Appendix of "Integration of Preferences in Decomposition Multi-Objective Optimization"

Appendix A
Proof of Theorem 1

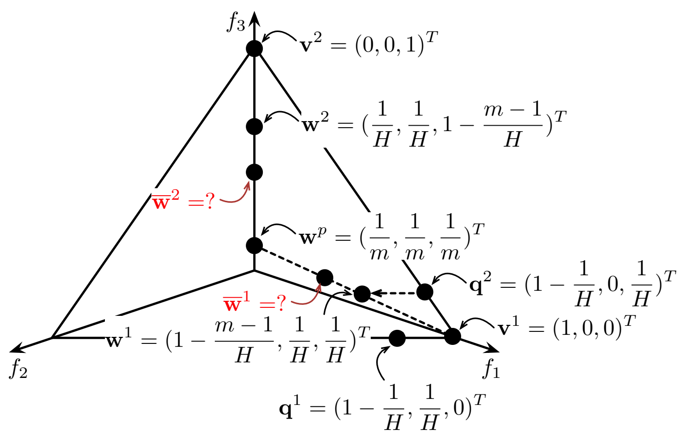


Fig. 1: Illustrative example for η computation.

Proof: Let us use a specific example shown in Fig. 1 to prove this theorem. Suppose the reference points are originally generated by the Das and Dennis's method [1]. Therefore, reference points are distributed on an unit m-simplex. Let the centroid of this simplex, i.e., $\mathbf{w}^p = (\frac{1}{m}, \cdots, \frac{1}{m})^T$, be the pivot point. Let us consider $\mathbf{v}^1 = (1, 0, 0)^T$ and $\mathbf{v}^2 = (0, 0, 1)^T$ as two vertices of an edge of this simplex. Obviously, the length of each edge of this simplex is the same, i.e., $\|\mathbf{v}^1 - \mathbf{v}^2\| = \sqrt{2}$. Suppose $\mathbf{w}^1$ and $\mathbf{w}^2$ are two reference points inside this simplex and closest to $\mathbf{v}^1$ and $\mathbf{v}^2$. Since reference points are generated in a structured manner, we can use a geometric method to find the coordinates of $\mathbf{w}^1$ and $\mathbf{w}^2$. As shown in Fig. 1, $\mathbf{q}^1$ and $\mathbf{q}^2$ are two reference points lying on the two edges and closest to $\mathbf{v}^1$. Obviously, $\mathbf{w}^1$ is a linear combination of $\mathbf{v}^1$, $\mathbf{q}^1$ and $\mathbf{q}^2$ as:

$$w_i^1 = (q_i^2 - v_i^1) + q_i^1 \tag{1}$$

where $i \in \{1, \cdots, m\}$. In summary, we can have $\mathbf{w}^1 = (1 - \frac{m-1}{H}, \cdots, \frac{1}{H})^T$ and $\mathbf{w}^2 = (\frac{1}{H}, \cdots, 1 - \frac{m-1}{H})^T$. Based on our non-uniform mapping scheme, we have the new locations of $\mathbf{w}^1$ and $\mathbf{w}^2$ can be calculated as:

$$\overline{\mathbf{w}}^i = \mathbf{w}^p + t \times \mathbf{u}^i \tag{2}$$

where $i \in \{1, 2\}$, $\mathbf{u}^i = \frac{\mathbf{v}^i - \mathbf{w}^p}{\|\mathbf{v}^i - \mathbf{w}^p\|}$ and

$$t = d - d\left(\frac{d - D}{d}\right)^{\frac{1}{\eta+1}} \tag{3}$$

where $d = \|\mathbf{v^i} - \mathbf{w^p}\|$ and $D = \|\mathbf{w}^i - \mathbf{w}^p\|$. Let $q = \frac{d-D}{d}$, we have:

$$t = d(1 - q^{\frac{1}{\eta+1}}) \tag{4}$$

In order to have the extent of ROI become size of τ of the EF, we have the following equation:

$$\frac{\|\overline{\mathbf{w}}^1 - \overline{\mathbf{w}}^2\|}{\|\mathbf{v}^1 - \mathbf{v}^2\|} = \tau \tag{5}$$

Since $\|\mathbf{v}^1 - \mathbf{v}^2\| = \sqrt{2}$, we have:

$$\|\overline{\mathbf{w}}^1 - \overline{\mathbf{w}}^2\| = \sqrt{2}\tau \tag{6}$$

Using equation (2) to substitute $\overline{\mathbf{w}}^1$ and $\overline{\mathbf{w}}^2$ in equation (6), we have:

$$\|t \times (\mathbf{u}^1 - \mathbf{u}^2)\| = \sqrt{2}\tau \tag{7}$$

Using equation (4) to substitute t in equation (7), we have:

$$d(1 - q^{\frac{1}{\eta+1}}) \times \|\mathbf{u}^1 - \mathbf{u}^2\| = \sqrt{2}\tau \tag{8}$$

By substitution, we have:

$$\begin{aligned} (1 - q^{\frac{1}{\eta+1}}) \times \|\mathbf{v}^1 - \mathbf{v}^2\| &= \sqrt{2}\tau \\ \implies q^{\frac{1}{\eta+1}} &= 1 - \tau \\ \implies \eta &= \frac{\log q}{\log(1-\tau)} - 1 \end{aligned} \tag{9}$$

Since the coordinates of $\mathbf{v}^1$, $\mathbf{w}^1$ and $\mathbf{w}^p$ are known, we have:

$$\begin{aligned} d &= \sqrt{(1 - \frac{1}{m})^2 + (m-1)\frac{1}{m^2}} \\ &= \sqrt{1 - \frac{1}{m}} \end{aligned} \tag{10}$$

and

$$\begin{aligned} D &= \sqrt{(1 - \frac{m-1}{H} - \frac{1}{m})^2 + (m-1)(\frac{1}{m} - \frac{1}{H})^2} \\ &= \sqrt{1 - \frac{1}{m}}(1 - \frac{m}{H}) \\ &= (1 - \frac{m}{H})d \end{aligned} \tag{11}$$

Based on equation (10) and equation (11), we have:

$$\begin{aligned} q &= \frac{d - D}{d} \\ &= \frac{d - (1 - \frac{m}{H})d}{d} \\ &= \frac{m}{H} \end{aligned} \tag{12}$$

■

Appendix B
Proof of Corollary 1

Proof: As discussed in Section III-D, we should set $\eta > 0$ in the NUMS. Thus, based on Theorem 1, we have:

$$\frac{\log \frac{m}{H}}{\log(1-\tau)} > 1 \tag{13}$$

Since $\frac{m}{H} < 1$ and $1 - \tau < 1$, we have:

$$\log \frac{m}{H} \leq \log(1-\tau) \implies 0 < \tau < 1 - \frac{m}{H} \tag{14}$$

■

Appendix C
Proof of Corollary 2

Proof: The proof of this corollary is similar to the Theorem 1. Let us use Fig. 1 for illustration again. As for the reference points $\mathbf{w}^1$ and $\mathbf{w}^2$, we should have the following relationship after the non-uniform mapping:

$$\frac{\|\overline{\mathbf{w}}^1 - \overline{\mathbf{w}}^2\|}{\|\mathbf{w}^1 - \mathbf{w}^2\|} = \tau \tag{15}$$

Since $\|\mathbf{w}^1 - \mathbf{w}^2\| = \sqrt{2}(1 - \frac{m}{H})$, we have:

$$\|\overline{\mathbf{w}}^1 - \overline{\mathbf{w}}^2\| = \sqrt{2}(1 - \frac{m}{H})\tau \tag{16}$$

Using equation (2) to substitute $\overline{\mathbf{w}}^1$ and $\overline{\mathbf{w}}^2$ in equation (16), we have:

$$\|t \times (\mathbf{u}^1 - \mathbf{u}^2)\| = \sqrt{2}(1 - \frac{m}{H})\tau \tag{17}$$

Using equation (4) to substitute t in equation (17), we have:

$$d(1 - q^{\frac{1}{\eta+1}}) \times \|\mathbf{u}^1 - \mathbf{u}^2\| = \sqrt{2}(1 - \frac{m}{H})\tau \tag{18}$$

Since $\|\mathbf{v}^1 - \mathbf{v}^2\| = \sqrt{2}$, by substitution, we have:

$$\begin{aligned} 1 - q^{\frac{1}{\eta+1}} &= (1 - \frac{m}{H})\tau \\ \implies \eta &= \frac{\log q}{\log[1 - (1 - \frac{m}{H})\tau]} - 1 \end{aligned} \tag{19}$$

where $q = \frac{m}{H}$ according to equation (12). ■

Appendix D
Proof of Corollary 3

Proof: Since $\eta > 0$, according to equation (19), we have:

$$\frac{\log q}{\log[1 - (1 - \frac{m}{H})\tau]} > 1 \tag{20}$$

Since $\frac{m}{H} < 1$ and $1 - (1 - \frac{m}{H})\tau < 1$, we have:

$$\log \frac{m}{H} < \log\left[1 - (1 - \frac{m}{H})\tau\right] \implies 0 < \tau < 1 \tag{21}$$

■

APPENDIX E
PARAMETER SETTINGS

TABLE I: Settings of Aspiration Level Vector

Problem	m	Unattainable	Attainable
DTLZ1	2	(0.3,0.5)^T	(0.5,0.6)^T
	3	(0.05,0.05,0.2)^T	(0.3,0.3,0.2)^T
	5	(0.05,0.05,0.1,0.08,0.03)^T	(0.2,0.1,0.1,0.3,0.4)^T
	8	(0.01,0.02,0.07,0.02,0.06,0.2,0.1,0.01)^T	(0.1,0.2,0.1,0.4,0.4,0.1,0.3,0.1)^T
	10	(0.02,0.01,0.06,0.04,0.04,0.01,0.02,0.03,0.05,0.08)^T	(0.05,0.1,0.1,0.05,0.1,0.2,0.08,0.03,0.3,0.1)^T
DTLZ2-4	2	(0.65,0.7)^T	(0.75,0.75)^T
	3	(0.2,0.5,0.6)^T	(0.7,0.8,0.5)^T
	5	(0.3,0.1,0.4,0.2,0.3)^T	(0.7,0.6,0.3,0.8,0.5)^T
	8	(0.3,0.1,0.4,0.25,0.1,0.15,0.4,0.25)^T	(0.6,0.5,0.75,0.2,0.3,0.55,0.7,0.6)^T
	10	(0.1,0.1,0.3,0.4,0.2,0.5,0.25,0.15,0.1,0.4)^T	(0.3,0.3,0.3,0.1,0.3,0.55,0.35,0.35,0.25,0.45)^T
WFG41	2	(0.65,0.7)^T	(0.75,0.75)^T
	3	(0.2,0.5,0.6)^T	(0.7,0.8,0.5)^T
	5	(0.3,0.1,0.4,0.2,0.3)^T	(0.7,0.6,0.3,0.8,0.5)^T
	8	(0.3,0.1,0.4,0.25,0.1,0.15,0.4,0.25)^T	(0.6,0.5,0.75,0.2,0.3,0.55,0.7,0.6)^T
	10	(0.1,0.1,0.3,0.4,0.2,0.5,0.25,0.15,0.1,0.4)^T	(0.3,0.3,0.3,0.1,0.3,0.55,0.35,0.35,0.25,0.45)^T
WFG42	2	(0.5,0.1)^T	(0.6,0.15)^T
	3	(0.05,0.05,0.2)^T	(0.15,0.15,0.25)^T
	5	(0.03,0.04,0.08,0.04,0.04)^T	(0.1,0.05,0.1,0.05,0.05)^T
	8	(0.01,0.02,0.03,0.01,0.01,0.05,0.01,0.01)^T	(0.2,0.2,0.1,0.3,0.05,0.15,0.2,0.15)^T
	10	(0.005,0.002,0.005,0.005,0.008,0.005,0.002,0.002,0.001,0.005)^T	(0.3,0.3,0.5,0.3,0.5,0.45,0.25,0.35,0.25,0.4)^T
WFG43	2	(0.9,0.8)^T	(0.95,0.93)^T
	3	(0.6,0.8,0.8)^T	(0.95,0.95,0.85)^T
	5	(0.4,0.4,0.5,0.4,0.4)^T	(0.5,0.7,1,0.9,0.6)^T
	8	(0.4,0.4,0.5,0.4,0.4,0.5,0.6,0.5)^T	(0.95,1,1,0.95,0.8,0.95,0.95,1)^T
	10	(0.5,0.2,0.4,0.3,0.4,0.5,0.6,0.7,0.6,0.7)^T	(1,0.85,1,0.95,0.85,0.95,0.95,1,0.8,0.9)^T
WFG44	2	(0.008,0.006)^T	(0.009,0.008)^T
	3	(0.0004,0.0004,0.0003)^T	(0.001,0.0001,0.0005)^T
	5	(5e-06,1e-05,1e-05,1e-05,5e-06)^T	(0.0005,0.001,0.0003,0.0006,0.0008)^T
	8	(5e-06,1e-06,1e-06,1.5e-06,5e-07,1e-06,5e-07,2e-07)^T	(0.001,0.0015,0.002,0.002,0.003,0.003,0.002,0.001)^T
	10	(2e-06,8e-07,3e-07,3e-07,7e-07,1e-07,2e-08,2e-07,5e-07,2e-07)^T	(0.0015,0.002,0.002,0.001,0.0015,0.002,0.002,0.003,0.003,0.002)^T
WFG45	2	(0.7,0.45)^T	(0.75,0.5)^T
	3	(0.3,0.7,0.3)^T	(0.5,0.8,0.5)^T
	5	(0.6,0.3,0.3,0.3,0.3)^T	(0.8,0.5,0.4,0.5,0.5)^T
	8	(0.2,0.3,0.3,0.3,0.3,0.2,0.1,0.2)^T	(0.8,0.5,0.4,0.5,0.5,0.7,0.7,0.9)^T
	10	(0.2,0.15,0.15,0.2,0.1,0.15,0.05,0.1,0.1,0.2)^T	(0.9,0.65,0.6,0.5,0.55,0.8,0.8,0.9,0.5,0.55)^T
WFG46	2	(0.55,0.4)^T	(0.65,0.5)^T
	3	(0.3,0.25,0.3)^T	(0.35,0.35,0.4)^T
	5	(0.1,0.1,0.2,0.1,0.15)^T	(0.2,0.3,0.4,0.2,0.3)^T
	8	(0.1,0.08,0.05,0.1,0.05,0.1,0.05,0.2)^T	(0.2,0.5,0.3,0.2,0.3,0.1,0.1,0.2)^T
	10	(0.05,0.08,0.05,0.02,0.05,0.04,0.05,0.08,0.1,0.05)^T	(0.25,0.5,0.35,0.5,0.35,0.6,0.5,0.5,0.4,0.35)^T
WFG47	2	(0.4,0.45)^T	(0.5,0.5)^T
	3	(0.7,0.4,0.4)^T	(0.9,0.5,0.6)^T
	5	(0.5,0.3,0.3,0.3,0.2)^T	(0.8,0.4,0.5,0.4,0.6)^T
	8	(0.08,0.15,0.1,0.1,0.12,0.1,0.1,0.1)^T	(0.5,0.4,0.7,0.4,0.8,0.8,0.75,0.7)^T
	10	(0.05,0.05,0.1,0.1,0.05,0.08,0.05,0.05,0.03,0.05)^T	(0.6,0.35,0.75,0.6,0.85,0.8,0.75,0.7,0.7,0.8)^T
WFG48	2	(0.6,0.4)^T	(0.7,0.5)^T
	3	(0.25,0.2,0.1)^T	(0.35,0.3,0.2)^T
	5	(0.05,0.08,0.05,0.06,0.05)^T	(0.1,0.2,0.08,0.15,0.1)^T
	8	(0.02,0.03,0.02,0.01,0.02,0.03,0.04,0.03)^T	(0.15,0.25,0.35,0.45,0.15,0.45,0.35,0.25)^T
	10	(0.005,0.008,0.01,0.01,0.02,0.03,0.04,0.03,0.02,0.01)^T	(0.25,0.45,0.3,0.6,0.2,0.5,0.35,0.5,0.6,0.7)^T

TABLE II: Settings of Number of Function Evaluations (FEs), N is the population size

Problem	m	# of FEs	Problem	m	# of FEs
DTLZ1	3	$400 \times N$	DTLZ3	3	$1000 \times N$
	5	$1000 \times N$		5	$1200 \times N$
	8	$1200 \times N$		8	$1500 \times N$
	10	$1500 \times N$		10	$1800 \times N$
DTLZ2	3	$250 \times N$	DTLZ4	3	$600 \times N$
	5	$800 \times N$		5	$1200 \times N$
	8	$1000 \times N$		8	$1500 \times N$
	10	$1360 \times N$		10	$1800 \times N$
WFG41-WFG48	2	$400 \times N$			
	3	$400 \times N$			
	5	$1000 \times N$			
	8	$1200 \times N$			
	10	$1500 \times N$			

APPENDIX F
DESCRIPTION OF STATISTICAL ANALYSIS FRAMEWORK

In this work, we employ the statistical analysis suggested in [2] to validate the statistical significance of the results obtained by different algorithms. Specifically, as shown in Fig. 2, we at first carry out a Kolmogorov-Smirnov test to check whether the results follow a normal distribution or not. If so, we use Levene test to check the homogeneity of the variances. Afterwards, we use ANOVA test to validate the significance if samples are with equal variance; otherwise we use Welch test instead. On the other hand, if the results do not follow a normal distribution, we use Kruskal-Wallis test to compare the median metric values obtained by different algorithms. Note that we set the confidence level as 95% (i.e., significance level of 5% or p-value under 0.05) in the statistical tests.

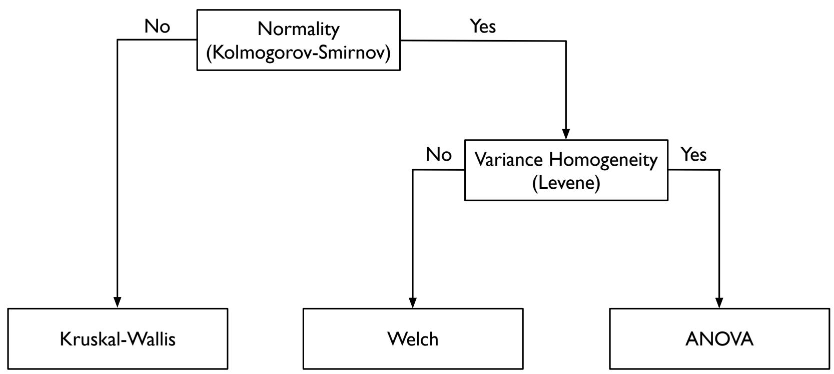


Fig. 2: Statistical analysis framework [2].

APPENDIX G
PLOTS OF FINAL POPULATIONS

This section provides the visual comparisons of different preference-based EMO algorithms. In particular, we plot the final solutions obtained by different algorithms that achieve the best R-HV value.

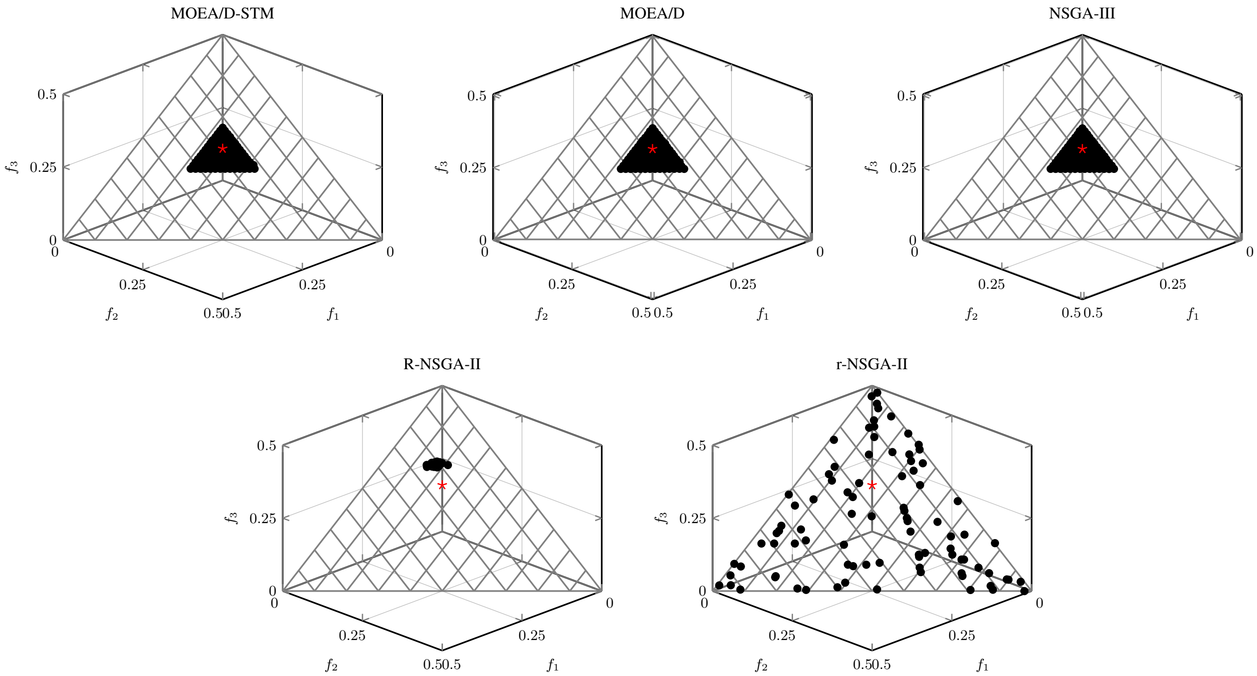


Fig. 3: Comparisons on 3-objective DTLZ1 where $\mathbf{z}^r = (0.05, 0.05, 0.2)$.

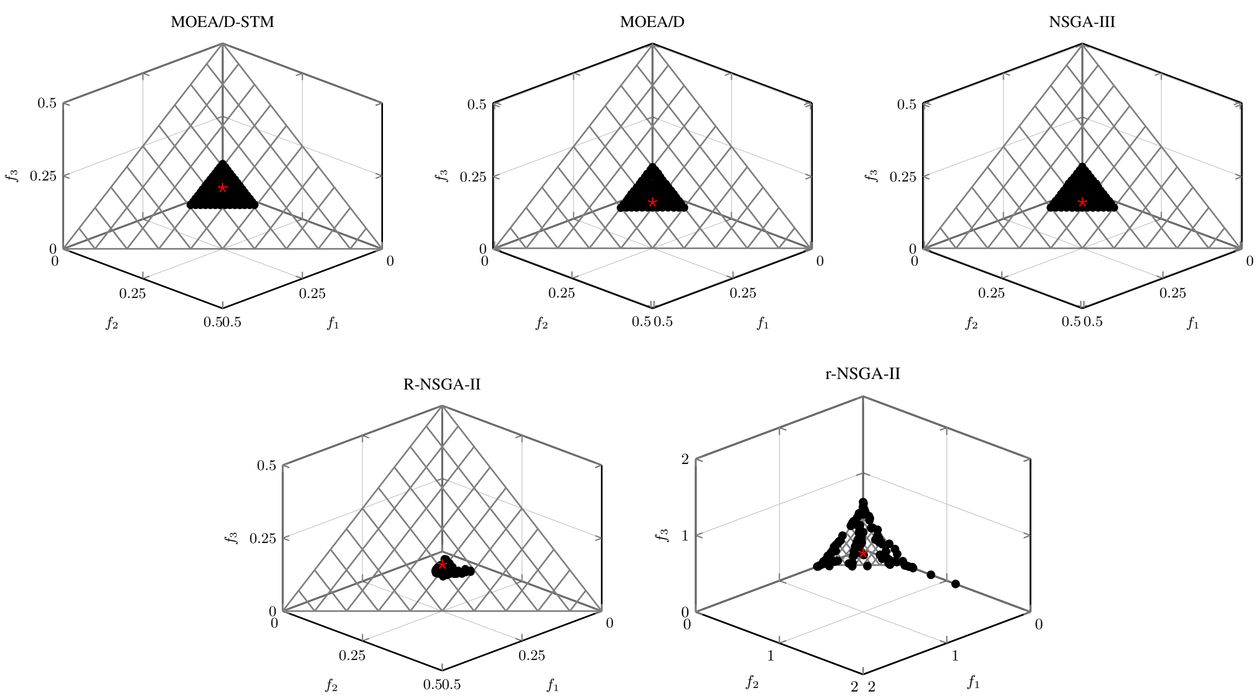


Fig. 4: Comparisons on 3-objective DTLZ1 where $\mathbf{z}^r = (0.3, 0.3, 0.2)$.

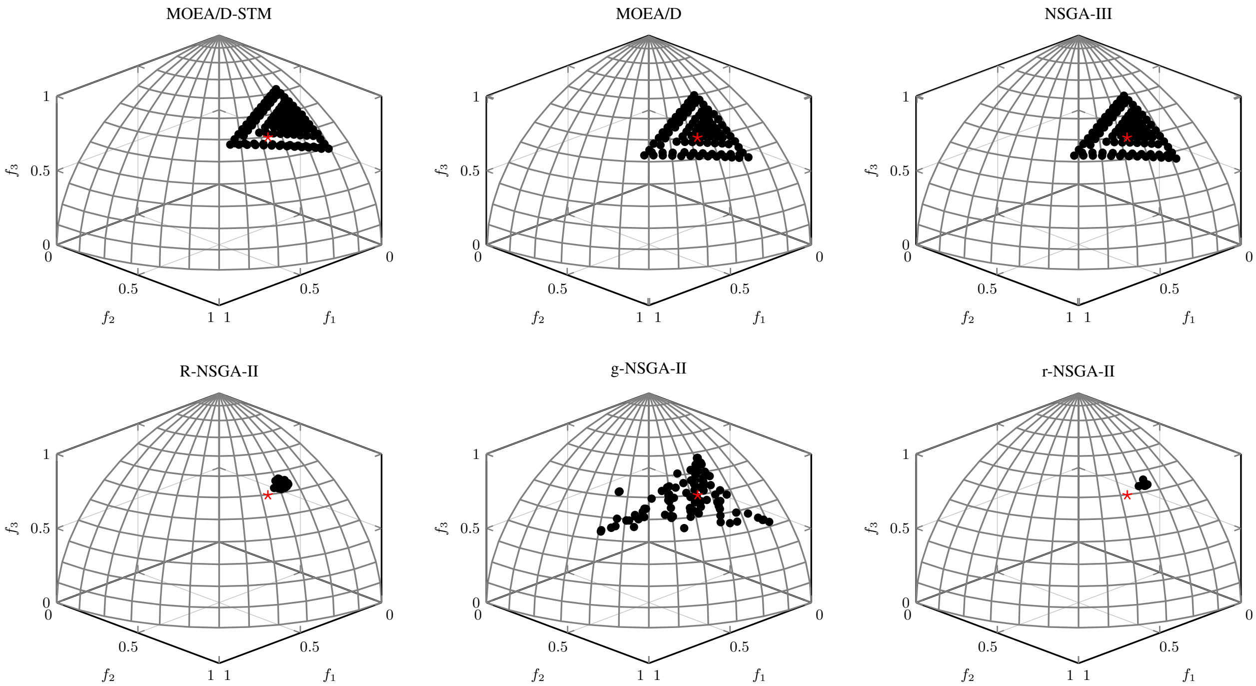


Fig. 5: Comparisons on 3-objective DTLZ2 where $\mathbf{z}^r = (0.2, 0.5, 0.6)$.

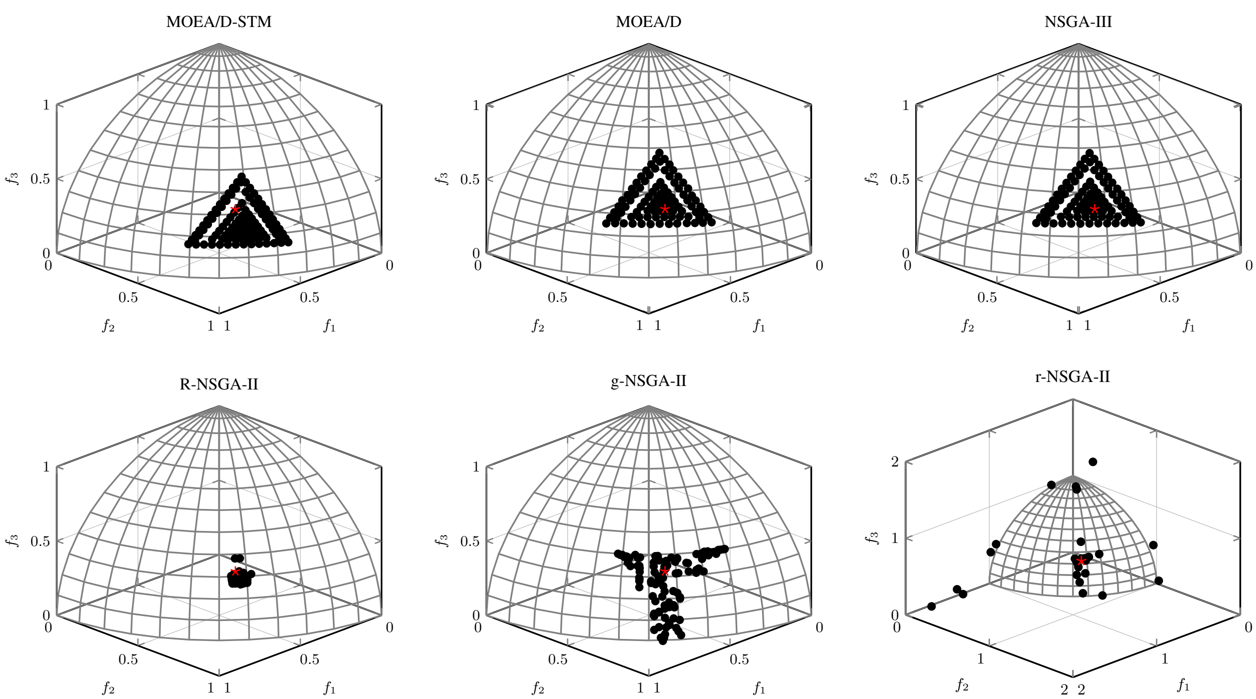


Fig. 6: Comparisons on 3-objective DTLZ2 where $\mathbf{z}^r = (0.7, 0.8, 0.5)$.

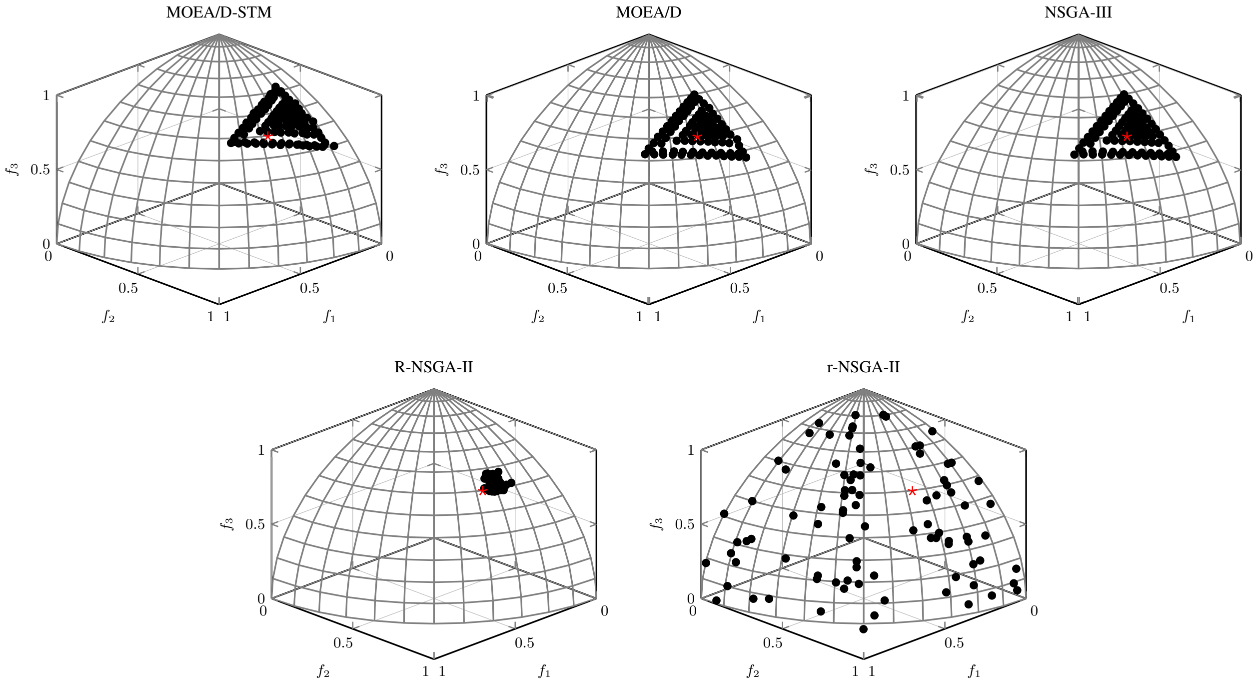


Fig. 7: Comparisons on 3-objective DTLZ3 where $\mathbf{z}^r = (0.2, 0.5, 0.6)$.

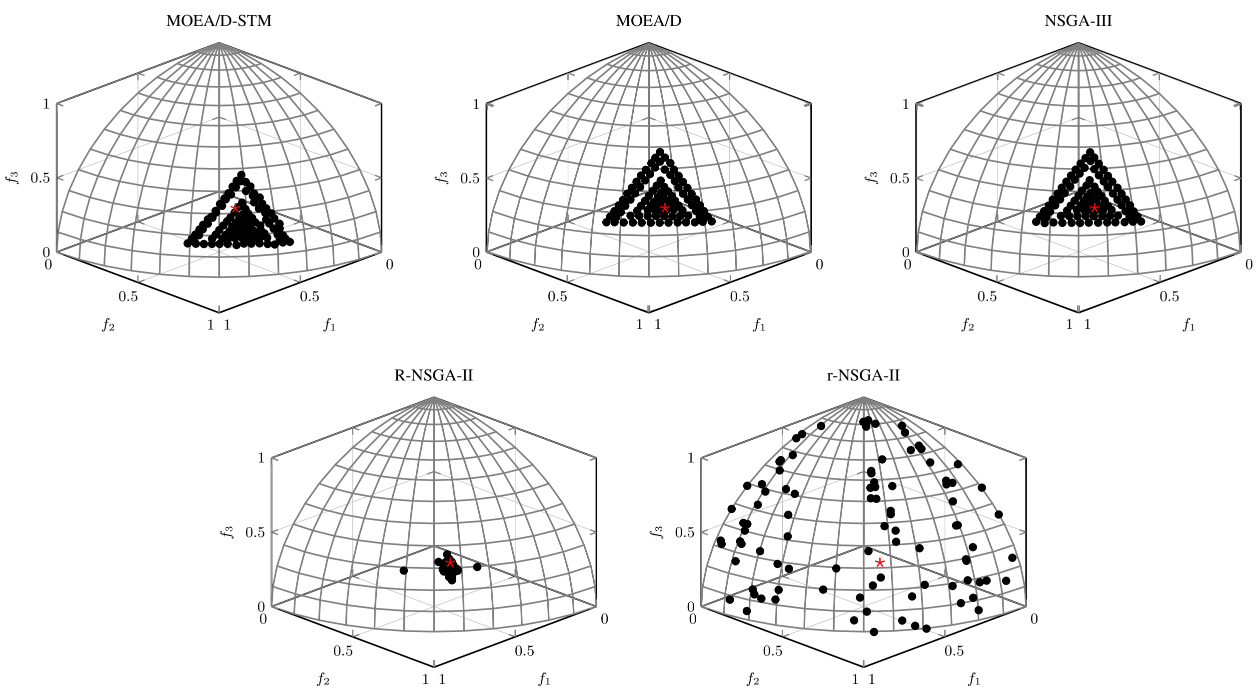


Fig. 8: Comparisons on 3-objective DTLZ3 where $\mathbf{z}^r = (0.7, 0.8, 0.5)$.

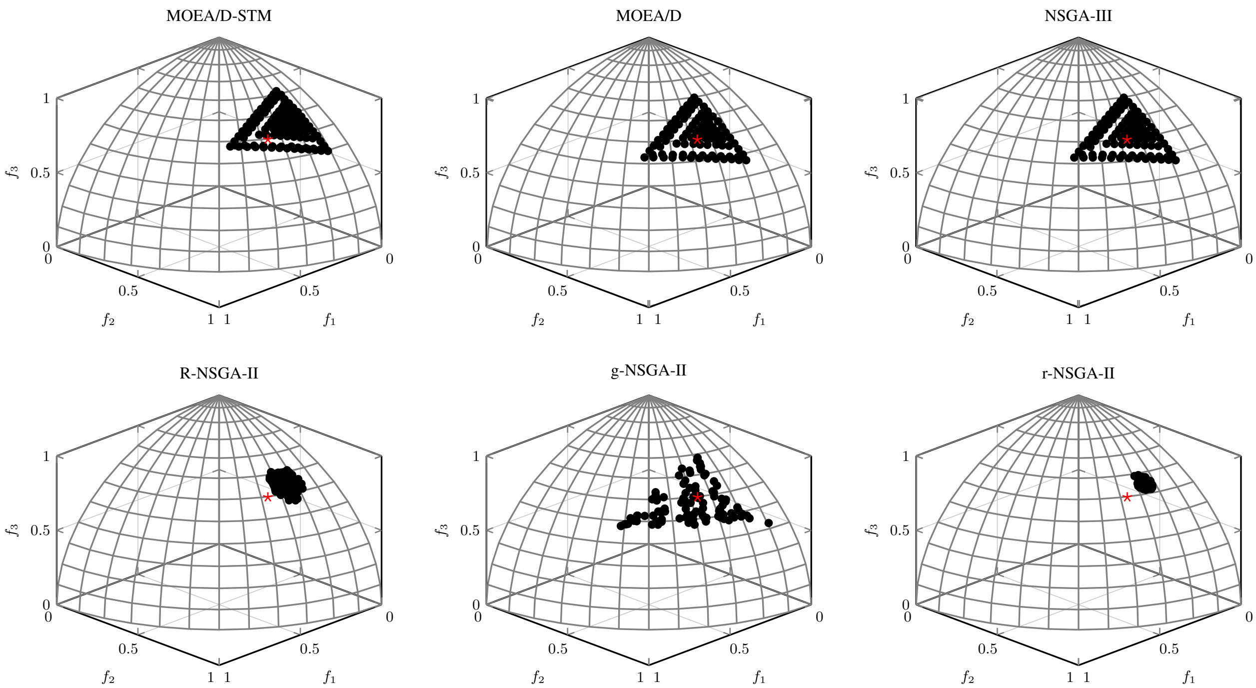


Fig. 9: Comparisons on 3-objective DTLZ4 where $\mathbf{z}^r = (0.2, 0.5, 0.6)$.

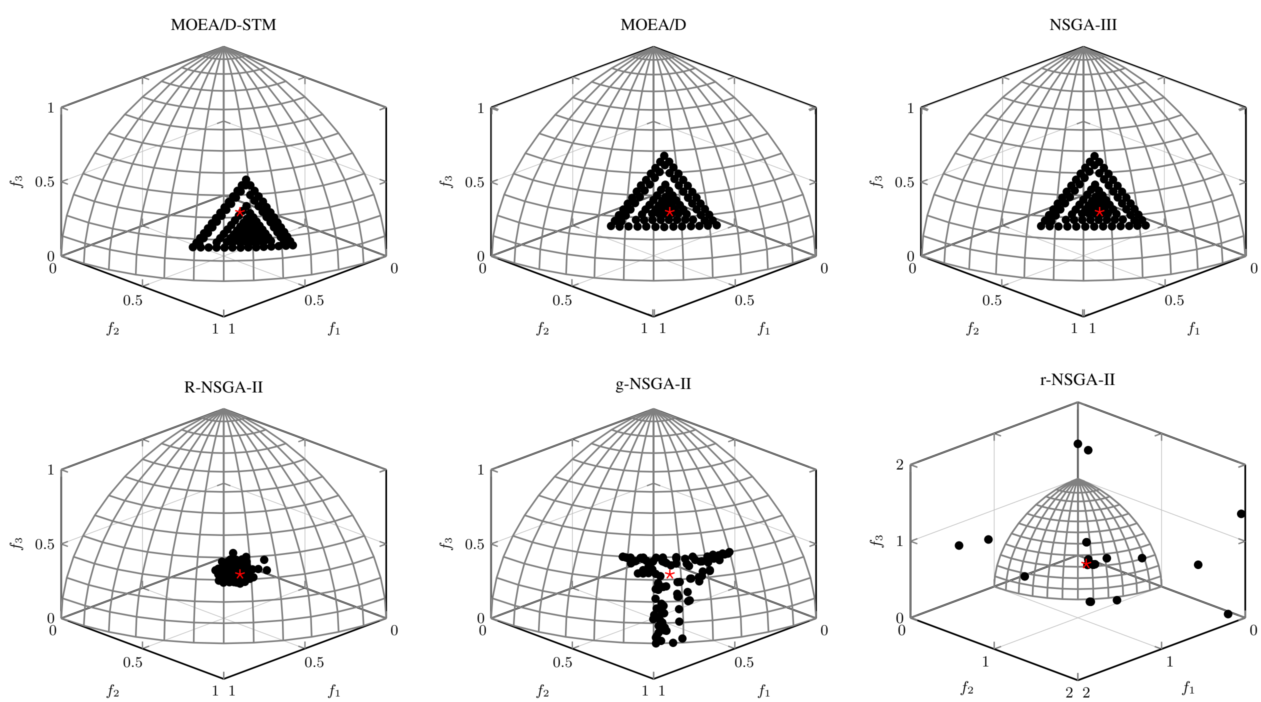
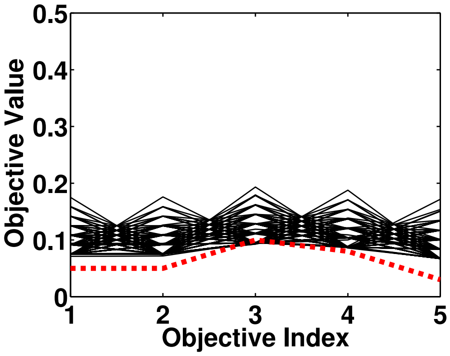
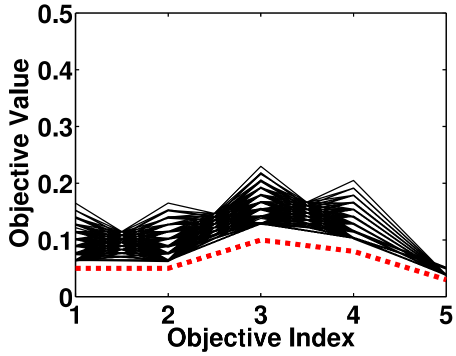


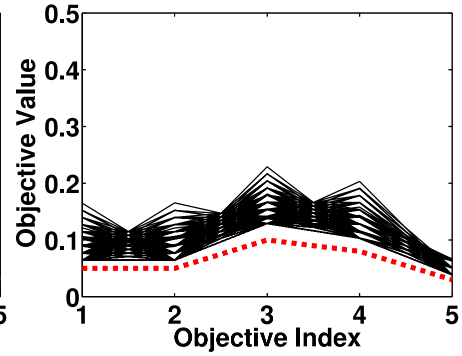
Fig. 10: Comparisons on 3-objective DTLZ4 where $\mathbf{z}^r = (0.7, 0.8, 0.5)$.



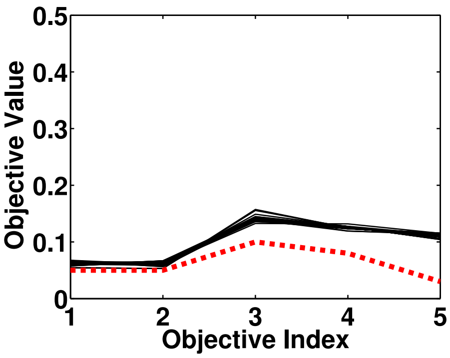
(a) MOEA/D-STM



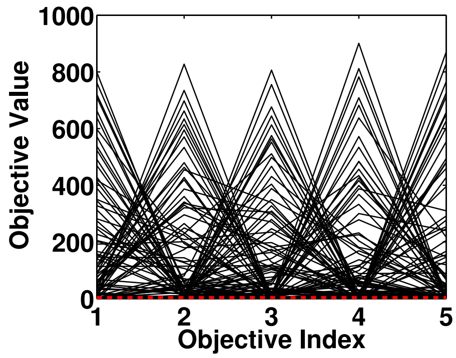
(b) MOEA/D



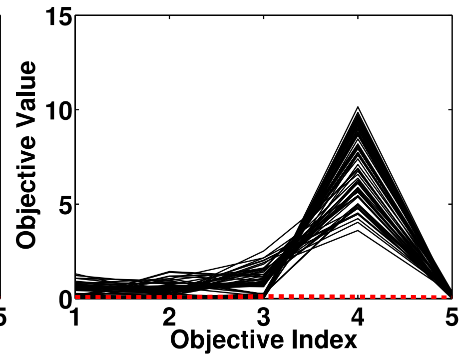
(c) NSGA-III



(d) R-NSGA-II

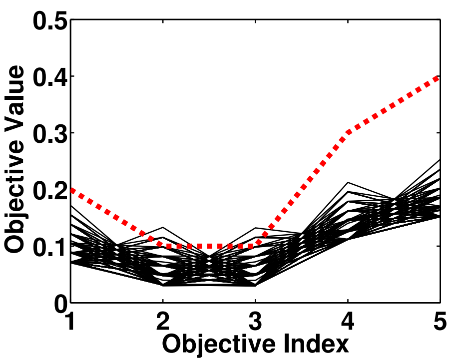


(e) g-NSGA-II

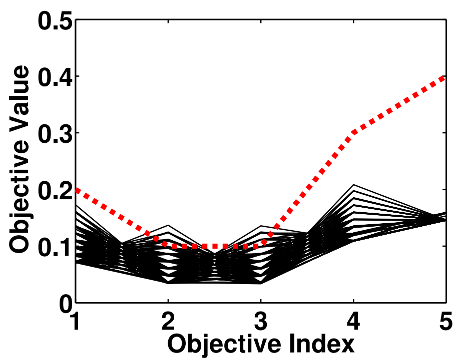


(f) r-NSGA-II

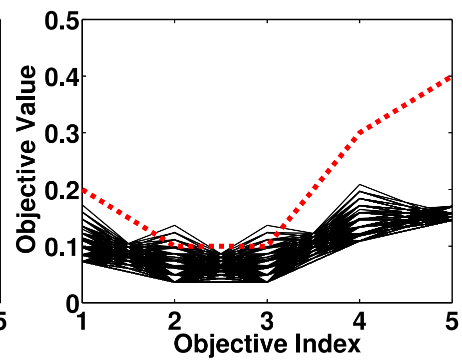
Fig. 11: Comparisons on 5-objective DTLZ1 where $\mathbf{z}^r = (0.05, 0.05, 0.1, 0.08, 0.03)^T$.



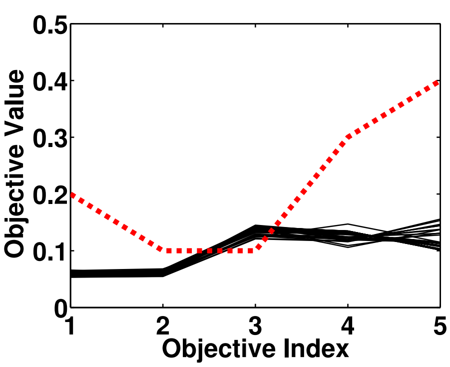
(a) MOEA/D-STM



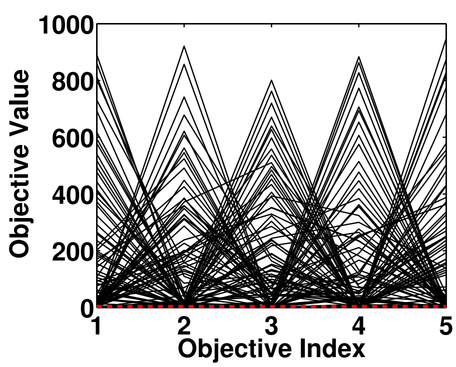
(b) MOEA/D



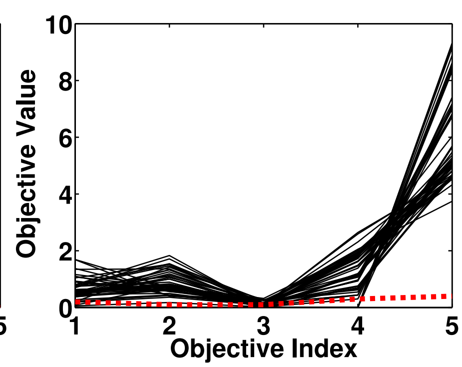
(c) NSGA-III



(d) R-NSGA-II

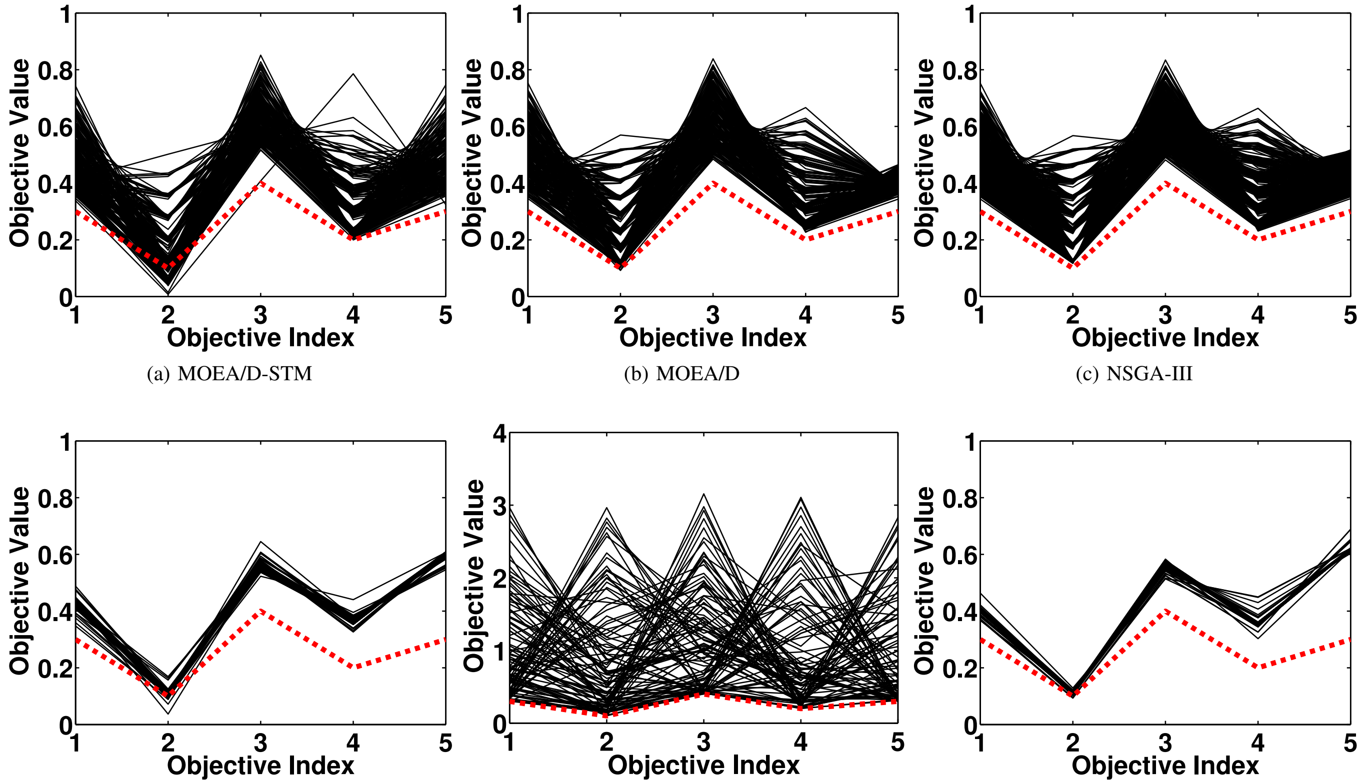


(e) g-NSGA-II



(f) r-NSGA-II

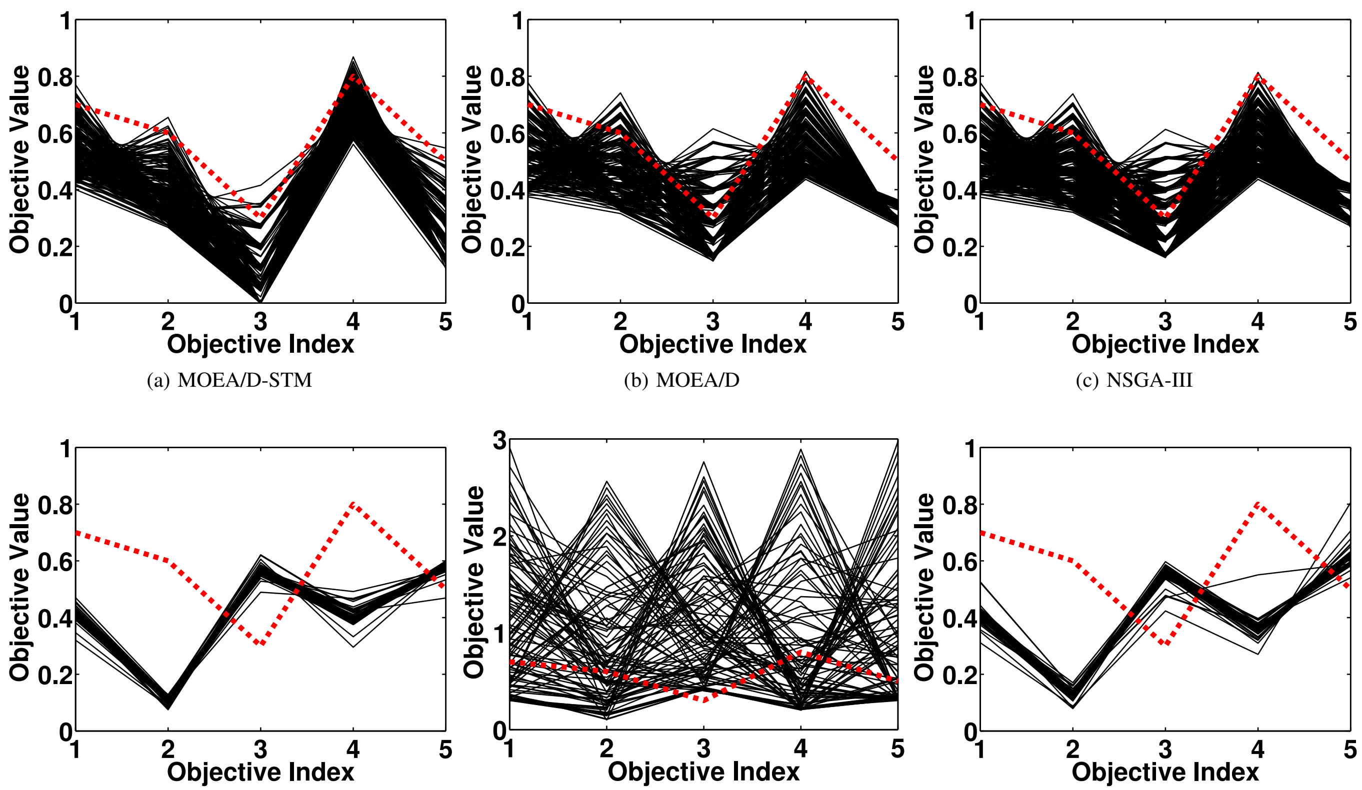
Fig. 12: Comparisons on 5-objective DTLZ1 where $\mathbf{z}^r = (0.2, 0.1, 0.1, 0.3, 0.4)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) R-NSGA-II (e) g-NSGA-II (f) r-NSGA-II

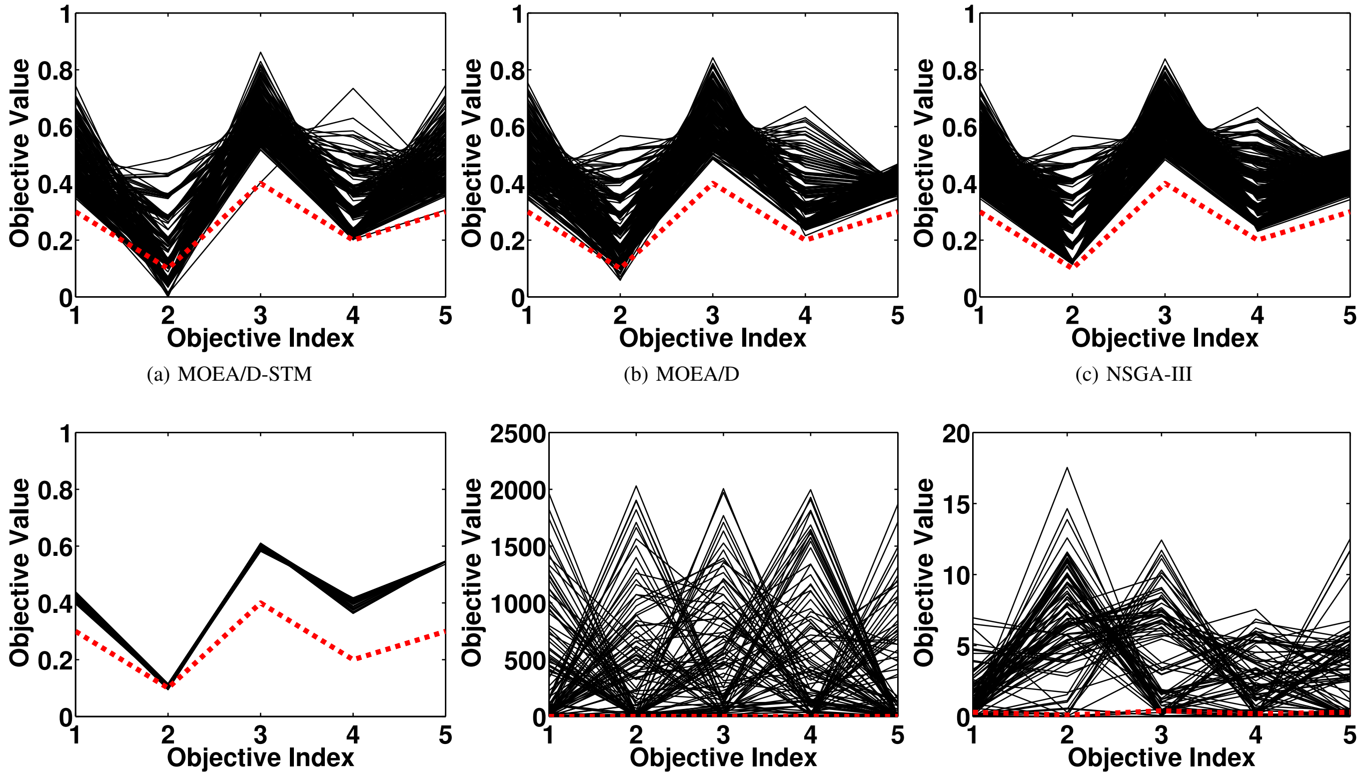
Fig. 13: Comparisons on 5-objective DTLZ2 where $\mathbf{z}^r = (0.3, 0.1, 0.4, 0.2, 0.3)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) R-NSGA-II (e) g-NSGA-II (f) r-NSGA-II

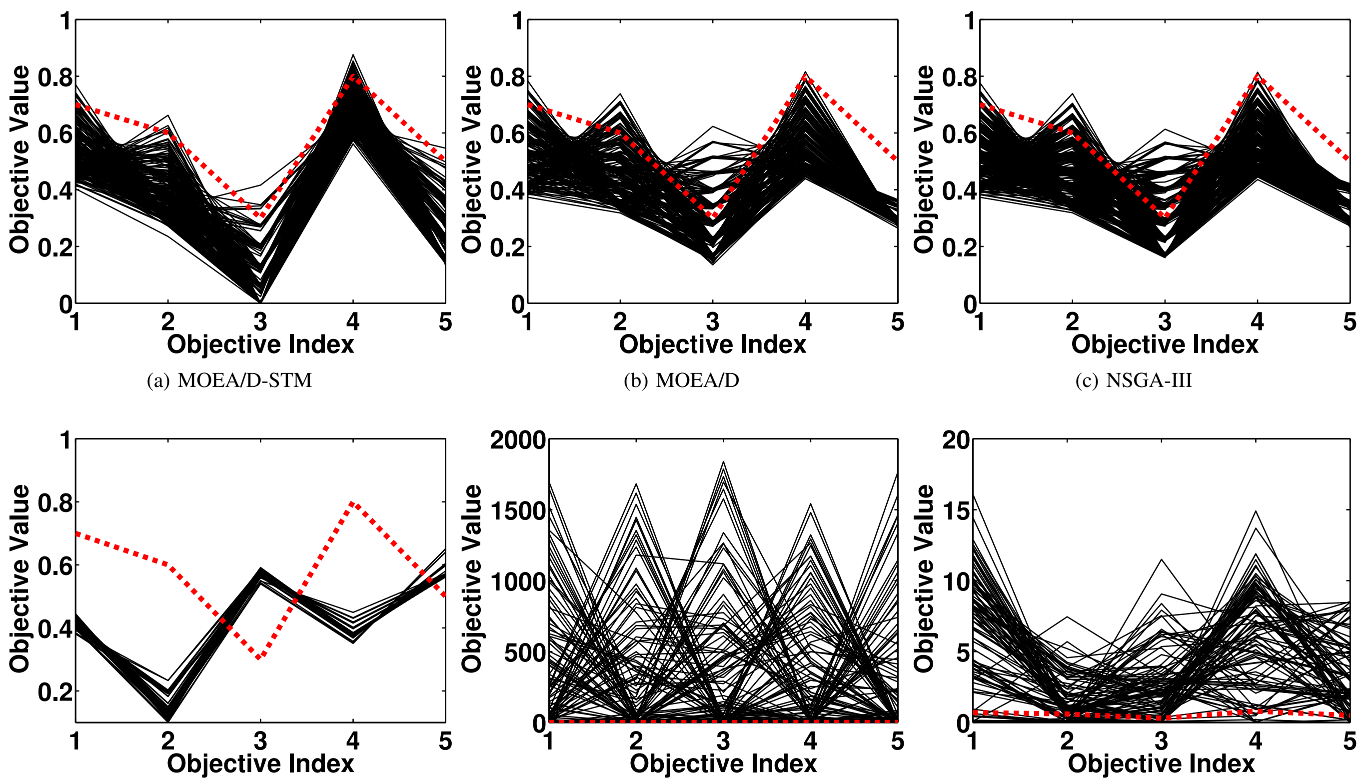
Fig. 14: Comparisons on 5-objective DTLZ2 where $\mathbf{z}^r = (0.7, 0.6, 0.3, 0.8, 0.5)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) R-NSGA-II (e) g-NSGA-II (f) r-NSGA-II

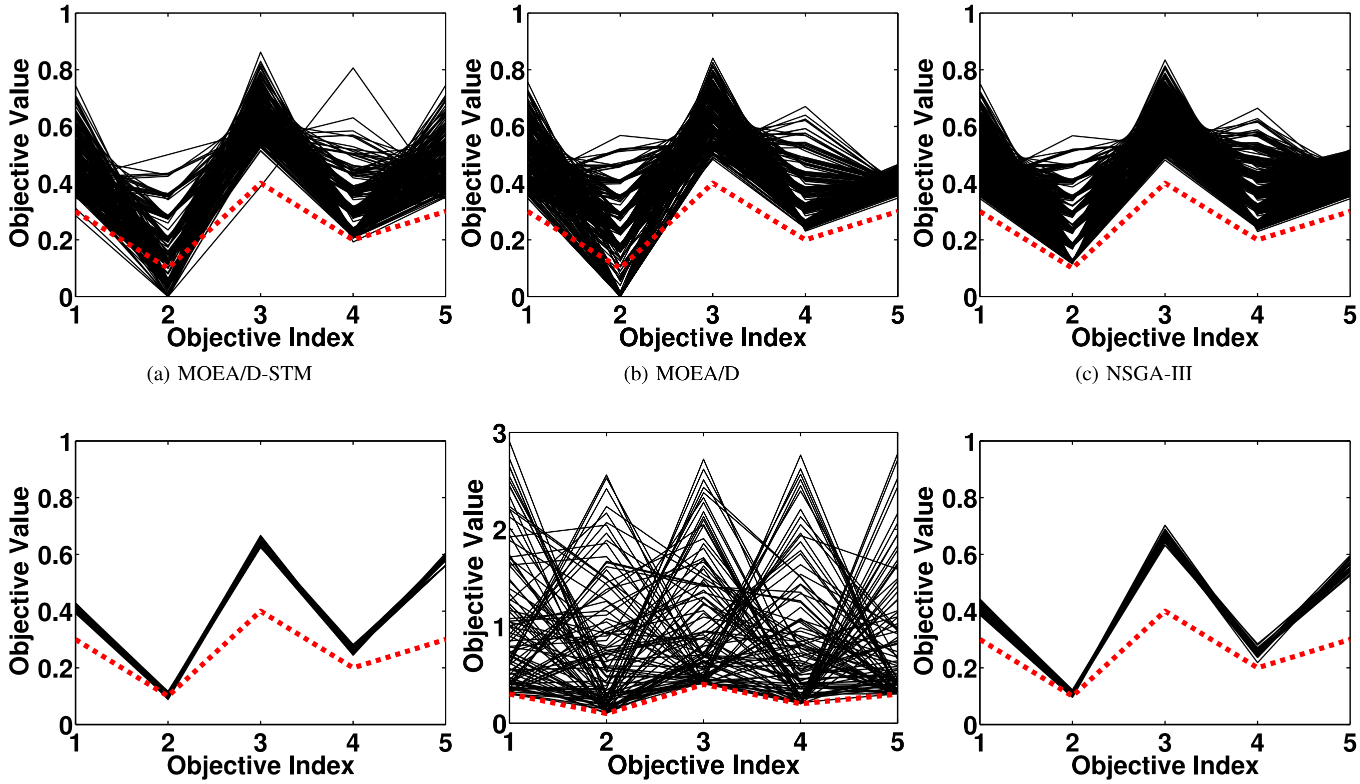
Fig. 15: Comparisons on 5-objective DTLZ3 where $\mathbf{z}^r = (0.3, 0.1, 0.4, 0.2, 0.3)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) R-NSGA-II (e) g-NSGA-II (f) r-NSGA-II

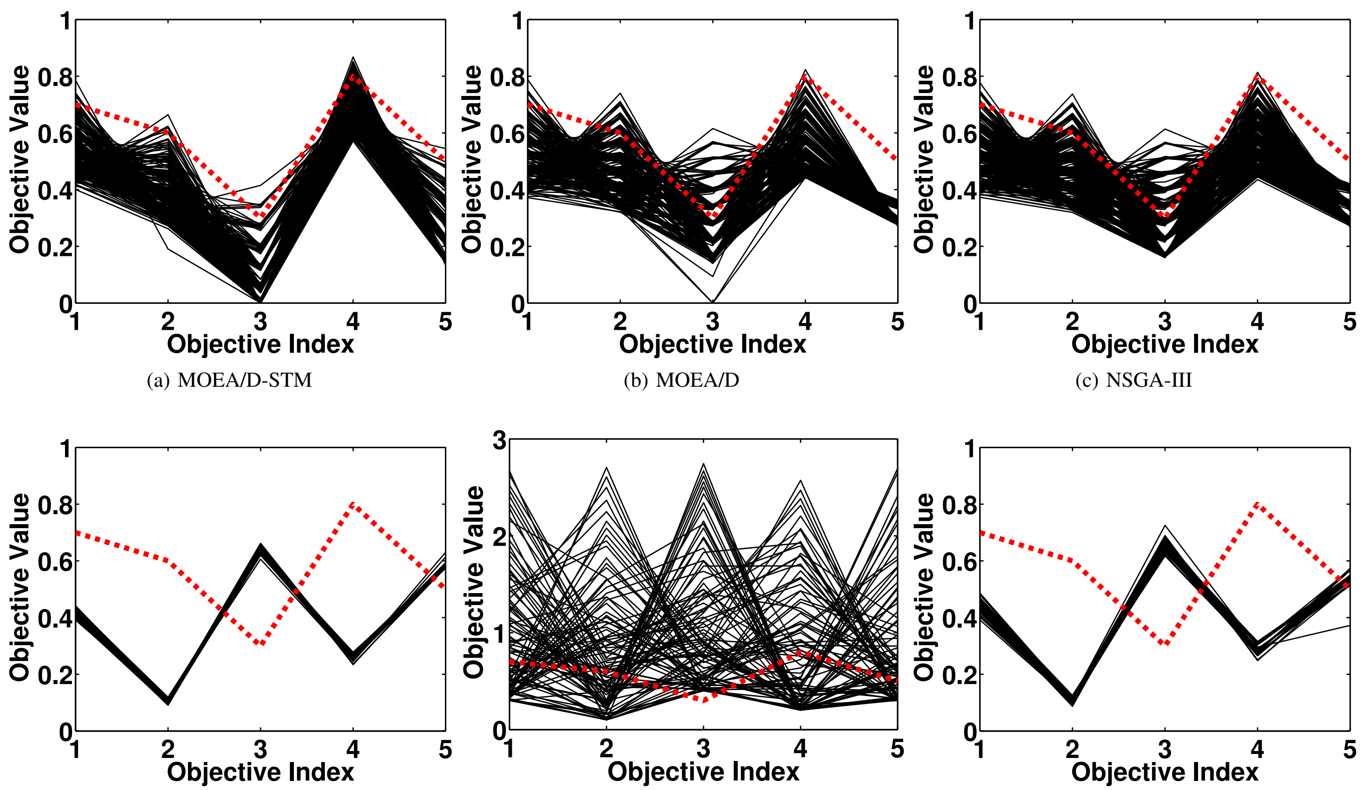
Fig. 16: Comparisons on 5-objective DTLZ3 where $\mathbf{z}^r = (0.7, 0.6, 0.3, 0.8, 0.5)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) R-NSGA-II (e) g-NSGA-II (f) r-NSGA-II

Fig. 17: Comparisons on 5-objective DTLZ4 where $\mathbf{z}^r = (0.3, 0.1, 0.4, 0.2, 0.3)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) R-NSGA-II (e) g-NSGA-II (f) r-NSGA-II

Fig. 18: Comparisons on 5-objective DTLZ4 where $\mathbf{z}^r = (0.7, 0.6, 0.3, 0.8, 0.5)^T$.

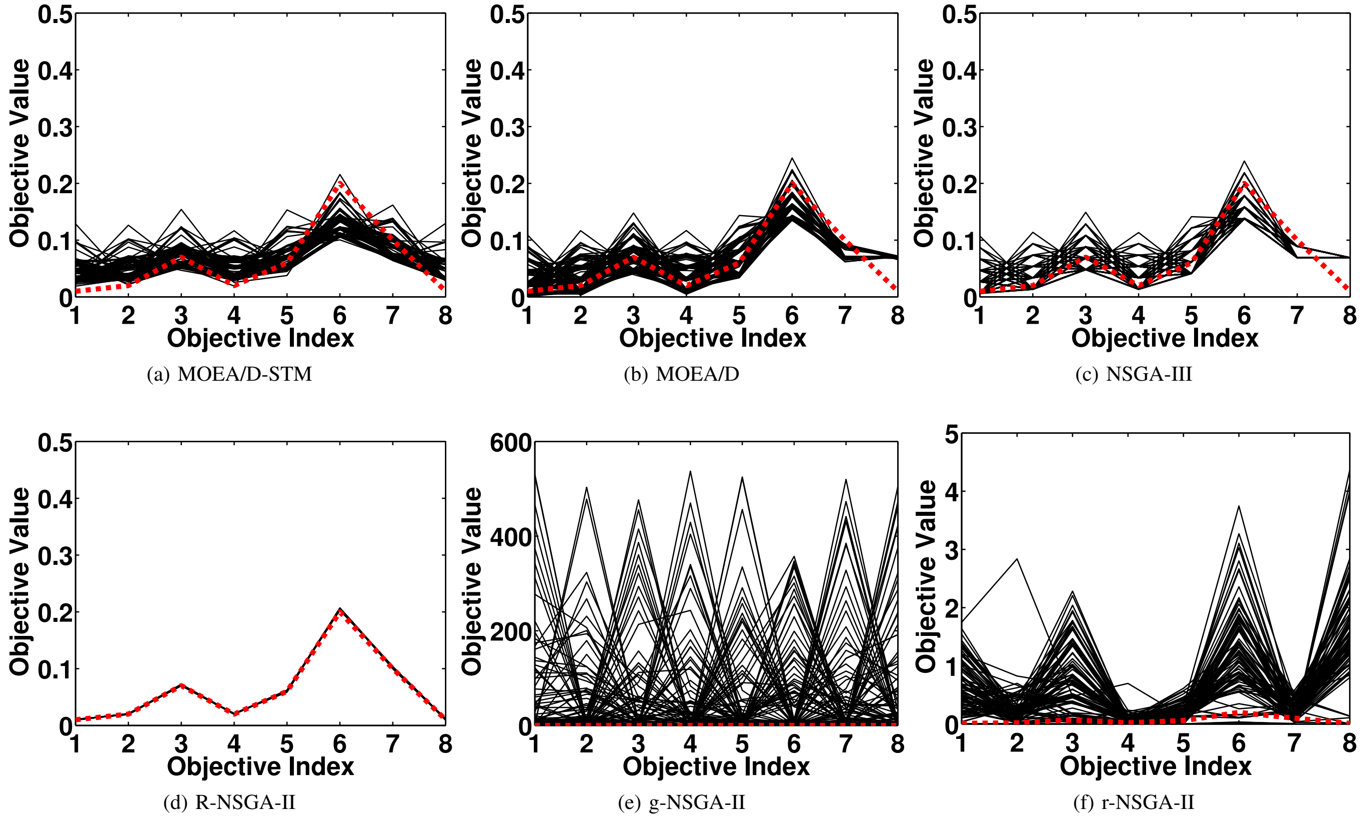


Fig. 19: Comparisons on 8-objective DTLZ1 where $\mathbf{z}^r = (0.01, 0.02, 0.07, 0.02, 0.06, 0.2, 0.1, 0.01)^T$.

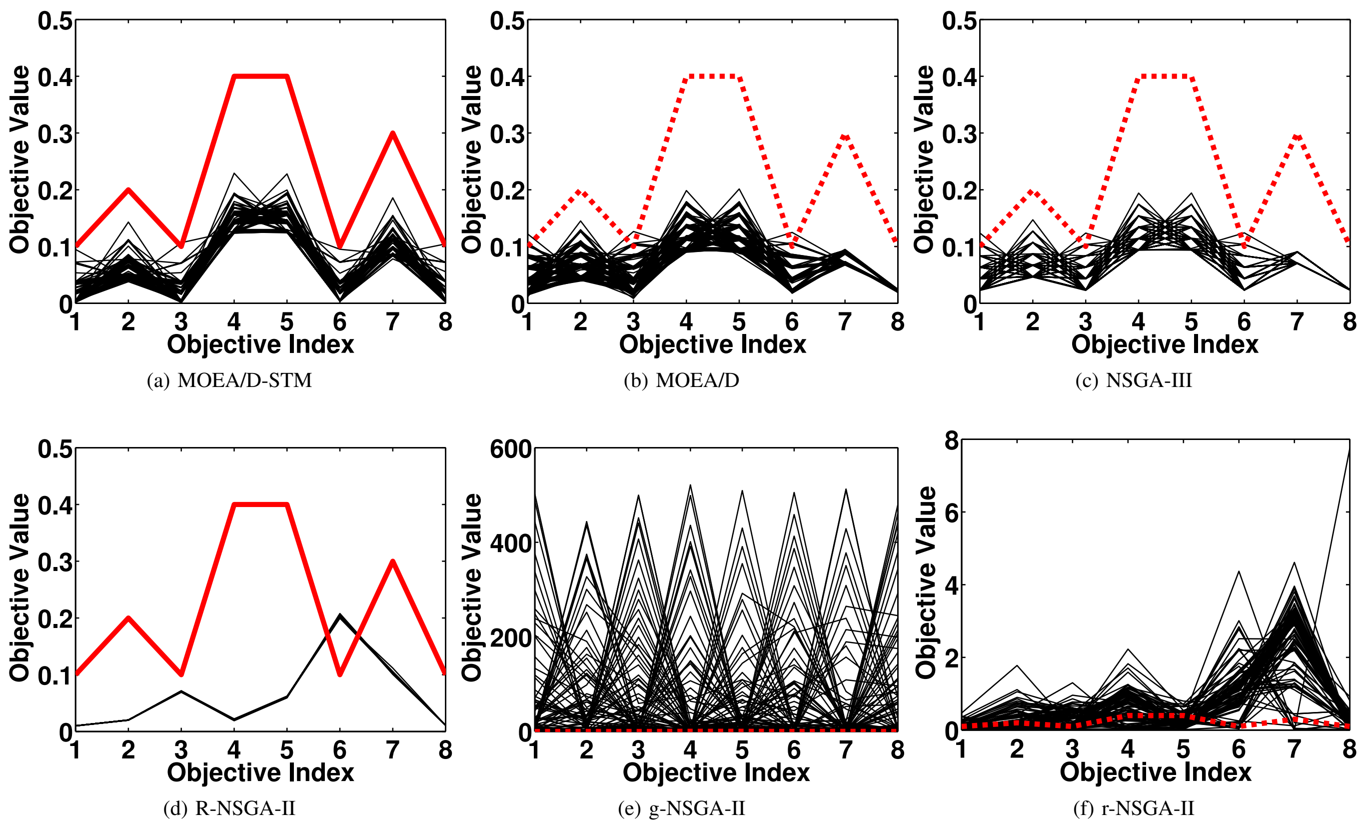
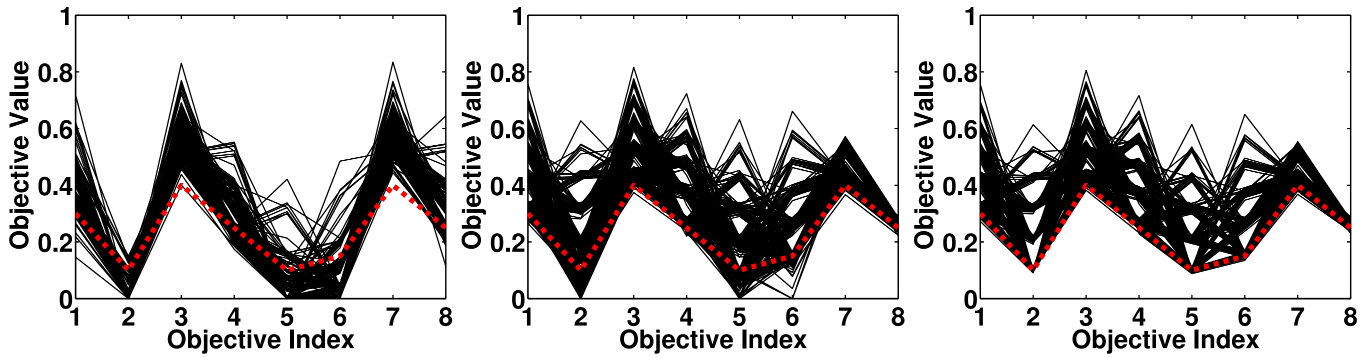
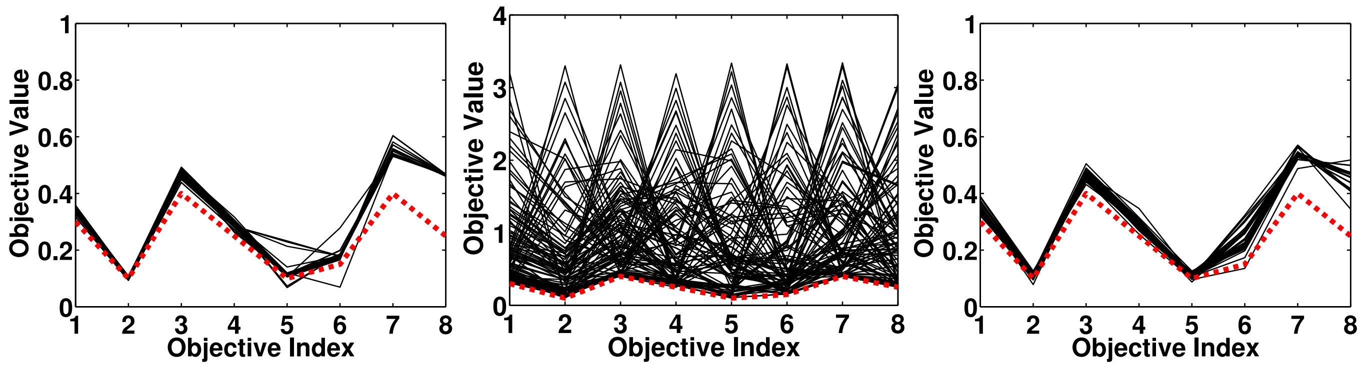


Fig. 20: Comparisons on 8-objective DTLZ1 where $\mathbf{z}^r = (0.1, 0.2, 0.1, 0.4, 0.4, 0.1, 0.3, 0.1)^T$.

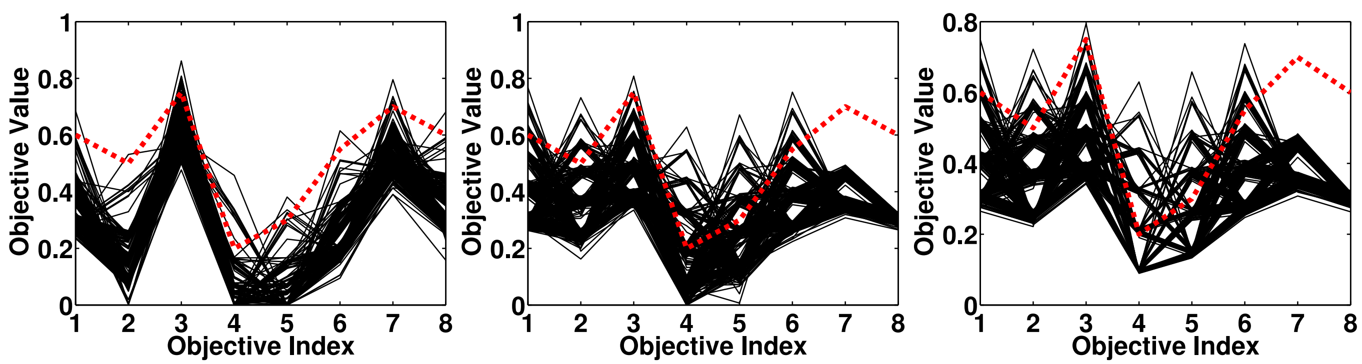


(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

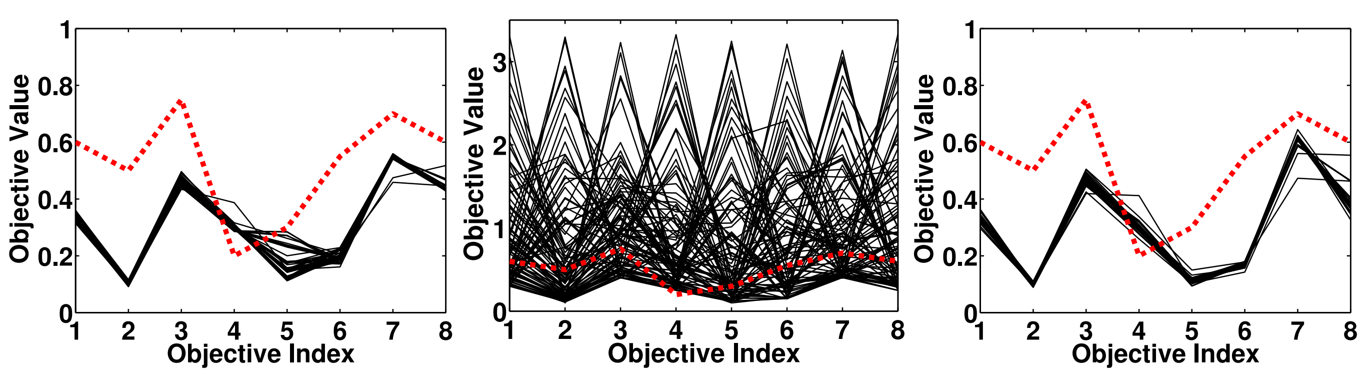


(d) R-NSGA-II (e) g-NSGA-II (f) r-NSGA-II

Fig. 21: Comparisons on 8-objective DTLZ2 where $\mathbf{z}^r = (0.3, 0.1, 0.4, 0.25, 0.1, 0.15, 0.4, 0.25)^T$.

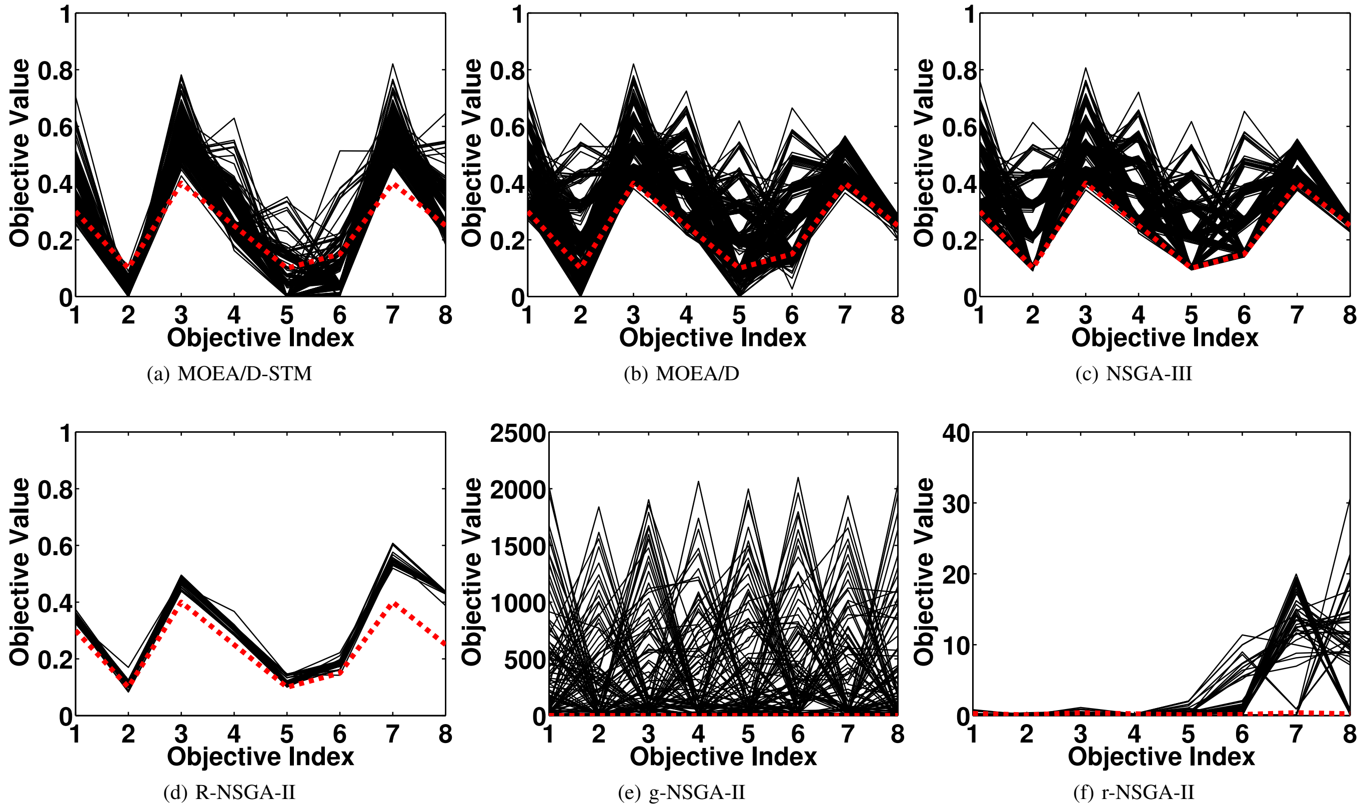


(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III



(d) R-NSGA-II (e) g-NSGA-II (f) r-NSGA-II

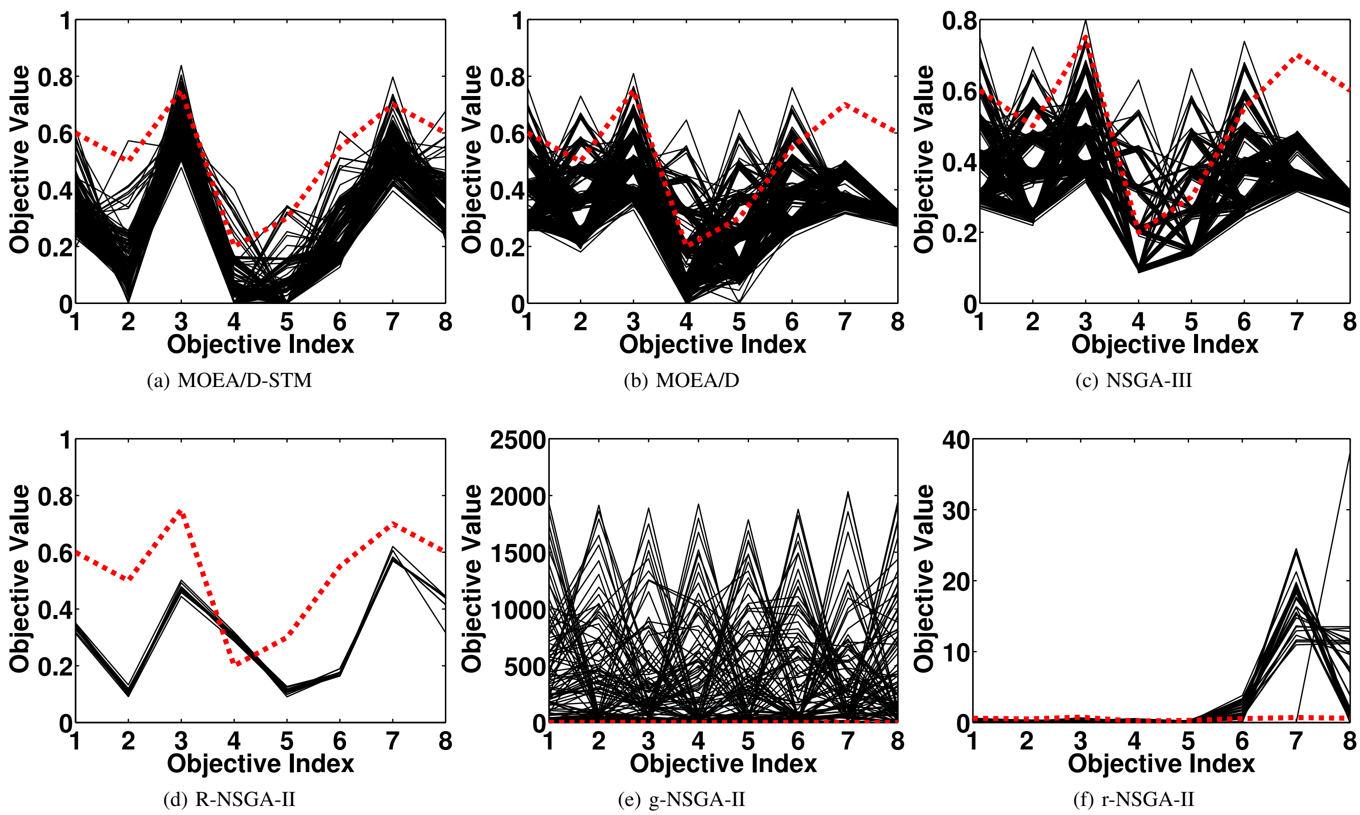
Fig. 22: Comparisons on 8-objective DTLZ2 where $\mathbf{z}^r = (0.6, 0.5, 0.75, 0.2, 0.3, 0.55, 0.7, 0.6)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) R-NSGA-II (e) g-NSGA-II (f) r-NSGA-II

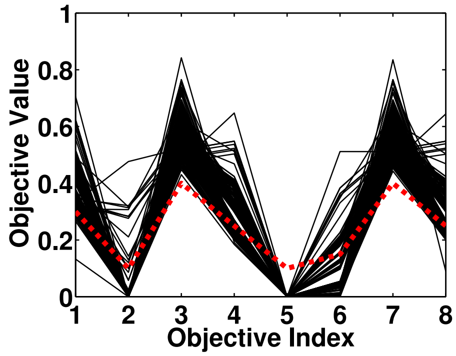
Fig. 23: Comparisons on 8-objective DTLZ3 where $\mathbf{z}^r = (0.3, 0.1, 0.4, 0.25, 0.1, 0.15, 0.4, 0.25)^T$.



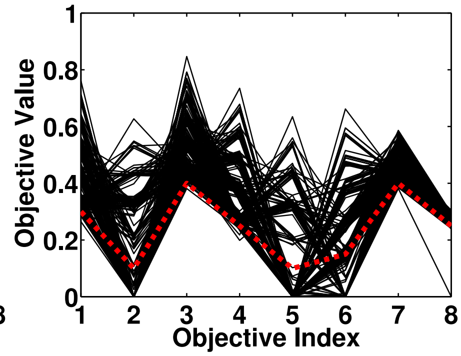
(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) R-NSGA-II (e) g-NSGA-II (f) r-NSGA-II

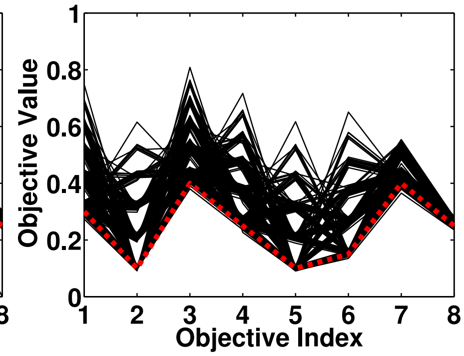
Fig. 24: Comparisons on 8-objective DTLZ3 where $\mathbf{z}^r = (0.6, 0.5, 0.75, 0.2, 0.3, 0.55, 0.7, 0.6)^T$.



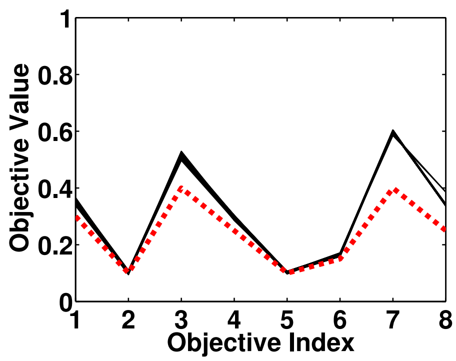
(a) MOEA/D-STM



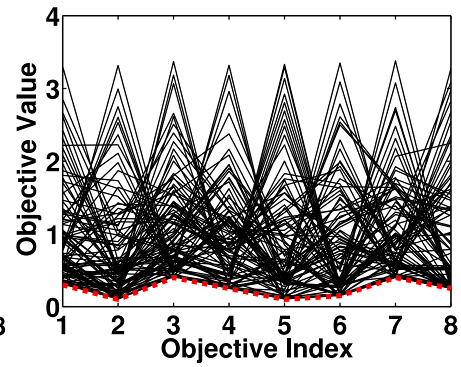
(b) MOEA/D



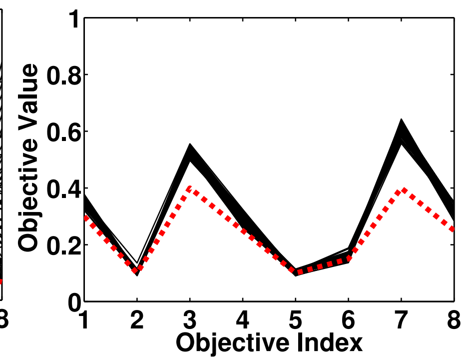
(c) NSGA-III



(d) R-NSGA-II

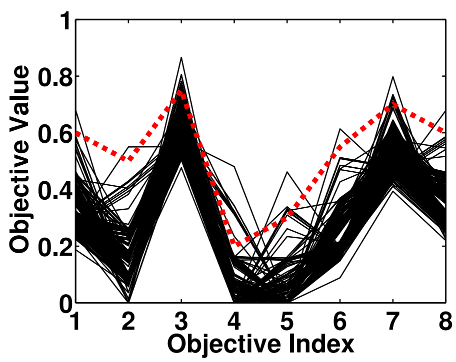


(e) g-NSGA-II

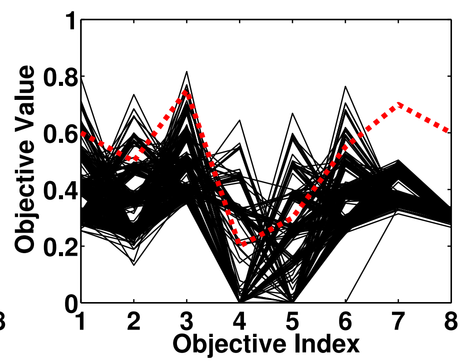


(f) r-NSGA-II

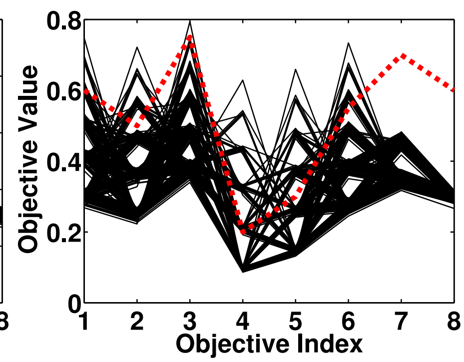
Fig. 25: Comparisons on 8-objective DTLZ4 where $\mathbf{z}^r = (0.3, 0.1, 0.4, 0.25, 0.1, 0.15, 0.4, 0.25)^T$.



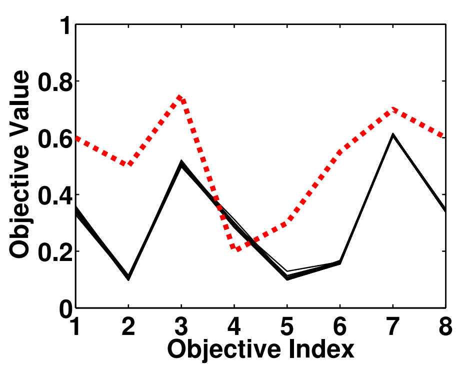
(a) MOEA/D-STM



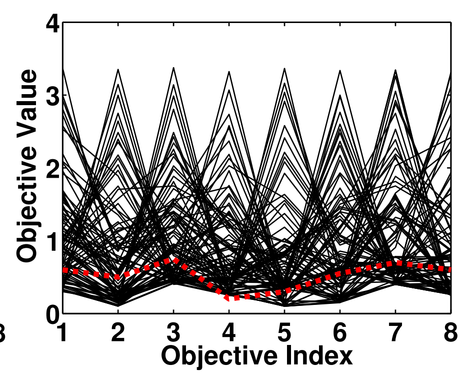
(b) MOEA/D



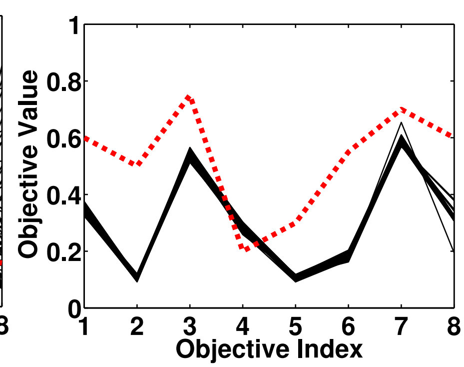
(c) NSGA-III



(d) R-NSGA-II

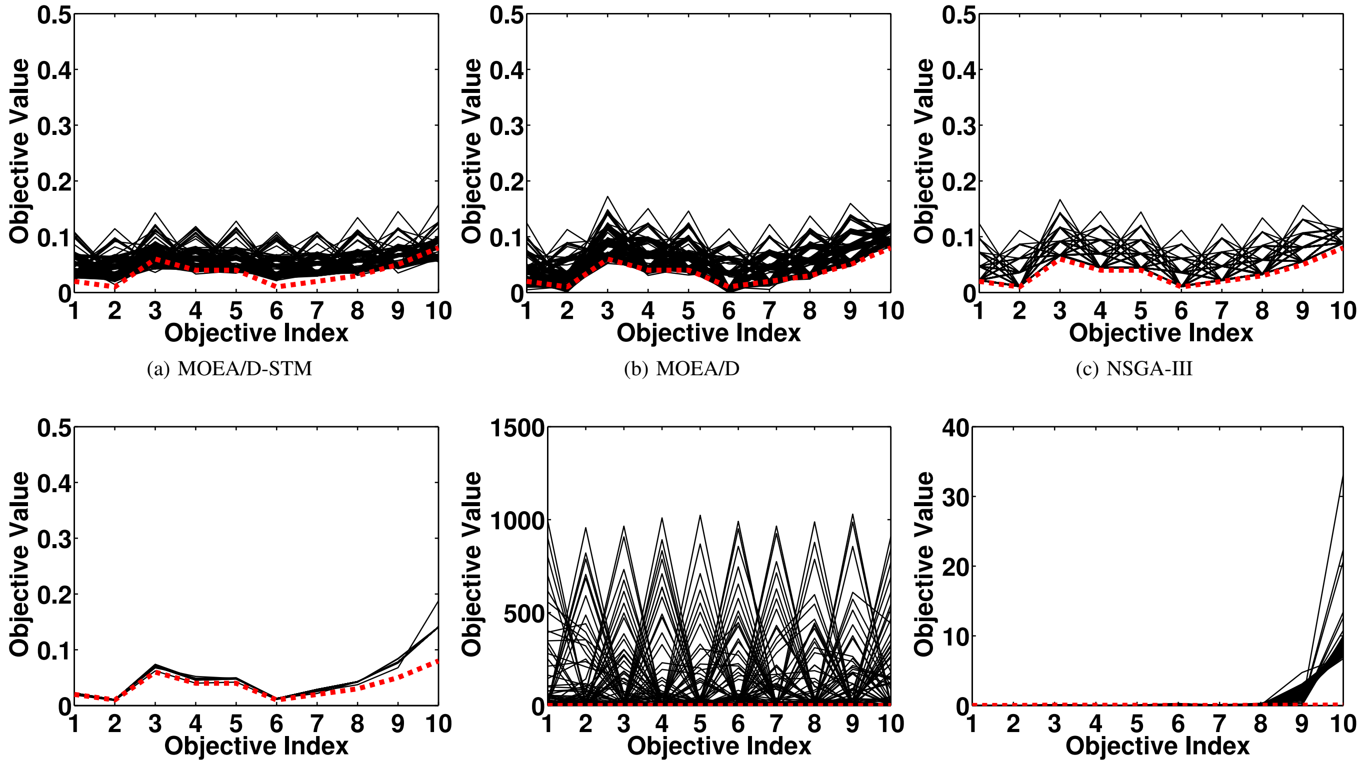


(e) g-NSGA-II



(f) r-NSGA-II

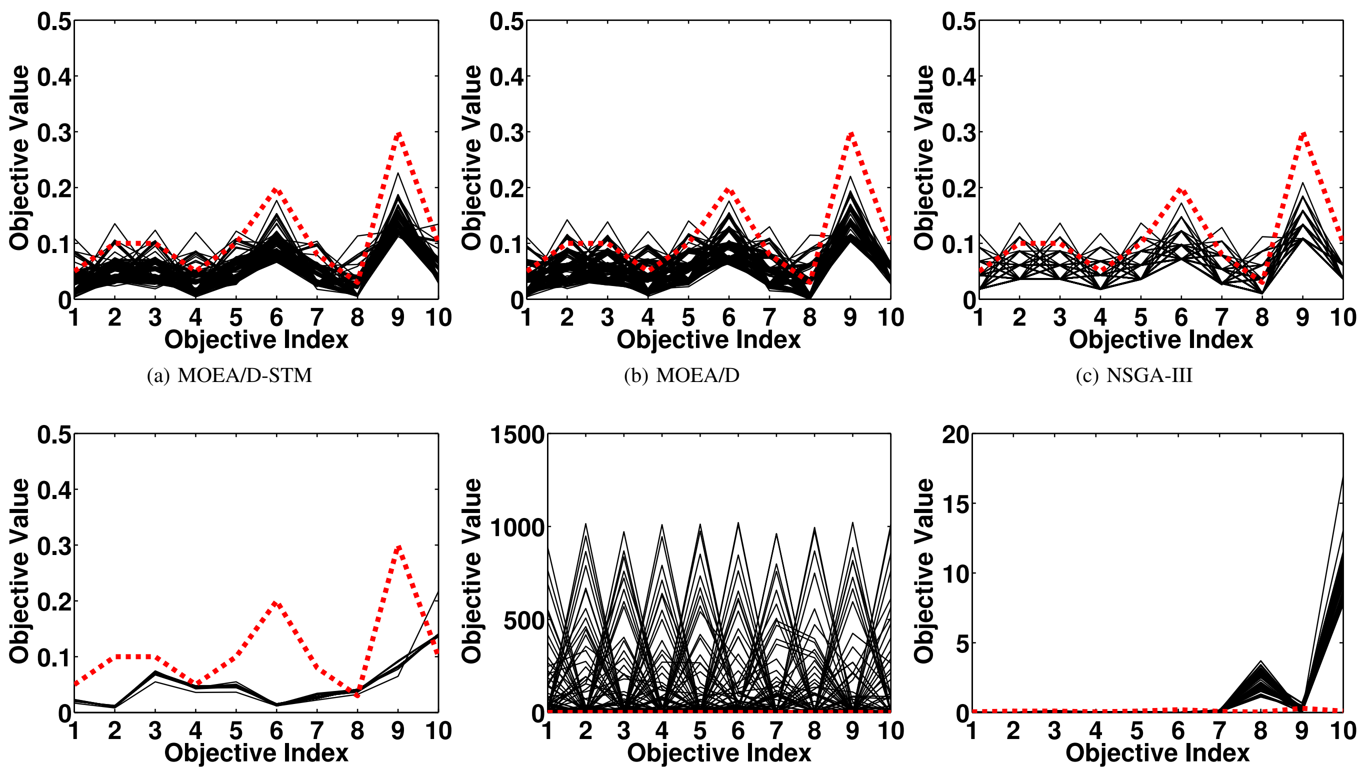
Fig. 26: Comparisons on 8-objective DTLZ4 where $\mathbf{z}^r = (0.6, 0.5, 0.75, 0.2, 0.3, 0.55, 0.7, 0.6)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) R-NSGA-II (e) g-NSGA-II (f) r-NSGA-II

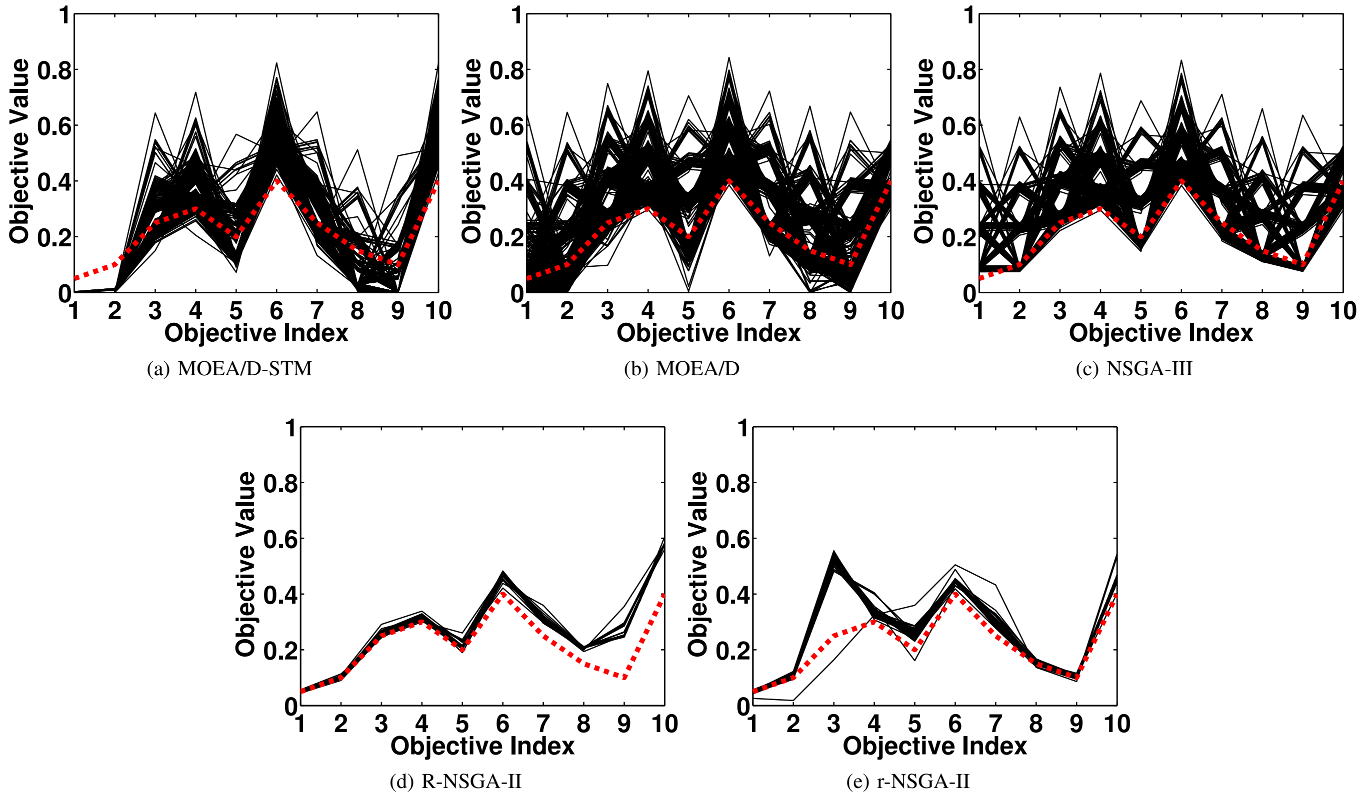
Fig. 27: Comparisons on 10-objective DTLZ1 where $\mathbf{z}^r = (0.02, 0.01, 0.06, 0.04, 0.04, 0.01, 0.02, 0.03, 0.05, 0.08)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) R-NSGA-II (e) g-NSGA-II (f) r-NSGA-II

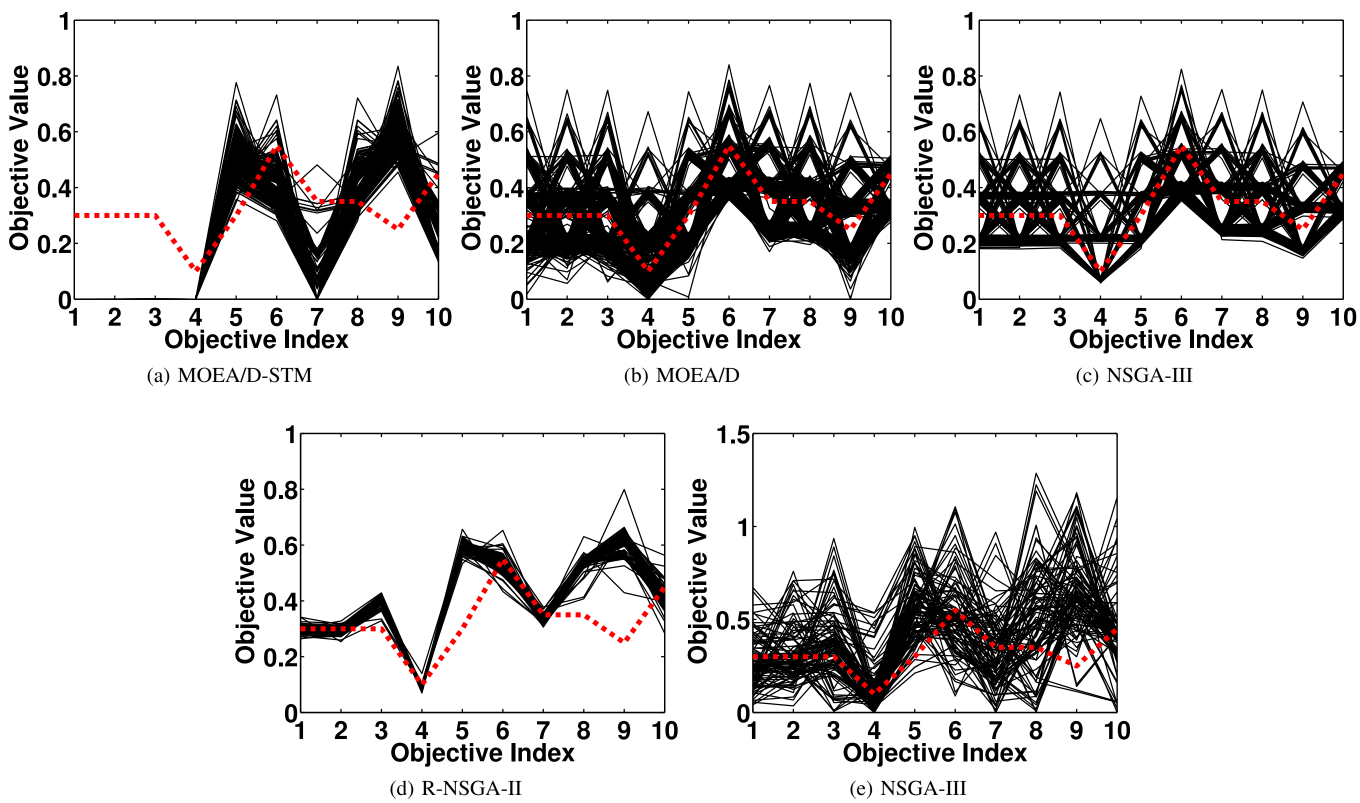
Fig. 28: Comparisons on 10-objective DTLZ1 where $\mathbf{z}^r = (0.05, 0.1, 0.1, 0.05, 0.1, 0.2, 0.08, 0.03, 0.3, 0.1)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) R-NSGA-II (e) r-NSGA-II

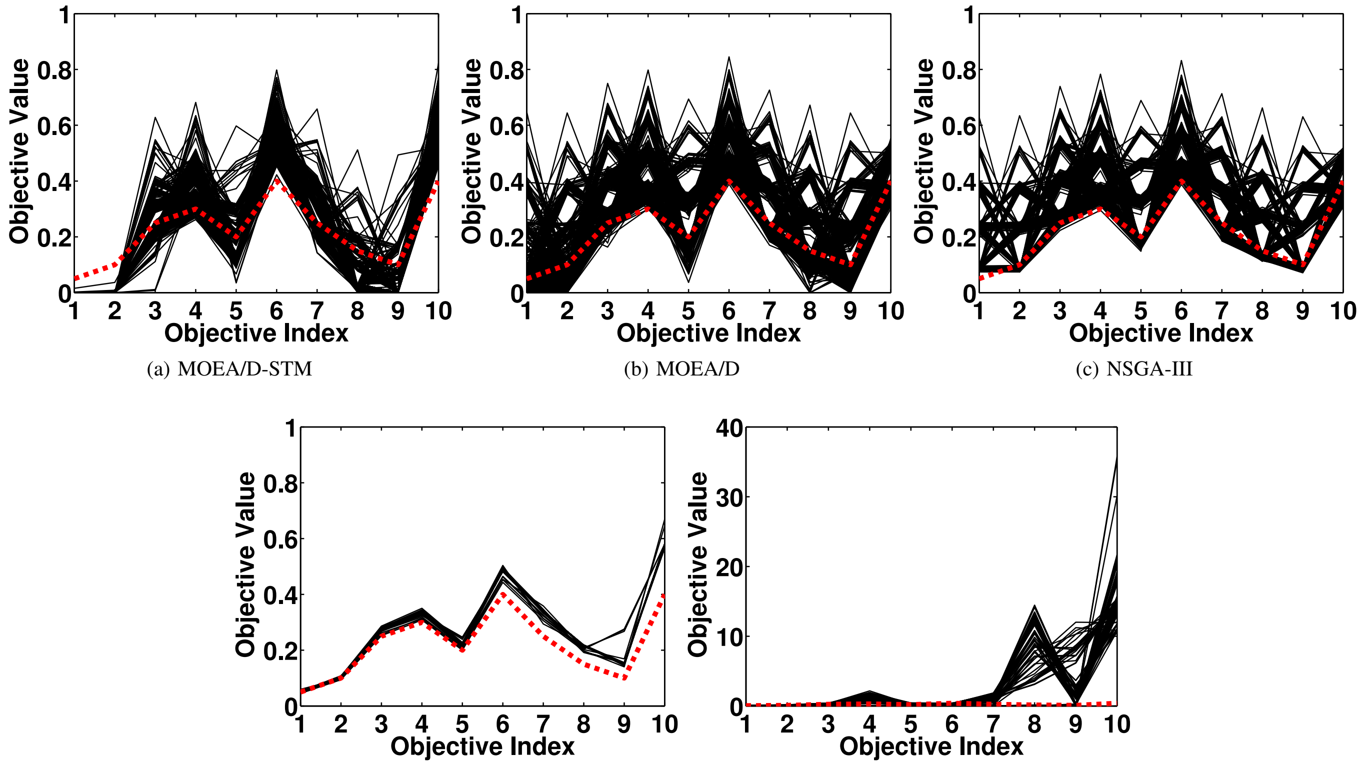
Fig. 29: Comparisons on 10-objective DTLZ2 where $\mathbf{z}^r = (0.05, 0.1, 0.25, 0.3, 0.2, 0.4, 0.25, 0.15, 0.1, 0.4)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) R-NSGA-II (e) NSGA-III

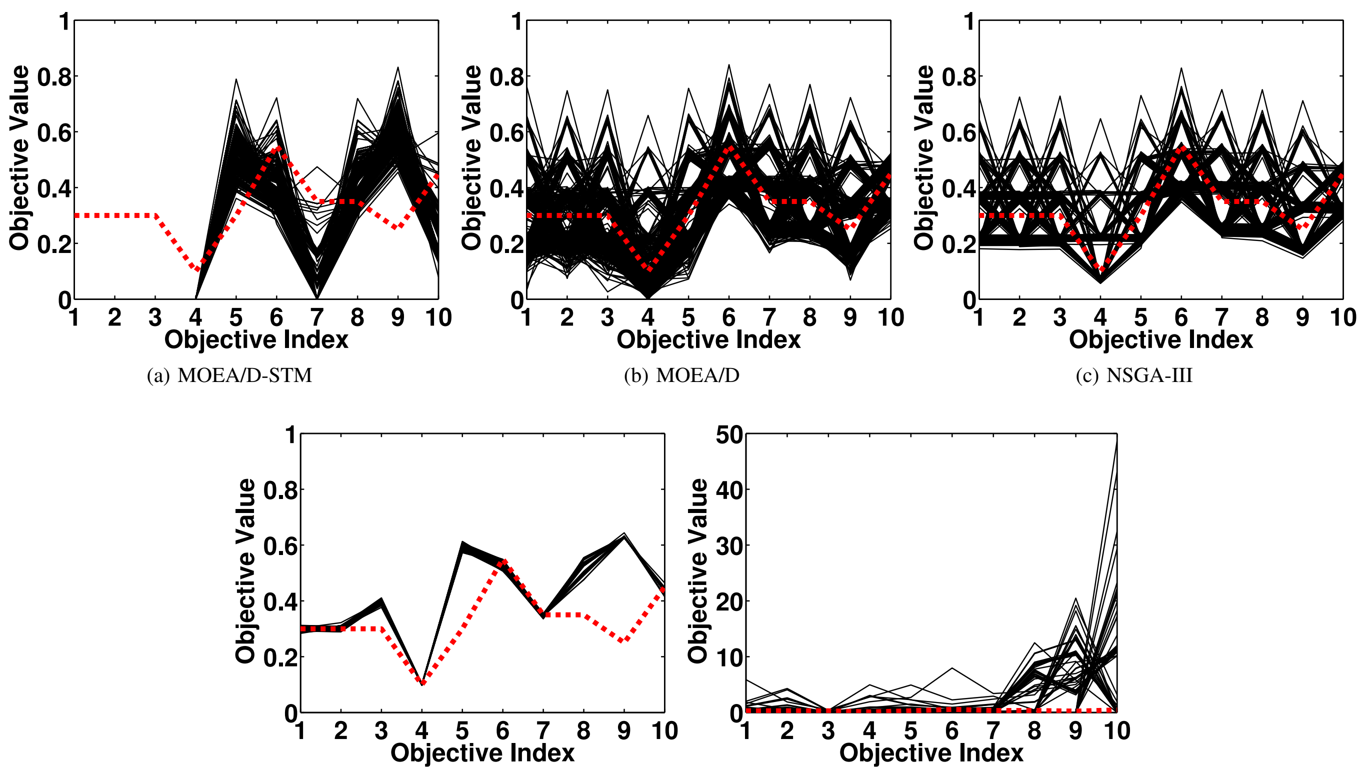
Fig. 30: Comparisons on 10-objective DTLZ2 where $\mathbf{z}^r = (0.3, 0.3, 0.3, 0.1, 0.3, 0.55, 0.35, 0.35, 0.25, 0.45)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) R-NSGA-II (e) r-NSGA-II

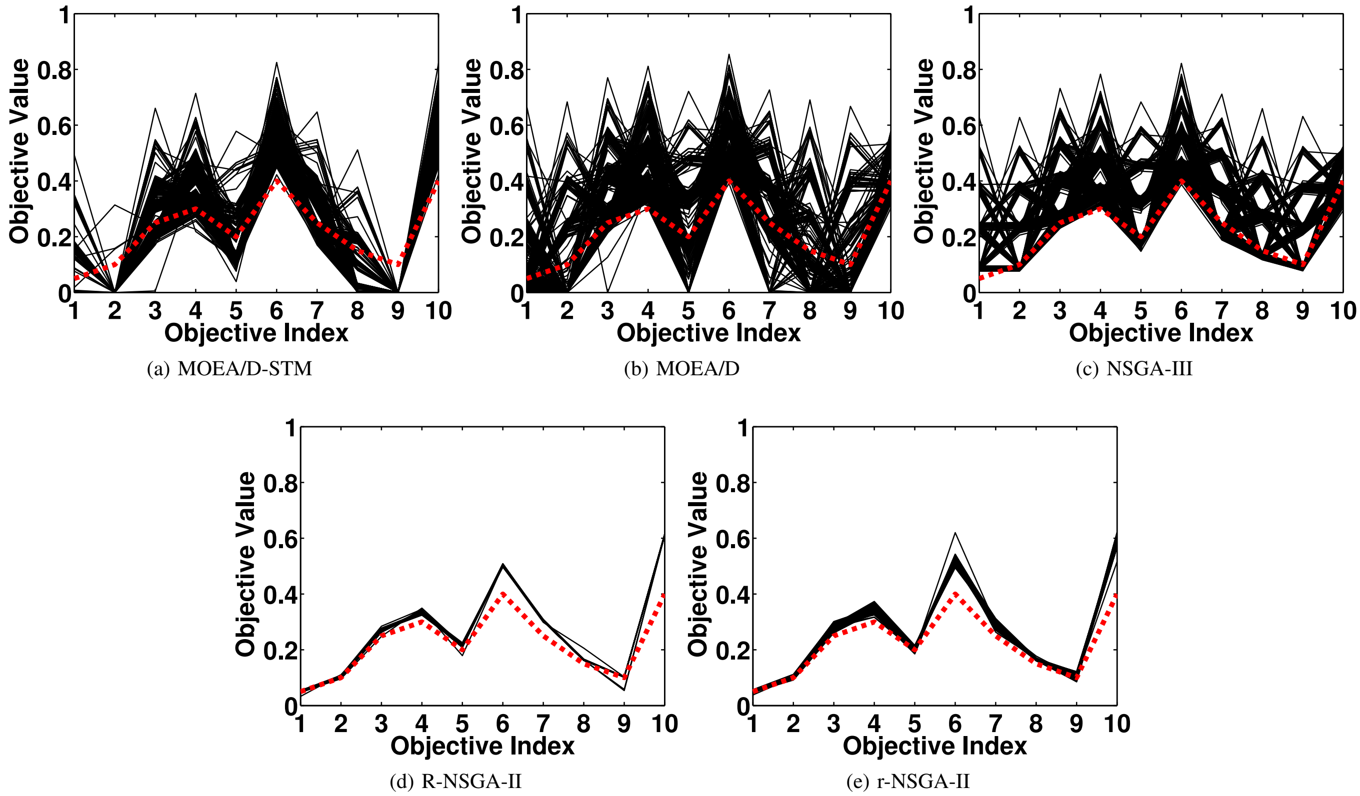
Fig. 31: Comparisons on 10-objective DTLZ3 where $\mathbf{z}^r = (0.05, 0.1, 0.25, 0.3, 0.2, 0.4, 0.25, 0.15, 0.1, 0.4)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) R-NSGA-II (e) r-NSGA-II

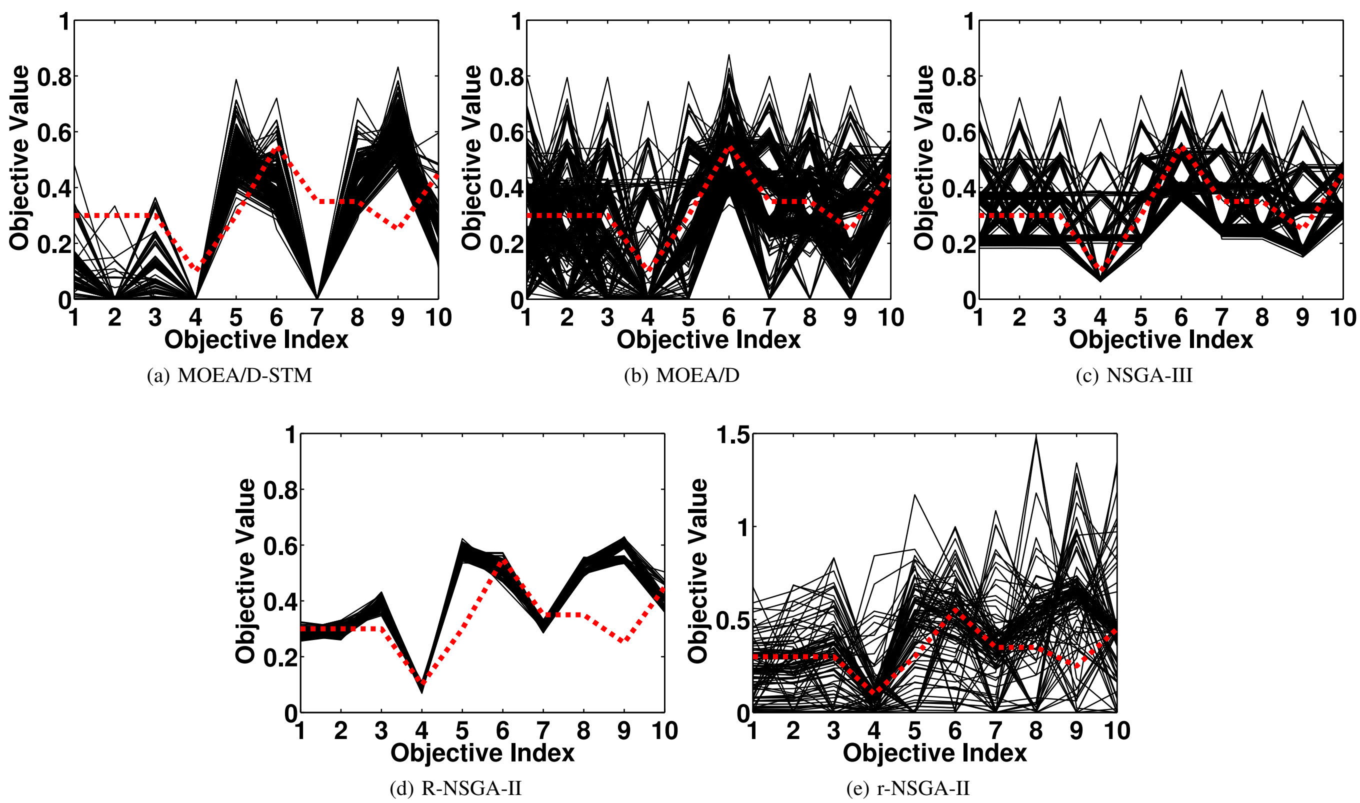
Fig. 32: Comparisons on 10-objective DTLZ3 where $\mathbf{z}^r = (0.3, 0.3, 0.3, 0.1, 0.3, 0.55, 0.35, 0.35, 0.25, 0.45)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) R-NSGA-II (e) r-NSGA-II

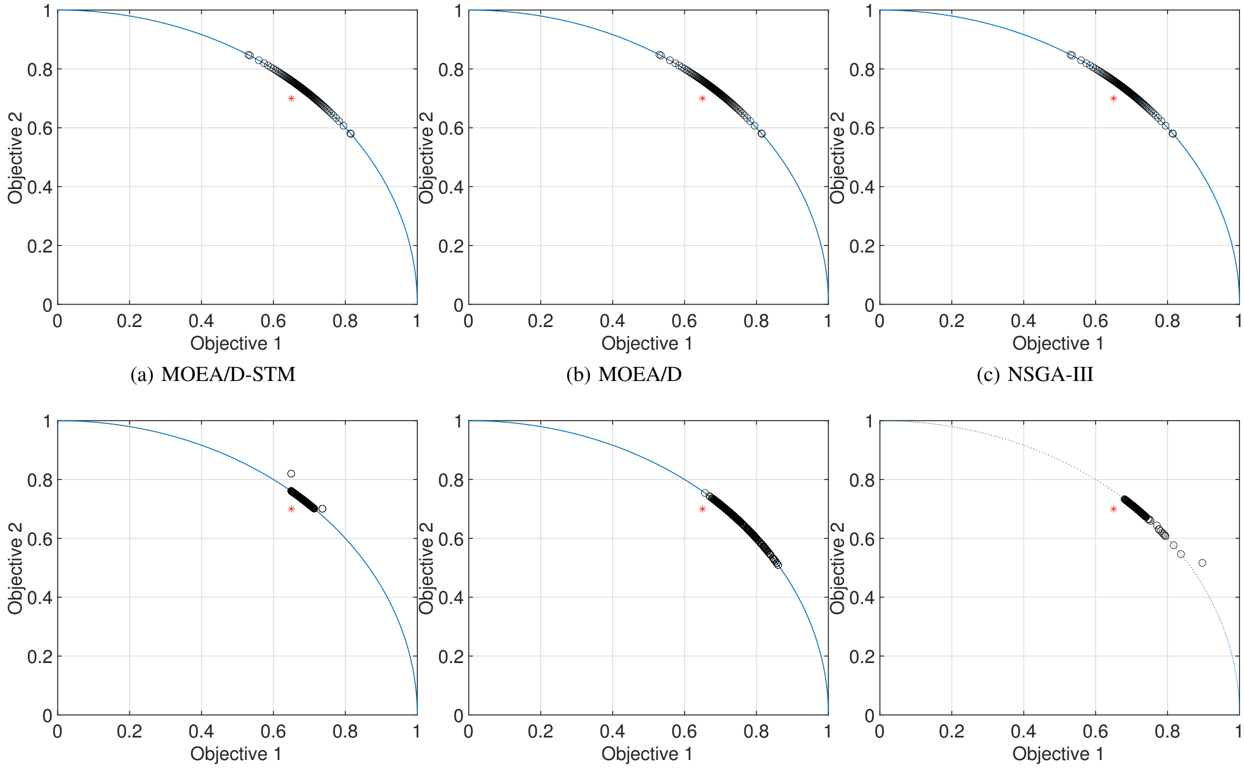
Fig. 33: Comparisons on 10-objective DTLZ4 where $\mathbf{z}^r = (0.05, 0.1, 0.25, 0.3, 0.2, 0.4, 0.25, 0.15, 0.1, 0.4)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) R-NSGA-II (e) r-NSGA-II

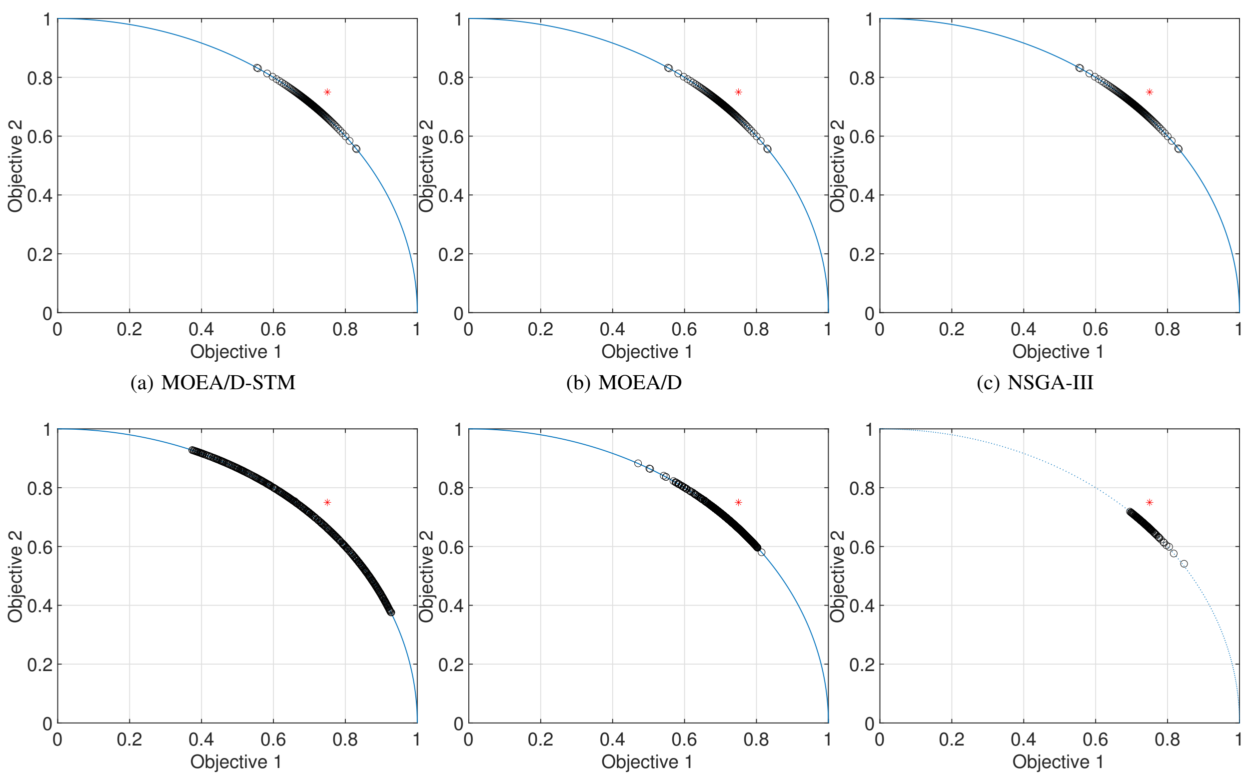
Fig. 34: Comparisons on 10-objective DTLZ4 where $\mathbf{z}^r = (0.3, 0.3, 0.3, 0.1, 0.3, 0.55, 0.35, 0.35, 0.25, 0.45)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

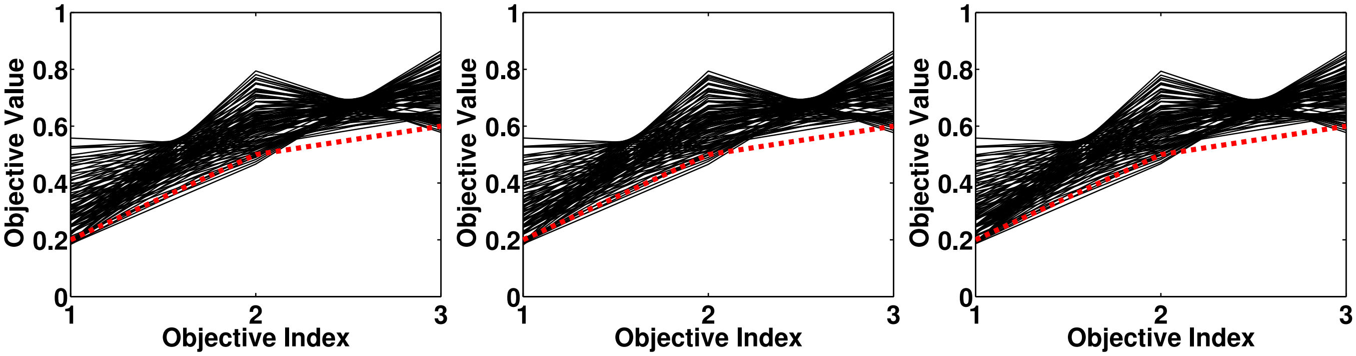
Fig. 35: Comparisons on 2-objective WFG41 where $\mathbf{z}^r = (0.65, 0.7)^T$.



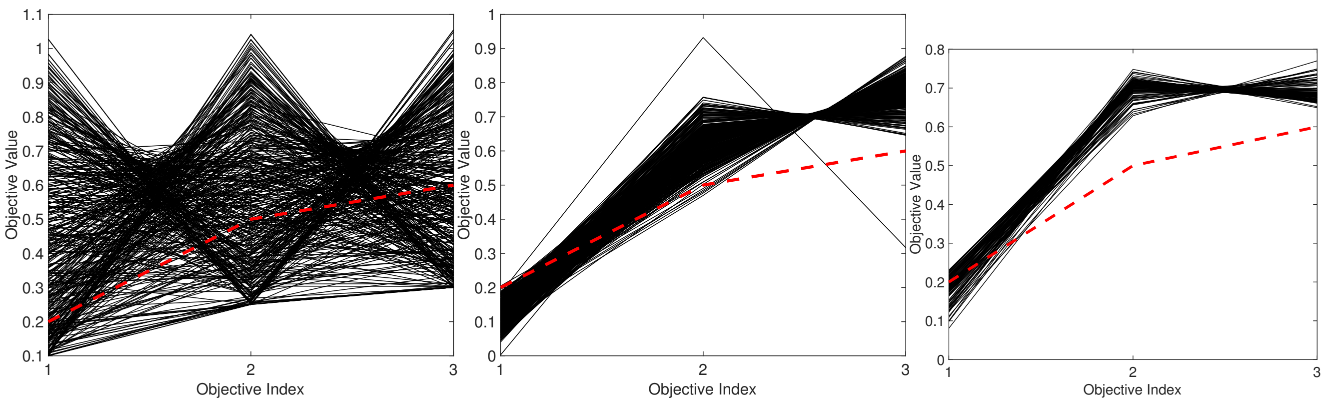
(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

Fig. 36: Comparisons on 2-objective WFG41 where $\mathbf{z}^r = (0.75, 0.75)^T$.

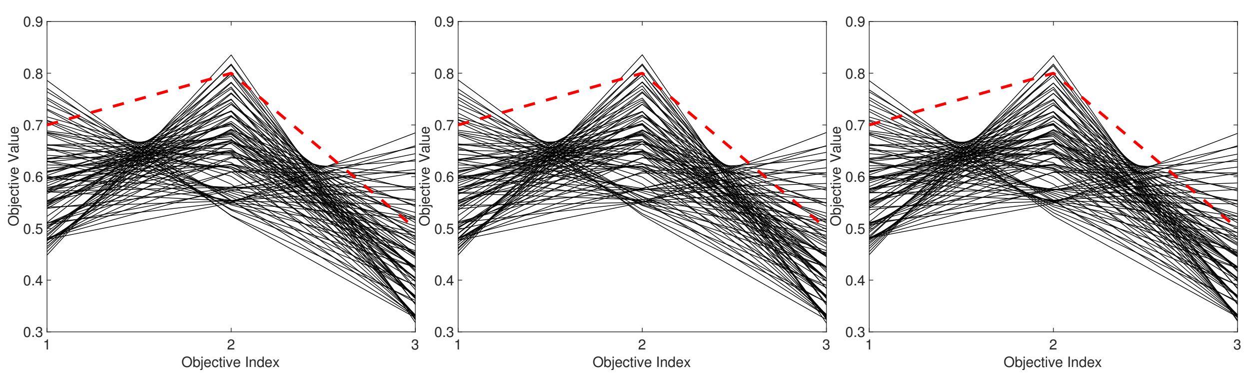


(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

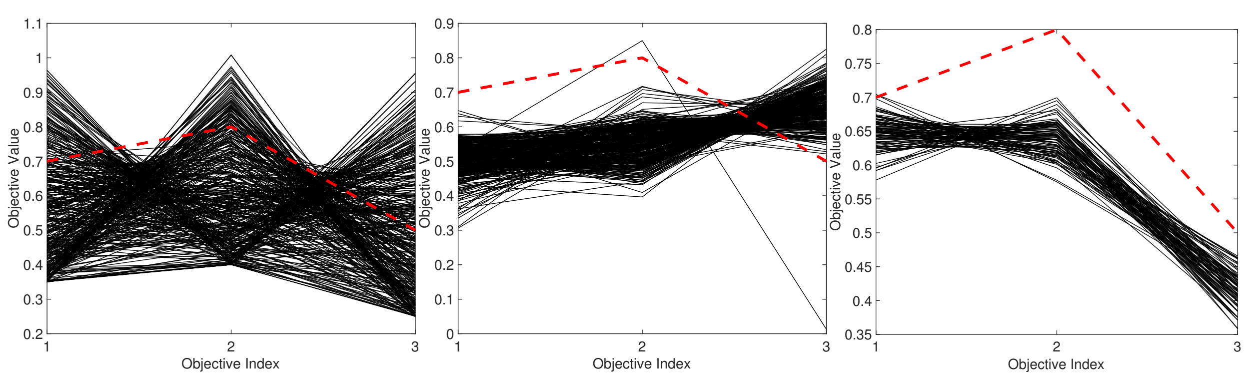


(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

Fig. 37: Comparisons on 3-objective WFG41 where $\mathbf{z}^r = (0.2, 0.5, 0.6)^T$.

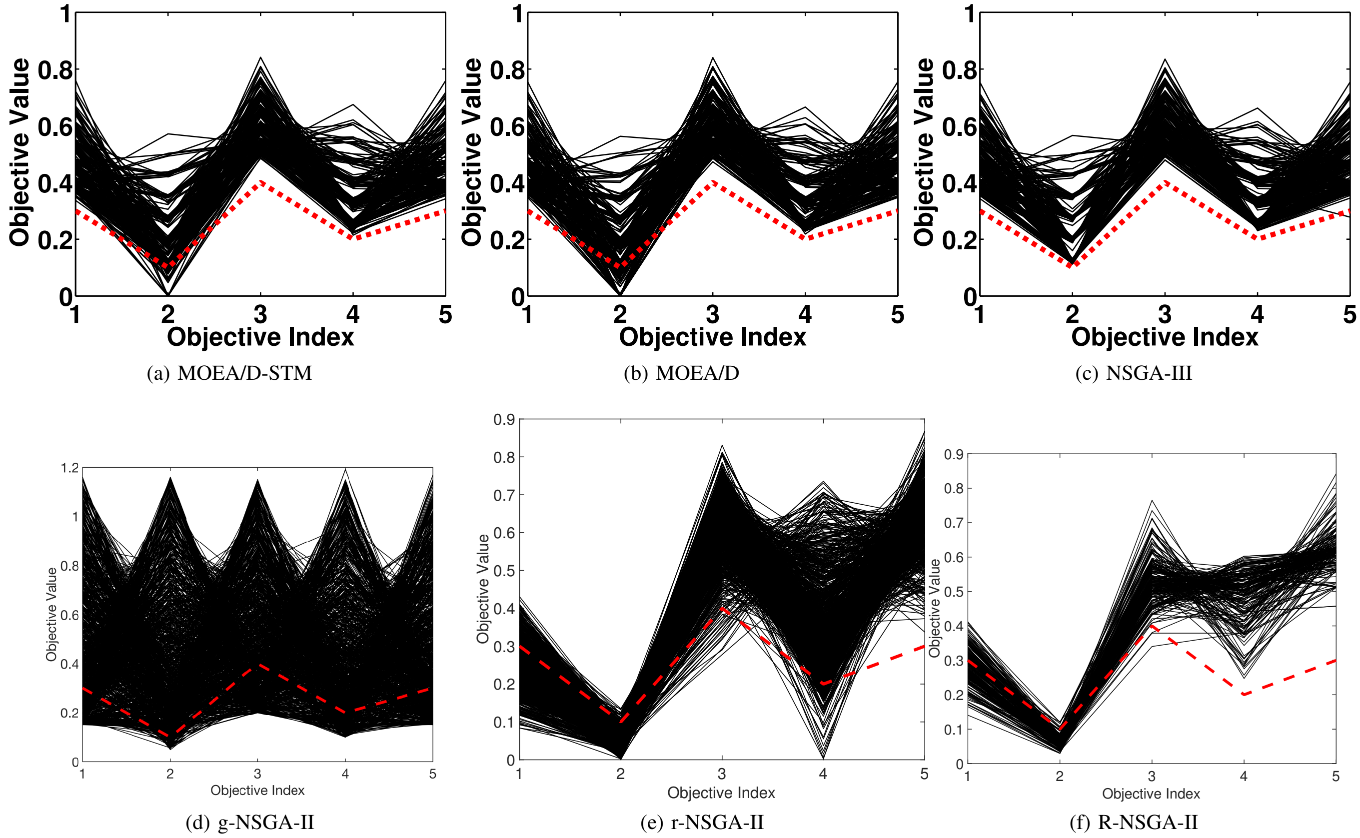


(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III



(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

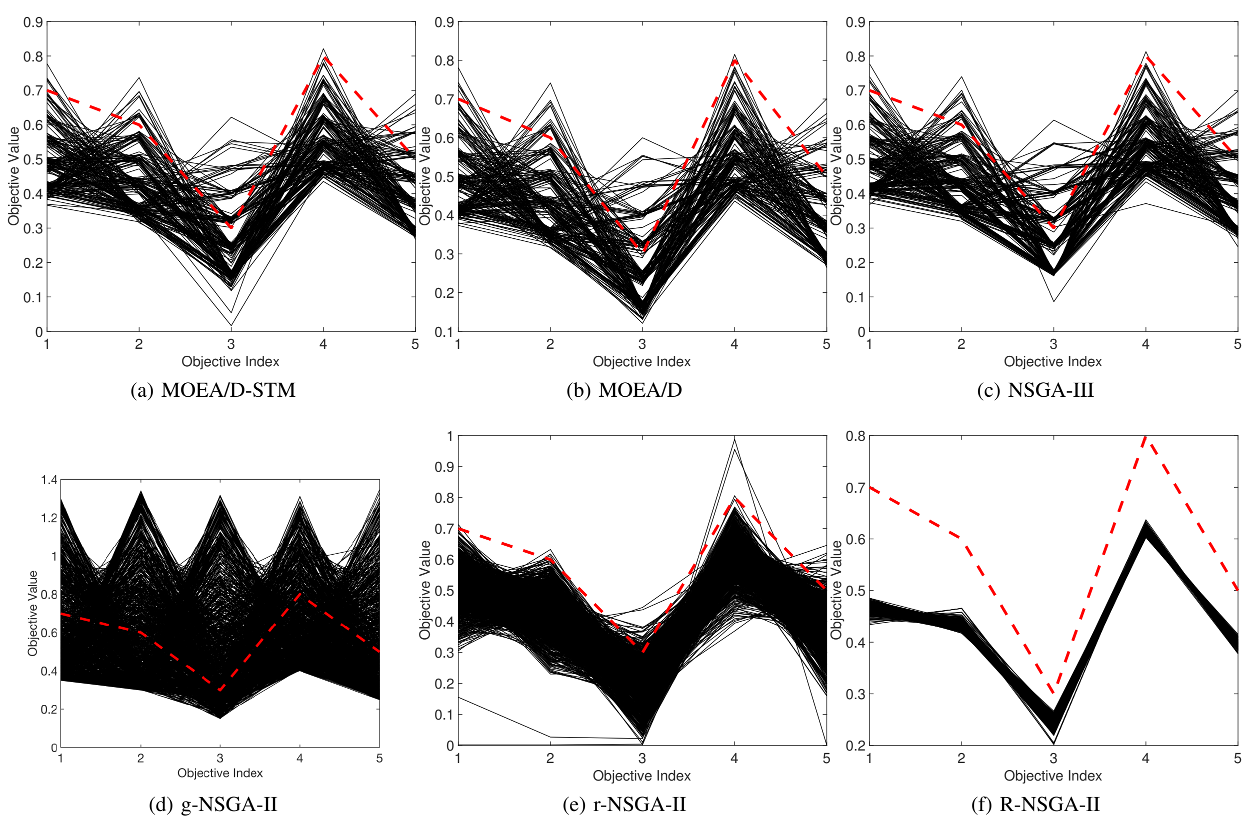
Fig. 38: Comparisons on 3-objective WFG41 where $\mathbf{z}^r = (0.7, 0.8, 0.5)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

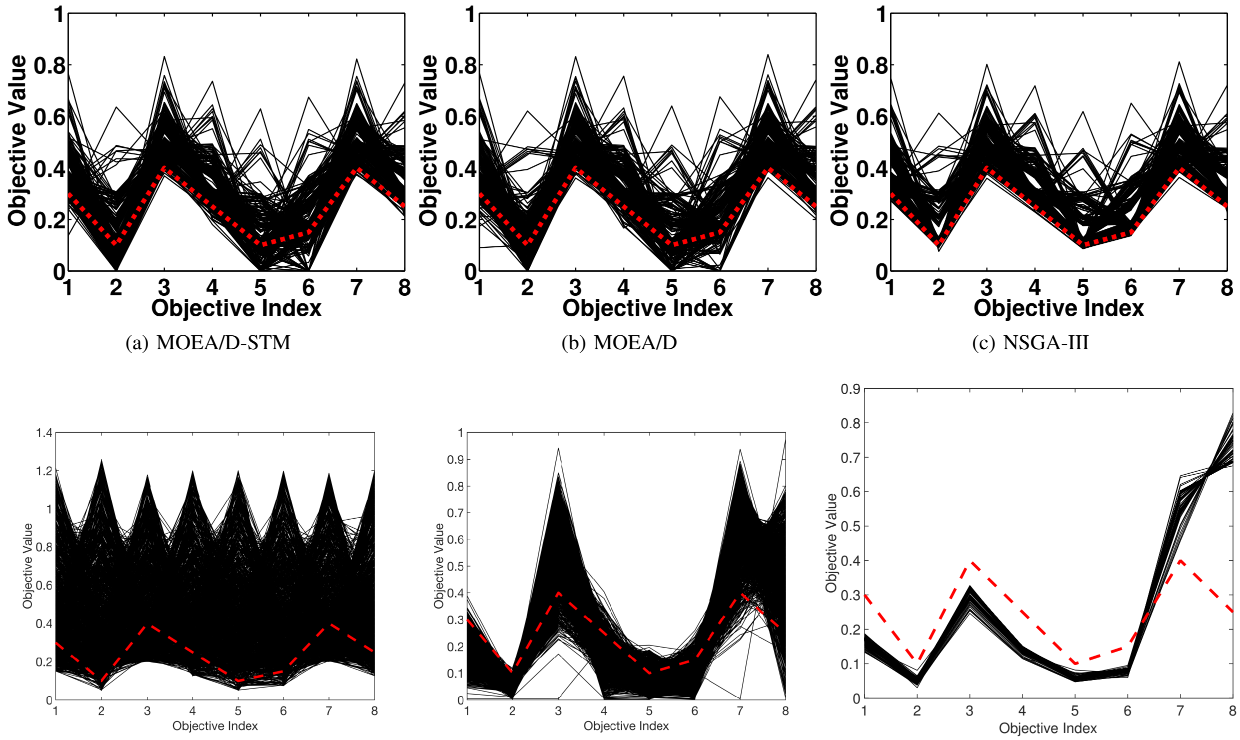
Fig. 39: Comparisons on 5-objective WFG41 where $\mathbf{z}^r = (0.3, 0.1, 0.4, 0.2, 0.3)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

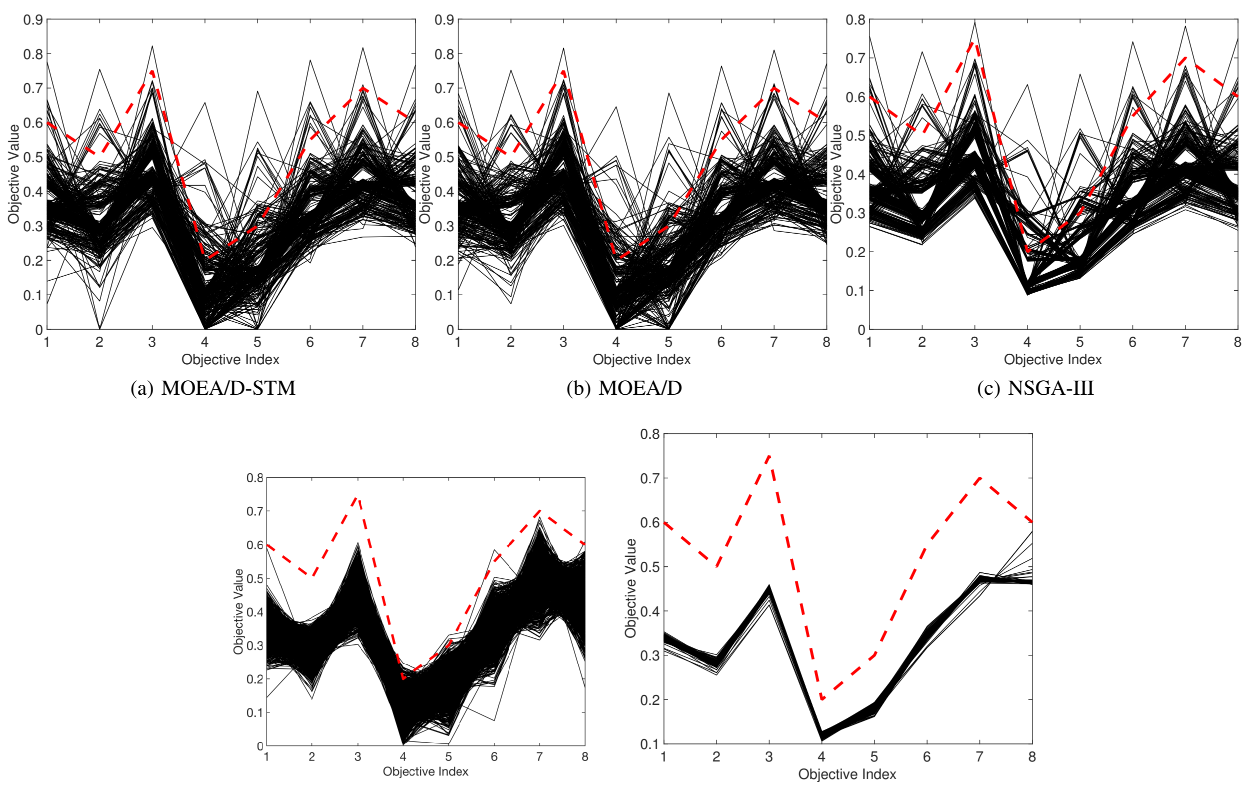
Fig. 40: Comparisons on 5-objective WFG41 where $\mathbf{z}^r = (0.7, 0.6, 0.3, 0.8, 0.5)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

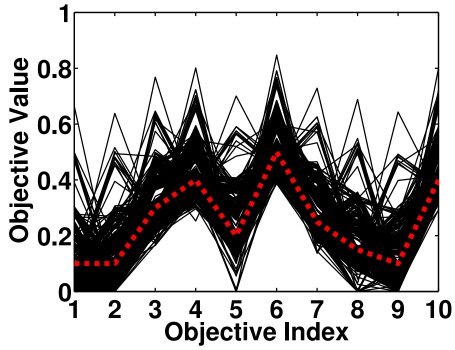
Fig. 41: Comparisons on 8-objective WFG41 where $\mathbf{z}^r = (0.3, 0.1, 0.4, 0.25, 0.1, 0.15, 0.4, 0.25)^T$.



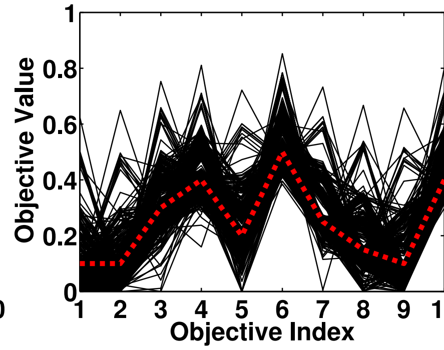
(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) r-NSGA-II (e) R-NSGA-II

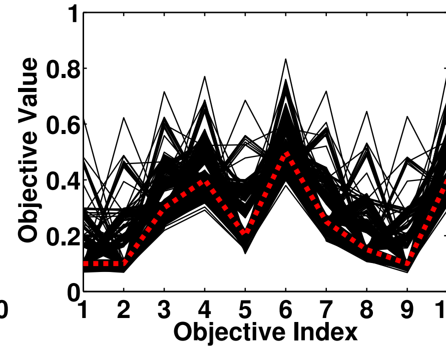
Fig. 42: Comparisons on 8-objective WFG41 where $\mathbf{z}^r = (0.6, 0.5, 0.75, 0.2, 0.3, 0.55, 0.7, 0.6)^T$.



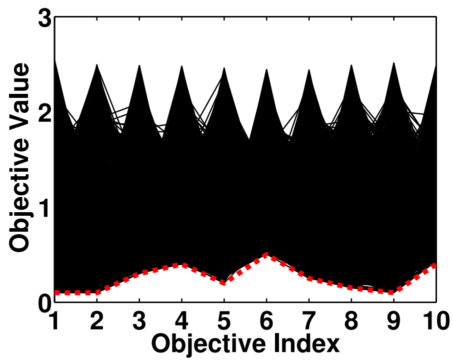
(a) MOEA/D-STM



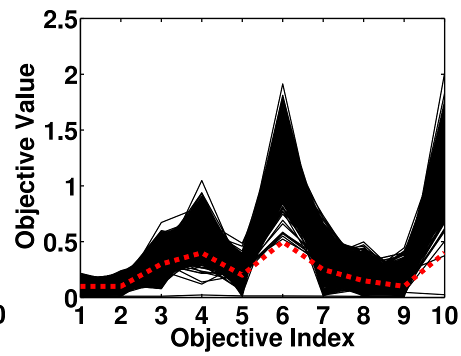
(b) MOEA/D



(c) NSGA-III

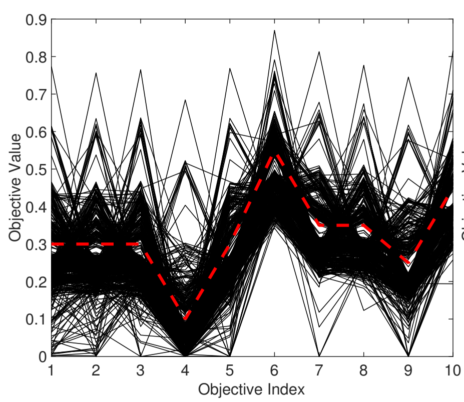


(d) g-NSGA-II

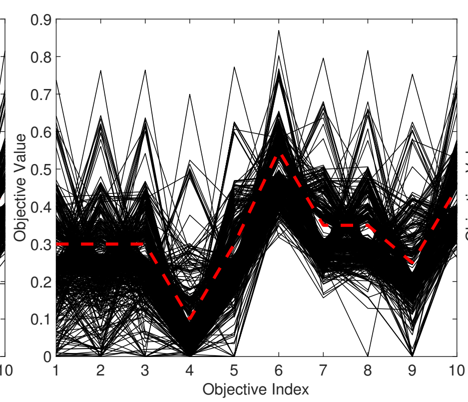


(e) r-NSGA-II

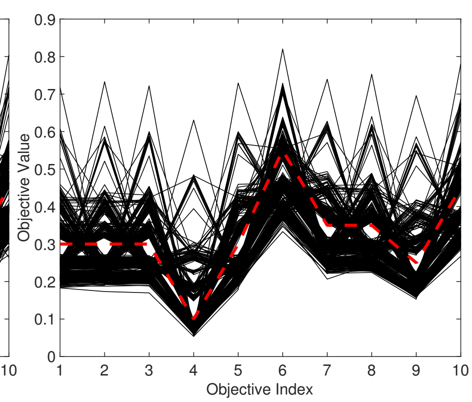
Fig. 43: Comparisons on 10-objective WFG41 where $\mathbf{z}^r = (0.1, 0.1, 0.3, 0.4, 0.2, 0.5, 0.25, 0.15, 0.1, 0.4)^T$.



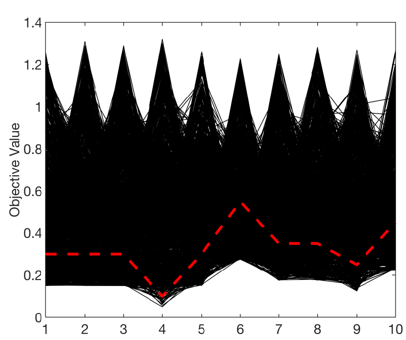
(a) MOEA/D-STM



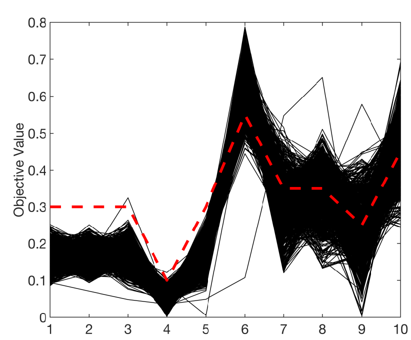
(b) MOEA/D



(c) NSGA-III

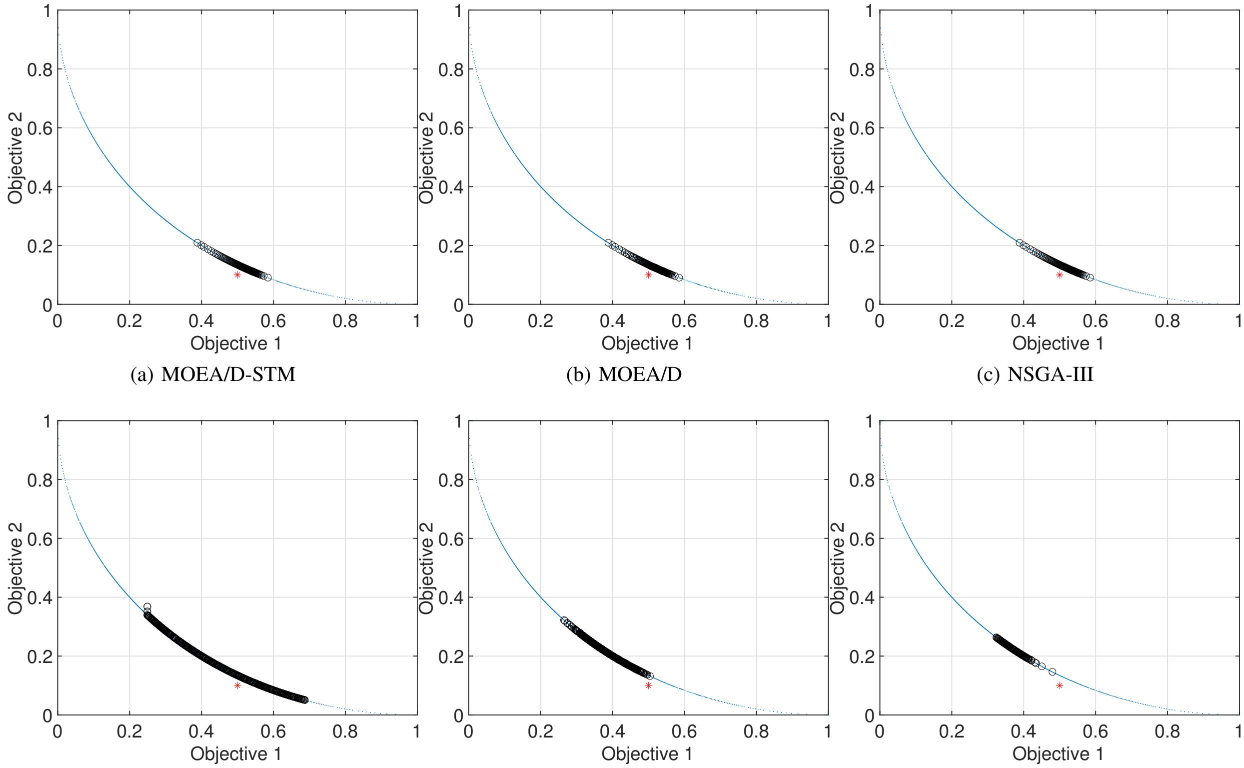


(d) g-NSGA-II



(e) r-NSGA-II

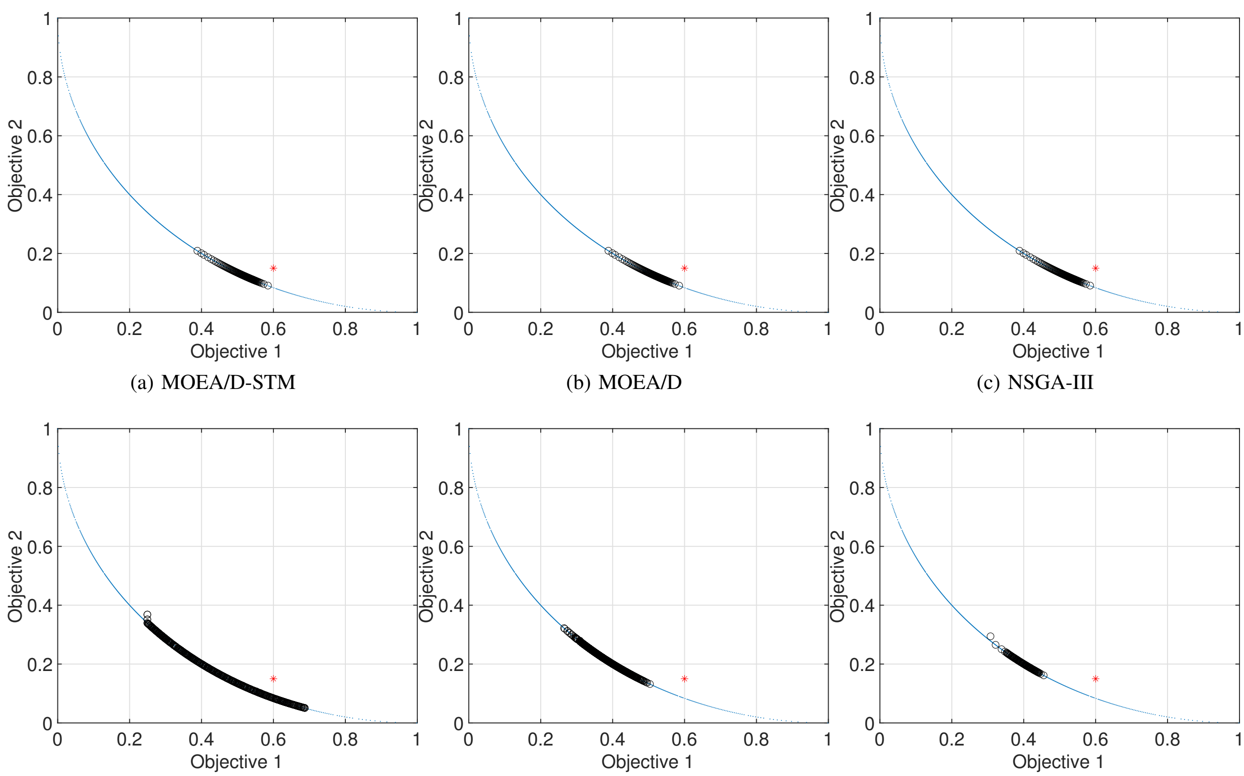
Fig. 44: Comparisons on 10-objective WFG41 where $\mathbf{z}^r = (0.3, 0.3, 0.3, 0.1, 0.3, 0.55, 0.35, 0.35, 0.25, 0.45)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

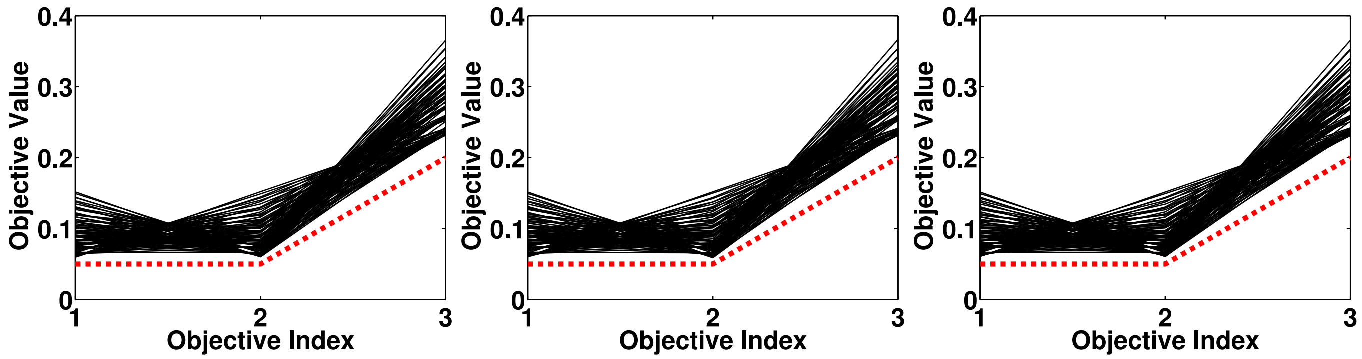
Fig. 45: Comparisons on 2-objective WFG42 where $\mathbf{z}^r = (0.5, 0.1)^T$.



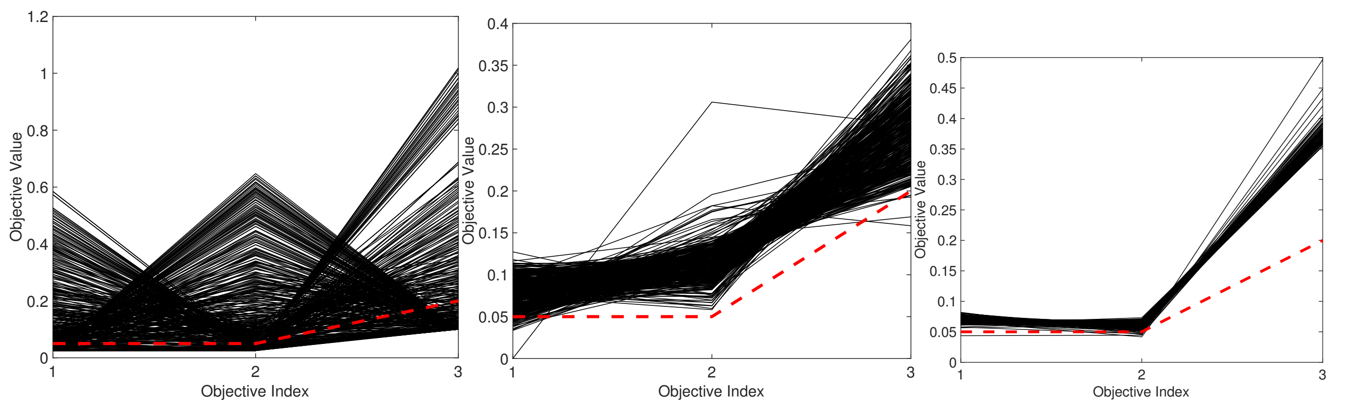
(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

Fig. 46: Comparisons on 2-objective WFG42 where $\mathbf{z}^r = (0.6, 0.15)^T$.

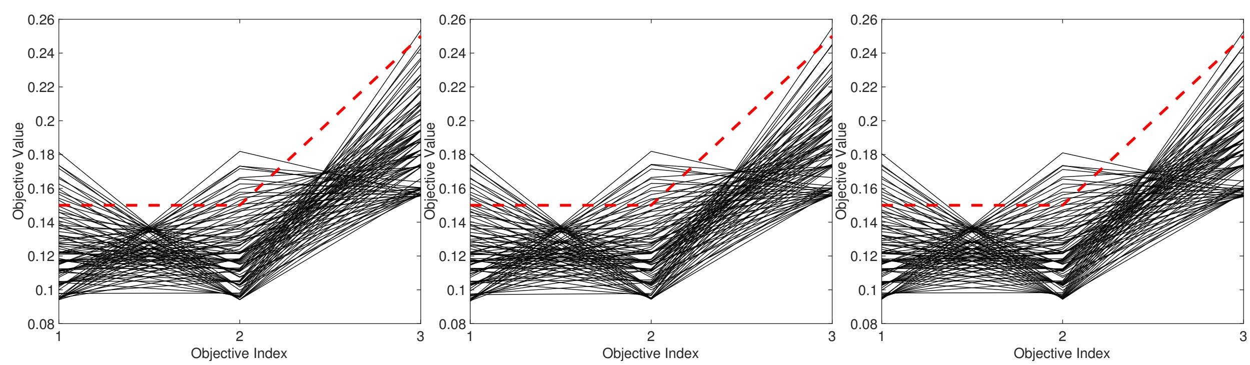


(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

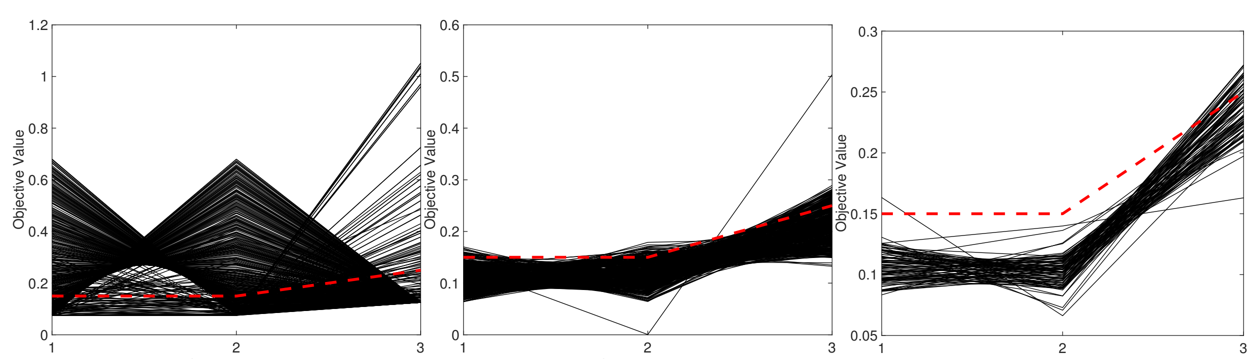


(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

Fig. 47: Comparisons on 3-objective WFG42 where $\mathbf{z}^r = (0.05, 0.05, 0.2)^T$.

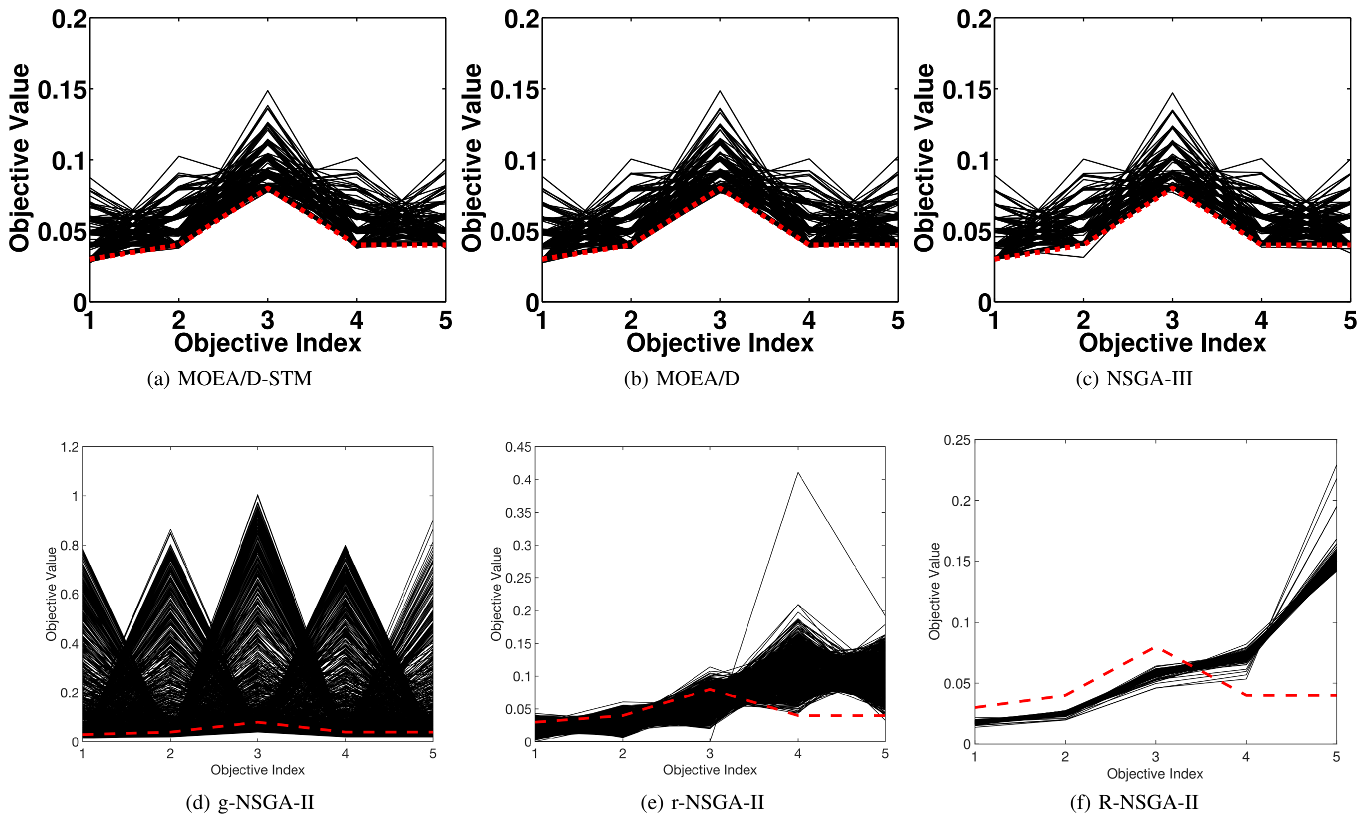


(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III



(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

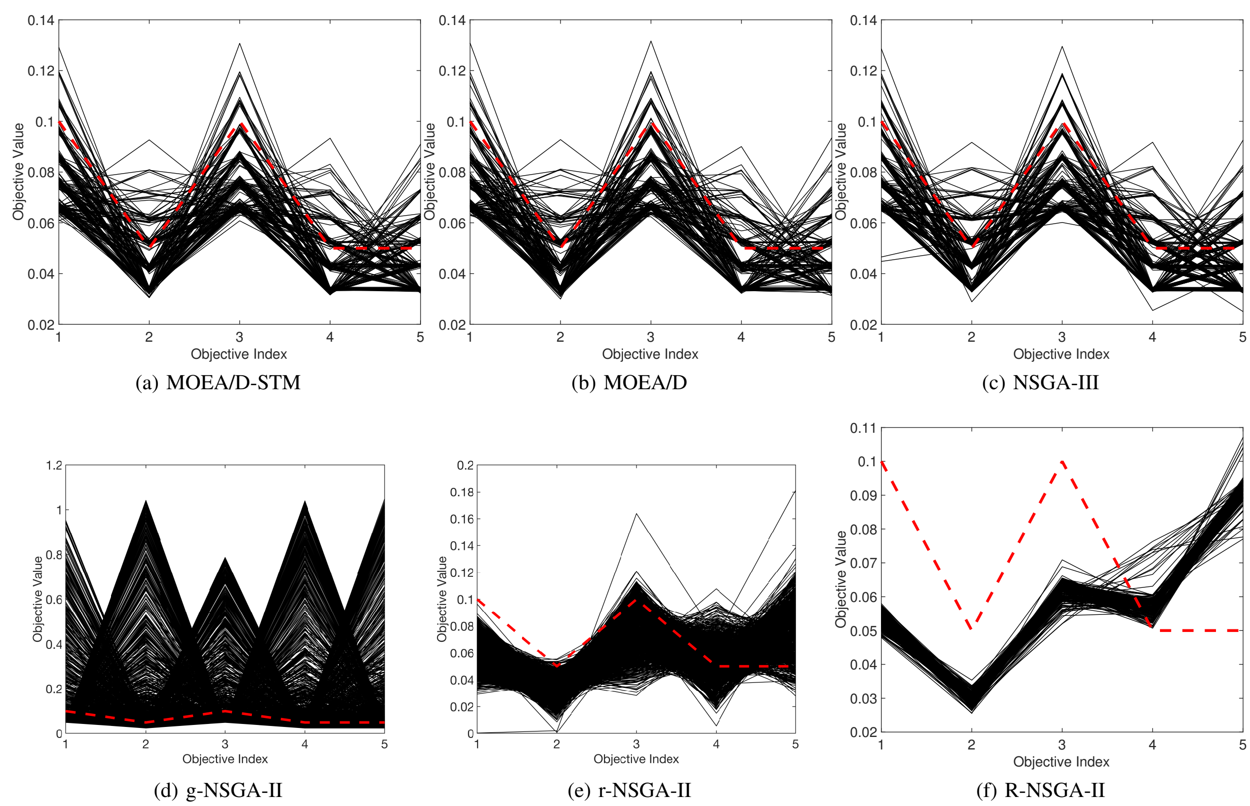
Fig. 48: Comparisons on 3-objective WFG42 where $\mathbf{z}^r = (0.15, 0.15, 0.25)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

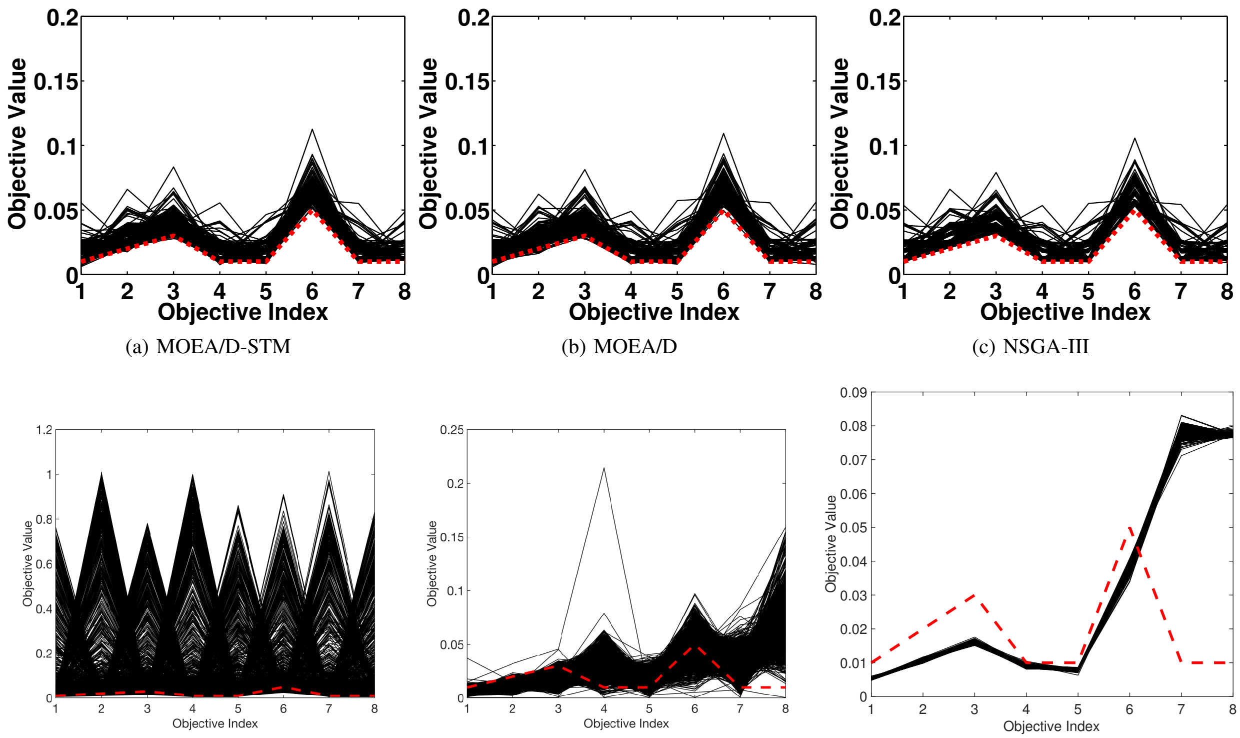
Fig. 49: Comparisons on 5-objective WFG42 where $\mathbf{z}^r = (0.03, 0.04, 0.08, 0.04, 0.04)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

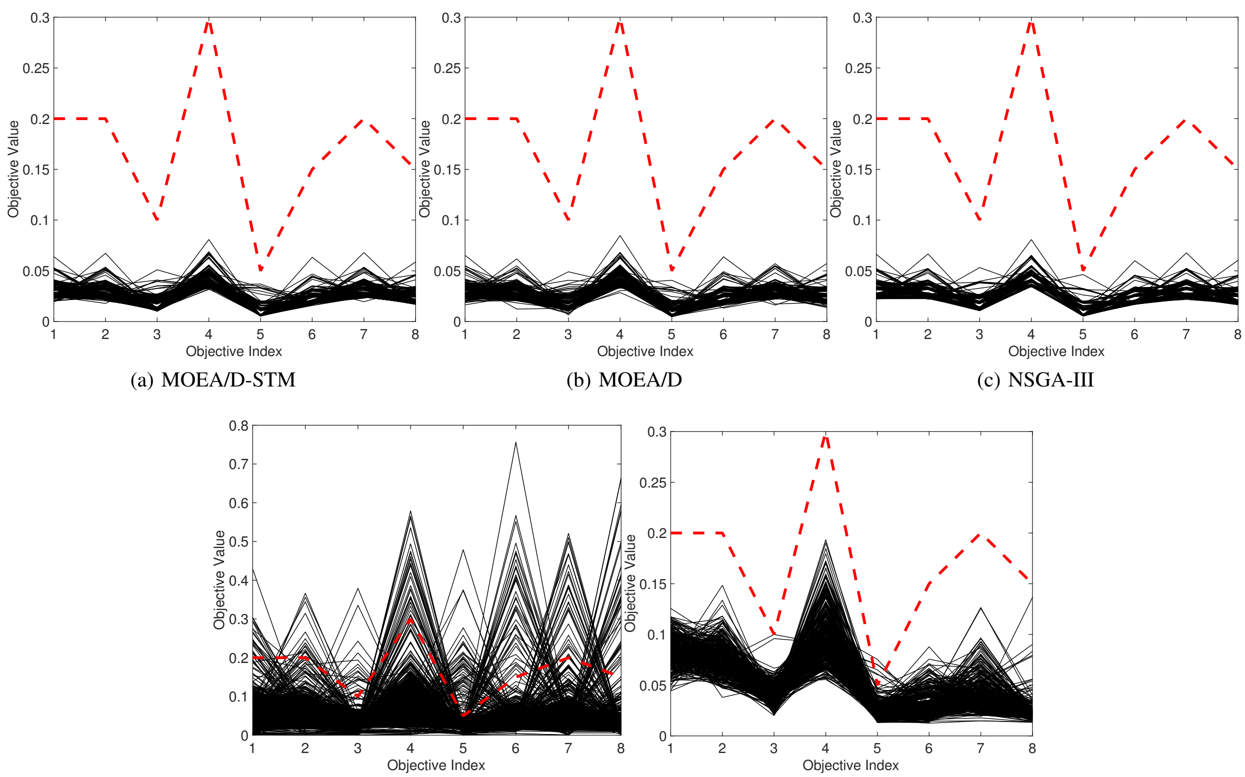
Fig. 50: Comparisons on 5-objective WFG42 where $\mathbf{z}^r = (0.1, 0.05, 0.1, 0.05, 0.05)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

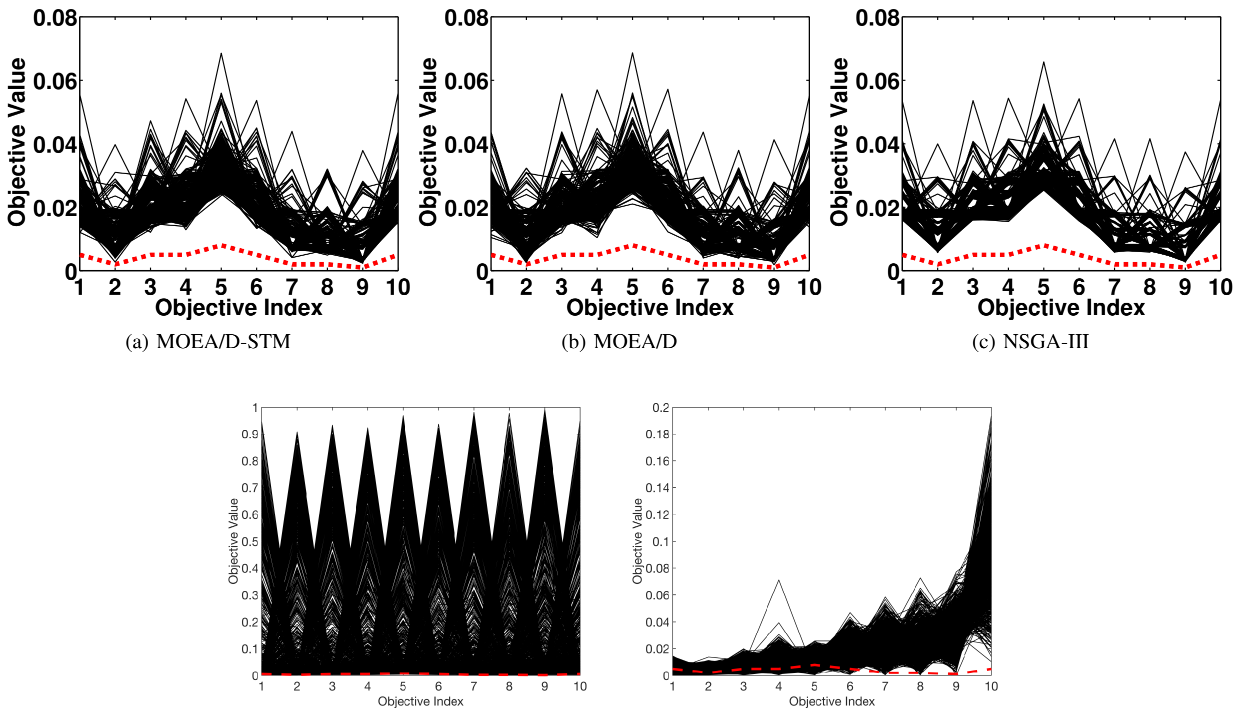
Fig. 51: Comparisons on 8-objective WFG42 where $\mathbf{z}^r = (0.01, 0.02, 0.03, 0.01, 0.01, 0.05, 0.01, 0.01)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) r-NSGA-II (e) R-NSGA-II

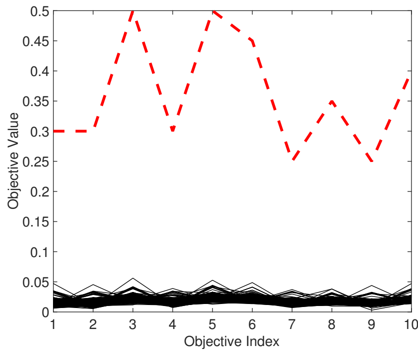
Fig. 52: Comparisons on 8-objective WFG42 where $\mathbf{z}^r = (0.2, 0.2, 0.1, 0.3, 0.05, 0.15, 0.2, 0.15)^T$.



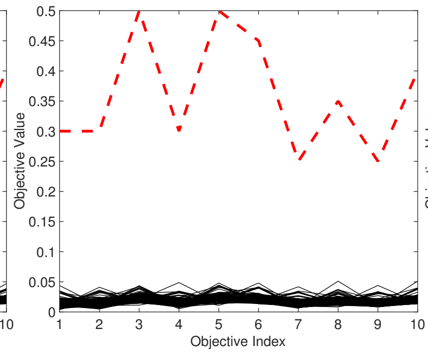
(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II

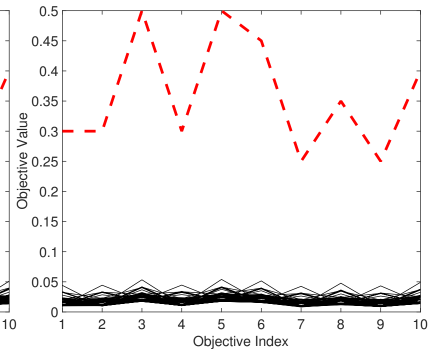
Fig. 53: Comparisons on 10-objective WFG42 where $\mathbf{z}^r = (0.005, 0.002, 0.005, 0.005, 0.008, 0.005, 0.002, 0.002, 0.001, 0.005)^T$.



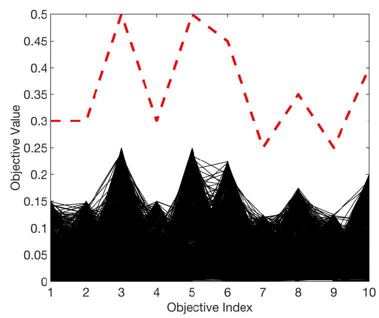
(a) MOEA/D-STM



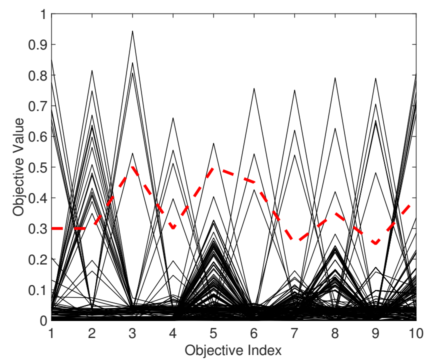
(b) MOEA/D



(c) NSGA-III

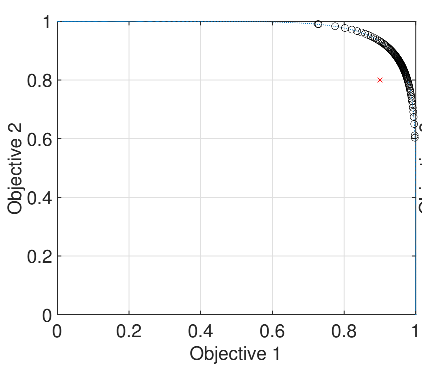


(d) g-NSGA-II

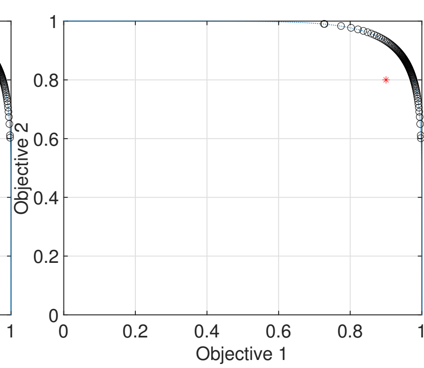


(e) r-NSGA-II

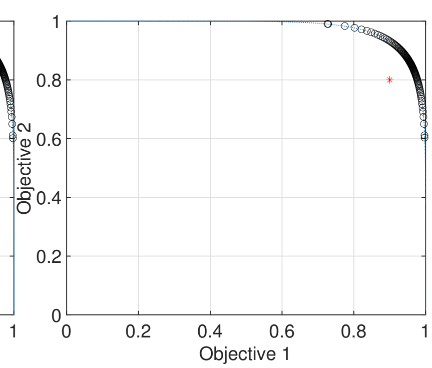
Fig. 54: Comparisons on 10-objective WFG42 where $\mathbf{z}^r = (0.3, 0.3, 0.5, 0.3, 0.5, 0.45, 0.25, 0.35, 0.25, 0.4)^T$.



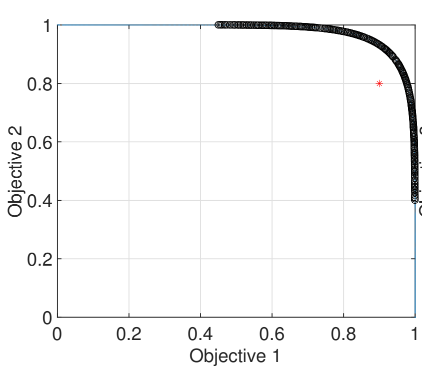
(a) MOEA/D-STM



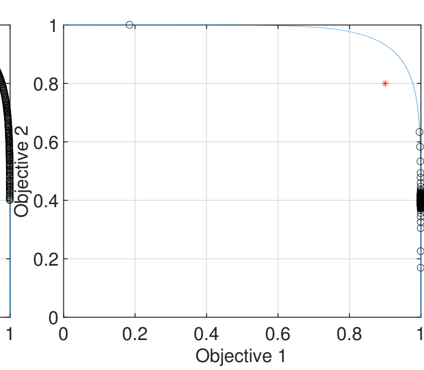
(b) MOEA/D



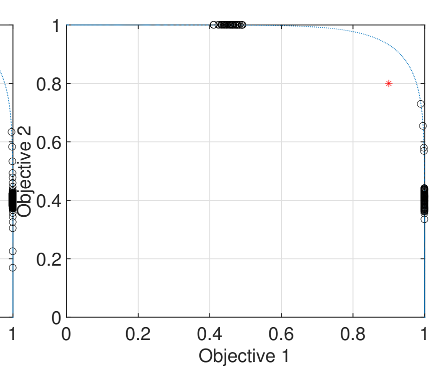
(c) NSGA-III



(d) g-NSGA-II

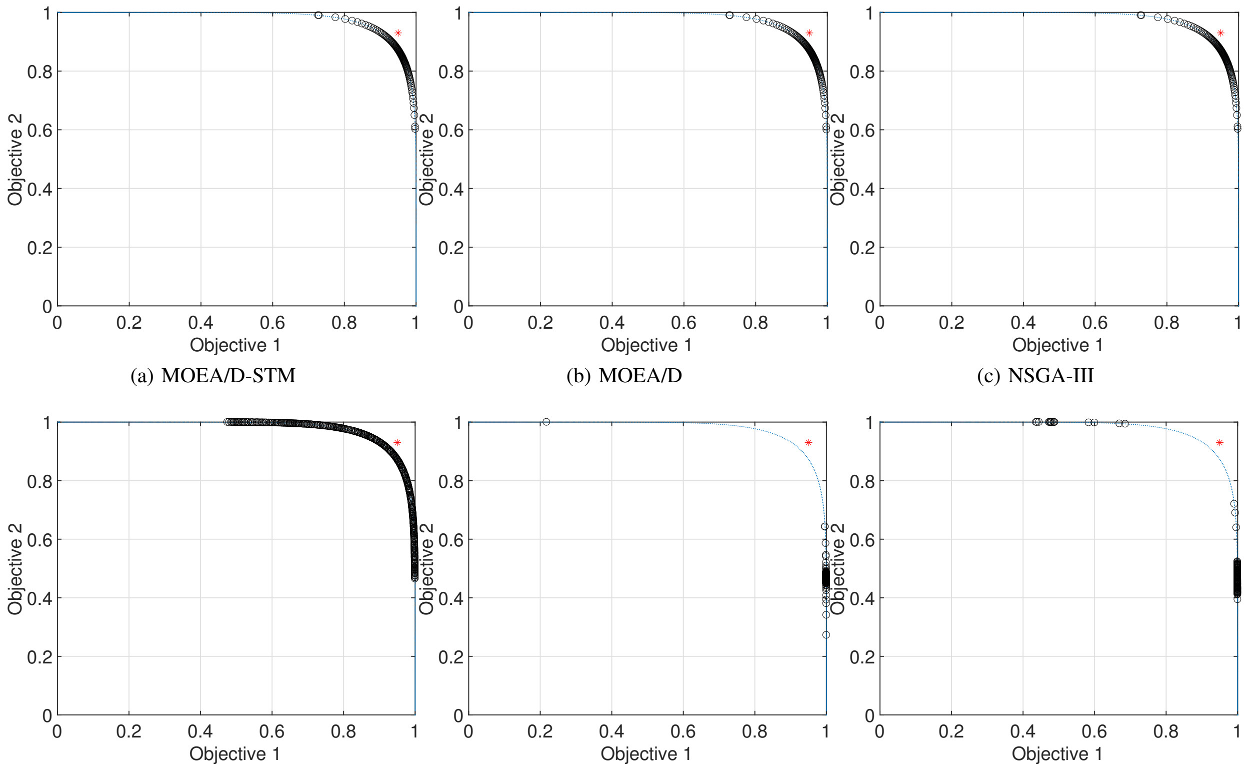


(e) r-NSGA-II



(f) R-NSGA-II

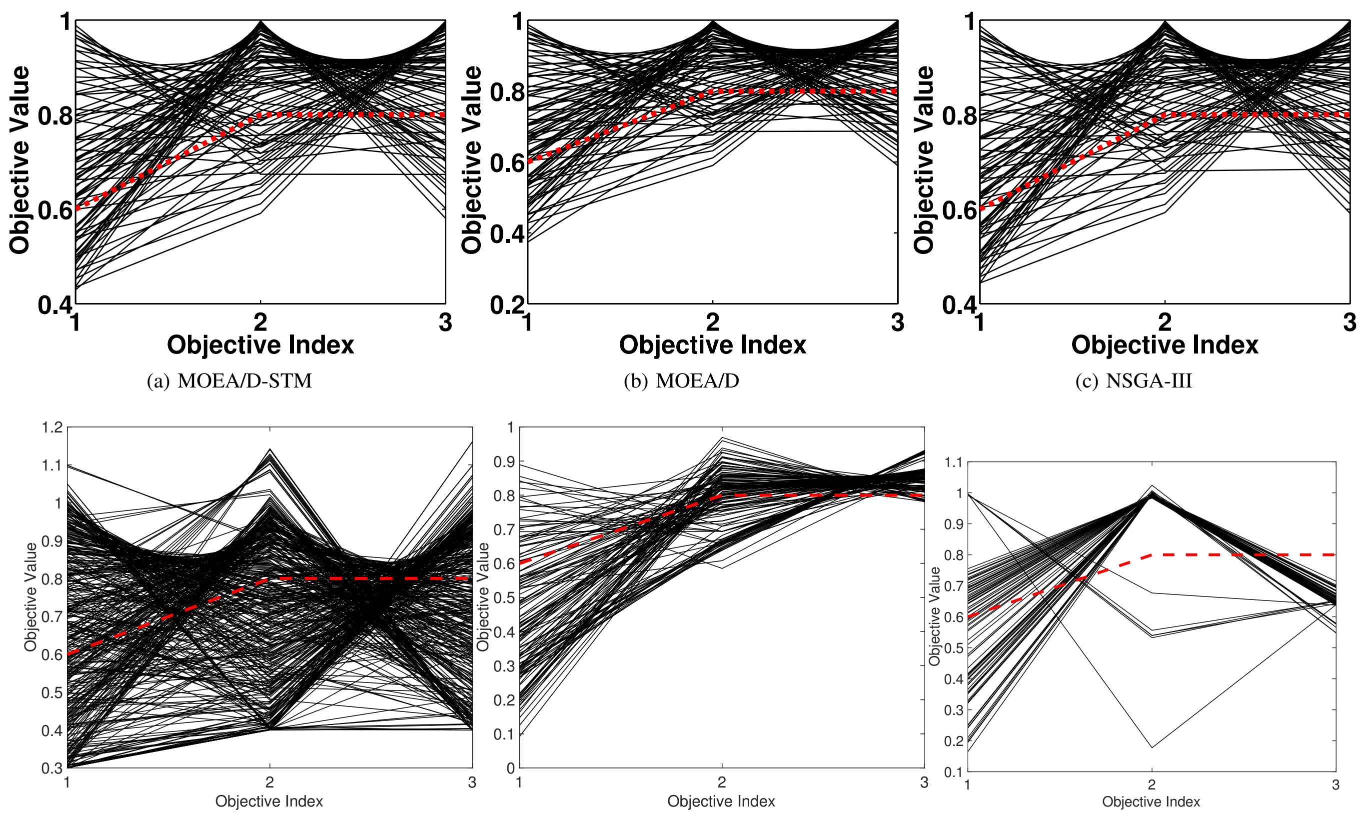
Fig. 55: Comparisons on 2-objective WFG43 where $\mathbf{z}^r = (0.9, 0.8)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

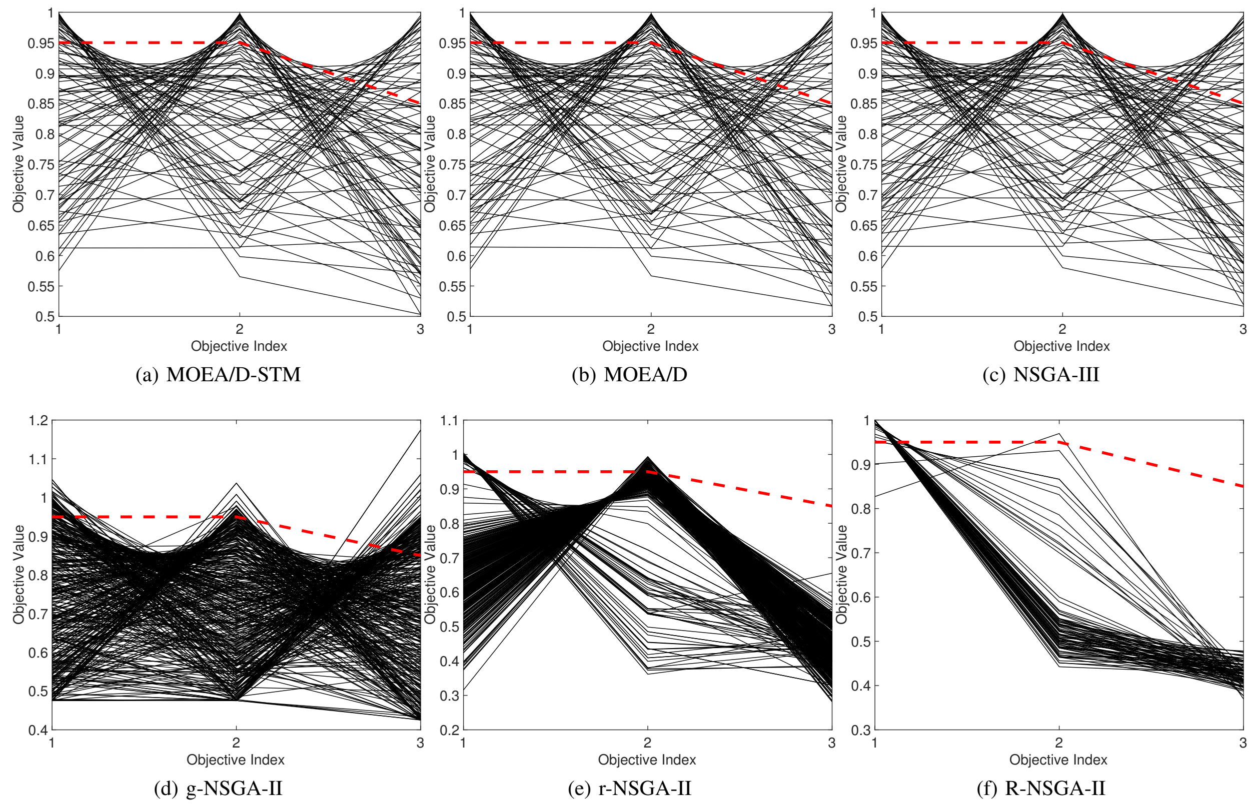
Fig. 56: Comparisons on 2-objective WFG43 where $\mathbf{z}^r = (0.95, 0.93)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

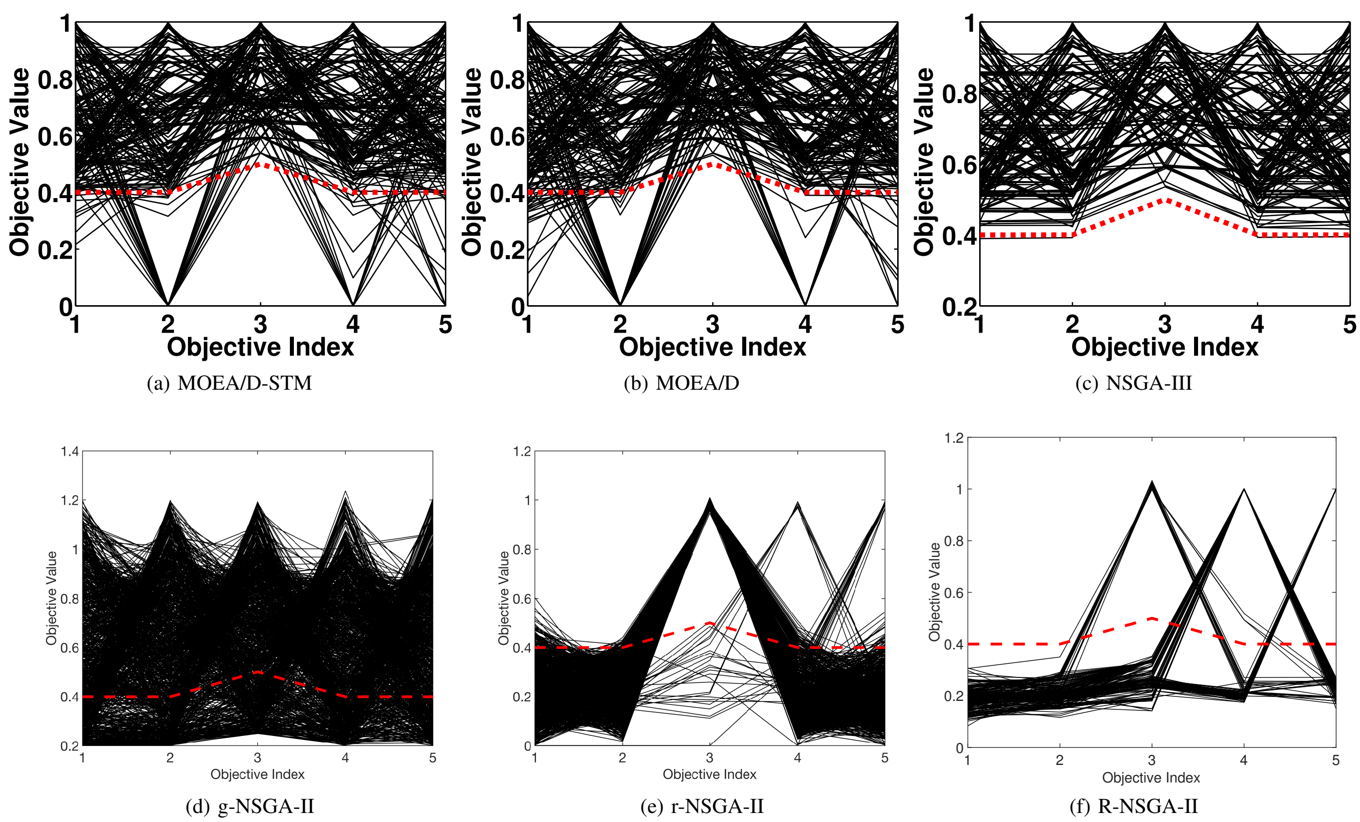
Fig. 57: Comparisons on 3-objective WFG43 where $\mathbf{z}^r = (0.6, 0.8, 0.8)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

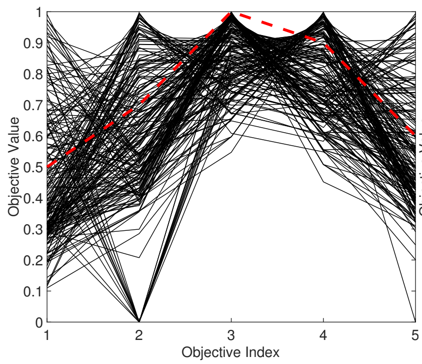
Fig. 58: Comparisons on 3-objective WFG43 where $\mathbf{z}^r = (0.95, 0.95, 0.85)^T$.



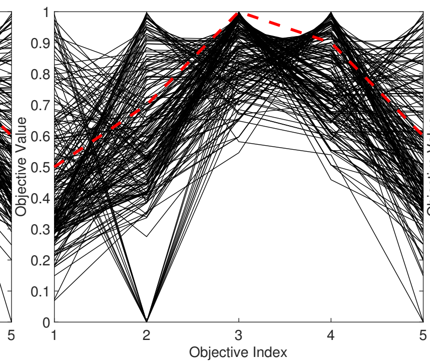
(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

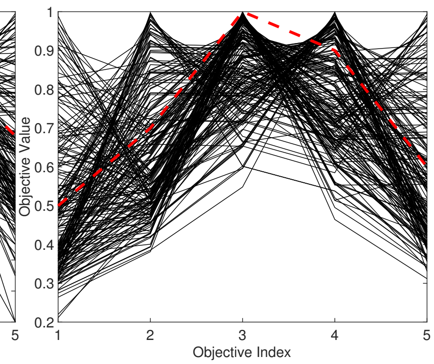
Fig. 59: Comparisons on 5-objective WFG43 where $\mathbf{z}^r = (0.4, 0.4, 0.5, 0.4, 0.4)^T$.



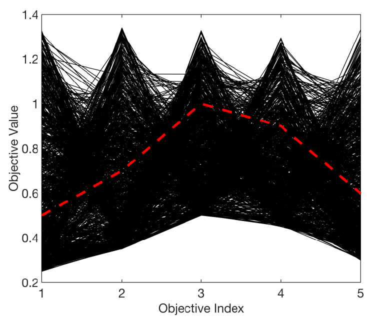
(a) MOEA/D-STM



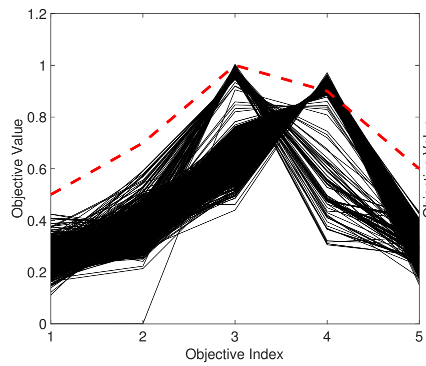
(b) MOEA/D



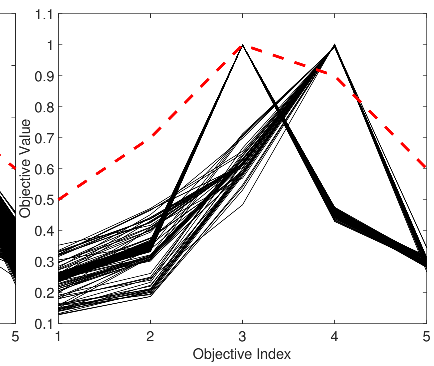
(c) NSGA-III



(d) g-NSGA-II

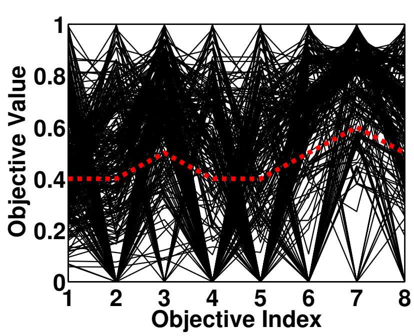


(e) r-NSGA-II

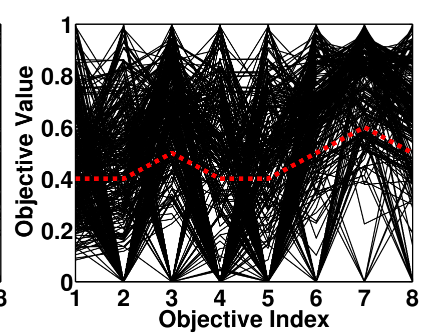


(f) R-NSGA-II

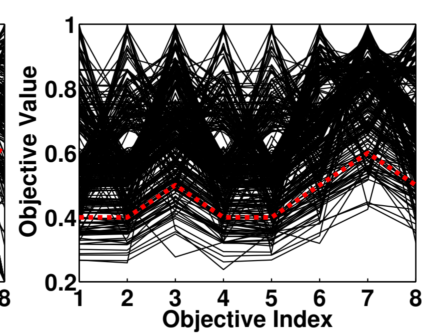
Fig. 60: Comparisons on 5-objective WFG43 where $\mathbf{z}^r = (0.5, 0.7, 1.0, 0.9, 0.6)^T$.



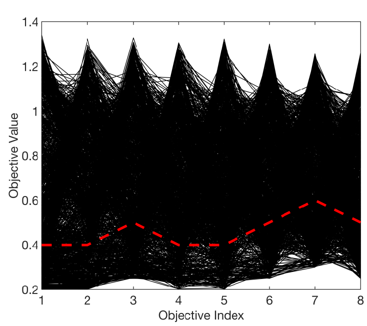
(a) MOEA/D-STM



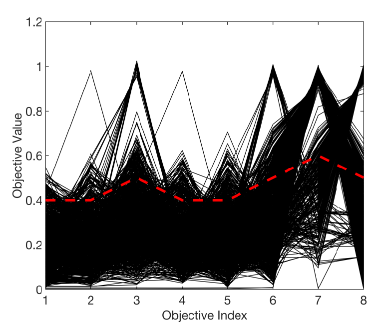
(b) MOEA/D



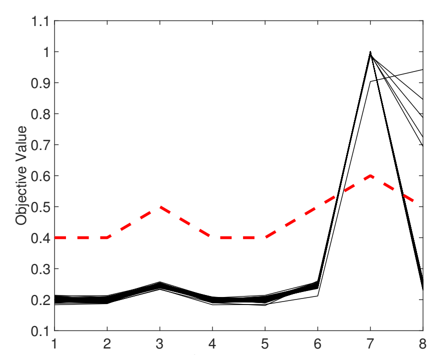
(c) NSGA-III



(d) g-NSGA-II

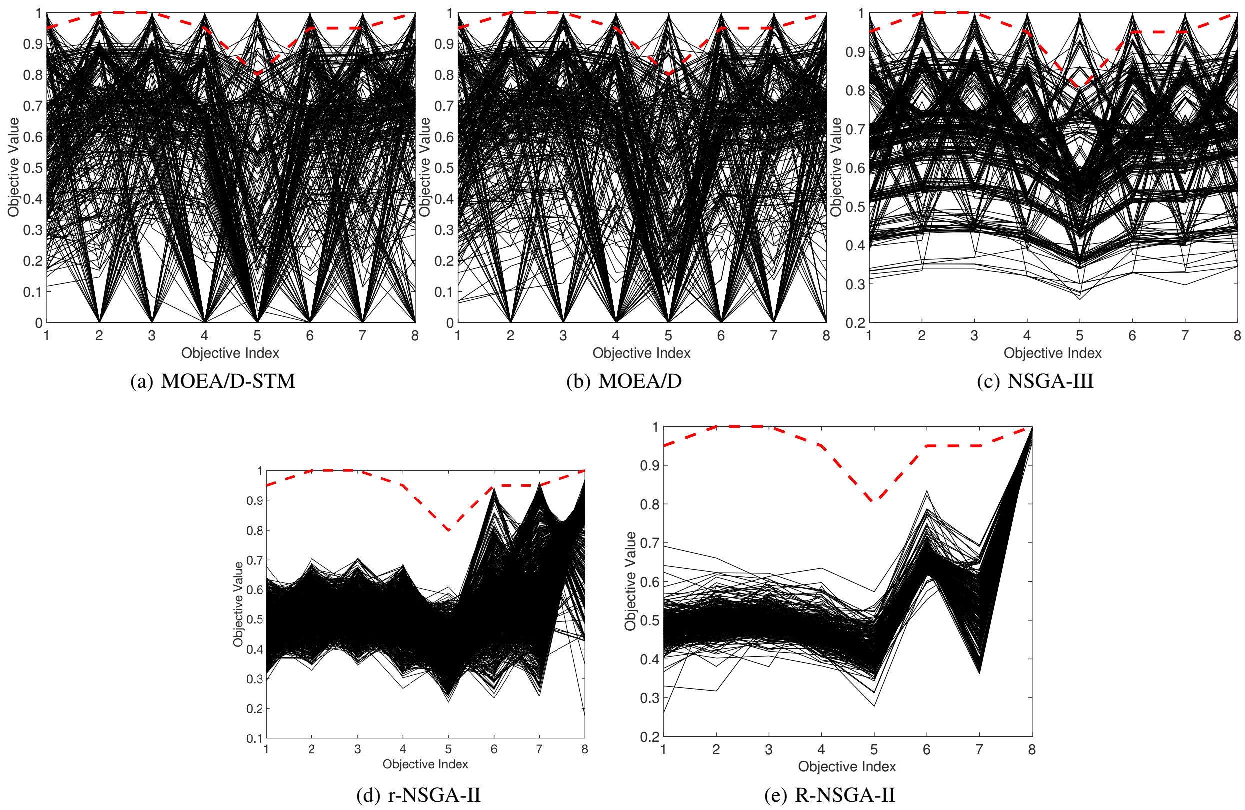


(e) r-NSGA-II



(f) R-NSGA-II

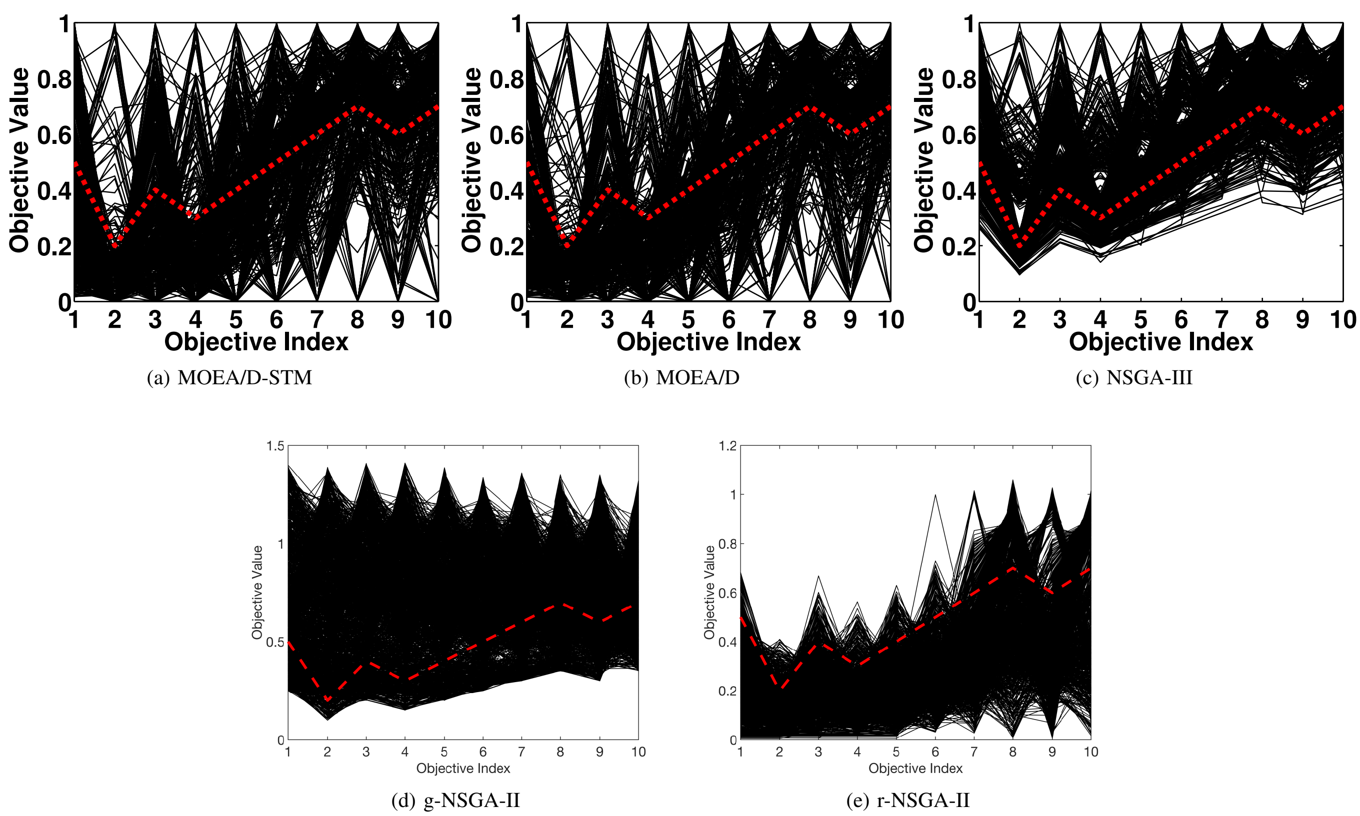
Fig. 61: Comparisons on 8-objective WFG43 where $\mathbf{z}^r = (0.4, 0.4, 0.5, 0.4, 0.4, 0.5, 0.6, 0.5)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) r-NSGA-II (e) R-NSGA-II

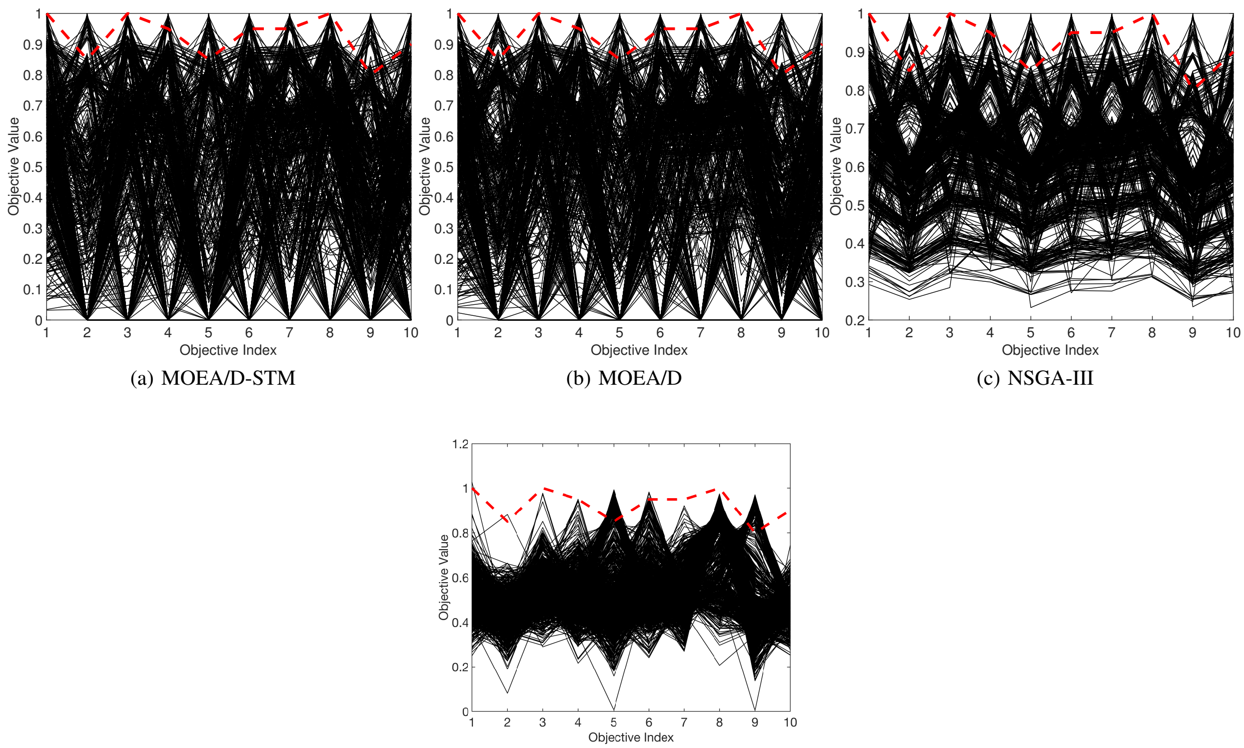
Fig. 62: Comparisons on 8-objective WFG43 where $\mathbf{z}^r = (0.95, 1.0, 1.0, 0.95, 0.8, 0.95, 0.95, 1.0)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II

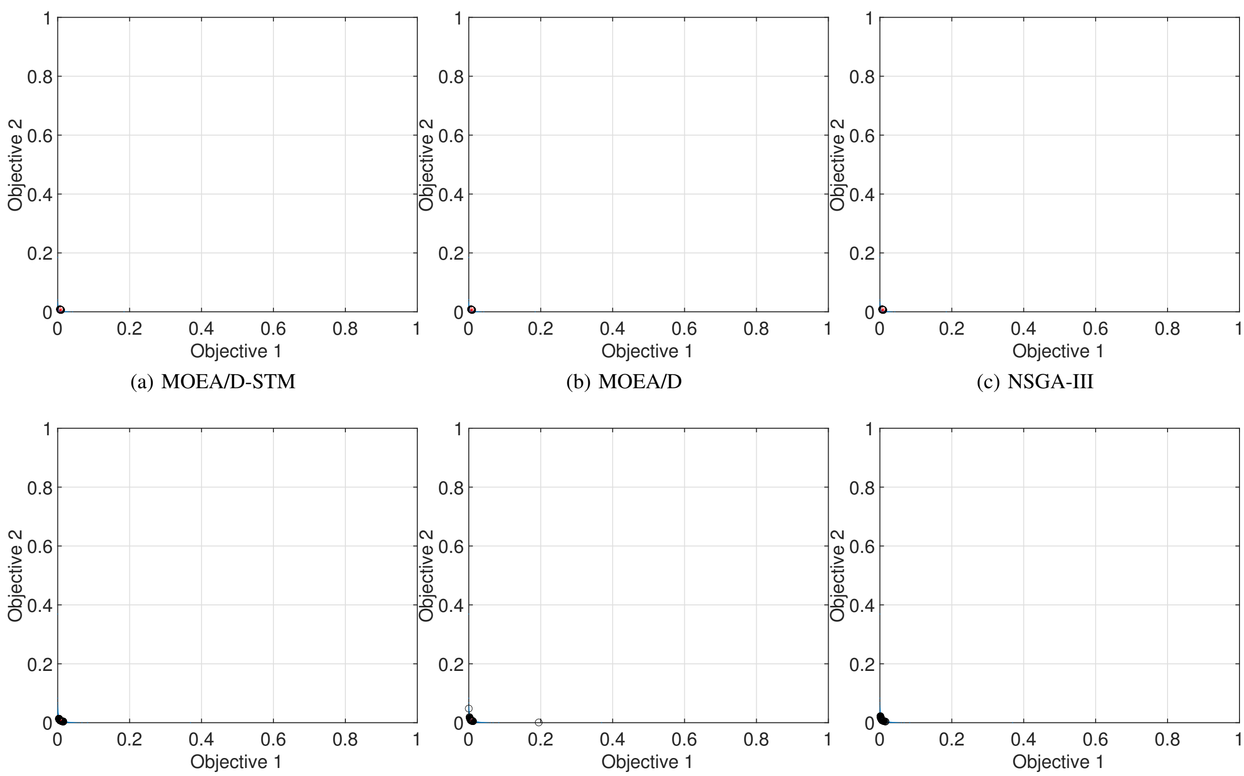
Fig. 63: Comparisons on 10-objective WFG43 where $\mathbf{z}^r = (0.5, 0.2, 0.4, 0.3, 0.4, 0.5, 0.6, 0.7, 0.6, 0.7)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) r-NSGA-II

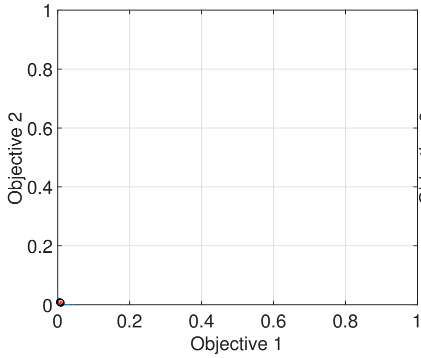
Fig. 64: Comparisons on 10-objective WFG43 where $\mathbf{z}^r = (1.0, 0.85, 1.0, 0.95, 0.85, 0.95, 0.95, 1.0, 0.8, 0.9)^T$.



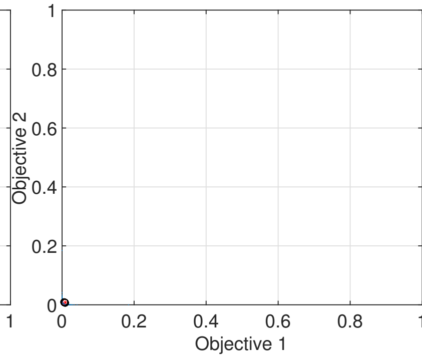
(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

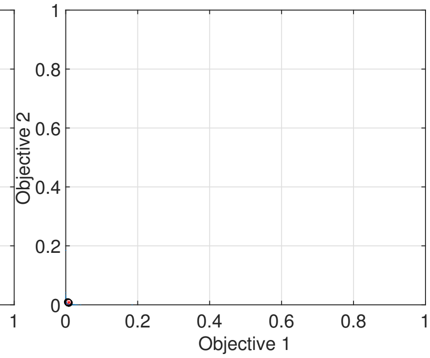
Fig. 65: Comparisons on 2-objective WFG44 where $\mathbf{z}^r = (0.008, 0.006)^T$.



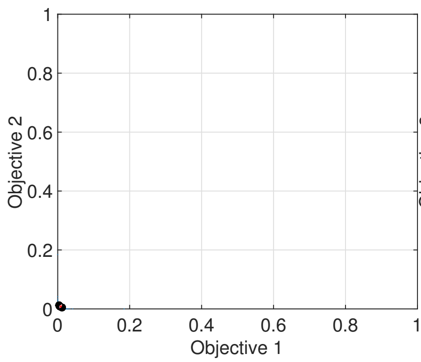
(a) MOEA/D-STM



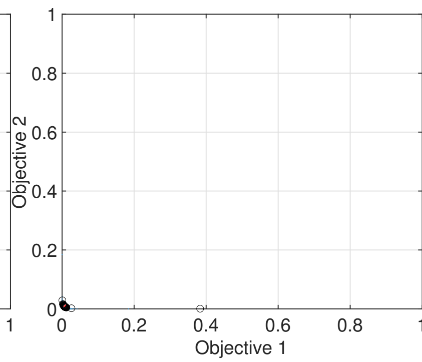
(b) MOEA/D



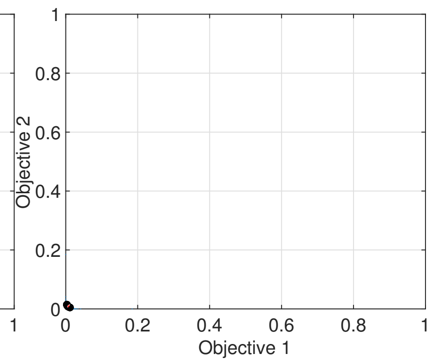
(c) NSGA-III



(d) g-NSGA-II

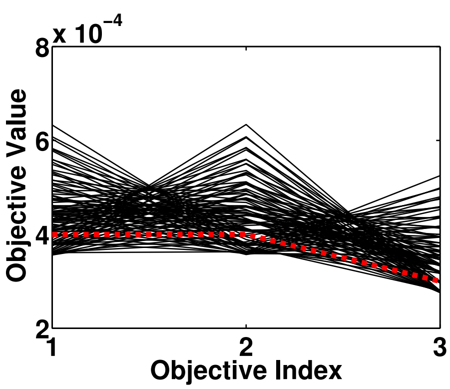


(e) r-NSGA-II

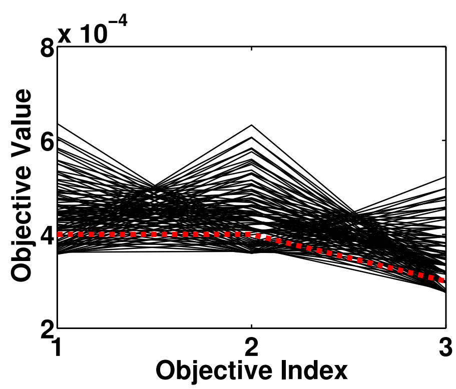


(f) R-NSGA-II

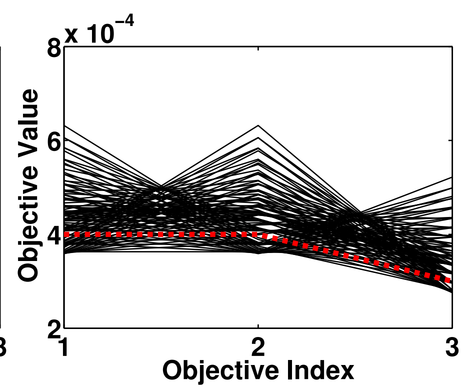
Fig. 66: Comparisons on 2-objective WFG44 where $\mathbf{z}^r = (0.009, 0.008)^T$.



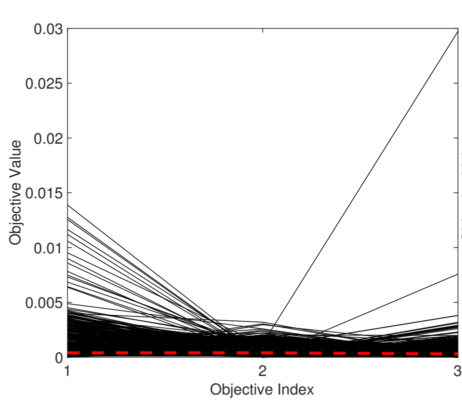
(a) MOEA/D-STM



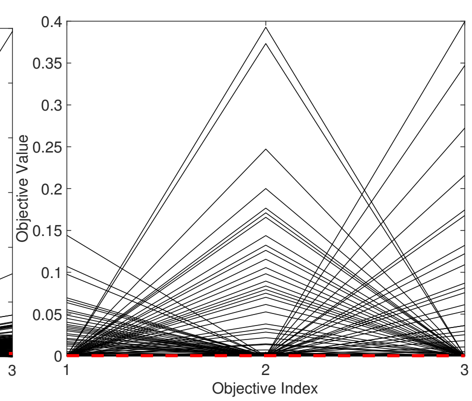
(b) MOEA/D



(c) NSGA-III

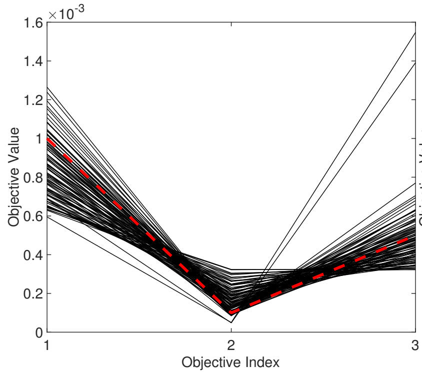


(d) r-NSGA-II

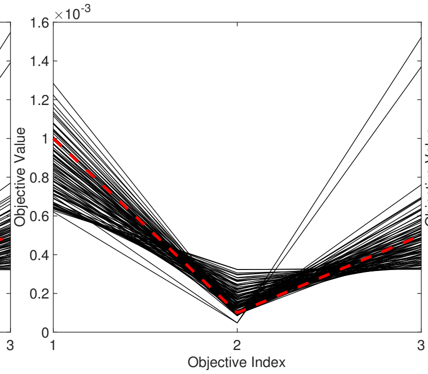


(e) R-NSGA-II

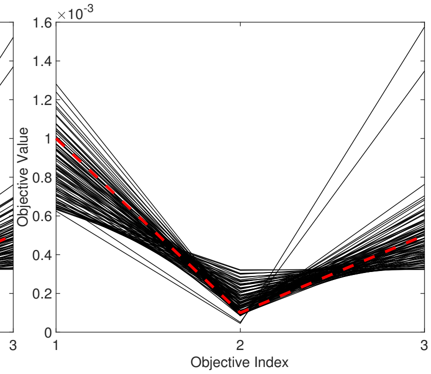
Fig. 67: Comparisons on 3-objective WFG44 where $\mathbf{z}^r = (0.0004, 0.0004, 0.0003)^T$.



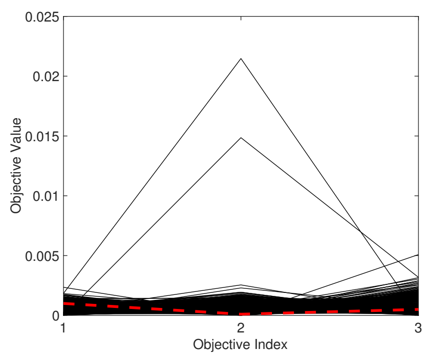
(a) MOEA/D-STM



(b) MOEA/D

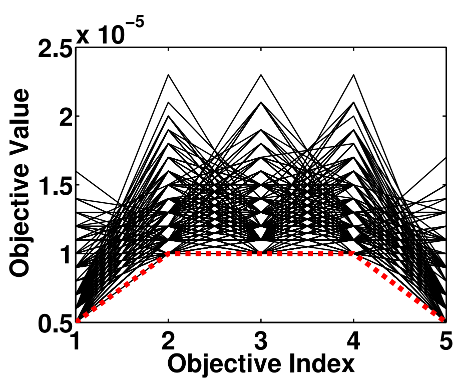


(c) NSGA-III

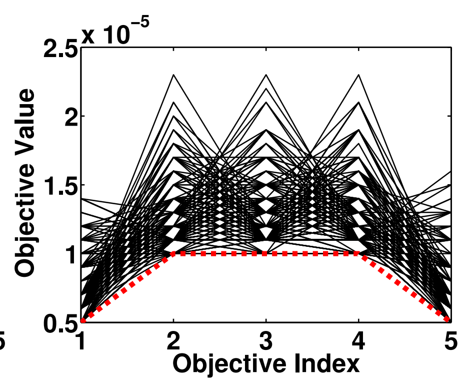


(d) r-NSGA-II

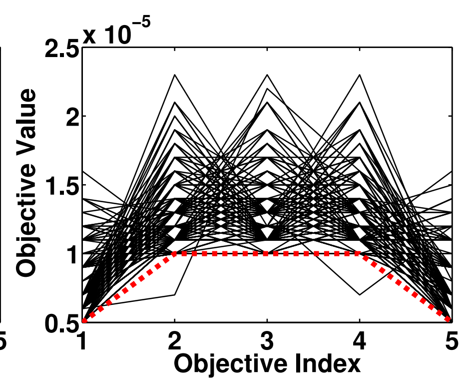
Fig. 68: Comparisons on 3-objective WFG44 where $\mathbf{z}^r = (0.001, 0.0001, 0.0005)^T$.



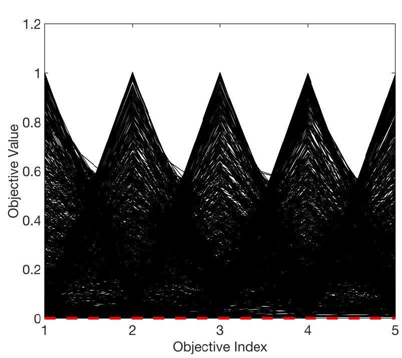
(a) MOEA/D-STM



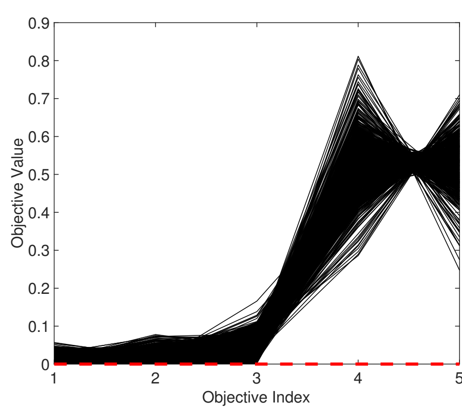
(b) MOEA/D



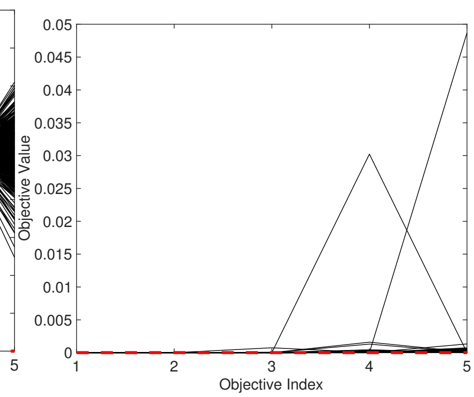
(c) NSGA-III



(d) g-NSGA-II

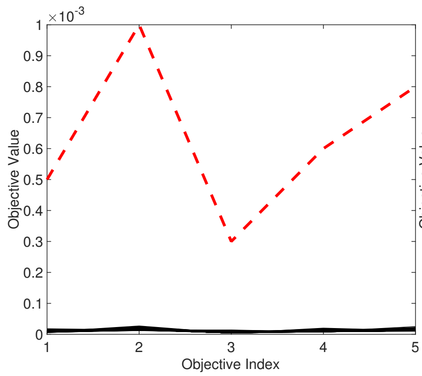


(e) r-NSGA-II

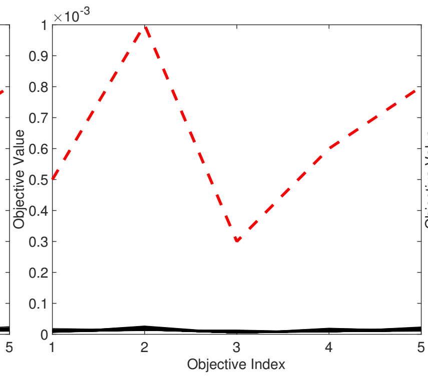


(f) R-NSGA-II

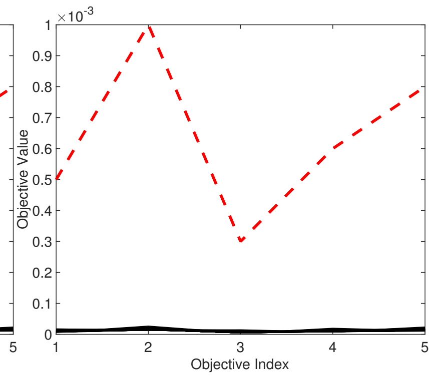
Fig. 69: Comparisons on 5-objective WFG44 where $\mathbf{z}^r = (0.000005, 0.00001, 0.00001, 0.00001, 0.000005)^T$.



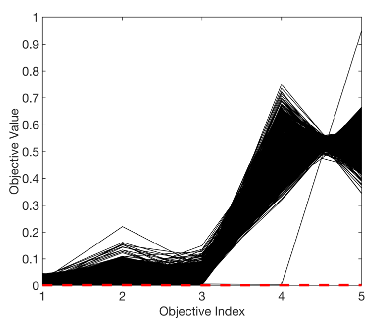
(a) MOEA/D-STM



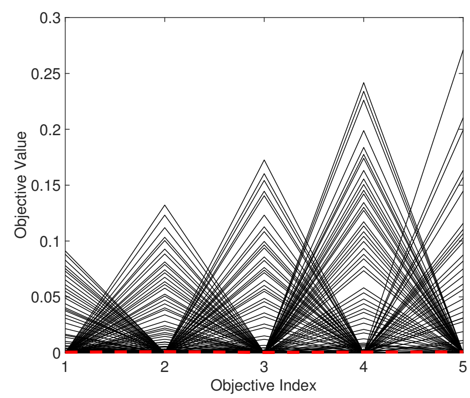
(b) MOEA/D



(c) NSGA-III

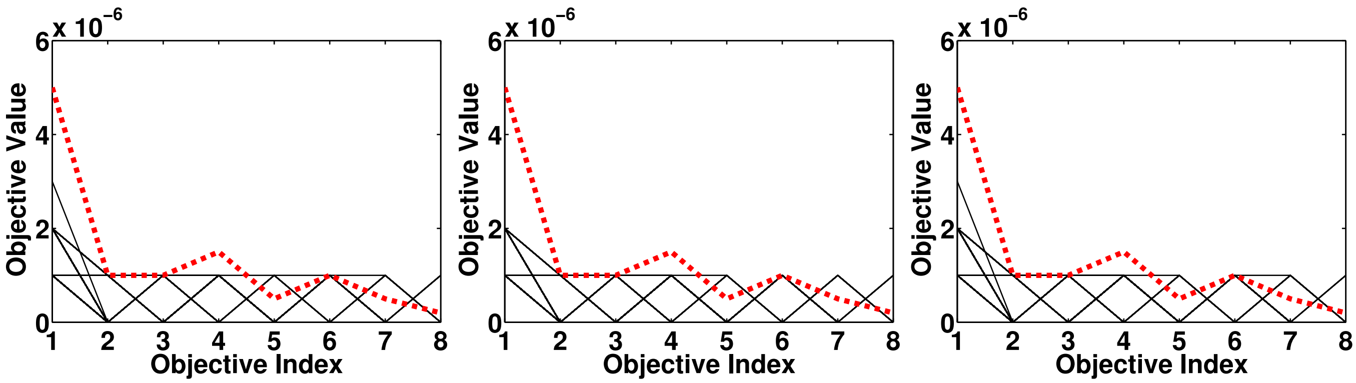


(d) r-NSGA-II

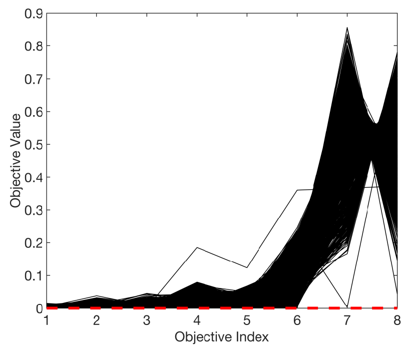


(e) R-NSGA-II

Fig. 70: Comparisons on 5-objective WFG44 where $\mathbf{z}^r = (0.0005, 0.001, 0.0003, 0.0006, 0.0008)^T$.

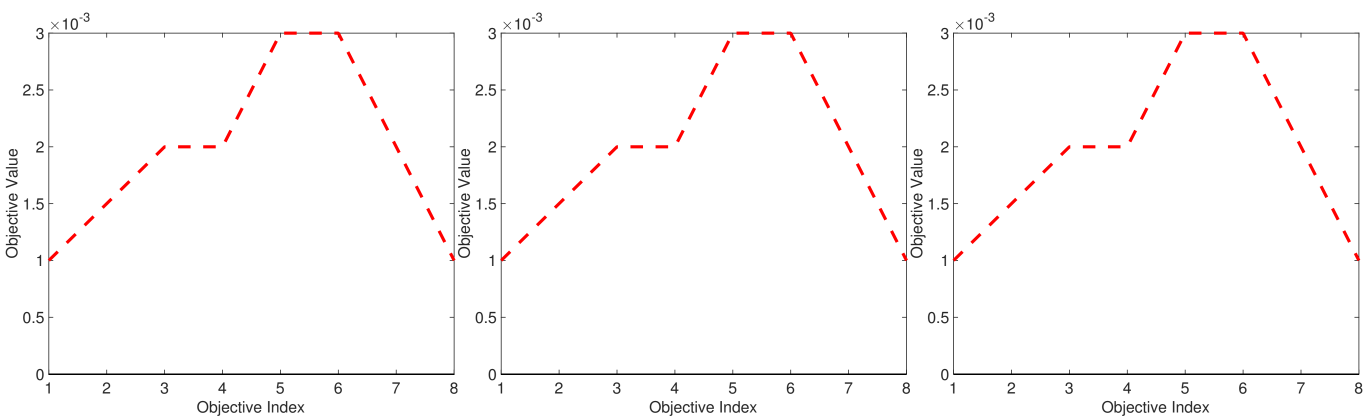


(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III



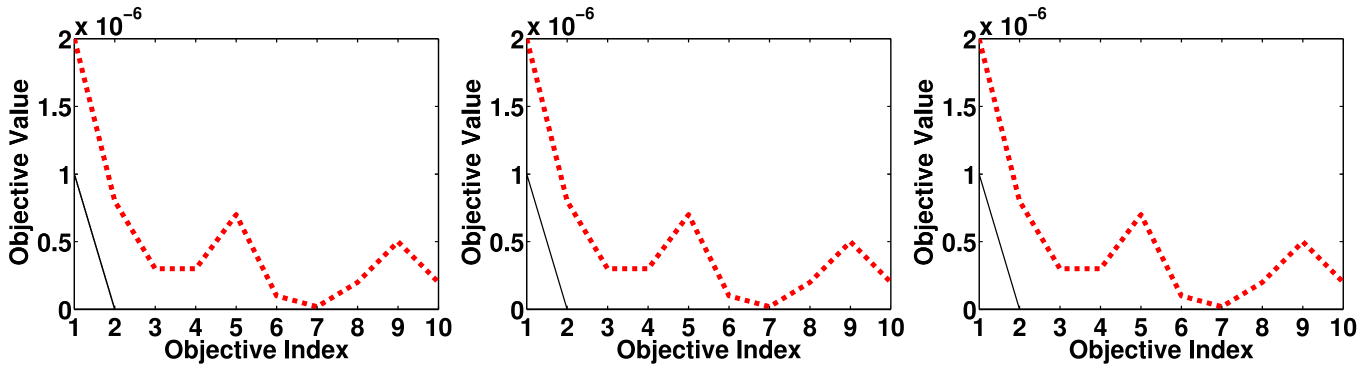
(d) r-NSGA-II

Fig. 71: Comparisons on 8-objective WFG44 where $\mathbf{z}^r = (0.000005, 0.000001, 0.000001, 0.0000015, 0.0000005, 0.000001, 0.0000005, 0.000$



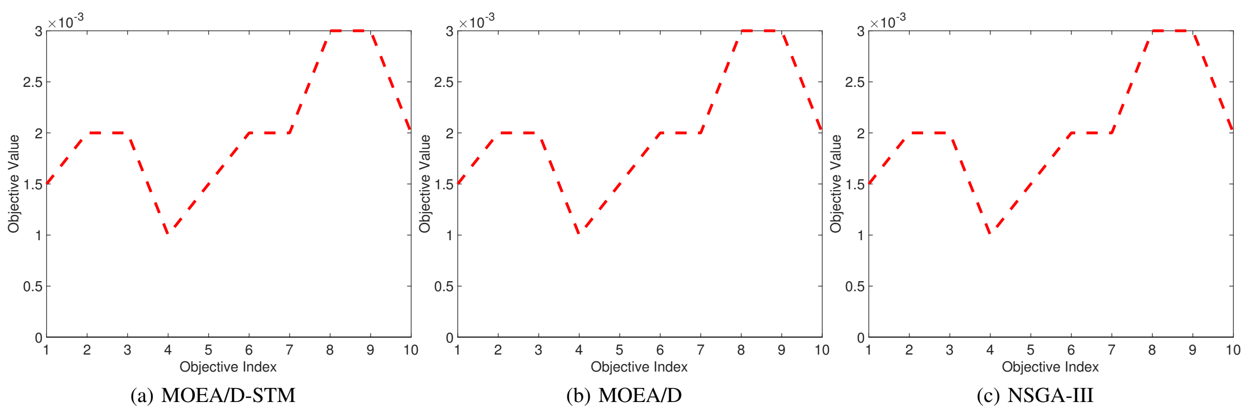
(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

Fig. 72: Comparisons on 8-objective WFG44 where $\mathbf{z}^r = (0.001, 0.0015, 0.002, 0.002, 0.003, 0.003, 0.002, 0.001)^T$.



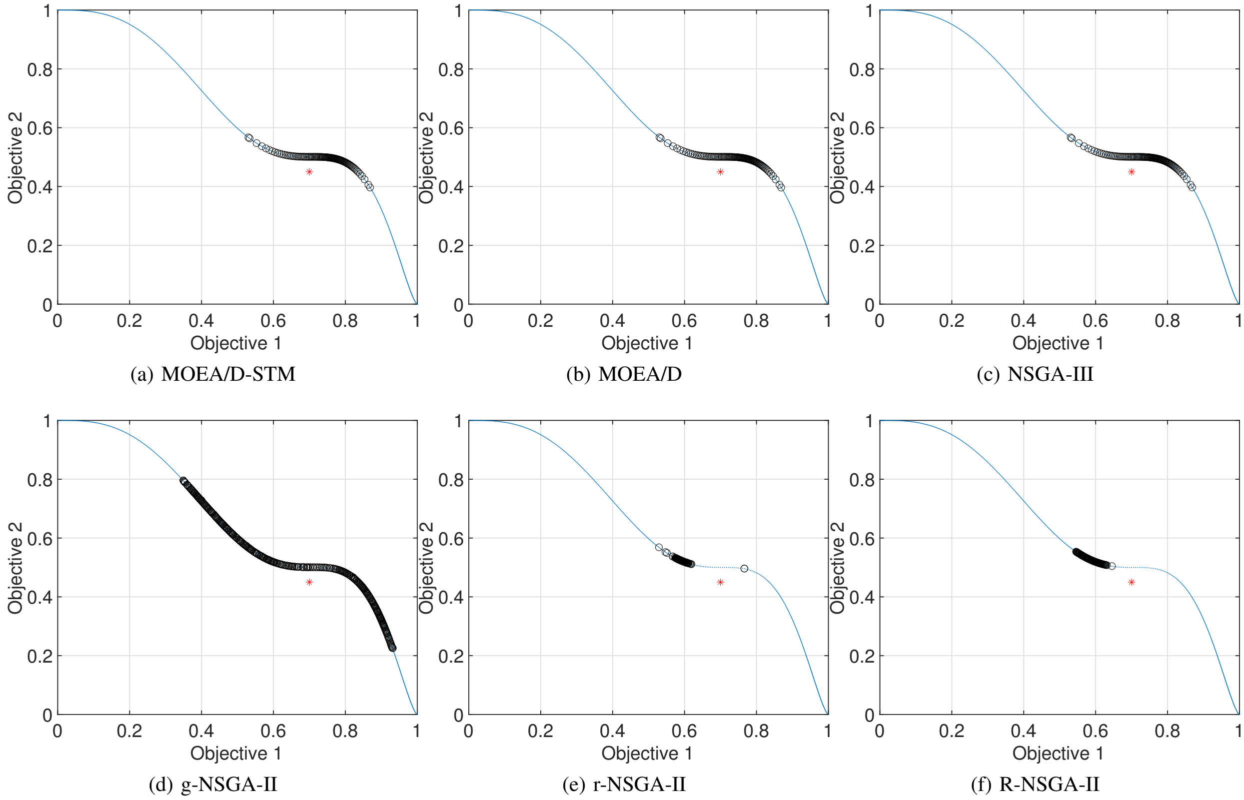
(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

Fig. 73: Comparisons on 10-objective WFG44 where $\mathbf{z}^r = (0.000002, 0.0000008, 0.0000003, 0.0000003, 0.0000007, 0.0000001, 0.00000002$



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

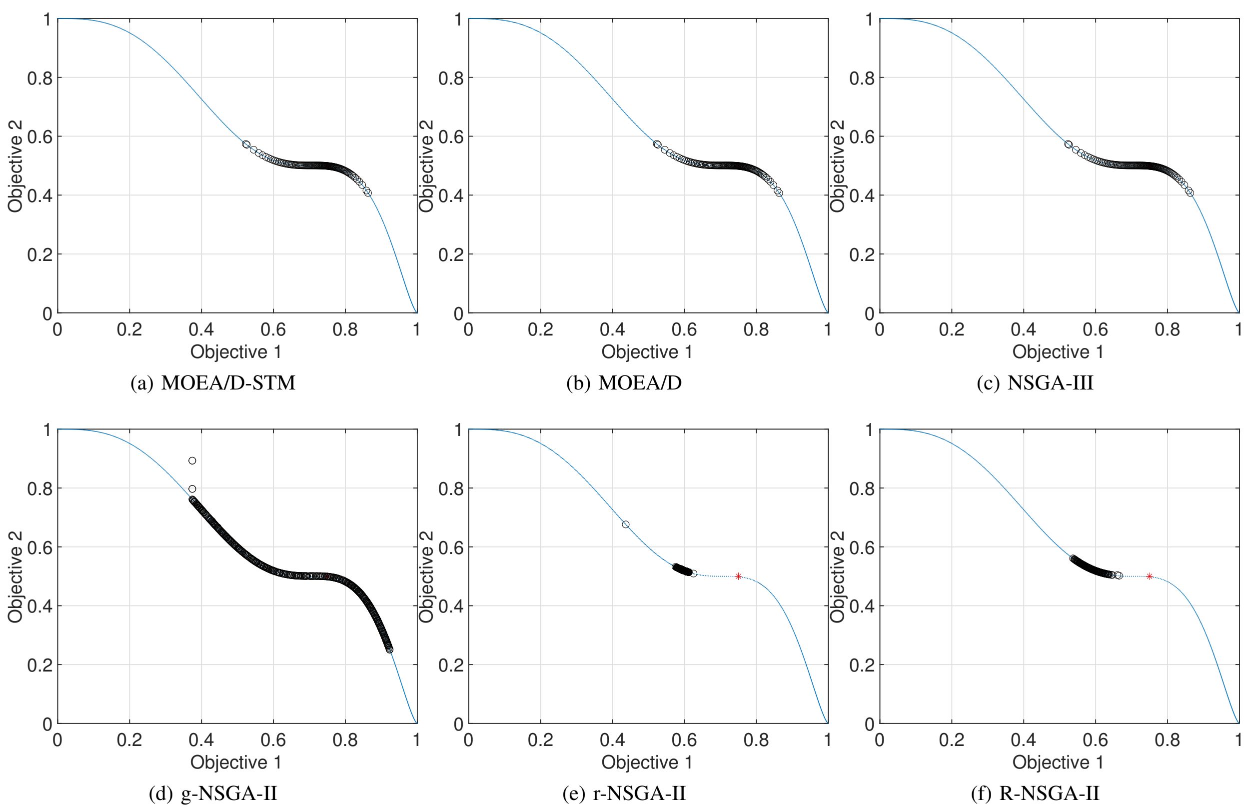
Fig. 74: Comparisons on 10-objective WFG44 where $\mathbf{z}^r = (0.0015, 0.002, 0.002, 0.001, 0.0015, 0.002, 0.002, 0.003, 0.003, 0.002)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

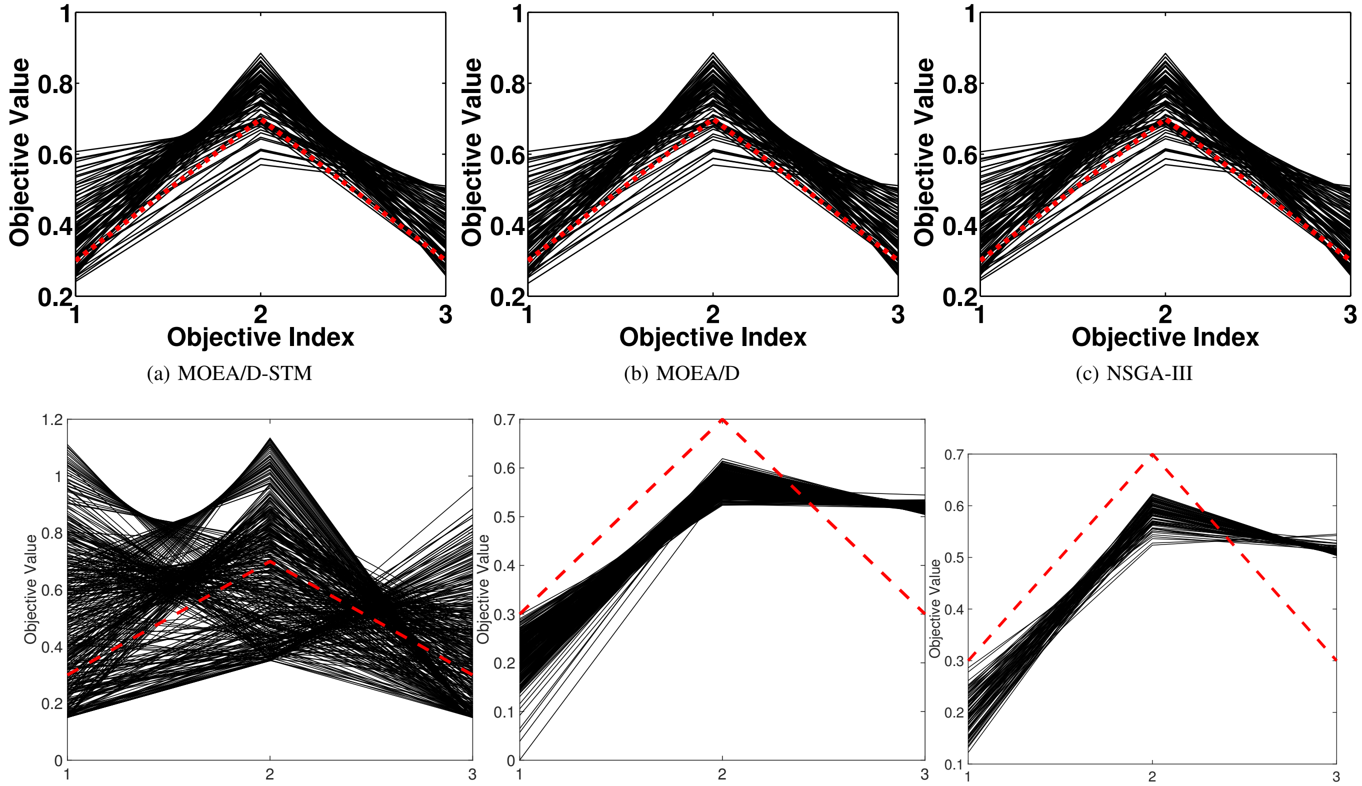
Fig. 75: Comparisons on 2-objective WFG45 where $\mathbf{z}^r = (0.7, 0.45)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

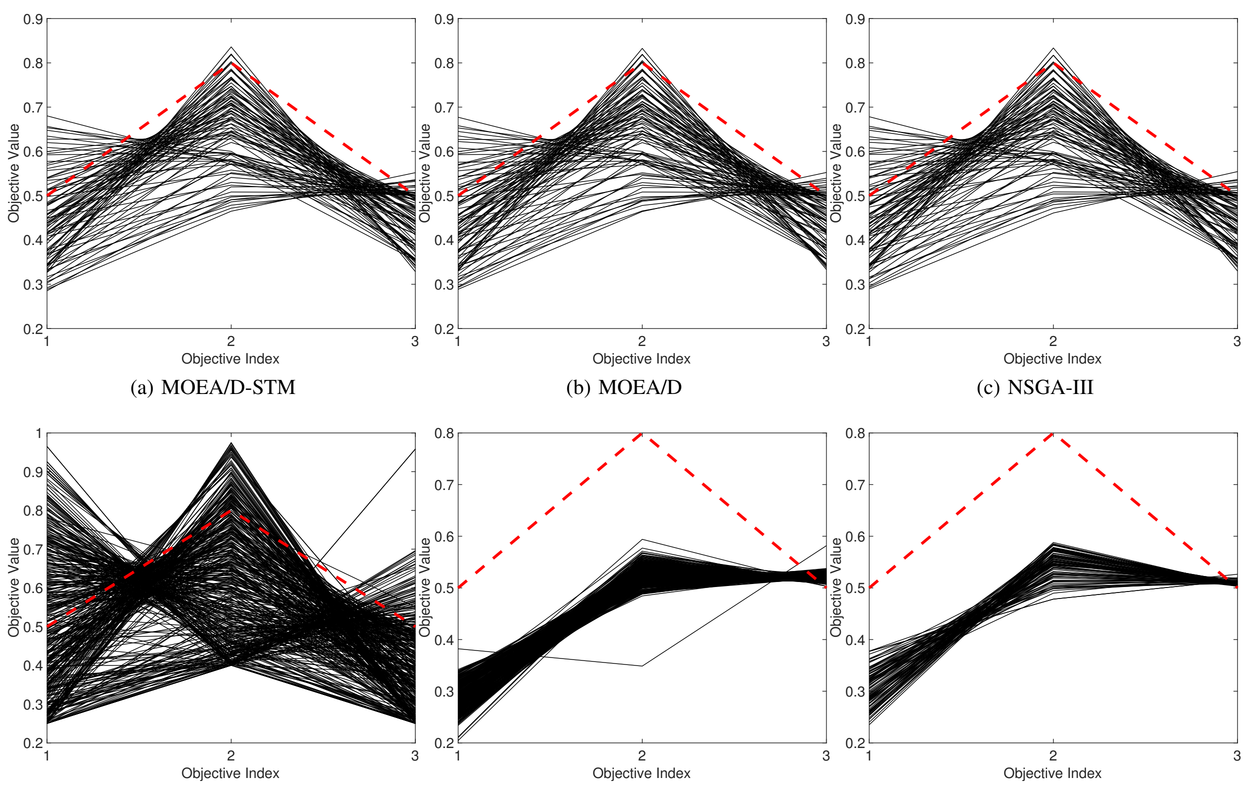
Fig. 76: Comparisons on 2-objective WFG45 where $\mathbf{z}^r = (0.75, 0.5)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

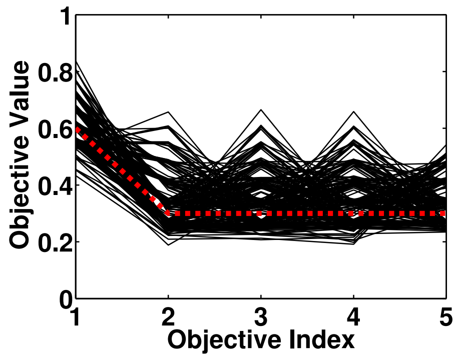
Fig. 77: Comparisons on 3-objective WFG45 where $\mathbf{z}^r = (0.3, 0.7, 0.3)^T$.



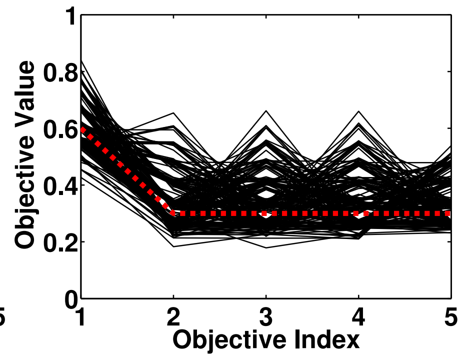
(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

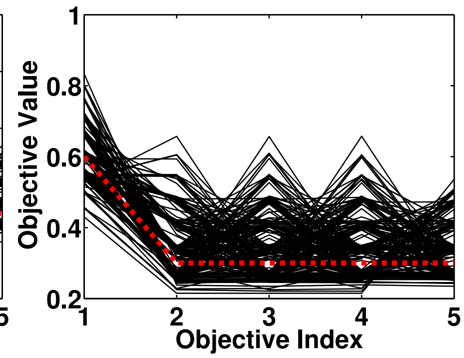
Fig. 78: Comparisons on 3-objective WFG45 where $\mathbf{z}^r = (0.5, 0.8, 0.5)^T$.



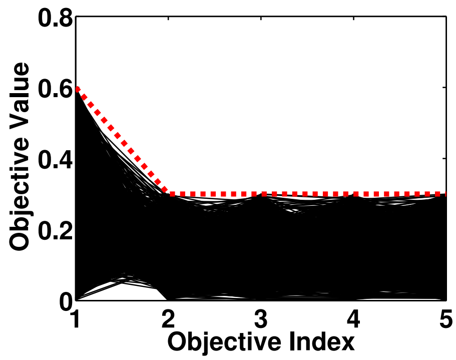
(a) MOEA/D-STM



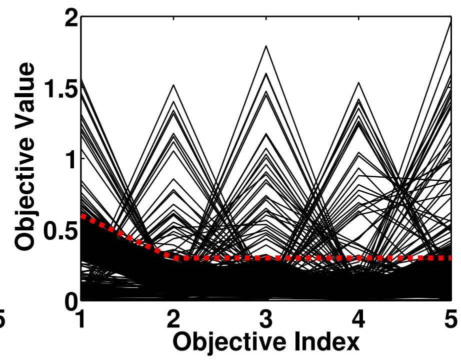
(b) MOEA/D



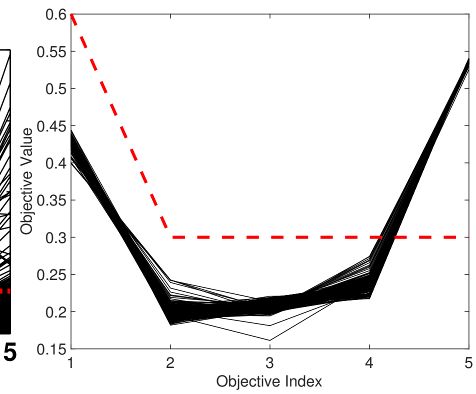
(c) NSGA-III



(d) g-NSGA-II

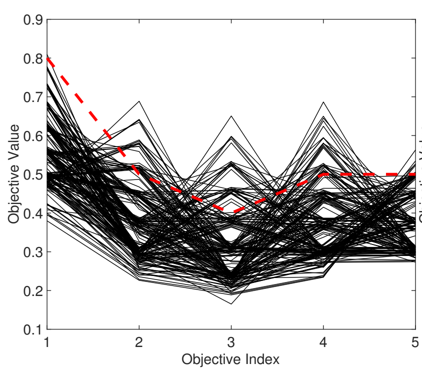


(e) r-NSGA-II

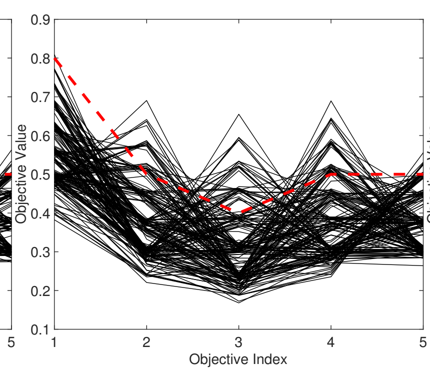


(f) R-NSGA-II

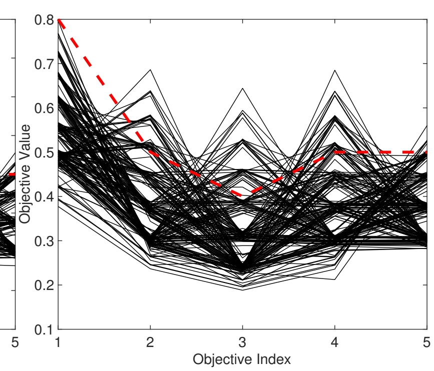
Fig. 79: Comparisons on 5-objective WFG45 where $\mathbf{z}^r = (0.6, 0.3, 0.3, 0.3, 0.3)^T$.



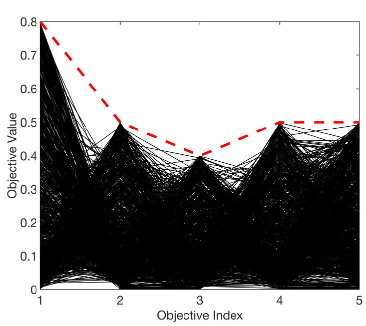
(a) MOEA/D-STM



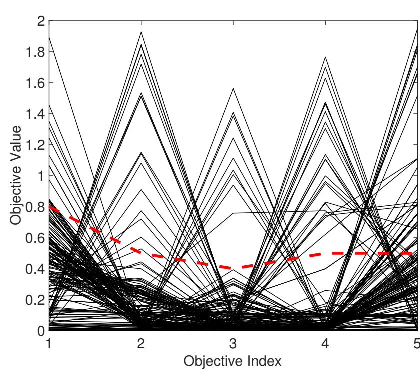
(b) MOEA/D



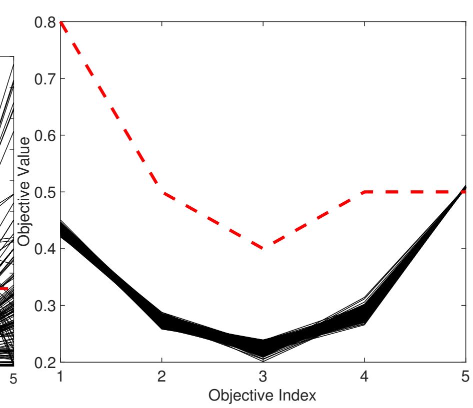
(c) NSGA-III



(d) g-NSGA-II

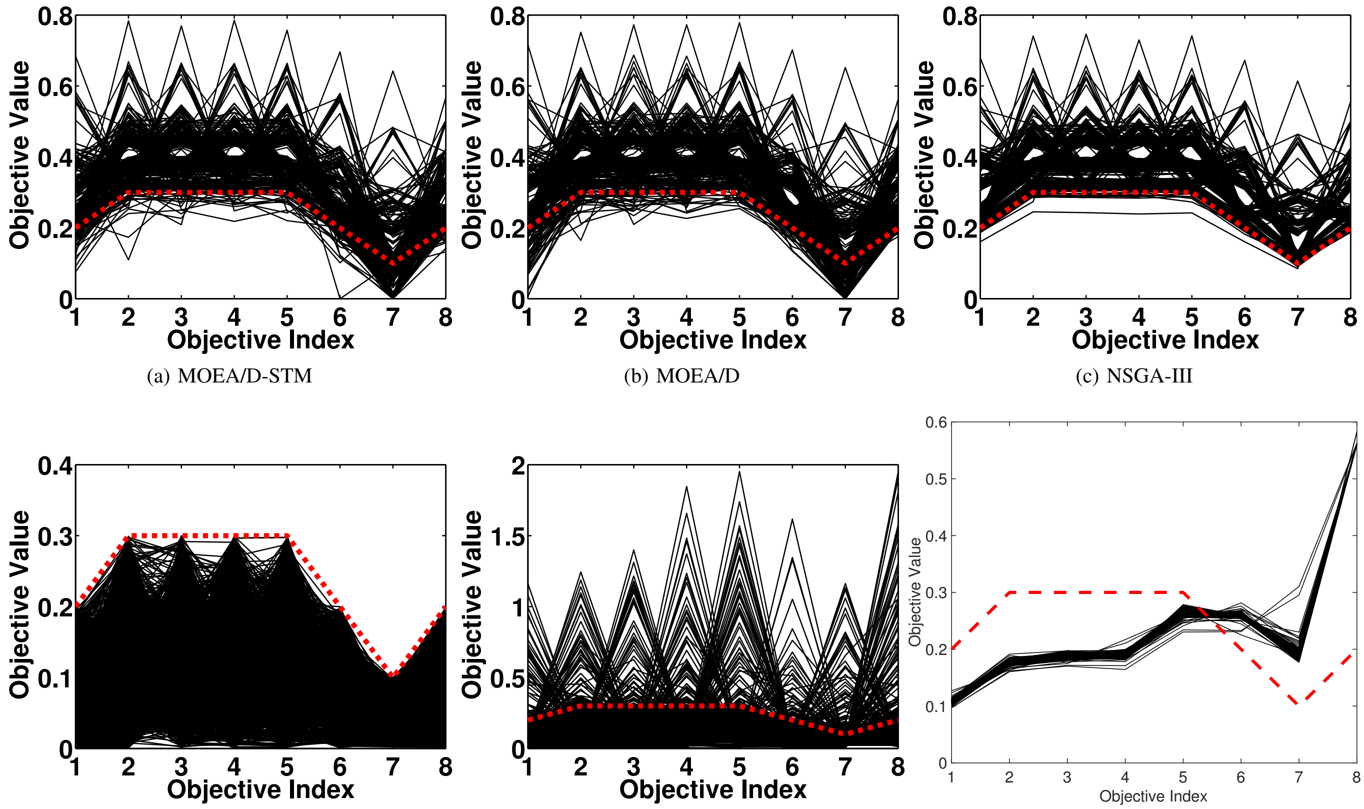


(e) r-NSGA-II



(f) R-NSGA-II

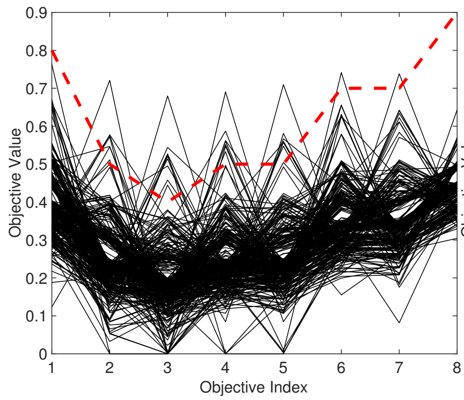
Fig. 80: Comparisons on 5-objective WFG45 where $\mathbf{z}^r = (0.8, 0.5, 0.4, 0.5, 0.5)^T$.



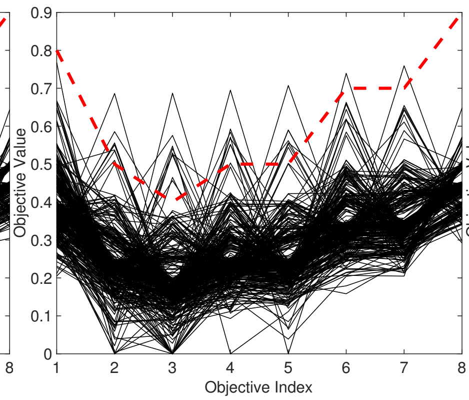
(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

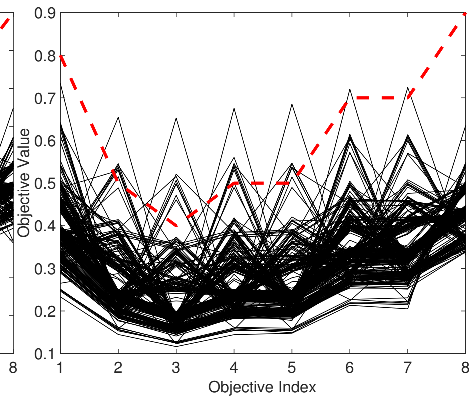
Fig. 81: Comparisons on 8-objective WFG45 where $\mathbf{z}^r = (0.2, 0.3, 0.3, 0.3, 0.3, 0.2, 0.1, 0.2)^T$.



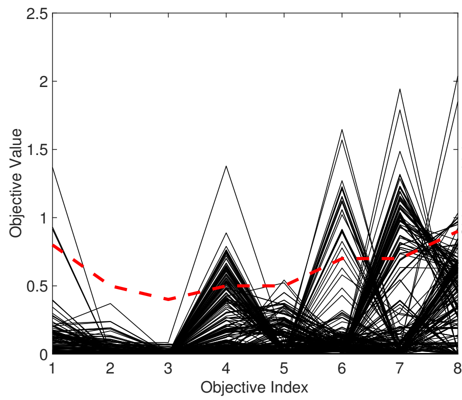
(a) MOEA/D-STM



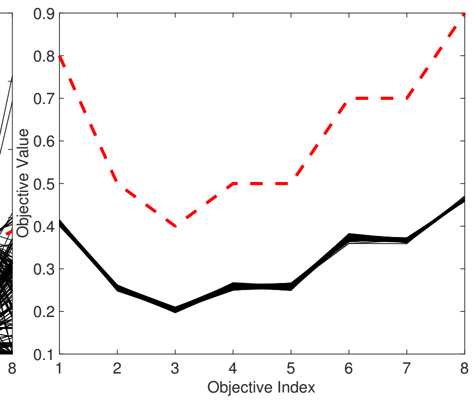
(b) MOEA/D



(c) NSGA-III

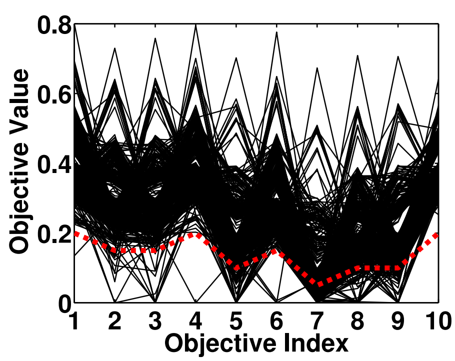


(d) r-NSGA-II

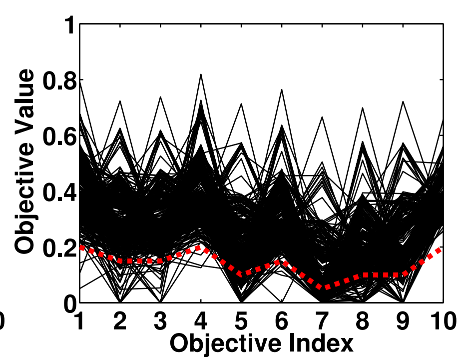


(e) R-NSGA-II

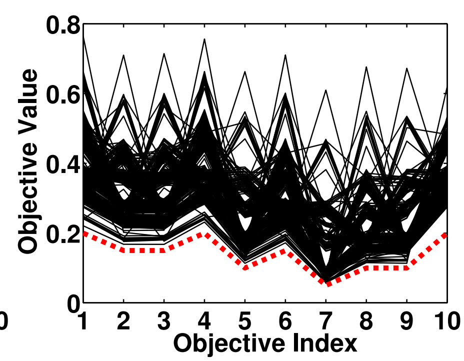
Fig. 82: Comparisons on 8-objective WFG45 where $\mathbf{z}^r = (0.8, 0.5, 0.4, 0.5, 0.5, 0.7, 0.7, 0.9)^T$.



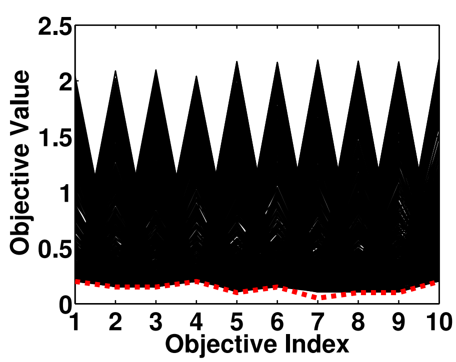
(a) MOEA/D-STM



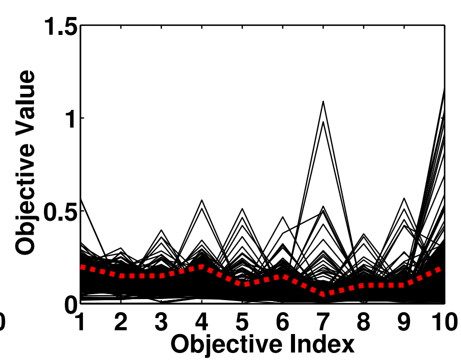
(b) MOEA/D



(c) NSGA-III



(d) g-NSGA-II



(e) r-NSGA-II

Fig. 83: Comparisons on 10-objective WFG45 where $\mathbf{z}^r = (0.2, 0.15, 0.15, 0.2, 0.1, 0.15, 0.05, 0.1, 0.1, 0.2)^T$.

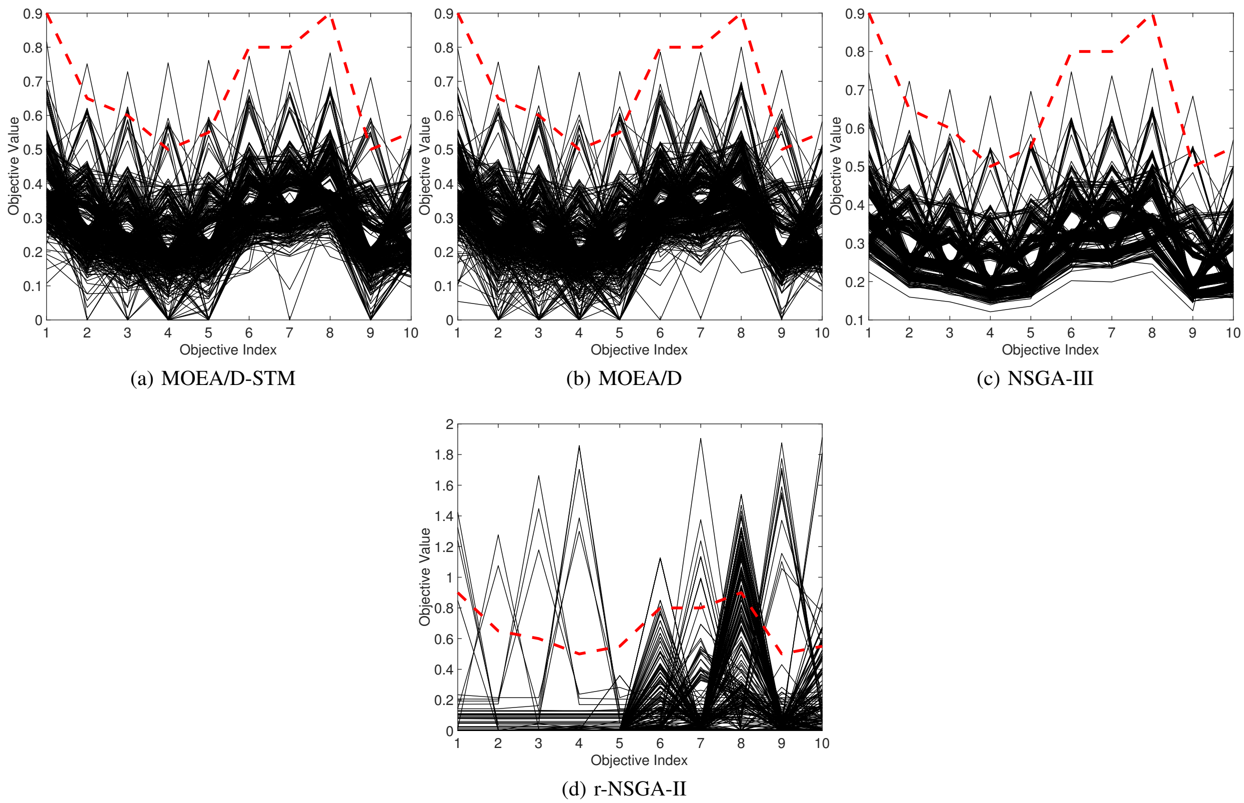


Fig. 84: Comparisons on 10-objective WFG45 where $\mathbf{z}^r = (0.9, 0.65, 0.6, 0.5, 0.55, 0.8, 0.8, 0.9, 0.5, 0.55)^T$.

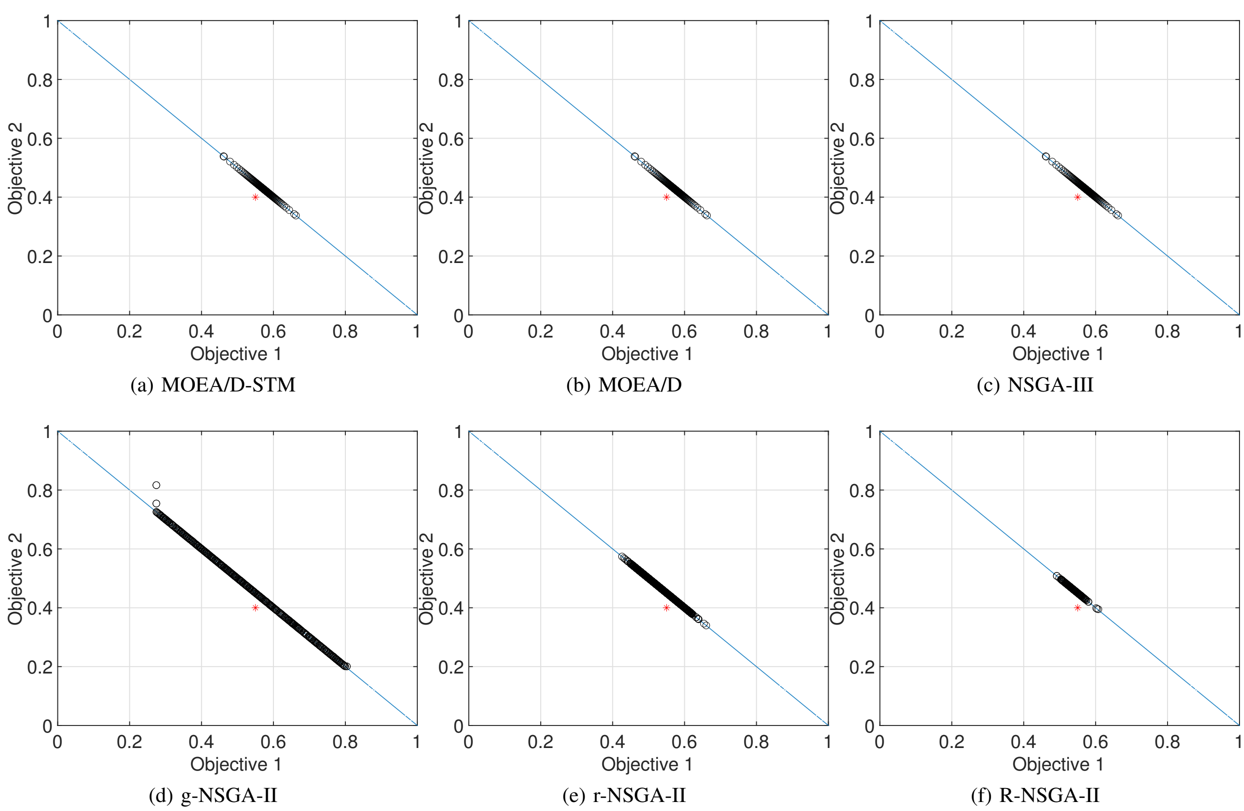
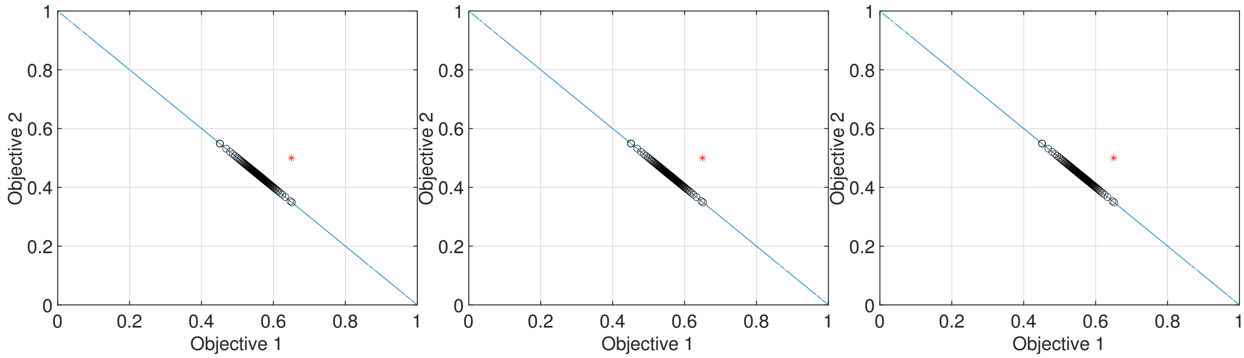
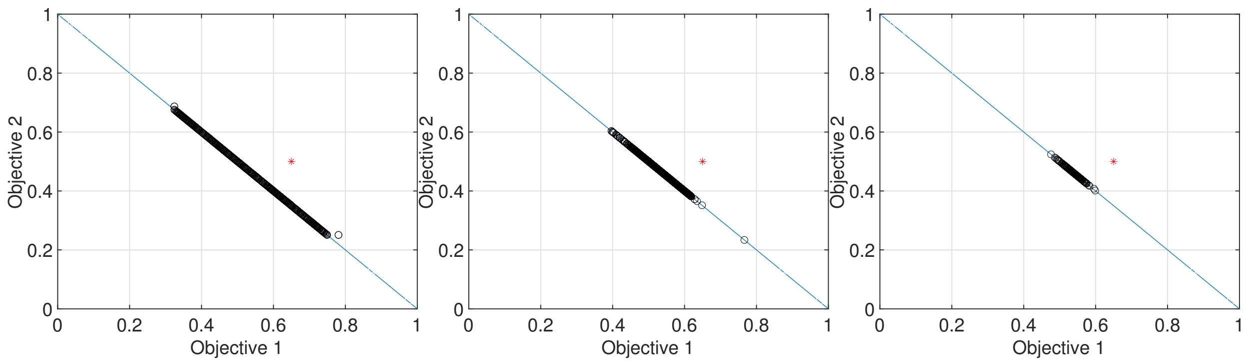


Fig. 85: Comparisons on 2-objective WFG46 where $\mathbf{z}^r = (0.55, 0.4)^T$.

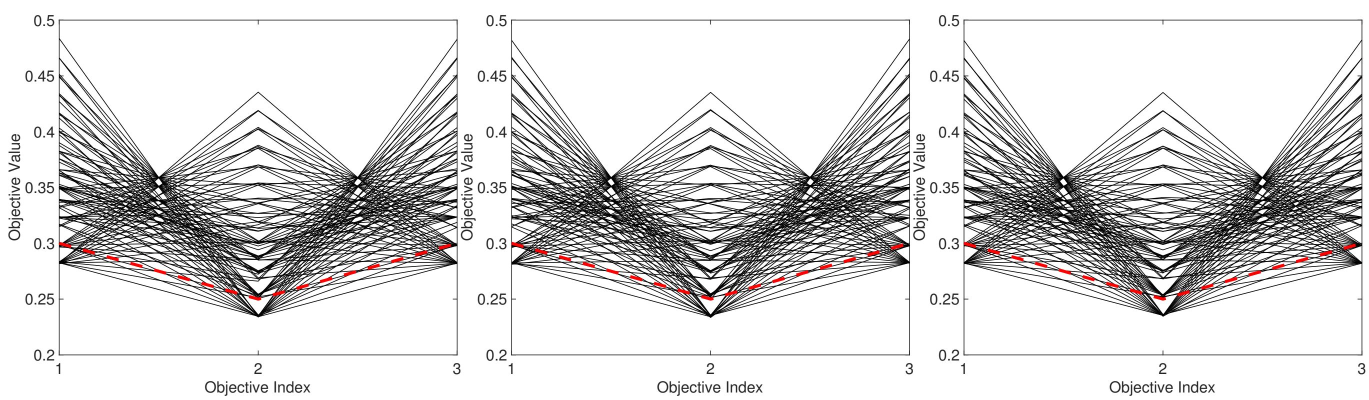


(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

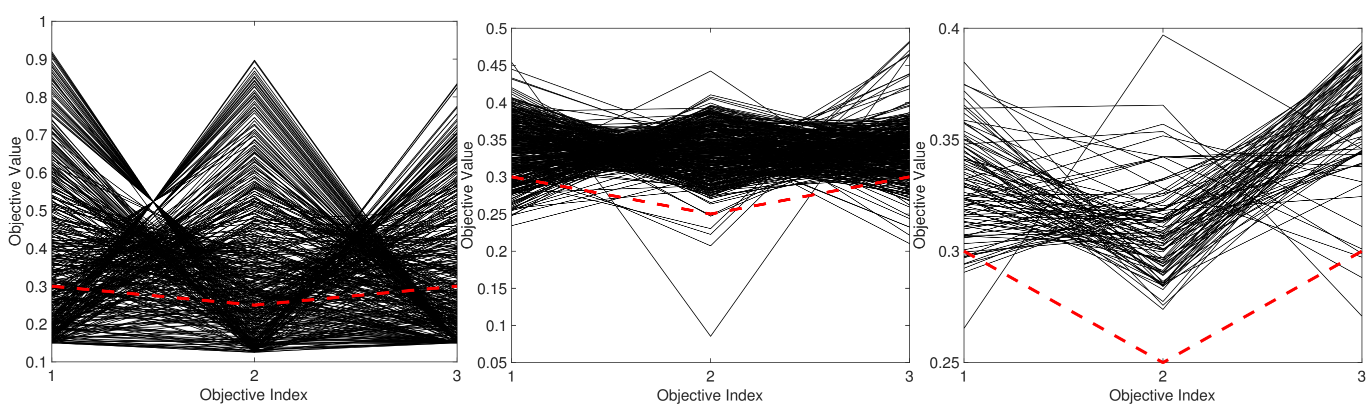


(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

Fig. 86: Comparisons on 2-objective WFG46 where $\mathbf{z}^r = (0.65, 0.5)^T$.

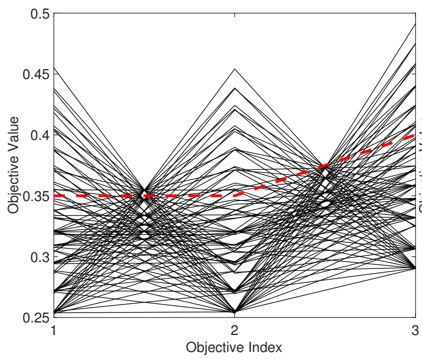


(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

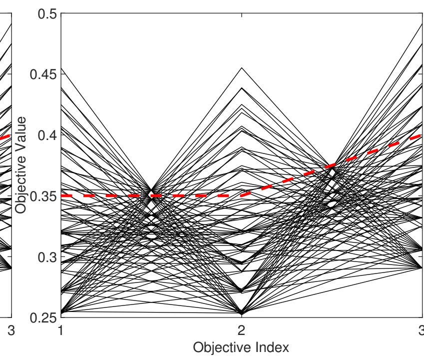


(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

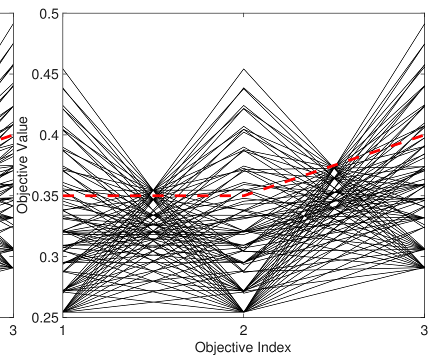
Fig. 87: Comparisons on 3-objective WFG46 where $\mathbf{z}^r = (0.3, 0.25, 0.3)^T$.



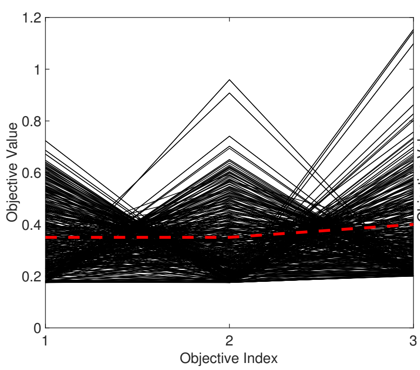
(a) MOEA/D-STM



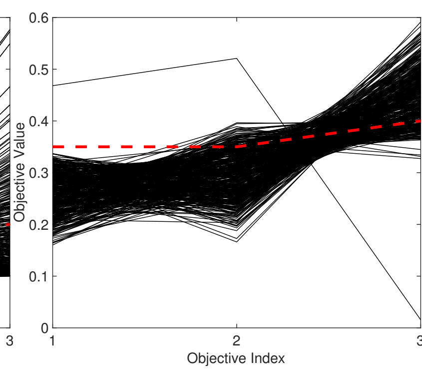
(b) MOEA/D



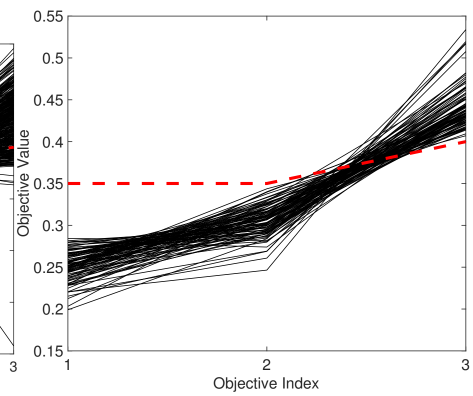
(c) NSGA-III



(d) g-NSGA-II

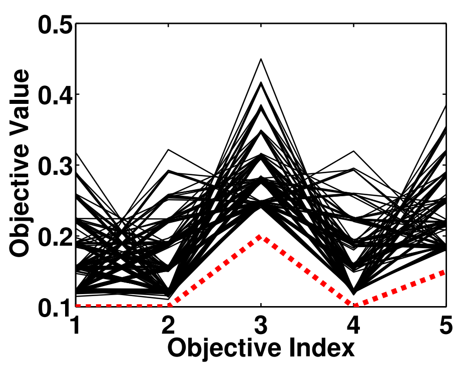


(e) r-NSGA-II

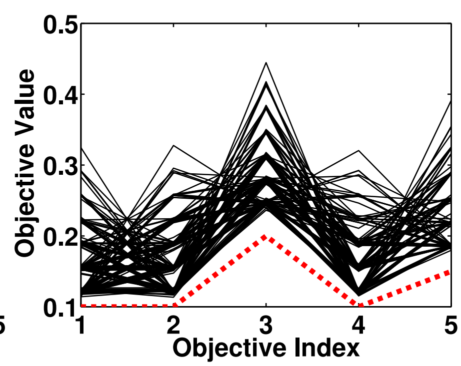


(f) R-NSGA-II

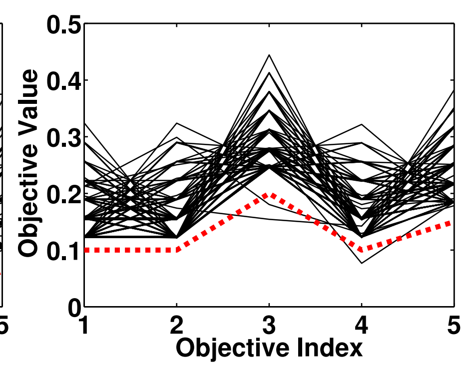
Fig. 88: Comparisons on 3-objective WFG46 where $\mathbf{z}^r = (0.35, 0.35, 0.4)^T$.



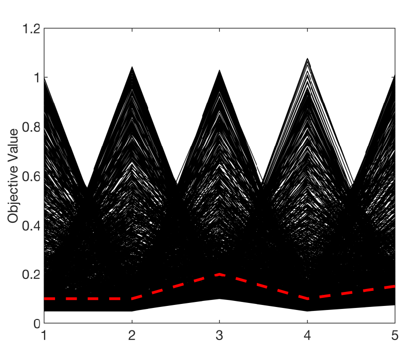
(a) MOEA/D-STM



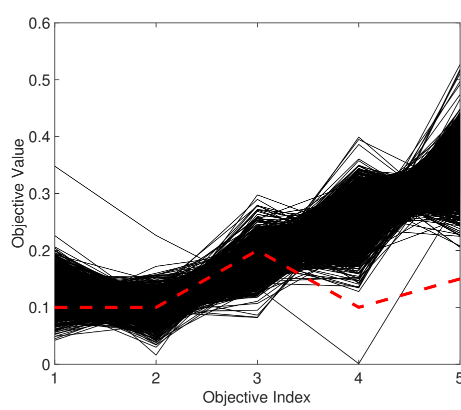
(b) MOEA/D



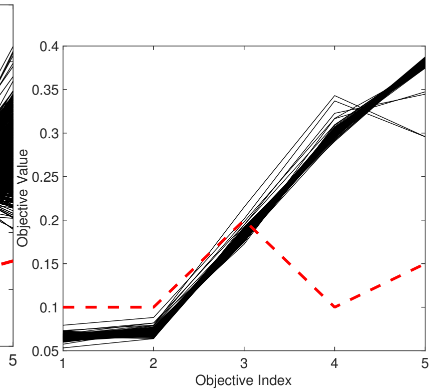
(c) NSGA-III



(d) g-NSGA-II

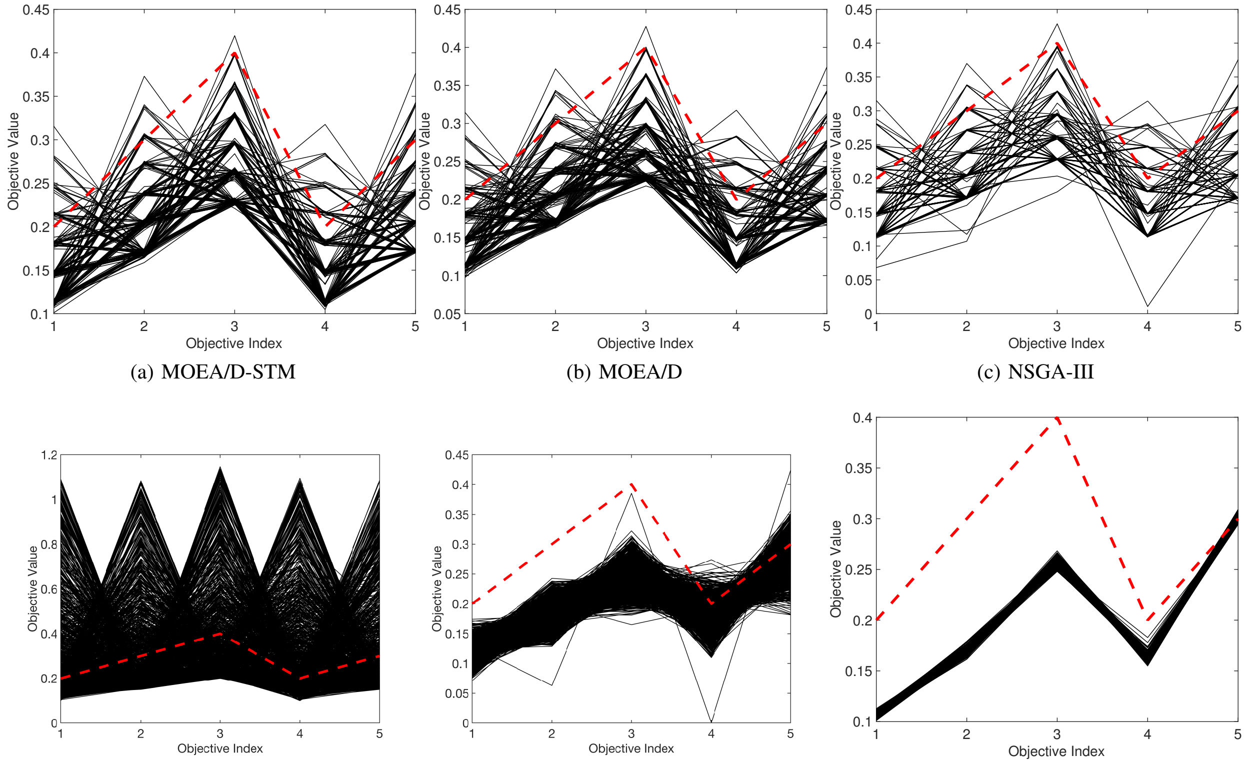


(e) r-NSGA-II



(f) R-NSGA-II

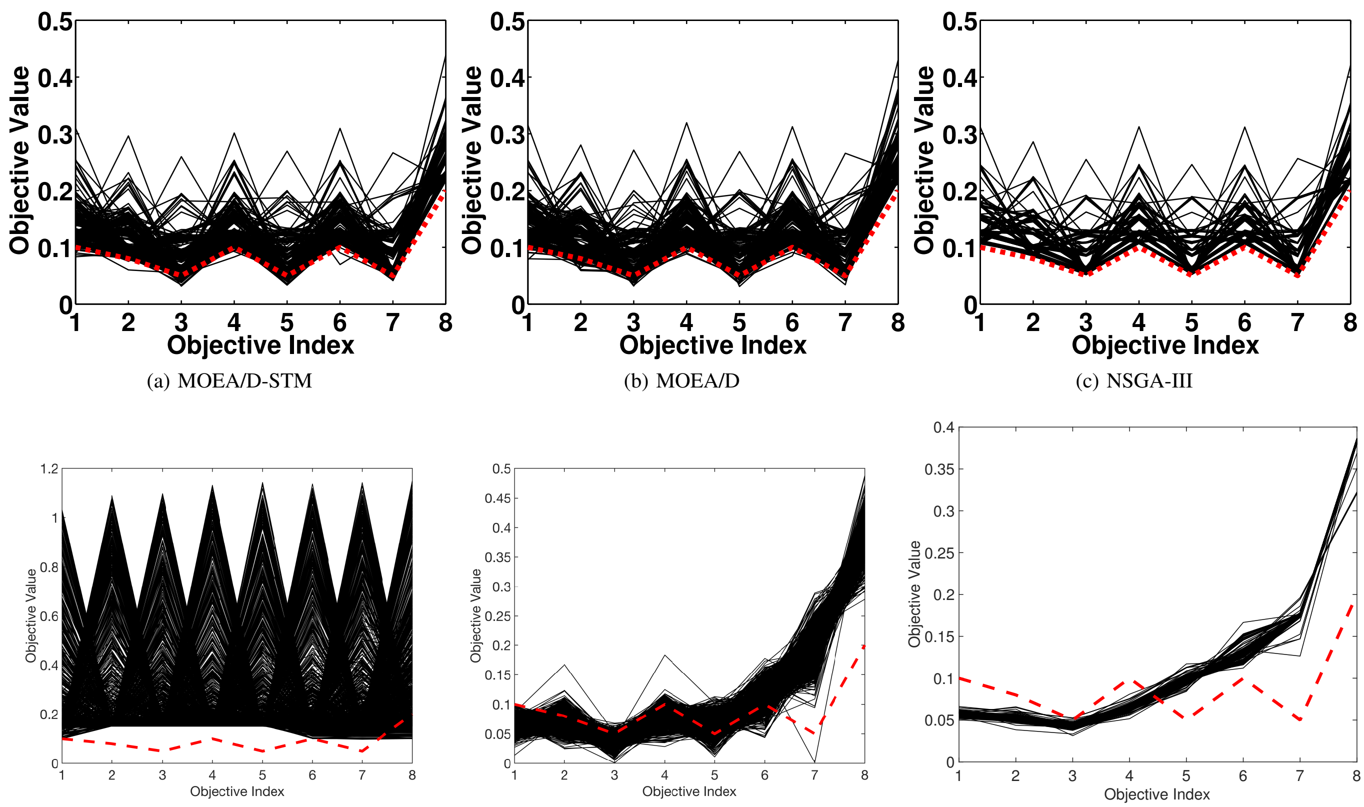
Fig. 89: Comparisons on 5-objective WFG46 where $\mathbf{z}^r = (0.1, 0.1, 0.2, 0.1, 0.15)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

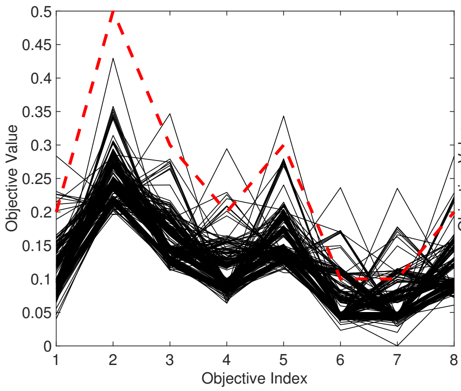
Fig. 90: Comparisons on 5-objective WFG46 where $\mathbf{z}^r = (0.2, 0.3, 0.4, 0.2, 0.3)^T$.



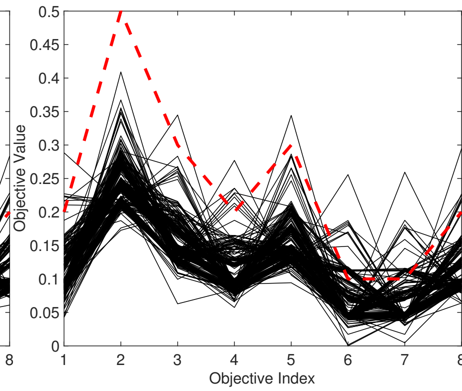
(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

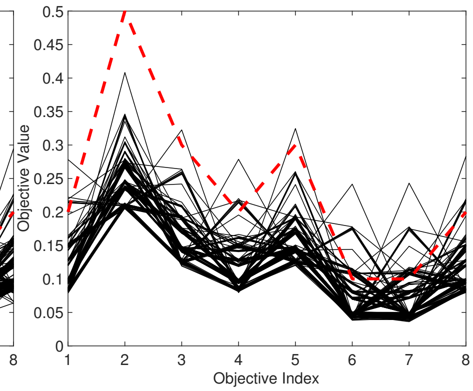
Fig. 91: Comparisons on 8-objective WFG46 where $\mathbf{z}^r = (0.1, 0.08, 0.05, 0.1, 0.05, 0.1, 0.05, 0.2)^T$.



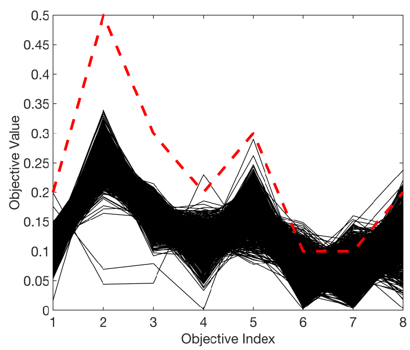
(a) MOEA/D-STM



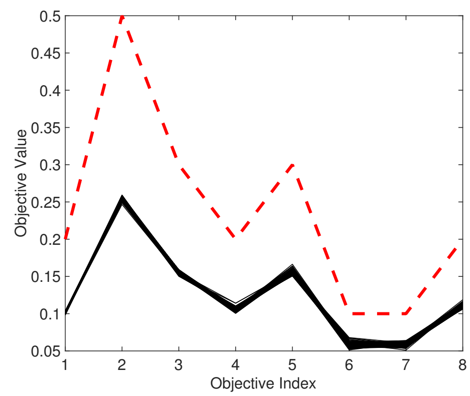
(b) MOEA/D



(c) NSGA-III

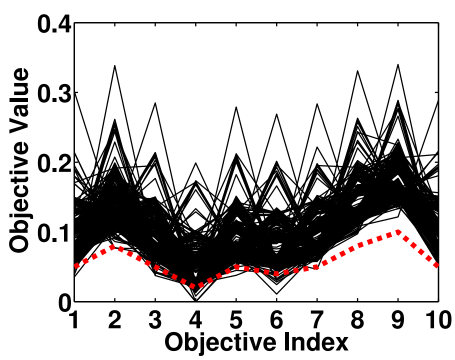


(d) r-NSGA-II

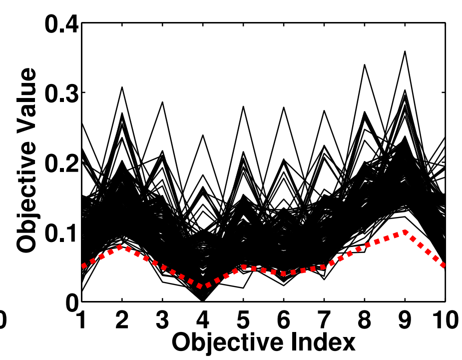


(e) R-NSGA-II

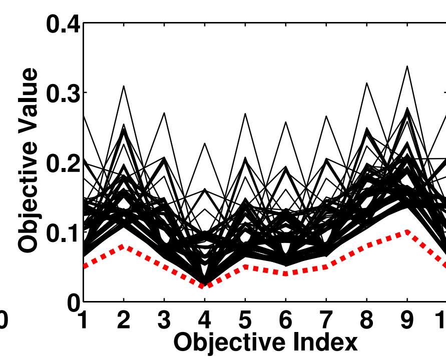
Fig. 92: Comparisons on 8-objective WFG46 where $\mathbf{z}^r = (0.2, 0.5, 0.3, 0.2, 0.3, 0.1, 0.1, 0.2)^T$.



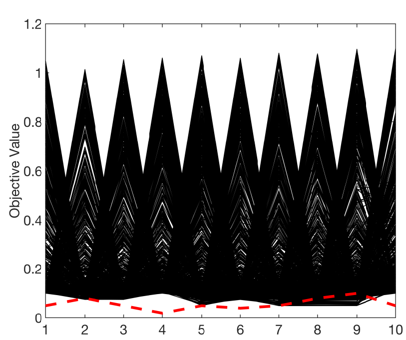
(a) MOEA/D-STM



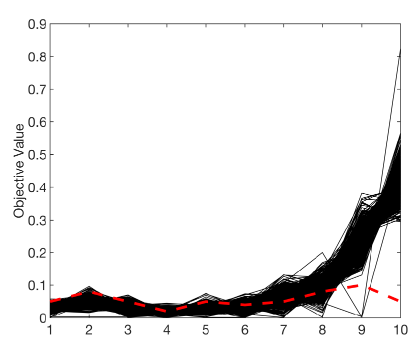
(b) MOEA/D



(c) NSGA-III

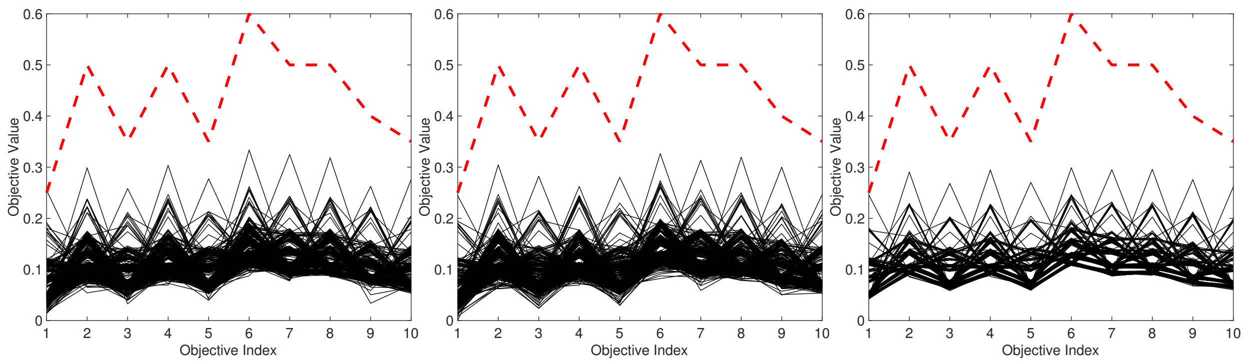


(d) g-NSGA-II

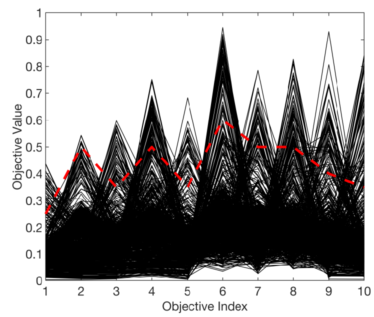


(e) r-NSGA-II

Fig. 93: Comparisons on 10-objective WFG46 where $\mathbf{z}^r = (0.05, 0.08, 0.05, 0.02, 0.05, 0.04, 0.05, 0.08, 0.1, 0.05)^T$.

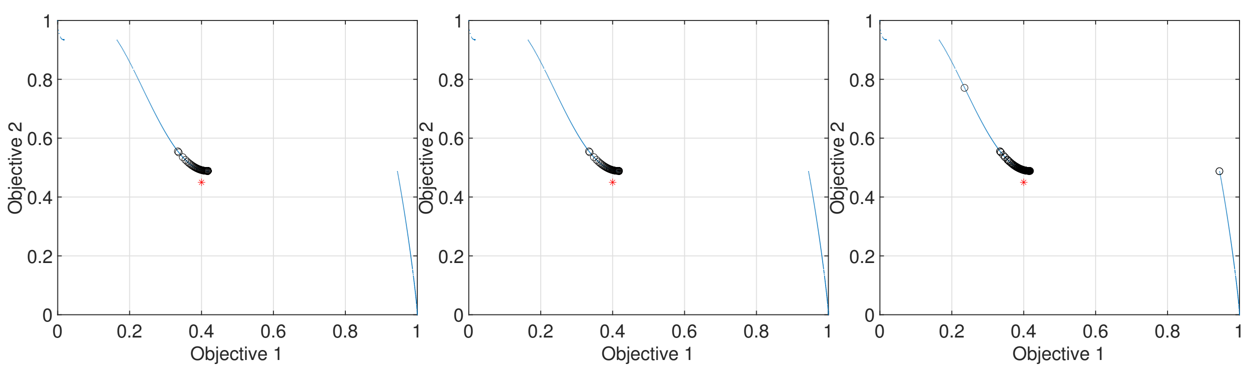


(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

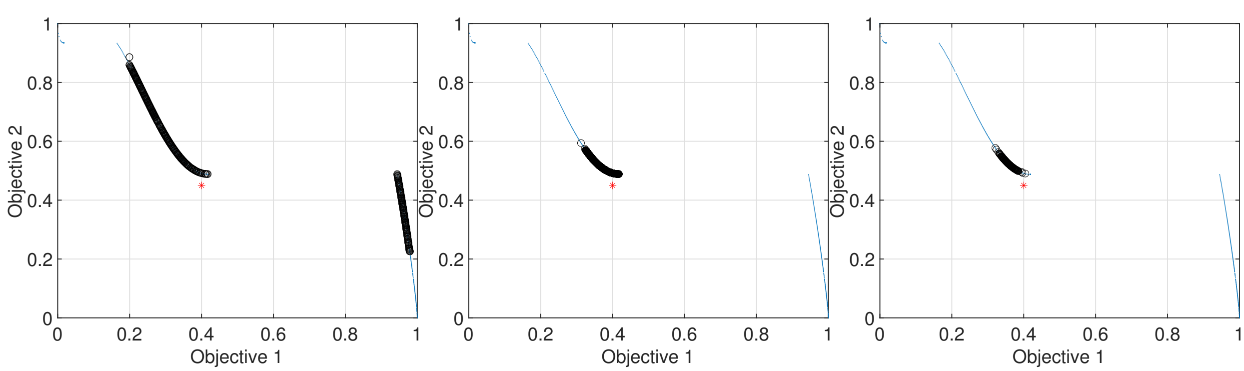


(d) r-NSGA-II

Fig. 94: Comparisons on 10-objective WFG46 where $\mathbf{z}^r = (0.25, 0.5, 0.35, 0.5, 0.35, 0.6, 0.5, 0.5, 0.4, 0.35)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III



(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

Fig. 95: Comparisons on 2-objective WFG47 where $\mathbf{z}^r = (0.4, 0.45)^T$.

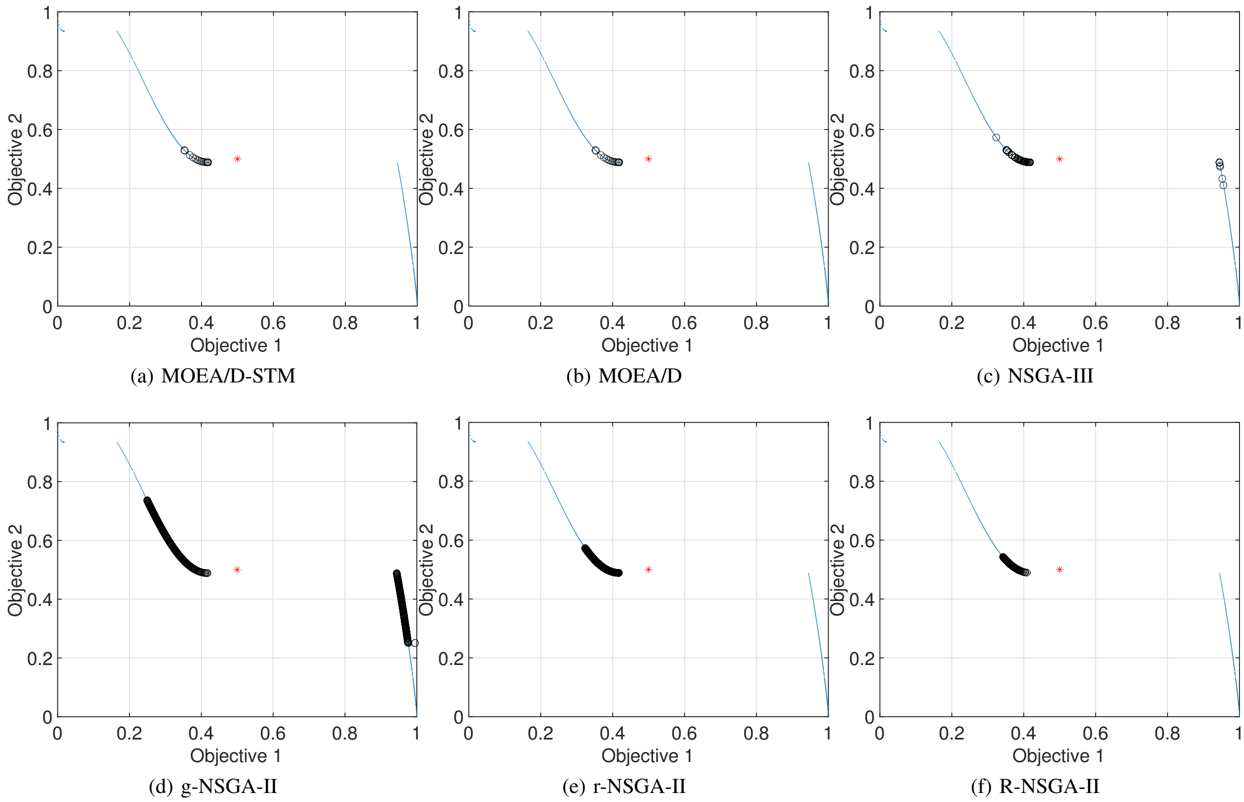


Fig. 96: Comparisons on 2-objective WFG47 where $\mathbf{z}^r = (0.5, 0.5)^T$.

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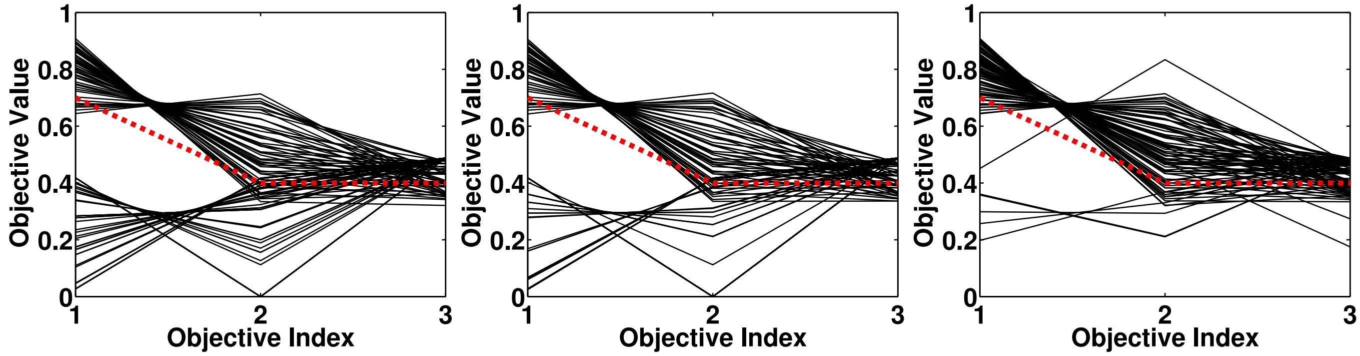
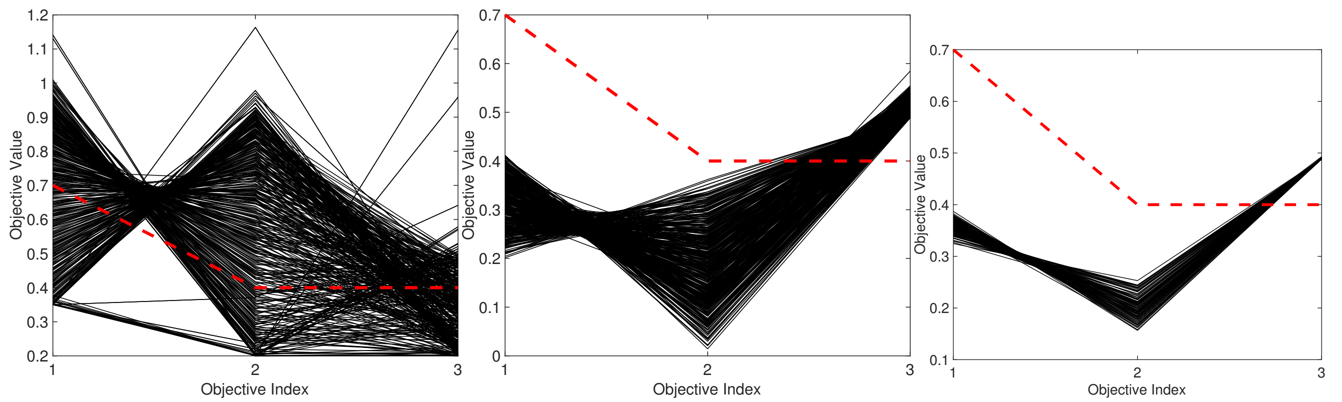
[1] I. Das and J. E. Dennis, "Normal-Boundary Intersection: A new method for generating the pareto surface in nonlinear multicriteria optimization problems," *SIAM Journal on Optimization*, vol. 8, pp. 631–657, 1998.

[2] J. J. Durillo, A. J. Nebro, C. A. C. Coello, J. García-Nieto, F. Luna, and E. Alba, "A study of multiobjective metaheuristics when solving parameter scalable problems," *IEEE Trans. Evolutionary Computation*, vol. 14, no. 4, pp. 618–635, 2010.

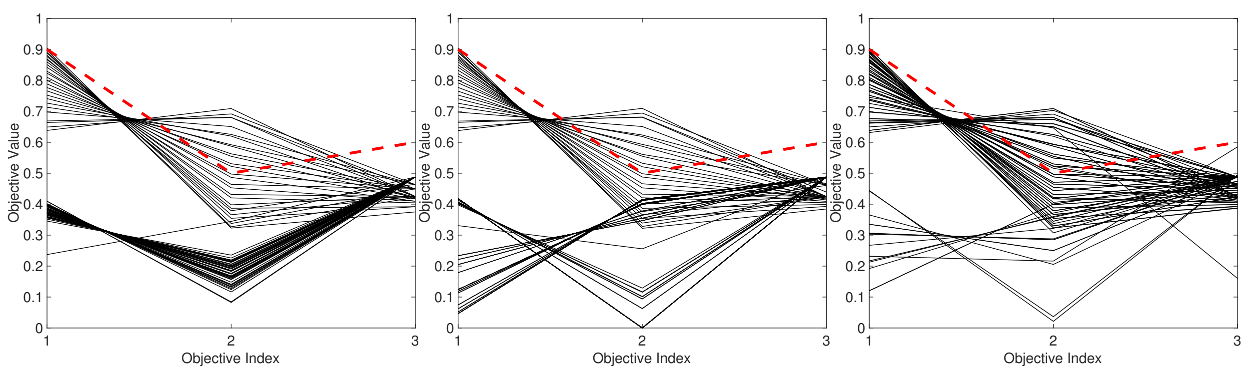


(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

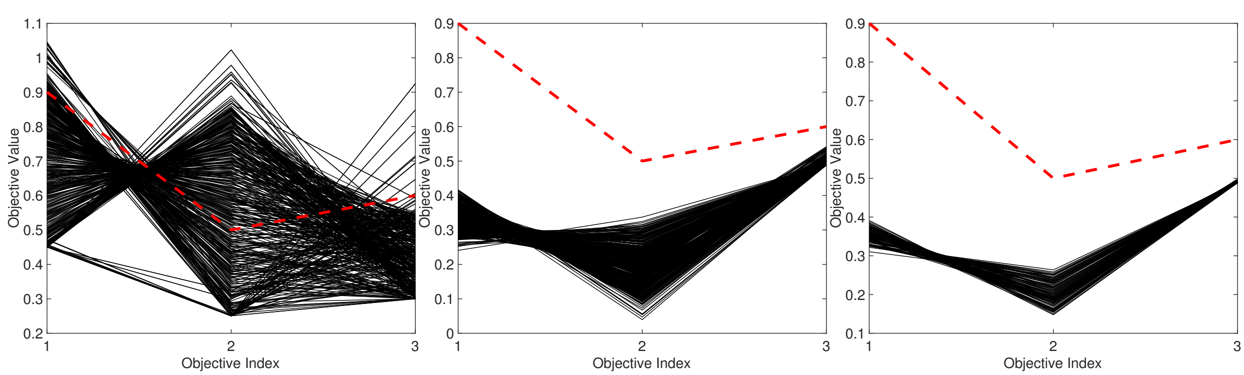


(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

Fig. 97: Comparisons on 3-objective WFG47 where $\mathbf{z}^r = (0.7, 0.4, 0.4)^T$.

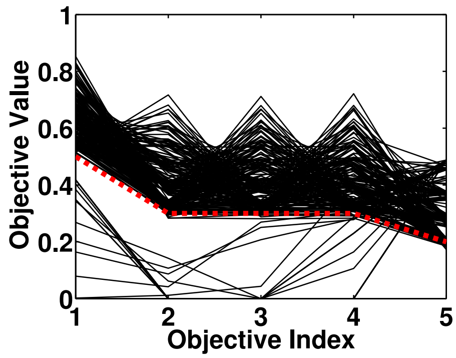


(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

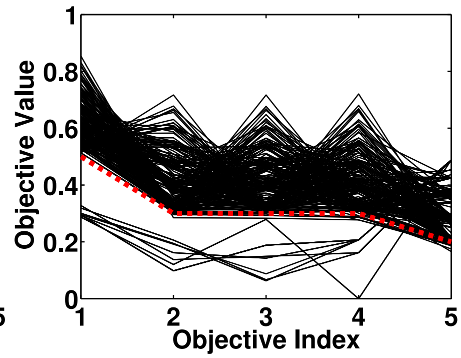


(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

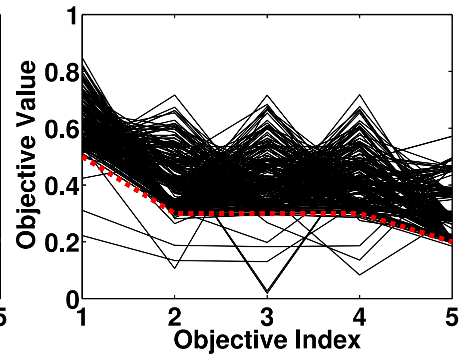
Fig. 98: Comparisons on 3-objective WFG47 where $\mathbf{z}^r = (0.9, 0.5, 0.6)^T$.



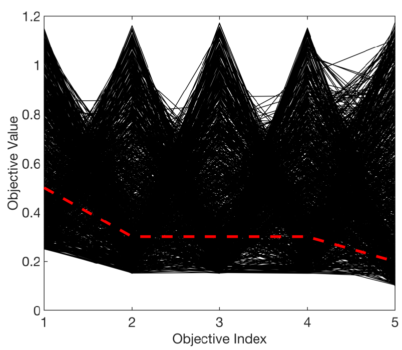
(a) MOEA/D-STM



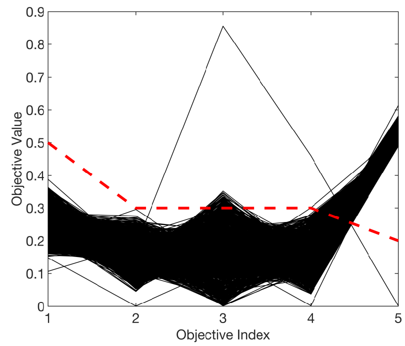
(b) MOEA/D



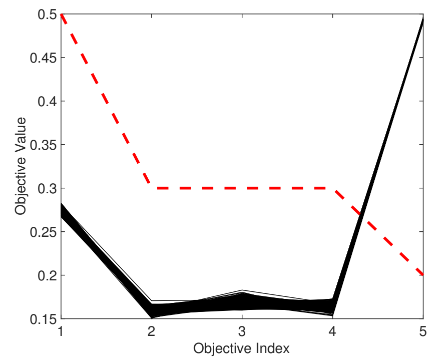
(c) NSGA-III



(d) g-NSGA-II

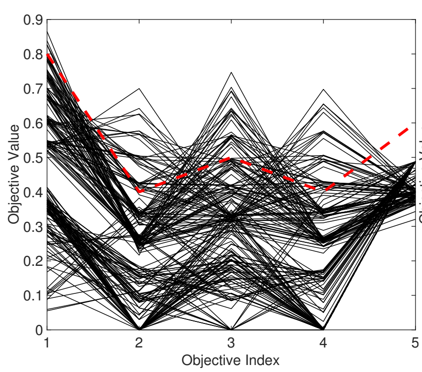


(e) r-NSGA-II

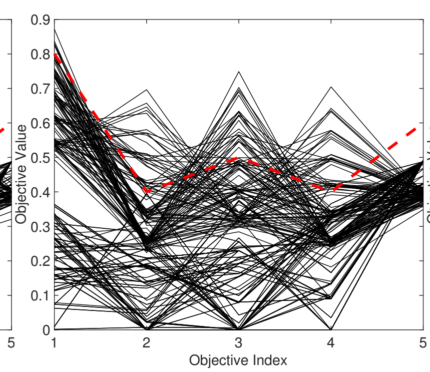


(f) R-NSGA-II

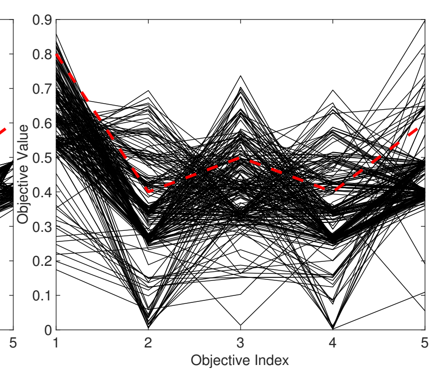
Fig. 99: Comparisons on 5-objective WFG47 where $\mathbf{z}^r = (0.5, 0.3, 0.3, 0.3, 0.2)^T$.



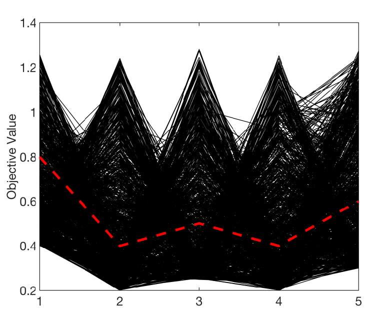
(a) MOEA/D-STM



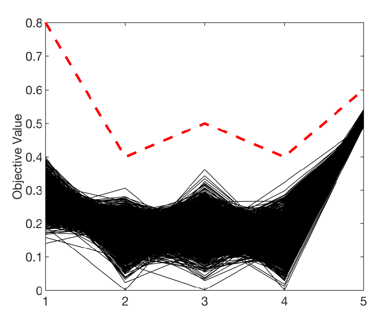
(b) MOEA/D



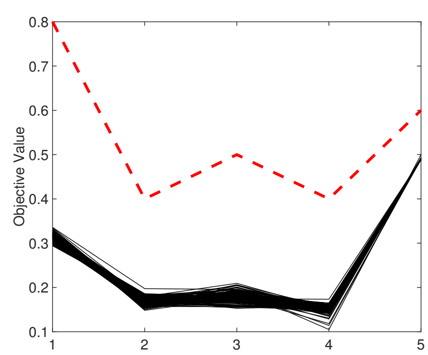
(c) NSGA-III



(d) g-NSGA-II

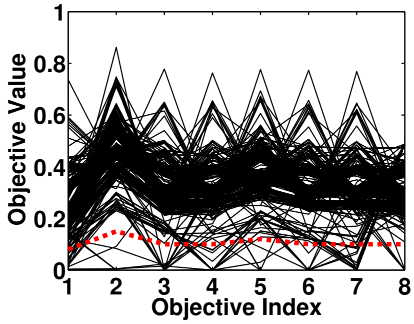


(e) r-NSGA-II

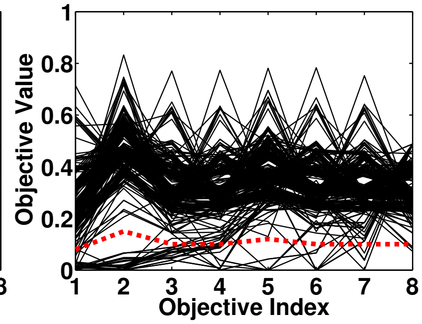


(f) R-NSGA-II

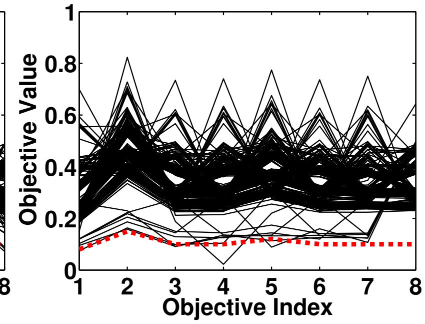
Fig. 100: Comparisons on 5-objective WFG47 where $\mathbf{z}^r = (0.8, 0.4, 0.5, 0.4, 0.6)^T$.



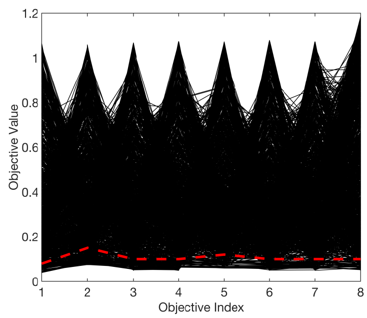
(a) MOEA/D-STM



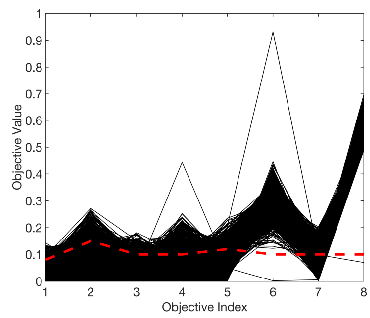
(b) MOEA/D



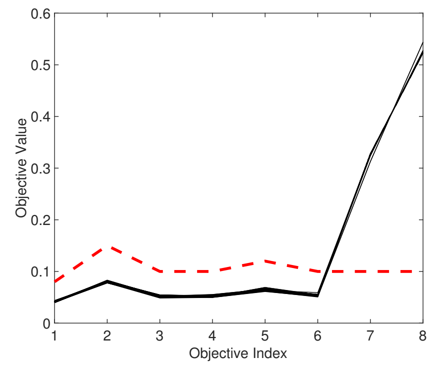
(c) NSGA-III



(d) g-NSGA-II

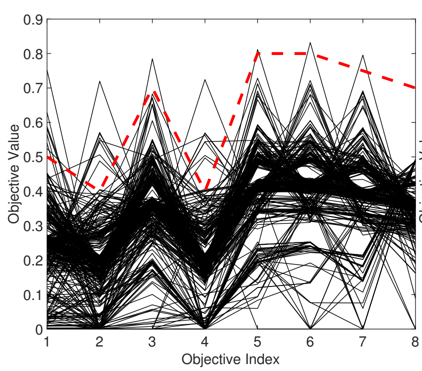


(e) r-NSGA-II

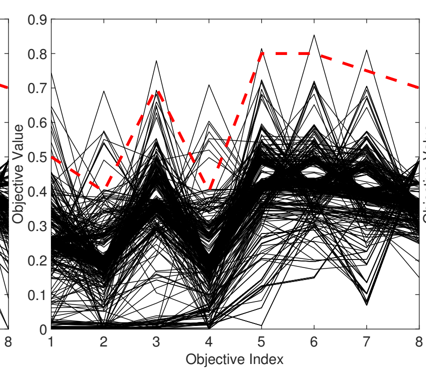


(f) R-NSGA-II

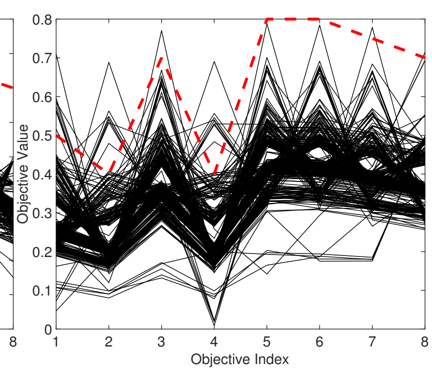
Fig. 101: Comparisons on 8-objective WFG47 where $\mathbf{z}^r = (0.08, 0.15, 0.1, 0.1, 0.12, 0.1, 0.1, 0.1)^T$.



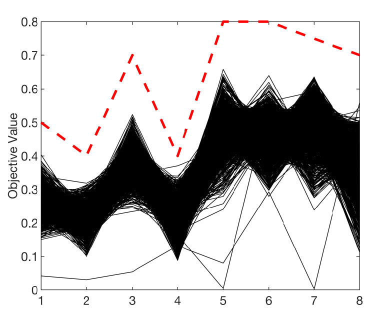
(a) MOEA/D-STM



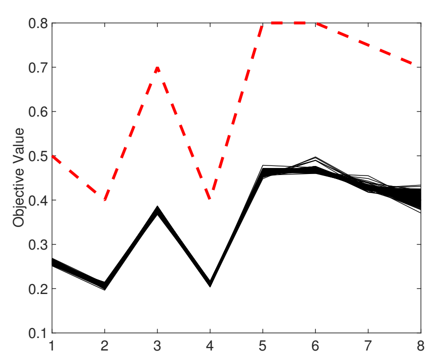
(b) MOEA/D



(c) NSGA-III

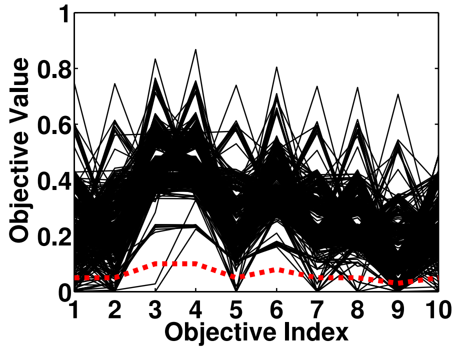


(d) r-NSGA-II

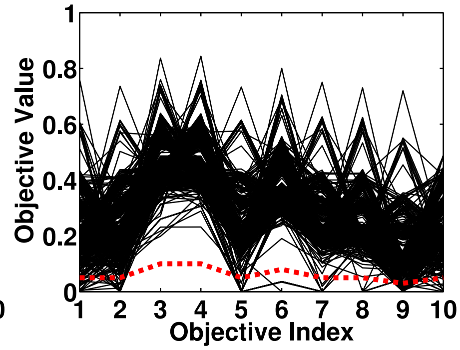


(e) R-NSGA-II

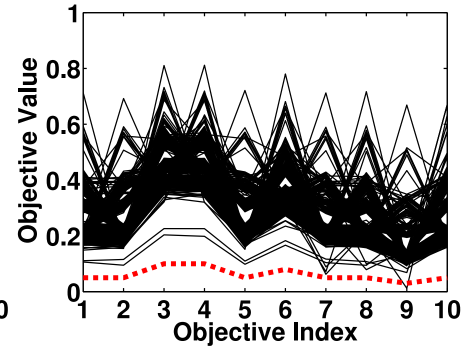
Fig. 102: Comparisons on 8-objective WFG47 where $\mathbf{z}^r = (0.5, 0.4, 0.7, 0.4, 0.8, 0.8, 0.75, 0.7)^T$.



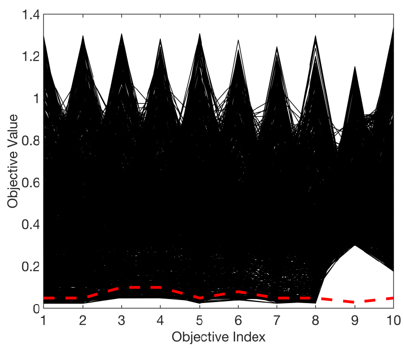
(a) MOEA/D-STM



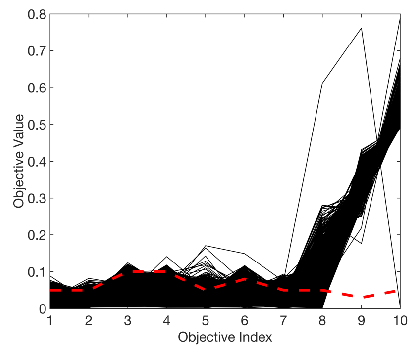
(b) MOEA/D



(c) NSGA-III

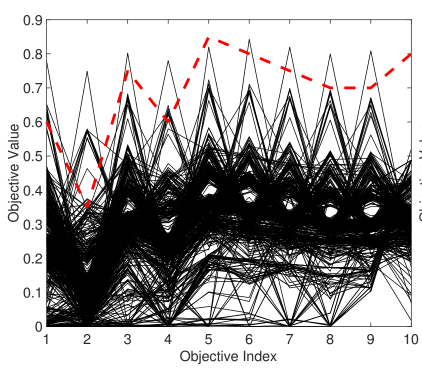


(d) g-NSGA-II

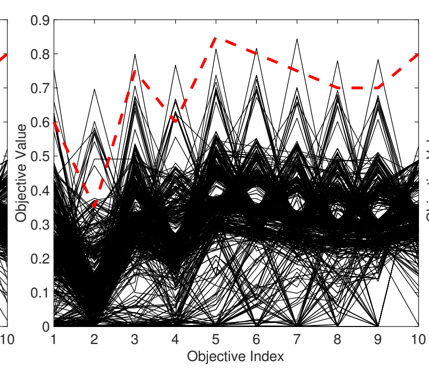


(e) r-NSGA-II

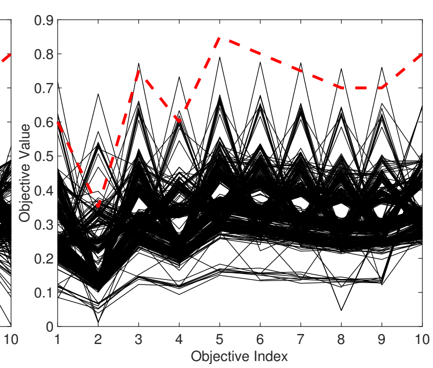
Fig. 103: Comparisons on 10-objective WFG47 where $\mathbf{z}^r = (0.05, 0.05, 0.1, 0.1, 0.05, 0.08, 0.05, 0.05, 0.03, 0.05)^T$.



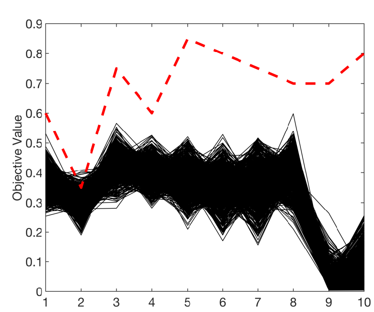
(a) MOEA/D-STM



(b) MOEA/D

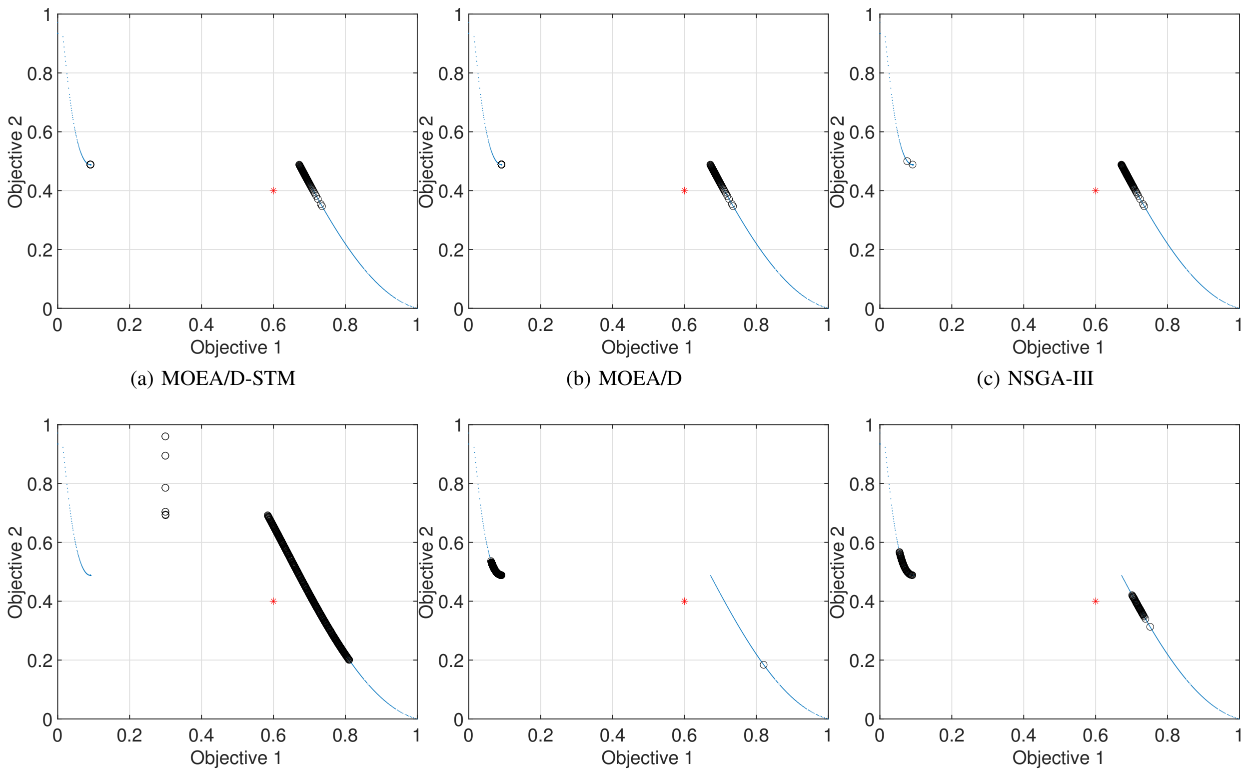


(c) NSGA-III



(d) r-NSGA-II

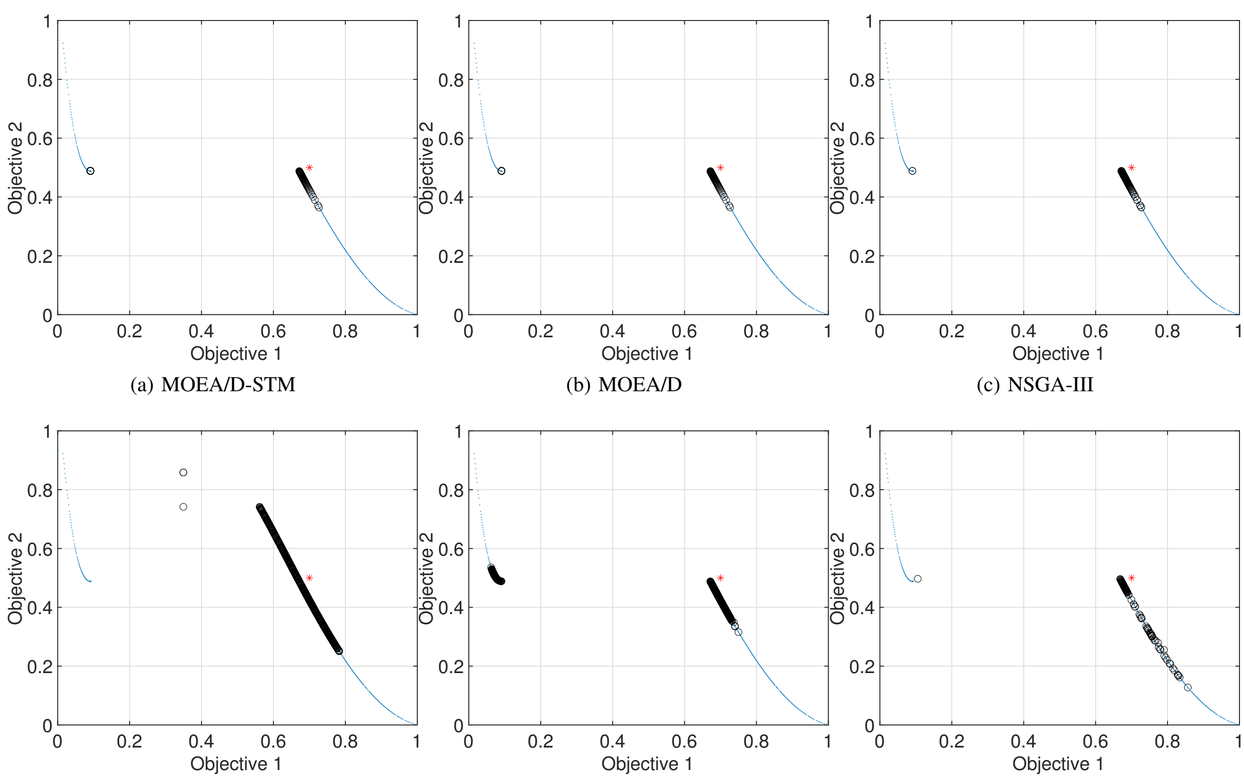
Fig. 104: Comparisons on 10-objective WFG47 where $\mathbf{z}^r = (0.6, 0.35, 0.75, 0.6, 0.85, 0.8, 0.75, 0.7, 0.7, 0.8)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

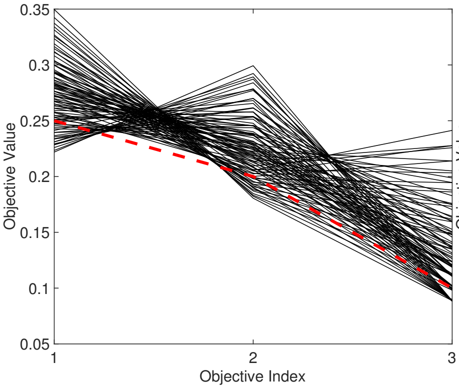
Fig. 105: Comparisons on 2-objective WFG48 where $\mathbf{z}^r = (0.6, 0.4)^T$.



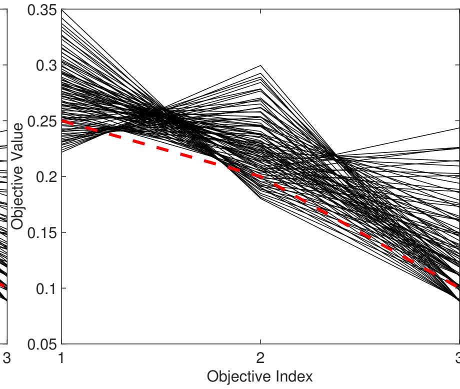
(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II (f) R-NSGA-II

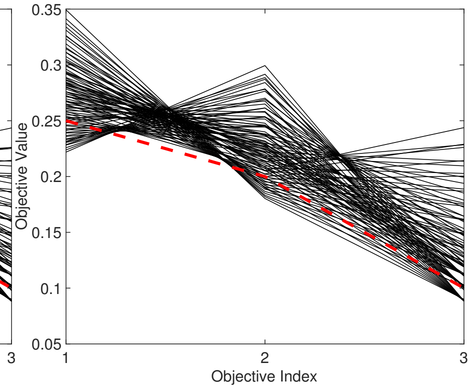
Fig. 106: Comparisons on 2-objective WFG48 where $\mathbf{z}^r = (0.7, 0.5)^T$.



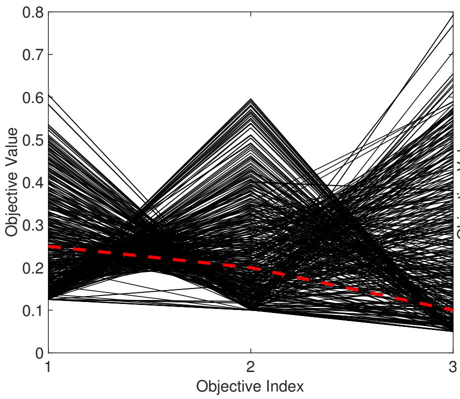
(a) MOEA/D-STM



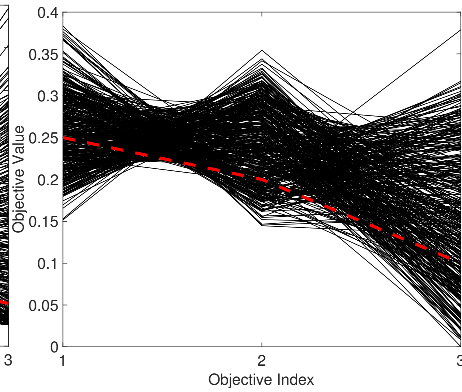
(b) MOEA/D



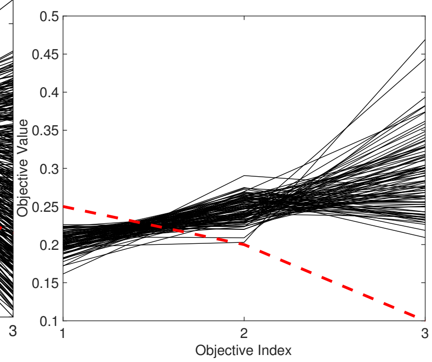
(c) NSGA-III



(d) g-NSGA-II

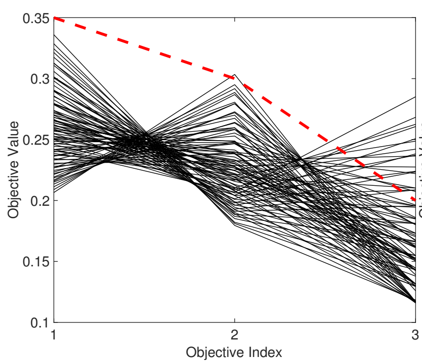


(e) r-NSGA-II

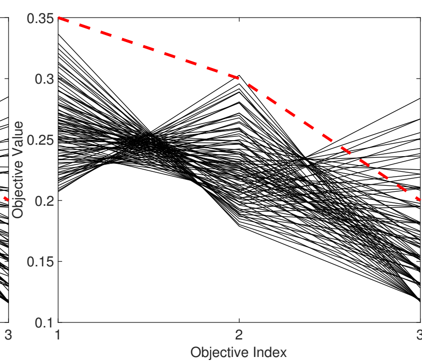


(f) R-NSGA-II

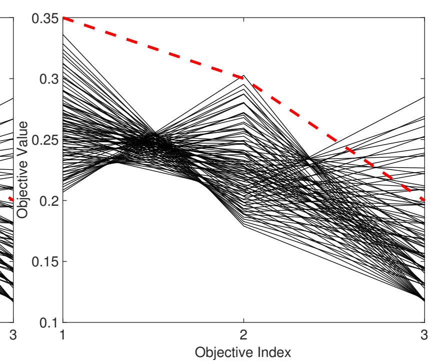
Fig. 107: Comparisons on 3-objective WFG48 where $\mathbf{z}^r = (0.25, 0.2, 0.1)^T$.



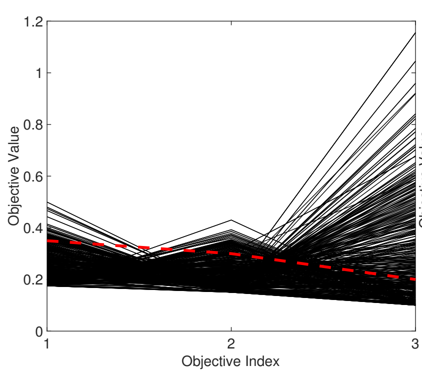
(a) MOEA/D-STM



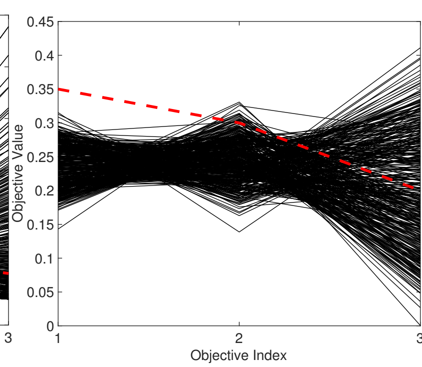
(b) MOEA/D



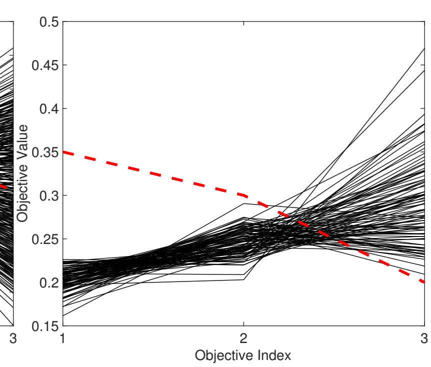
(c) NSGA-III



(d) g-NSGA-II

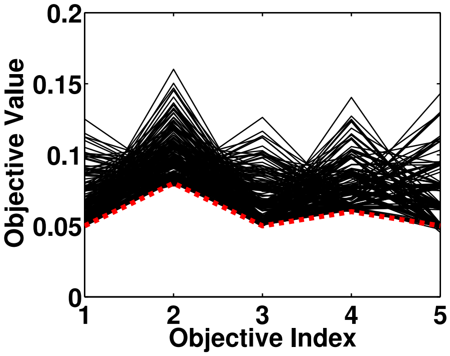


(e) r-NSGA-II

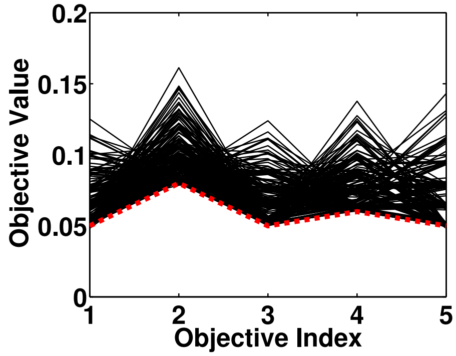


(f) R-NSGA-II

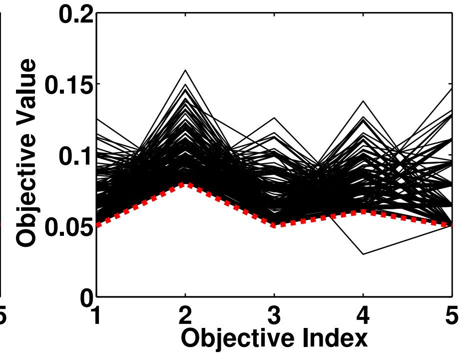
Fig. 108: Comparisons on 3-objective WFG48 where $\mathbf{z}^r = (0.35, 0.3, 0.2)^T$.



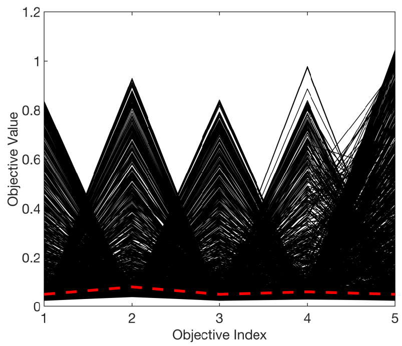
(a) MOEA/D-STM



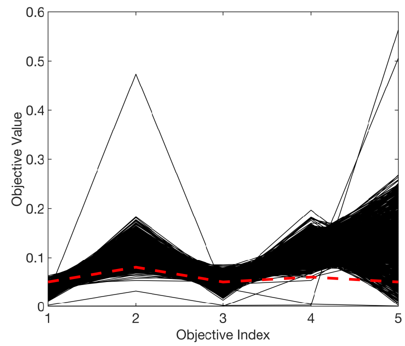
(b) MOEA/D



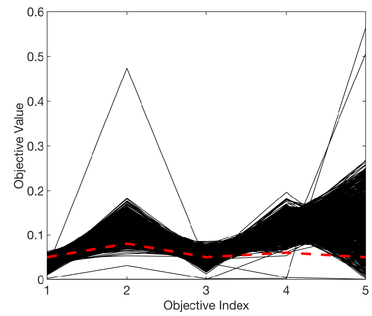
(c) NSGA-III



(d) g-NSGA-II

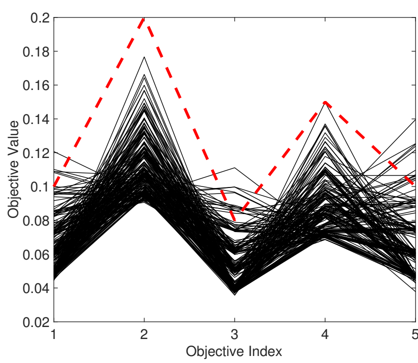


(e) r-NSGA-II

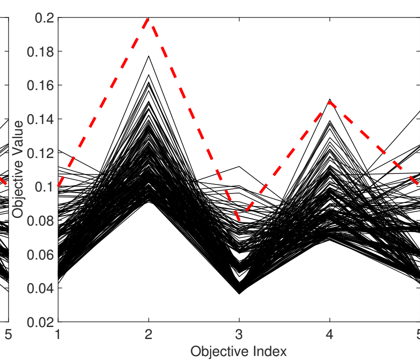


(f) R-NSGA-II

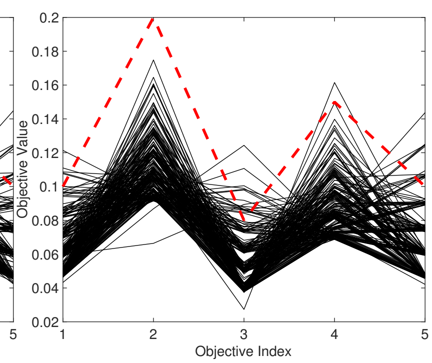
Fig. 109: Comparisons on 5-objective WFG48 where $\mathbf{z}^r = (0.05, 0.08, 0.05, 0.06, 0.05)^T$.



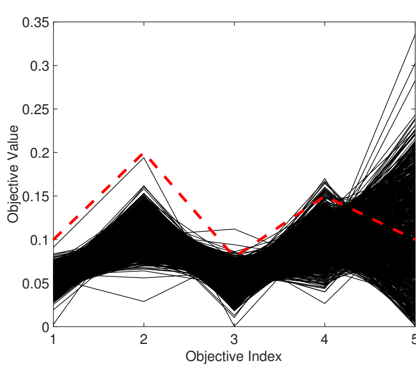
(a) MOEA/D-STM



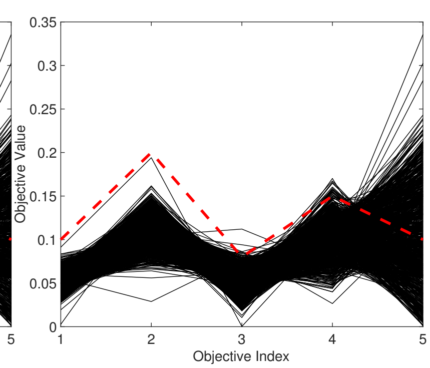
(b) MOEA/D



(c) NSGA-III

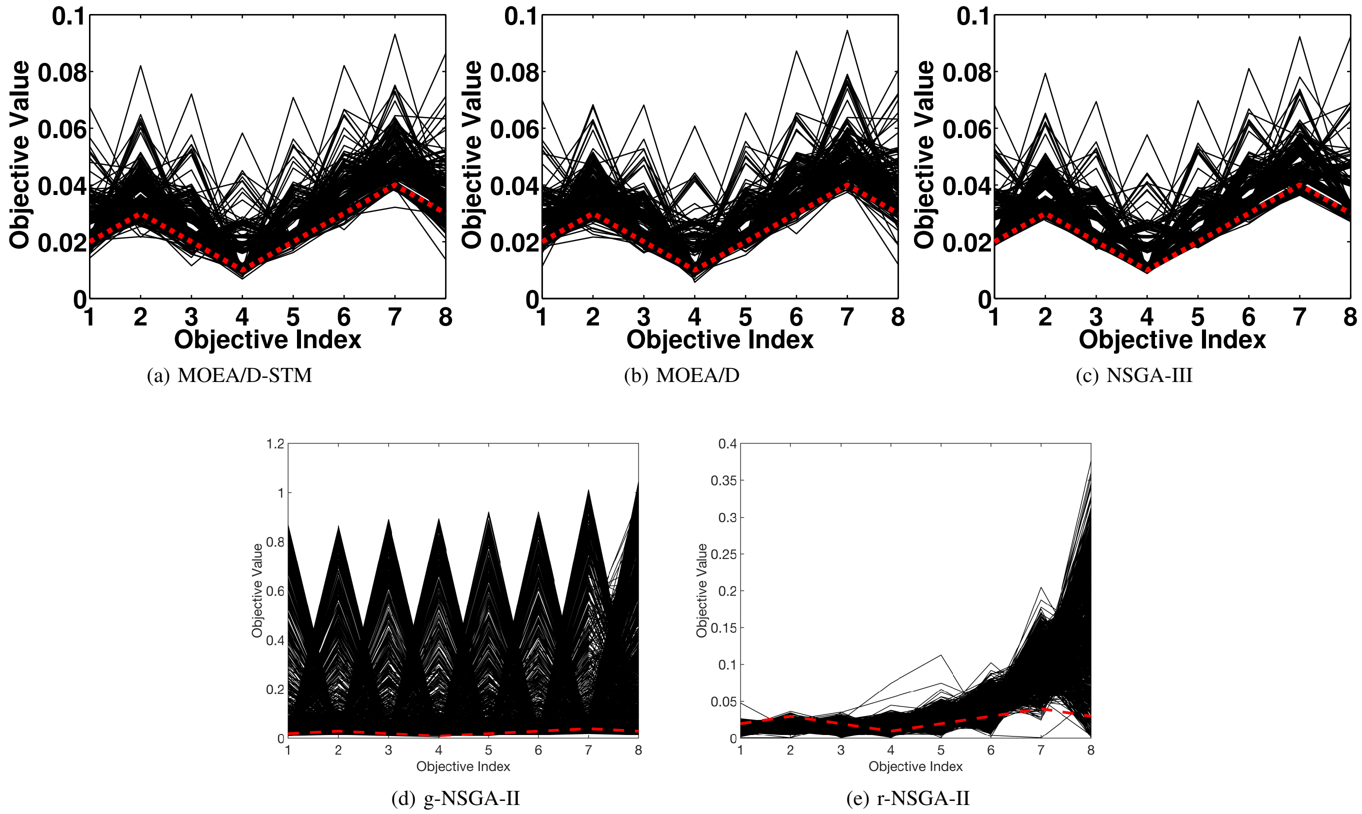


(d) r-NSGA-II



(e) R-NSGA-II

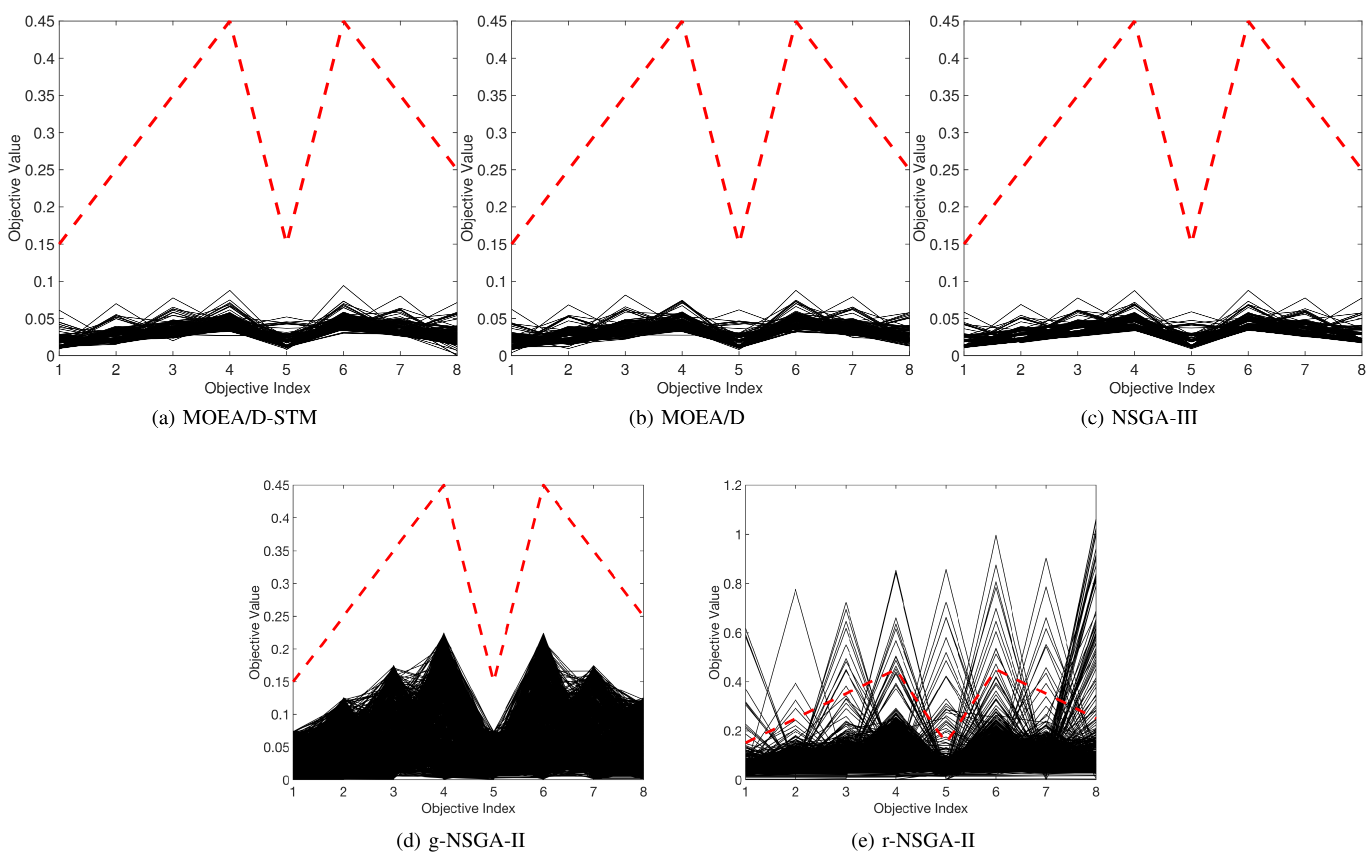
Fig. 110: Comparisons on 5-objective WFG48 where $\mathbf{z}^r = (0.1, 0.2, 0.08, 0.15, 0.1)^T$.



(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II

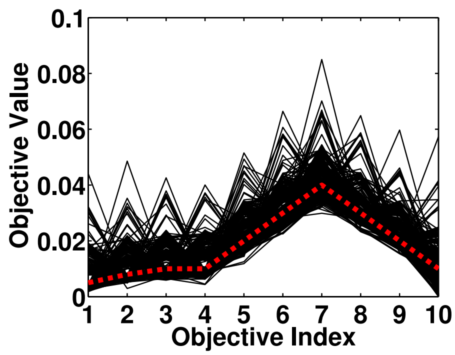
Fig. 111: Comparisons on 8-objective WFG48 where $\mathbf{z}^r = (0.02, 0.03, 0.02, 0.01, 0.02, 0.03, 0.04, 0.03)^T$.



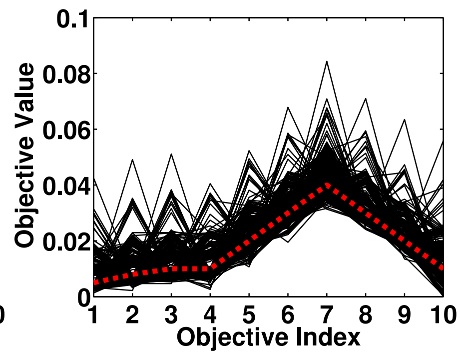
(a) MOEA/D-STM (b) MOEA/D (c) NSGA-III

(d) g-NSGA-II (e) r-NSGA-II

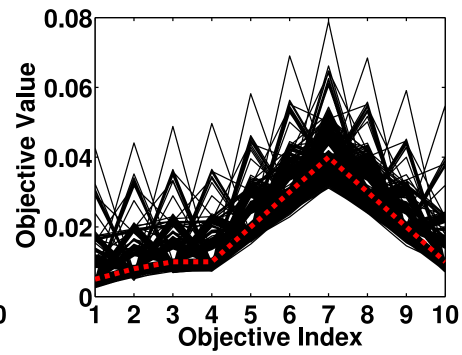
Fig. 112: Comparisons on 8-objective WFG48 where $\mathbf{z}^r = (0.15, 0.25, 0.35, 0.45, 0.15, 0.45, 0.35, 0.25)^T$.



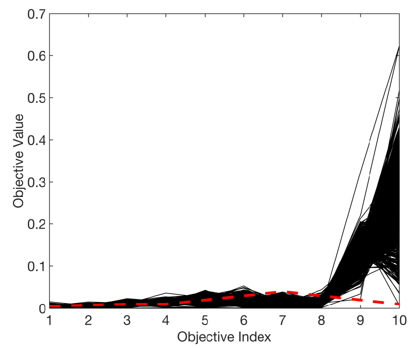
(a) MOEA/D-STM



(b) MOEA/D

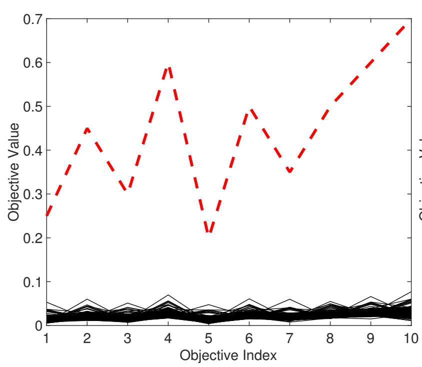


(c) NSGA-III

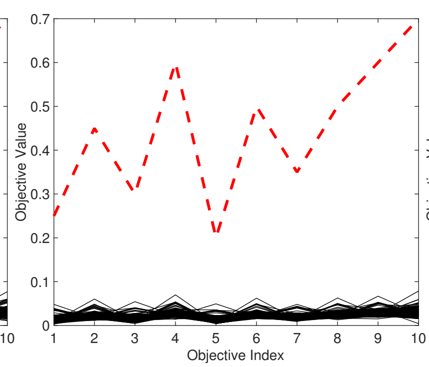


(d) r-NSGA-II

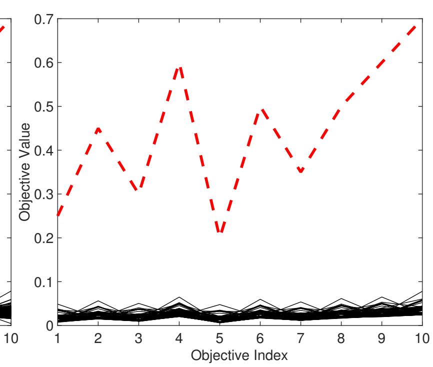
Fig. 113: Comparisons on 10-objective WFG48 where $\mathbf{z}^r = (0.005, 0.008, 0.01, 0.01, 0.02, 0.03, 0.04, 0.03, 0.02, 0.01)^T$.



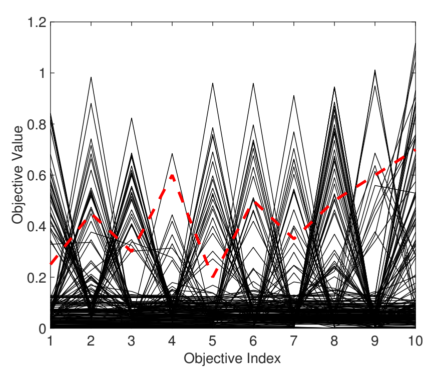
(a) MOEA/D-STM



(b) MOEA/D



(c) NSGA-III



(d) r-NSGA-II

Fig. 114: Comparisons on 10-objective WFG48 where $\mathbf{z}^r = (0.25, 0.45, 0.3, 0.6, 0.2, 0.5, 0.35, 0.5, 0.6, 0.7)^T$.