y*p.y;
 friend Double abs(const Point &p){
    return sqrt( abs2(p) );
};
typedef Point Vector;
```

```
struct Line{
  Point P; Vector v;
  bool operator < (const Line &b)const{
    return atan2(v.y,v.x) < atan2(b.v.y,b.v.x);
  }
};</pre>
```

4.2 外心 Circumcentre

```
#include "2Dpoint.cpp"

Point circumcentre(Point &p0, Point &p1, Point &p2){
    Point a = p1-p0;
    Point b = p2-p0;
    Double c1 = abs2(a)*0.5;
    Double c2 = abs2(b)*0.5;
    Double d = a % b;
    Double x = p0.x + (c1*b.y - c2*a.y) / d;
    Double y = p0.y + (c2*a.x - c1*b.x) / d;
    return {x,y};
}
```

4.3 Convex Hull

```
#include "2Dpoint.cpp"
// retunr H, 第一個點會在 H 出現兩次
void ConvexHull(vector<Point> &P, vector<Point> &H){
    int n = P.size(), m=0;
    sort(P.begin(),P.end());
    H.clear();
    for (int i=0; i<n; i++){</pre>
        while (m>=2 && (P[i]-H[m-2]) % (H[m-1]-H[m-2])
            <0)H.pop_back(), m--;
        H.push_back(P[i]), m++;
    }
    for (int i=n-2; i>=0; i--){
        while (m>=2 && (P[i]-H[m-2]) % (H[m-1]-H[m-2])
            <0)H.pop_back(), m--;
        H.push_back(P[i]), m++;
    }
}
```

4.4 半平面交

```
bool OnLeft(const Line& L,const Point& p){
  return Cross(L.v,p-L.P)>0;
Point GetIntersection(Line a, Line b){
  Vector u = a.P-b.P;
  Double t = Cross(b.v,u)/Cross(a.v,b.v);
  return a.P + a.v*t;
int HalfplaneIntersection(Line* L, int n, Point* poly){
  sort(L,L+n);
  int first,last;
  Point *p = new Point[n];
  Line *q = new Line[n];
  q[first=last=0] = L[0];
  for(int i=1;i<n;i++){</pre>
    while(first < last && !OnLeft(L[i],p[last-1])) last</pre>
    while(first < last && !OnLeft(L[i],p[first])) first</pre>
        ++;
    q[++last]=L[i];
    if(fabs(Cross(q[last].v,q[last-1].v))<EPS){</pre>
      last--;
      if(OnLeft(q[last],L[i].P)) q[last]=L[i];
```

4.5 圓交

4.6 線段交

4.7 Smallest Covering Circle

```
#include "circumcentre.cpp"
pair<Point,Double> SmallestCircle(int n, Point _p[]){
  Point *p = new Point[n];
  memcpy(p,_p,sizeof(Point)*n);
  random_shuffle(p,p+n);
  Double r2=0;
  Point cen;
  for (int i=0; i<n; i++){</pre>
    if ( abs2(cen-p[i]) <= r2)continue;</pre>
    cen = p[i], r2=0;
    for (int j=0; j<i; j++){</pre>
      if ( abs2(cen-p[j]) <= r2)continue;</pre>
      cen = (p[i]+p[j])*0.5;
      r2 = abs2(cen-p[i]);
      for (int k=0; k<j; k++){</pre>
        if ( abs2(cen-p[k]) <= r2)continue;</pre>
        cen = circumcentre(p[i],p[j],p[k]);
        r2 = abs2(cen-p[k]);
    }
  }
```

```
delete[] p;
  return {cen,r2};
}
// auto res = SmallestCircle(,);
```

5 Mathmatics

5.1 ax+by=gcd(a,b)

```
typedef pair<int, int> pii;
pii extgcd(int a, int b){
  if(b == 0) return make_pair(1, 0);
  else{
   int p = a / b;
   pii q = extgcd(b, a % b);
   return make_pair(q.second, q.first - q.second * p);
  }
}
```

5.2 BigInt

```
struct Bigint{
 static const int LEN = 60;
  static const int BIGMOD = 10000;
 int s:
 int v1, v[LEN];
 // vector<int> v;
 Bigint() : s(1) \{ vl = 0; \}
 Bigint(long long a) {
    s = 1; v1 = 0;
    if (a < 0) { s = -1; a = -a; }
    while (a) {
     push_back(a % BIGMOD);
      a /= BIGMOD;
   }
 Bigint(string str) {
   s = 1; v1 = 0;
   int stPos = 0, num = 0;
    if (!str.empty() && str[0] == '-') {
      stPos = 1;
     s = -1:
    for (int i=SZ(str)-1, q=1; i>=stPos; i--) {
     num += (str[i] - '0') * q;
     if ((q *= 10) >= BIGMOD) {
        push_back(num);
        num = 0; q = 1;
     }
   if (num) push_back(num);
 int len() const { return vl; /* return SZ(v); */ }
 bool empty() const { return len() == 0; }
 void push_back(int x) { v[vl++] = x; /* v.PB(x); */ }
 void pop_back() { vl--; /* v.pop_back(); */ }
 int back() const { return v[vl-1]; /* return v.back()
 void n() { while (!empty() && !back()) pop_back(); }
 void resize(int nl) {
   vl = nl; fill(v, v+vl, 0);
         v.resize(nl); // fill(ALL(v), 0);
 void print() const {
   if (empty()) { putchar('0'); return; }
   if (s == -1) putchar('-');
    printf("%d", back());
    for (int i=len()-2; i>=0; i--) printf("%.4d",v[i]);
 friend std::ostream& operator << (std::ostream& out,</pre>
      const Bigint &a) {
```

```
if (a.empty()) { out << "0"; return out; }</pre>
  if (a.s == -1) out << "-";</pre>
  out << a.back();</pre>
  for (int i=a.len()-2; i>=0; i--) {
    char str[10];
    snprintf(str, 5, "%.4d", a.v[i]);
    out << str:
 return out;
int cp3(const Bigint &b)const {
  if (s != b.s) return s > b.s ? 1 : -1;
  if (s == -1) return -(-*this).cp3(-b);
  if (len() != b.len()) return len()>b.len()?1:-1;
  for (int i=len()-1; i>=0; i--)
    if (v[i]!=b.v[i]) return v[i]>b.v[i]?1:-1;
  return 0;
bool operator < (const Bigint &b)const{ return cp3(b)</pre>
bool operator <= (const Bigint &b)const{ return cp3(b</pre>
    )<=0; }
bool operator >= (const Bigint &b)const{ return cp3(b
    )>=0; }
bool operator == (const Bigint &b)const{ return cp3(b
    )==0; }
bool operator != (const Bigint &b)const{ return cp3(b
    )!=0; }
bool operator > (const Bigint &b)const{ return cp3(b)
Bigint operator - () const {
  Bigint r = (*this);
  r.s = -r.s;
 return r;
Bigint operator + (const Bigint &b) const {
  if (s == -1) return -(-(*this)+(-b));
  if (b.s == -1) return (*this)-(-b);
  Bigint r;
  int nl = max(len(), b.len());
  r.resize(nl + 1);
  for (int i=0; i<nl; i++) {</pre>
    if (i < len()) r.v[i] += v[i];</pre>
    if (i < b.len()) r.v[i] += b.v[i];</pre>
    if(r.v[i] >= BIGMOD) {
      r.v[i+1] += r.v[i] / BIGMOD;
      r.v[i] %= BIGMOD;
 r.n();
 return r;
Bigint operator - (const Bigint &b) const {
  if (s == -1) return -(-(*this)-(-b));
  if (b.s == -1) return (*this)+(-b);
  if ((*this) < b) return -(b-(*this));</pre>
  Bigint r;
  r.resize(len());
  for (int i=0; i<len(); i++) {</pre>
    r.v[i] += v[i];
    if (i < b.len()) r.v[i] -= b.v[i];</pre>
    if (r.v[i] < 0) {</pre>
      r.v[i] += BIGMOD;
      r.v[i+1]--;
    }
  }
 r.n();
 return r;
Bigint operator * (const Bigint &b) {
  Bigint r;
  r.resize(len() + b.len() + 1);
  r.s = s * b.s;
  for (int i=0; i<len(); i++) {</pre>
    for (int j=0; j<b.len(); j++) {</pre>
      r.v[i+j] += v[i] * b.v[j];
      if(r.v[i+j] >= BIGMOD) {
```

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```
r.v[i+j+1] += r.v[i+j] / BIGMOD;
          r.v[i+j] %= BIGMOD;
      }
    }
    r.n();
    return r;
  Bigint operator / (const Bigint &b) {
    Bigint r;
    r.resize(max(1, len()-b.len()+1));
    int oriS = s;
    Bigint b2 = b; // b2 = abs(b)
    s = b2.s = r.s = 1;
    for (int i=r.len()-1; i>=0; i--) {
      int d=0, u=BIGMOD-1;
      while(d<u) {</pre>
        int m = (d+u+1)>>1;
        r.v[i] = m;
        if((r*b2) > (*this)) u = m-1;
        else d = m;
      }
      r.v[i] = d;
    }
    s = oriS;
    r.s = s * b.s;
    r.n();
    return r;
  Bigint operator % (const Bigint &b) {
    return (*this)-(*this)/b*b;
};
```

5.3 FFT

```
const double pi = atan(1.0)*4;
struct Complex {
    double x,y;
    Complex(double _x=0, double _y=0)
        :x(_x),y(_y) {}
    Complex operator + (Complex &tt) { return Complex(x
        +tt.x,y+tt.y); }
    Complex operator - (Complex &tt) { return Complex(x
    -tt.x,y-tt.y); }
Complex operator * (Complex &tt) { return Complex(x
        *tt.x-y*tt.y,x*tt.y+y*tt.x); }
void fft(Complex *a, int n, int rev) {
    // n是大于等于相乘的两个数组长度的2的幂次
    // 从0开始表示长度,对a进行操作
    // rev==1进行DFT,==-1进行IDFT
    for (int i = 1, j = 0; i < n; ++ i) {
        for (int k = n > 1; k > (j^k); k > 1);
        if (i<j) std::swap(a[i],a[j]);</pre>
    for (int m = 2; m <= n; m <<= 1) {
        Complex wm(cos(2*pi*rev/m),sin(2*pi*rev/m));
        for (int i = 0; i < n; i += m) {</pre>
            Complex w(1.0,0.0);
            for (int j = i; j < i+m/2; ++ j) {</pre>
                Complex t = w*a[j+m/2];
                a[j+m/2] = a[j] - t;
                a[j] = a[j] + t;
                W = W * Wm;
            }
        }
    if (rev==-1) {
        for (int i = 0; i < n; ++ i) a[i].x /= n,a[i].y</pre>
}
```

5.4 FWHT

```
const int MAXN = 1<<20;

void FWHT(int a[], int l=0, int r=MAXN-1){
   if (l==r)return;

   int mid = (l+r)>>1+1, n = r-l+1;
   FWHT(a,l,mid-1);
   FWHT(a,mid,r);

for (int i=0; i<(n>>1); i++){
   int a1=a[l+i], a2=a[mid+i];
   a[l+i] = a1+a2;
   a[mid+i] = a1-a2;
}
}
```

5.5 GaussElimination

```
// by bcw_codebook
const int MAXN = 300;
const double EPS = 1e-8;
int n;
double A[MAXN][MAXN];
void Gauss() {
  for(int i = 0; i < n; i++) {</pre>
    bool ok = 0;
     for(int j = i; j < n; j++) {</pre>
       if(fabs(A[j][i]) > EPS) {
         swap(A[j], A[i]);
         ok = 1:
         break:
      }
     if(!ok) continue;
     double fs = A[i][i];
     for(int j = i+1; j < n; j++) {</pre>
       double r = A[j][i] / fs;
       for(int k = i; k < n; k++) {</pre>
         A[j][k] -= A[i][k] * r;
    }
  }
}
```

5.6 Inverse

```
int inverse[100000];
void invTable(int b, int p) {
  inverse[1] = 1;
  for( int i = 2; i <= b; i++ ) {
    inverse[i] = (long long)inverse[p%i] * (p-p/i) % p;
  }
}
int inv(int b, int p) {
  return b == 1 ? 1 : ((long long)inv(p % b, p) * (p-p/b) % p);
}</pre>
```

5.7 LinearPrime

```
const int MAXP = 100; //max prime
vector<int> P; // primes
void build_prime(){
    static bitset<MAXP> ok;
    int np=0;
    for (int i=2; i<MAXP; i++){
        if (ok[i]==0)P.push_back(i), np++;
        for (int j=0; j<np && i*P[j]<MAXP; j++){
            ok[ i*P[j]] = 1;
            if ( i%P[j]==0 )break;
        }
    }
}</pre>
```

5.8 Miller Rabin

```
typedef long long LL;
inline LL bin_mul(LL a, LL n,const LL& MOD){
 LL re=0;
  while (n>0){
    if (n&1) re += a;
    a += a; if (a>=MOD) a-=MOD;
   n>>=1;
  return re%MOD;
inline LL bin_pow(LL a, LL n,const LL& MOD){
 LL re=1:
  while (n>0){
   if (n&1) re = bin_mul(re,a,MOD);
    a = bin_mul(a,a,MOD);
   n>>=1;
 }
  return re;
}
bool is_prime(LL n){
 //static LL sprp[3] = { 2LL, 7LL, 61LL};
  static LL sprp[7] = { 2LL, 325LL, 9375LL,
    28178LL, 450775LL, 9780504LL,
    1795265022LL };
  if (n==1 || (n&1)==0 ) return n==2;
  int u=n-1, t=0;
  while ( (u&1)==0 ) u>>=1, t++;
  for (int i=0; i<3; i++){</pre>
    LL x = bin_pow(sprp[i]%n, u, n);
    if (x==0 || x==1 || x==n-1)continue;
    for (int j=1; j<t; j++){</pre>
      x=x*x%n;
      if (x==1 || x==n-1)break;
    if (x==n-1)continue;
    return 0;
 }
  return 1;
```

5.9 Pollard's rho

```
// from PEC
// does not work when n is prime
Int f(Int x, Int mod){
  return add(mul(x, x, mod), 1, mod);
}
Int pollard_rho(Int n) {
  if ( !(n & 1) ) return 2;
  while (true) {
    Int y = 2, x = rand()%(n-1) + 1, res = 1;
    for ( int sz = 2 ; res == 1 ; sz *= 2 ) {
      for ( int i = 0 ; i < sz && res <= 1 ; i++) {</pre>
```

```
x = f(x, n);
res = __gcd(abs(x-y), n);
}
y = x;
}
if ( res != 0 && res != n ) return res;
}
}
```

5.10 數論基本工具

```
Int POW(Int a, Int n, Int mod){
    Int re=1;
    while (n>0){
        if (n&1LL) re = re*a%mod;
        a = a*a%mod;
        n>>=1;
    }
    return re;
}

Int C(Int n, Int m){
    if (m<0 || m>n)return 0;
    return J[n] * inv(J[m]*J[n-m]%MOD) %MOD;
}
```

5.11 Mobius

5.12 Simplex

```
// Two-phase simplex algorithm for solving linear
   programs of the form
//
       maximize
                    c^T x
//
       subject to Ax <= b
//
                    x >= 0
11
// INPUT: A -- an m x n matrix
//
         b -- an m-dimensional vector
//
          c -- an n-dimensional vector
//
         x -- a vector where the optimal solution will
     be stored
// OUTPUT: value of the optimal solution (infinity if
    unbounded
//
           above, nan if infeasible)
//
// To use this code, create an LPSolver object with A,
   b, and c as
// arguments. Then, call Solve(x).
#include <iostream>
#include <iomanip>
#include <vector>
```

```
#include <cmath>
#include <limits>
using namespace std;
typedef long double DOUBLE;
typedef vector<DOUBLE> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;
const DOUBLE EPS = 1e-9;
struct LPSolver {
 int m, n;
  VI B, N;
  VVD D;
  LPSolver(const VVD &A, const VD &b, const VD &c) :
    m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2,
    for (int i = 0; i < m; i++) for (int j = 0; j < n;
        j++) D[i][j] = A[i][j];
    for (int i = 0; i < m; i++) { B[i] = n + i; D[i][n]</pre>
         = -1; D[i][n + 1] = b[i]; }
    for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -
        c[j]; }
    N[n] = -1; D[m + 1][n] = 1;
  void Pivot(int r, int s) {
    double inv = 1.0 / D[r][s];
    for (int i = 0; i < m + 2; i++) if (i != r)</pre>
      for (int j = 0; j < n + 2; j++) if (j != s)
        D[i][j] -= D[r][j] * D[i][s] * inv;
    for (int j = 0; j < n + 2; j++) if (j != s) D[r][j]</pre>
         *= inv;
    for (int i = 0; i < m + 2; i++) if (i != r) D[i][s]
         *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
  bool Simplex(int phase) {
    int x = phase == 1 ? m + 1 : m;
    while (true) {
      int s = -1;
      for (int j = 0; j <= n; j++) {</pre>
        if (phase == 2 && N[j] == -1) continue;
        if (s == -1 || D[x][j] < D[x][s] || D[x][j] ==
            D[x][s] \&\& N[j] < N[s]) s = j;
      if (D[x][s] > -EPS) return true;
      int r = -1;
      for (int i = 0; i < m; i++) {</pre>
        if (D[i][s] < EPS) continue;</pre>
        if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n +</pre>
             1] / D[r][s] ||
          (D[i][n + 1] / D[i][s]) == (D[r][n + 1] / D[r]
               ][s]) && B[i] < B[r]) r = i;
      if (r == -1) return false;
      Pivot(r, s);
  }
  DOUBLE Solve(VD &x) {
    int r = 0;
    for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r][</pre>
        n + 1) r = i;
    if (D[r][n + 1] < -EPS) {</pre>
      Pivot(r, n);
      if (!Simplex(1) || D[m + 1][n + 1] < -EPS) return</pre>
            -numeric_limits<DOUBLE>::infinity();
      for (int i = 0; i < m; i++) if (B[i] == -1) {
        int s = -1;
        for (int j = 0; j <= n; j++)
```

```
(November 6, 2020) 8
          if (s == -1 || D[i][j] < D[i][s] || D[i][j]</pre>
              == D[i][s] \&\& N[j] < N[s]) s = j;
        Pivot(i, s);
      }
    if (!Simplex(2)) return numeric_limits<DOUBLE>::
        infinity();
    x = VD(n);
    for (int i = 0; i < m; i++) if (B[i] < n) x[B[i]] =</pre>
         D[i][n + 1];
    return D[m][n + 1];
 }
};
int main() {
  const int m = 4;
  const int n = 3;
  \{ -1, -5, 0 \},
    { 1, 5, 1 },
    \{-1, -5, -1\}
  };
  DOUBLE _b[m] = { 10, -4, 5, -5 };
DOUBLE _c[n] = { 1, -1, 0 };
  VVD A(m);
  VD b(_b, _b + m);
  VD c(_c, _c + n);
for (int i = 0; i < m; i++) A[i] = VD(_A[i], _A[i] +
  LPSolver solver(A, b, c);
  VD x;
  DOUBLE value = solver.Solve(x);
  cerr << "VALUE: " << value << endl; // VALUE: 1.29032</pre>
  cerr << "SOLUTION:"; // SOLUTION: 1.74194 0.451613 1</pre>
  for (size_t i = 0; i < x.size(); i++) cerr << " " <<</pre>
      x[i];
  cerr << endl;
  return 0;
5.13 SG
Anti Nim (取走最後一個石子者敗)
先手必勝 if and only if
1. 「所有」堆的石子數都為 1 且遊戲的 SG 值為 0。
   「有些」堆的石子數大於 1 且遊戲的 SG 值不為 0。
```

```
先手必勝 if and only if
1. 「所有」堆的石子數都為 1 且遊戲的 SG 值為 0。
2. 「有些」堆的石子數大於 1 且遊戲的 SG 值不為 0。
Anti-SG (決策集合為空的遊戲者贏)
定義 SG 值為 0 時,遊戲結束,則先手必勝 if and only if
1. 遊戲中沒有單一遊戲的 SG 函數大於 1 且遊戲的 SG 函數為 0。
2. 遊戲中某個單一遊戲的 SG 函數大於 1 且遊戲的 SG 函數不為 0。
Sprague-Grundy
1. 雙人、回合制
2. 資訊完全公開
3. 無隨機因素
```

4. 可在有限步內結束

6. 雙方可採取的行動相同

|SG(S) 的值為 0:後手(P)必勝

5. 沒有和局

```
不為 0: 先手(N)必勝
int mex(set S) {
    // find the min number >= 0 that not in the S
    // e.g. S = {0, 1, 3, 4} mex(S) = 2
}

state = []
int SG(A) {
    if (A not in state) {
        S = sub_states(A)
        if( len(S) > 1 ) state[A] = reduce(operator.xor, [
            SG(B) for B in S])
    else state[A] = mex(set(SG(B) for B in next_states(A)))
    }
    return state[A]
}
```

5.14 Theorem

```
Lucas's Theorem
 For non-negative integer n,m and prime P,
 C(m,n) \mod P = C(m/M,n/M) * C(m%M,n%M) \mod P
 = mult_i ( C(m_i,n_i) )
 where m_i is the i-th digit of m in base P.
Pick's Theorem
 A = i + b/2 - 1
Kirchhoff's theorem
 A_{ii} = deg(i), A_{ij} = (i,j) \in E? -1:0
 Deleting any one row, one column, and cal the det(A)
 -----
Nth Catalan recursive function:
C_0 = 1, C_{n+1} = C_n * 2(2n + 1)/(n+2)
Mobius Formula
u(n) = 1 , if n = 1
(-1)^m , 若 n 無平方數因數,且 n = p1*p2*p3
          *...*pk
              ,若 n 有大於 1 的平方數因數
- Property
1. (積性函數) u(a)u(b) = u(ab)
2. \sum \{d|n\} \ u(d) = [n == 1]
Mobius Inversion Formula
if f(n) = \sum \{d|n\} g(d)
       g(n) = \sum \{d|n\} \ u(n/d)f(d)
            = \sum \{d|n\} \ u(d)f(n/d)
- Application
the number/power of gcd(i, j) = k
- Trick
分塊, O(sqrt(n))
Chinese Remainder Theorem (m_i 兩兩互質)
 x = a_1 \pmod{m_1}
 x = a_2 \pmod{m_2}
 x = a_i \pmod{m_i}
construct a solution:
 Let M = m_1 * m_2 * m_3 * ... * m_n
 Let M_i = M / m_i
 t_i = 1 / M_i
 t_i * M_i = 1 \pmod{m_i}
 solution x = a_1 * t_1 * M_1 + a_2 * t_2 * M_2 + ...
     + a_n * t_n * M_n + k * M
 = k*M + \sum a_i * t_i * M_i, k is positive integer.
```

6 Graph

6.1 BCC

```
邊雙連通
```

```
任意兩點間至少有兩條不重疊的路徑連接,找法:
1. 標記出所有的橋
2. 對全圖進行 DFS,不走橋,每一次 DFS 就是一個新的邊雙
// from BCW
struct BccEdge {
  static const int MXN = 100005;
  struct Edge { int v,eid; };
  int n,m,step,par[MXN],dfn[MXN],low[MXN];
  vector<Edge> E[MXN];
  DisjointSet djs;
  void init(int _n) {
    n = _n; m = 0;
for (int i=0; i<n; i++) E[i].clear();</pre>
    djs.init(n);
  void add_edge(int u, int v) {
    E[u].PB({v, m});
    E[v].PB({u, m});
    m++;
  void DFS(int u, int f, int f_eid) {
    par[u] = f;
    dfn[u] = low[u] = step++;
    for (auto it:E[u]) {
      if (it.eid == f_eid) continue;
      int v = it.v;
      if (dfn[v] == -1) {
        DFS(v, u, it.eid);
        low[u] = min(low[u], low[v]);
      } else {
        low[u] = min(low[u], dfn[v]);
      }
    }
  }
  void solve() {
    step = 0;
    memset(dfn, -1, sizeof(int)*n);
    for (int i=0; i<n; i++) {</pre>
      if (dfn[i] == -1) DFS(i, i, -1);
    djs.init(n);
    for (int i=0; i<n; i++) {</pre>
      if (low[i] < dfn[i]) djs.uni(i, par[i]);</pre>
}graph;
```

6.2 Dijkstra

```
typedef struct Edge{
   int v; long long len;
   bool operator > (const Edge &b)const { return len>b
        .len; }
```

```
} State;
const long long INF = 1LL<<60;</pre>
void Dijkstra(int n, vector<Edge> G[], long long d[],
    int s, int t=-1){
    static priority_queue<State, vector<State>, greater
         <State> > pq;
    while ( pq.size() )pq.pop();
    for (int i=1; i<=n; i++)d[i]=INF;</pre>
    d[s]=0; pq.push( (State){s,d[s]} );
    while ( pq.size() ){
        auto x = pq.top(); pq.pop();
        int u = x.v;
        if (d[u]<x.len)continue;</pre>
        if (u==t)return;
        for (auto &e:G[u]){
             if (d[e.v] > d[u]+e.len){
                 d[e.v] = d[u]+e.len;
                 pq.push( (State) {e.v,d[e.v]} );
             }
        }
    }
}
```

6.3 Theorm - Domination

```
Maximum Independent Set
General: [NPC] maximum clique of complement of G
Tree: [P] Greedy
Bipartite Graph: [P] Maximum Cardinality Bipartite
   Matching
Minimum Dominating Set
General: [NPC]
Tree: [P] DP
Bipartite Graph: [NPC]
Minimum Vertex Cover
General: [NPC] (?)maximum clique of complement of G
Tree: [P] Greedy, from leaf to root
Bipartite Graph: [P] Maximum Cardinality Bipartite
    Matching
Minimum Edge Cover
General: [P] V - Maximum Matching
Bipartite Graph: [P] Greedy, strategy: cover small
    degree node first.
(Min/Max)Weighted: [P]: Minimum/Minimum Weight Matching
```

6.4 Strongly Connected Component(SCC)

6.5 DominatorTree

```
// PEC VER
// idom[n] is the unique node that strictly dominates n
// not strictly dominate any other node that strictly
    dominates n.
// idom[n] = 0 if n is entry or the entry cannot reach
struct DominatorTree{
 static const int MAXN = 200010;
  int n,s;
 vector<int> g[MAXN],pred[MAXN];
  vector<int> cov[MAXN];
  int dfn[MAXN],nfd[MAXN],ts;
  int par[MAXN];
  int sdom[MAXN],idom[MAXN];
  int mom[MAXN],mn[MAXN];
  inline bool cmp(int u,int v) { return dfn[u] < dfn[v</pre>
      ]; }
```

```
int eval(int u) {
    if(mom[u] == u) return u;
    int res = eval(mom[u]);
    if(cmp(sdom[mn[mom[u]]],sdom[mn[u]]))
      mn[u] = mn[mom[u]];
    return mom[u] = res;
  void init(int _n, int _s) {
    n = _n;
    s = _s;
    REP1(i,1,n) {
      g[i].clear();
      pred[i].clear();
      idom[i] = 0;
    }
  void add_edge(int u, int v) {
    g[u].push_back(v);
    pred[v].push_back(u);
  void DFS(int u) {
    ts++;
    dfn[u] = ts;
    nfd[ts] = u;
    for(int v:g[u]) if(dfn[v] == 0) {
      par[v] = u;
      DFS(v);
  void build() {
    ts = 0;
    REP1(i,1,n) {
      dfn[i] = nfd[i] = 0;
      cov[i].clear();
      mom[i] = mn[i] = sdom[i] = i;
    DFS(s);
    for (int i=ts; i>=2; i--) {
      int u = nfd[i];
      if(u == 0) continue;
      for(int v:pred[u]) if(dfn[v]) {
        eval(v);
        if(cmp(sdom[mn[v]],sdom[u])) sdom[u] = sdom[mn[
            v]];
      cov[sdom[u]].push_back(u);
      mom[u] = par[u];
      for(int w:cov[par[u]]) {
        eval(w);
        if(cmp(sdom[mn[w]],par[u])) idom[w] = mn[w];
        else idom[w] = par[u];
      cov[par[u]].clear();
    REP1(i,2,ts) {
      int u = nfd[i];
      if(u == 0) continue ;
      if(idom[u] != sdom[u]) idom[u] = idom[idom[u]];
 }
}dom;
#define MXN 100005
#define PB push_back
#define FZ(s) memset(s,0,sizeof(s))
struct Scc{
int n, nScc, vst[MXN], bln[MXN];
vector<int> E[MXN], rE[MXN], vec;
void init(int _n){
  n = n;
  for (int i=0; i<MXN; i++){</pre>
    E[i].clear();
    rE[i].clear();
```

```
void add_edge(int u, int v){
  E[u].PB(v);
  rE[v].PB(u);
void DFS(int u){
  vst[u]=1;
  for (auto v : E[u])
    if (!vst[v]) DFS(v);
  vec.PB(u);
void rDFS(int u){
  vst[u] = 1;
  bln[u] = nScc;
  for (auto v : rE[u])
    if (!vst[v]) rDFS(v);
void solve(){
  nScc = 0;
  vec.clear();
  FZ(vst);
  for (int i=0; i<n; i++)</pre>
    if (!vst[i]) DFS(i);
  reverse(vec.begin(),vec.end());
  FZ(vst);
  for (auto v : vec){
    if (!vst[v]){
      rDFS(v);
      nScc++;
  }
}
};
```

6.6 Manhattan MST

```
#include <bits/stdc++.h>
using namespace std;
const int MAXN = 100005;
const int OFFSET = 2000; // y-x may < 0, offset it, if</pre>
    y-x too large, please write a unique function
const int INF = 0xFFFFFFF;
int n;
int x[MAXN], y[MAXN], p[MAXN];
typedef pair<int, int> pii;
pii bit[MAXN]; // [ val, pos ]
struct P {
    int x, y, id;
    bool operator<(const P&b ) const {</pre>
        if (x == b.x) return y > b.y;
        else return x > b.x;
};
vector<P> op;
struct E {
    int x, y, cost;
    bool operator<(const E&b ) const {</pre>
        return cost < b.cost;</pre>
    }
};
vector<E> edges;
int find(int x) {
    return p[x] == x ? x : p[x] = find(p[x]);
}
void update(int i, int v, int p) {
    while ( i ) {
        if ( bit[i].first > v ) bit[i] = {v, p};
        i -= i & (-i);
    }
}
```

```
pii query(int i) {
    pii res = {INF, INF};
    while ( i < MAXN ) {</pre>
        if ( bit[i].first < res.first ) res = {bit[i].</pre>
             first, bit[i].second};
         i += i & (-i);
    }
    return res;
}
void input() {
    cin >> n;
    for ( int i = 0 ; i < n ; i++ ) cin >> x[i] >> y[i
        ), op.push_back((P) {x[i], y[i], i});
void mst() {
    for ( int i = 0 ; i < MAXN ; i++ ) p[i] = i;</pre>
    int res = 0;
    sort(edges.begin(), edges.end());
    for ( auto e : edges ) {
        int x = find(e.x), y = find(e.y);
        if ( x != y ) {
            p[x] = y;
             res += e.cost;
        }
    cout << res << endl;</pre>
}
void construct() {
    sort(op.begin(), op.end());
    for ( int i = 0 ; i < n ; i++ ) {</pre>
        pii q = query(op[i].y - op[i].x + OFFSET);
         update(op[i].y - op[i].x + OFFSET, op[i].x + op
             [i].y, op[i].id);
         if ( q.first == INF ) continue;
         edges.push_back((E) {op[i].id, q.second, abs(x[
             op[i].id]-x[q.second]) + abs(y[op[i].id]-y[
             q.second]) });
    }
}
void solve() {
    // [45 ~ 90 deg]
    for ( int i = 0 ; i < MAXN ; i++ ) bit[i] = {INF,</pre>
         INF };
    construct();
    // [0 ~ 45 deg]
    for ( int i = 0 ; i < MAXN ; i++ ) bit[i] = {INF,</pre>
        INF };
    for ( int i = 0 ; i < n ; i++ ) swap(op[i].x, op[i</pre>
        ].y);
    construct();
    for ( int i = 0 ; i < n ; i++ ) swap(op[i].x, op[i</pre>
         ].y);
    // [-90 ~ -45 deg]
    for ( int i = 0 ; i < MAXN ; i++ ) bit[i] = {INF,</pre>
        INF };
    for ( int i = 0 ; i < n ; i++ ) op[i].y *= -1;</pre>
    construct();
    // [-45 ~ 0 deg]
    for ( int i = 0 ; i < MAXN ; i++ ) bit[i] = {INF,</pre>
    for ( int i = 0 ; i < n ; i++ ) swap(op[i].x, op[i</pre>
        ].y);
    construct();
    // mst
    mst();
}
```

```
int main () {
    input();
    solve();
    return 0;
}
```

6.7 Hungarian

```
// Maximum Cardinality Bipartite Matching
struct Graph {
    static const int MAXN = 5005;
    vector<int> G[MAXN];
    int match[MAXN]; // Matching Result
    int vis[MAXN];
    void init(int _n) {
        n = _n;
        for ( int i = 0 ; i < n ; i++ ) G[i].clear();</pre>
    }
    bool dfs(int u) {
        for ( auto v:G[u] ) {
            if (!vis[v]) {
                 vis[v] = true;
                 if (match[v] == -1 || dfs(match[v])) {
                     match[v] = u;
                     match[u] = v;
                     return true:
                 }
            }
        }
        return false;
    }
    int solve() {
        int res = 0;
        memset(match, -1, sizeof(match));
        for (int i = 0; i < n; i++) {</pre>
             if (match[i] == -1) {
                 memset(vis, 0, sizeof(vis));
                 if (dfs(i)) res += 1;
             }
        return res;
    }
} graph;
```

6.8 KM

```
Detect non-perfect-matching:

    set all edge[i][j] as INF

2. if solve() >= INF, it is not perfectmatching.
// Maximum Weight Perfect Bipartite Matching
// allow negative weight!
typedef long long Int;
struct KM {
    static const int MAXN = 1050;
    static const int INF = 1LL<<60;</pre>
    int n, match[MAXN], vx[MAXN], vy[MAXN];
    Int edge[MAXN][MAXN], lx[MAXN], ly[MAXN], slack[
        MAXN];
    void init(int _n){
        n = _n;
        for ( int i = 0 ; i < n ; i++ )</pre>
            for ( int j = 0; j < n ; j++ )</pre>
                 edge[i][j] = 0;
    void add_edge(int x, int y, Int w){
        edge[x][y] = w;
```

```
bool DFS(int x){
         vx[x] = 1;
        for ( int y = 0 ; y < n ; y++ ) {
             if ( vy[y] ) continue;
             if (lx[x] + ly[y] > edge[x][y]) {
                 slack[y] = min(slack[y], lx[x] + ly[y]
                      - edge[x][y]);
             } else {
                 vy[y] = 1;
                 if ( match[y] == -1 || DFS(match[y]) ){
                     match[y] = x;
                     return true;
                 }
             }
        }
        return false;
    Int solve() {
         fill(match, match + n, -1);
        fill(lx, lx + n, -INF);
         fill(ly, ly + n, 0);
         for ( int i = 0; i < n; i++ )
             for ( int j = 0; j < n; j++ )
                 lx[i] = max(lx[i], edge[i][j]);
        for ( int i = 0 ; i < n; i++ ) {
             fill(slack, slack + n, INF);
             while (true){
                 fill(vx, vx + n, 0);
                 fill(vy, vy + n, 0);
                 if ( DFS(i) ) break;
                 Int d = INF;
                 for ( int j = 0 ; j < n ; j++ )</pre>
                     if ( !vy[j] ) d = min(d, slack[j]);
                 for ( int j = 0 ; j < n ; j++ ) {</pre>
                     if (vx[j]) lx[j] -= d;
                     if (vy[j]) ly[j] += d;
                     else slack[j] -= d;
                 }
             }
        Int res = 0;
        for ( int i = 0 ; i < n ; i++ ) {</pre>
             res += edge[ match[i] ][i];
        return res;
    }
} graph;
```

6.9 Theorm - Matching

```
|最大匹配 + 最小邊覆蓋 = V
|最大獨立集 + 最小點覆蓋 = V
|最大匹配 = 最小點覆蓋
|最小路徑覆蓋數 = V - 最大匹配數
```

6.10 Maximum General Matching

```
int lca(int u, int v){
    for ( ++t ; ; swap(u, v) ) {
      if ( u == 0 ) continue;
      if ( vis[u] == t ) return u;
      vis[u] = t;
      u = st[ pa[ match[u] ] ];
    }
  void flower(int u, int v, int l, queue<int> &q) {
    while ( st[u] != 1 ) {
      pa[u] = v;
      if ( S[ v = match[u] ] == 1 ) {
        q.push(v);
        S[v] = 0;
      st[u] = st[v] = 1;
      u = pa[v];
    }
  bool bfs(int u){
    for ( int i = 1 ; i <= n ; i++ ) st[i] = i;
    memset(S, -1, sizeof(S));
    queue<int>q;
    q.push(u);
    S[u] = 0;
    while ( !q.empty() ) {
      u = q.front(); q.pop();
      for ( int i = 0 ; i < (int)G[u].size(); i++) {</pre>
        int v = G[u][i];
        if ( S[v] == -1 ) {
          pa[v] = u;
          S[v] = 1;
          if ( !match[v] ) {
            for ( int lst ; u ; v = lst, u = pa[v] ) {
              lst = match[u];
              match[u] = v;
              match[v] = u;
            }
            return 1;
          q.push(match[v]);
          S[ match[v] ] = 0;
        } else if ( !S[v] && st[v] != st[u] ) {
          int 1 = lca(st[v], st[u]);
          flower(v, u, l, q);
          flower(u, v, 1, q);
        }
      }
    }
    return 0;
  int solve(){
    memset(pa, 0, sizeof(pa));
    memset(match, 0, sizeof(match));
    int ans = 0;
    for ( int i = 1 ; i <= n ; i++ )</pre>
      if ( !match[i] && bfs(i) ) ans++;
    return ans;
  }
} graph;
```

6.11 Minimum General Weighted Matching

```
void add_edge(int u, int v, int w) {
        e[u][v] = e[v][u] = w;
    bool SPFA(int u){
        if (onstk[u]) return true;
         stk.push_back(u);
        onstk[u] = 1;
         for ( int v = 0 ; v < n ; v++ ) {</pre>
             if (u != v && match[u] != v && !onstk[v] )
                 int m = match[v];
                 if ( d[m] > d[u] - e[v][m] + e[u][v] )
                     d[m] = d[u] - e[v][m] + e[u][v];
                     onstk[v] = 1;
                     stk.push_back(v);
                     if (SPFA(m)) return true;
                     stk.pop_back();
                     onstk[v] = 0;
                 }
             }
        onstk[u] = 0;
        stk.pop_back();
         return false;
    int solve() {
        for ( int i = 0 ; i < n ; i += 2 ) {</pre>
             match[i] = i+1;
             match[i+1] = i;
        while (true){
             int found = 0;
             for ( int i = 0 ; i < n ; i++ )</pre>
                 onstk[ i ] = d[ i ] = 0;
             for ( int i = 0 ; i < n ; i++ ) {</pre>
                 stk.clear();
                 if ( !onstk[i] && SPFA(i) ) {
                     found = 1:
                     while ( stk.size() >= 2 ) {
                          int u = stk.back(); stk.
                              pop_back();
                          int v = stk.back(); stk.
                              pop_back();
                          match[u] = v;
                         match[v] = u;
                     }
                 }
             if (!found) break;
         int ret = 0;
         for ( int i = 0 ; i < n ; i++ )</pre>
            ret += e[i][match[i]];
         ret /= 2;
        return ret;
} graph;
```

6.12 Maximum Clique

```
const int MAXN = 105;
int best;
int m ,n;
int num[MAXN];
// int x[MAXN];
int path[MAXN];
int g[MAXN][MAXN];
bool dfs( int *adj, int total, int cnt ){
   int i, j, k;
   int t[MAXN];
   if( total == 0 ){
        if( best < cnt ){</pre>
```

```
// for( i = 0; i < cnt; i++) path[i] = x[i
             best = cnt; return true;
        return false;
    for( i = 0; i < total; i++){</pre>
        if( cnt+(total-i) <= best ) return false;</pre>
        if( cnt+num[adj[i]] <= best ) return false;</pre>
        // x[cnt] = adj[i];
        for( k = 0, j = i+1; j < total; j++ )</pre>
             if( g[ adj[i] ][ adj[j] ] )
                 t[ k++ ] = adj[j];
                 if( dfs( t, k, cnt+1 ) ) return true;
    } return false;
int MaximumClique(){
    int i, j, k;
    int adj[MAXN];
    if( n <= 0 ) return 0;
    best = 0;
    for( i = n-1; i >= 0; i--){
        // x[0] = i;
        for( k = 0, j = i+1; j < n; j++)
             if( g[i][j] ) adj[k++] = j;
        dfs( adj, k, 1 );
        num[i] = best;
    return best;
}
```

6.13 Steiner Tree

```
// Minimum Steiner Tree
// O(V 3^T + V^2 2^T)
struct SteinerTree{
#define V 33
#define T 8
#define INF 1023456789
  int n , dst[V][V] , dp[1 \leftrightarrow T][V] , tdst[V];
  void init( int _n ){
    n = n;
    for( int i = 0 ; i < n ; i ++ ){</pre>
       for( int j = 0 ; j < n ; j ++ )</pre>
       dst[ i ][ j ] = INF;
dst[ i ][ i ] = 0;
    }
  void add_edge( int ui , int vi , int wi ){
    dst[ ui ][ vi ] = min( dst[ ui ][ vi ] , wi );
    dst[ vi ][ ui ] = min( dst[ vi ][ ui ] , wi );
  void shortest_path(){
    for( int k = 0 ; k < n ; k ++ )</pre>
       for( int i = 0 ; i < n ; i ++ )</pre>
         for( int j = 0 ; j < n ; j ++ )</pre>
           dst[ i ][ j ] = min( dst[ i ][ j ],
                 dst[ i ][ k ] + dst[ k ][ j ] );
  int solve( const vector<int>& ter ){
    int t = (int)ter.size();
    for( int i = 0 ; i < ( 1 << t ) ; i ++ )</pre>
       for( int j = 0 ; j < n ; j ++ )</pre>
         dp[i][j] = INF;
     for( int i = 0 ; i < n ; i ++ )</pre>
       dp[0][i] = 0;
     for( int msk = 1 ; msk < ( 1 << t ) ; msk ++ ){</pre>
      if( msk == ( msk & (-msk) ) ){
         int who = __lg( msk );
         for( int i = 0 ; i < n ; i ++ )</pre>
           dp[ msk ][ i ] = dst[ ter[ who ] ][ i ];
         continue;
       for( int i = 0 ; i < n ; i ++ )</pre>
         for( int submsk = ( msk - 1 ) & msk ; submsk ;
                   submsk = (submsk - 1) \& msk)
```

```
dp[ msk ][ i ] = min( dp[ msk ][ i ],
                              dp[ submsk ][ i ] +
                              dp[ msk ^ submsk ][ i ] );
       for( int i = 0 ; i < n ; i ++ ){</pre>
         tdst[ i ] = INF;
         for( int j = 0 ; j < n ; j ++ )</pre>
           tdst[ i ] = min( tdst[ i ],
                      dp[ msk ][ j ] + dst[ j ][ i ] );
       for( int i = 0 ; i < n ; i ++ )</pre>
         dp[ msk ][ i ] = tdst[ i ];
     int ans = INF;
     for( int i = 0 ; i < n ; i ++ )</pre>
       ans = min(ans, dp[(1 << t) - 1][i]);
    return ans;
  }
} solver;
```

6.14 最小平均環

```
// from BCW
/* minimum mean cycle */
const int MAXE = 1805;
const int MAXN = 35;
const double inf = 1029384756;
const double eps = 1e-6;
struct Edge {
  int v,u;
  double c;
int n,m,prv[MAXN][MAXN], prve[MAXN][MAXN], vst[MAXN];
Edge e[MAXE];
vector<int> edgeID, cycle, rho;
double d[MAXN][MAXN];
inline void bellman_ford() {
  for(int i=0; i<n; i++) d[0][i]=0;</pre>
  for(int i=0; i<n; i++) {</pre>
     fill(d[i+1], d[i+1]+n, inf);
     for(int j=0; j<m; j++) {</pre>
       int v = e[j].v, u = e[j].u;
       if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
         d[i+1][u] = d[i][v]+e[j].c;
         prv[i+1][u] = v;
         prve[i+1][u] = j;
      }
    }
  }
double karp_mmc() {
  // returns inf if no cycle, mmc otherwise
  double mmc=inf;
  int st = -1;
  bellman_ford();
  for(int i=0; i<n; i++) {</pre>
     double avg=-inf;
     for(int k=0; k<n; k++) {</pre>
       if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i])</pre>
       else avg=max(avg,inf);
    if (avg < mmc) tie(mmc, st) = tie(avg, i);</pre>
  for(int i=0; i<n; i++) vst[i] = 0;</pre>
  edgeID.clear(); cycle.clear(); rho.clear();
   for (int i=n; !vst[st]; st=prv[i--][st]) {
     vst[st]++;
    edgeID.PB(prve[i][st]);
    rho.PB(st);
  while (vst[st] != 2) {
    int v = rho.back(); rho.pop_back();
     cycle.PB(v);
     vst[v]++;
```

```
reverse(ALL(edgeID));
edgeID.resize(SZ(cycle));
return mmc;
}
```

6.15 偵測負環

```
#include <bits/stdc++.h>
using namespace std;
const int INF = 1000000;
const int MAXN = 200;
int n, m, q;
int d[MAXN][MAXN];
int main () {
    while ( cin >> n >> m >> q && n) {
         for ( int i = 0 ; i <= n ; i++ ) {</pre>
             for ( int j = 0 ; j <= n ; j++ ) d[i][j] =</pre>
                  (i==j ? 0 : INF);
         }
         for ( int i = 0 ; i < m ; i++ ) {</pre>
             int a, b, c;
             cin >> a >> b >> c;
             d[a][b] = min(d[a][b], c);
         }
         for ( int k = 0 ; k < n ; k++ ) {
             for ( int i = 0 ; i < n ; i++ ) {</pre>
                  for ( int j = 0 ; j < n ; j++ ) {
                      if ( d[i][j] > d[i][k] + d[k][j] &&
                            d[i][k] < INF && d[k][j] < INF
                           //printf("%d > %d + %d\n", d[i]
                               ][j], d[i][k], d[k][j]);
                           //if ( d[i][k] >= INF || d[k][j
                               ] >= INF ) cout << "NO : " << i << " " << j << " " <<
                               k << "--":
                          d[i][j] = min(d[i][j], d[i][k]
                               + d[k][j]);
                      }
                  }
             }
         }
         for ( int i = 0 ; i < n ; i++ ) {</pre>
             for ( int j = 0 ; j < n ; j++ ) {</pre>
                  for ( int k = 0 ; k < n && d[i][j] != -</pre>
                      INF ; k++ ) {
                      if ( d[k][k] < 0 && d[i][k] != INF</pre>
                           && d[k][j] != INF )
                           d[i][j] = -INF;
                  }
             }
         int u, v;
         for (int i=0;i<q;i++){</pre>
             scanf("%d%d",&u,&v);
             if (d[u][v] == INF) printf("Impossible\n");
             else if (d[u][v] == -INF) printf("-Infinity
                  \n");
             else printf("%d\n",d[u][v]);
         puts("");
    return 0;
}
```

6.16 Tarjan

```
割點
點 u 為割點 if and only if 滿足 1. or 2.
1. u 爲樹根,且 u 有多於一個子樹。
2. u 不爲樹根,且滿足存在 (u,v) 爲樹枝邊 (或稱父子邊,
    即 u 爲 v 在搜索樹中的父親),使得 DFN(u) <= Low(v)
一條無向邊 (u,v) 是橋 if and only if (u,v) 爲樹枝邊,且
    滿足 DFN(u) < Low(v)。
// 0 base
struct TarjanSCC{
  static const int MAXN = 1000006;
  int n, dfn[MAXN], low[MAXN], scc[MAXN], scn, count;
  vector<int> G[MAXN];
  stack<int> stk;
  bool ins[MAXN];
  void tarjan(int u){
    dfn[u] = low[u] = ++count;
    stk.push(u);
    ins[u] = true;
    for(auto v:G[u]){
      if(!dfn[v]){
        tarjan(v);
        low[u] = min(low[u], low[v]);
      }else if(ins[v]){
        low[u] = min(low[u], dfn[v]);
    }
    if(dfn[u] == low[u]){
     int v;
      do {
      v = stk.top();
      stk.pop();
      scc[v] = scn;
      ins[v] = false;
      } while(v != u);
      scn++:
   }
  }
  void getSCC(){
    memset(dfn,0,sizeof(dfn));
    memset(low,0,sizeof(low));
    memset(ins,0,sizeof(ins));
    memset(scc,0,sizeof(scc));
    count = scn = 0;
    for(int i = 0 ; i < n ; i++ ){</pre>
      if(!dfn[i]) tarjan(i);
  }
}SCC;
```

7 Data Structure

7.1 2D Range Tree

```
// remember sort x !!!!!
typedef int T;
const int LGN = 20;
const int MAXN = 100005;
struct Point{
    T x, y;
    friend bool operator < (Point a, Point b){</pre>
```

```
return tie(a.x,a.y) < tie(b.x,b.y);</pre>
    }
};
struct TREE{
    Point pt;
    int toleft;
}tree[LGN][MAXN];
struct SEG{
    T mx, Mx;
    int sz;
    TREE *st;
}seg[MAXN*4];
vector<Point> P:
void build(int 1, int r, int o, int deep){
    seg[o].mx = P[1].x;
    seg[o].Mx = P[r].x;
    seg[o].sz = r-l+1;;
    if(1 == r){
        tree[deep][r].pt = P[r];
        tree[deep][r].toleft = 0;
        seg[o].st = &tree[deep][r];
        return:
    int mid = (l+r)>>1;
    build(l,mid,o+o,deep+1);
    build(mid+1,r,o+o+1,deep+1);
    TREE *ptr = &tree[deep][1];
    TREE *pl = &tree[deep+1][l], *nl = &tree[deep+1][
        mid+1];
    TREE *pr = &tree[deep+1][mid+1], *nr = &tree[deep
        +1][r+1];
    int cnt = 0;
    while(pl != nl && pr != nr) {
         *(ptr) = pl->pt.y <= pr->pt.y ? cnt++, *(pl++):
              *(pr++);
        ptr -> toleft = cnt; ptr++;
    while(pl != nl) *(ptr) = *(pl++), ptr -> toleft =
         ++cnt, ptr++;
    while(pr != nr) *(ptr) = *(pr++), ptr -> toleft =
        cnt, ptr++;
int main(){
    int n; cin >> n;
    for(int i = 0 ;i < n; i++){</pre>
        T x,y; cin >> x >> y;
        P.push_back((Point){x,y});
    sort(P.begin(),P.end());
    build(0,n-1,1,0);
}
```

7.2 ext heap

```
#include <bits/extc++.h>
typedef __gnu_pbds::priority_queue<int> heap_t;
heap_t a,b;

int main() {
    a.clear();
    b.clear();
    a.push(1);
    a.push(3);
    b.push(2);
    b.push(4);
    assert(a.top() == 3);
    assert(b.top() == 4);
    // merge two heap
    a.join(b);
    assert(a.top() == 4);
```

```
assert(b.empty());
return 0;
}
```

7.3 KD tree

```
// from BCW
const int MXN = 100005;
struct KDTree {
  struct Node {
    int x,y,x1,y1,x2,y2;
    int id,f;
    Node *L, *R;
  }tree[MXN];
  int n:
  Node *root;
  long long dis2(int x1, int y1, int x2, int y2) {
    long long dx = x1-x2;
    long long dy = y1-y2;
    return dx*dx+dy*dy;
  static bool cmpx(Node& a, Node& b){ return a.x<b.x; }</pre>
  static bool cmpy(Node& a, Node& b){ return a.y<b.y; }</pre>
  void init(vector<pair<int,int>> ip) {
    n = ip.size();
    for (int i=0; i<n; i++) {</pre>
      tree[i].id = i;
      tree[i].x = ip[i].first;
      tree[i].y = ip[i].second;
    root = build_tree(0, n-1, 0);
  Node* build_tree(int L, int R, int dep) {
    if (L>R) return nullptr;
    int M = (L+R)/2;
    tree[M].f = dep%2;
    nth_element(tree+L, tree+M, tree+R+1, tree[M].f ?
        cmpy : cmpx);
    tree[M].x1 = tree[M].x2 = tree[M].x;
    tree[M].y1 = tree[M].y2 = tree[M].y;
    tree[M].L = build_tree(L, M-1, dep+1);
    if (tree[M].L) {
      tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
      tree[M].x2 = max(tree[M].x2, tree[M].L->x2);
      tree[M].y1 = min(tree[M].y1, tree[M].L->y1);
      tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
    tree[M].R = build_tree(M+1, R, dep+1);
    if (tree[M].R) {
      tree[M].x1 = min(tree[M].x1, tree[M].R->x1);
      tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
      tree[M].y1 = min(tree[M].y1, tree[M].R->y1);
      tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
    return tree+M;
  int touch(Node* r, int x, int y, long long d2){
    long long dis = sqrt(d2)+1;
    if (x<r->x1-dis || x>r->x2+dis || y<r->y1-dis || y>
        r->y2+dis)
      return 0;
    return 1;
  void nearest(Node* r, int x, int y, int &mID, long
      long &md2) {
    if (!r || !touch(r, x, y, md2)) return;
    long long d2 = dis2(r->x, r->y, x, y);
    if (d2 < md2 || (d2 == md2 \&\& mID < r->id)) {
      mID = r \rightarrow id;
```

```
md2 = d2;
    // search order depends on split dim
    if ((r->f == 0 \&\& x < r->x) ||
        (r->f == 1 && y < r->y)) {
      nearest(r->L, x, y, mID, md2);
      nearest(r->R, x, y, mID, md2);
    } else {
      nearest(r->R, x, y, mID, md2);
      nearest(r->L, x, y, mID, md2);
  int query(int x, int y) {
    int id = 1029384756;
    long long d2 = 102938475612345678LL;
    nearest(root, x, y, id, d2);
    return id;
  }
}tree;
```

7.4 Link Cut tree

```
// from bcw codebook
const int MXN = 100005;
const int MEM = 100005;
struct Splay {
  static Splay nil, mem[MEM], *pmem;
  Splay *ch[2], *f;
  int val, rev, size;
  Splay () : val(-1), rev(0), size(0) {
    f = ch[0] = ch[1] = &nil;
  Splay (int _val) : val(_val), rev(0), size(1) {
    f = ch[0] = ch[1] = &nil;
  bool isr() {
    return f->ch[0] != this && f->ch[1] != this;
  int dir() {
    return f->ch[0] == this ? 0 : 1;
  void setCh(Splay *c, int d) {
    ch[d] = c;
    if (c != &nil) c->f = this;
    pull();
  void push() {
    if (rev) {
      swap(ch[0], ch[1]);
      if (ch[0] != &nil) ch[0]->rev ^= 1;
      if (ch[1] != &nil) ch[1]->rev ^= 1;
      rev=0;
  void pull() {
    size = ch[0] -> size + ch[1] -> size + 1;
    if (ch[0] != &nil) ch[0]->f = this;
    if (ch[1] != &nil) ch[1]->f = this;
} Splay::nil, Splay::mem[MEM], *Splay::pmem = Splay::
    mem;
Splay *nil = &Splay::nil;
void rotate(Splay *x) {
  Splay *p = x->f;
  int d = x->dir();
  if (!p->isr()) p->f->setCh(x, p->dir());
  else x->f = p->f;
  p->setCh(x->ch[!d], d);
  x->setCh(p, !d);
  p->pull(); x->pull();
vector<Splay*> splayVec;
```

```
void splay(Splay *x) {
  splayVec.clear();
  for (Splay *q=x;; q=q->f) {
    splayVec.push_back(q);
    if (q->isr()) break;
 reverse(begin(splayVec), end(splayVec));
  for (auto it : splayVec) it->push();
  while (!x->isr()) {
    if (x->f->isr()) rotate(x);
    else if (x->dir()==x->f->dir()) rotate(x->f),rotate
        (x);
    else rotate(x), rotate(x);
 }
}
Splay* access(Splay *x) {
 Splay *q = nil;
 for (;x!=nil;x=x->f) {
   splay(x);
   x->setCh(q, 1);
   q = x;
 }
 return q;
void evert(Splay *x) {
 access(x);
  splay(x);
 x->rev ^= 1;
 x->push(); x->pull();
void link(Splay *x, Splay *y) {
// evert(x);
 access(x);
  splay(x);
  evert(v):
 x->setCh(y, 1);
void cut(Splay *x, Splay *y) {
// evert(x);
 access(y);
 splay(y);
 y->push();
 y - ch[0] = y - ch[0] - f = nil;
int N, Q;
Splay *vt[MXN];
int ask(Splay *x, Splay *y) {
 access(x);
  access(y);
  splay(x);
  int res = x->f->val;
 if (res == -1) res=x->val;
  return res;
int main(int argc, char** argv) {
  scanf("%d%d", &N, &Q);
  for (int i=1; i<=N; i++)</pre>
    vt[i] = new (Splay::pmem++) Splay(i);
  while (Q--) {
    char cmd[105];
    int u, v;
    scanf("%s", cmd);
    if (cmd[1] == 'i') {
      scanf("%d%d", &u, &v);
      link(vt[v], vt[u]);
    } else if (cmd[0] == 'c') {
      scanf("%d", &v);
      cut(vt[1], vt[v]);
   } else {
      scanf("%d%d", &u, &v);
      int res=ask(vt[u], vt[v]);
      printf("%d \setminus n", res);
 }
```

```
return 0;
```

7.5 Sparse Table

```
const int MAXN = 200005;
const int lgN = 20;
struct SP{ //sparse table
  int Sp[MAXN][lgN];
  function<int(int,int)> opt;
  void build(int n, int *a){ // 0 base
     for (int i=0 ;i<n; i++) Sp[i][0]=a[i];</pre>
    for (int h=1; h<lgN; h++){</pre>
       int len = 1<<(h-1), i=0;</pre>
       for (; i+len<n; i++)</pre>
         Sp[i][h] = opt(Sp[i][h-1], Sp[i+len][h-1]);
       for (; i<n; i++)</pre>
         Sp[i][h] = Sp[i][h-1];
  int query(int 1, int r){
    int h = __lg(r-l+1);
     int len = 1<<h;</pre>
     return opt( Sp[l][h] , Sp[r-len+1][h] );
  }
};
```

7.6 Treap Lin

```
#include <cstdio>
#include <cstdlib>
#include <algorithm>
#include <string.h>
using namespace std;
const int INF = 999999999;
int ran(){
    static unsigned x = 20170928;
    return x = 0xdefaced*x+1;
struct Treap{
    Treap *1,*r;
    int num,m,sz,tag,ra,ad;
    Treap(int a){
        1=r=NULL;
        num=m=a;
        sz=1;
        tag=ad=0;
        ra = ran();
}*head,*tp;
int size(Treap *a){
    return a ? a->sz : 0;
int min(Treap *a){
    return a ? a->m+a->ad : INF;
int add(Treap *a){
    return a ? a->ad : 0;
void push(Treap *a){
    if(!a) return;
    if(a->tag){
        swap(a->1,a->r);
        if(a->1)a->1->tag ^= 1;
        if(a->r)a->r->tag ^= 1;
        a->tag=0;
    if(a->1)a->1->ad += a->ad;
    if(a->r)a->r->ad += a->ad;
```

```
a - num += a - ad;
    a->m += a->ad:
    a \rightarrow ad = 0;
void pull(Treap *a){
    if(!a) return;
    a->sz=1+size(a->l)+size(a->r);
    a->m = min(a->num, min(min(a->1), min(a->r));
Treap* merge(Treap *a, Treap *b){
    if(!a || !b) return a ? a : b;
    if(a->ra > b->ra){
        push(a);
        a->r = merge(a->r,b);
        pull(a);
        return a;
    }else{
        push(b);
        b->1 = merge(a,b->1);
        pull(b);
        return b;
void split (Treap *o, Treap *&a, Treap *&b,int k){
    if(!k) a=NULL, b=o;
    else if(size(o)==k) a=o, b=NULL;
    else{
        push(o);
        if(k <= size(o->1)){
            b = o;
             split(o->1, a, b->1,k);
            pull(b);
        }else{
            split(o->r, a->r, b, k-size(o->l)-1);
             pull(a);
        }
    }
int main(){
    int n,tmp;
    scanf("%d",&n);
    for(int i = 0;i < n;i++){
    scanf("%d",&tmp);</pre>
        tp = new Treap(tmp);
        head = merge(head,tp);
    int Q;
    scanf("%d\n",&Q);
    char ss[50];
    int a, b, c;
    Treap *ta, *tb, *tc, *td;
    while(Q--){
        scanf("%s",ss);
        if(strcmp(ss,"ADD")==0){
             scanf("%d %d %d",&a,&b,&c);
             split(head,tb,tc,b);
             split(tb,ta,tb,a-1);
             tb -> ad += c;
             head = merge(ta, merge(tb, tc));
        }else if(strcmp(ss,"REVERSE")==0){
            scanf("%d %d",&a,&b);
             split(head,tb,tc,b);
             split(tb,ta,tb,a-1);
             tb -> tag ^= 1;
        head = merge(ta, merge(tb, tc));
}else if(strcmp(ss,"REVOLVE")==0){
            scanf("%d %d %d",&a,&b,&c);
             split(head,tb,tc,b);
             split(tb,ta,tb,a-1);
            int szz = size(tb);
            c %= szz;
            split(tb,tb,td,szz-c);
             tb=merge(td,tb);
             head = merge(ta, merge(tb, tc));
        }else if(strcmp(ss,"INSERT")==0){
```

```
scanf("%d %d",&a,&b);
              split(head,ta,tc,a);
              tb = new Treap(b);
         head = merge(ta, merge(tb, tc));
}else if(strcmp(ss,"DELETE")==0){
              scanf("%d",&a);
              split(head,ta,tc,a-1);
              split(tc,tb,tc,1);
              delete tb;
              head = merge(ta,tc);
         }else if(strcmp(ss,"MIN")==0){
              scanf("%d %d",&a,&b);
              split(head,tb,tc,b);
              split(tb,ta,tb,a-1);
              printf("%d \ n", min(tb));
              head = merge(ta, merge(tb, tc));
         }
    }
}
```

8 String

8.1 AC 自動機

```
// remember make_fail() !!!
// notice MLE
const int sigma = 62;
const int MAXC = 200005;
inline int idx(char c){
    if ('A'<= c && c <= 'Z')return c-'A';</pre>
    if ('a'<= c && c <= 'z')return c-'a' + 26;
if ('0'<= c && c <= '9')return c-'0' + 52;
}
struct ACautomaton{
    struct Node{
        Node *next[sigma], *fail;
        int cnt; // dp
        Node(){
             memset(next,0,sizeof(next));
             fail=0;
             cnt=0;
    } buf[MAXC], *bufp, *ori, *root;
    void init(){
        bufp = buf;
        ori = new (bufp++) Node();
        root = new (bufp++) Node();
    void insert(int n, char *s){
        Node *ptr = root;
        for (int i=0; s[i]; i++){
             int c = idx(s[i]);
             if (ptr->next[c]==NULL)
                 ptr->next[c] = new (bufp++) Node();
             ptr = ptr->next[c];
        ptr->cnt=1;
    Node* trans(Node *o, int c){
        while (o->next[c]==NULL) o = o->fail;
        return o->next[c];
    void make_fail(){
        static queue<Node*> que;
        for (int i=0; i<sigma; i++)</pre>
             ori->next[i] = root;
```

```
root->fail = ori;

que.push(root);
while ( que.size() ){
    Node *u = que.front(); que.pop();
    for (int i=0; i<sigma; i++){
        if (u->next[i]==NULL)continue;
        u->next[i]->fail = trans(u->fail,i);
        que.push(u->next[i]);
    }
    u->cnt += u->fail->cnt;
}
}
ac;
```

8.2 KMP

```
template<typename T>
void build_KMP(int n, T *s, int *f){ // 1 base
  f[0]=-1, f[1]=0;
  for (int i=2; i<=n; i++){</pre>
    int w = f[i-1];
    while (w>=0 \&\& s[w+1]!=s[i])w = f[w];
    f[i]=w+1;
  }
}
template<typename T>
int KMP(int n, T *a, int m, T *b){
  build_KMP(m,b,f);
  int ans=0;
  for (int i=1, w=0; i<=n; i++){</pre>
    while ( w \ge 0 \&\& b[w+1]! = a[i] )w = f[w];
    w++;
    if (w==m){
      ans++:
      w=f[w];
    }
  }
  return ans;
```

8.3 迴文字動機

```
// remember init()
// remember make_fail() !!!
// insert s need 1 base !!!
// notice MLE
const int sigma = 62;
const int MAXC = 1000006;
inline int idx(char c){
    if ('a'<= c && c <= 'z')return c-'a';
    if ('A'<= c && c <= 'Z')return c-'A'+26;
    if ('0'<= c && c <= '9')return c-'0'+52;
struct PalindromicTree{
    struct Node{
        Node *next[sigma], *fail;
        int len, cnt; // for dp
        Node(){
            memset(next,0,sizeof(next));
            fail=0:
            len = cnt = 0;
    } buf[MAXC], *bufp, *even, *odd;
    void init(){
        bufp = buf;
        even = new (bufp++) Node();
        odd = new (bufp++) Node();
        even->fail = odd;
        odd \rightarrow len = -1;
```

```
void insert(char *s){
         Node* ptr = even;
         for (int i=1; s[i]; i++){
             ptr = extend(ptr,s+i);
    }
    Node* extend(Node *o, char *ptr){
         int c = idx(*ptr);
         while ( *ptr != *(ptr-1-o->len) )o=o->fail;
         Node *&np = o->next[c];
         if (!np){
             np = new (bufp++) Node();
             np \rightarrow len = o \rightarrow len + 2;
             Node *f = o->fail;
             if (f){
                 while ( *ptr != *(ptr-1-f->len) )f=f->
                      fail;
                 np->fail = f->next[c];
             else {
                 np->fail = even;
             np->cnt = np->fail->cnt:
         np->cnt++;
         return np:
} PAM;
```

8.4 Suffix Automaton

```
// par : fail link
// val : a topological order ( useful for DP )
// go[x] : automata edge ( x is integer in [0,26) )
struct SAM{
  struct State{
    int par, go[26], val;
    State () : par(0), val(0){ FZ(go); }
    State (int _val) : par(0), val(_val){ FZ(go); }
  vector<State> vec;
  int root, tail;
  void init(int arr[], int len){
    vec.resize(2);
    vec[0] = vec[1] = State(0);
    root = tail = 1;
    for (int i=0; i<len; i++)</pre>
      extend(arr[i]);
  void extend(int w){
    int p = tail, np = vec.size();
    vec.PB(State(vec[p].val+1));
    for ( ; p && vec[p].go[w]==0; p=vec[p].par)
      vec[p].go[w] = np;
    if (p == 0){
      vec[np].par = root;
    } else {
      if (vec[vec[p].go[w]].val == vec[p].val+1){
        vec[np].par = vec[p].go[w];
      } else {
        int q = vec[p].go[w], r = vec.size();
        vec.PB(vec[q]);
        vec[r].val = vec[p].val+1;
        vec[q].par = vec[np].par = r;
        for ( ; p && vec[p].go[w] == q; p=vec[p].par)
          vec[p].go[w] = r;
      }
    tail = np;
  }
};
```

8.5 smallest rotation

```
string mcp(string s){
   int n = s.length();
   s += s;
   int i=0, j=1;
   while (i<n && j<n){
      int k = 0;
      while (k < n && s[i+k] == s[j+k]) k++;
      if (s[i+k] <= s[j+k]) j += k+1;
      else i += k+1;
      if (i == j) j++;
   }
   int ans = i < n ? i : j;
   return s.substr(ans, n);
}
Contact GitHub API Training Shop Blog About</pre>
```

8.6 Suffix Array

```
/*he[i]保存了在後綴數組中相鄰兩個後綴的最長公共前綴長度
 *sa[i]表示的是字典序排名為i的後綴是誰(字典序越小的排
      名越靠前)
 *rk[i]表示的是後綴我所對應的排名是多少 */
const int MAX = 1020304;
int ct[MAX], he[MAX], rk[MAX];
int sa[MAX], tsa[MAX], tp[MAX][2];
void suffix_array(char *ip){
  int len = strlen(ip);
  int alp = 256;
  memset(ct, 0, sizeof(ct));
  for(int i=0;i<len;i++) ct[ip[i]+1]++;</pre>
  for(int i=1;i<alp;i++) ct[i]+=ct[i-1];</pre>
  for(int i=0;i<len;i++) rk[i]=ct[ip[i]];</pre>
  for(int i=1;i<len;i*=2){</pre>
    for(int j=0;j<len;j++){</pre>
      if(j+i>=len) tp[j][1]=0;
      else tp[j][1]=rk[j+i]+1;
      tp[j][0]=rk[j];
    memset(ct, 0, sizeof(ct));
    for(int j=0;j<len;j++) ct[tp[j][1]+1]++;</pre>
    for(int j=1;j<len+2;j++) ct[j]+=ct[j-1];</pre>
    for(int j=0;j<len;j++) tsa[ct[tp[j][1]]++]=j;</pre>
    memset(ct, 0, sizeof(ct));
    for(int j=0;j<len;j++) ct[tp[j][0]+1]++;</pre>
    for(int j=1;j<len+1;j++) ct[j]+=ct[j-1];</pre>
    for(int j=0;j<len;j++)</pre>
      sa[ct[tp[tsa[j]][0]]++]=tsa[j];
    rk[sa[0]]=0;
    for(int j=1;j<len;j++){</pre>
      if( tp[sa[j]][0] == tp[sa[j-1]][0] &&
        tp[sa[j]][1] == tp[sa[j-1]][1])
        rk[sa[j]] = rk[sa[j-1]];
      else
        rk[sa[j]] = j;
  for(int i=0,h=0;i<len;i++){</pre>
    if(rk[i]==0) h=0;
    else{
      int j=sa[rk[i]-1];
      h=max(0,h-1);
      for(;ip[i+h]==ip[j+h];h++);
    he[rk[i]]=h;
  }
}
```

8.7 Z-value

```
z[0] = 0;
for ( int bst = 0, i = 1; i < len ; i++ ) {</pre>
  if ( z[bst] + bst <= i ) z[i] = 0;</pre>
  else z[i] = min(z[i - bst], z[bst] + bst - i);
  while ( str[i + z[i]] == str[z[i]] ) z[i]++;
  if ( i + z[i] > bst + z[bst] ) bst = i;
// 回文版
void Zpal(const char *s, int len, int *z) {
    // Only odd palindrome len is considered
    // z[i] means that the longest odd palindrom
        centered at
    // i is [i-z[i] .. i+z[i]]
    z[0] = 0;
    for (int b=0, i=1; i<len; i++) {</pre>
        if (z[b] + b >= i) z[i] = min(z[2*b-i], b+z[b]-
            i);
        else z[i] = 0;
        while (i+z[i]+1 < len and i-z[i]-1 >= 0 and
               s[i+z[i]+1] == s[i-z[i]-1]) z[i] ++;
        if(z[i] + i > z[b] + b) b = i;
}
```

9 Others

9.1 矩陣數定理

```
新的方法介绍
下面我们介绍一种新的方法——Matrix-Tree定理(Kirchhoff矩
   阵-树定理)。
Matrix-Tree定理是解决生成树计数问题最有力的武器之一。它
   首先于1847年被Kirchhoff证明。在介绍定理之前,我们首
   先明确几个概念:
1、G的度数矩阵D[G]是一个n*n的矩阵,并且满足: 当i≠j时,
  dij=0;当i=j时,dij等于vi的度数。
2、G的邻接矩阵A[G]也是一个n*n的矩阵, 并且满足:如果vi
   、vi之间有边直接相连,则aii=1,否则为0。
我们定义G的Kirchhoff矩阵(也称为拉普拉斯算子)C[G]为C[G]=
  D[G]-A[G],
则Matrix-Tree定理可以描述为:G的所有不同的生成树的个数
   等于其Kirchhoff矩阵C[G]任何一个n-1阶主子式的行列式
   的绝对值。
所谓n-1阶主子式,就是对于r(1≤r≤n),将C[G]的第r行、第r列
   同时去掉后得到的新矩阵,用Cr[G]表示。
生成树计数
算法步骤:
1、 构建拉普拉斯矩阵
   Matrix[i][j] =
degree(i) , i==j
       -1, i-j有边
       0,其他情况
2、 去掉第r行,第r列(r任意)
3、 计算矩阵的行列式
MYID
     : Chen Fan
I ANG
     : G++
#include <stdio.h>
#include <string.h>
#include <algorithm>
#include <iostream>
#include <math.h>
using namespace std;
```

```
const double eps = 1e-8;
const int MAXN = 110;
int sgn(double x)
    if(fabs(x) < eps)return 0;</pre>
    if(x < 0) return -1;
    else return 1;
double b[MAXN][MAXN];
double det(double a[][MAXN],int n)
    int i, j, k, sign = 0;
    double ret = 1;
    for(i = 0;i < n;i++)</pre>
    for(j = 0;j < n;j++) b[i][j] = a[i][j];</pre>
    for(i = 0;i < n;i++)</pre>
         if(sgn(b[i][i]) == 0)
         {
             for(j = i + 1; j < n; j++)
             if(sgn(b[j][i]) != 0) break;
             if(j == n)return 0;
             for(k = i;k < n;k++) swap(b[i][k],b[j][k]);</pre>
             sign++;
         ret *= b[i][i];
         for(k = i + 1;k < n;k++) b[i][k]/=b[i][i];</pre>
         for(j = i+1; j < n; j++)
         for(k = i+1; k < n; k++) b[j][k] -= b[j][i]*b[i][
             k];
    if(sign & 1)ret = -ret;
    return ret;
double a[MAXN][MAXN];
int g[MAXN][MAXN];
int main()
    int T;
    int n,m;
    int u,v;
    scanf("%d",&T);
    while(T--)
         scanf("%d%d",&n,&m);
         memset(g,0,sizeof(g));
         while(m--)
         {
             scanf("%d%d",&u,&v);
             u--;v--;
             g[u][v] = g[v][u] = 1;
         memset(a,0,sizeof(a));
         for(int i = 0;i < n;i++)</pre>
         for(int j = 0; j < n; j++)</pre>
         if(i != j && g[i][j])
             a[i][i]++;
             a[i][j] = -1;
         double ans = det(a,n-1);
         printf("%.0lf \setminus n",ans);
    return 0;
}
```

9.2 CYK

```
// 2016 NCPC from sunmoon

// 轉換

#define MAXN 55

struct CNF{
  int s,x,y;//s->xy | s->x, if y==-1
```

```
int cost;
  CNF(){}
  CNF(int s,int x,int y,int c):s(s),x(x),y(y),cost(c){}
int state;//規則數量
map<char,int> rule;//每個字元對應到的規則,小寫字母為終
    端字符
vector<CNF> cnf;
inline void init(){
  state=0:
  rule.clear();
  cnf.clear();
inline void add_to_cnf(char s,const string &p,int cost)
  if(rule.find(s)==rule.end())rule[s]=state++;
  for(auto c:p)if(rule.find(c)==rule.end())rule[c]=
      state++;
  if(p.size()==1){
    cnf.push_back(CNF(rule[s],rule[p[0]],-1,cost));
  }else{
    int left=rule[s];
    int sz=p.size();
    for(int i=0;i<sz-2;++i){</pre>
      cnf.push_back(CNF(left,rule[p[i]],state,0));
      left=state++;
    cnf.push_back(CNF(left,rule[p[sz-2]],rule[p[sz-1]],
        cost));
  }
}
// 計算
vector<long long> dp[MAXN][MAXN];
vector<bool> neg_INF[MAXN][MAXN];//如果花費是負的可能會
    有無限小的情形
inline void relax(int 1,int r,const CNF &c,long long
    cost,bool neg_c=0){
  if(!neg_INF[1][r][c.s]&&(neg_INF[1][r][c.x]||cost<dp[</pre>
      1][r][c.s])){
    if(neg_c||neg_INF[1][r][c.x]){
      dp[1][r][c.s]=0;
      neg_INF[1][r][c.s]=true;
    }else dp[l][r][c.s]=cost;
inline void bellman(int l,int r,int n){
  for(int k=1;k<=state;++k)</pre>
    for(auto c:cnf)
      if(c.y==-1)relax(l,r,c,dp[l][r][c.x]+c.cost,k==n)
inline void cyk(const vector<int> &tok){
  for(int i=0;i<(int)tok.size();++i){</pre>
    for(int j=0;j<(int)tok.size();++j){</pre>
      dp[i][j]=vector<long long>(state+1,INT_MAX);
      neg_INF[i][j]=vector<bool>(state+1, false);
    dp[i][i][tok[i]]=0;
    bellman(i,i,tok.size());
  for(int r=1;r<(int)tok.size();++r){</pre>
    for(int l=r-1;l>=0;--1){
      for(int k=1;k<r;++k)</pre>
        for(auto c:cnf)
          if(~c.y)relax(1,r,c,dp[1][k][c.x]+dp[k+1][r][
              c.y]+c.cost);
      bellman(l,r,tok.size());
  }
}
```

```
9.3 數位統計
```

```
int dfs(int pos, int state1, int state2 ...., bool
    limit, bool zero) {
    if ( pos == -1 ) return 是否符合條件;
    int &ret = dp[pos][state1][state2][....];
    if ( ret != -1 && !limit ) return ret;
    int ans = 0;
    int upper = limit ? digit[pos] : 9;
    for ( int i = 0 ; i <= upper ; i++ ) {</pre>
        ans += dfs(pos - 1, new_state1, new_state2,
            limit & ( i == upper), ( i == 0) && zero);
    if (!limit ) ret = ans;
   return ans;
int solve(int n) {
   int it = 0;
    for ( ; n ; n /= 10 ) digit[it++] = n % 10;
    return dfs(it - 1, 0, 0, 1, 1);
```

9.4 1D/1D dp 優化

```
#include<bits/stdc++.h>
int t, n, L;
int p;
char s[MAXN][35];
ll sum[MAXN] = \{0\};
long double dp[MAXN] = {0};
int prevd[MAXN] = {0};
long double pw(long double a, int n) {
    if ( n == 1 ) return a;
    long double b = pw(a, n/2);
    if ( n & 1 ) return b*b*a;
    else return b*b;
long double f(int i, int j) {
      cout << `(sum[i] - sum[j]+i-j-1-L) << endl;
    return pw(abs(sum[i] - sum[j]+i-j-1-L), p) + dp[j];
}
struct INV {
    int L, R, pos;
INV stk[MAXN*10];
int top = 1, bot = 1;
void update(int i) {
    while ( top > bot && i < stk[top].L && f(stk[top].L</pre>
         , i) < f(stk[top].L, stk[top].pos) ) {</pre>
        stk[top - 1].R = stk[top].R;
        top--;
    int lo = stk[top].L, hi = stk[top].R, mid, pos =
        stk[top].pos;
    //if ( i >= lo ) lo = i + 1;
    while ( lo != hi ) {
        mid = lo + (hi - lo) / 2;
if ( f(mid, i) < f(mid, pos) ) hi = mid;
        else lo = mid + 1;
    if ( hi < stk[top].R ) {</pre>
        stk[top + 1] = (INV) { hi, stk[top].R, i };
        stk[top++].R = hi;
    }
int main() {
    cin >> t;
    while ( t-- ) {
        cin >> n >> L >> p;
        dp[0] = sum[0] = 0;
        for ( int i = 1 ; i <= n ; i++ ) {
             cin >> s[i];
             sum[i] = sum[i-1] + strlen(s[i]);
```

```
dp[i] = numeric_limits<long double>::max();
}
stk[top] = (INV) {1, n + 1, 0};
for ( int i = 1 ; i <= n ; i++ ) {
    if ( i >= stk[bot].R ) bot++;
    dp[i] = f(i, stk[bot].pos);
    update(i);
    cout << (ll) f(i, stk[bot].pos) << endl;
}
if ( dp[n] > 1e18 ) {
    cout << "Too hard to arrange" << endl;
} else {
    vector<PI> as;
    cout << (ll)dp[n] << endl;
}
return 0;
}</pre>
```

9.5 Theorm - DP optimization

```
Monotonicity & 1D/1D DP & 2D/1D DP
Definition xD/yD
1D/1D DP[j] = min(0 \le i < j) \{ DP[i] + w(i, j) \}; DP[0] = k
2D/1D DP[i][j] = min(i < k \le j) \{ DP[i][k - 1] + DP[k][j] \}
     + w(i, j); DP[i][i] = 0
Monotonicity
      C
a | w(a, c) w(a, d)
b \mid w(b, c) w(b, d)
Monge Condition
Concave(凹四邊形不等式): w(a, c) + w(b, d) >= w(a, d) +
     w(b, c)
Convex (凸四邊形不等式): w(a, c) + w(b, d) <= w(a, d) +
     w(b, c)
Totally Monotone
Concave(凹單調): w(a, c) <= w(b, d) ----> w(a, d) <= w
Convex (凸單調): w(a, c) >= w(b, d) ----> w(a, d) >= w
    (b, c)
1D/1D DP O(n^2) \rightarrow O(nlgn)
**CONSIDER THE TRANSITION POINT**
Solve 1D/1D Concave by Stack
Solve 1D/1D Convex by Deque
2D/1D Convex DP (Totally Monotone) O(n^3) -> O(n^2)
h(i, j - 1) \le h(i, j) \le h(i + 1, j)
```

9.6 Stable Marriage

```
// normal stable marriage problem
// input:
//3

//Albert Laura Nancy Marcy
//Brad Marcy Nancy Laura
//Chuck Laura Marcy Nancy
//Laura Chuck Albert Brad
//Marcy Albert Chuck Brad
//Nancy Brad Albert Chuck

#include<bits/stdc++.h>
using namespace std;
const int MAXN = 505;
int n;
int favor[MAXN][MAXN]; // favor[boy_id][rank] = girl_id
;
```

```
int order[MAXN][MAXN]; // order[girl_id][boy_id] = rank
int current[MAXN]; // current[boy_id] = rank; boy_id
    will pursue current[boy_id] girl.
int girl_current[MAXN]; // girl[girl_id] = boy_id;
void initialize() {
  for ( int i = 0 ; i < n ; i++ ) {</pre>
    current[i] = 0;
    girl_current[i] = n;
    order[i][n] = n;
  }
}
map<string, int> male, female;
string bname[MAXN], gname[MAXN];
int fit = 0;
void stable_marriage() {
  queue<int> que;
  for ( int i = 0 ; i < n ; i++ ) que.push(i);</pre>
  while ( !que.empty() ) {
    int boy_id = que.front();
    que.pop();
    int girl_id = favor[boy_id][current[boy_id]];
    current[boy_id] ++;
    if ( order[girl_id][boy_id] < order[girl_id][</pre>
         girl_current[girl_id]] ) {
       if ( girl_current[girl_id] < n ) que.push(</pre>
           girl_current[girl_id]); // if not the first
       girl_current[girl_id] = boy_id;
    } else {
      que.push(boy_id);
}
int main() {
  cin >> n:
  for ( int i = 0 ; i < n; i++ ) {</pre>
    string p, t;
    cin >> p;
    male[p] = i;
    bname[i] = p;
    for ( int j = 0 ; j < n ; j++ ) {</pre>
      cin >> t;
      if (!female.count(t)) {
         gname[fit] = t;
         female[t] = fit++;
       favor[i][j] = female[t];
  }
  for ( int i = 0 ; i < n ; i++ ) {</pre>
    string p, t;
    cin >> p;
    for ( int j = 0 ; j < n ; j++ ) {
      cin >> t;
       order[female[p]][male[t]] = j;
  initialize();
  stable_marriage();
  for ( int i = 0 ; i < n ; i++ ) {
  cout << bname[i] << " " << gname[favor[i][current[i]</pre>
         ] - 1]] << endl;
```

```
|}
```

9.7 Mo's algorithm

```
int 1 = 0, r = 0, nowAns = 0, BLOCK_SIZE, n, m;
int ans[];
struct QUE{
    int 1, r, id;
    friend bool operator < (QUE a, QUE b){</pre>
         if(a.1 / BLOCK_SIZE != b.1 / BLOCK_SIZE)
             return a.1 / BLOCK_SIZE < b.1 / BLOCK_SIZE;</pre>
         return a.r < b.r;</pre>
    }
}querys[];
inline void move(int pos, int sign) {
    // update nowAns
void solve() {
    BLOCK_SIZE = int(ceil(pow(n, 0.5)));
    sort(querys, querys + m);
    for (int i = 0; i < m; ++i) {</pre>
         const QUE &q = querys[i];
         while (1 > q.1) move(--1, 1);
         while (r < q.r) move(r++, 1);</pre>
         while (1 < q.1) move(1++, -1);</pre>
         while (r > q.r) move(--r, -1);
         ans[q.id] = nowAns;
    }
}
```

9.8 parser

```
#include <bits/stdc++.h>
using namespace std;
typedef long long T;
bool GG;
T Eval2(char *&end) {
    T Eval0(char *&);
    T res=0;
    if ( *end=='(' ){
        res = Eval0(++end);
        if (*(end++)==')') return res;
else { GG = true; return -1; }
    else if( isdigit(*end) ){
        return strtol(end, &end, 10);
       // 可改成 {strtol ,strtoll strtod}
    else { GG = true; return -1; }
T Evalx(char *&end){
    if(GG) return -1;
    T res = Eval2(end); if(GG) return -1;
    while (*end == '%'){
        end++;
        res = ( res % Eval2(end) );
        if(GG) return -1;
    return res;
T Eval1(char *&end) {
    if(GG) return -1;
    T res = Evalx(end); if(GG) return -1;
    while (*end=='*' || *end == '/'){
        end++:
        if(*(end-1) == '*')res = ( res * Evalx(end) );
        else if(*(end-1) == '/')res = ( res / Evalx(end
            ));
```

```
if(GG) return -1;
    return res;
}
T Eval12(char *&end){
    if(GG) return -1;
    T res=1;
    if(*end == '-'){
        end++;
        res = -1;
    res *= Evalx(end);
    while (*end=='*' || *end == '/'){
        end++;
        if(*(end-1) == '*')res = ( res * Evalx(end) );
        else if(*(end-1) == '/')res = ( res / Evalx(end
        if(GG) return -1;
    return res;
T Eval0(char *&end) {
    if(GG) return -1;
    T res;
    res = Eval12(end); if(GG) return -1;
    while (*end=='+' || *end == '-'){
        end++;
        if(*(end-1) == '+')res = (res + Eval1(end));
        else res = ( res - Eval1(end) );
        if(GG) return -1;
    return res;
}
T parse(char *s){
    GG = false;
    T res = Eval0(s);
    while(*s != '\0'){
   if(*s != ' ')GG = true;
        s++;
    return res;
int main() {
    char expr[3003];
    string str;
    int cnt = 0;
    while (getline (cin,str)){
        printf("case %d:\n",++cnt);
        strcpy(expr,str.c_str());
        T ans = parse(expr);
        if(GG) puts("syntactically incorrect\n");
        else printf("%lld\n\n", ans);
}
E0 = E1' (+-E1)*
E1 = Ex (/*Ex)*
Ex = E2 (\%E2)*
E2 = (E0) or R+
E1' = Ex'(/*Ex)* or -Ex(/*Ex)*
```

9.9 python 小抄

```
| #!/usr/bin/env python3
| # 帕斯卡三角形
| n = 10
| dp = [ [1 for j in range(n)] for i in range(n) ]
| for i in range(1,n):
```

```
for j in range(1,n):
        dp[i][j] = dp[i][j-1] + dp[i-1][j]
for i in range(n):
    print( '
             '.join( '{:5d}'.format(x) for x in dp[i] )
# EOF
while True:
        n, m = map(int, input().split())
    except:
        break
    print( min(n,m), max(n,m) )
# input a sequence of number
a = [ int(x) for x in input().split() ]
a.sort()
print( ''.join( str(x)+' ' for x in a ) )
# LCS
ncase = int( input() )
for _ in range(ncase):
   n, m = [int(x) for x in input().split()]
    a, b = "$"+input(), "$"+input()
    dp = [ [int(0) for j in range(m+1)] for i in range(
        n+1) ]
    for i in range(1,n+1):
        for j in range(1,m+1):
            dp[i][j] = max(dp[i-1][j],dp[i][j-1])
            if a[i]==b[j]:
                dp[i][j] = max(dp[i][j],dp[i-1][j-1]+1)
    for i in range(1,n+1):
        print(dp[i][1:])
    print('a=\{:s\}, b=\{:s\}, |LCS(a,b)|=\{:d\}'.format(a
        [1:],b[1:],dp[n][m]))
# Basic operator
a, b = 10, 20
a/b # 0.5
a//b # 0
a%b # 10
a**b # 10^20
# if, else if, else
if a==0:
    print('zero')
elif a>0:
   print('postive')
else:
   print('negative')
# stack
                # C++
stack = [3,4,5]
stack.append(6) # push()
stack.pop()
               # pop()
stack[-1]
                # top()
               # size() 0(1)
len(stack)
               # C++
# queue
from collections import deque
queue = deque([3,4,5])
queue.append(6) # push()
queue.popleft() # pop()
             # front()
queue[0]
                # size() 0(1)
len(queue)
```

10 Persistence