Contents

1 Basic

1.1 compile

```
# preset before coding
echo "cd ~/Desktop" >> ~/.bashrc
gedit -> preference -> tab width: 4

# Editor
gedit a.cpp

# Compile
g++ a.cpp -std=c++11

**All file will be compiled to a.out unless you use -o(not recommanded, just use a.out)**
# Run
./a.out

# Run with file input
./a.out < input.txt

# Run with file input and output
./a.out < input.txt > output.txt

# Python Run
python3 a.py < input.txt > output.txt

# Copy Paste In Ubuntu
* copy: ctrl+insert
* paste: shift+insert
```

1.2 default code

```
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef pair<int,int> pii;
#ifdef ONLINE_JUDGE
#define cerr if(false) cerr
#endif

int main(){
#ifndef ONLINE_JUDGE
  //freopen("input.txt","r",stdin);
  freopen("output.txt","w",stdout);
  freopen("debug.txt","w",stderr);
#else
  ios_base::sync_with_stdio(0);
  cin.tie(false);
#endif
}
```

1.3 debug list

```
模板要記得 init
priority_queue 要清空
把邊界條件都加入測資
邊界條件 (過程溢位, 題目數據範圍), 會不會爆 long long
是否讀錯題目, 想不到時可以自己讀一次題目
比較容易有問題的地方換人寫
注意公式有沒有推錯或抄錯
精度誤差 sqrt(大大的東西) + EPS
測試 %lld or %I64d
喇分 random_suffle 隨機演算法
```

2 Dark Code

2.1 IO optimization

```
*if output to much, consider put all output in array first, then output the array.
getchar() -> getchar_unlocked()
fread() -> fread_unlocked()
-------------------------
inline char readchar() {
  const int S = 1<<20; // buffer size
  static char buf[S], *p = buf, *q = buf;
  if(p == q && (q = (p=buf)+fread(buf,1,S,stdin)) == buf) return EOF;
  return *p++;
}

inline int nxtint() {
  // if readchar can't use, change readchar() to getchar()
  int x = 0;
  int c = readchar(), neg = false;
  if (c == EOF) return -1;
  while (('0' > c || c > '9') && c != '-' && c != EOF)
      c = readchar();
  if (c == '-')neg = true, c = readchar();
  while ('0' <= c && c <= '9') x = x * 10 + (c ^ '0'),
      c = readchar();
  if (neg) x = -x;
  return x;
}
```

3 Flow

3.1 Dinic

```
(a) Bounded Maxflow Construction:
1. add two node ss, tt
2. add_edge(ss, tt, INF)
3. for each edge u -> v with capacity [l, r]:
        add_edge(u, tt, l)
        add_edge(ss, v, l)
        add_edge(u, v, r-l)
4. see (b), check if it is possible.
5. answer is maxflow(ss, tt) + maxflow(s, t)
-------------------------------------------------------
(b) Bounded Possible Flow:
1. same construction method as (a)
2. run maxflow(ss, tt)
3. for every edge connected with ss or tt:
        rule: check if their rest flow is exactly 0
4. answer is possible if every edge do satisfy the rule
    ;
5. otherwise, it is NOT possible.
-------------------------------------------------------
(c) Bounded Minimum Flow:
1. same construction method as (a)
2. answer is maxflow(ss, tt)
-------------------------------------------------------
(d) Bounded Minimum Cost Flow:
* the concept is somewhat like bounded possible flow.
1. same construction method as (a)
2. answer is maxflow(ss, tt) + (∑ l * cost for every
    edge)
-------------------------------------------------------
(e) Minimum Cut:
1. run maxflow(s, t)
2. run cut(s)
3. ss[i] = 1: node i is at the same side with s.
-------------------------------------------------------

const long long INF = 1LL<<60;
struct Dinic {  //O(VVE), with minimum cut
    static const int MAXN = 5003;
    struct Edge{
        int u, v;
        long long cap, rest;
    };

    int n, m, s, t, d[MAXN], cur[MAXN];
    vector<Edge> edges;
    vector<int> G[MAXN];

    void init(){
        edges.clear();
        for ( int i = 0 ; i < MAXN ; i++ ) G[i].clear()
            ;
    }

    // min cut start
    bool side[MAXN];
    void cut(int u) {
        side[u] = 1;
        for ( int i : G[u] ) {
            if ( !side[ edges[i].v ] && edges[i].rest )
                 cut(edges[i].v);
        }
    }
    // min cut end

    void add_edge(int u, int v, long long cap){
        edges.push_back( {u, v, cap, cap} );
        edges.push_back( {v, u, 0, 0LL} );
        m = edges.size();
        G[u].push_back(m-2);
        G[v].push_back(m-1);
    }

    bool bfs(){
        memset(d, -1, sizeof(d));
        queue<int> que;
        que.push(s); d[s]=0;
        while (!que.empty()){
            int u = que.front(); que.pop();
            for (int ei : G[u]){
                Edge &e = edges[ei];
                if (d[e.v] < 0 && e.rest > 0){
                    d[e.v] = d[u] + 1;
                    que.push(e.v);
                }
            }
        }
        return d[t] >= 0;
    }

    long long dfs(int u, long long a){
        if ( u == t || a == 0 ) return a;
        long long flow = 0, f;
        for ( int &i=cur[u]; i < (int)G[u].size() ; i++
             ) {
            Edge &e = edges[ G[u][i] ];
            if ( d[u] + 1 != d[e.v] ) continue;
            f = dfs(e.v, min(a, e.rest) );
            if ( f > 0 ) {
                e.rest -= f;
                edges[ G[u][i]^1 ].rest += f;
                flow += f;
                a -= f;
                if ( a == 0 )break;
            }
        }
        return flow;
    }

    long long maxflow(int s, int t){
        this->s = s, this->t = t;
        long long flow = 0, mf;
        while ( bfs() ){
            memset(cur, 0, sizeof(cur));
            while ( (mf = dfs(s, INF)) ) flow += mf;
        }
        return flow;
    }
} dinic;
```

3.2 min cost flow

```
// long long version
typedef pair<long long, long long> pll;
struct CostFlow {
    static const int MAXN = 350;
    static const long long INF = 1LL<<60;
    struct Edge {
        int to, r;
        long long rest, c;
    };
    int n, pre[MAXN], preL[MAXN]; bool inq[MAXN];
    long long dis[MAXN], fl, cost;
    vector<Edge> G[MAXN];
    void init() {
        for ( int i = 0 ; i < MAXN ; i++) G[i].clear();
    }
    void add_edge(int u, int v, long long rest, long
        long c) {
        G[u].push_back({v, (int)G[v].size()  , rest,  c
            });
        G[v].push_back({u, (int)G[u].size()-1, 0, -c});
    }
    pll flow(int s, int t) {
        fl = cost = 0;
        while (true) {
            fill(dis, dis+MAXN, INF);
            fill(inq, inq+MAXN, 0);
            dis[s] = 0;
            queue<int> que;
            que.push(s);
            while ( !que.empty() ) {
                int u = que.front(); que.pop();
                inq[u] = 0;
                for ( int i = 0 ; i < (int)G[u].size()
                    ; i++) {
                    int v = G[u][i].to;
                    long long w = G[u][i].c;
                    if ( G[u][i].rest > 0 && dis[v] >
                        dis[u] + w) {
                        pre[v] = u; preL[v] = i;
                        dis[v] = dis[u] + w;
                        if (!inq[v]) {
                            inq[v] = 1;
                            que.push(v);
                        }
                    }
                }
            }

            if (dis[t] == INF) break;
            long long tf = INF;
            for (int v = t, u, l ; v != s ; v = u ) {
                u = pre[v]; l = preL[v];
                tf = min(tf, G[u][l].rest);
            }
            for (int v = t, u, l ; v != s ; v = u ) {
                u = pre[v]; l = preL[v];
                G[u][l].rest -= tf;
                G[v][G[u][l].r].rest += tf;
            }
            cost += tf * dis[t];
            fl += tf;
        }
        return {fl, cost};
    }
} flow;
```

4 Mathmatics

4.1 ax+by=gcd(a,b)

```
typedef pair<int, int> pii;
pii extgcd(int a, int b){
  if(b == 0) return make_pair(1, 0);
  else{
    int p = a / b;
    pii q = extgcd(b, a % b);
    return make_pair(q.second, q.first - q.second * p);
  }
}
```

4.2 BigInt

```
struct Bigint{
  static const int LEN = 60;
  static const int BIGMOD = 10000;
  int s;
  int vl, v[LEN];
  //  vector<int> v;
  Bigint() : s(1) { vl = 0; }
  Bigint(long long a) {
    s = 1; vl = 0;
    if (a < 0) { s = -1; a = -a; }
    while (a) {
      push_back(a % BIGMOD);
      a /= BIGMOD;
    }
  }
  Bigint(string str) {
    s = 1; vl = 0;
    int stPos = 0, num = 0;
    if (!str.empty() && str[0] == '-') {
      stPos = 1;
      s = -1;
    }
    for (int i=SZ(str)-1, q=1; i>=stPos; i--) {
      num += (str[i] - '0') * q;
      if ((q *= 10) >= BIGMOD) {
        push_back(num);
        num = 0; q = 1;
      }
    }
    if (num) push_back(num);
  }
  int len() const { return vl; /* return SZ(v); */ }
  bool empty() const { return len() == 0; }
  void push_back(int x) { v[vl++] = x; /* v.PB(x); */ }
  void pop_back() { vl--; /* v.pop_back(); */ }
  int back() const { return v[vl-1]; /* return v.back()
      ; */ }
  void n() { while (!empty() && !back()) pop_back(); }
  void resize(int nl) {
    vl = nl; fill(v, v+vl, 0);
    //    v.resize(nl); // fill(ALL(v), 0);
  }
  void print() const {
    if (empty()) { putchar('0'); return; }
    if (s == -1) putchar('-');
    printf("%d", back());
    for (int i=len()-2; i>=0; i--) printf("%.4d",v[i]);
  }
  friend std::ostream& operator << (std::ostream& out,
      const Bigint &a) {
    if (a.empty()) { out << "0"; return out; }
    if (a.s == -1) out << "-";
    out << a.back();
    for (int i=a.len()-2; i>=0; i--) {
      char str[10];
      snprintf(str, 5, "%.4d", a.v[i]);
      out << str;
    }
    return out;
  }
  int cp3(const Bigint &b)const {
    if (s != b.s) return s > b.s ? 1 : -1;
    if (s == -1) return -(-*this).cp3(-b);
    if (len() != b.len()) return len()>b.len()?1:-1;
    for (int i=len()-1; i>=0; i--)
      if (v[i]!=b.v[i]) return v[i]>b.v[i]?1:-1;
    return 0;
  }
  bool operator < (const Bigint &b)const{ return cp3(b)
      ==-1; }
  bool operator <= (const Bigint &b)const{ return cp3(b
      )<=0; }
  bool operator >= (const Bigint &b)const{ return cp3(b
      )>=0; }
  bool operator == (const Bigint &b)const{ return cp3(b
      )==0; }
  bool operator != (const Bigint &b)const{ return cp3(b
      )!=0; }
  bool operator > (const Bigint &b)const{ return cp3(b)
      ==1; }
  Bigint operator - () const {
    Bigint r = (*this);
    r.s = -r.s;
    return r;
  }
  Bigint operator + (const Bigint &b) const {
    if (s == -1) return -(-(*this)+(-b));
    if (b.s == -1) return (*this)-(-b);
    Bigint r;
    int nl = max(len(), b.len());
    r.resize(nl + 1);
    for (int i=0; i<nl; i++) {
      if (i < len()) r.v[i] += v[i];
      if (i < b.len()) r.v[i] += b.v[i];
      if(r.v[i] >= BIGMOD) {
        r.v[i+1] += r.v[i] / BIGMOD;
        r.v[i] %= BIGMOD;
      }
    }
    r.n();
    return r;
  }
  Bigint operator - (const Bigint &b) const {
    if (s == -1) return -(-(*this)-(-b));
    if (b.s == -1) return (*this)+(-b);
    if ((*this) < b) return -(b-(*this));
    Bigint r;
    r.resize(len());
    for (int i=0; i<len(); i++) {
      r.v[i] += v[i];
      if (i < b.len()) r.v[i] -= b.v[i];
      if (r.v[i] < 0) {
        r.v[i] += BIGMOD;
        r.v[i+1]--;
      }
    }
    r.n();
    return r;
  }
  Bigint operator * (const Bigint &b) {
    Bigint r;
    r.resize(len() + b.len() + 1);
    r.s = s * b.s;
    for (int i=0; i<len(); i++) {
      for (int j=0; j<b.len(); j++) {
        r.v[i+j] += v[i] * b.v[j];
        if(r.v[i+j] >= BIGMOD) {
          r.v[i+j+1] += r.v[i+j] / BIGMOD;
          r.v[i+j] %= BIGMOD;
        }
      }
    }
    r.n();
    return r;
  }
  Bigint operator / (const Bigint &b) {
    Bigint r;
    r.resize(max(1, len()-b.len()+1));
    int oriS = s;
    Bigint b2 = b; // b2 = abs(b)
    s = b2.s = r.s = 1;
    for (int i=r.len()-1; i>=0; i--) {
      int d=0, u=BIGMOD-1;
      while(d<u) {
        int m = (d+u+1)>>1;
        r.v[i] = m;
        if((r*b2) > (*this)) u = m-1;
        else d = m;
      }
      r.v[i] = d;
    }
    s = oriS;
    r.s = s * b.s;
    r.n();
    return r;
  }
  Bigint operator % (const Bigint &b) {
    return (*this)-(*this)/b*b;
  }
```

```
};
```

4.3 FFT

```
const double pi = atan(1.0)*4;
struct Complex {
    double x,y;
    Complex(double _x=0,double _y=0)
        :x(_x),y(_y) {}
    Complex operator + (Complex &tt) { return Complex(x
        +tt.x,y+tt.y); }
    Complex operator - (Complex &tt) { return Complex(x
        -tt.x,y-tt.y); }
    Complex operator * (Complex &tt) { return Complex(x
        *tt.x-y*tt.y,x*tt.y+y*tt.x); }
};
void fft(Complex *a, int n, int rev) {
    // n是大于等于相乘的两个数组长度的2的幂次
    // 从0开始表示长度，对a进行操作
    // rev==1进行DFT，==-1进行IDFT
    for (int i = 1,j = 0; i < n; ++ i) {
        for (int k = n>>1; k > (j^=k); k >>= 1);
        if (i<j) std::swap(a[i],a[j]);
    }
    for (int m = 2; m <= n; m <<= 1) {
        Complex wm(cos(2*pi*rev/m),sin(2*pi*rev/m));
        for (int i = 0; i < n; i += m) {
            Complex w(1.0,0.0);
            for (int j = i; j < i+m/2; ++ j) {
                Complex t = w*a[j+m/2];
                a[j+m/2] = a[j] - t;
                a[j] = a[j] + t;
                w = w * wm;
            }
        }
    }
    if (rev==-1) {
        for (int i = 0; i < n; ++ i) a[i].x /= n,a[i].y
             /= n;
    }
}
```

4.4 FWHT

```
// FWHT template

const int MAXN = 1<<20;

void FWHT(int a[], int l=0, int r=MAXN-1){
  if (l==r)return;

  int mid = (l+r)>>1+1, n = r-l+1;
  FWHT(a,l,mid-1);
  FWHT(a,mid,r);

  for (int i=0; i<(n>>1); i++){
    int a1=a[l+i], a2=a[mid+i];
    a[l+i] = a1+a2;
    a[mid+i] = a1-a2;
  }
}
```

4.5 GaussElimination

```
// by bcw_codebook

const int MAXN = 300;
const double EPS = 1e-8;

int n;
double A[MAXN][MAXN];
```

```
void Gauss() {
  for(int i = 0; i < n; i++) {
    bool ok = 0;
    for(int j = i; j < n; j++) {
      if(fabs(A[j][i]) > EPS) {
        swap(A[j], A[i]);
        ok = 1;
        break;
      }
    }
    if(!ok) continue;

    double fs = A[i][i];
    for(int j = i+1; j < n; j++) {
      double r = A[j][i] / fs;
      for(int k = i; k < n; k++) {
        A[j][k] -= A[i][k] * r;
      }
    }
  }
}
```

4.6 Inverse

```
int inverse[100000];
void invTable(int b, int p) {
  inverse[1] = 1;
  for( int i = 2; i <= b; i++ ) {
    inverse[i] = (long long)inverse[p%i] * (p-p/i) % p;
  }
}

int inv(int b, int p) {
  return b == 1 ? 1 : ((long long)inv(p % b, p) * (p-p/
      b) % p);
}
```

4.7 LinearPrime

```
const int MAXP = 100; //max prime
vector<int> P;  // primes
void build_prime(){
  static bitset<MAXP> ok;
  int np=0;
  for (int i=2; i<MAXP; i++){
    if (ok[i]==0)P.push_back(i), np++;
    for (int j=0; j<np && i*P[j]<MAXP; j++){
      ok[ i*P[j] ] = 1;
      if ( i%P[j]==0 )break;
    }
  }
}
```

4.8 Miller Rabin

```
typedef long long LL;

inline LL bin_mul(LL a, LL n,const LL& MOD){
  LL re=0;
  while (n>0){
    if (n&1) re += a;
    a += a; if (a>=MOD) a-=MOD;
    n>>=1;
  }
  return re%MOD;
}

inline LL bin_pow(LL a, LL n,const LL& MOD){
  LL re=1;
  while (n>0){
    if (n&1) re = bin_mul(re,a,MOD);
    a = bin_mul(a,a,MOD);
    n>>=1;
  }
  return re;
}

bool is_prime(LL n){
  //static LL sprp[3] = { 2LL, 7LL, 61LL};
  static LL sprp[7] = { 2LL, 325LL, 9375LL,
    28178LL, 450775LL, 9780504LL,
    1795265022LL };
  if (n==1 || (n&1)==0 ) return n==2;
  int u=n-1, t=0;
  while ( (u&1)==0 ) u>>=1, t++;
  for (int i=0; i<3; i++){
    LL x = bin_pow( sprp[i]%n, u, n);
    if (x==0 || x==1 || x==n-1)continue;

    for (int j=1; j<t; j++){
      x=x*x%n;
      if (x==1 || x==n-1)break;
    }
    if (x==n-1)continue;
    return 0;
  }
  return 1;
}
```

4.9 Pollard’s rho

```
// from PEC
// does not work when n is prime
Int f(Int x, Int mod){
  return add(mul(x, x, mod), 1, mod);
}
Int pollard_rho(Int n) {
  if ( !(n & 1) ) return 2;
  while (true) {
    Int y = 2, x = rand()%(n-1) + 1, res = 1;
    for ( int sz = 2 ; res == 1 ; sz *= 2 ) {
      for ( int i = 0 ; i < sz && res <= 1 ; i++) {
        x = f(x, n);
        res = __gcd(abs(x-y), n);
      }
      y = x;
    }
    if ( res != 0 && res != n ) return res;
  }
}
```

4.10 數論基本工具

```
Int POW(Int a, Int n, Int mod){
  Int re=1;
  while (n>0){
    if (n&1LL) re = re*a%mod;
    a = a*a%mod;
    n>>=1;
  }
  return re;
}

Int C(Int n, Int m){
  if (m<0 || m>n)return 0;
  return J[n] * inv(J[m]*J[n-m]%MOD) %MOD;
}
```

4.11 Mobius

```
void mobius() {
    fill(isPrime, isPrime + MAXN, 1);
    mu[1] = 1, num = 0;
    for (int i = 2; i < MAXN; ++i) {
        if (isPrime[i]) primes[num++] = i, mu[i] = -1;
        static int d;
        for (int j = 0; j < num && (d = i * primes[j])
            < MAXN; ++j) {
            isPrime[d] = false;
            if (i % primes[j] == 0) {
                mu[d] = 0; break;
            } else mu[d] = -mu[i];
        }
    }
}
```

4.12 Simplex

```
// Two-phase simplex algorithm for solving linear
    programs of the form
//
//     maximize     c^T x
//     subject to   Ax <= b
//                  x >= 0
//
// INPUT: A -- an m x n matrix
//        b -- an m-dimensional vector
//        c -- an n-dimensional vector
//        x -- a vector where the optimal solution will
     be stored
//
// OUTPUT: value of the optimal solution (infinity if
    unbounded
//         above, nan if infeasible)
//
// To use this code, create an LPSolver object with A,
    b, and c as
// arguments.  Then, call Solve(x).

#include <iostream>
#include <iomanip>
#include <vector>
#include <cmath>
#include <limits>

using namespace std;

typedef long double DOUBLE;
typedef vector<DOUBLE> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;

const DOUBLE EPS = 1e-9;

struct LPSolver {
  int m, n;
  VI B, N;
  VVD D;

  LPSolver(const VVD &A, const VD &b, const VD &c) :
    m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2,
        VD(n + 2)) {
    for (int i = 0; i < m; i++) for (int j = 0; j < n;
        j++) D[i][j] = A[i][j];
    for (int i = 0; i < m; i++) { B[i] = n + i; D[i][n]
         = -1; D[i][n + 1] = b[i]; }
    for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -
        c[j]; }
    N[n] = -1; D[m + 1][n] = 1;
  }

  void Pivot(int r, int s) {
    double inv = 1.0 / D[r][s];
    for (int i = 0; i < m + 2; i++) if (i != r)
      for (int j = 0; j < n + 2; j++) if (j != s)
        D[i][j] -= D[r][j] * D[i][s] * inv;
    for (int j = 0; j < n + 2; j++) if (j != s) D[r][j]
         *= inv;
    for (int i = 0; i < m + 2; i++) if (i != r) D[i][s]
         *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
  }

  bool Simplex(int phase) {
    int x = phase == 1 ? m + 1 : m;
    while (true) {
      int s = -1;
      for (int j = 0; j <= n; j++) {
        if (phase == 2 && N[j] == -1) continue;
        if (s == -1 || D[x][j] < D[x][s] || D[x][j] ==
            D[x][s] && N[j] < N[s]) s = j;
      }
      if (D[x][s] > -EPS) return true;
      int r = -1;
      for (int i = 0; i < m; i++) {
        if (D[i][s] < EPS) continue;
        if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n +
             1] / D[r][s] ||
          (D[i][n + 1] / D[i][s]) == (D[r][n + 1] / D[r
              ][s]) && B[i] < B[r]) r = i;
      }
      if (r == -1) return false;
      Pivot(r, s);
    }
  }

  DOUBLE Solve(VD &x) {
    int r = 0;
    for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r][
        n + 1]) r = i;
    if (D[r][n + 1] < -EPS) {
      Pivot(r, n);
      if (!Simplex(1) || D[m + 1][n + 1] < -EPS) return
           -numeric_limits<DOUBLE>::infinity();
      for (int i = 0; i < m; i++) if (B[i] == -1) {
        int s = -1;
        for (int j = 0; j <= n; j++)
          if (s == -1 || D[i][j] < D[i][s] || D[i][j]
              == D[i][s] && N[j] < N[s]) s = j;
        Pivot(i, s);
      }
    }
    if (!Simplex(2)) return numeric_limits<DOUBLE>::
        infinity();
    x = VD(n);
    for (int i = 0; i < m; i++) if (B[i] < n) x[B[i]] =
         D[i][n + 1];
    return D[m][n + 1];
  }
};

int main() {

  const int m = 4;
  const int n = 3;
  DOUBLE _A[m][n] = {
    { 6, -1, 0 },
    { -1, -5, 0 },
    { 1, 5, 1 },
    { -1, -5, -1 }
  };
  DOUBLE _b[m] = { 10, -4, 5, -5 };
  DOUBLE _c[n] = { 1, -1, 0 };

  VVD A(m);
  VD b(_b, _b + m);
  VD c(_c, _c + n);
  for (int i = 0; i < m; i++) A[i] = VD(_A[i], _A[i] +
      n);

  LPSolver solver(A, b, c);
  VD x;
  DOUBLE value = solver.Solve(x);

  cerr << "VALUE: " << value << endl; // VALUE: 1.29032
  cerr << "SOLUTION:"; // SOLUTION: 1.74194 0.451613 1
  for (size_t i = 0; i < x.size(); i++) cerr << " " << x[i];
  cerr << endl;
  return 0;
}
```

4.13 SG

```
Anti Nim (取走最後一個石子者敗)

先手必勝 if and only if
1. 「所有」堆的石子數都為 1 且遊戲的 SG 值為 0。
2. 「有些」堆的石子數大於 1 且遊戲的 SG 值不為 0。

-------------------------------------------------------
Anti-SG (決策集合為空的遊戲者贏)

定義 SG 值為 0 時，遊戲結束，
則先手必勝 if and only if
1. 遊戲中沒有單一遊戲的 SG 函數大於 1 且遊戲的 SG 函數為 0。
2. 遊戲中某個單一遊戲的 SG 函數大於 1 且遊戲的 SG 函數不為 0。

-------------------------------------------------------
Sprague-Grundy

1. 雙人、回合制
2. 資訊完全公開
3. 無隨機因素
4. 可在有限步內結束
5. 沒有和局
6. 雙方可採取的行動相同

SG(S) 的值為 0：後手(P)必勝
不為 0：先手(N)必勝

int mex(set S) {
  // find the min number >= 0 that not in the S
  // e.g. S = {0, 1, 3, 4} mex(S) = 2
}

state = []
int SG(A) {
  if (A not in state) {
    S = sub_states(A)
    if( len(S) > 1 ) state[A] = reduce(operator.xor, [SG(B) for B in S])
    else state[A] = mex(set(SG(B) for B in next_states(A)))
  }
  return state[A]
}
```

4.14 Theorem

```
/*
Lucas's Theorem
  For non-negative integer n,m and prime P,
  C(m,n) mod P = C(m/M,n/M) * C(m%M,n%M) mod P
  = mult_i ( C(m_i,n_i) )
  where m_i is the i-th digit of m in base P.
-------------------------------------------------------
Pick's Theorem
  A = i + b/2 - 1
-------------------------------------------------------
Kirchhoff's theorem
  A_{ii} = deg(i), A_{ij} = (i,j) \in E ? -1 : 0
  Deleting any one row, one column, and cal the det(A)
-------------------------------------------------------
Nth Catalan recursive function:
C_0 = 1, C_{n+1} = C_n * 2(2n + 1)/(n+2)
-------------------------------------------------------
Mobius Formula
u(n) = 1          , if n = 1
       (-1)^m     , 若 n 無平方數因數，且 n = p1*p2*p3*...*pk
       0          , 若 n 有大於 1 的平方數因數
- Property
1. (積性函數) u(a)u(b) = u(ab)
2. ∑_{d|n} u(d) = [n == 1]
-------------------------------------------------------
Mobius Inversion Formula
if      f(n) = ∑_{d|n} g(d)
then    g(n) = ∑_{d|n} u(n/d)f(d)
             = ∑_{d|n} u(d)f(n/d)
- Application
the number/power of gcd(i, j) = k
- Trick
分塊, O(sqrt(n))
-------------------------------------------------------
Chinese Remainder Theorem (m_i 兩兩互質)

  x = a_1 (mod m_1)
  x = a_2 (mod m_2)
  ....
  x = a_i (mod m_i)

construct a solution:

  Let M = m_1 * m_2 * m_3 * ... * m_n
  Let M_i = M / m_i

  t_i = 1 / M_i
  t_i * M_i = 1 (mod m_i)

  solution x = a_1 * t_1 * M_1 + a_2 * t_2 * M_2 + ... + a_n * t_n * M_n + k * M
  = k*M + ∑ a_i * t_i * M_i, k is positive integer.

  under mod M, there is one solution x = ∑ a_i * t_i * M_i
-------------------------------------------------------
Burnside's lemma
|G| * |X/G|  = sum( |X^g| ) where g in G
總方法數: 每一種旋轉下不動點的個數總和 除以 旋轉的方法數
*/
```

5 Graph

5.1 BCC

```
邊雙連通

任意兩點間至少有兩條不重疊的路徑連接，找法：
1. 標記出所有的橋
2. 對全圖進行 DFS，不走橋，每一次 DFS 就是一個新的邊雙連通

// from BCW

struct BccEdge {
  static const int MXN = 100005;
  struct Edge { int v,eid; };
  int n,m,step,par[MXN],dfn[MXN],low[MXN];
  vector<Edge> E[MXN];
  DisjointSet djs;
  void init(int _n) {
    n = _n; m = 0;
    for (int i=0; i<n; i++) E[i].clear();
    djs.init(n);
  }
  void add_edge(int u, int v) {
    E[u].PB({v, m});
    E[v].PB({u, m});
    m++;
  }
  void DFS(int u, int f, int f_eid) {
    par[u] = f;
    dfn[u] = low[u] = step++;
    for (auto it:E[u]) {
      if (it.eid == f_eid) continue;
      int v = it.v;
      if (dfn[v] == -1) {
        DFS(v, u, it.eid);
        low[u] = min(low[u], low[v]);
      } else {
        low[u] = min(low[u], dfn[v]);
      }
    }
  }
  void solve() {
    step = 0;
    memset(dfn, -1, sizeof(int)*n);
    for (int i=0; i<n; i++) {
      if (dfn[i] == -1) DFS(i, i, -1);
    }
    djs.init(n);
    for (int i=0; i<n; i++) {
      if (low[i] < dfn[i]) djs.uni(i, par[i]);
    }
  }
}graph;
```

5.2 Dijkstra

```
typedef struct Edge{
    int v; long long len;
    bool operator > (const Edge &b)const { return len>b
        .len; }
} State;

const long long INF = 1LL<<60;

void Dijkstra(int n, vector<Edge> G[], long long d[],
    int s, int t=-1){
    static priority_queue<State, vector<State>, greater
        <State> > pq;
    while ( pq.size() )pq.pop();
    for (int i=1; i<=n; i++)d[i]=INF;
    d[s]=0; pq.push( (State){s,d[s]} );
    while ( pq.size() ){
        auto x = pq.top(); pq.pop();
        int u = x.v;
        if (d[u]<x.len)continue;
        if (u==t)return;
        for (auto &e:G[u]){
            if (d[e.v] > d[u]+e.len){
                d[e.v] = d[u]+e.len;
                pq.push( (State) {e.v,d[e.v]} );
            }
        }
    }
}
```

5.3 Strongly Connected Component(SCC)

```
#define MXN 100005
#define PB push_back
#define FZ(s) memset(s,0,sizeof(s))
struct Scc{
int n, nScc, vst[MXN], bln[MXN];
vector<int> E[MXN], rE[MXN], vec;
void init(int _n){
  n = _n;
  for (int i=0; i<MXN; i++){
    E[i].clear();
    rE[i].clear();
  }
}
void add_edge(int u, int v){
  E[u].PB(v);
  rE[v].PB(u);
}
void DFS(int u){
  vst[u]=1;
  for (auto v : E[u])
    if (!vst[v]) DFS(v);
  vec.PB(u);
}
void rDFS(int u){
  vst[u] = 1;
  bln[u] = nScc;
  for (auto v : rE[u])
    if (!vst[v]) rDFS(v);
}
void solve(){
  nScc = 0;
  vec.clear();
  FZ(vst);
  for (int i=0; i<n; i++)
    if (!vst[i]) DFS(i);
  reverse(vec.begin(),vec.end());
  FZ(vst);
  for (auto v : vec){
    if (!vst[v]){
      rDFS(v);
      nScc++;
    }
  }
}
};
```

5.4 Hungarian

```
// Maximum Cardinality Bipartite Matching

struct Graph {
    static const int MAXN = 5005;
    vector<int> G[MAXN];
    int n;
    int match[MAXN]; // Matching Result
    int vis[MAXN];

    void init(int _n) {
        n = _n;
        for ( int i = 0 ; i < n ; i++ ) G[i].clear();
    }

    bool dfs(int u) {
        for ( auto v:G[u] ) {
            if (!vis[v]) {
                vis[v] = true;
                if (match[v] == -1 || dfs(match[v])) {
                    match[v] = u;
                    match[u] = v;
                    return true;
                }
            }
        }
        return false;
    }

    int solve() {
        int res = 0;
        memset(match, -1, sizeof(match));
        for (int i = 0; i < n; i++) {
            if (match[i] == -1) {
                memset(vis, 0, sizeof(vis));
                if (dfs(i)) res += 1;
            }
        }
        return res;
    }
} graph;
```

5.5 KM

```
Detect non-perfect-matching:
1. set all edge[i][j] as INF
2. if solve() >= INF, it is not perfectmatching.
-------------------------------------------------------
// Maximum Weight Perfect Bipartite Matching
// allow negative weight!

typedef long long Int;
struct KM {
    static const int MAXN = 1050;
    static const int INF = 1LL<<60;
    int n, match[MAXN], vx[MAXN], vy[MAXN];
    Int edge[MAXN][MAXN], lx[MAXN], ly[MAXN], slack[
        MAXN];
    void init(int _n){
        n = _n;
        for ( int i = 0 ; i < n ; i++ )
            for ( int j = 0; j < n ; j++ )
                edge[i][j] = 0;
    }
    void add_edge(int x, int y, Int w){
        edge[x][y] = w;
    }
    bool DFS(int x){
        vx[x] = 1;
        for ( int y = 0 ; y < n ; y++ ) {
            if ( vy[y] ) continue;
            if ( lx[x] + ly[y] > edge[x][y] ) {
                slack[y] = min(slack[y], lx[x] + ly[y]
                    - edge[x][y]);
            } else {
                vy[y] = 1;
                if ( match[y] == -1 || DFS(match[y]) ){
                    match[y] = x;
                    return true;
                }
            }
        }
        return false;
    }
    Int solve() {
        fill(match, match + n, -1);
        fill(lx, lx + n, -INF);
        fill(ly, ly + n, 0);
        for ( int i = 0; i < n; i++ )
            for ( int j = 0; j < n; j++ )
                lx[i] = max(lx[i], edge[i][j]);
        for ( int i = 0 ; i < n; i++ ) {
            fill(slack, slack + n, INF);
            while (true){
                fill(vx, vx + n, 0);
                fill(vy, vy + n, 0);
                if ( DFS(i) ) break;
                Int d = INF;
                for ( int j = 0 ; j < n ; j++ )
                    if ( !vy[j] ) d = min(d, slack[j]);
                for ( int j = 0 ; j < n ; j++ ) {
                    if (vx[j]) lx[j] -= d;
                    if (vy[j]) ly[j] += d;
                    else slack[j] -= d;
                }
            }
        }
        Int res = 0;
        for ( int i = 0 ; i < n ; i++ ) {
            res += edge[ match[i] ][i];
        }
        return res;
    }
} graph;
```

5.6 最小平均環

```
// from BCW

/* minimum mean cycle */
const int MAXE = 1805;
const int MAXN = 35;
const double inf = 1029384756;
const double eps = 1e-6;
struct Edge {
  int v,u;
  double c;
};
int n,m,prv[MAXN][MAXN], prve[MAXN][MAXN], vst[MAXN];
Edge e[MAXE];
vector<int> edgeID, cycle, rho;
double d[MAXN][MAXN];
inline void bellman_ford() {
  for(int i=0; i<n; i++) d[0][i]=0;
  for(int i=0; i<n; i++) {
    fill(d[i+1], d[i+1]+n, inf);
    for(int j=0; j<m; j++) {
      int v = e[j].v, u = e[j].u;
      if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
        d[i+1][u] = d[i][v]+e[j].c;
        prv[i+1][u] = v;
        prve[i+1][u] = j;
      }
    }
  }
}
double karp_mmc() {
  // returns inf if no cycle, mmc otherwise
  double mmc=inf;
  int st = -1;
  bellman_ford();
  for(int i=0; i<n; i++) {
    double avg=-inf;
    for(int k=0; k<n; k++) {
      if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i])
          /(n-k));
      else avg=max(avg,inf);
    }
    if (avg < mmc) tie(mmc, st) = tie(avg, i);
  }
  for(int i=0; i<n; i++) vst[i] = 0;
  edgeID.clear(); cycle.clear(); rho.clear();
  for (int i=n; !vst[st]; st=prv[i--][st]) {
    vst[st]++;
    edgeID.PB(prve[i][st]);
    rho.PB(st);
  }
  while (vst[st] != 2) {
    int v = rho.back(); rho.pop_back();
    cycle.PB(v);
    vst[v]++;
  }
  reverse(ALL(edgeID));
  edgeID.resize(SZ(cycle));
  return mmc;
}
```

5.7 偵測負環

```
#include <bits/stdc++.h>
using namespace std;
```

```
const int INF = 1000000;
const int MAXN = 200;
int n, m, q;
int d[MAXN][MAXN];

int main () {
    while ( cin >> n >> m >> q && n) {

        for ( int i = 0 ; i <= n ; i++ ) {
            for ( int j = 0 ; j <= n ; j++ ) d[i][j] = (i==j ? 0 : INF);
        }

        for ( int i = 0 ; i < m ; i++ ) {
            int a, b, c;
            cin >> a >> b >> c;
            d[a][b] = min(d[a][b], c);
        }

        for ( int k = 0 ; k < n ; k++ ) {
            for ( int i = 0 ; i < n ; i++ ) {
                for ( int j = 0 ; j < n ; j++ ) {

                    if ( d[i][j] > d[i][k] + d[k][j] && d[i][k] < INF && d[k][j] < INF ) {
                        //printf("%d > %d + %d\n", d[i][j], d[i][k], d[k][j]);
                        //if ( d[i][k] >= INF || d[k][j] >= INF ) cout << "NO : " << i << " " << j << " " << k << "--";
                        d[i][j] = min(d[i][j], d[i][k] + d[k][j]);
                    }
                }
            }
        }

        for ( int i = 0 ; i < n ; i++ ) {
            for ( int j = 0 ; j < n ; j++ ) {
                for ( int k = 0 ; k < n && d[i][j] != -INF ; k++ ) {
                    if ( d[k][k] < 0 && d[i][k] != INF && d[k][j] != INF  )
                        d[i][j] = -INF;
                }
            }
        }
        int u, v;
        for (int i=0;i<q;i++){
            scanf("%d%d",&u,&v);

            if (d[u][v] == INF) printf("Impossible\n");
            else if (d[u][v] == -INF) printf("-Infinity\n");
            else printf("%d\n",d[u][v]);
        }
        puts("");
    }
    return 0;
}
```

5.8 Tarjan

```
割點
點 u 為割點 if and only if 滿足 1. or 2.
1. u 爲樹根，且 u 有多於一個子樹。
2. u 不爲樹根，且滿足存在 (u,v) 爲樹枝邊 (或稱父子邊，即 u 爲 v 在搜索樹中的父親)，使得 DFN(u) <= Low(v)。

-------------------------------------------------------
橋
一條無向邊 (u,v) 是橋 if and only if (u,v) 爲樹枝邊，且滿足 DFN(u) < Low(v)。
```

```
// 0 base
struct TarjanSCC{
  static const int MAXN = 1000006;
  int n, dfn[MAXN], low[MAXN], scc[MAXN], scn, count;
  vector<int> G[MAXN];
  stack<int> stk;
  bool ins[MAXN];

  void tarjan(int u){
    dfn[u] = low[u] = ++count;
    stk.push(u);
    ins[u] = true;

    for(auto v:G[u]){
      if(!dfn[v]){
        tarjan(v);
        low[u] = min(low[u], low[v]);
      }else if(ins[v]){
        low[u] = min(low[u], dfn[v]);
      }
    }

    if(dfn[u] == low[u]){
      int v;
      do {
      v = stk.top();
      stk.pop();
      scc[v] = scn;
      ins[v] = false;
      } while(v != u);
      scn++;
    }
  }

  void getSCC(){
    memset(dfn,0,sizeof(dfn));
    memset(low,0,sizeof(low));
    memset(ins,0,sizeof(ins));
    memset(scc,0,sizeof(scc));
    count = scn = 0;
    for(int i = 0 ; i < n ; i++ ){
      if(!dfn[i]) tarjan(i);
    }
  }

}SCC;
```

6 Data Structure

6.1 2D Range Tree

```
// remember sort x !!!!!
typedef int T;
const int LGN = 20;
const int MAXN = 100005;

struct Point{
    T x, y;
    friend bool operator < (Point a, Point b){
        return tie(a.x,a.y) < tie(b.x,b.y);
    }
};
struct TREE{
    Point pt;
    int toleft;
}tree[LGN][MAXN];
struct SEG{
    T mx, Mx;
    int sz;
    TREE *st;
}seg[MAXN*4];
```

```
vector<Point> P;

void build(int l, int r, int o, int deep){
    seg[o].mx = P[l].x;
    seg[o].Mx = P[r].x;
    seg[o].sz = r-l+1;;

    if(l == r){
        tree[deep][r].pt = P[r];
        tree[deep][r].toleft = 0;
        seg[o].st = &tree[deep][r];
        return;
    }
    int mid = (l+r)>>1;
    build(l,mid,o+o,deep+1);
    build(mid+1,r,o+o+1,deep+1);

    TREE *ptr = &tree[deep][l];
    TREE *pl = &tree[deep+1][l], *nl = &tree[deep+1][
        mid+1];
    TREE *pr = &tree[deep+1][mid+1], *nr = &tree[deep
        +1][r+1];

    int cnt = 0;
    while(pl != nl && pr != nr) {
        *(ptr) = pl->pt.y <= pr->pt.y ? cnt++, *(pl++):
             *(pr++);
        ptr -> toleft = cnt; ptr++;
    }
    while(pl != nl) *(ptr) = *(pl++), ptr -> toleft =
        ++cnt, ptr++;
    while(pr != nr) *(ptr) = *(pr++), ptr -> toleft =
        cnt, ptr++;

}
int main(){
    int n; cin >> n;
    for(int i = 0 ;i < n; i++){
        T x,y; cin >> x >> y;
        P.push_back((Point){x,y});
    }
    sort(P.begin(),P.end());
    build(0,n-1,1,0);
}
```

6.2 Sparse Table

```
const int MAXN = 200005;
const int lgN = 20;

struct SP{ //sparse table
  int Sp[MAXN][lgN];
  function<int(int,int)> opt;
  void build(int n, int *a){ // 0 base
    for (int i=0 ;i<n; i++) Sp[i][0]=a[i];

    for (int h=1; h<lgN; h++){
      int len = 1<<(h-1), i=0;
      for (; i+len<n; i++)
        Sp[i][h] = opt( Sp[i][h-1] , Sp[i+len][h-1] );
      for (; i<n; i++)
        Sp[i][h] = Sp[i][h-1];
    }
  }
  int query(int l, int r){
    int h = __lg(r-l+1);
    int len = 1<<h;
    return opt( Sp[l][h] , Sp[r-len+1][h] );
  }
};
```

7 String

7.1 AC 自動機

```
// remember make_fail() !!!
// notice MLE

const int sigma = 62;
const int MAXC = 200005;

inline int idx(char c){
    if ('A'<= c && c <= 'Z')return c-'A';
    if ('a'<= c && c <= 'z')return c-'a' + 26;
    if ('0'<= c && c <= '9')return c-'0' + 52;
}

struct ACautomaton{
    struct Node{
        Node *next[sigma], *fail;
        int cnt; // dp
        Node(){
            memset(next,0,sizeof(next));
            fail=0;
            cnt=0;
        }
    } buf[MAXC], *bufp, *ori, *root;

    void init(){
        bufp = buf;
        ori = new (bufp++) Node();
        root = new (bufp++) Node();
    }

    void insert(int n, char *s){
        Node *ptr = root;
        for (int i=0; s[i]; i++){
            int c = idx(s[i]);
            if (ptr->next[c]==NULL)
                ptr->next[c] = new (bufp++) Node();
            ptr = ptr->next[c];
        }
        ptr->cnt=1;
    }

    Node* trans(Node *o, int c){
        while (o->next[c]==NULL) o = o->fail;
        return o->next[c];
    }

    void make_fail(){
        static queue<Node*> que;

        for (int i=0; i<sigma; i++)
            ori->next[i] = root;
        root->fail = ori;

        que.push(root);
        while ( que.size() ){
            Node *u = que.front(); que.pop();
            for (int i=0; i<sigma; i++){
                if (u->next[i]==NULL)continue;
                u->next[i]->fail = trans(u->fail,i);
                que.push(u->next[i]);
            }
            u->cnt += u->fail->cnt;
        }
    }
} ac;
```

7.2 KMP

```
template<typename T>
void build_KMP(int n, T *s, int *f){ // 1 base
  f[0]=-1, f[1]=0;
  for (int i=2; i<=n; i++){
    int w = f[i-1];
    while (w>=0 && s[w+1]!=s[i])w = f[w];
    f[i]=w+1;
  }
}

template<typename T>
int KMP(int n, T *a, int m, T *b){
  build_KMP(m,b,f);
  int ans=0;

  for (int i=1, w=0; i<=n; i++){
    while ( w>=0 && b[w+1]!=a[i] )w = f[w];
    w++;
    if (w==m){
      ans++;
      w=f[w];
    }
  }
  return ans;
}
```

7.3 迴文字動機

```
// remember init()      !!!
// remember make_fail() !!!
// insert s need 1 base !!!
// notice MLE
const int sigma = 62;
const int MAXC = 1000006;
inline int idx(char c){
    if ('a'<= c && c <= 'z')return c-'a';
    if ('A'<= c && c <= 'Z')return c-'A'+26;
    if ('0'<= c && c <= '9')return c-'0'+52;
}
struct PalindromicTree{
    struct Node{
        Node *next[sigma], *fail;
        int len, cnt; // for dp
        Node(){
            memset(next,0,sizeof(next));
            fail=0;
            len = cnt = 0;
        }
    } buf[MAXC], *bufp, *even, *odd;

    void init(){
        bufp = buf;
        even = new (bufp++) Node();
        odd  = new (bufp++) Node();
        even->fail = odd;
        odd->len = -1;
    }

    void insert(char *s){
        Node* ptr = even;
        for (int i=1; s[i]; i++){
            ptr = extend(ptr,s+i);
        }
    }

    Node* extend(Node *o, char *ptr){
        int c = idx(*ptr);
        while ( *ptr != *(ptr-1-o->len) )o=o->fail;
        Node *&np = o->next[c];
        if (!np){
            np = new (bufp++) Node();
            np->len = o->len+2;
            Node *f = o->fail;
            if (f){
                while ( *ptr != *(ptr-1-f->len) )f=f->
                    fail;
                np->fail = f->next[c];
            }
            else {
                np->fail = even;
            }
            np->cnt = np->fail->cnt;
        }
        np->cnt++;
        return np;
    }
} PAM;
```

7.4 Suffix Automaton

```
// par : fail link
// val : a topological order ( useful for DP )
// go[x] : automata edge ( x is integer in [0,26) )

struct SAM{
  struct State{
    int par, go[26], val;
    State () : par(0), val(0){ FZ(go); }
    State (int _val) : par(0), val(_val){ FZ(go); }
  };
  vector<State> vec;
  int root, tail;

  void init(int arr[], int len){
    vec.resize(2);
    vec[0] = vec[1] = State(0);
    root = tail = 1;
    for (int i=0; i<len; i++)
      extend(arr[i]);
  }
  void extend(int w){
    int p = tail, np = vec.size();
    vec.PB(State(vec[p].val+1));
    for ( ; p && vec[p].go[w]==0; p=vec[p].par)
      vec[p].go[w] = np;
    if (p == 0){
      vec[np].par = root;
    } else {
      if (vec[vec[p].go[w]].val == vec[p].val+1){
        vec[np].par = vec[p].go[w];
      } else {
        int q = vec[p].go[w], r = vec.size();
        vec.PB(vec[q]);
        vec[r].val = vec[p].val+1;
        vec[q].par = vec[np].par = r;
        for ( ; p && vec[p].go[w] == q; p=vec[p].par)
          vec[p].go[w] = r;
      }
    }
    tail = np;
  }
};
```

7.5 smallest rotation

```
string mcp(string s){
  int n = s.length();
  s += s;
  int i=0, j=1;
  while (i<n && j<n){
    int k = 0;
    while (k < n && s[i+k] == s[j+k]) k++;
    if (s[i+k] <= s[j+k]) j += k+1;
    else i += k+1;
    if (i == j) j++;
  }
  int ans = i < n ? i : j;
  return s.substr(ans, n);
}
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```

7.6 Suffix Array

```
/*he[i]保存了在後綴數組中相鄰兩個後綴的最長公共前綴長度
 *sa[i]表示的是字典序排名為i的後綴是誰（字典序越小的排
     名越靠前）
 *rk[i]表示的是後綴我所對應的排名是多少  */

const int MAX = 1020304;
int ct[MAX], he[MAX], rk[MAX];
int sa[MAX], tsa[MAX], tp[MAX][2];
void suffix_array(char *ip){
  int len = strlen(ip);
  int alp = 256;
  memset(ct, 0, sizeof(ct));
  for(int i=0;i<len;i++) ct[ip[i]+1]++;
  for(int i=1;i<alp;i++) ct[i]+=ct[i-1];
  for(int i=0;i<len;i++) rk[i]=ct[ip[i]];
  for(int i=1;i<len;i*=2){
    for(int j=0;j<len;j++){
      if(j+i>=len) tp[j][1]=0;
      else tp[j][1]=rk[j+i]+1;
      tp[j][0]=rk[j];
    }
    memset(ct, 0, sizeof(ct));
    for(int j=0;j<len;j++) ct[tp[j][1]+1]++;
    for(int j=1;j<len+2;j++) ct[j]+=ct[j-1];
    for(int j=0;j<len;j++) tsa[ct[tp[j][1]]++]=j;
    memset(ct, 0, sizeof(ct));
    for(int j=0;j<len;j++) ct[tp[j][0]+1]++;
    for(int j=1;j<len+1;j++) ct[j]+=ct[j-1];
    for(int j=0;j<len;j++)
      sa[ct[tp[tsa[j]][0]]++]=tsa[j];
    rk[sa[0]]=0;
    for(int j=1;j<len;j++){
      if( tp[sa[j]][0] == tp[sa[j-1]][0] &&
        tp[sa[j]][1] == tp[sa[j-1]][1] )
        rk[sa[j]] = rk[sa[j-1]];
      else
        rk[sa[j]] = j;
    }
  }
  for(int i=0,h=0;i<len;i++){
    if(rk[i]==0) h=0;
    else{
      int j=sa[rk[i]-1];
      h=max(0,h-1);
      for(;ip[i+h]==ip[j+h];h++);
    }
    he[rk[i]]=h;
  }
}
```

7.7 Z-value

```
z[0] = 0;
for ( int bst = 0, i = 1; i < len ; i++ ) {
  if ( z[bst] + bst <= i ) z[i] = 0;
  else z[i] = min(z[i - bst], z[bst] + bst - i);
  while ( str[i + z[i]] == str[z[i]] ) z[i]++;
  if ( i + z[i] > bst + z[bst] ) bst = i;
}
```

```
// 回文版

void Zpal(const char *s, int len, int *z) {
    // Only odd palindrome len is considered
    // z[i] means that the longest odd palindrom
        centered at
    // i is [i-z[i] .. i+z[i]]
    z[0] = 0;
    for (int b=0, i=1; i<len; i++) {
        if (z[b] + b >= i) z[i] = min(z[2*b-i], b+z[b]-
            i);
        else z[i] = 0;
        while (i+z[i]+1 < len and i-z[i]-1 >= 0 and
               s[i+z[i]+1] == s[i-z[i]-1]) z[i] ++;
        if (z[i] + i > z[b] + b) b = i;
    }
}
```

8 Others

8.1 矩陣數定理

新的方法介绍

下面我们介绍一种新的方法——Matrix-Tree定理(Kirchhoff矩阵-树定理)。

Matrix-Tree定理是解决生成树计数问题最有力的武器之一。它首先于1847年被Kirchhoff证明。在介绍定理之前，我们首先明确几个概念：

1、G的度数矩阵D[G]是一个n*n的矩阵，并且满足：当i≠j时,dij=0；当i=j时，dij等于vi的度数。

2、G的邻接矩阵A[G]也是一个n*n的矩阵， 并且满足：如果vi、vj之间有边直接相连，则aij=1，否则为0。

我们定义G的Kirchhoff矩阵(也称为拉普拉斯算子)C[G]为C[G]=D[G]-A[G]，

则Matrix-Tree定理可以描述为：G的所有不同的生成树的个数等于其Kirchhoff矩阵C[G]任何一个n-1阶主子式的行列式的绝对值。

所谓n-1阶主子式，就是对于r(1≤r≤n)，将C[G]的第r行、第r列同时去掉后得到的新矩阵，用Cr[G]表示。

```
生成树计数
算法步骤：
1、 构建拉普拉斯矩阵
     Matrix[i][j] =
degree(i) , i==j
          -1，i-j有边
           0，其他情况
2、 去掉第r行，第r列 (r任意)
3、 计算矩阵的行列式
```

```
/* **********************************************
MYID    : Chen Fan
LANG    : G++
PROG    : Count_Spaning_Tree_From_Kuangbin
********************************************** */
#include <stdio.h>
#include <string.h>
#include <algorithm>
#include <iostream>
#include <math.h>
using namespace std;
const double eps = 1e-8;
const int MAXN = 110;
int sgn(double x)
{
    if(fabs(x) < eps)return 0;
    if(x < 0)return -1;
    else return 1;
}
double b[MAXN][MAXN];
double det(double a[][MAXN],int n)
{
    int i, j, k, sign = 0;
    double ret = 1;
    for(i = 0;i < n;i++)
    for(j = 0;j < n;j++) b[i][j] = a[i][j];
    for(i = 0;i < n;i++)
    {
        if(sgn(b[i][i]) == 0)
        {
            for(j = i + 1; j < n;j++)
```

```
      if(sgn(b[j][i]) != 0) break;
      if(j == n)return 0;
      for(k = i;k < n;k++) swap(b[i][k],b[j][k]);
      sign++;
    }
    ret *= b[i][i];
    for(k = i + 1;k < n;k++) b[i][k]/=b[i][i];
    for(j = i+1;j < n;j++)
    for(k = i+1;k < n;k++) b[j][k] -= b[j][i]*b[i][
      k];
  }
  if(sign & 1)ret = -ret;
  return ret;
}
double a[MAXN][MAXN];
int g[MAXN][MAXN];
int main()
{
  int T;
  int n,m;
  int u,v;
  scanf("%d",&T);
  while(T--)
  {
    scanf("%d%d",&n,&m);
    memset(g,0,sizeof(g));
    while(m--)
    {
      scanf("%d%d",&u,&v);
      u--;v--;
      g[u][v] = g[v][u] = 1;
    }
    memset(a,0,sizeof(a));
    for(int i = 0;i < n;i++)
    for(int j = 0;j < n;j++)
    if(i != j && g[i][j])
    {
      a[i][i]++;
      a[i][j] = -1;
    }
    double ans = det(a,n-1);
    printf("%.0lf\n",ans);
  }
  return 0;
}
```

8.2 CYK

```
// 2016 NCPC from sunmoon
```

```
// 轉換

#define MAXN 55
struct CNF{
 int s,x,y;//s->xy | s->x, if y==-1
 int cost;
 CNF(){}
 CNF(int s,int x,int y,int c):s(s),x(x),y(y),cost(c){}
};
int state;//規則數量
map<char,int> rule;//每個字元對應到的規則，小寫字母為終
  端字符
vector<CNF> cnf;
inline void init(){
 state=0;
 rule.clear();
 cnf.clear();
}
inline void add_to_cnf(char s,const string &p,int cost)
  {
 if(rule.find(s)==rule.end())rule[s]=state++;
 for(auto c:p)if(rule.find(c)==rule.end())rule[c]=
   state++;
 if(p.size()==1){
  cnf.push_back(CNF(rule[s],rule[p[0]],-1,cost));
 }else{
  int left=rule[s];
  int sz=p.size();
  for(int i=0;i<sz-2;++i){
   cnf.push_back(CNF(left,rule[p[i]],state,0));
   left=state++;
  }
  cnf.push_back(CNF(left,rule[p[sz-2]],rule[p[sz-1]],
    cost));
 }
}
```

```
// 計算

vector<long long> dp[MAXN][MAXN];
vector<bool> neg_INF[MAXN][MAXN];//如果花費是負的可能會
  有無限小的情形
inline void relax(int l,int r,const CNF &c,long long
  cost,bool neg_c=0){
 if(!neg_INF[l][r][c.s]&&(neg_INF[l][r][c.x]||cost<dp[
   l][r][c.s])){
  if(neg_c||neg_INF[l][r][c.x]){
   dp[l][r][c.s]=0;
   neg_INF[l][r][c.s]=true;
  }else dp[l][r][c.s]=cost;
 }
}
inline void bellman(int l,int r,int n){
 for(int k=1;k<=state;++k)
  for(auto c:cnf)
   if(c.y==-1)relax(l,r,c,dp[l][r][c.x]+c.cost,k==n)
     ;
}
inline void cyk(const vector<int> &tok){
 for(int i=0;i<(int)tok.size();++i){
  for(int j=0;j<(int)tok.size();++j){
   dp[i][j]=vector<long long>(state+1,INT_MAX);
   neg_INF[i][j]=vector<bool>(state+1,false);
  }
  dp[i][i][tok[i]]=0;
  bellman(i,i,tok.size());
 }
 for(int r=1;r<(int)tok.size();++r){
  for(int l=r-1;l>=0;--l){
   for(int k=l;k<r;++k)
    for(auto c:cnf)
     if(~c.y)relax(l,r,c,dp[l][k][c.x]+dp[k+1][r][
       c.y]+c.cost);
   bellman(l,r,tok.size());
  }
 }
}
```

8.3 數位統計

```
int dfs(int pos, int state1, int state2 ....., bool
  limit, bool zero) {
  if ( pos == -1 ) return 是否符合條件;
  int &ret = dp[pos][state1][state2][....];
  if ( ret != -1 && !limit ) return ret;
  int ans = 0;
  int upper = limit ? digit[pos] : 9;
  for ( int i = 0 ; i <= upper ; i++ ) {
    ans += dfs(pos - 1, new_state1, new_state2,
      limit & ( i == upper), ( i == 0) && zero);
  }
  if ( !limit ) ret = ans;
  return ans;
}

int solve(int n) {
  int it = 0;
  for ( ; n ; n /= 10 ) digit[it++] = n % 10;
  return dfs(it - 1, 0, 0, 1, 1);
}
```

8.4 1D/1D dp 優化

```
#include<bits/stdc++.h>

int t, n, L;
int p;
char s[MAXN][35];
ll sum[MAXN] = {0};
long double dp[MAXN] = {0};
int prevd[MAXN] = {0};

long double pw(long double a, int n) {
    if ( n == 1 ) return a;
    long double b = pw(a, n/2);
    if ( n & 1 ) return b*b*a;
    else return b*b;
}
long double f(int i, int j) {
//    cout << (sum[i] - sum[j]+i-j-1-L) << endl;
    return pw(abs(sum[i] - sum[j]+i-j-1-L), p) + dp[j];
}
struct INV {
    int L, R, pos;
};
INV stk[MAXN*10];
int top = 1, bot = 1;
void update(int i) {
    while ( top > bot && i < stk[top].L && f(stk[top].L
        , i) < f(stk[top].L, stk[top].pos) ) {
        stk[top - 1].R = stk[top].R;
        top--;
    }
    int lo = stk[top].L, hi = stk[top].R, mid, pos =
        stk[top].pos;
    //if ( i >= lo ) lo = i + 1;
    while ( lo != hi ) {
        mid = lo + (hi - lo) / 2;
        if ( f(mid, i) < f(mid, pos) ) hi = mid;
        else lo = mid + 1;
    }
    if ( hi < stk[top].R ) {
        stk[top + 1] = (INV) { hi, stk[top].R, i };
        stk[top++].R = hi;
    }
}

int main() {
    cin >> t;
    while ( t-- ) {
        cin >> n >> L >> p;
        dp[0] = sum[0] = 0;
        for ( int i = 1 ; i <= n ; i++ ) {
            cin >> s[i];
            sum[i] = sum[i-1] + strlen(s[i]);
            dp[i] = numeric_limits<long double>::max();
        }
        stk[top] = (INV) {1, n + 1, 0};
        for ( int i = 1 ; i <= n ; i++ ) {
            if ( i >= stk[bot].R ) bot++;
            dp[i] = f(i, stk[bot].pos);
            update(i);
//            cout << (ll) f(i, stk[bot].pos) << endl;
        }
        if ( dp[n] > 1e18 ) {
            cout << "Too hard to arrange" << endl;
        } else {
            vector<PI> as;
            cout << (ll)dp[n] << endl;
        }
    }
    return 0;
}
```

8.5 Theorm - DP optimization

```
Monotonicity & 1D/1D DP & 2D/1D DP
------------------------------------------------------
Definition xD/yD
1D/1D DP[j] = min(0≤i<j) { DP[i] + w(i, j) }; DP[0] = k
2D/1D DP[i][j] = min(i<k≤j) { DP[i][k - 1] + DP[k][j] }
     + w(i, j); DP[i][i] = 0
------------------------------------------------------
Monotonicity
       c       d
  ----------------
a | w(a, c) w(a, d)
b | w(b, c) w(b, d)

Monge Condition
Concave(凹四邊形不等式): w(a, c) + w(b, d) >= w(a, d) +
     w(b, c)
Convex (凸四邊形不等式): w(a, c) + w(b, d) <= w(a, d) +
     w(b, c)

Totally Monotone
Concave(凹單調): w(a, c) <= w(b, d) -----> w(a, d) <= w
    (b, c)
Convex (凸單調): w(a, c) >= w(b, d) -----> w(a, d) >= w
    (b, c)
------------------------------------------------------
1D/1D DP O(n^2) -> O(nlgn)
**CONSIDER THE TRANSITION POINT**
Solve 1D/1D Concave by Stack
Solve 1D/1D Convex by Deque
------------------------------------------------------
2D/1D Convex DP (Totally Monotone) O(n^3) -> O(n^2)
h(i, j − 1) ≤ h(i, j) ≤ h(i + 1, j)
```

8.6 Stable Marriage

```
// normal stable marriage problem
// input:
//3
//Albert Laura Nancy Marcy
//Brad Marcy Nancy Laura
//Chuck Laura Marcy Nancy
//Laura Chuck Albert Brad
//Marcy Albert Chuck Brad
//Nancy Brad Albert Chuck

#include<bits/stdc++.h>
using namespace std;
const int MAXN = 505;

int n;
int favor[MAXN][MAXN]; // favor[boy_id][rank] = girl_id
    ;
int order[MAXN][MAXN]; // order[girl_id][boy_id] = rank
    ;
int current[MAXN]; // current[boy_id] = rank; boy_id
    will pursue current[boy_id] girl.
int girl_current[MAXN]; // girl[girl_id] = boy_id;

void initialize() {
  for ( int i = 0 ; i < n ; i++ ) {
    current[i] = 0;
    girl_current[i] = n;
    order[i][n] = n;
  }
}

map<string, int> male, female;
string bname[MAXN], gname[MAXN];
int fit = 0;

void stable_marriage() {

  queue<int> que;
  for ( int i = 0 ; i < n ; i++ ) que.push(i);
  while ( !que.empty() ) {
    int boy_id = que.front();
    que.pop();

    int girl_id = favor[boy_id][current[boy_id]];
    current[boy_id] ++;

    if ( order[girl_id][boy_id] < order[girl_id][
        girl_current[girl_id]] ) {
      if ( girl_current[girl_id] < n ) que.push(
          girl_current[girl_id]); // if not the first
          time
      girl_current[girl_id] = boy_id;
    } else {
      que.push(boy_id);
    }
  }

}

int main() {
  cin >> n;

  for ( int i = 0 ; i < n;  i++ ) {
    string p, t;
    cin >> p;
    male[p] = i;
    bname[i] = p;
    for ( int j = 0 ; j < n ; j++ ) {
      cin >> t;
      if ( !female.count(t) ) {
        gname[fit] = t;
        female[t] = fit++;
      }
      favor[i][j] = female[t];
    }
  }

  for ( int i = 0 ; i < n ; i++ ) {
    string p, t;
    cin >> p;
    for ( int j = 0 ; j < n ; j++ ) {
      cin >> t;
      order[female[p]][male[t]] = j;
    }
  }

  initialize();
  stable_marriage();

  for ( int i = 0 ; i < n ; i++ ) {
    cout << bname[i] << " " << gname[favor[i][current[i
        ] - 1]] << endl;
  }

}
```

8.7 parser

```
#include <bits/stdc++.h>
using namespace std;

typedef long long T;
bool GG;

T Eval2(char *&end) {
    T Eval0(char *&);
    T res=0;
    if ( *end=='(' ){
        res = Eval0(++end);
        if (*(end++)==')') return res;
        else { GG = true; return -1; }
    }
    else if( isdigit(*end) ){
        return strtol(end, &end, 10);
    }   // 可改成  {strtol ,strtoll strtod}
    else { GG = true; return -1; }
}

T Evalx(char *&end){
    if(GG) return -1;
    T res = Eval2(end); if(GG) return -1;
    while (*end == '%'){
        end++;
        res = ( res % Eval2(end) );
        if(GG) return -1;
    }
    return res;
}

T Eval1(char *&end) {
    if(GG) return -1;
    T res = Evalx(end); if(GG) return -1;
    while (*end=='*' || *end == '/'){
        end++;
        if(*(end-1) == '*')res = ( res * Evalx(end) );
        else if(*(end-1) == '/')res = ( res / Evalx(end
            ) );
        if(GG) return -1;
    }
    return res;
}

T Eval12(char *&end){
    if(GG) return -1;
    T res=1;
    if(*end == '-'){
        end++;
        res = -1;
    }
    res *= Evalx(end);
    while (*end=='*' || *end == '/'){
        end++;
        if(*(end-1) == '*')res = ( res * Evalx(end) );
        else if(*(end-1) == '/')res = ( res / Evalx(end
            ) );
        if(GG) return -1;
    }
    return res;
}
T Eval0(char *&end) {
    if(GG) return -1;
    T res;
    res = Eval12(end); if(GG) return -1;
    while (*end=='+' || *end == '-'){
        end++;
        if(*(end-1) == '+')res = ( res + Eval1(end) );
        else res = ( res - Eval1(end) );
        if(GG) return -1;
    }
    return res;
}

T parse(char *s){
    GG = false;
    T res = Eval0(s);
    while(*s != '\0'){
        if(*s != ' ')GG = true;
        s++;
    }
    return res;
}

int main() {
    char expr[3003];
    string str;
    int cnt = 0;
    while (getline (cin,str)){
        printf("case %d:\n",++cnt);
        strcpy(expr,str.c_str());
        T ans = parse(expr);
        if(GG) puts("syntactically incorrect\n");
        else printf("%lld\n\n", ans);
    }
}

/*
E0 = E1' (+-E1)*
E1 = Ex (/*Ex)*
Ex = E2 (%E2)*
E2 = (E0) or R+
E1' = Ex (/* Ex)*  or  -Ex (/* Ex)*

*/
```

8.8 python 小抄

```
#!/usr/bin/env python3

# 帕斯卡三角形
n = 10
dp = [ [1 for j in range(n)] for i in range(n) ]
for i in range(1,n):
    for j in range(1,n):
        dp[i][j] = dp[i][j-1] + dp[i-1][j]

for i in range(n):
    print( ' '.join( '{:5d}'.format(x) for x in dp[i] )
         )

# EOF
while True:
    try:
        n, m = map(int, input().split())
    except:
        break
    print( min(n,m), max(n,m) )

# input a sequence of number
a = [ int(x) for x in input().split() ]
a.sort()
print( ''.join( str(x)+' ' for x in a ) )

# LCS
ncase = int( input() )
for _ in range(ncase):
    n, m = [int(x) for x in input().split()]
    a, b = "$"+input(), "$"+input()

    dp = [ [int(0) for j in range(m+1)] for i in range(
        n+1) ]

    for i in range(1,n+1):
        for j in range(1,m+1):
            dp[i][j] = max(dp[i-1][j],dp[i][j-1])
            if a[i]==b[j]:
                dp[i][j] = max(dp[i][j],dp[i-1][j-1]+1)

    for i in range(1,n+1):
        print(dp[i][1:])

    print('a={:s}, b={:s}, |LCS(a,b)|={:d}'.format(a
        [1:],b[1:],dp[n][m]))

# Basic operator
a, b = 10, 20
a/b  # 0.5
a//b # 0
a%b  # 10
a**b # 10^20

# if, else if, else
if a==0:
    print('zero')
elif a>0:
    print('postive')
else:
    print('negative')

# stack         # C++
stack = [3,4,5]
stack.append(6) # push()
stack.pop()     # pop()
stack[-1]       # top()
len(stack)      # size() O(1)

# queue         # C++
from collections import deque
queue = deque([3,4,5])
queue.append(6) # push()
queue.popleft() # pop()
queue[0]        # front()
len(queue)      # size() O(1)
```

9 Persistence